High-Performance Computing for High-Precision Processes at High-Energy Colliders

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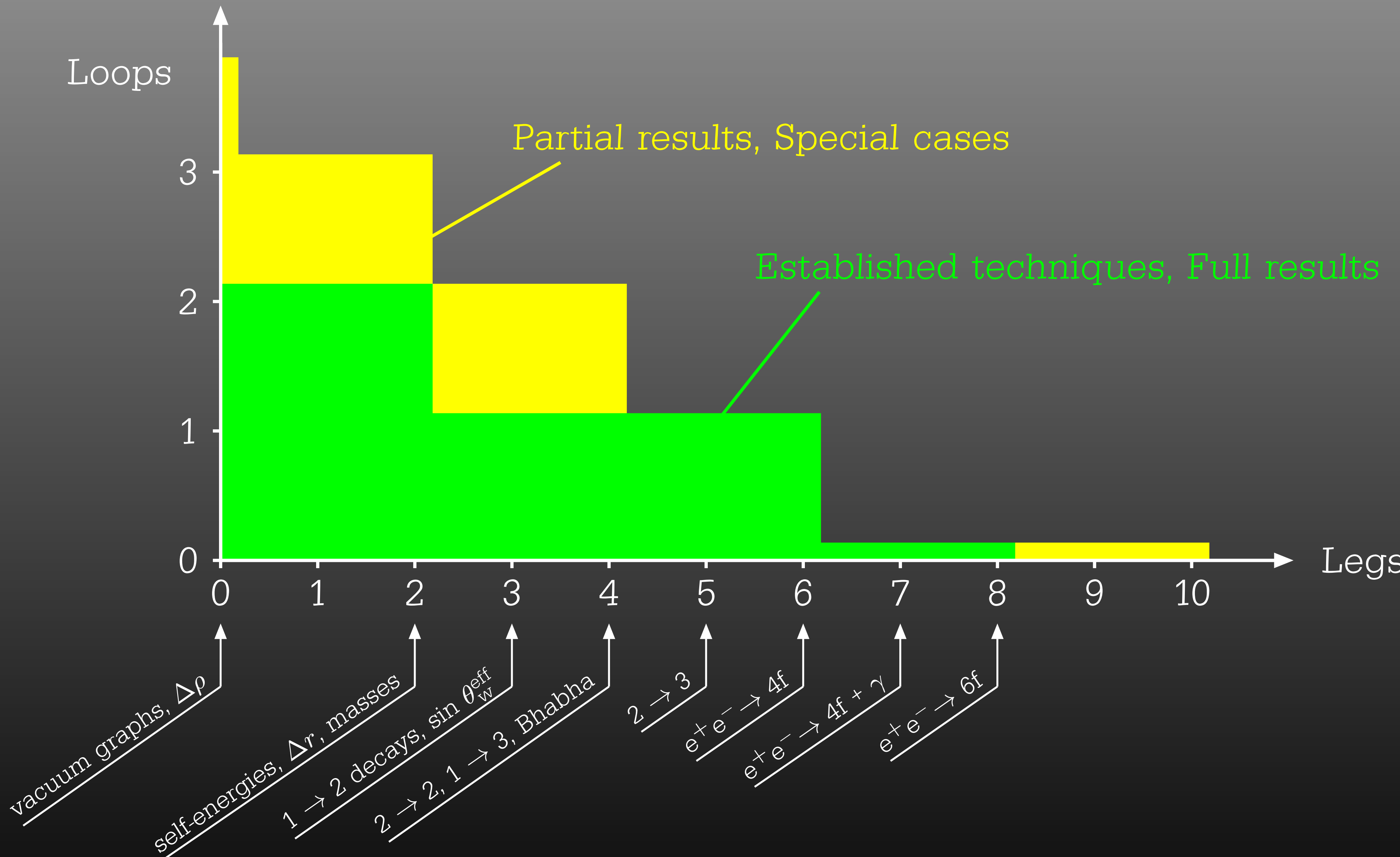
for the MPP Pheno Group

The Frontier?



Components of a Calculation

- **Need all pieces up to desired order in α_s or α_{em},**

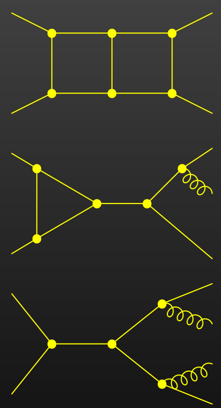
LO $\sigma_0 \propto |\mathcal{M}_0|^2 \,,$

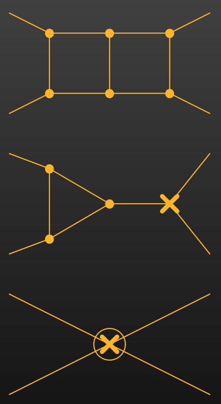
NLO $\sigma_1 \propto |\mathcal{M}_0|^2 + 2\,\mathrm{Re}\,\mathcal{M}_0^* \mathcal{M}_1 \,,$

NNLO $\sigma_2 \propto |\mathcal{M}_0|^2 + 2\,\mathrm{Re}\,\mathcal{M}_0^* \mathcal{M}_1 + |\mathcal{M}_1|^2 + 2\,\mathrm{Re}\,\mathcal{M}_0^* \mathcal{M}_2 \quad \ldots$

- **IR divergences: need real radiation to cancel poles.**
- **UV divergences: need counter-terms (unless $\overline{\text{MS}}$).**

Loops	Legs	CT-Order
N	$m \to n$	0
$N-1$	$m \to n+1$	1
$\vdots$	$\vdots$	$\vdots$
0	$m \to n+N$	N





Each Component…

- Model → Model file → Diagrams — Diagram Generation

- Fermion algebra (traces, Dirac eq) — Symbolic Simplification
- Color algebra (SU(N) traces)
- Perform tensor reduction, *or*
  Isolate integrals for OPP
- Other simplifications (e.g. Fierz, abbreviations)

- Code generation (Fortran, C/++) — Numerical Evaluation
- Phase-space integration
- Evaluation of loop integrals

Disclaimer: 'traditional' Feynman-diagrammatic approach, some steps may be absent or different when using recursion relations, the OpenLoops method, etc.

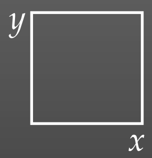
Identify Showstoppers/Bottlenecks

- Master integrals not all known (2L+).
- Renormalization unknown (BSM).
- Computational complexity:
  - Tensor reduction,
  - Phase-space integration (real radiation),
  - Numerical precision (subtraction terms).

Sector Decomposition

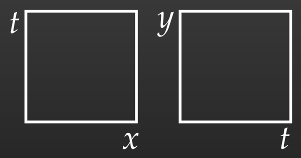
Sector Decomposition makes numerical evaluation of master integrals feasible.

- Dissect integration region into 'sectors,' chosen so that divergences can be easily be extracted. Simple example:



$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, \frac{1}{(x+y)^{2+\varepsilon}}$$

$$= \int_0^1 \mathrm{d}x \int_0^x \frac{\mathrm{d}y}{(x+y)^{2+\varepsilon}} + \int_0^1 \mathrm{d}y \int_0^y \frac{\mathrm{d}x}{(x+y)^{2+\varepsilon}}$$



$$= \int_0^1 \mathrm{d}x \int_0^1 \frac{x \, \mathrm{d}t}{(x+xt)^{2+\varepsilon}} + \int_0^1 \mathrm{d}y \int_0^1 \frac{y \, \mathrm{d}t}{(yt+y)^{2+\varepsilon}}$$

- Numerically integrate remaining 'well-behaved' function.

Sector Decomposition Problems

- No. of sectors can be 'large,' $\mathcal{O}(100)$ for 2L diagram.
  - Automate: e.g. SecDec.

Heinrich, Borowka, Jones, Kerner, Schlenk, Zirke

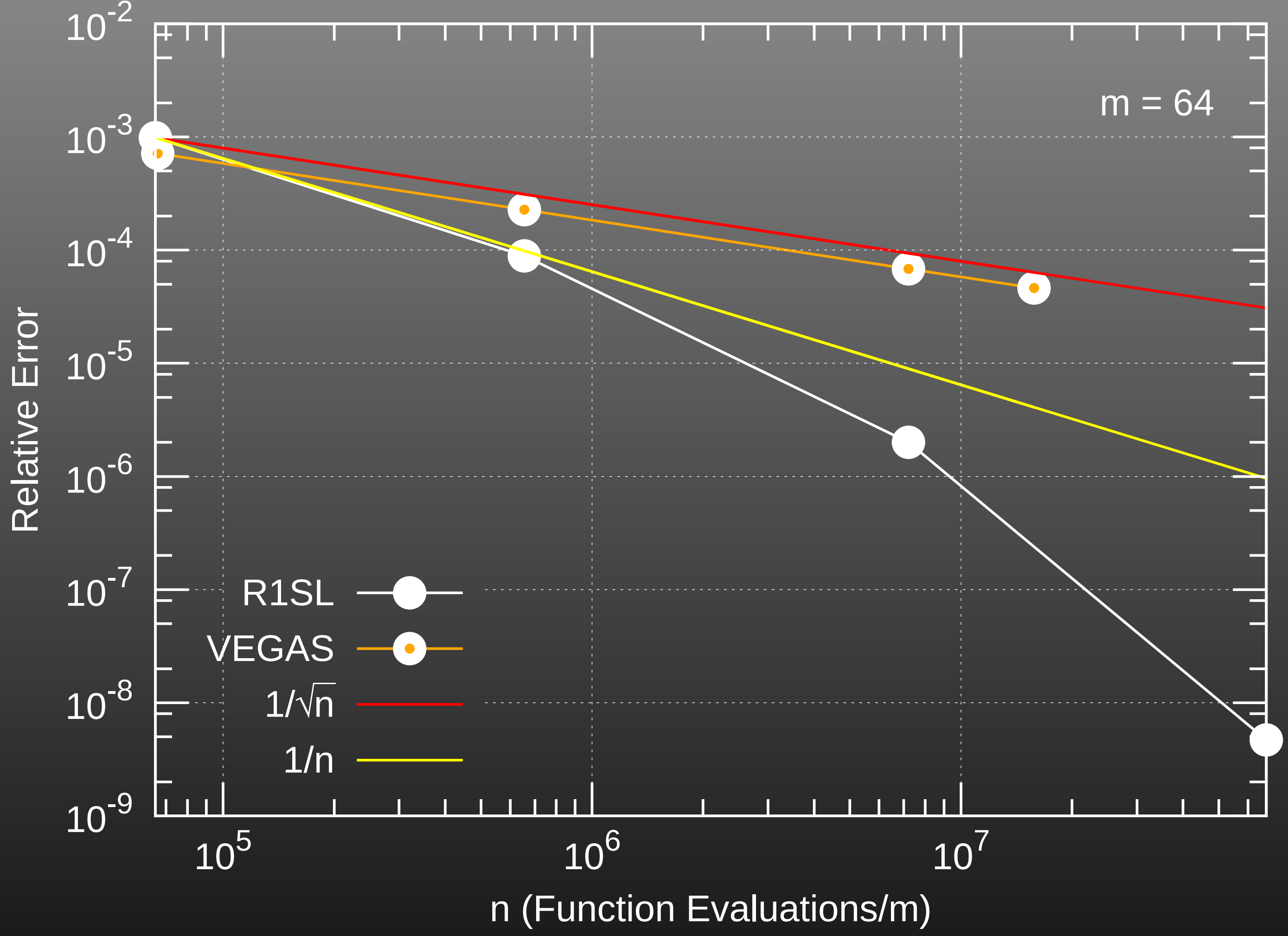
- Completely numerical evaluation slower by factor $\gtrsim$ 100.
  - Apply Sector Decomposition only to unknown integrals $\Leftrightarrow$ Database of known integrals.
  - Develop more efficient integration methods.
  - Use concurrent technologies (CPUs, GPUs, MICs).

Prototype Large-scale Project:

- $gg \to HH$ @ NNLO.

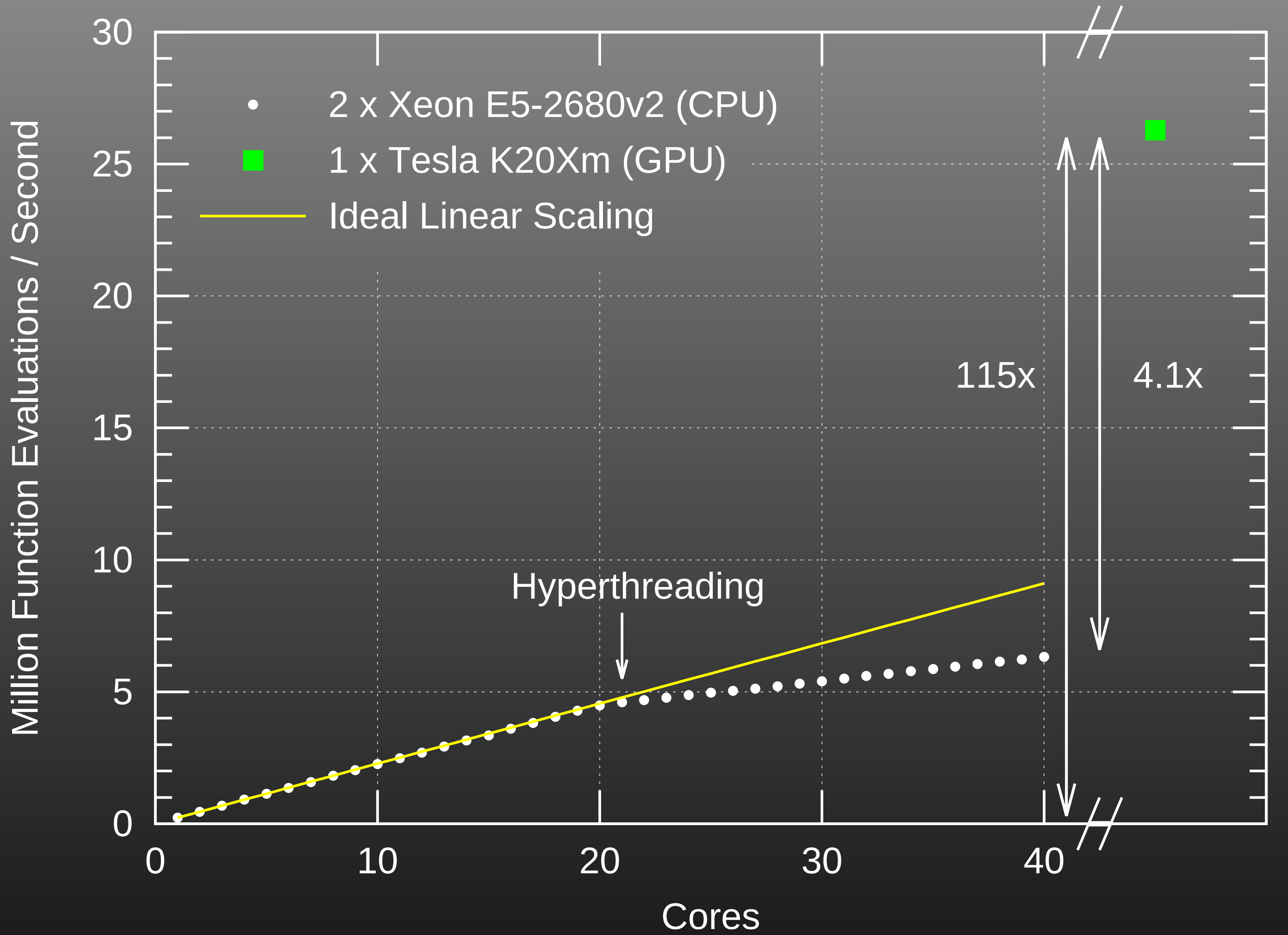
Heinrich, Jones, Kerner, Mastrolia, Schlenk, Schubert, Stoyanov, Di Vita, Zirke

More Efficient Integration Methods



Jones

CPUs, GPUs, MICs, and All That



Jones

Other Advances in Loop Integrals

- New techniques for solving loop integrals analytically are constantly discovered (beat any numerical evaluation).

  Puhlfürst, Stieberger

- Advances in unitarity methods beyond 1L.

  Mastrolia, Schubert

Loopedia Database

Question: Which master integrals are presently known?

- Vast literature.
- No central repository yet.
- Loopedia first attempt at community-driven database.
- Infrastructure by Loopedia group, integrals to be added by external researchers.

Papara, Heinrich, Hahn, Borowka

- Mixture of
  - Literature references,
  - Analytic results (Maple, Mathematica),
  - Numeric results (Fortran, C++ programs).

Paradigm Change for High-Precision Calculations

So far:

- Mostly driven by developments in master integrals.
- Slow pace, $\lesssim$ 1 loop order per decade.

Now seeing transition:

- Numerical evaluation of integrals makes computation feasible as much as compute power allows.
- Prediction for the Future of Loop Calculations in QCD:
  - Identify 'high-value targets' (Les Houches wish-list).
  - Acquire compute resources (WLCG, Cloud services).
  - Possibly derive 'fast' parameterizations from the full results (e.g. Padé approximations).

More Complexity in BSM

Outside of QCD often special requirements:

- Resummations (e.g. hbb in MSSM),
- Approximations (e.g. gaugeless limit),
- K-factors,
- Nontrivial renormalization.

Software design so far:

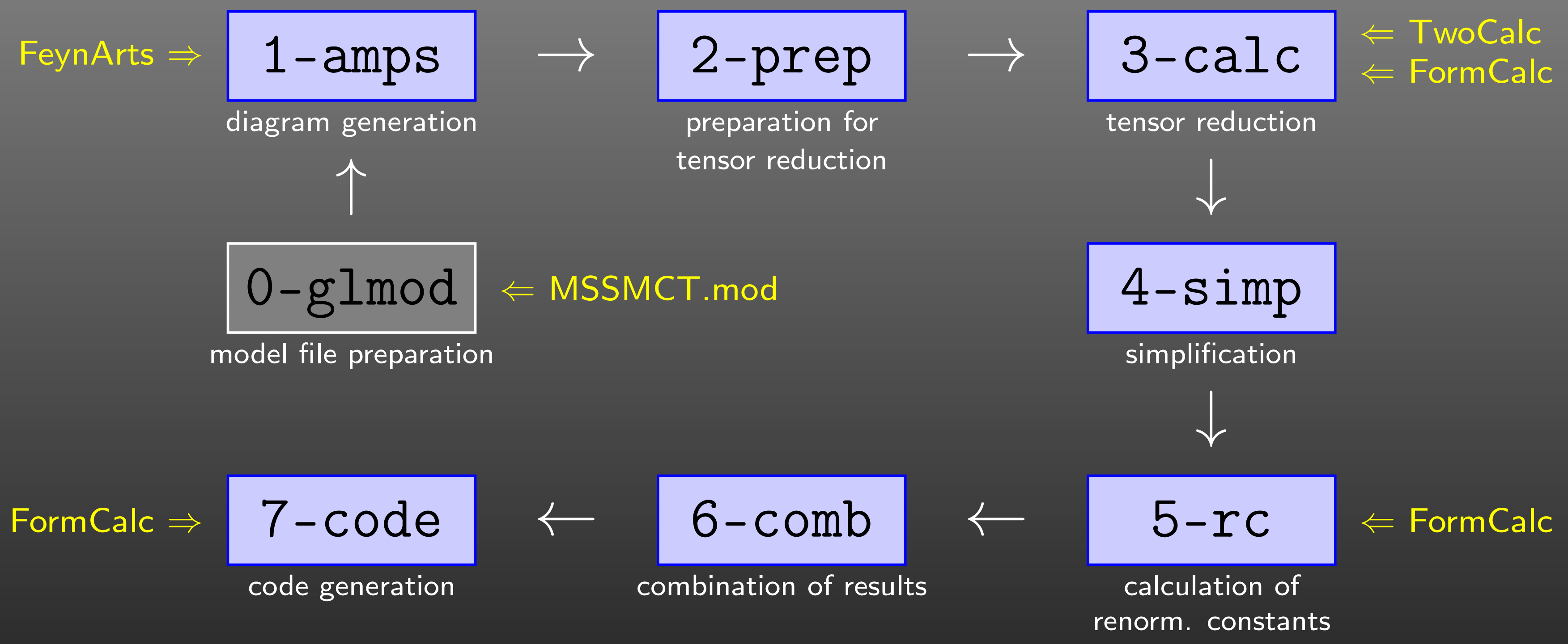
- Mostly 'monolithic' (one package does everything).
- Often controlled by parameter cards, not easy to use beyond intended purpose.
- May want to/must use other packages.

Universality w.r.t. Loop Order

- ✔ **Model → Model file → Diagrams** — Diagram Generation
- ✔ **Fermion algebra (traces, Dirac eq)** — Symbolic Simplification
- ✔ **Color algebra ($\mathrm{SU}(N)$ traces)**
- ✘ **Perform tensor reduction, *or* Isolate integrals for OPP**
- ✔ **Other simplifications (e.g. Fierz, abbreviations)**
- ✔ **Code generation (Fortran, C/++)** — Numerical Evaluation
- ✓ **Phase-space integration**
- ✘ **Evaluation of loop integrals**

$\mathcal{O}(\alpha_t^2)$ Higgs-mass corrections 'Organization chart'

Calculation split into 7 (8) steps:



Implemented as shell scripts + Mathematica packages.
Systematic input/output, log files, makefile, etc.

Scripting Solution

As a by-product of the implementation of the $\mathcal{O}(\alpha_t^2)$ corrections in FeynHiggs we have a

- Suite of scripts which can be used as a
- Template for similar calculations,
  - Two-loop,
  - Nontrivial model (MSSM),
  - Nontrivial renormalization,
  - Specific approximations (gaugeless, $p^2 = 0$),
  - Optimized output.

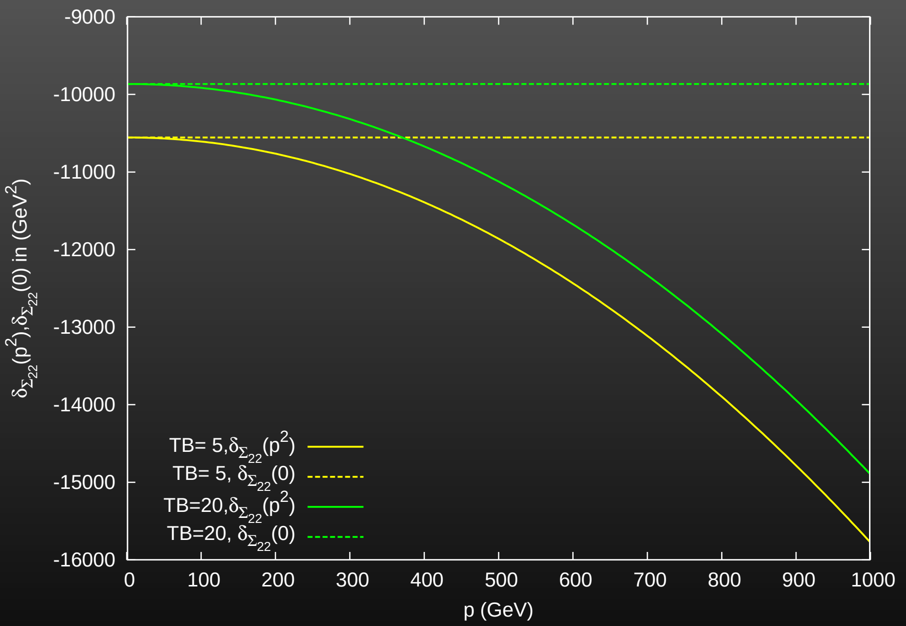
Hahn, Paßehr

Further BSM Application

k^2-dependent $\mathcal{O}(\alpha_s\alpha_t)$ Higgs-mass corrections

Borowka, Di Vita, Hahn, Heinrich, Hollik

- Relevant 2L integrals cannot be expressed by logs and dilogs (as for $k^2 = 0$), use SecDec or TSIL for evaluation.
- Available for SUSY analyses as a FeynHiggs extension.



MPP Pheno Group as of Dec 2015

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Long-term members:
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Henning Bahl, Tao-Li Cheng, Stephan Hessenberger, Stephan Jahn, Viktor Papara, Cyril Pietsch, Johannes Schlenk, Felix von Soden-Fraunhofen

The Last Episode

DAS WAR'S