Towards a Quantum Theory of Solitons

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Our Group

Lena Funke: Gravitational Anomalies

Lukas Gründing: Corpuscular Theories of Solitons, Instantons and AdS

Tehseen Rug: Corpuscular Theories of Solitons, Instantons and AdS

Sebastian Zell: Corpuscular Theory of dS

Gia Dvali: Everything

Contents

An Example

Classical electric field:

$$\partial_i E_i = \rho \tag{1}$$

- given charge distribution $\rho \Rightarrow$ classical electric field ρ
- QED $\Rightarrow$ particles are fundamental, classical field as emergent phenomenon.
- classical field E_i is coherent state of N (longitudinal) photons
- semi-classical limit: $N \to \infty$

Semiclassical Approach

Quantum effects encoded in fluctuations around background

- expand $E_i \to E_i + \delta E_i$
- compute loops of "δE_i-particles" in classical background E_i
- classical limit: $\hbar \to 0$

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Levels of Quantumness

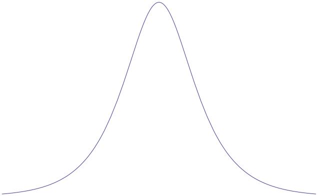
$$\begin{aligned} \text{Fully Quantum} &: N, \hbar \text{ finite} \\ \text{Semi-classical} &: N \to \infty,\ \hbar \text{ finite} \\ \text{Classical Limit} &: N \to \infty, \hbar \to 0 \end{aligned} \tag{2}$$

Non-topological Soliton

Definition: Soliton is a static, localized, finite energy solution

non-topological soliton:

$$\mathscr{L} = (\partial_\mu \phi)^2 - m^2\phi^2 + g^2\phi^4 \;\rightarrow\; \phi_c(x) = \frac{1}{\sqrt{2}}\frac{m}{g}\mathrm{sech}(mx)$$

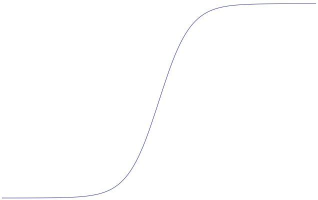


characteristic size: $L = m^{-1}$, Energy: $E_{nt} = 2\int dx(\partial_x \phi)^2 = \frac{2m^3}{3g^2}$

Topological Soliton

topological soliton:

$$\mathscr{L} = (\partial_\mu \phi)^2 - g^2(\phi^2 - m^2/g^2)^2 \rightarrow \phi_c(x) = \frac{m}{g}\tanh(xm)$$



characteristic size: $L = m^{-1}$, Energy: $E_{top} = \frac{8m^3}{3g^2}$

new characteristic: topological charge Q

Basic Idea (Dvali, Gomez, Gruending, Rug; Nucl. Phys. B)

Idea: Understand Solitons and their properties quantum mechanically!

- expand $\phi_c(x) = \sqrt{R} \int \frac{dk}{\sqrt{4\pi|k|}} \left(e^{ikx}\alpha_k + e^{-ikx}\alpha_k^*\right)$

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- construct quantum soliton state self-consistently as coherent state:
  $|\mathrm{sol}\rangle = \prod_k e^{-\frac{1}{2}|\alpha_k|^2} \sum_{n_k=0}^{\infty} \frac{\alpha_k^{n_k}}{\sqrt{n_k!}} |n_k\rangle \;\rightarrow\; \hat{a}_k|\mathrm{sol}\rangle = \sqrt{N_k}|\mathrm{sol}\rangle$

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- classical limit: $\langle \text{sol}|\hat{\phi}_c|\text{sol}\rangle = \phi_c$,
  $|\alpha_k|^2 \equiv N_k$: occupation of corpuscles in mode k.

Implications: Non-Topological Soliton

- total constituent number: $N = \int_k N_k \propto \frac{m^2}{g^2}$
- energy: $E_{non-top} = \int_k |k| N_k = \frac{4m^3}{3g^2}$

  dominant contribution to total number and energy: $k \sim m$!

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- vacuum instability: $\langle 0|sol\rangle = e^{-\frac{N}{2}} \neq 0$

  vacuum can decay into soliton quantum mechanically!

Implications: Topological Soliton

- total constituent number: $N = \int_k N_k \sim \log(k_0)|_{k_o \to 0} \to \infty$

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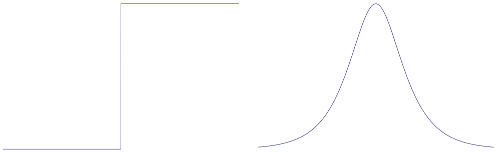
- attractive potential: interaction of corpuscles

Topological Soliton as Convolution

Idea: use convolution to disentangle energy and topology

- $\phi_c(x) = \frac{m}{g}\left(\mathrm{sign} \star \mathrm{sech}^2\right)(mx)$



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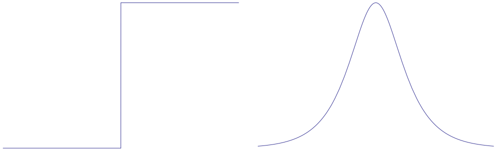

- convolution theorem: $\alpha_k = t_k\, c_k$

  $c_k \equiv \sqrt{\pi m}\frac{k}{g}\text{csch}\left(\frac{\pi k}{2m}\right),\ t_k \equiv \frac{i}{\sqrt{k}}$

  energy encoded in c_k quanta: $H = \int c_k^\dagger c_k$

  topological charge encoded in pole of t_k at $k=0$!

Summary and Outlook

Summary:

- represent solitons quantum mechanically as coherent states
- energy and topology encoded in distribution of corpuscles
- attractive potential due to interaction of corpuscles.

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Outlook:

- include $1/N$ corrections
- corpuscular theory of instantons
- coherent state picture of gravitational backgrounds

Thank You for Your Attention