A quantum picture of de Sitter spacetime

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Work with Gia Dvali and César Gomez

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Corpuscular approach

- Idea: The world is fundamentally quantum

⇒ Classical solution = collective effect of appropriate quanta (corpuscules)[1]

[1] G. Dvali and C. Gomez, *Quantum Compositeness of Gravity: Black Holes, AdS and Inflation*, `arXiv:1312.4795`.

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- Tehseen's talk: Solitons as corpuscular bound states[2]

⇒ Topological properties determined by number of corpuscules

[1] G. Dvali and C. Gomez, *Quantum Compositeness of Gravity: Black Holes, AdS and Inflation*, arXiv:1312.4795.

[2] G. Dvali, C. Gomez, L. Gründing and T. Rug, *Towards a Quantum Theory of Solitons*, arXiv:1508.03074.

Outline

De Sitter metric

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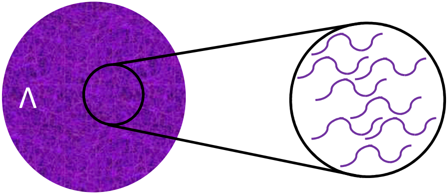
- Goal: Obtain Φ as classical limit of a graviton bound state



Bound-state gravitons

- Two different Fock spaces:
  - $\hat{a}^{\dagger}_{\vec{k}}$ creates **free** gravitons.
  - $\hat{b}^{\dagger}_{\vec{k}}$ creates **bound-state** gravitons.

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- Conditions on the quantum state $|N_\Lambda\rangle$:
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- Only free parameter left: $N \propto \langle N_\Lambda | b_{\vec{0}} b^{\dagger}_{\vec{0}} | N_\Lambda \rangle$

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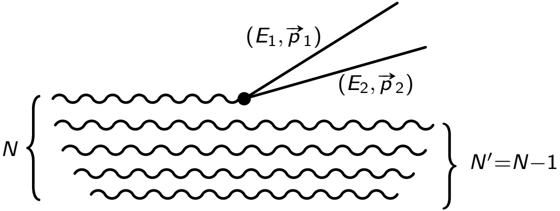
- Representation of Φ independent of source

Decay constant



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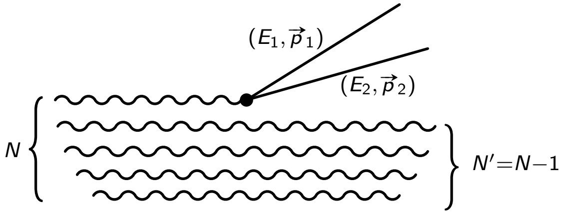

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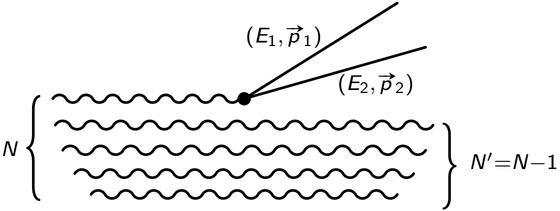

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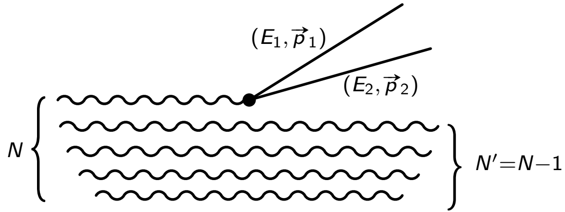

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- Quantum correction because of back-reaction $(N' \neq N)$

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  $\Rightarrow$ Quantum break time[3]:

$$\Delta t \approx N\Gamma^{-1} = \frac{M_p^2}{\Lambda^{1.5}}$$

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$\Rightarrow$ Final state without classical metric description?

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- Particle production because of graviton decay
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Future research

- Minkowski as graviton state
- Model final de Sitter state
- Inflationary scenarios
- Other metrics such as AdS