

Precise QCD predictions for Higgs boson pair production

Javier Mazzitelli

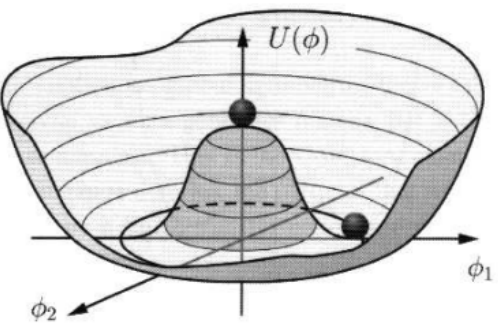
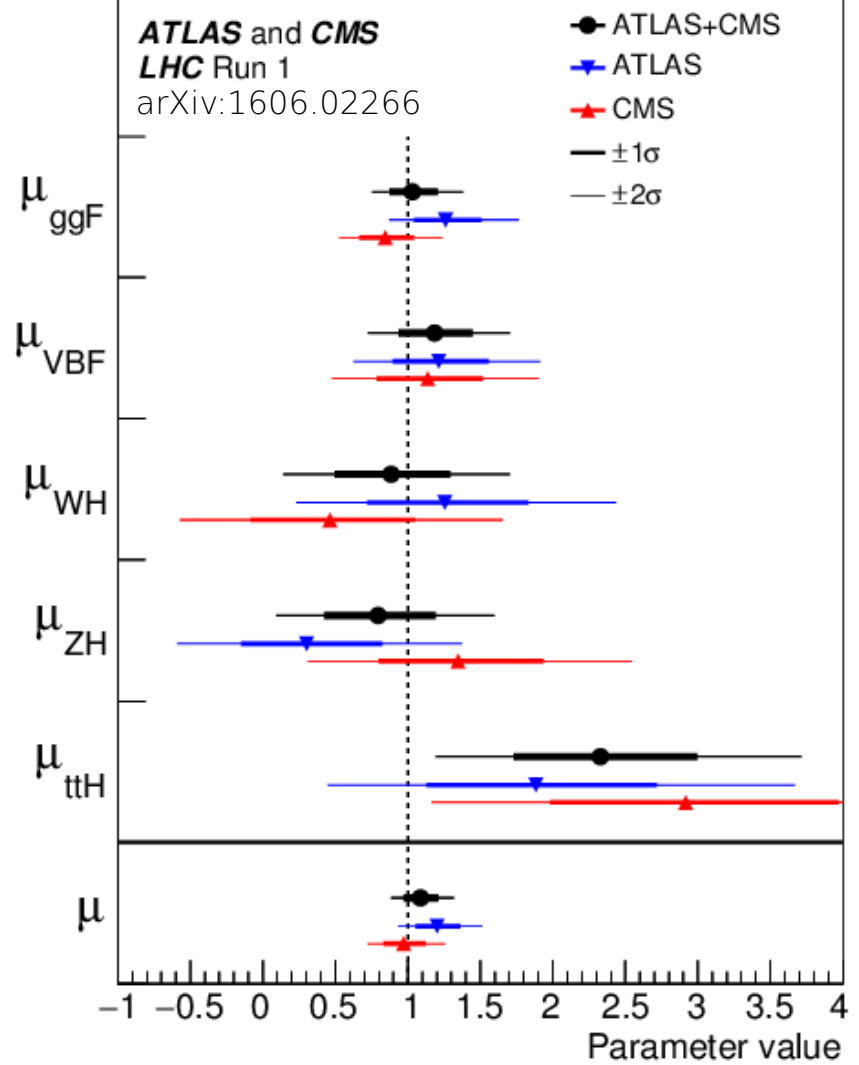


**Universität
Zürich^{UZH}**

MPI Munich, March 2018

Multi-Higgs production

- Higgs couplings to fermions and gauge bosons so far compatible with SM
- What happens for the Higgs self-couplings?
Are they relevant?
How can we measure them?



- Self-couplings determined by the Higgs potential

$$V(H) = \frac{1}{2}M_H^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda' H^4$$

In the SM: $\lambda = \lambda' = M_H^2 / (2v^2)$

- Crucial to understand EWSB

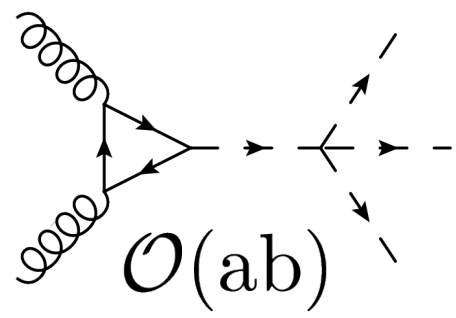
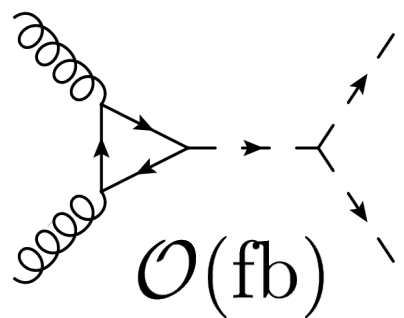
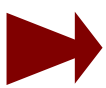
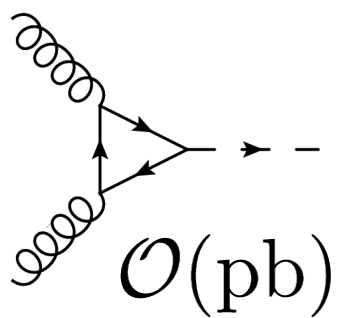
Multi-Higgs production \longrightarrow Direct access to Higgs self-couplings

Produce an off-shell Higgs boson that decays into:

Trilinear coupling
 $H^* \rightarrow HH$

Quartic coupling
 $H^* \rightarrow HHH$

Experimentally very challenging!



At the LHC:

Double Higgs production: challenging

Triple Higgs production: impossible

Outline

- **Introduction:**

 - Motivation, main production and decay modes

 - Status and prospects for the LHC

- **QCD corrections in the large M_t limit:**

 - NNLO cross section

 - Threshold resummation at NNLL

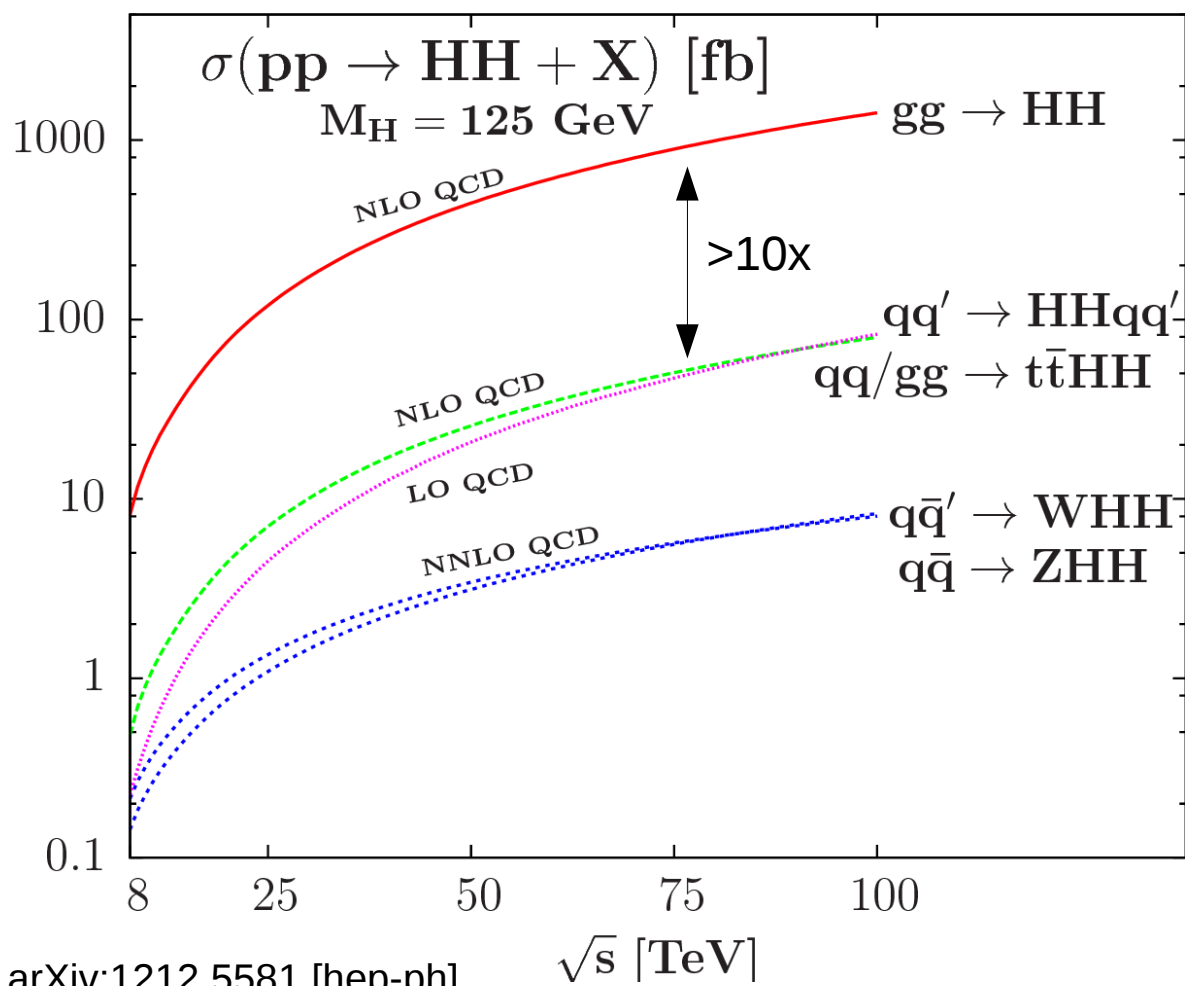
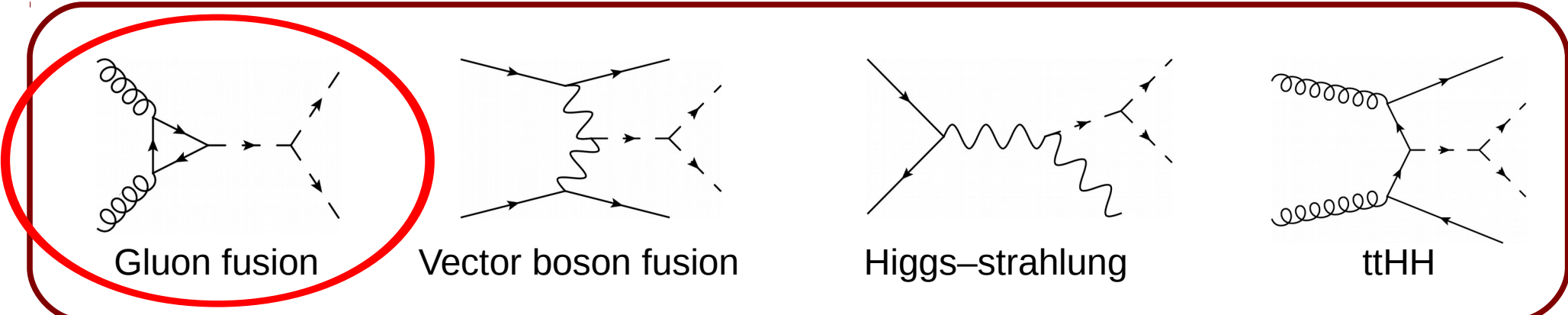
- **NNLO predictions with finite M_t effects:**

 - Technical ingredients, NNLO approximations

 - Total cross sections and differential distributions

- **Conclusions**

Double Higgs production mechanisms



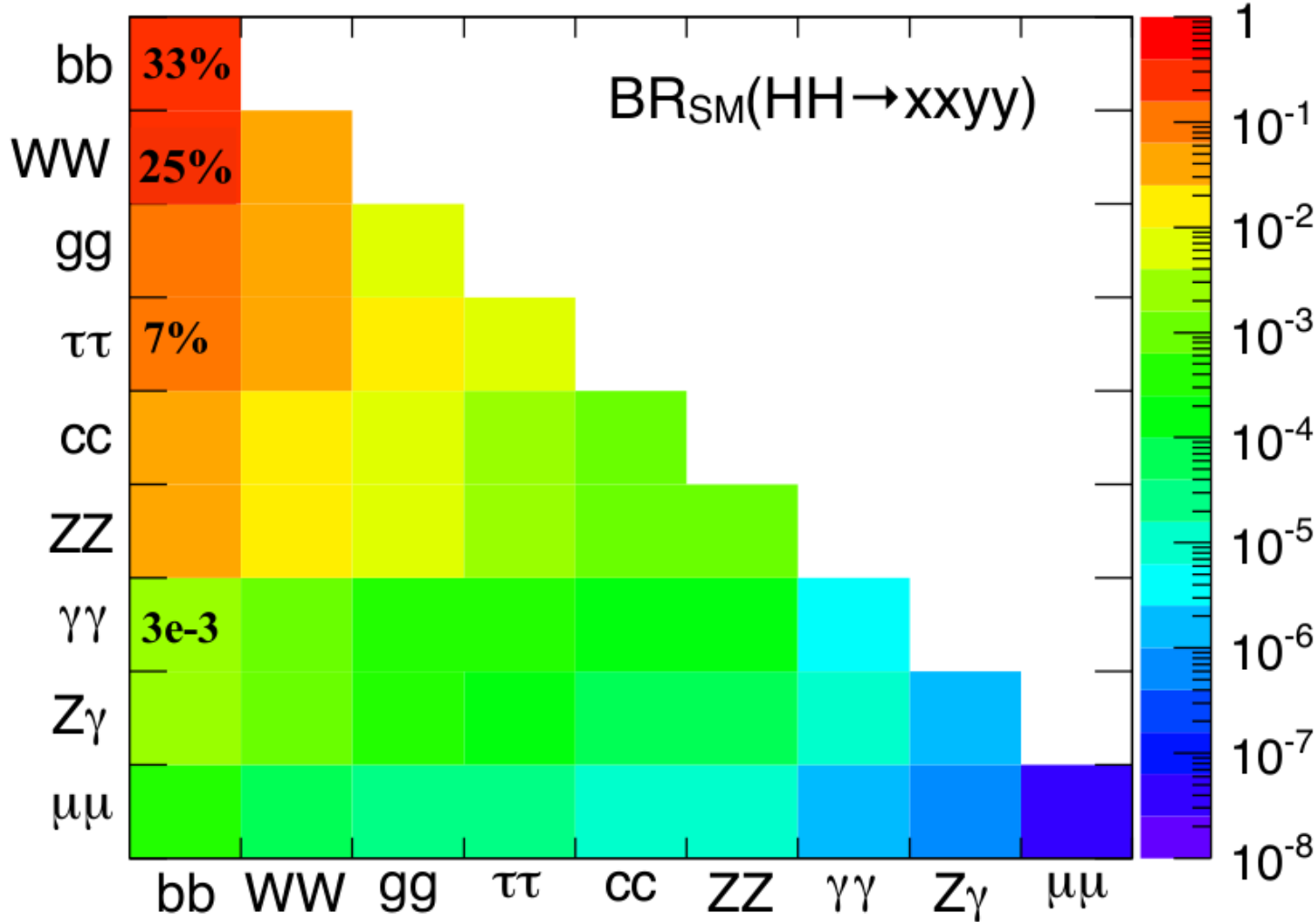
Gluon fusion
 ↓
 Main production channel

~1000 times smaller than single Higgs XS

Measurement of subleading channels is very difficult

From arXiv:1212.5581 [hep-ph]

Di-Higgs decay channels



Relevant channels: in general at least one H → bb to have large BR

- bbbb:** highest BR, high QCD and tt contamination
- bbWW:** high BG, large irreducible tt background
- bbττ:** relatively low background and low BR
- bbyy:** high purity, very low BR

LHC results

BSM scenarios can substantially enhance the HH cross section or produce a resonance



Both resonant and non-resonant searches have been performed at ATLAS and CMS

Results from non-resonant searches, upper limits:

- Run-I

ATLAS combined: 70 x SM
 CMS bbyy: 74 x SM

- Run-II

Reaching O(10) xSM sensitivity

- SM sensitivity: full HL-LHC statistics

σ/σ_{SM} 95% C.L. (exp)

	ATLAS	CMS
bbbb	<29 (38)	<342 (308)
bbWW		<79 (89)
bb $\tau\tau$		<28 (25)
bbyy	<117 (161)	<19 (17)
WWyy	<747 (386)	

P. Meridiani, EPS17

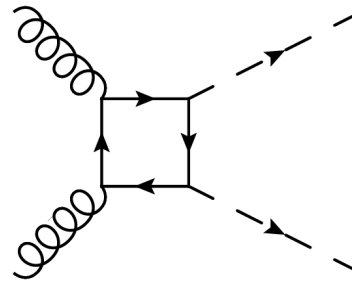
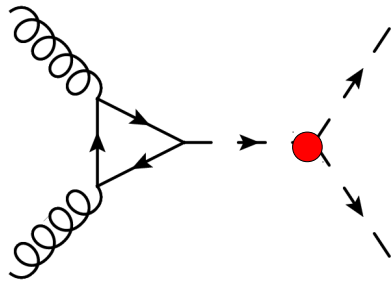
3 fb⁻¹ 13 fb⁻¹ 36 fb⁻¹

Prospects for the LHC and beyond

- Assuming a SM rate, HH production should be observed at the HL-LHC
- Expected uncertainty on the signal yield: $O(50\%)$ using $b\bar{b}\gamma\gamma$ and $b\bar{b}\tau\tau$
- Combination with other decay channels (specially $4b$) will reduce this uncertainty

[ATL-PHYS-PUB-2014-019, ATL-PHYS-PUB-2015-046, CMS PAS FTR-15-002]

Higgs pair production should be observed at the HL-LHC... **but we also want to measure λ**



Not all the contributions are sensitive!

Assuming a SM-like scenario

- Determination of λ will require full HL-LHC integrated luminosity and the combination of the different channels
- Even then, uncertainties on λ will be large
- Complementary information from loop effects in single Higgs and EW precision observables
- Precision determination of λ : one motivation for a 100TeV collider

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 - Threshold resummation at NNLL

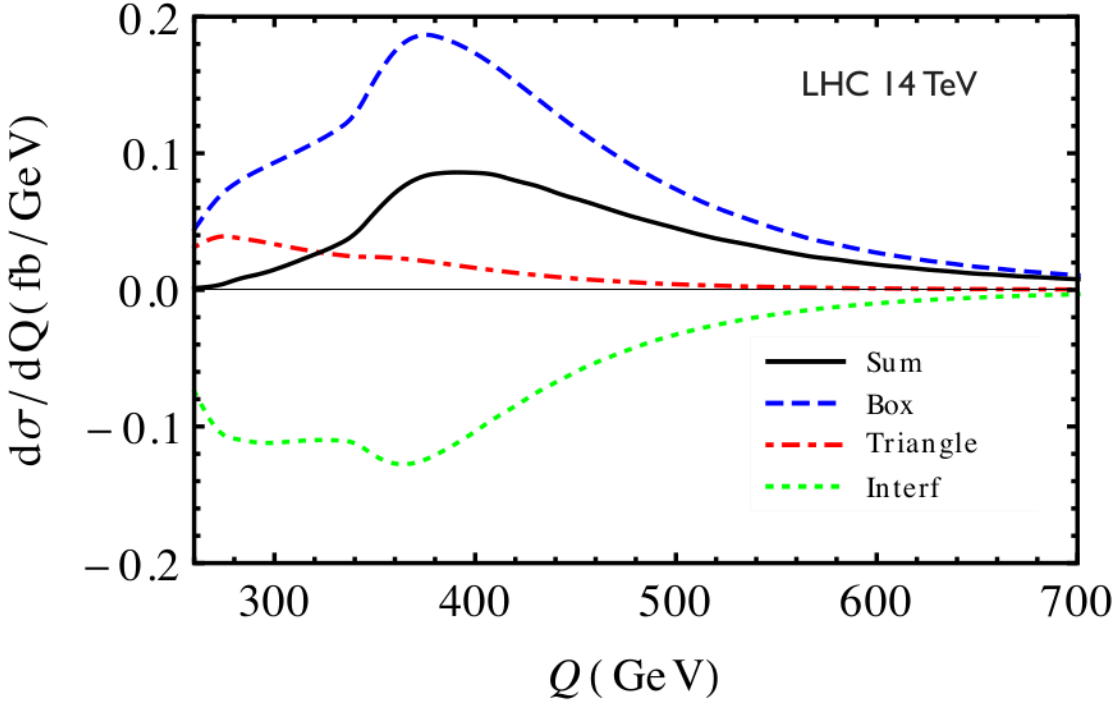
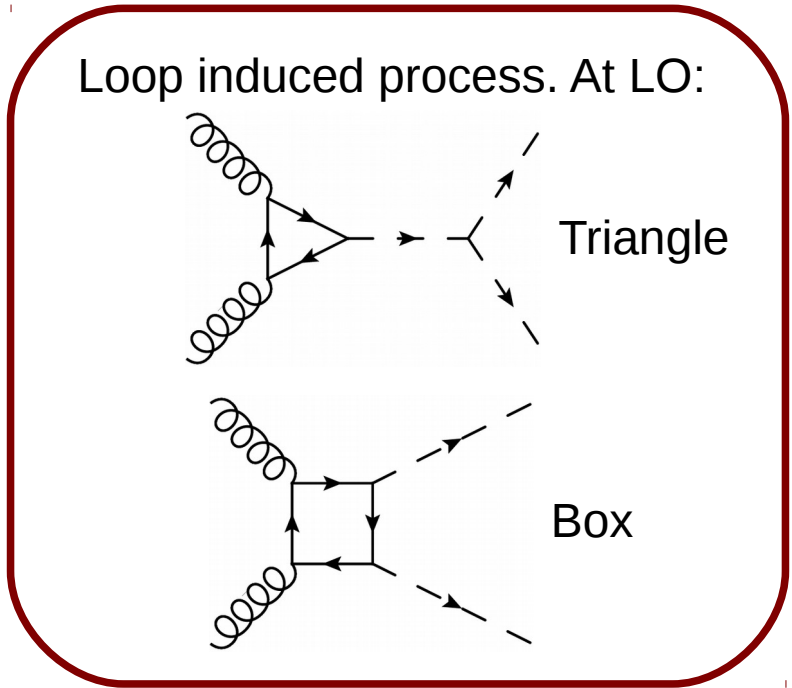
- **NNLO predictions with finite M_t effects:**

 - Technical ingredients, NNLO approximations

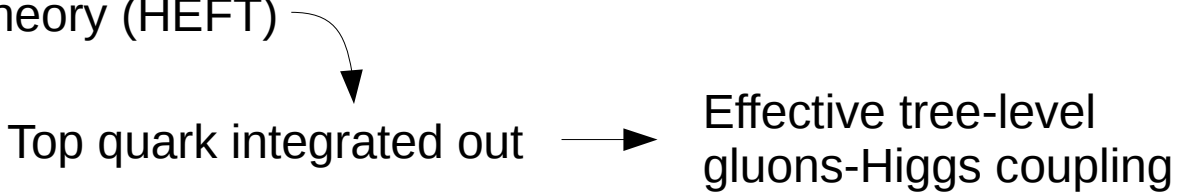
 - Total cross sections and differential distributions

- **Conclusions**

HH production via gluon fusion



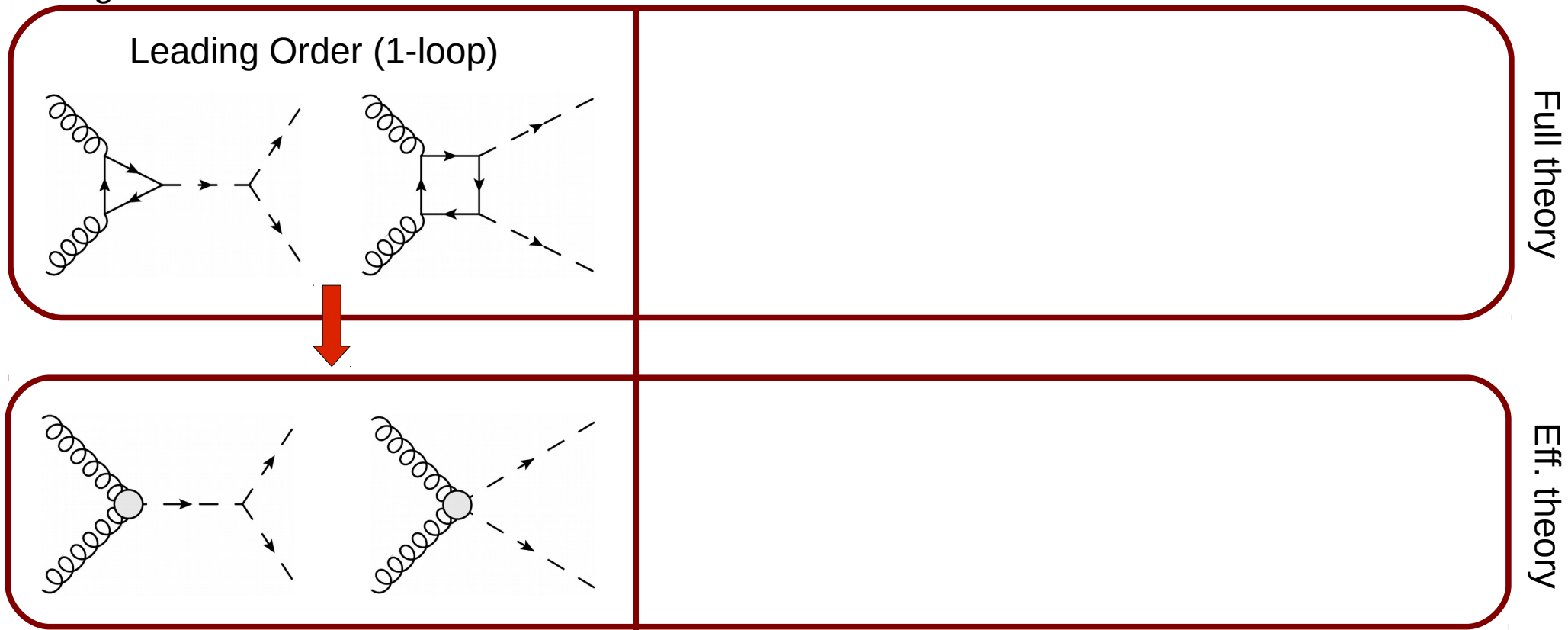
- LO has large uncertainties → QCD higher order corrections are needed!
- Their calculation is really difficult: exact NLO only became available in 2016
S. Borowka et al. arXiv:1604.06447
- NLO corrections are large (~66% at 14TeV), and with still sizeable uncertainties (~±13%)
- Beyond that: Higgs Effective Field Theory (HEFT)



- Corrections computed in the HEFT and normalized by exact LO differentially in M_{hh} (16% overestimation at NLO – further improvements also possible, but more about this later)

QCD corrections in the HEFT

E.g.: virtual corrections



- The effective vertices have the same structure!

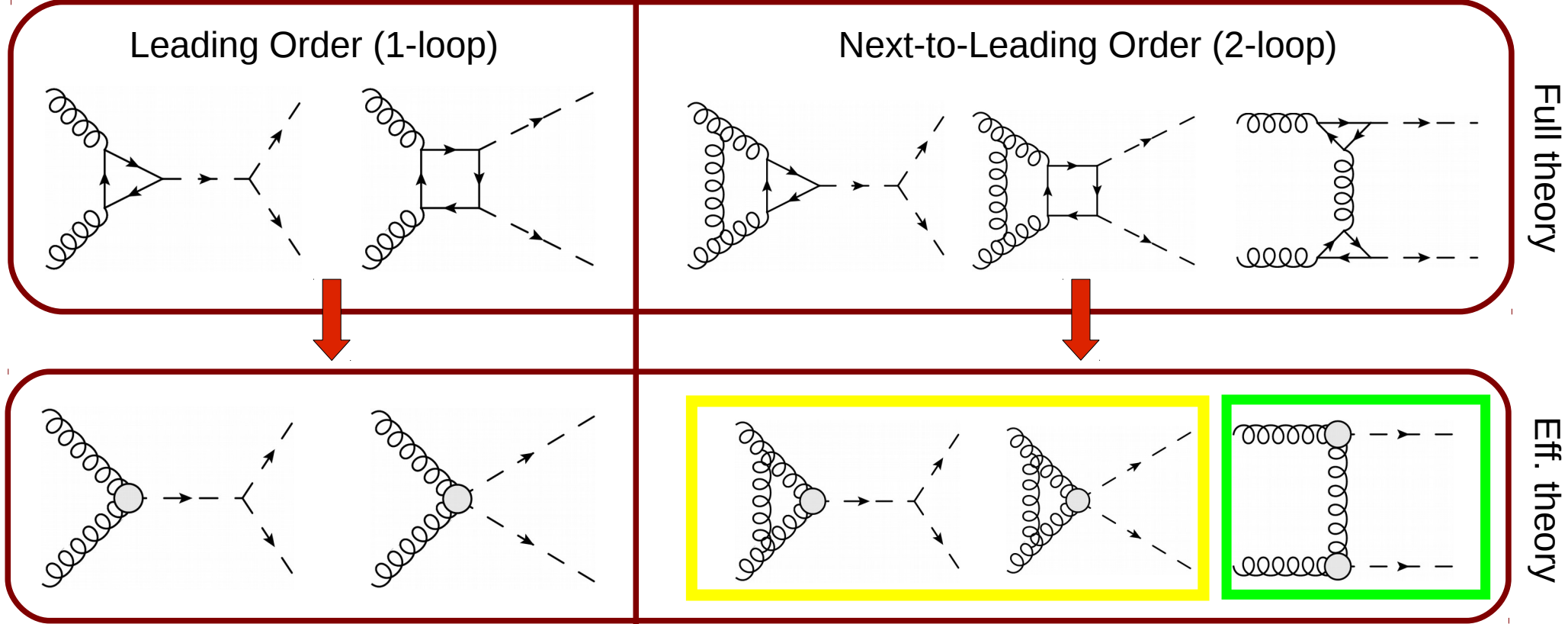
$$\mathcal{L}_{ggH} \propto G_{\mu\nu} G^{\mu\nu} H/v$$

$$\mathcal{L}_{ggHH} \propto G_{\mu\nu} G^{\mu\nu} (H/v)^2$$

- Profit from the single Higgs production results!

QCD corrections in the HEFT

E.g.: virtual corrections



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$$\mathcal{L}_{ggH} \propto G_{\mu\nu} G^{\mu\nu} H/v$$

$$\mathcal{L}_{ggHH} \propto G_{\mu\nu} G^{\mu\nu} (H/v)^2$$

- Profit from the single Higgs production results!

- We can split the calculation

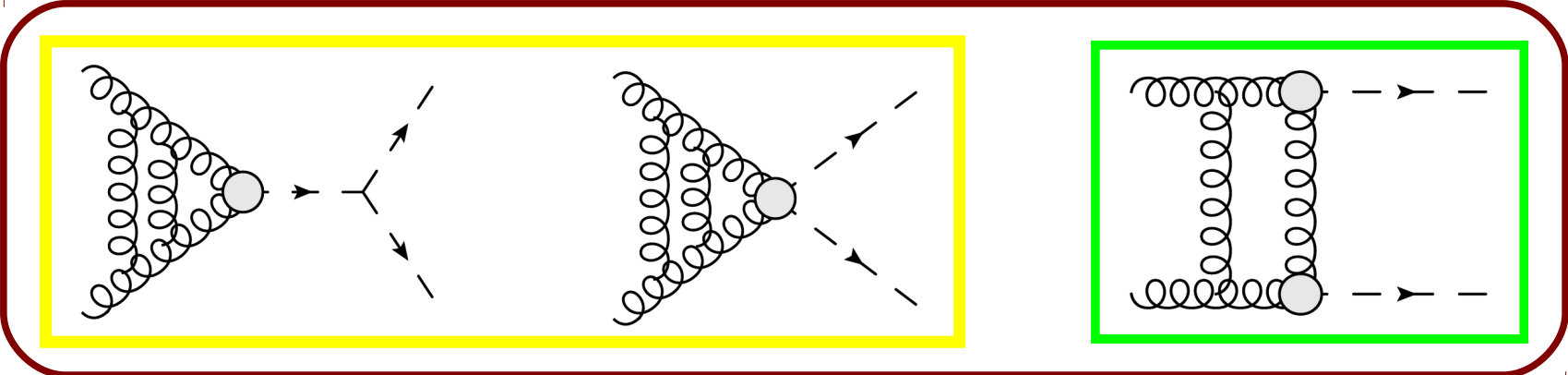
$$Q^2 \frac{d\hat{\sigma}}{dQ^2} = \hat{\sigma}^a + \hat{\sigma}^b$$

Single-Higgs like

Starts at NLO
New topologies with two effective vertices

QCD corrections in the HEFT

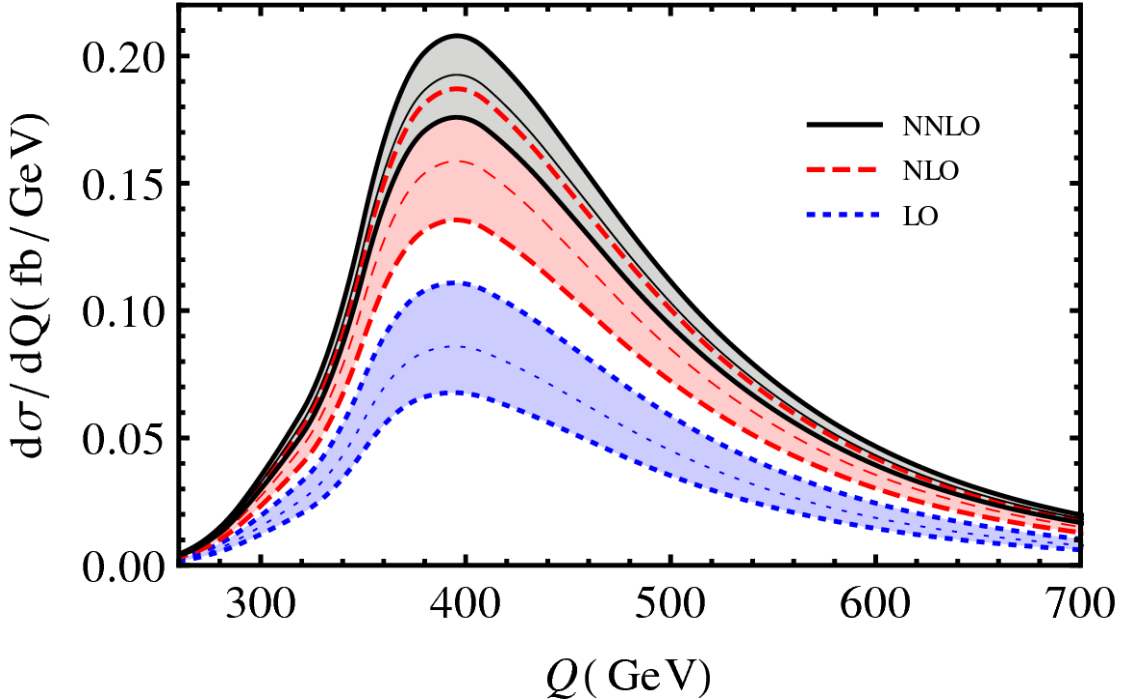
- At NNLO we have



- Similar idea for real-virtual and double-real corrections

We obtained analytical results for the NNLO total cross section, differential only in the HH invariant mass

- Extended to include BSM effects via EFT dimension 6 operators (backup slides)



Threshold Resummation

- All-order summation of threshold enhanced contributions

Higgs pair invariant mass

$$z = \frac{Q^2}{\hat{s}}$$

Threshold:

$$z \rightarrow 1 \iff N \rightarrow \infty$$

(Resummation performed in Mellin space)

- Originated by soft gluon emissions
- Threshold enhanced contributions: $(\ln N)^k$ (plus distributions in z space)

$$G_{gg,N}^{(\text{res})} = \Delta_N \times C_{gg} + \mathcal{O}(1/N)$$

Partonic cross section (in Mellin space)

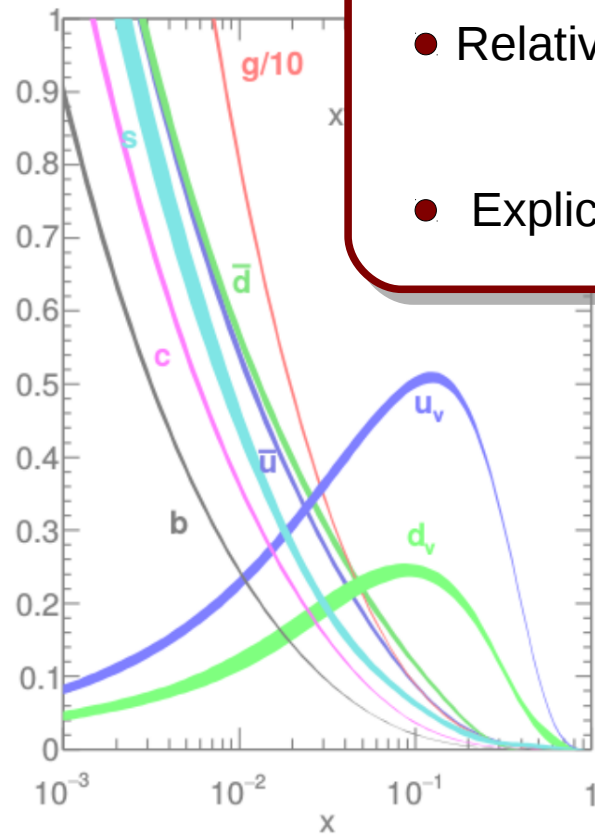
Sudakov factor
Exponentiates the large log corrections

Constant contributions
Virtual and non-logarithmic soft terms

Threshold Resummation

- Resummed contributions should account for the dominant effect of the uncalculated missing higher orders

- Parton distributions prefer lower partonic center of mass energies
- Relatively large invariant mass \longrightarrow Corrections dominated by threshold contributions
- Explicitly checked for HH up to NNLO via soft-virtual approximation



Threshold Resummation

$$G_{gg,N}^{(\text{res})} = \Delta_N \times C_{gg} + \mathcal{O}(1/N)$$

- **Sudakov factor:**

Known (same as for single Higgs prod)

$$\Delta_N = \exp \left(\underbrace{\ln N}_{\text{LL}} \underbrace{g^{(1)}(\lambda)}_{\text{NLL}} + \underbrace{g^{(2)}(\lambda)}_{\text{NLL}} + \underbrace{\alpha_S g^{(3)}(\lambda)}_{\text{NNLL}} + \dots \right), \quad \lambda = \beta_0 \alpha_S \ln N$$

Leading Log $\longrightarrow \alpha_S^n \ln^{n+1} N$

Next-to-Leading Log $\longrightarrow \alpha_S^n \ln^n N$

Next-to-Next-to-Leading Log $\longrightarrow \alpha_S^n \ln^{n-1} N$

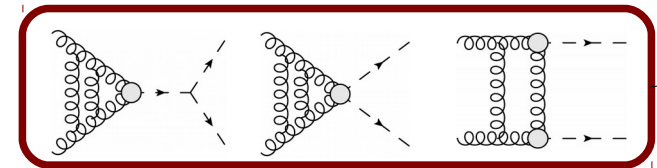
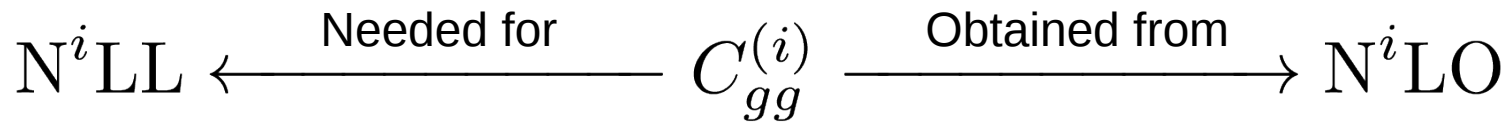
Threshold Resummation

$$G_{gg,N}^{(\text{res})} = \Delta_N \times C_{gg} + \mathcal{O}(1/N)$$

- **Constant contributions:**

Virtual and non-logarithmic
soft contributions

$$\longrightarrow C_{gg} = 1 + \sum_{n=1} \left(\frac{\alpha_S}{2\pi} \right)^n C_{gg}^{(n)}$$



- **Universal structure:** only process dependence encoded in FO virtual corrections

D. de Florian, JM, arXiv:1209.0673

Threshold Resummation

$$G_{gg,N}^{(\text{res})} = \Delta_N \times C_{gg} + \mathcal{O}(1/N)$$

- Inverse Mellin transform performed numerically
- Matching with the FO:

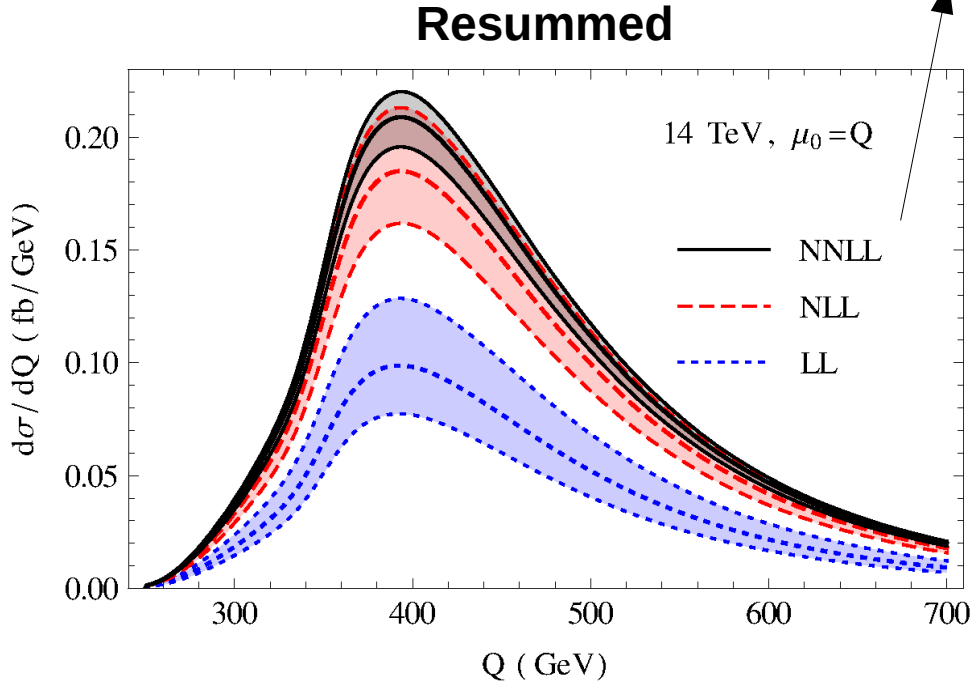
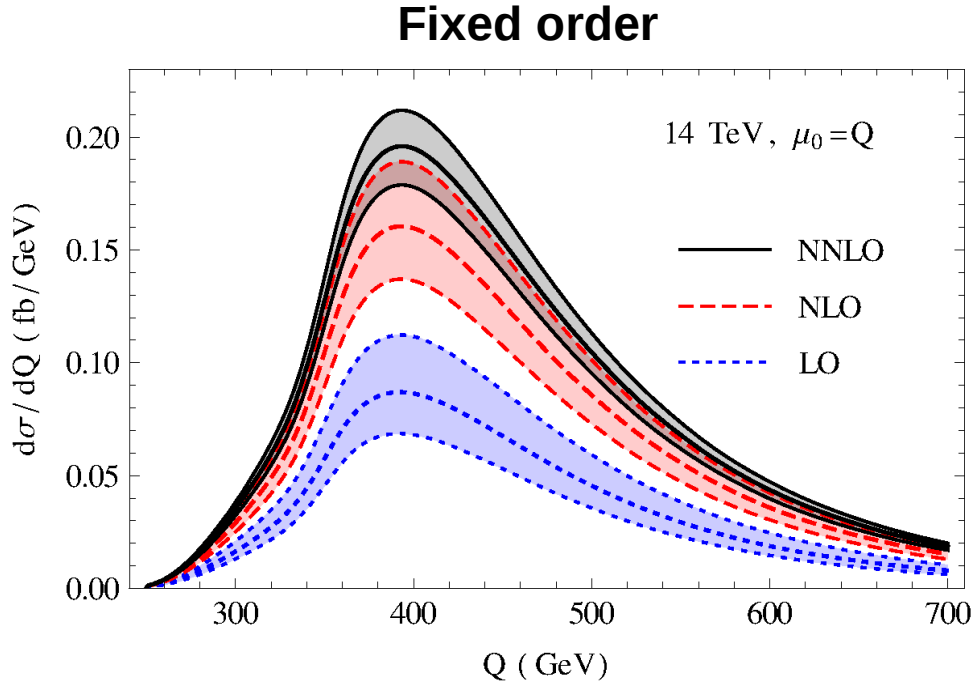
$$\sigma^{NNLL} = \underbrace{\sigma^{res} - \sigma^{res}|_{\mathcal{O}(\alpha_S^4)}} + \sigma^{NNLO}$$

Resummed contributions starting at $\mathcal{O}(\alpha_S^5)$

$$\Rightarrow \sigma^{NNLL}|_{\mathcal{O}(\alpha_S^4)} = \sigma^{NNLO}$$

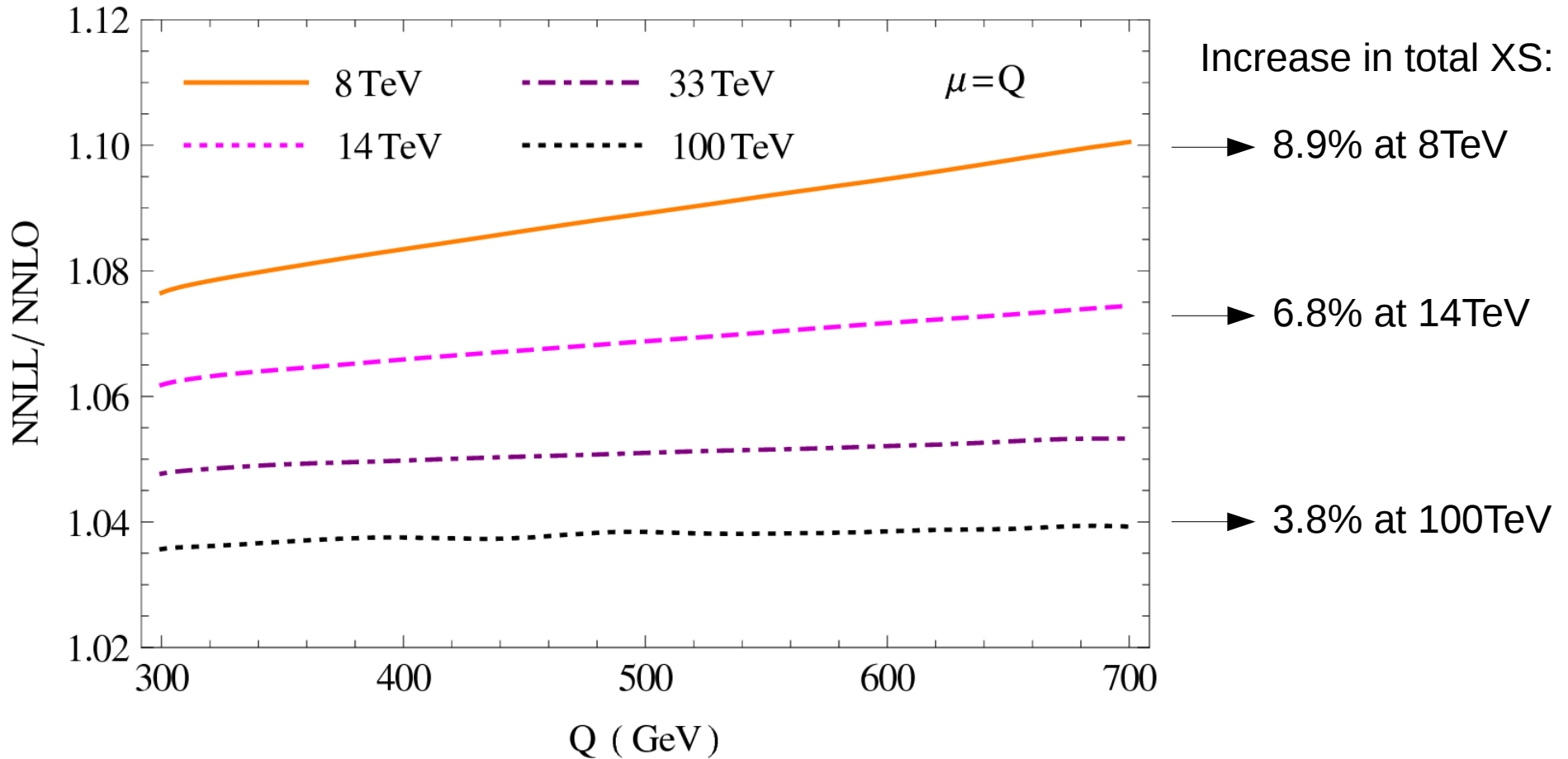
NNLO+NNLL numerical results

Here NNLL means NNLL+NNLO, etc



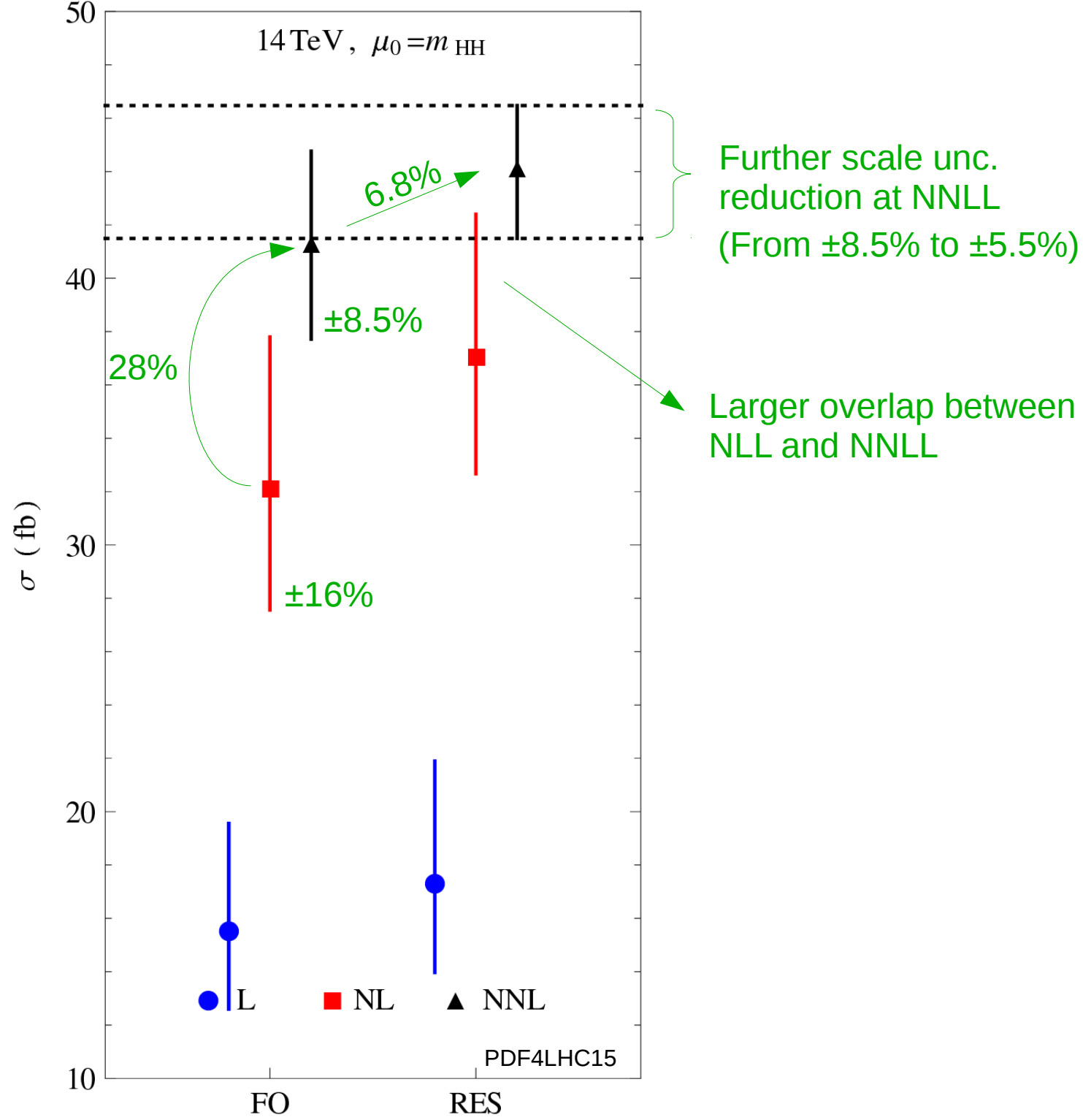
- Shape: small differences between FO and resummed distributions
- Uncertainty reduction from NNLO to NNLL
- Resummed contributions \longrightarrow increase of the cross section

- NNLL/NNLO ratio vs. HH invariant mass

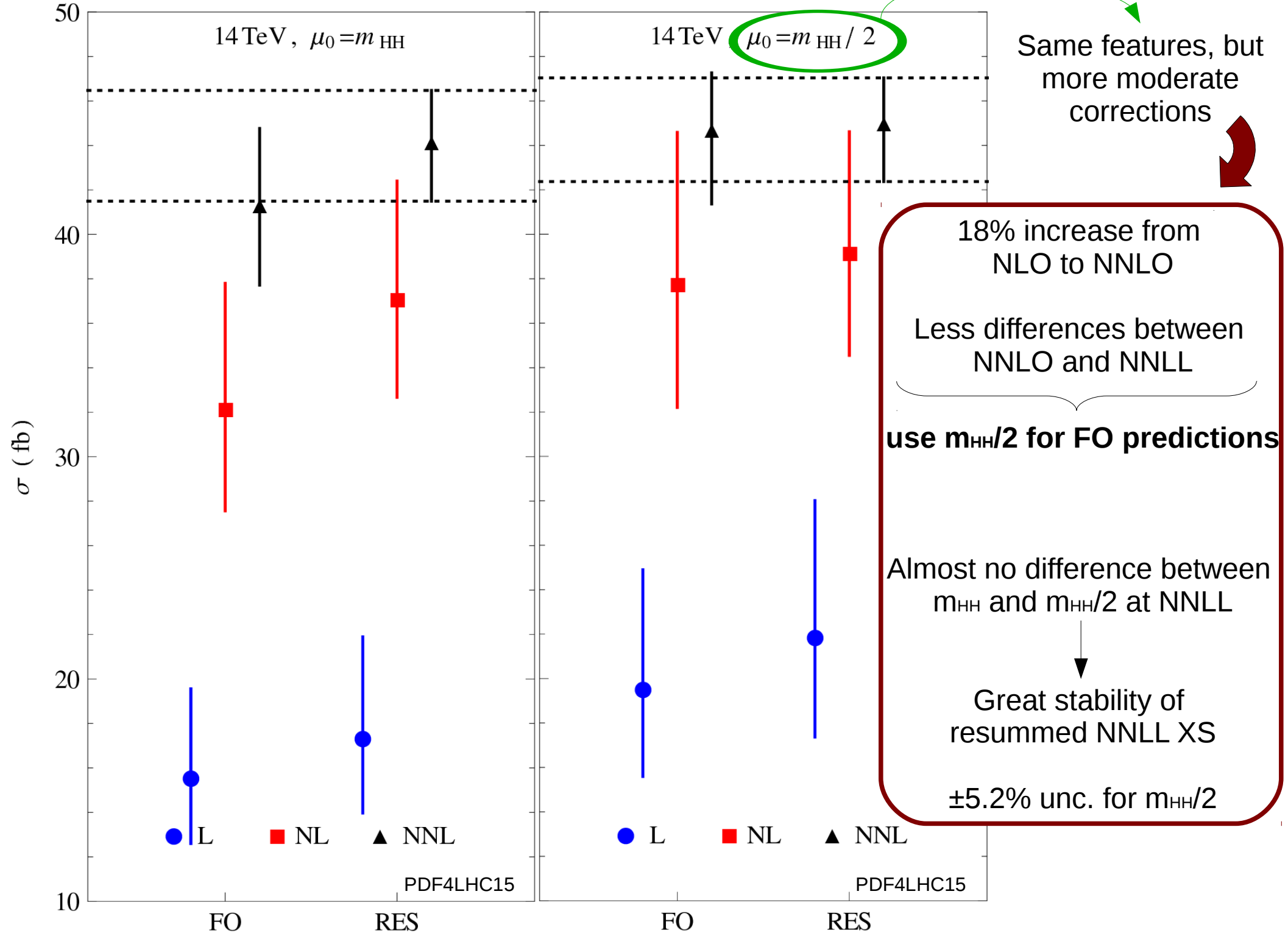


- NNLL always larger than NNLO, ratio is almost linear in Q
- Ratio increases for larger invariant masses → Closer to partonic threshold
- Larger collider energies → Smaller resummation effects (further from threshold)

● Total cross section



● Total cross section



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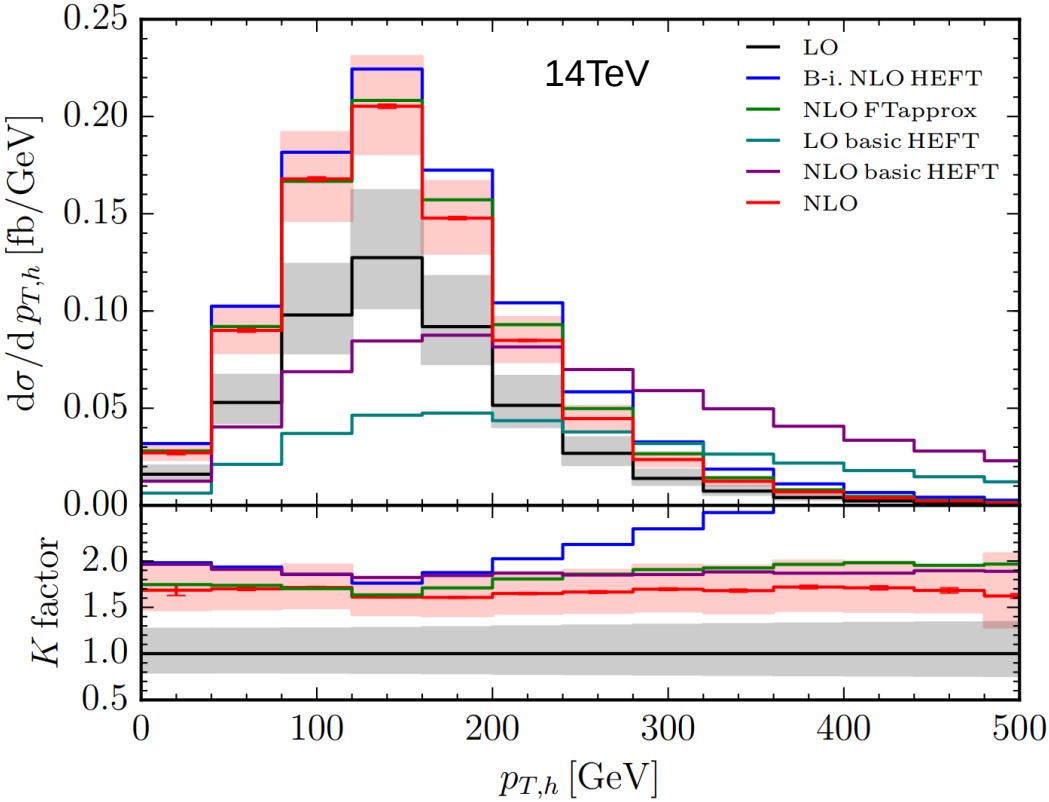
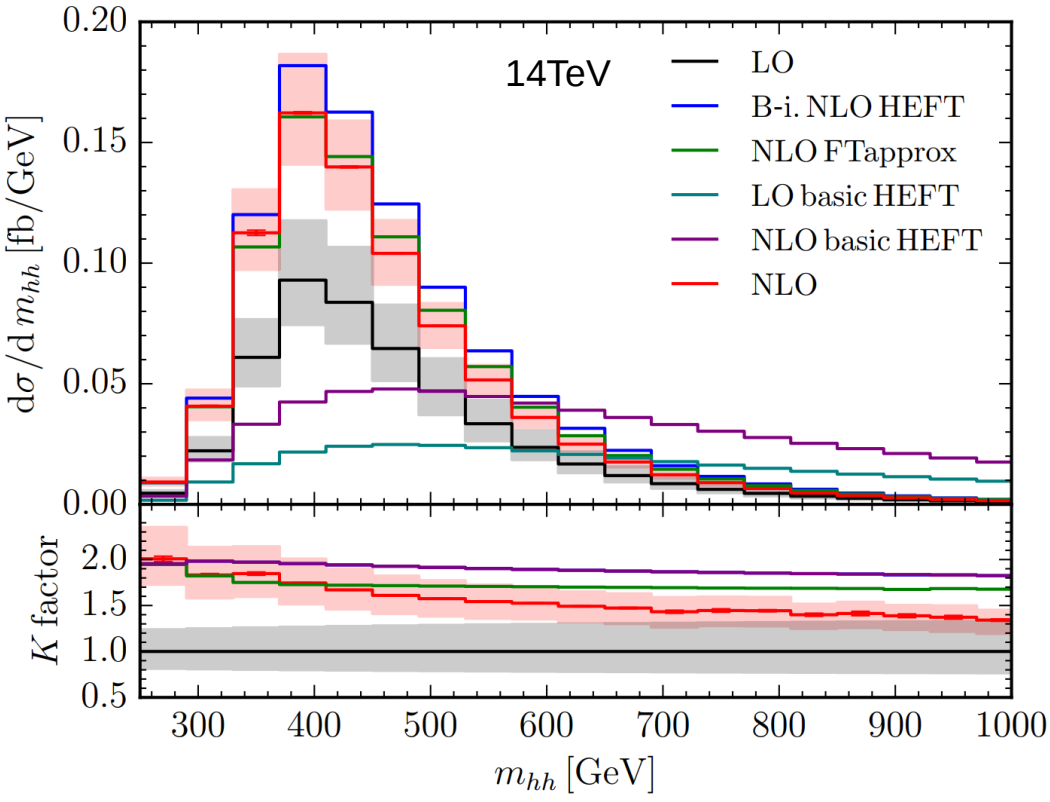
 - Total cross sections and differential distributions

- **Conclusions**

HEFT vs full theory

- HEFT: large M_t limit \rightarrow Worse than for single Higgs (larger invariant mass)
- Born improved overestimates the NLO total XS by a 15%
- Poor description of the tail of some distributions (associated with hard radiation)

NLO distributions [S. Borowka et al., arXiv:1608.04798]



- To obtain accurate NNLO results, we need to combine the HEFT NNLO with the full NLO
- Moreover, we need to include finite M_t effects in the NNLO corrections

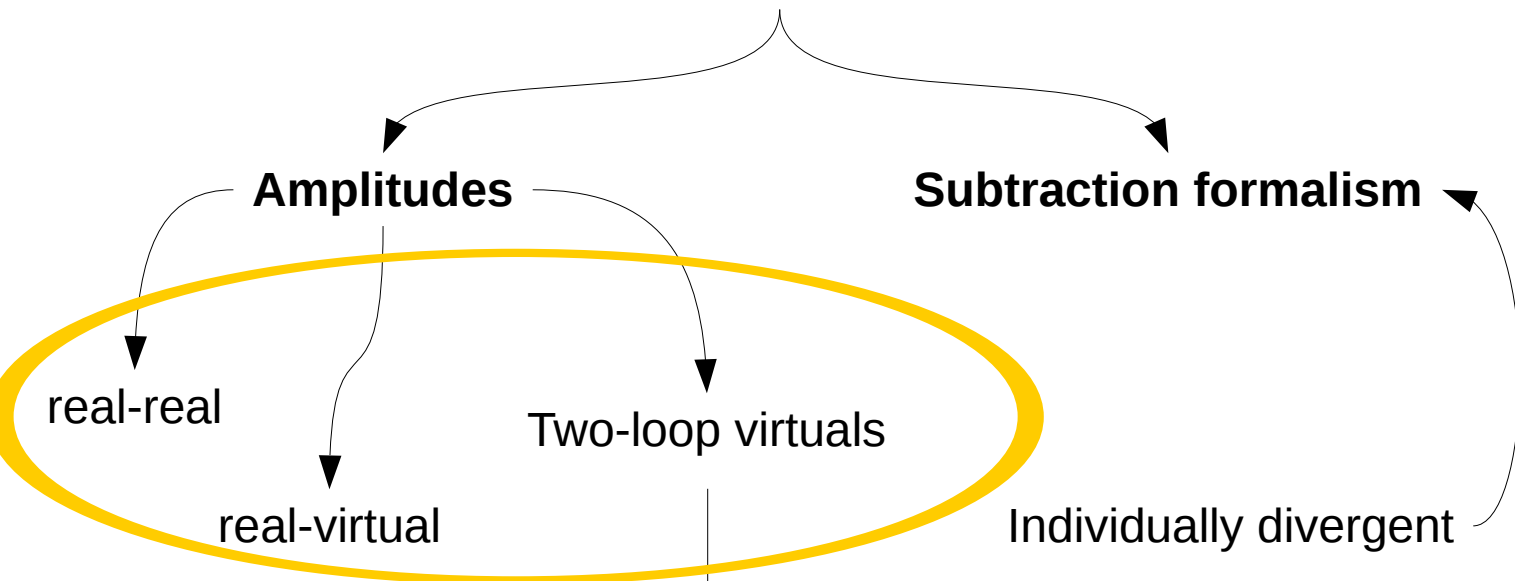
HH at NNLO with M_t effects

Higgs boson pair production at NNLO with top quark mass effects

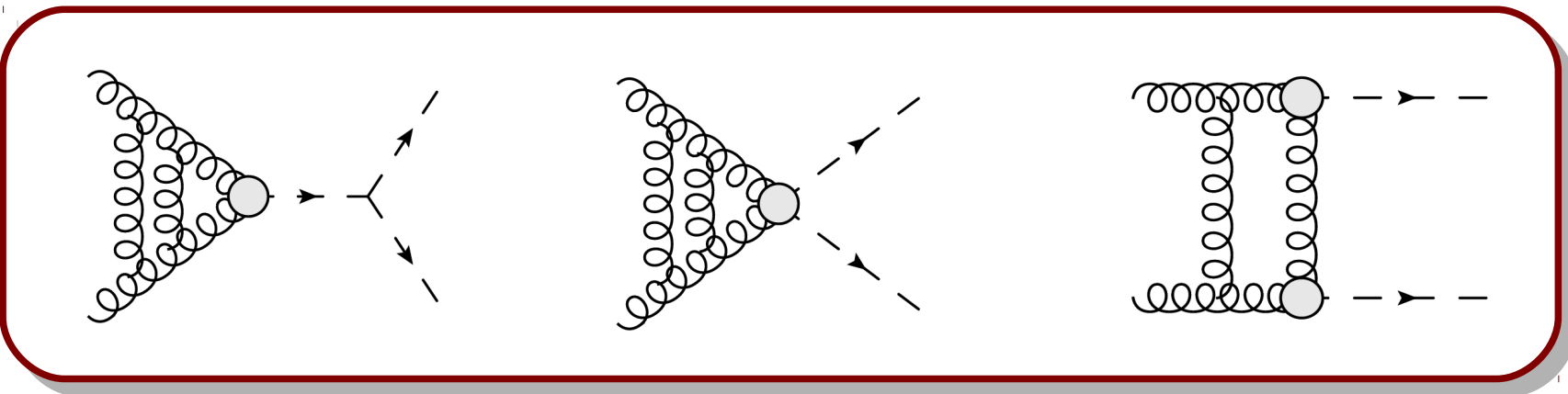
M. Grazzini, G. Heinrich, S. Jones, S. Kallweit, M. Kerner, J. Lindert, JM [arXiv:1803.02463]

- Fully differential predictions for Higgs boson pair production via gluon fusion
- Combination of full NLO with large- M_t NNLO
- NNLO piece improved with different reweighting techniques to account for finite- M_t effects
- Estimation of remaining M_t uncertainty at NNLO
- Most advanced perturbative prediction available to date

Differential HH production at NNLO (in the large M_t limit)



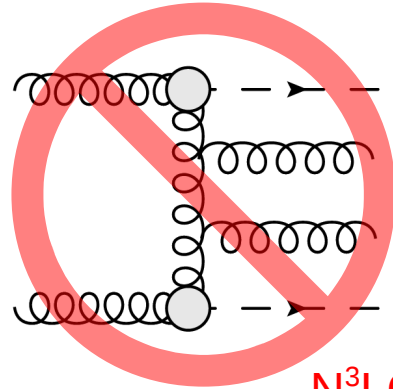
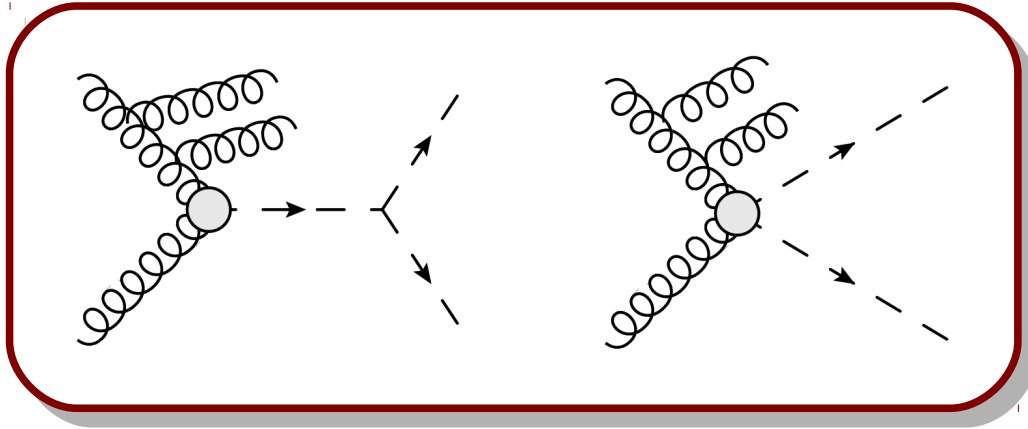
Obtained from the NNLO inclusive result



Double real and real-virtual

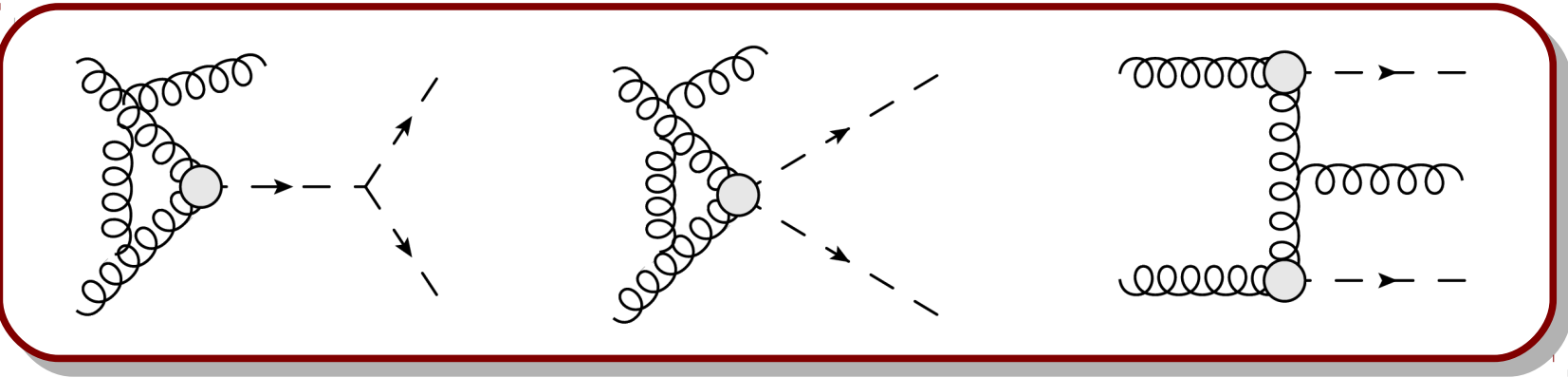
(in the large M_t limit)

real-real



N^3LO contribution

real-virtual



+ qg and qq channels

- Obtained from OpenLoops Cascioli, Lindert, Maierhöfer and Pozzorini, [arXiv:1111.5206]
- Fast and stable numerical evaluation of tree and one-loop matrix elements

qT subtraction

$$d\sigma_{\text{NNLO}}^{HH} \Big|_{q_T \neq 0} = d\sigma_{\text{NLO}}^{HH+\text{jet}} \longrightarrow \text{Computed with any NLO subtraction formalism}$$

real-real and real-virtual
of inclusive HH@NNLO

- Only $q_T \rightarrow 0$ infrared divergencies remain
- But small q_T behavior known from q_T resummation!

Process indep.
counterterm

$$d\sigma_{\text{NNLO}}^{HH} = \mathcal{H}_{\text{NNLO}}^{HH} \otimes d\sigma_{\text{LO}}^{HH} + \underbrace{\left[d\sigma_{\text{NLO}}^{HH+\text{jet}} - d\sigma_{\text{NNLO}}^{\text{CT}} \right]}_{\text{Finite for } q_T \rightarrow 0}$$

Hard coefficient that
includes the two-loop corrections

Finite for $q_T \rightarrow 0$

Our implementation is based on the public code MATRIX [Kallweit, Grazzini, Wiesemann]

qT subtraction

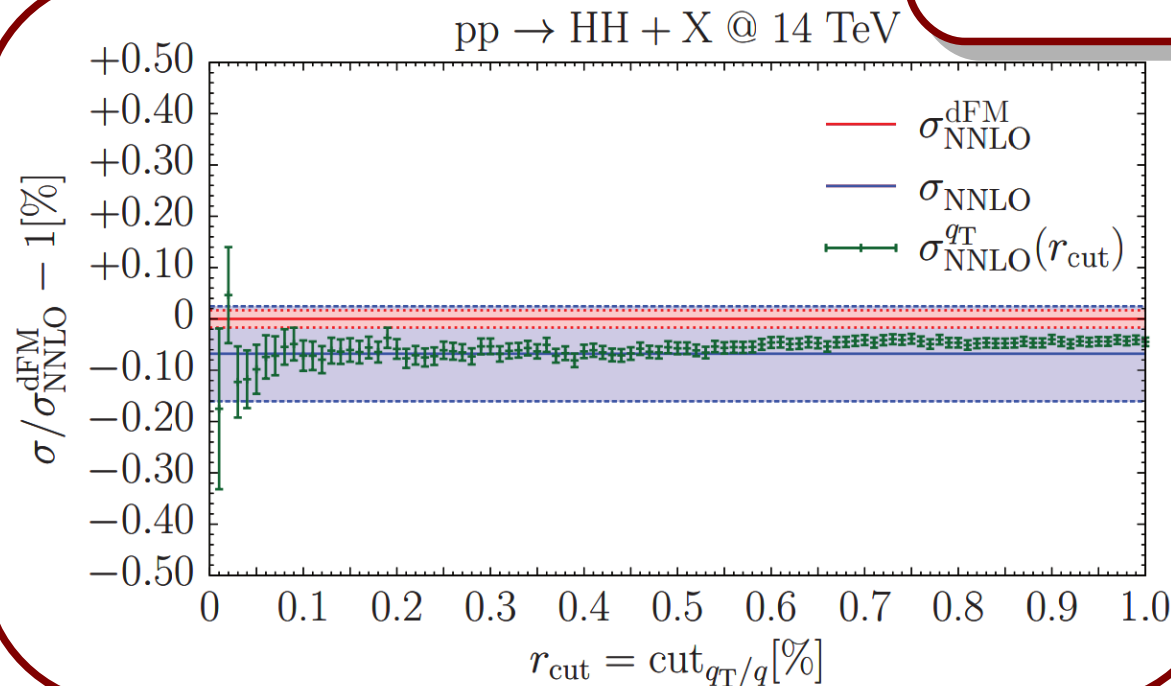
Finite for $q_T \rightarrow 0$

$$d\sigma_{\text{NNLO}}^{HH} = \mathcal{H}_{\text{NNLO}}^{HH} \otimes d\sigma_{\text{LO}}^{HH} + \left[d\sigma_{\text{NLO}}^{HH+\text{jet}} - d\sigma_{\text{NNLO}}^{\text{CT}} \right]$$

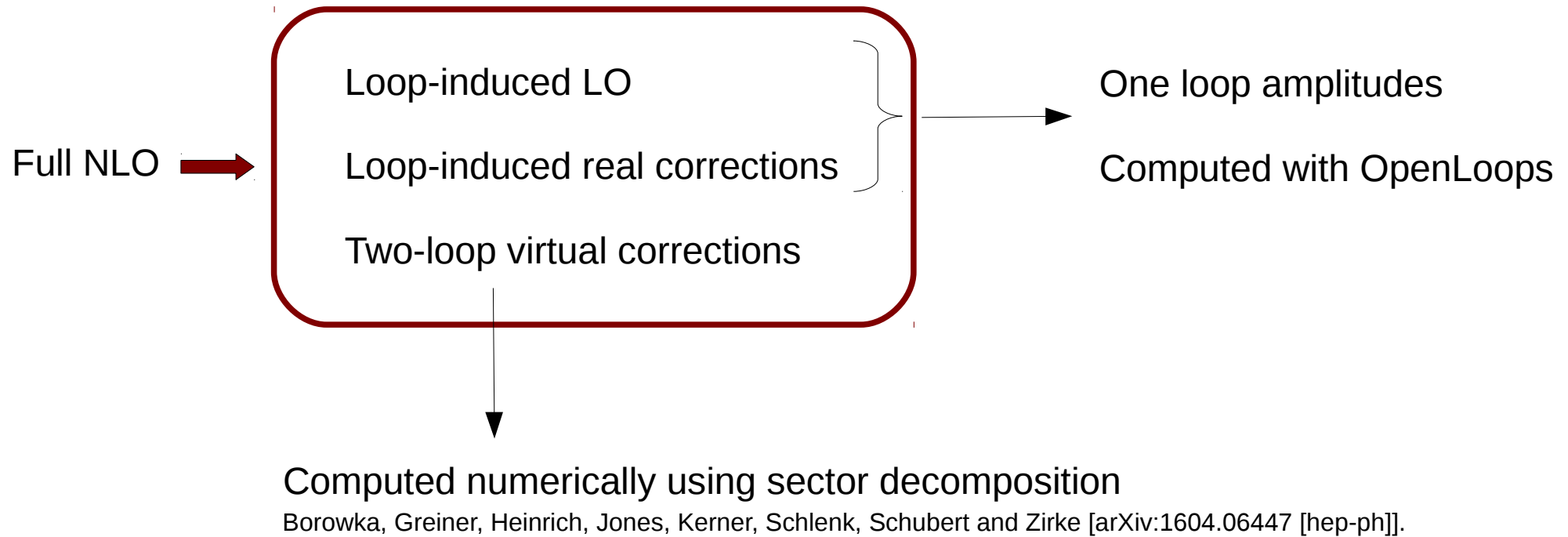
We need to introduce a cutoff ← Individually divergent when $q_T \rightarrow 0$

$$r = q_T / Q$$

- Introduce an r_{cut}
- Check that results are independent of r_{cut}
- Extrapolate $r_{\text{cut}} \rightarrow 0$ result



Combination with full NLO



Two-loop numerical integration is computationally very costly



Two dimensional grid + interpolation framework

Top mass effects at NNLO

Just adding the HEFT NNLO piece is not good enough!



We need to account for finite M_t effects at NNLO

We worked with three different approximations for the pure NNLO piece:

- NLO-improved approximation – $\text{NNLO}_{\text{NLO-i}}$
- Born-projected approximation – $\text{NNLO}_{\text{B-proj}}$
- Full-theory approximation – $\text{NNLO}_{\text{FTapprox}}$

NLO-improved approximation - NNLO_{NLO-i}

Done originally in Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk and Zirke, arXiv:1608.04798 [hep-ph]

Simplest approach: for **each bin** of each histogram we do

$$\text{NNLO}_{\text{NLO-i}} = \text{NLO} \times \left(\frac{\text{NNLO}}{\text{NLO}} \right)_{\text{HEFT}}$$

- Observable level reweighting, technically simple
- Finite M_t effects in the NNLO piece enter via the full NLO
- Has to be repeated for each observable and binning (bin size dependent!)
- We compute the total cross section based on the M_{hh} distribution

Born-projected approximation - NNLO_{B-proj}

Reweight each NNLO event by the ratio of the full and HEFT Born squared amplitudes

Different multiplicities (double real and real-virtual corrections)



Projection to Born kinematics needed

We make use of the qT-recoil procedure:

Catani, de Florian, Ferrera and Grazzini, arXiv:1507.06937 [hep-ph]

- Momenta of the Higgs bosons remain unchanged
- The new initial state partons momenta absorb the qT due to the additional radiation
- Initial state momenta remain massless, and their transverse component goes to zero when qT goes to zero (and then qT-cancellation is not spoiled)

Finite M_t effects entering only via the Born amplitude: no information about real radiation

Full-theory approximation - NNLO_{FTapprox}

- Double real corrections can be computed in the full theory (one-loop amplitudes)
- Idea: construct an approximation in which they are treated in an exact way

We perform a subprocess-wise reweighting: for each n-loop squared amplitude

$$\mathcal{A}_{\text{HEFT}}^{(n)}(ij \rightarrow HH + X)$$

we apply the reweighting

$$\mathcal{R}(ij \rightarrow HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \rightarrow HH + X)}{\mathcal{A}_{\text{HEFT}}^{(0)}(ij \rightarrow HH + X)}$$

- Same partonic subprocess used for reweighting: no need for a projection
- Amplitudes that are tree-level in the HEFT are treated exactly
- At NLO this agrees with the FTapprox in Maltoni, Vryonidou and Zaro, arXiv:1408.6542 [hep-ph]
- Great performance at NLO (4% difference with full NLO) + full M_t dependence in double reals

Our best NNLO prediction

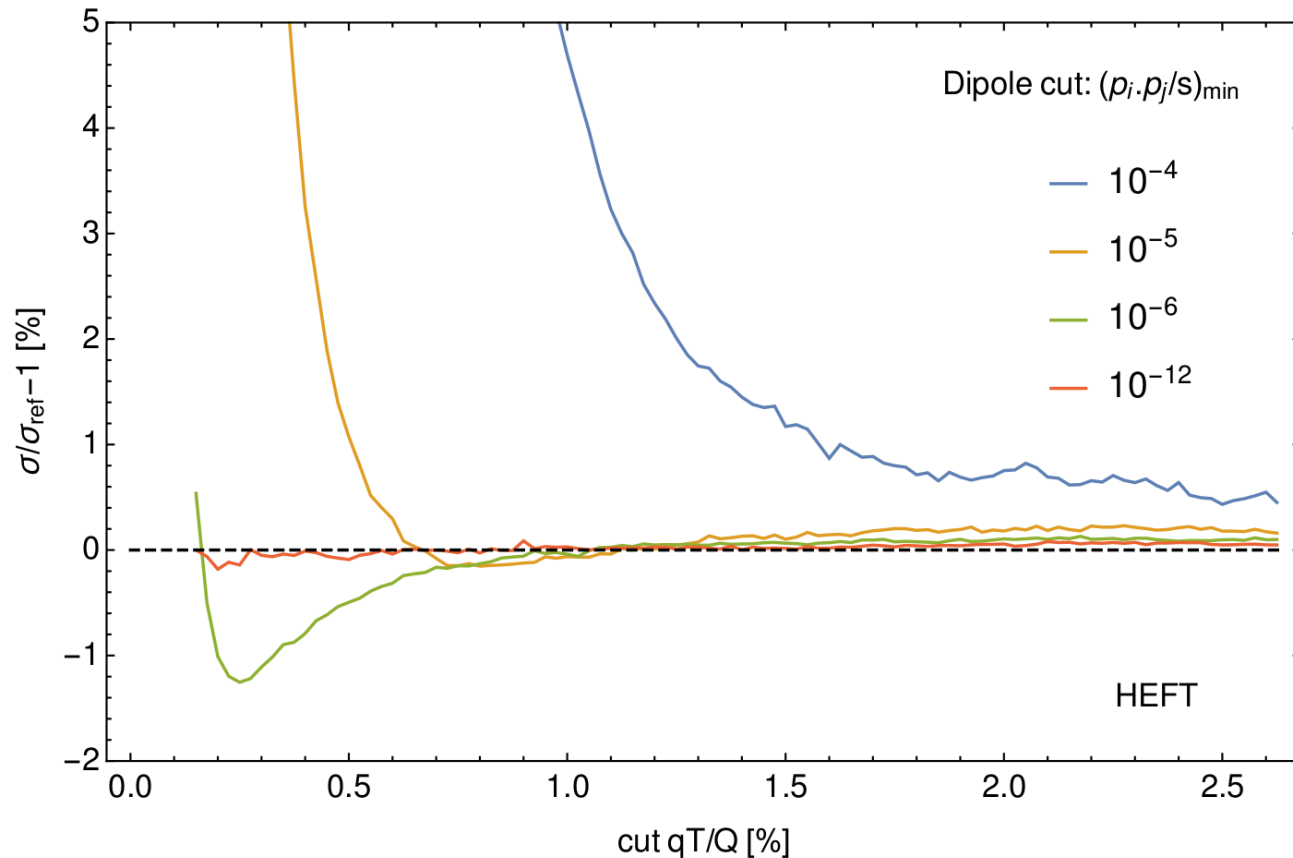


Numerical stability

- Loop-induced double real amplitudes can become unstable close to *dipole singularities*

$$\text{Small } \alpha = \frac{p_i \cdot p_j}{\hat{s}}, \text{ i and j emitters}$$

- Quadruple precision rescue non viable (~10 minutes per PS point for $gg \rightarrow HHgg$)
- Using a too large cut on α spoils the qT-cancellation



Numerical stability

Solution: we introduced a new parameter, $\alpha_{L-i,cut}$, below which we approximate the loop-induced amplitudes by the Born reweighted HEFT

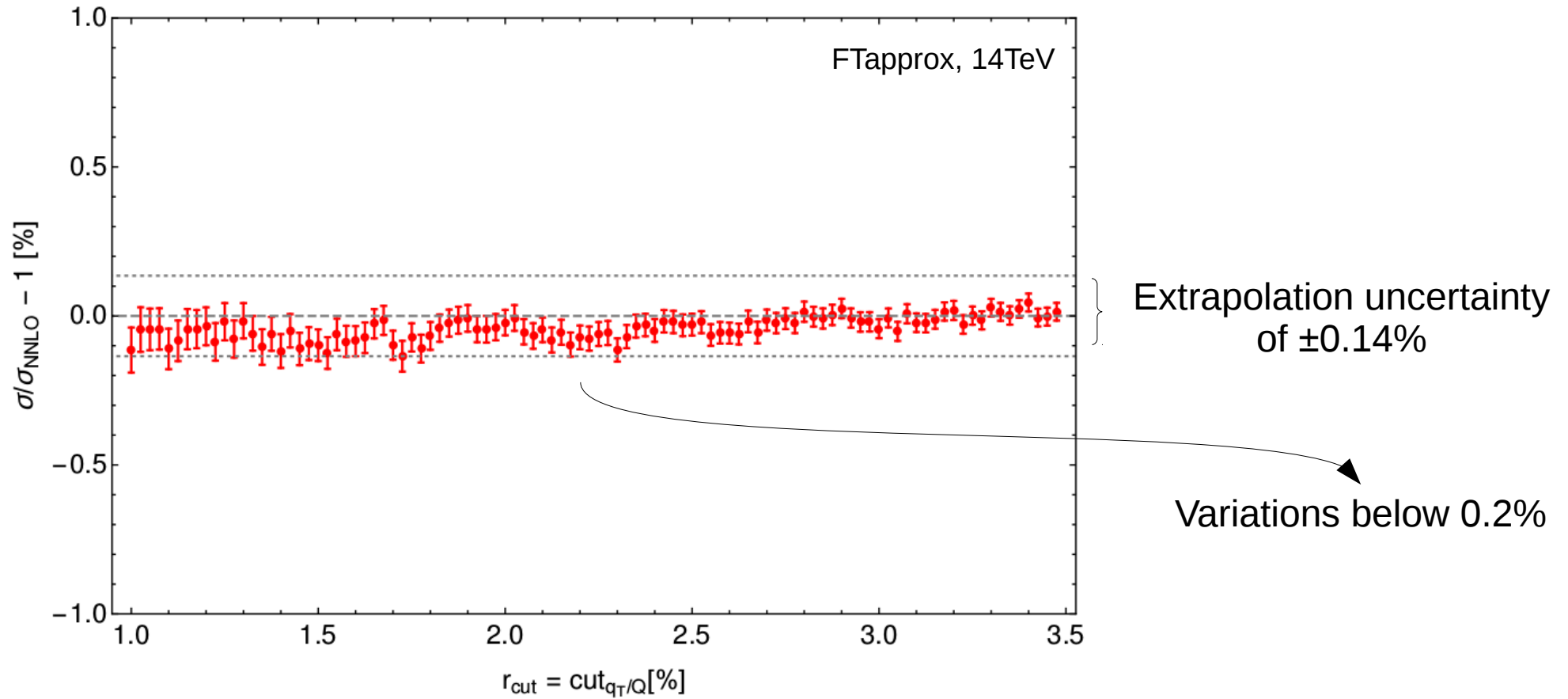
- We avoid evaluating the double real loop induced amplitudes in the unstable regions
- We can use a lower overall dipole cut \rightarrow we don't spoil the qT-cancellation

$$\alpha_{L-i,cut} = \underbrace{10^{-3} \text{ to } 10^{-5}}$$

$$\alpha_{cut} = 10^{-10}$$

Results independent in this range

Numerical stability



- Extrapolation to $r_{\text{cut}} \rightarrow 0$ via linear least χ^2 fit (vs quadratic in default MATRIX)
- Upper bound of the interval varied to get the best fit and uncertainty estimation

Numerical results

Setup of the calculation:

- $M_h = 125\text{GeV}$ $M_t = 173\text{GeV}$
- PDF4LHC15 sets at each corresponding order
- Central scale value $\mu_0 = M_{hh}/2$
- Scale uncertainties: 7-point variation
- Results for 13, 14, 27 and 100TeV
- No bottom quark contributions (effect below 1% at LO)
- No top quark width effects (2% at LO for the total cross section)

Total cross sections

\sqrt{s}	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	27.78 $^{+13.8\%}_{-12.8\%}$	32.88 $^{+13.5\%}_{-12.5\%}$	127.7 $^{+11.5\%}_{-10.4\%}$	1147 $^{+10.7\%}_{-9.9\%}$
NLO _{FTapprox} [fb]	28.91 $^{+15.0\%}_{-13.4\%}$	34.25 $^{+14.7\%}_{-13.2\%}$	134.1 $^{+12.7\%}_{-11.1\%}$	1220 $^{+11.9\%}_{-10.6\%}$
NNLO _{NLO-i} [fb]	32.69 $^{+5.3\%}_{-7.7\%}$	38.66 $^{+5.3\%}_{-7.7\%}$	149.3 $^{+4.8\%}_{-6.7\%}$	1337 $^{+4.1\%}_{-5.4\%}$
NNLO _{B-proj} [fb]	33.42 $^{+1.5\%}_{-4.8\%}$	39.58 $^{+1.4\%}_{-4.7\%}$	154.2 $^{+0.7\%}_{-3.8\%}$	1406 $^{+0.5\%}_{-2.8\%}$
NNLO _{FTapprox} [fb]	31.05 $^{+2.2\%}_{-5.0\%}$	36.69 $^{+2.1\%}_{-4.9\%}$	139.9 $^{+1.3\%}_{-3.9\%}$	1224 $^{+0.9\%}_{-3.2\%}$
M_t unc. NNLO _{FTapprox}	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
NNLO _{FTapprox} /NLO	1.118	1.116	1.096	1.067

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NLO _{FTapprox} [fb]	28.91 $^{+15.0\%}_{-13.4\%}$	34.25 $^{+14.7\%}_{-13.2\%}$	134.1 $^{+12.7\%}_{-11.1\%}$	1220 $^{+11.9\%}_{-10.6\%}$
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NNLO _{B-proj} [fb]	33.42 $^{+1.5\%}_{-4.8\%}$	39.58 $^{+1.4\%}_{-4.7\%}$	154.2 $^{+0.7\%}_{-3.8\%}$	1406 $^{+0.5\%}_{-2.8\%}$
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M_t unc. NNLO _{FTapprox}	$\pm 2.6\%$	$\pm 1.7\%$		
NNLO _{FTapprox} /NLO	1.118	1.116		

B-proj > NLO-i > FTapprox

Increase with respect to NLO at 14TeV:

- B-proj: 20%
- NLO-i: 18%
- FTapprox: 12%

Total cross sections

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M_t unc. NNLO _{FTapprox}	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
NNLO _{FTapprox} /NLO	1.118	1.116	1.096	1.067



- Size of perturbative corrections decreases with the energy for the FTapprox
- This doesn't happen for the other two approximations
- Not fully surprising: similar behavior for NLO K-factor

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- Strong reduction of the scale uncertainties at NNLO
- About a factor of 3 for the FTapprox at 14TeV

Even stronger reduction at 100TeV

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- At NLO the FTapprox overestimates full NLO by 4% \longrightarrow 11% for the pure NLO contribution
- Assuming a $\pm 11\%$ uncertainty for the pure NNLO piece \longrightarrow $\pm 1.2\%$ uncertainty at NNLO
- Multiply by a factor of 2 to be more conservative (14TeV)

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We can repeat the procedure for the Born-projected approximation

\longrightarrow Compatible results even without the factor of 2

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- But the difference between FTapprox and NLO-i increases with the collider energy faster than this uncertainty estimate
- To be more conservative, take half the difference between FTapprox and NLO-i

Top quark mass uncertainties

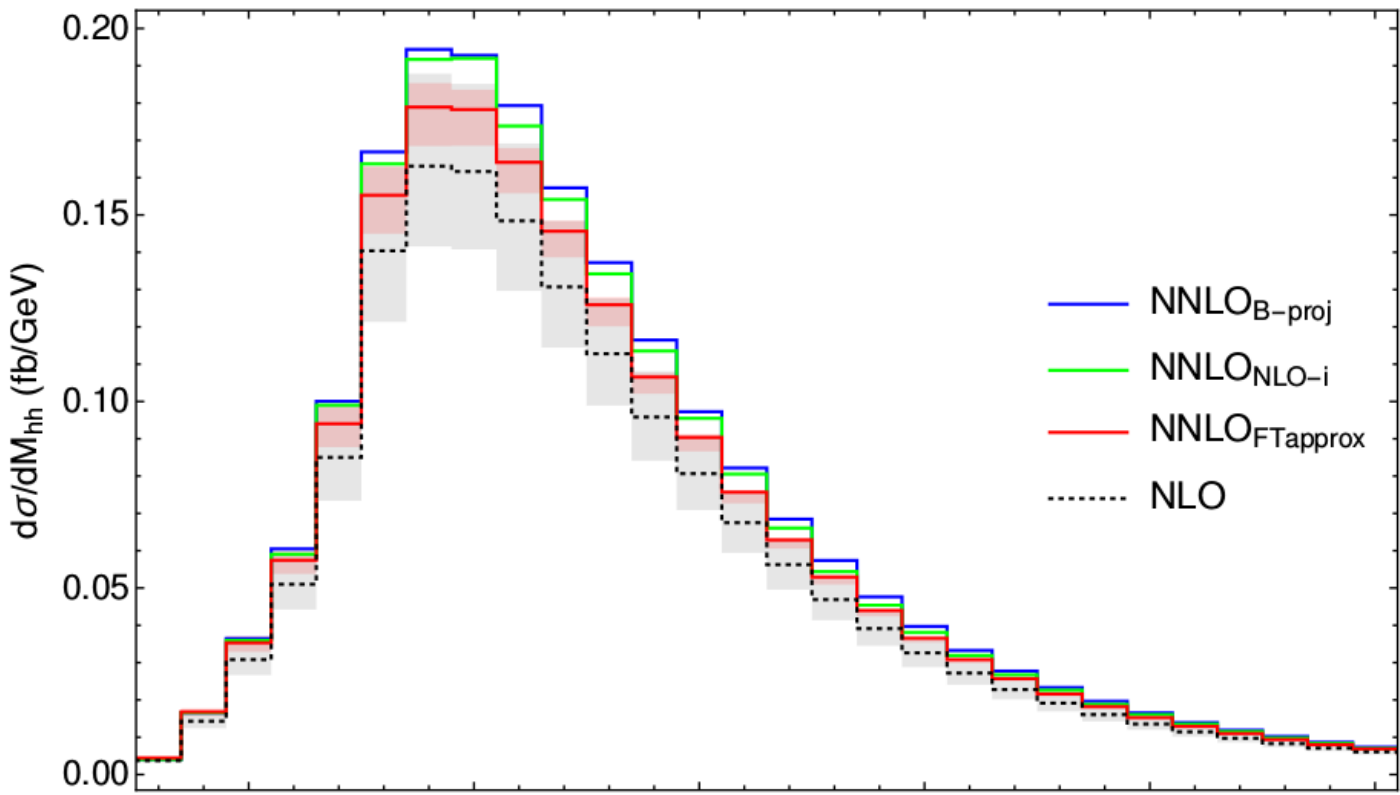
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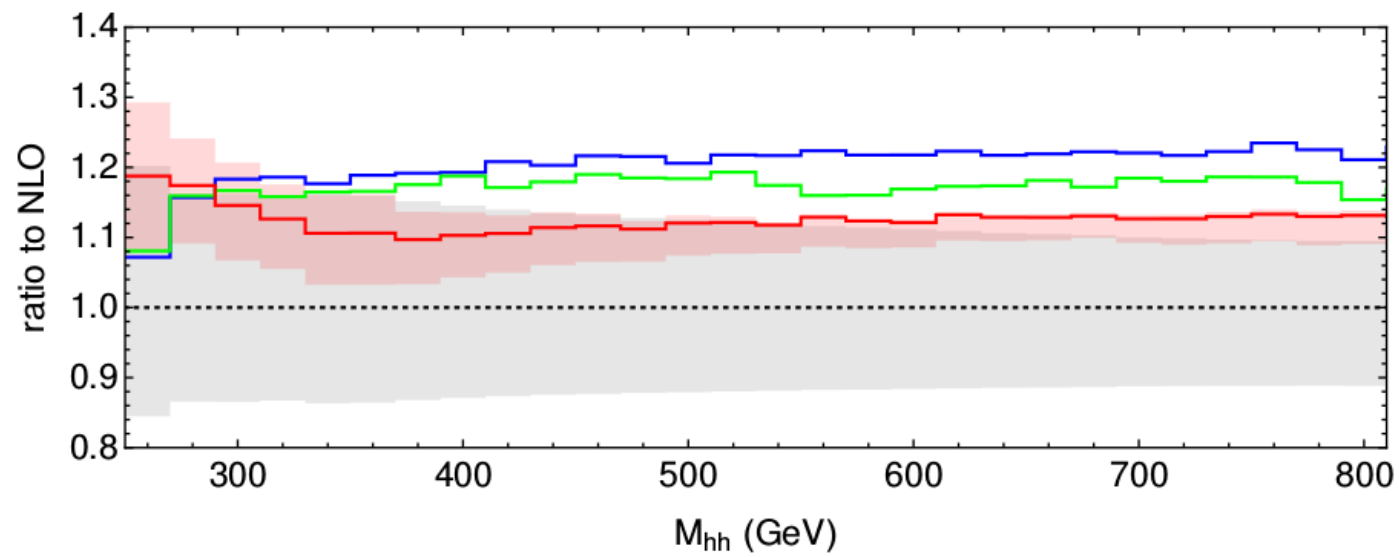
Small difference for LHC, more conservative for larger energies

Differential distributions - M_{hh}

$\sqrt{s} = 14 \text{ TeV}$

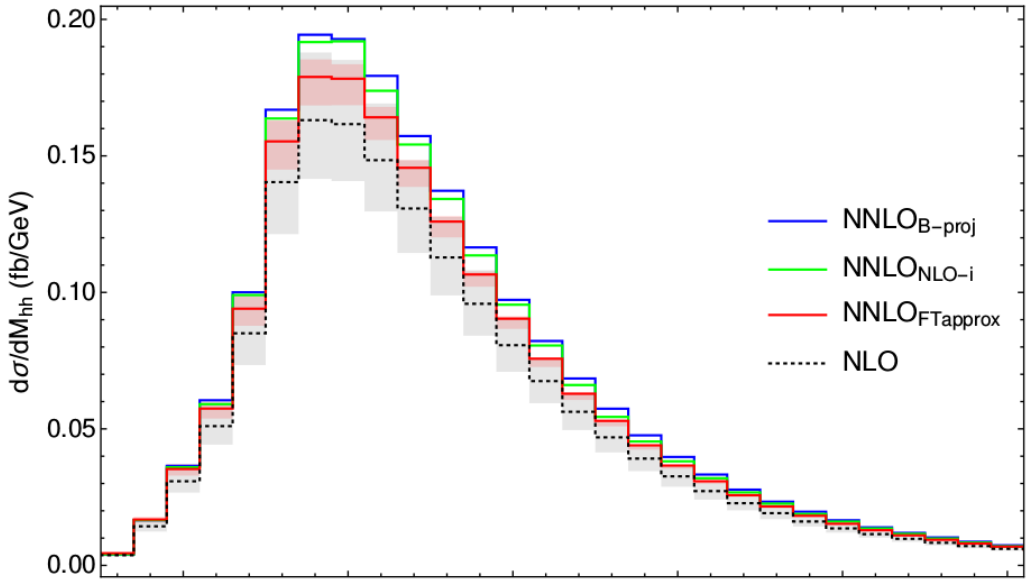


- B-proj and NLO-i have similar behaviors
- FTapprox presents larger corrections at threshold, minimum corrections at $M_{hh} \sim 400 \text{ GeV}$, slow increase towards the tail
- Scale uncertainties are substantially reduced in the whole range
- Overlap with the NLO band

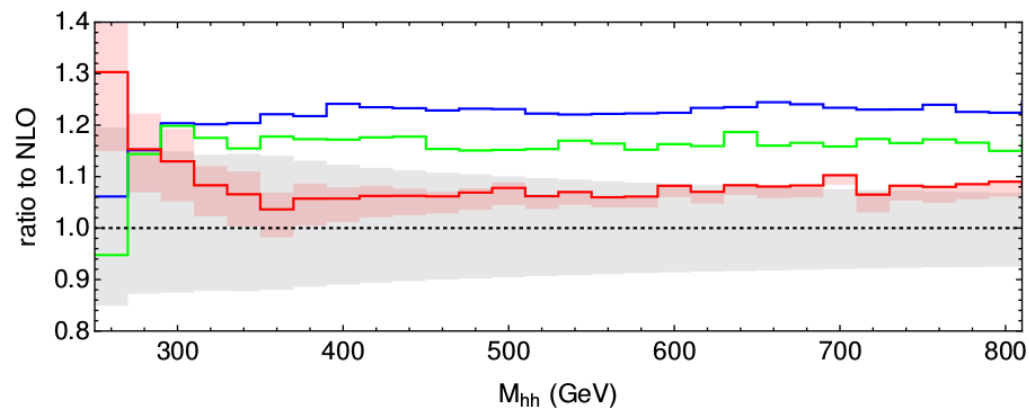
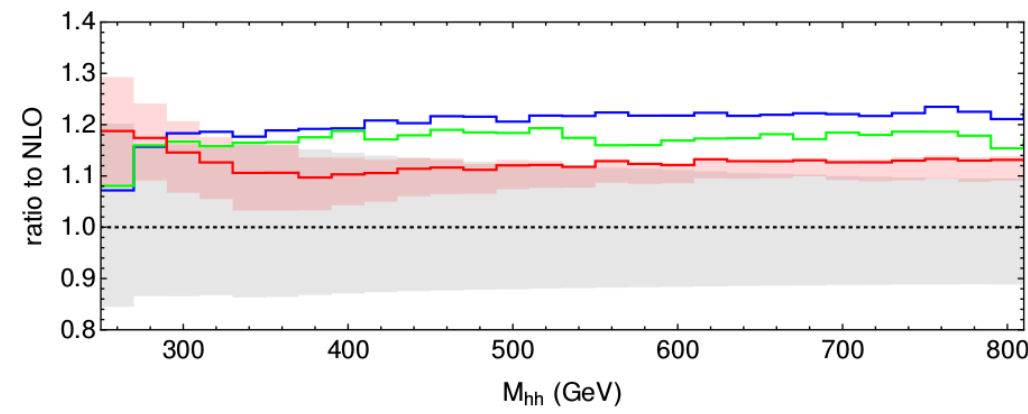
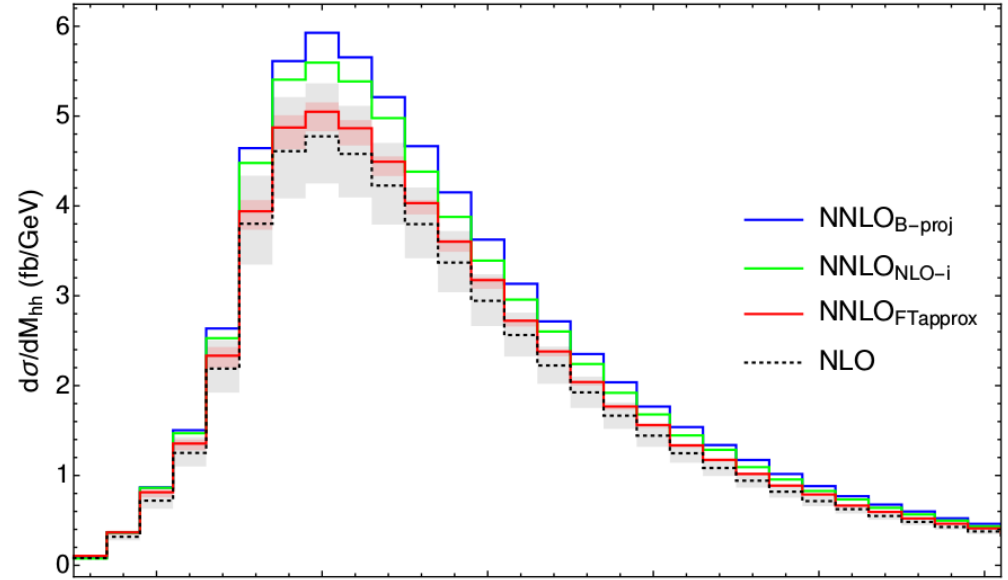


Differential distributions - M_{hh}

$\sqrt{s} = 14 \text{ TeV}$



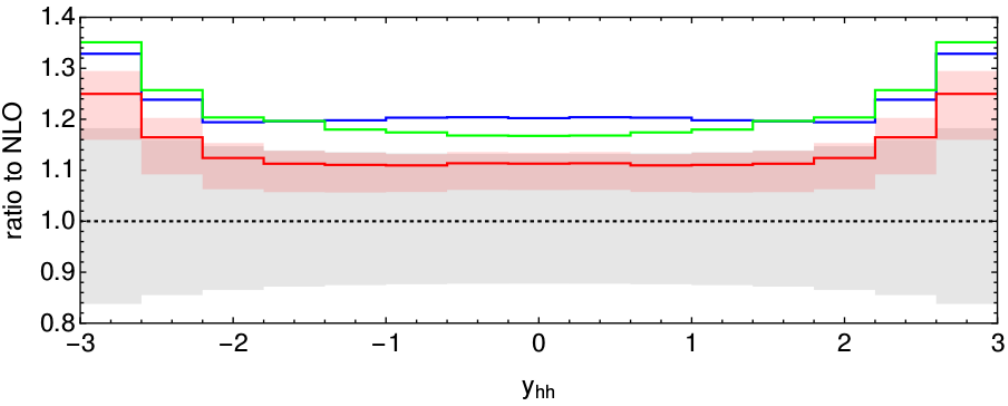
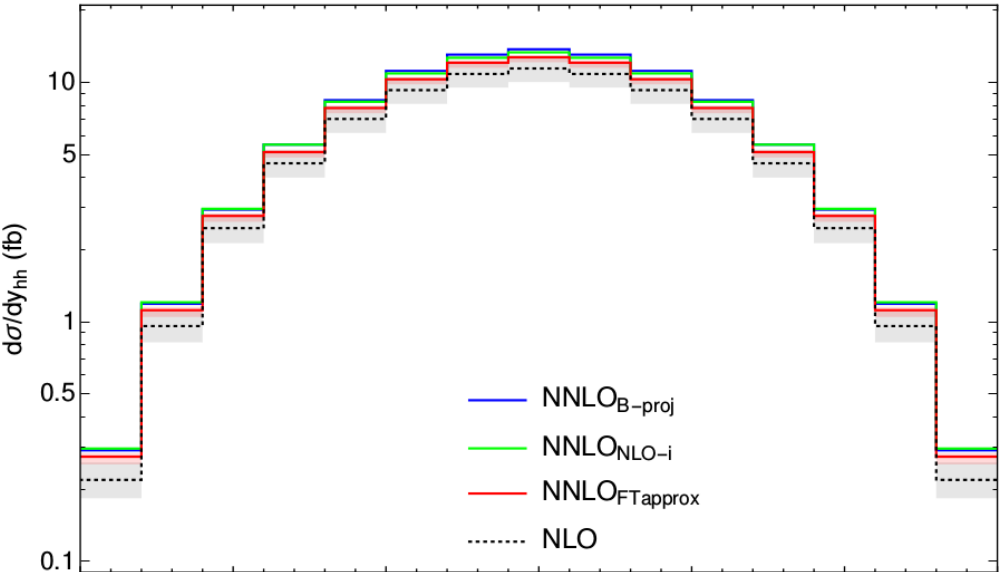
$\sqrt{s} = 100 \text{ TeV}$



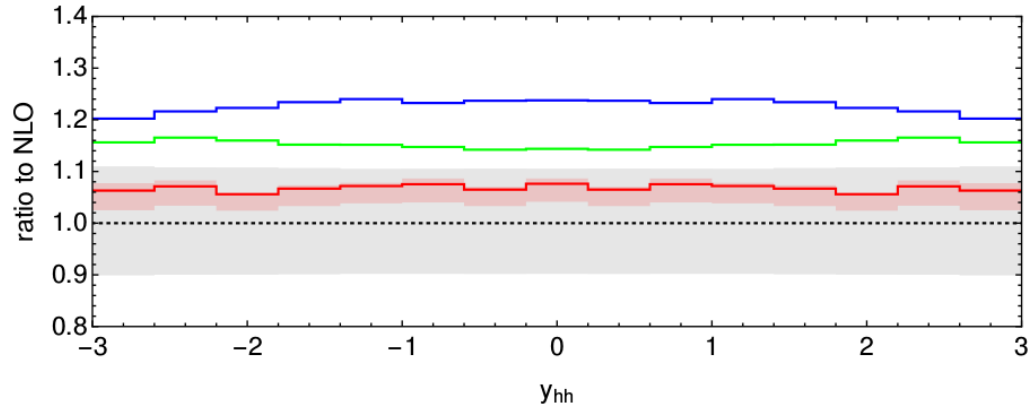
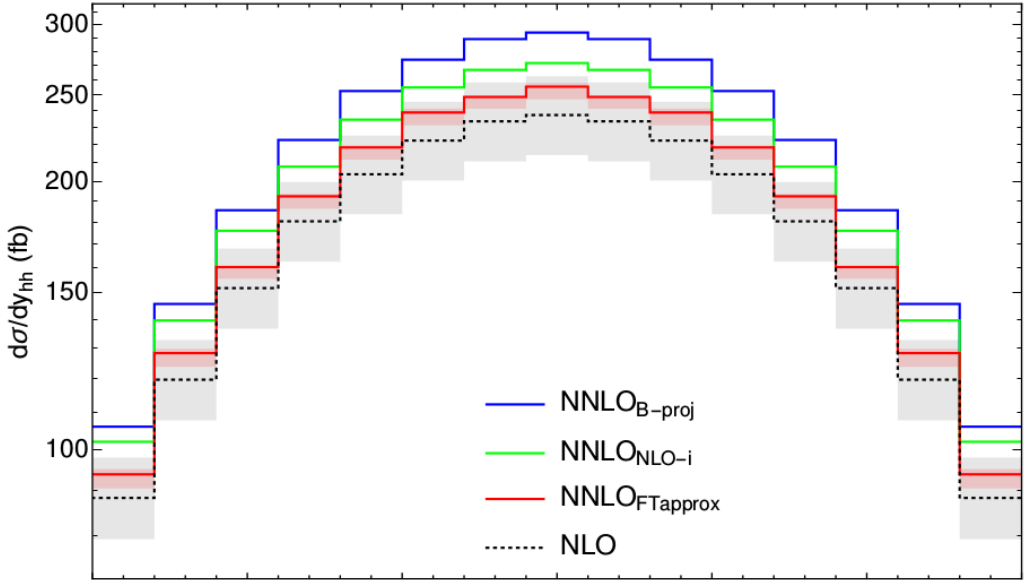
- Previous features enhanced at 100TeV
- Slower decrease in the tail of the distribution
- Larger separation between the different NNLO predictions, smaller corrections for the FTapprox
- FTapprox different behavior at threshold even stronger: due to contributions from events with hard radiation

Differential distributions - y_{hh}

$\sqrt{s} = 14 \text{ TeV}$

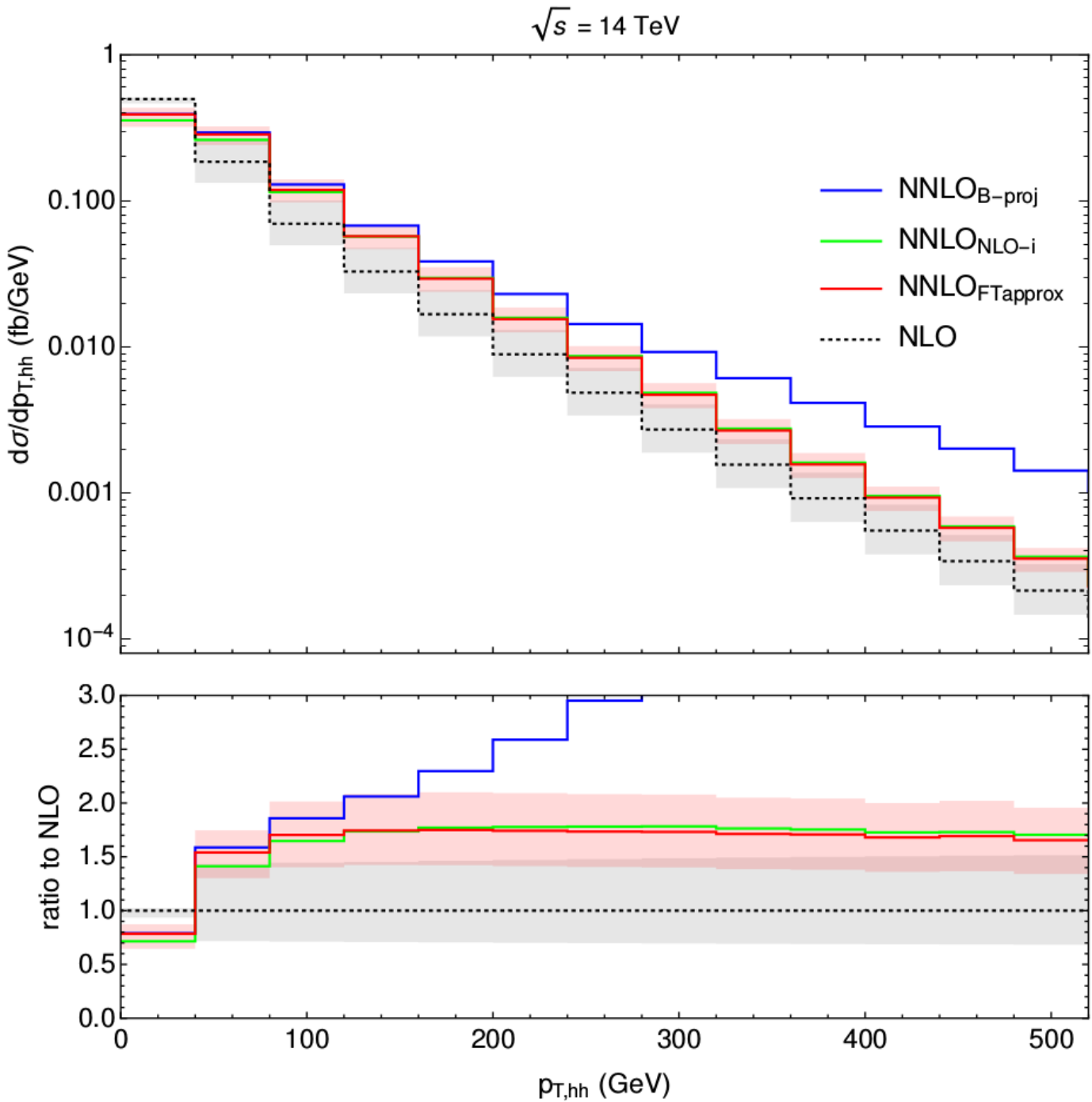


$\sqrt{s} = 100 \text{ TeV}$



- Not very different behaviors between the different approximations (besides normalization)
- Largest shape difference in the central region for NLO-i

Differential distributions - $p_{T,hh}$

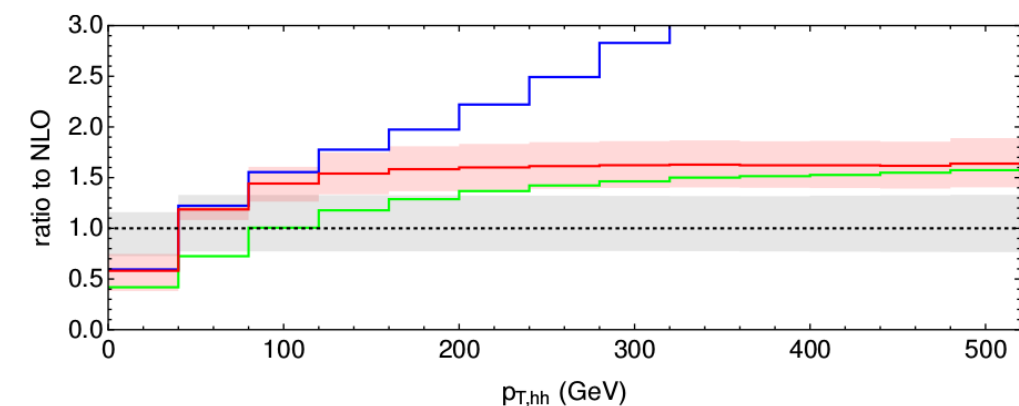
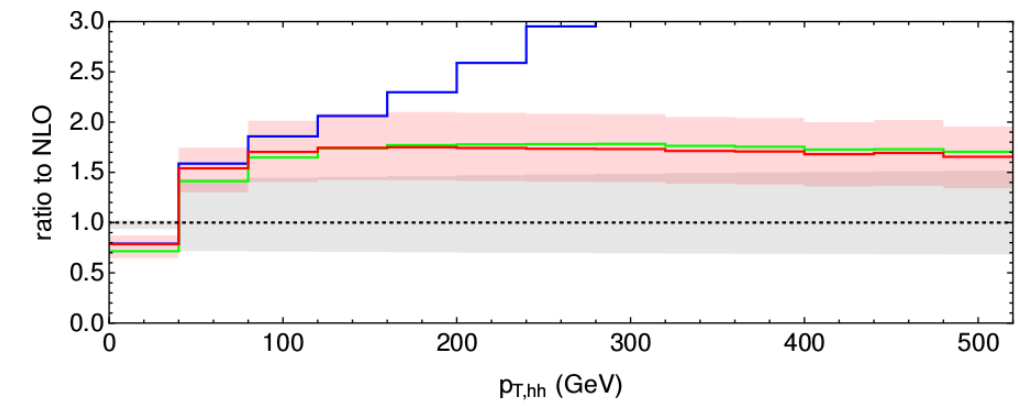
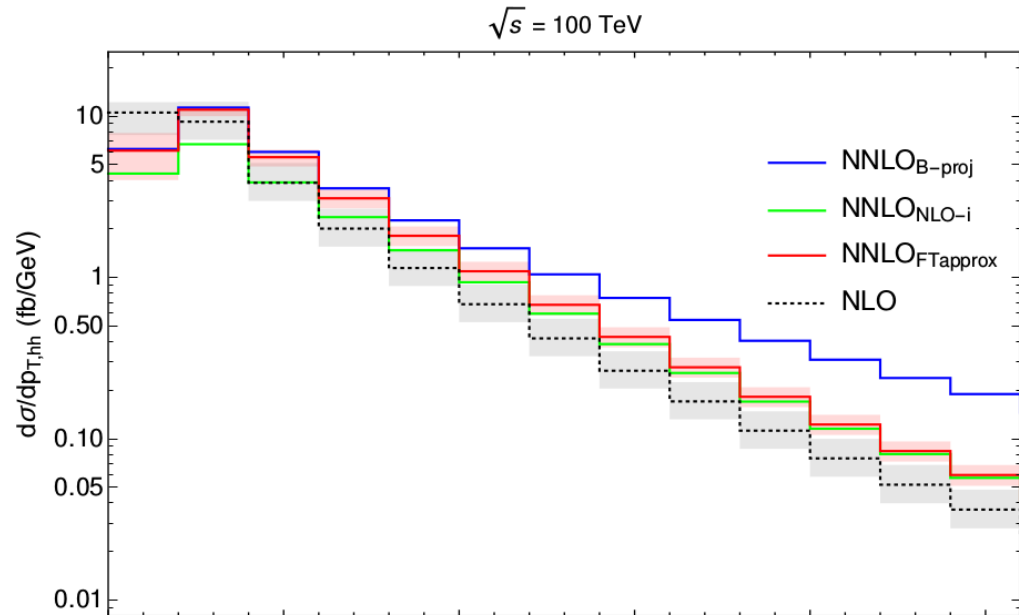
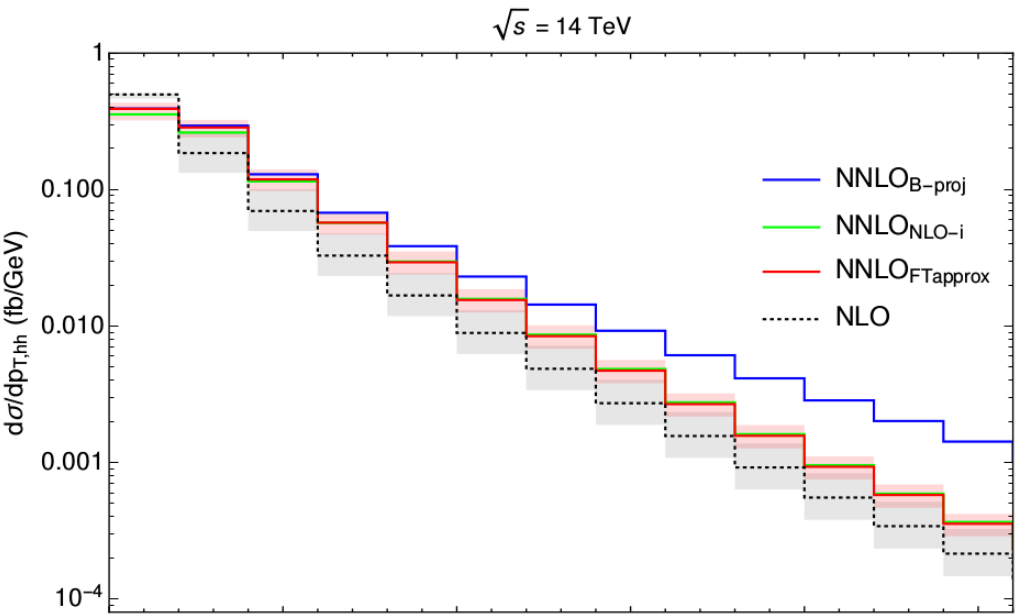


- B-proj corrections huge in the tail
↓
No surprise: no information about the lowest-order prediction for this observable
- Other two predictions in very good agreement
- Distribution trivial at LO: NNLO is effectively NLO

↓

Very large corrections
Sizeable scale uncertainties

Differential distributions - $p_{T,hh}$

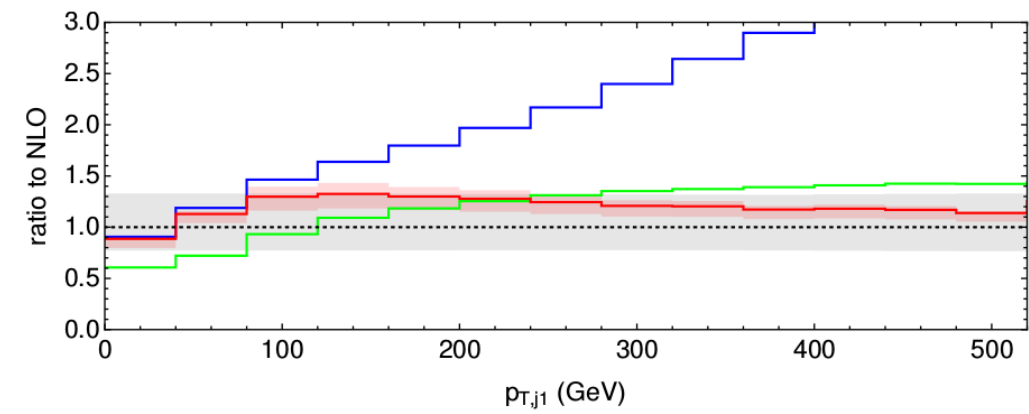
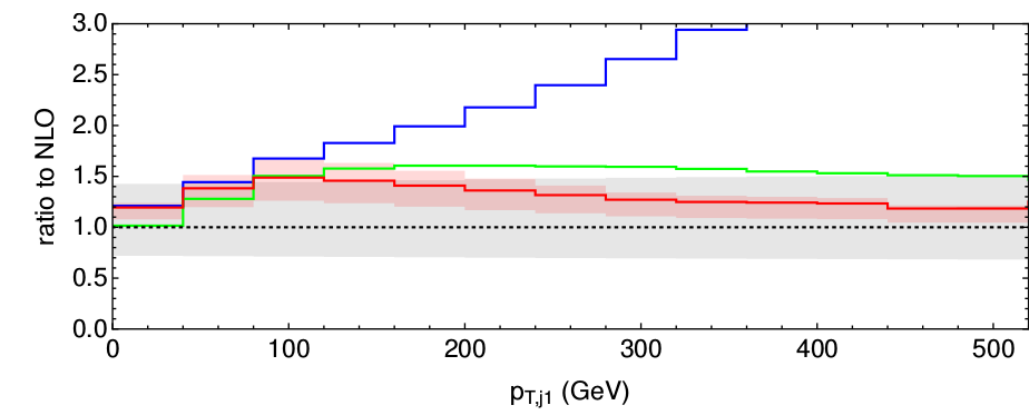
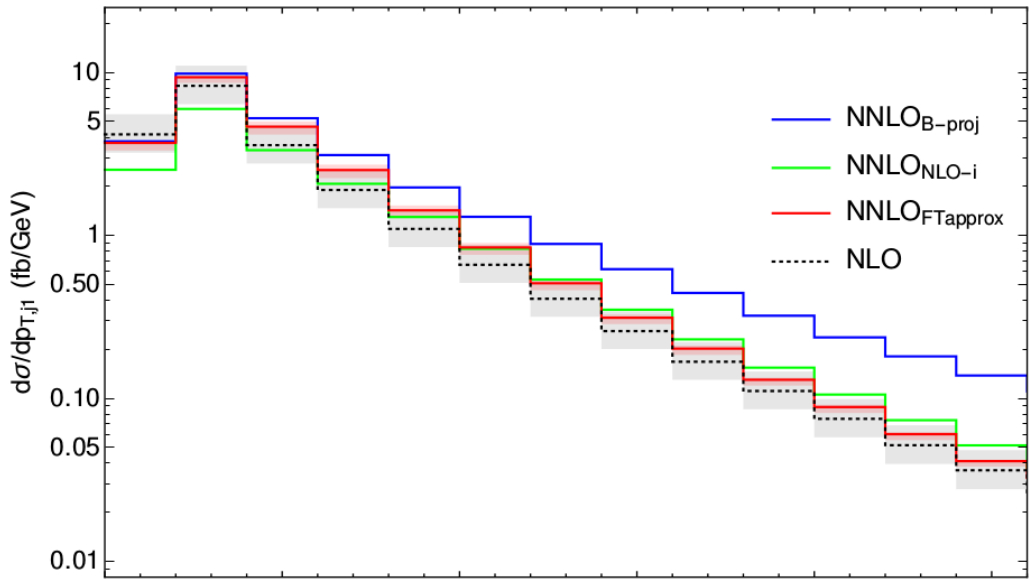
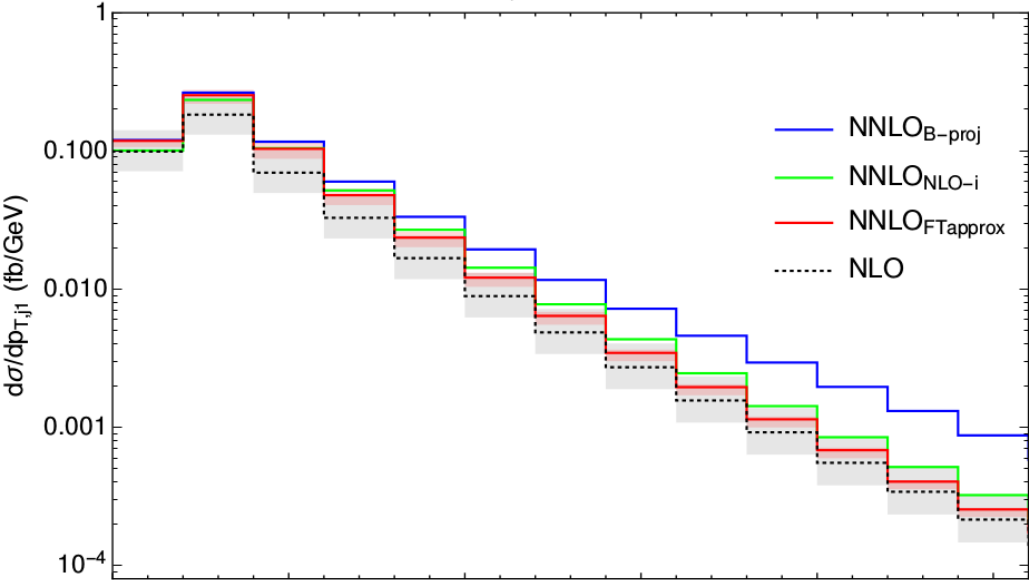


- Different behaviors are more pronounced at 100TeV
- Larger separation between FTapprox and NLO-i (almost full agreement in the tail)
- FTapprox agrees with B-proj for low $p_{T,hh}$

Differential distributions - $p_{T,j1}$

$\sqrt{s} = 14 \text{ TeV}$

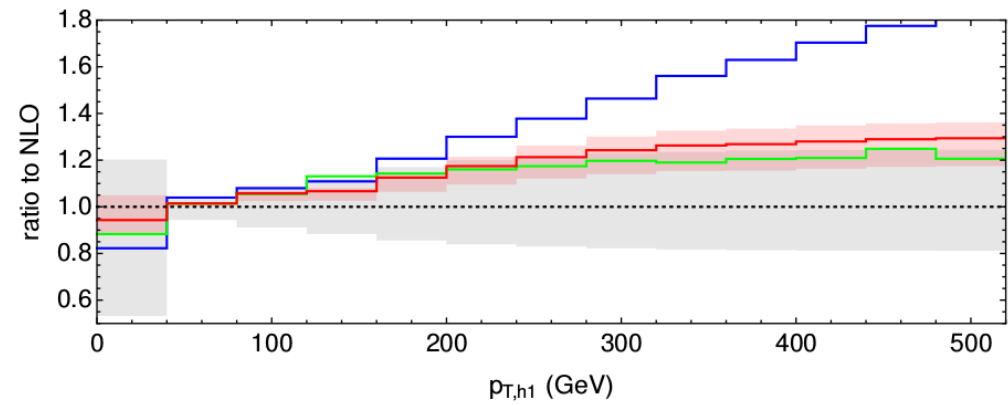
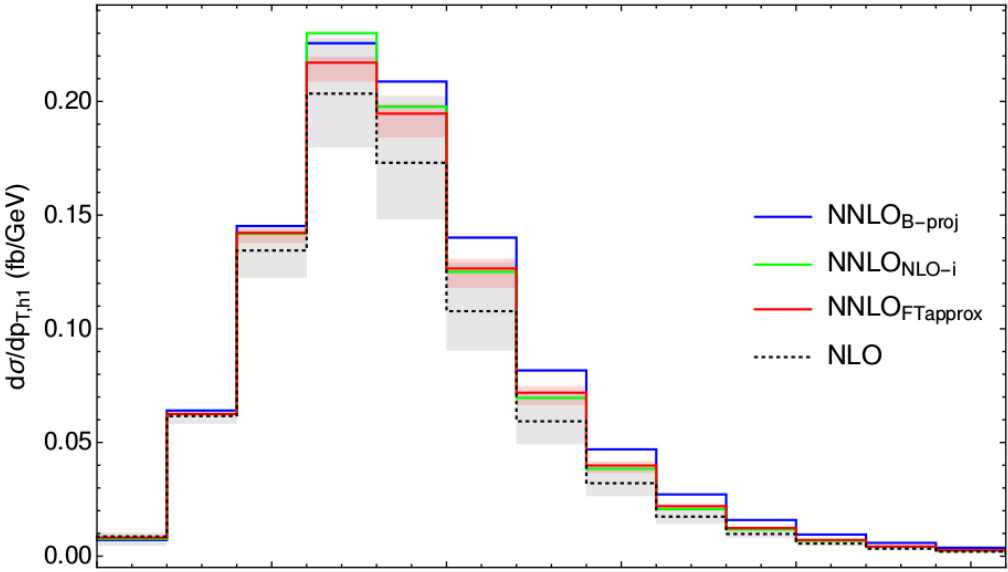
$\sqrt{s} = 100 \text{ TeV}$



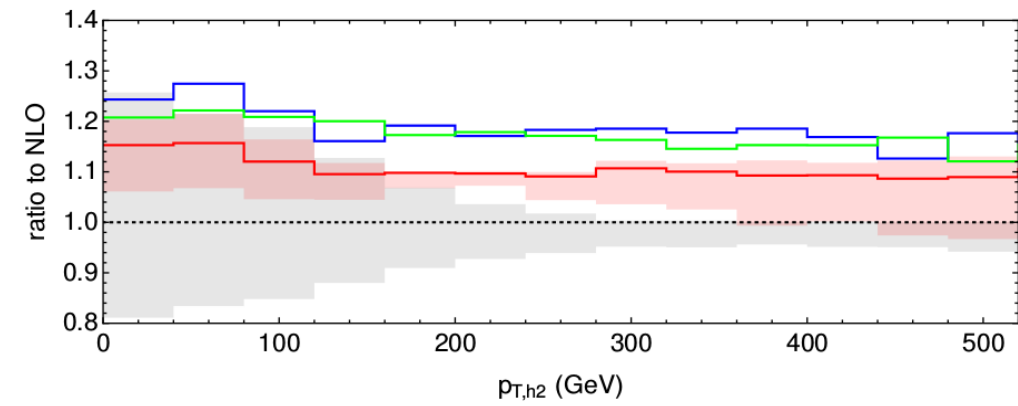
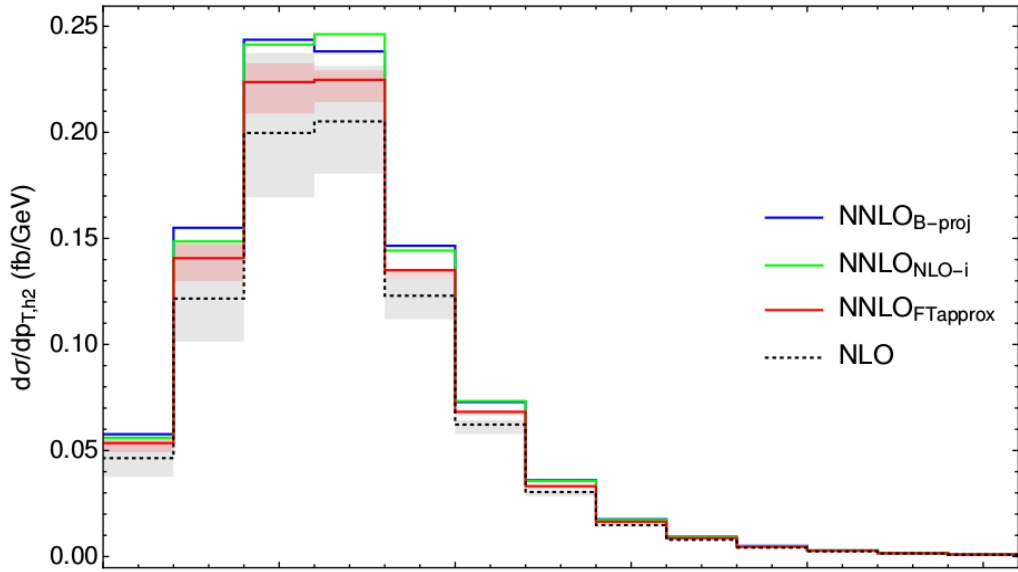
- Huge unphysical corrections in the tail for the B-proj approximation
- More pronounced differences between FTapprox and NLO-i compared to $p_{T,hh}$
- FTapprox predicts a softer spectrum, corrections contained in the NLO uncertainty band

Differential distributions - $p_{T,h1}$ and $p_{T,h2}$

$\sqrt{s} = 14 \text{ TeV}$



$\sqrt{s} = 14 \text{ TeV}$

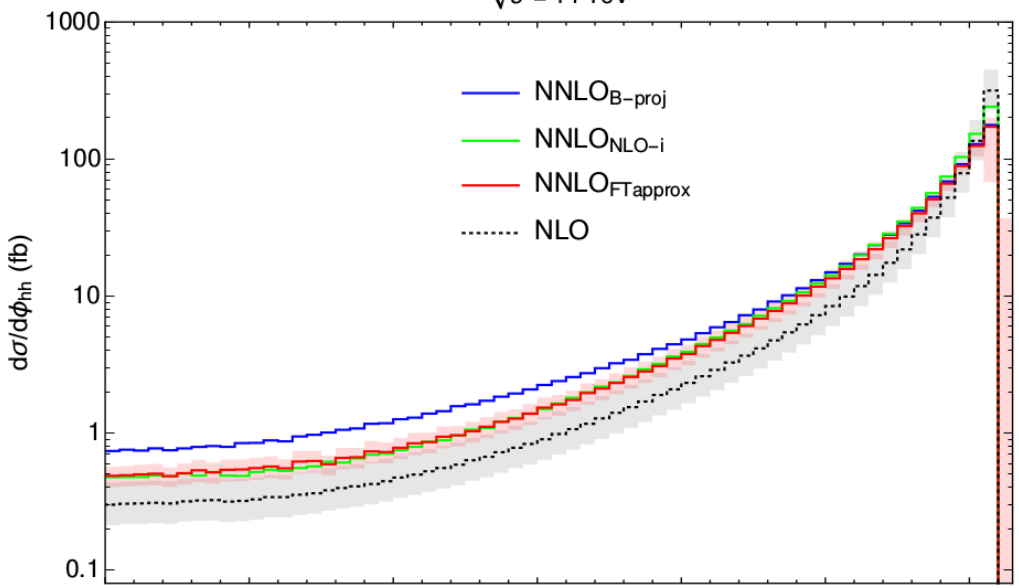


- Hardest Higgs p_T spectrum:
 Large corrections in the tail of the B-proj approximation
 Good agreement between FTapprox and NLO-i

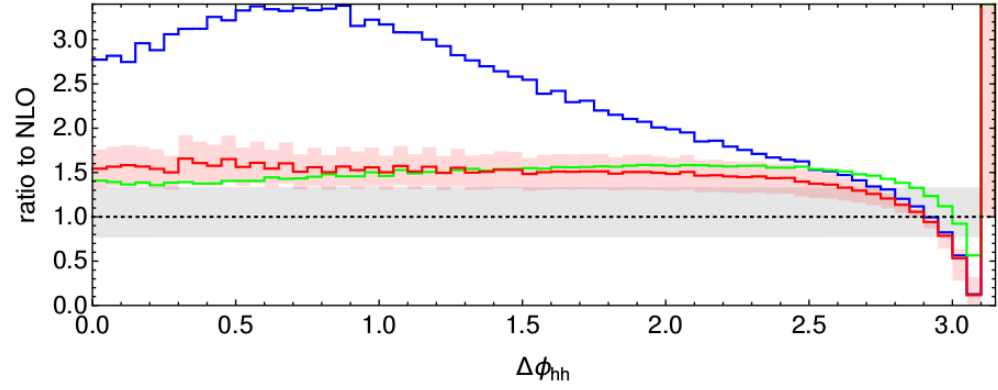
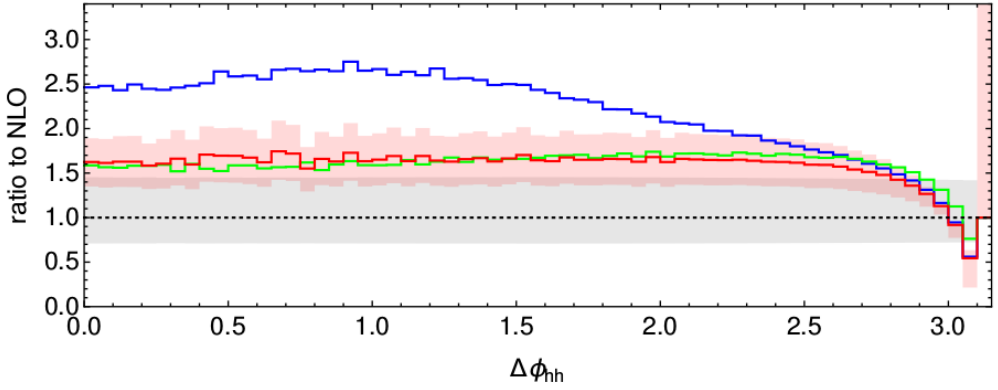
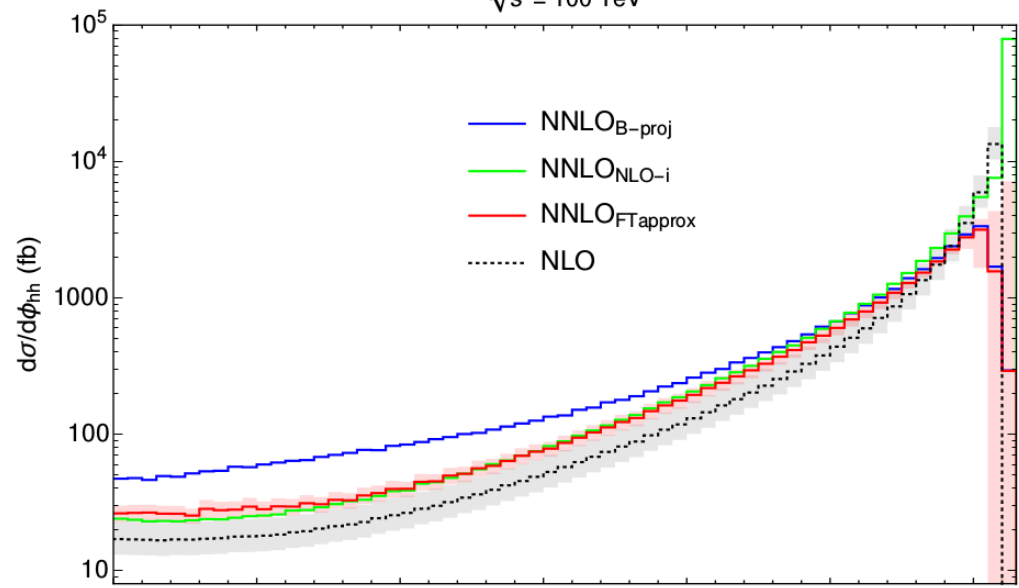
- Softer Higgs p_T spectrum:
 Similar shape for all approximations
 Larger NNLO scale uncertainties in the tail

Differential distributions - $\Delta\phi_{hh}$

$\sqrt{s} = 14 \text{ TeV}$



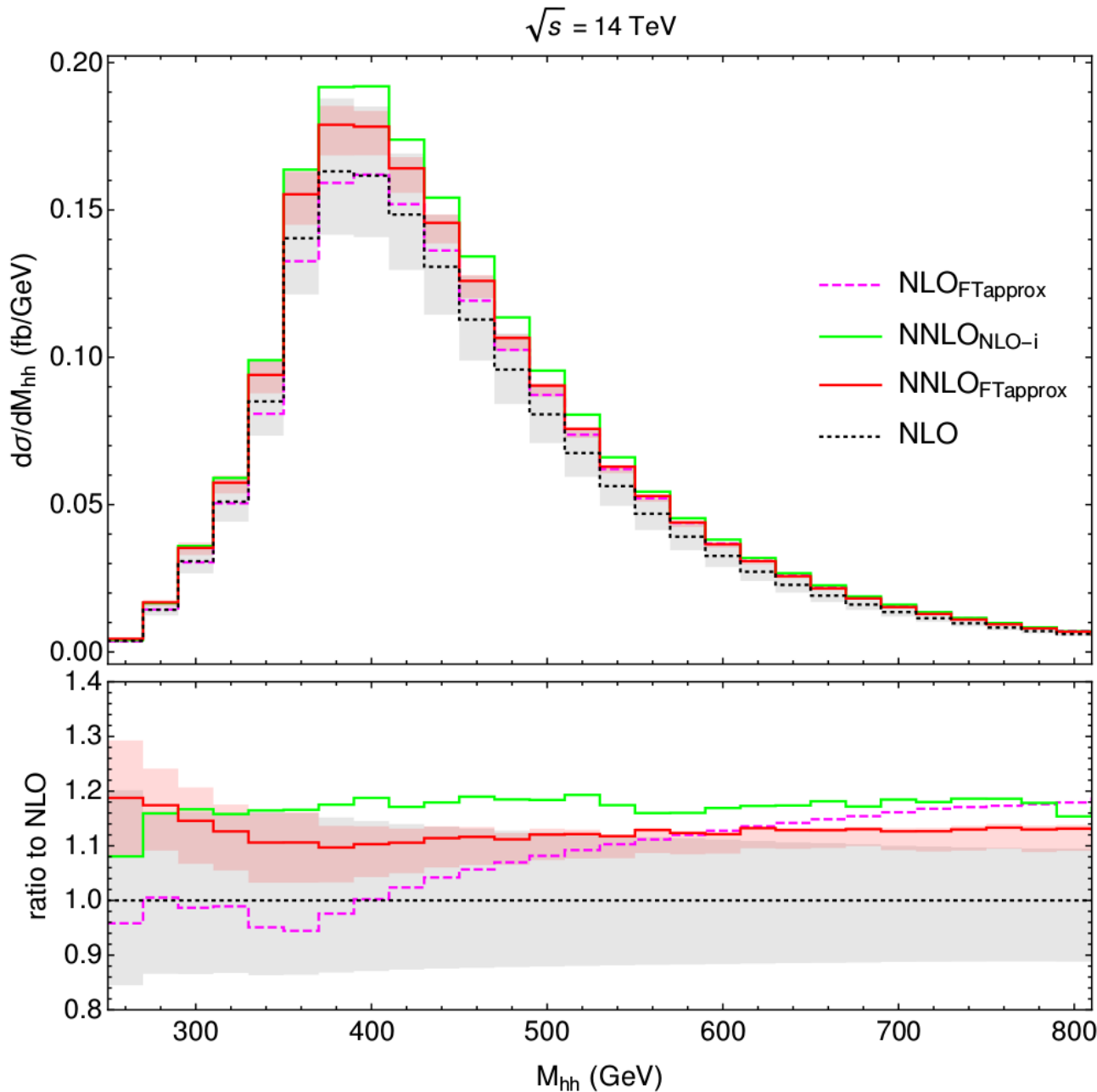
$\sqrt{s} = 100 \text{ TeV}$



- Trivial at LO: back-to-back. NNLO effectively NLO
- Large corrections above 50%, sizable scale uncertainties
- B-proj approximations predicts larger corrections in the region dominated by hard radiation
- Good general agreement between FTapprox and NLO-i, larger differences close to π

M_t uncertainties for distributions

Based on the performance of the FTapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the M_t uncertainties for distributions



For the Higgs pair invariant mass we can look at the previous order:

- Below $M_{hh} \sim 500 \text{ GeV}$ good accuracy at NLO, similar to inclusive cross section

↓

$\sim \pm 3\%$ uncertainty at NNLO

- Quality decreases in the tail

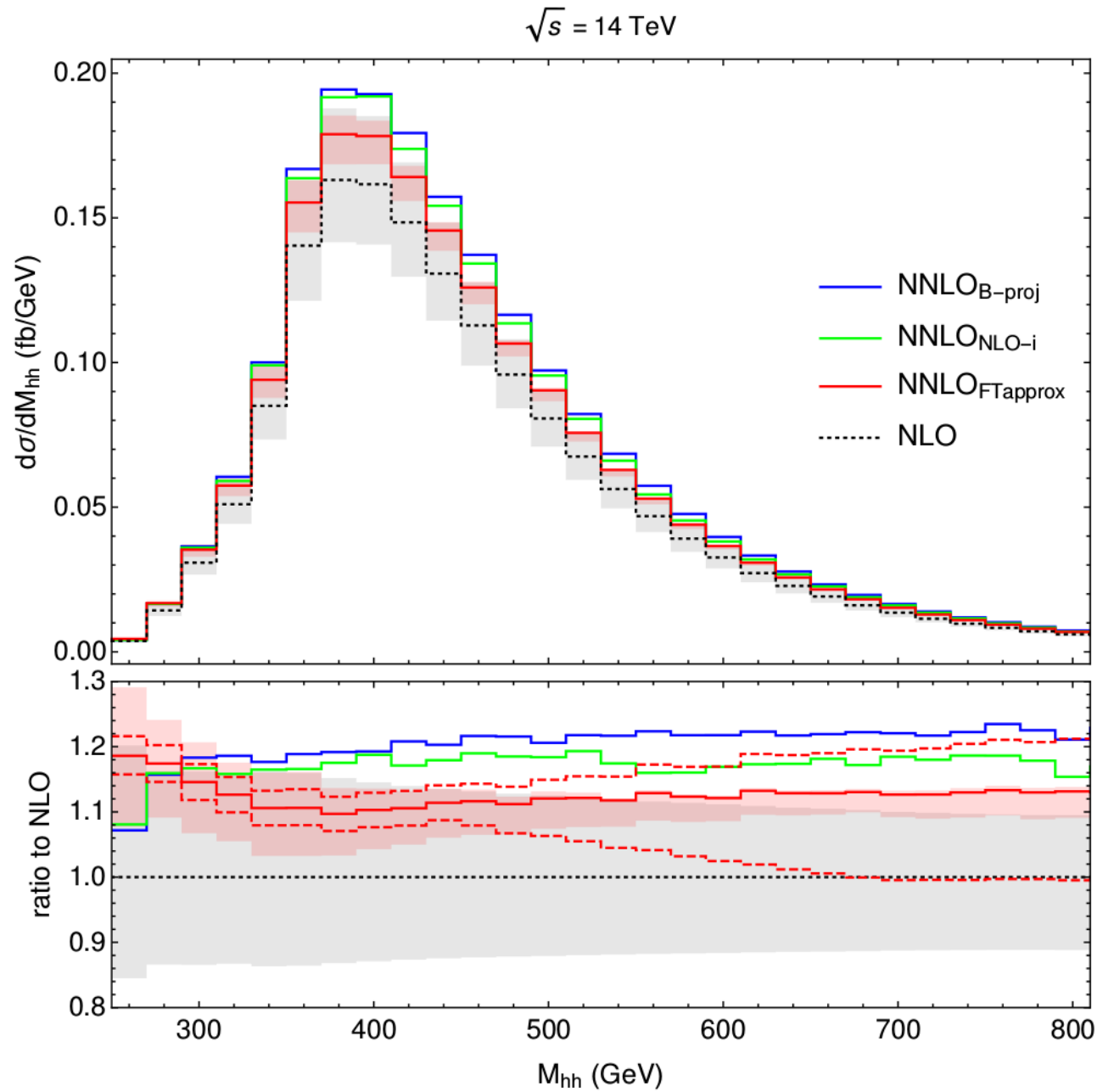
↓

$O(\pm 10\%)$ uncertainty at NNLO

Something similar can be done for y_{hh} , $p_{T,h1}$ and $p_{T,h2}$

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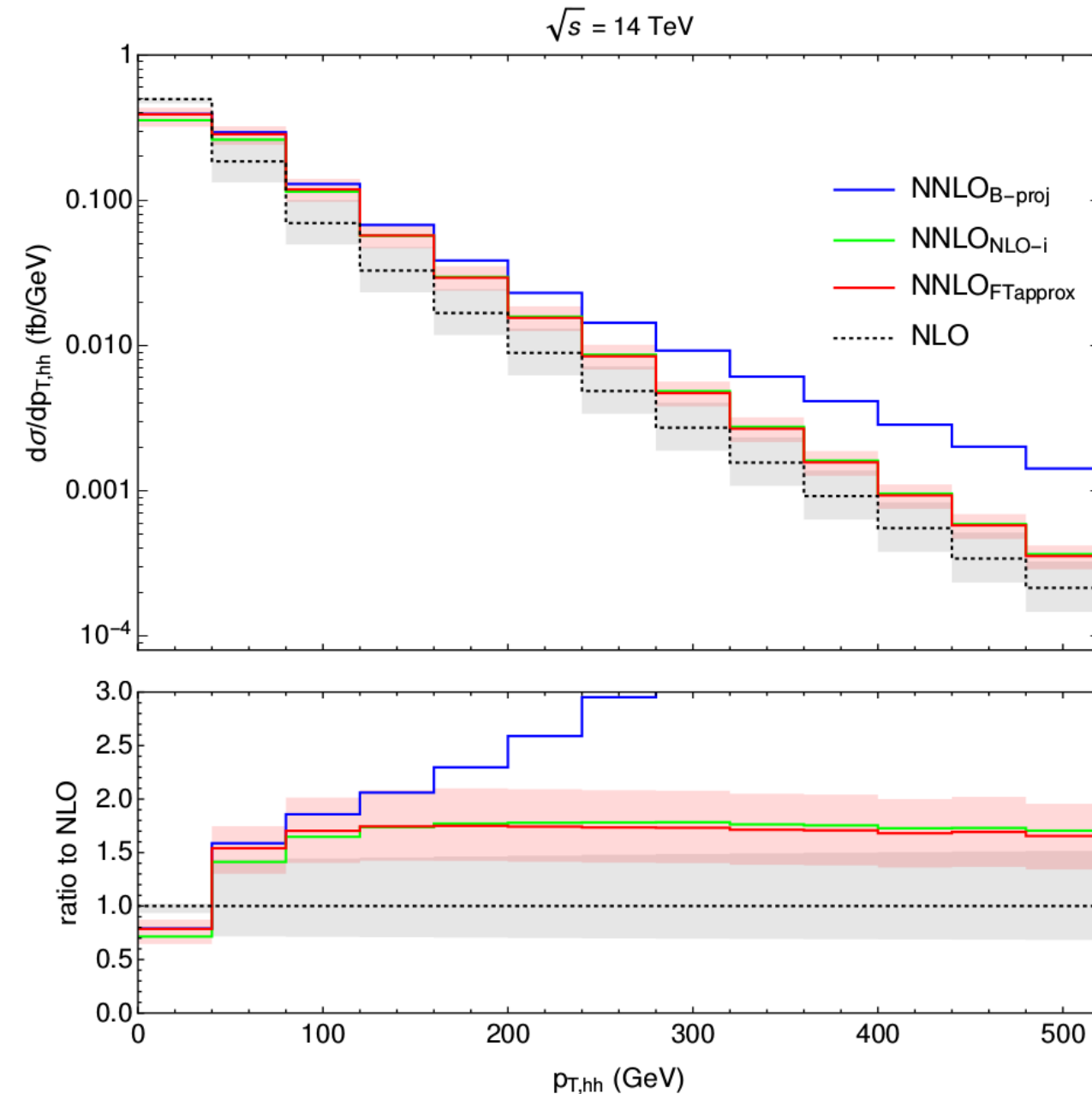
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M_t uncertainties for distributions

Based on the performance of the FTAapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the M_t uncertainties for distributions



- Distributions not defined/trivial at LO are exactly reproduced by FTAapprox at NLO \rightarrow more difficult to estimate uncertainties!

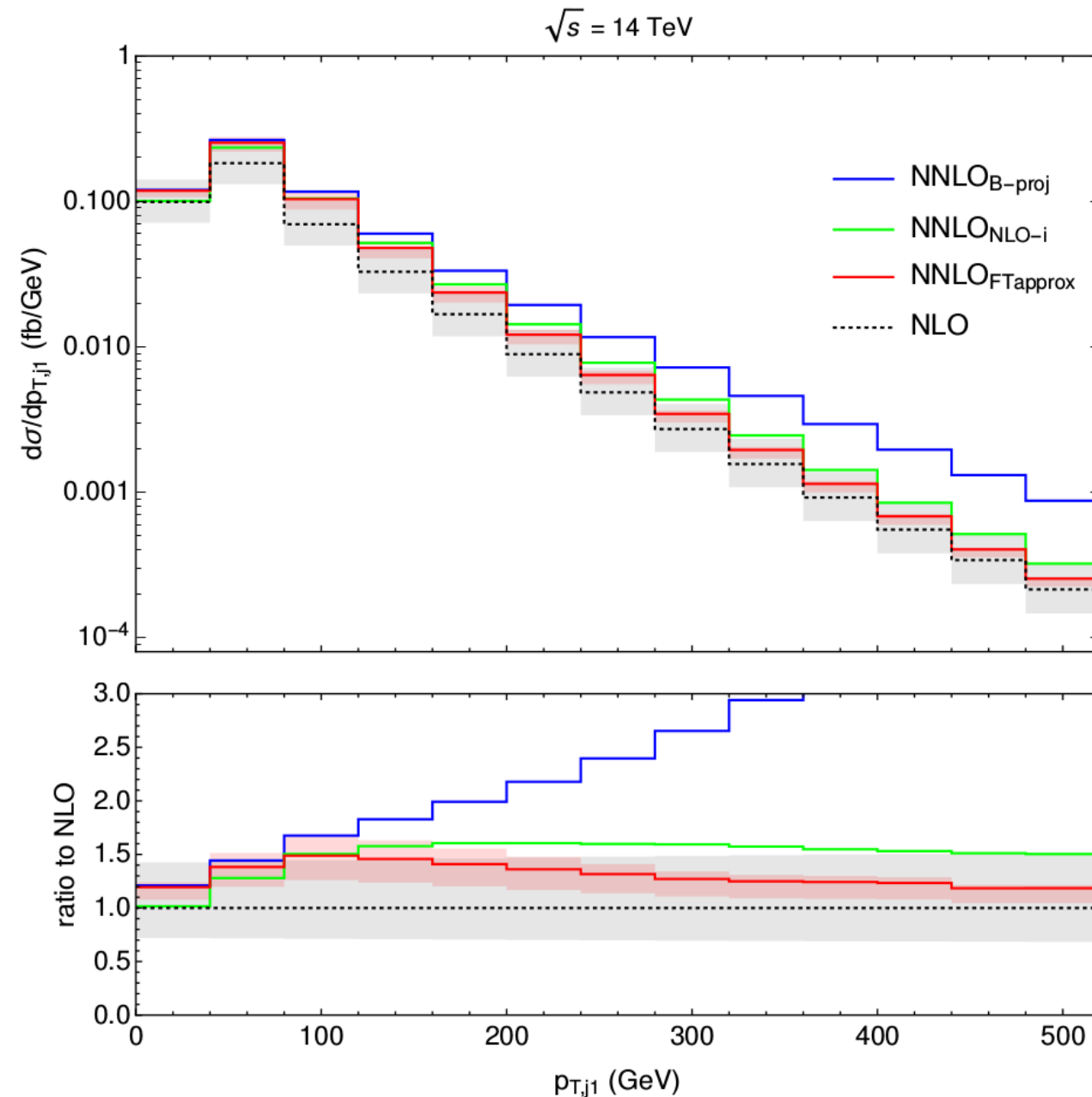


We can use the difference between FTAapprox and NLO-i as an estimate

- Relatively low uncertainties for hh transverse momentum and azimuthal separation, larger for leading jet p_T

M_t uncertainties for distributions

Based on the performance of the FTAprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the M_t uncertainties for distributions



- Distributions not defined/trivial at LO are exactly reproduced by FTAprox at NLO → more difficult to estimate uncertainties!



We can use the difference between FTAprox and NLO-i as an estimate

- Relatively low uncertainties for hh transverse momentum and azimuthal separation, larger for leading jet p_T

Conclusions

- **HH production** is the main way of measuring Higgs **self-coupling** → scalar potential
- Current limit: $\sim O(10)$ x SM cross section
- Should be **observed** in the **HL-LHC**
- Precision measurement of λ → future collider

- Lot of recent progress in the **theoretical predictions**:

NLO: full M_t dependence

Beyond: Large- M_t limit

- **NNLO** in the large- M_t limit
- Threshold resummation at **NNLL**

Further reduction of scale uncertainties

Suggests to use $\mu_0 = M_{hh}/2$ for fixed order predictions

Conclusions

- We **combined** the full NLO with the NNLO corrections computed in the HEFT
- **Fully differential** results, using qT-subtraction
- NNLO piece improved via different **reweightings** to account for **finite Mt effects**
- Our best prediction includes the **full double-real loop-induced** amplitudes
- Increase with respect to NLO from 12% at 13TeV to 7% at 100TeV
- Remaining **Mt uncertainty: few percent** level
- **Most advanced** perturbative prediction for HH available to date
- **Outlook:** combination with threshold resummation, results for non-SM self-couplings, inclusion of Higgs decays

Thanks!

Backup slides

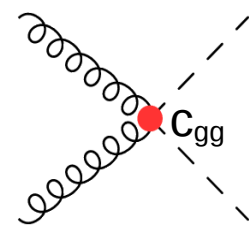
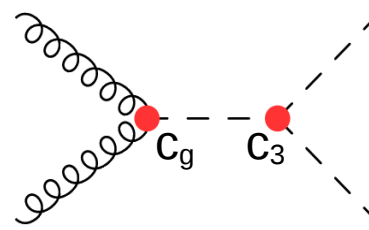
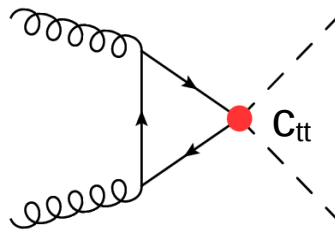
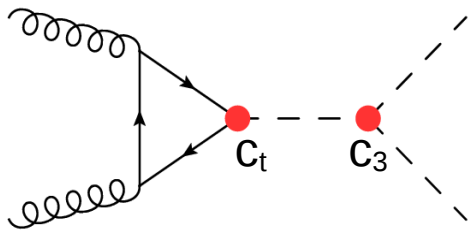
NNLO including dim 6 operators

D. de Florian, I. Fabre and JM
[arXiv:1704.05700]

We extended the computation to include BSM effects via EFT dimension 6 operators

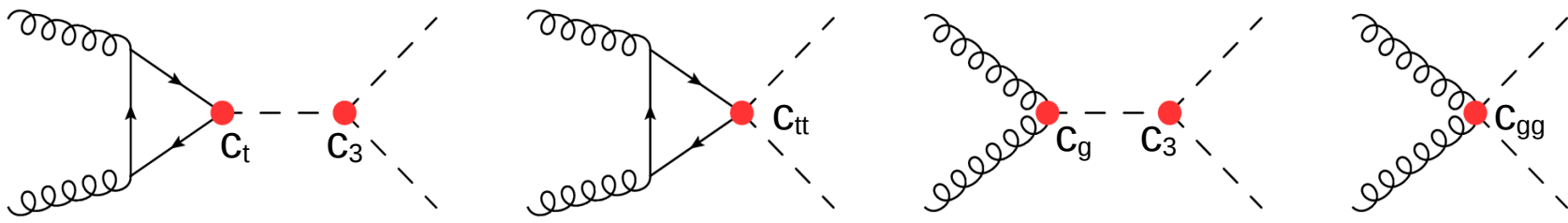
$$\mathcal{L}_{\text{eff}} = -M_t \bar{t}t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{2v^2} \right) - c_3 \frac{1}{6} \left(\frac{3M_h^3}{v} \right) h^3 + \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a \left(c_g \frac{h}{v} + c_{gg} \frac{h^2}{2v^2} \right)$$

- All relevant dimension 6 operators that vanish when $h=0$
- The SM corresponds to $c_t=c_3=1$ and $c_{tt}=c_g=c_{gg}=0$



NNLO including dim 6 operators

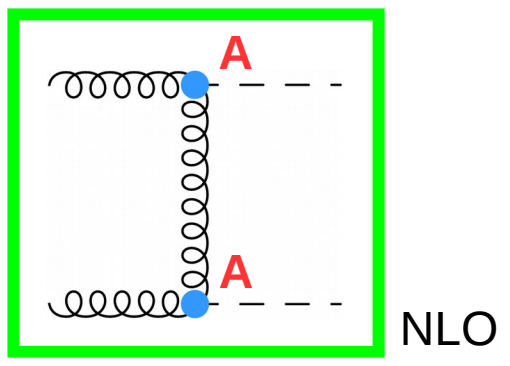
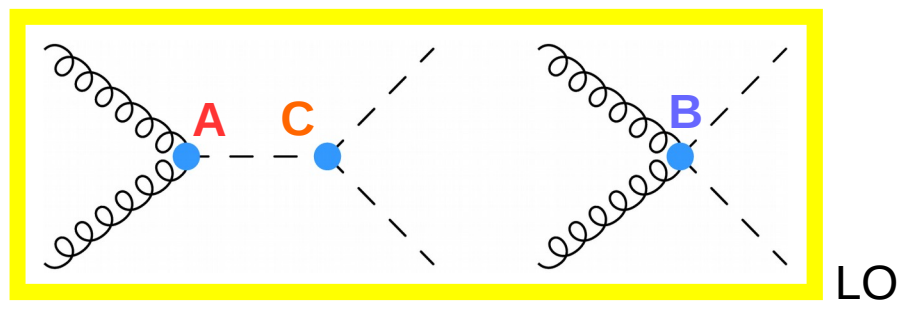
$$\mathcal{L}_{\text{eff}} = -M_t \bar{t}t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{2v^2} \right) - c_3 \frac{1}{6} \left(\frac{3M_h^3}{v} \right) h^3 + \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a \left(c_g \frac{h}{v} + c_{gg} \frac{h^2}{2v^2} \right)$$



↓ Large top quark mass limit

$$\mathcal{L}_{\text{eff}}^{\text{HTL}} = \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a \left\{ \underbrace{\frac{h}{v} \left[\frac{c_t}{12} C_H + c_g \right]}_A + \frac{h^2}{2v^2} \left[\underbrace{-\frac{c_t^2}{12} C_{HH} + \frac{c_{tt}}{12} C_H + c_{gg}}_B \right] \right\} - \underbrace{c_3 \frac{1}{6} \left(\frac{3M_h^3}{v} \right) h^3}_C$$

- Same couplings we have already in the SM HTL → easy to obtain the NNLO corrections
- Non trivial interplay between the different couplings:



NNLO including dim 6 operators

Does the size of the corrections change when we move away from the SM?

- We vary one coupling at a time:

$$c_3 = 1 + 10 \xi,$$

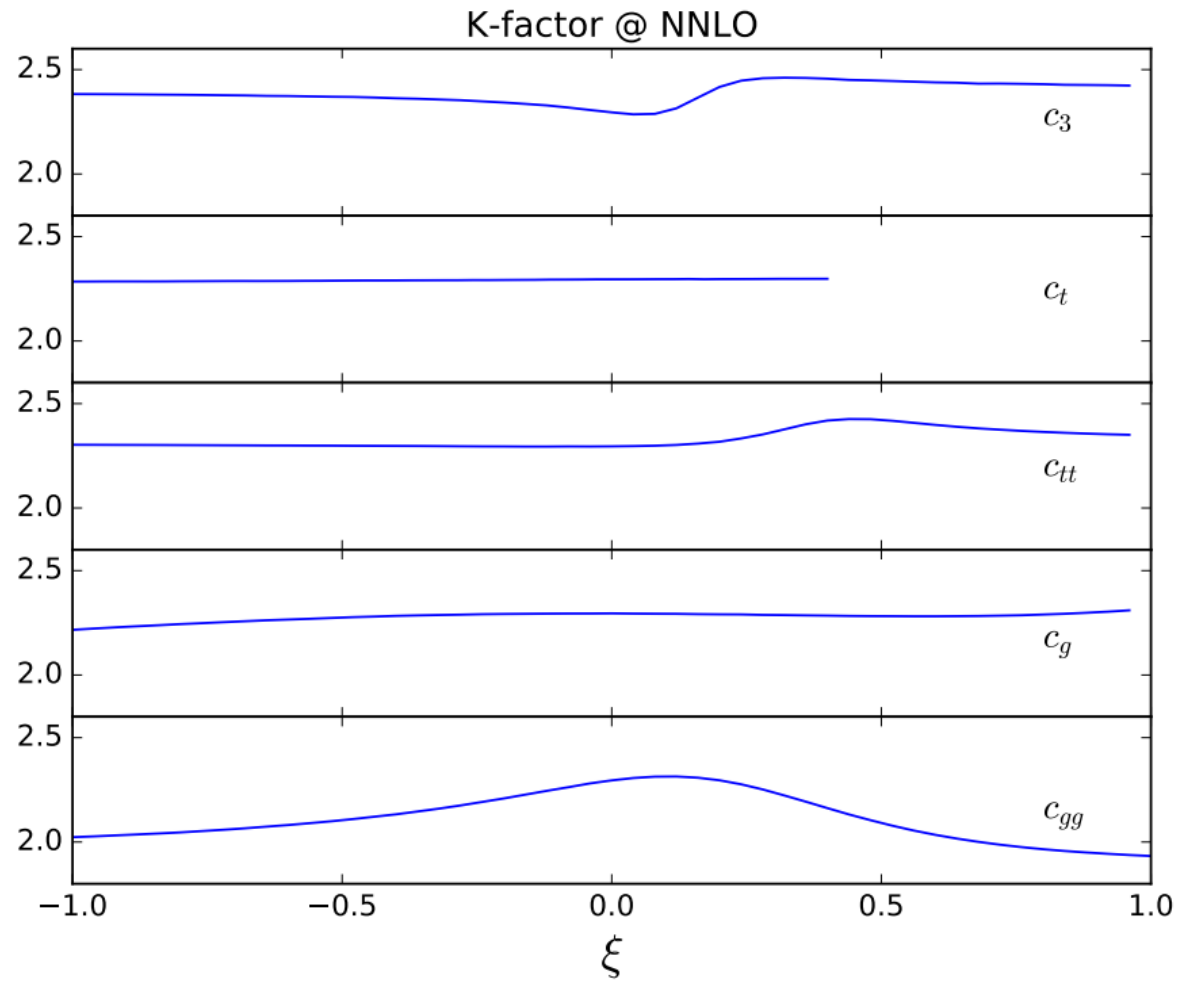
$$c_t = 1 + 0.35 \xi,$$

$$c_{tt} = 1.5 \xi,$$

$$c_g = 0.15 \xi,$$

$$c_{gg} = 0.15 \xi.$$

- Large corrections, but small dependence on the couplings



$\Delta K^{c_{gg}} = \frac{\max |K(c_{gg}) - K_{SM}|}{K_{SM}} \approx 15.8\%$ at $c_{gg} = 0.15$.

$\Delta K^{c_3} \approx 7.2\%$ at $c_3 = 4.20$,

$\Delta K^{c_{tt}} \approx 5.7\%$ at $c_{tt} = 0.66$,

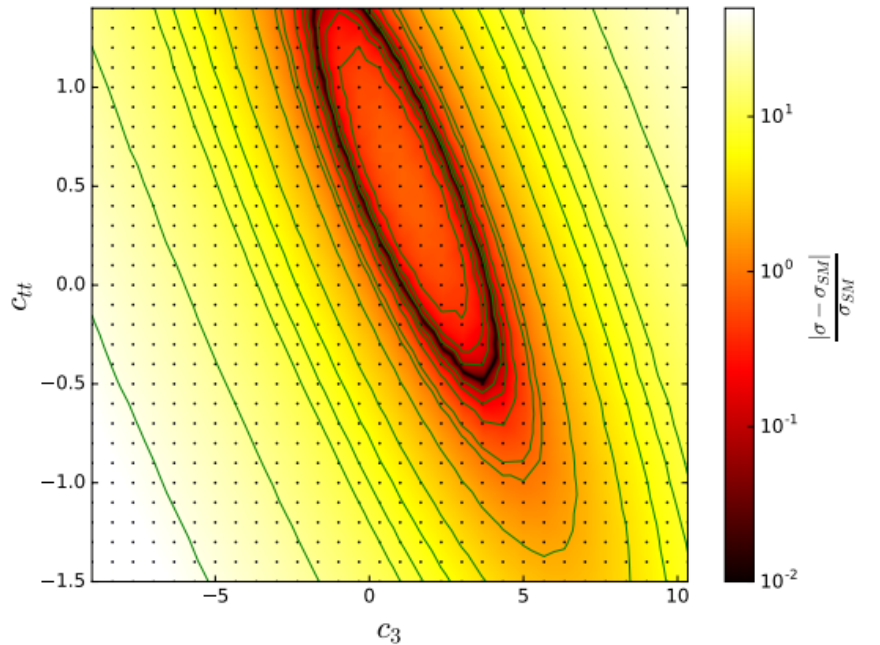
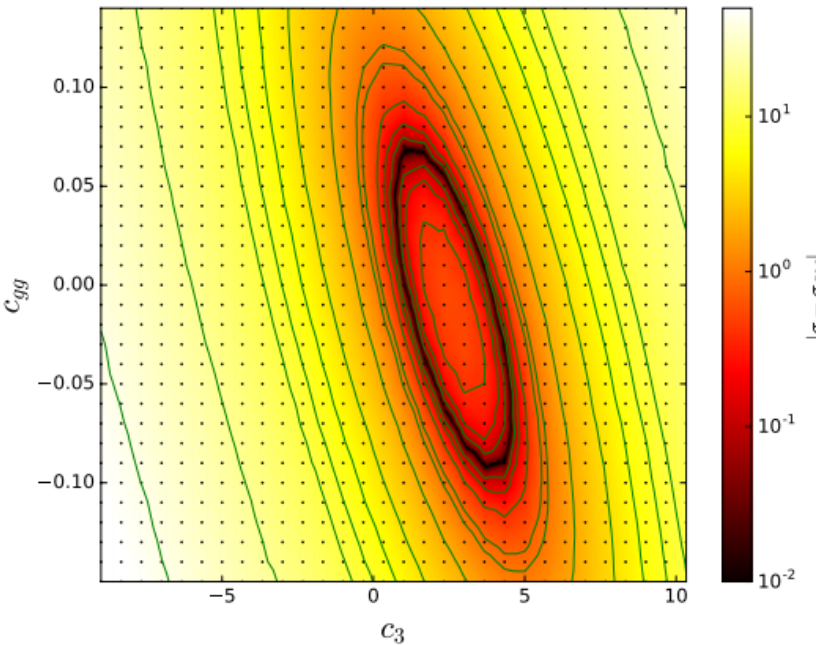
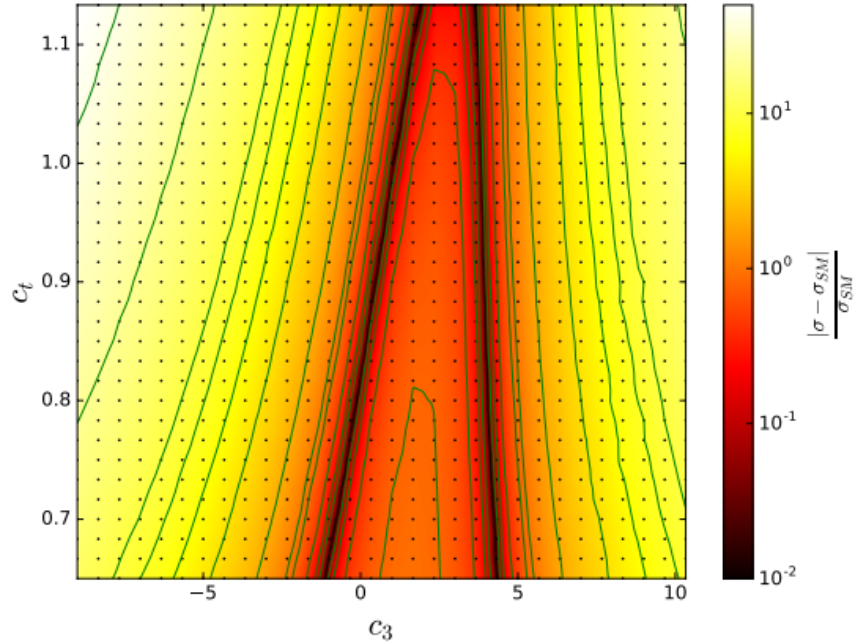
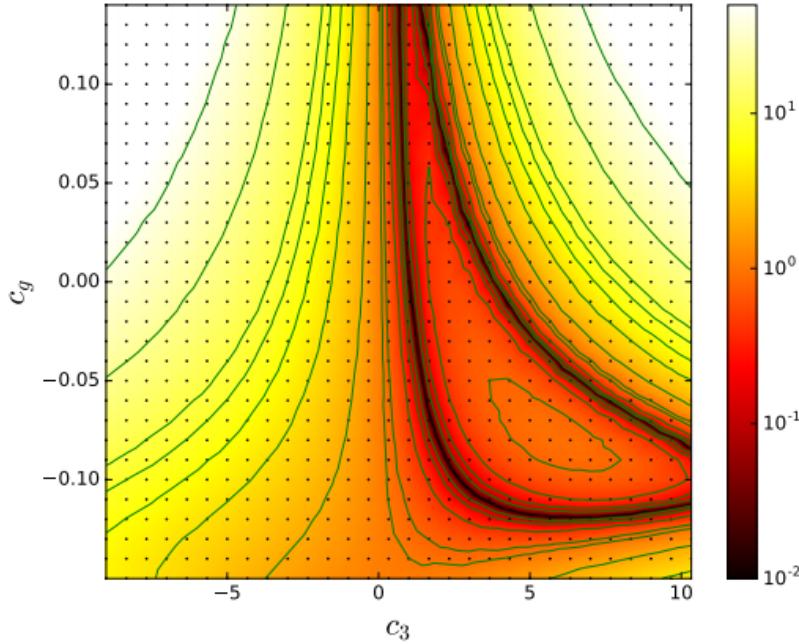
$\Delta K^{c_g} \approx 3.4\%$ at $c_g = -0.15$,

$\Delta K^{c_t} \approx 0.5\%$ at $c_t = 0.65$.

Larger deviations when doing simultaneous variations, but mainly in regions where the cross section is small

Degeneracy and M_{hh} distribution

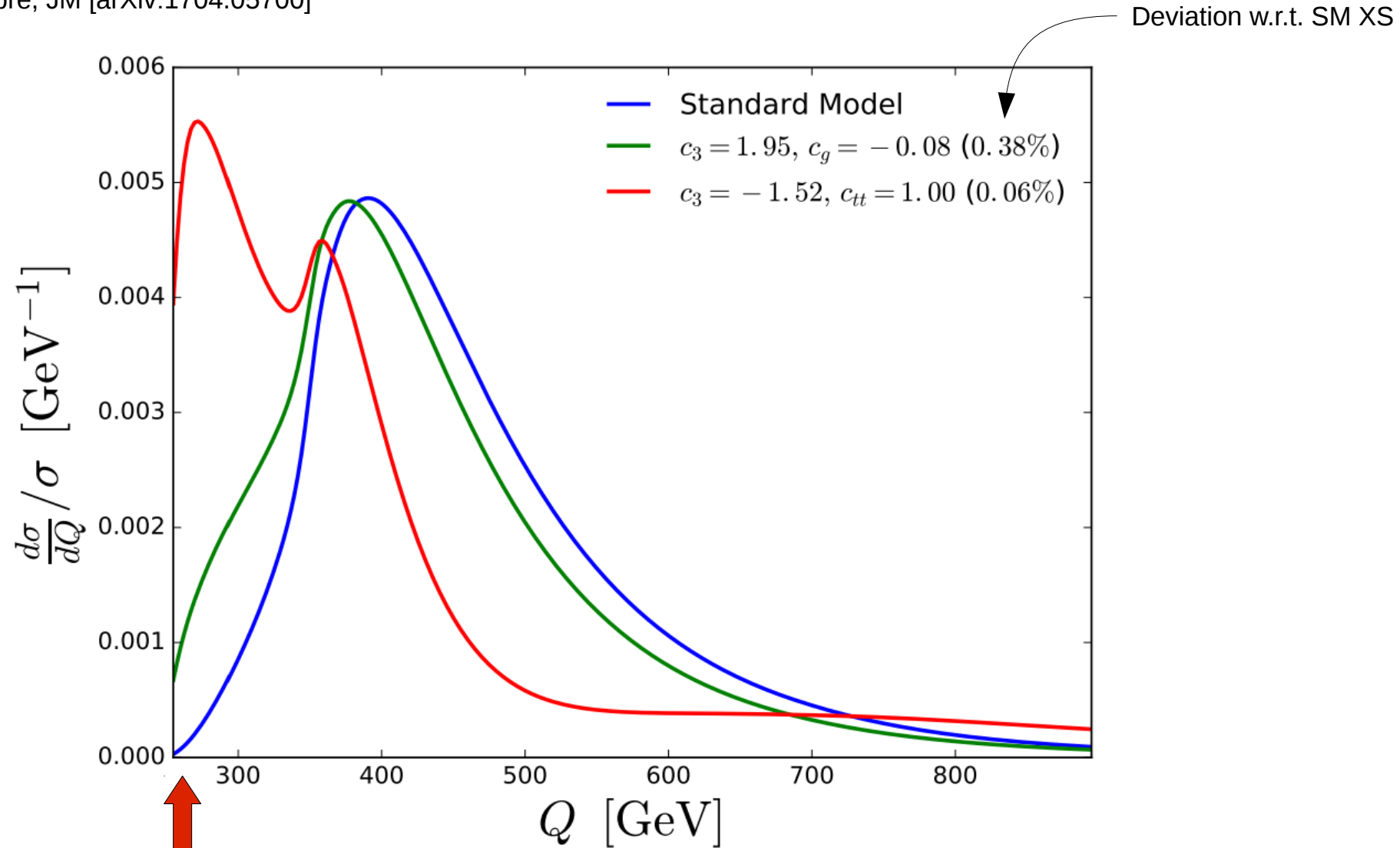
Different combinations of couplings can give a total XS similar to SM



Degeneracy and M_{hh} distribution

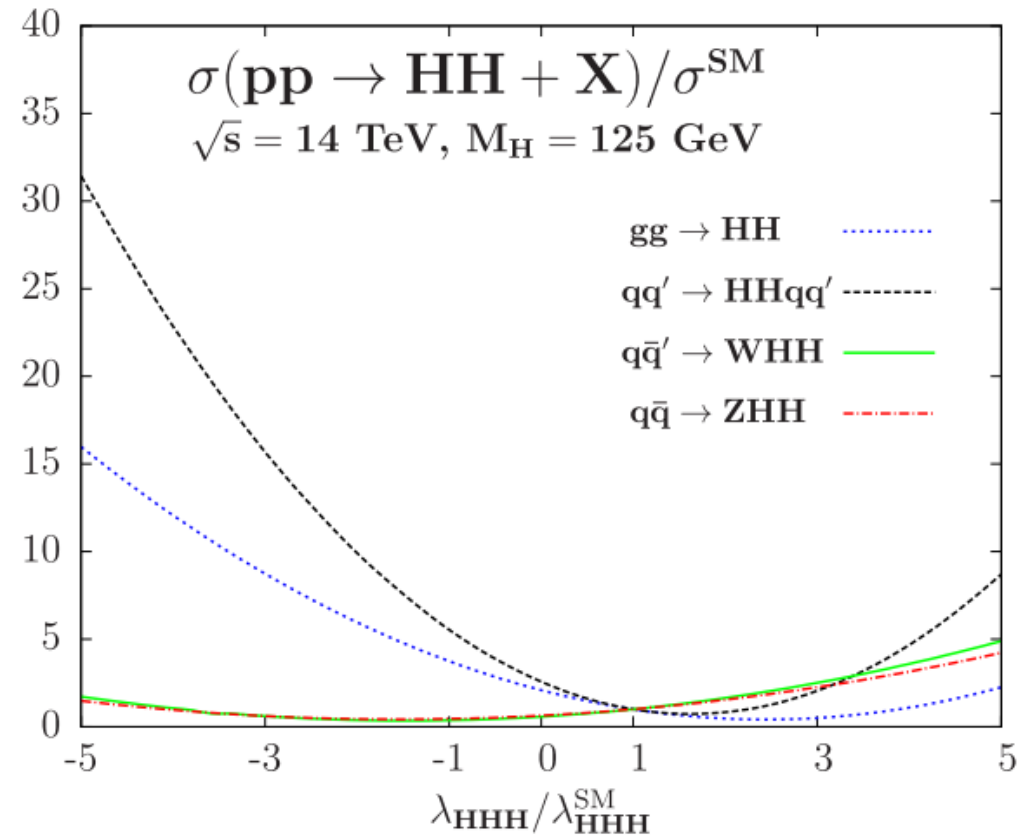
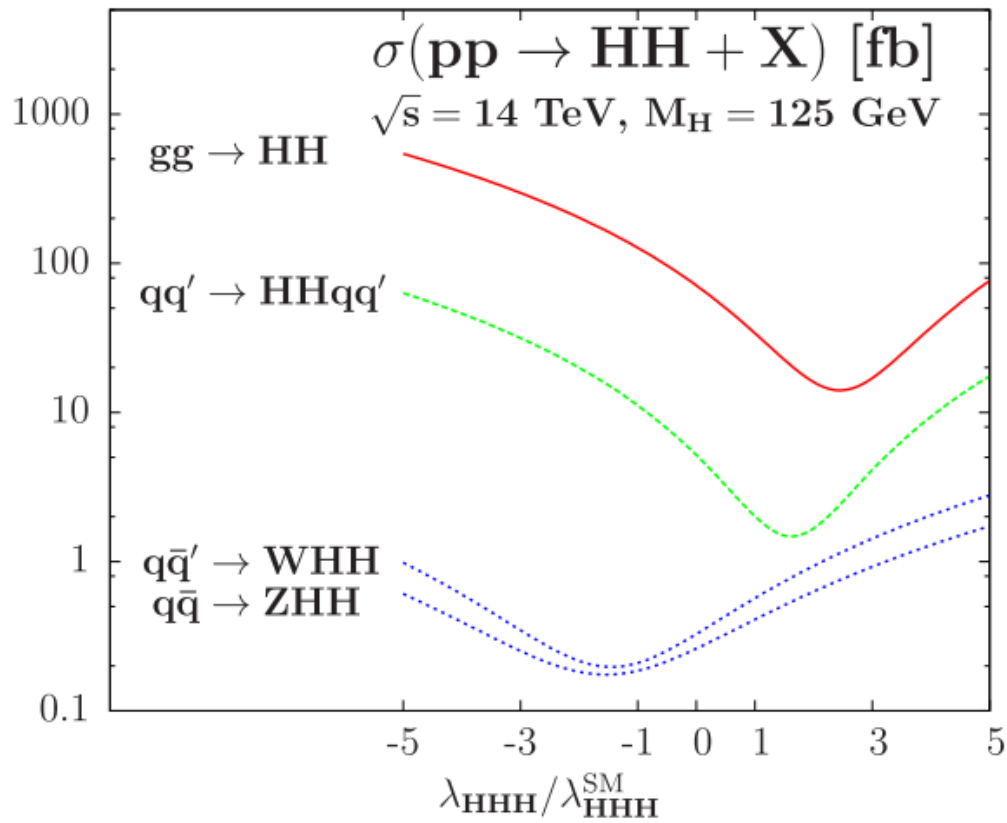
The invariant mass distribution can help disentangling the degeneracy

S. Borowka et al. [arXiv:1608.04798]
de Florian, Fabre, JM [arXiv:1704.05700]



Threshold is particularly sensitive due to triangle-box cancellations in the SM

Sensitivity to self-coupling for the different production mechanisms



Scale variation for fixed order and resummed total XS

