



Max-Planck-Institut für Physik

Duality

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Young Scientist Workshop 2018 - Schloss Ringberg

Outline

Dualities

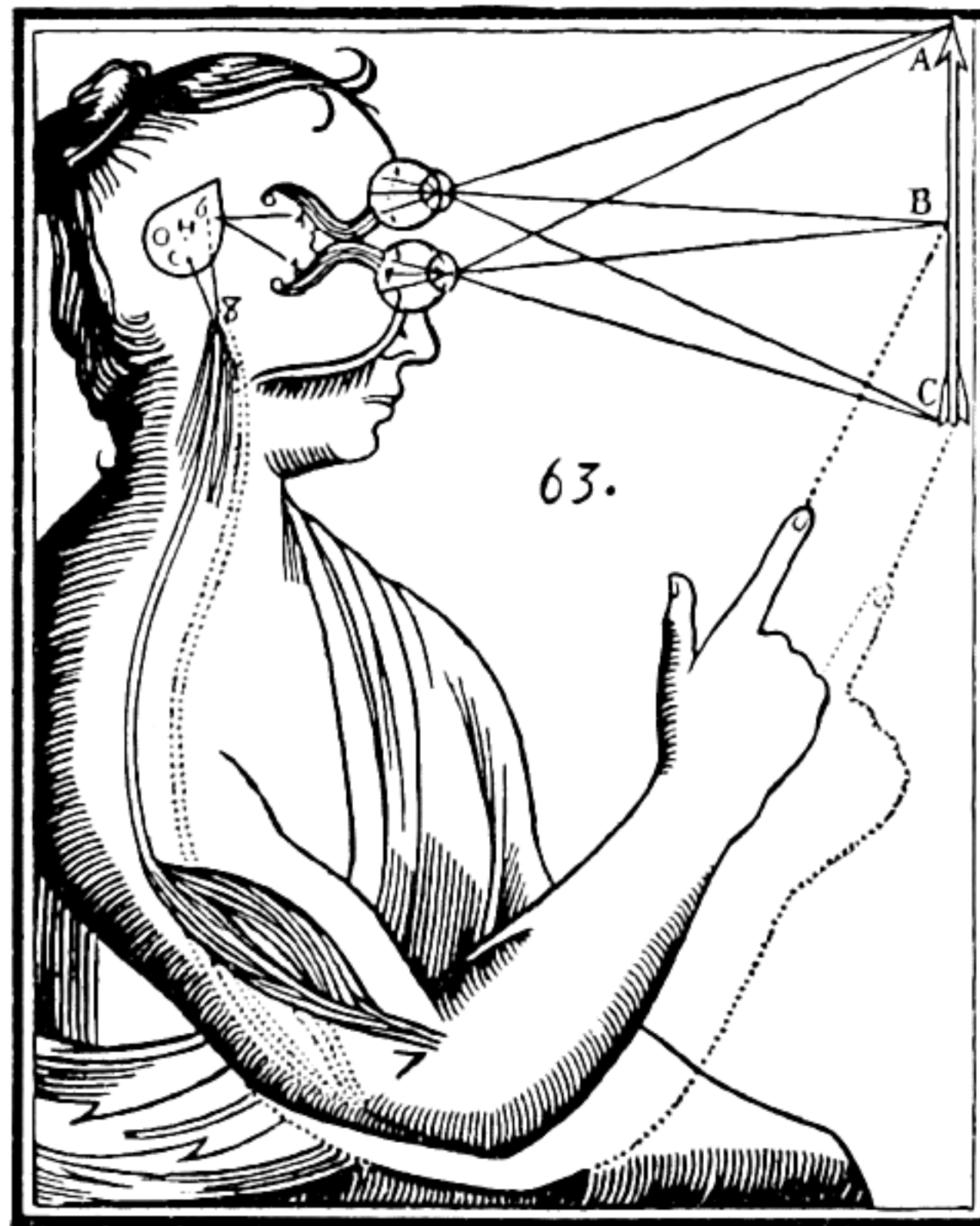
Where?

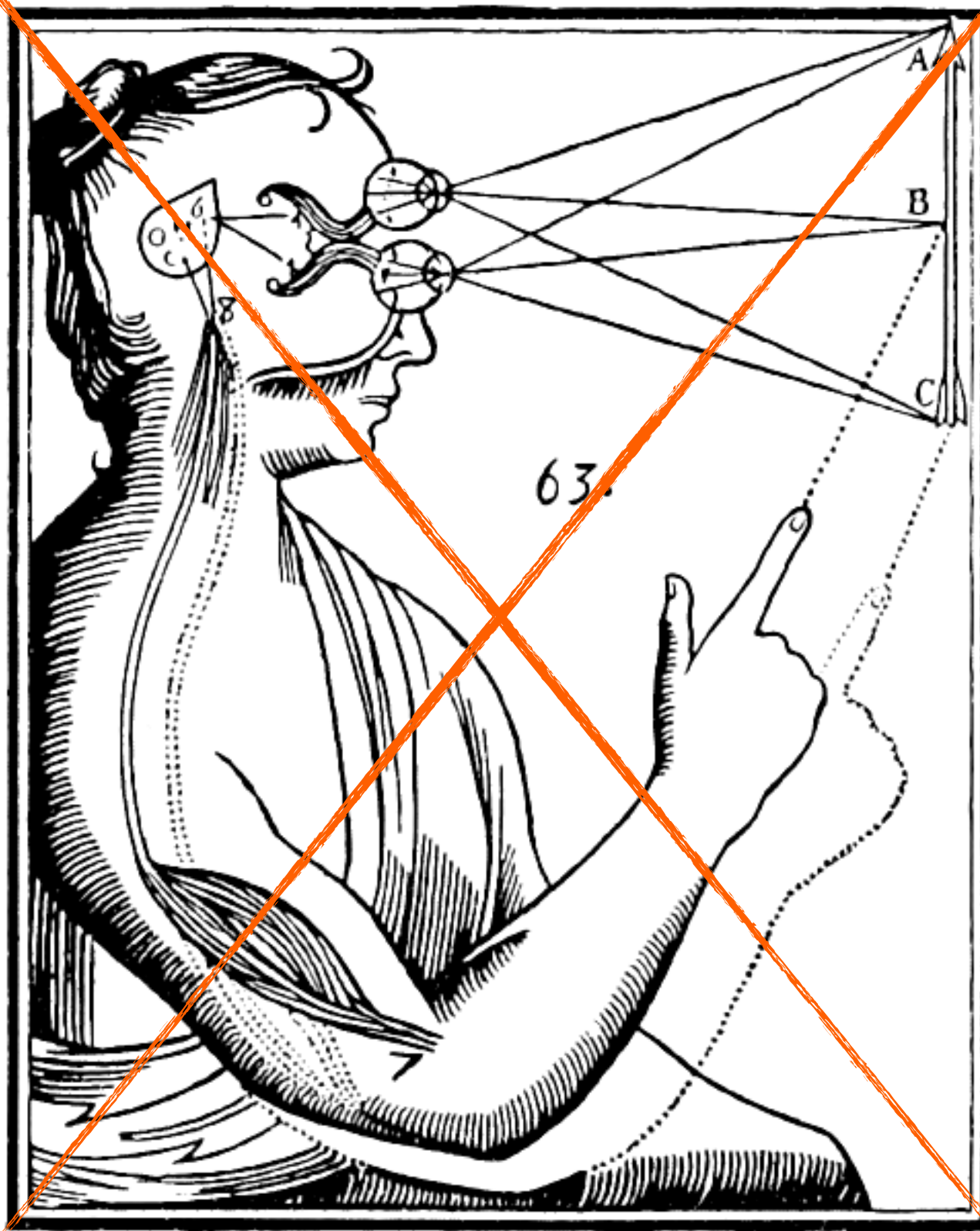
What?

Why?

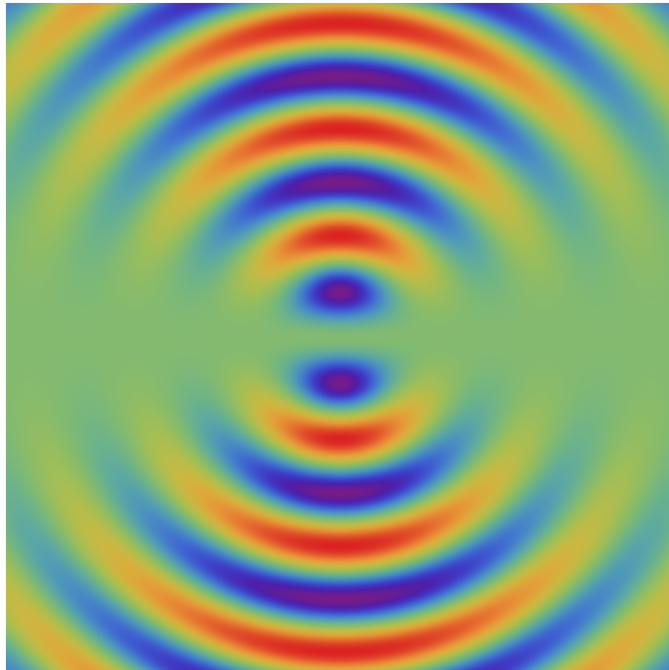
How?




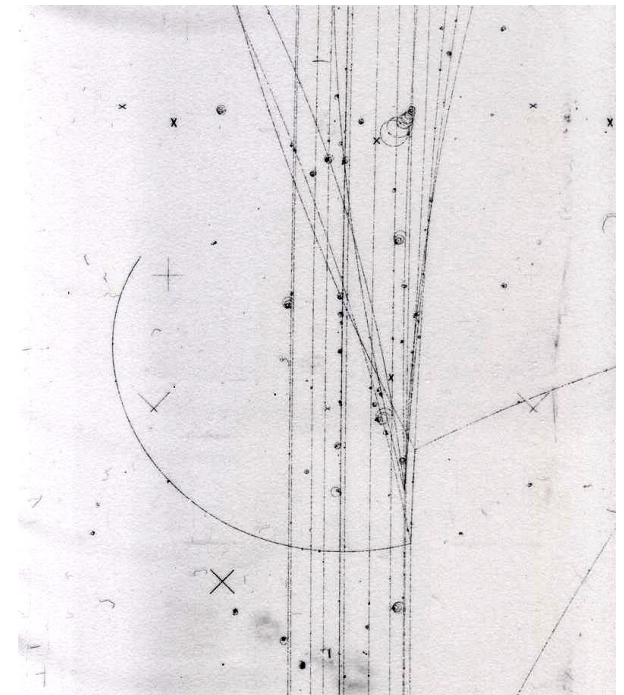




Dualities




$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$$



- Physical system has different descriptions in different limits
- True nature of the phenomenon not captured by limits
- Not quite what we mean by duality nowadays...

Two exactly equivalent physical theories



In a deep and nontrivial way...

Dualities are Useful!

Dualities can map...

quantum to classical
high energy to low energy
strong to weak coupling
gravitational to non-gravitational
gauge group G to gauge group H
spacetime X to spacetime Y

- Often one description easier to tackle than the other!
- Can be computational tool for real world physics

Dualities in (Quantum-) Field Theory

$$F = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\mathcal{L} = \int d^4x \frac{1}{4g^2} F^2 = \int d^4x \frac{1}{4g^2} (E^2 - B^2)$$

gauge coupling

Maxwell EQNs w/o source:

$$\begin{aligned} \nabla \cdot E &= 0 \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\partial_t B \\ \nabla \times B &= +\partial_t E \end{aligned}$$

Electromagnetic Duality

$$\begin{pmatrix} E \\ B \end{pmatrix} \rightarrow \begin{pmatrix} B \\ -E \end{pmatrix}$$

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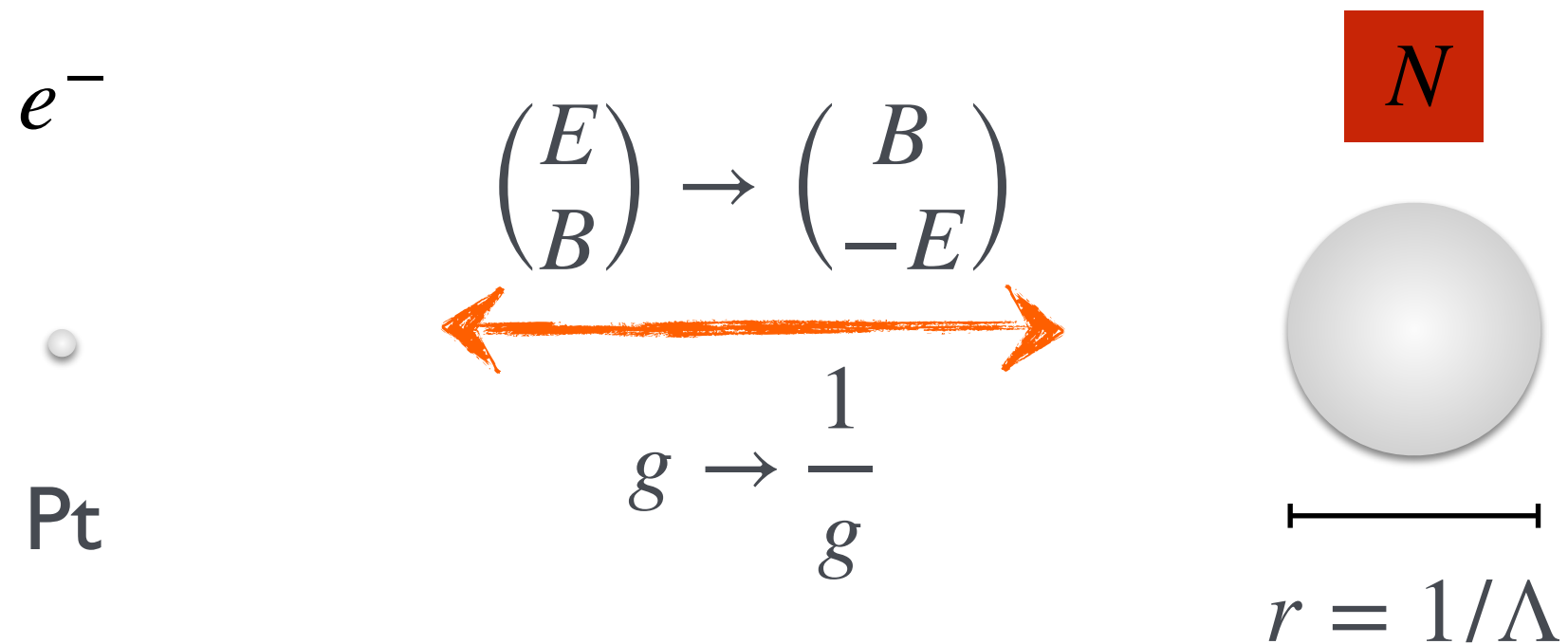
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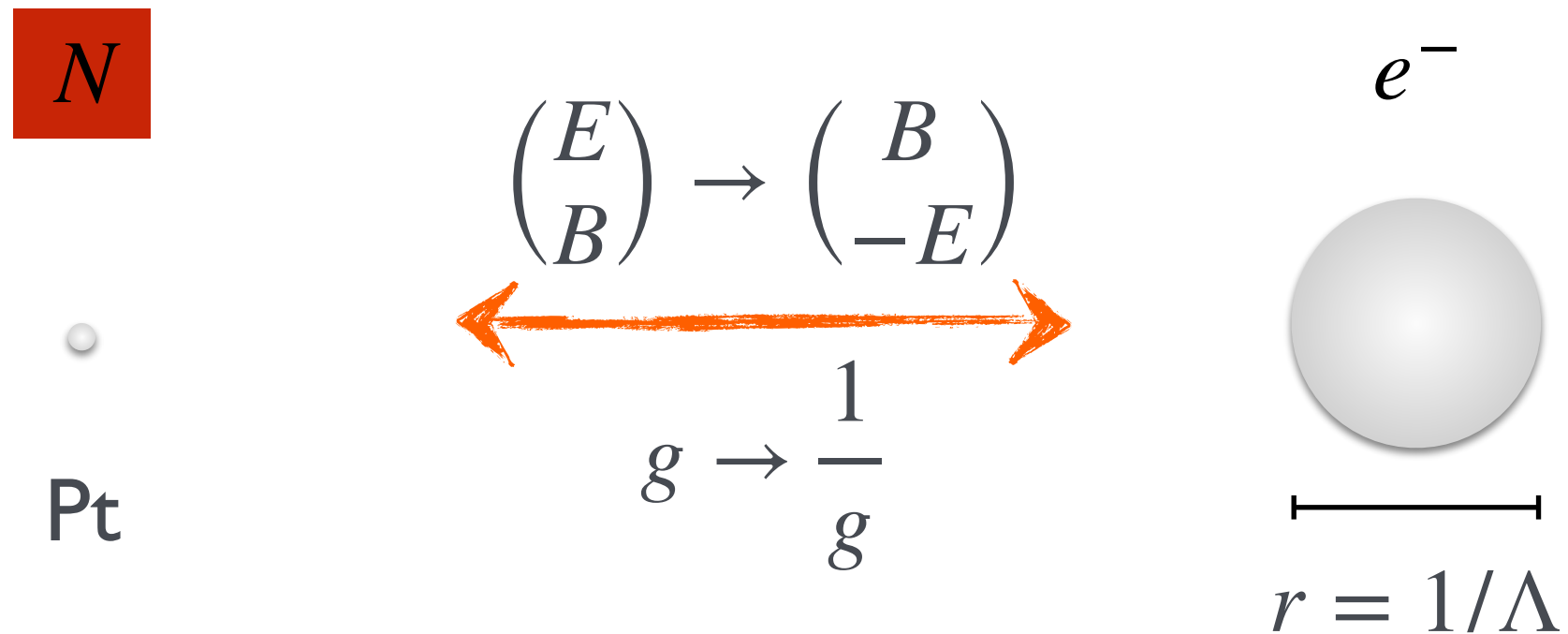
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Dualities in (Quantum-) Field Theory



Quantum mechanically **exact** in $\mathcal{N} = 4$ SUSY Yang-Mills

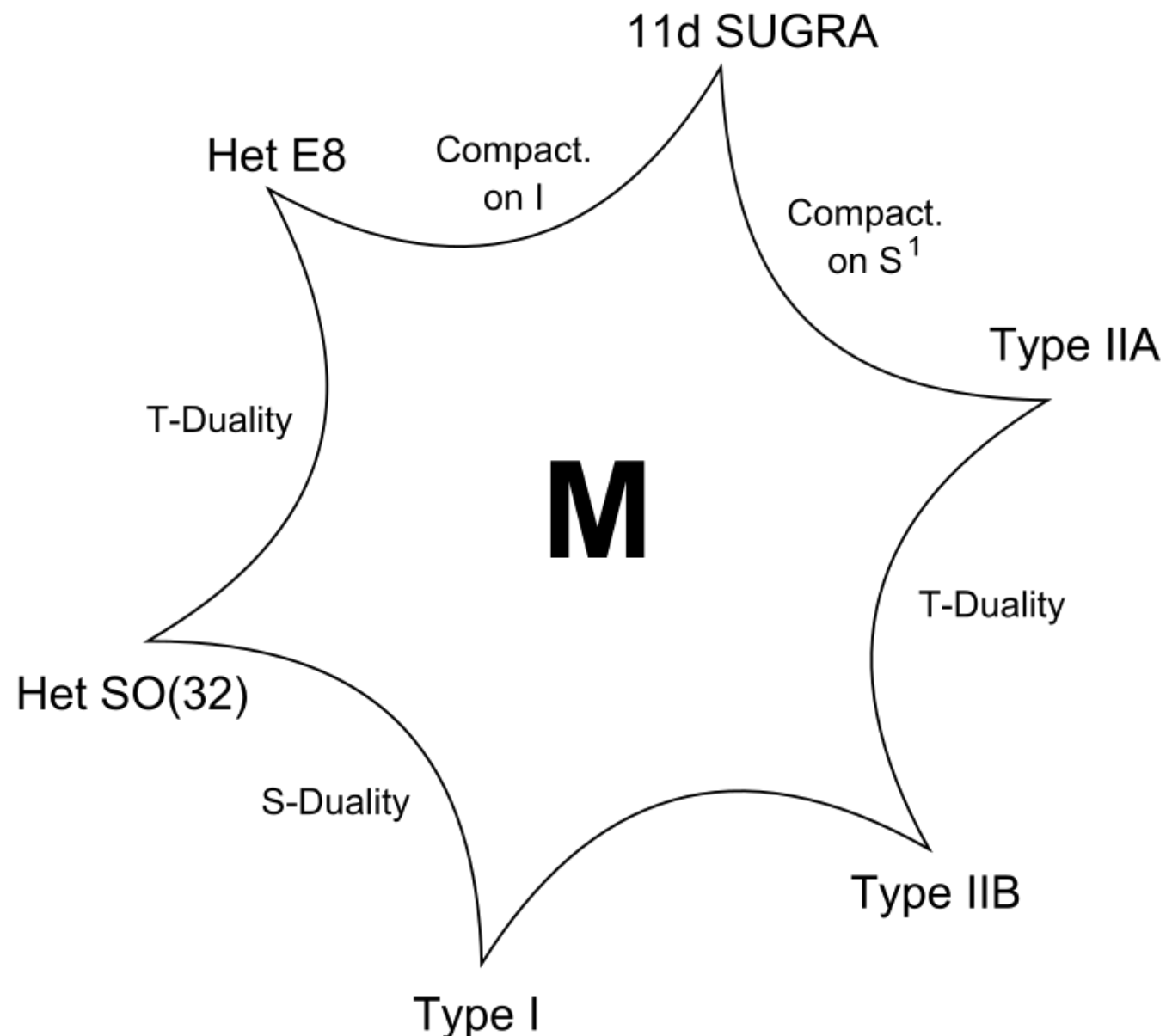
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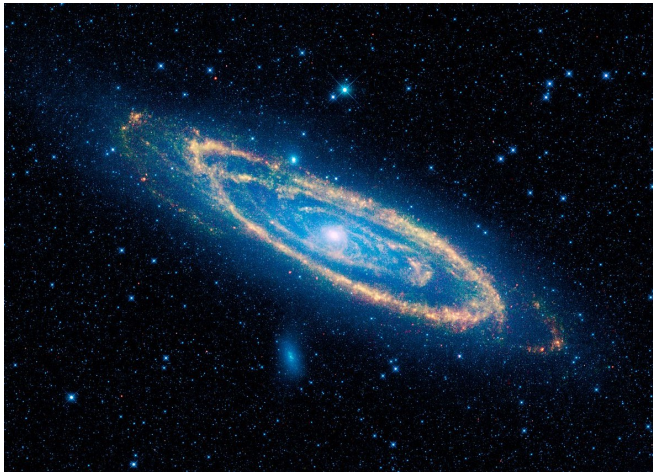
Dualities in Quantum Gravity

- “Different” String Theories in 10 dimensions are all different sides of the same coin! Exists a **duality web**

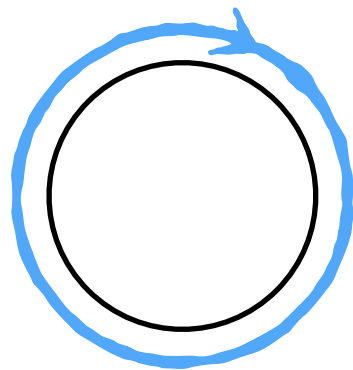


T is for Target-space

$$10^{-10} \text{ m} \equiv 10^{-60} \text{ m}$$




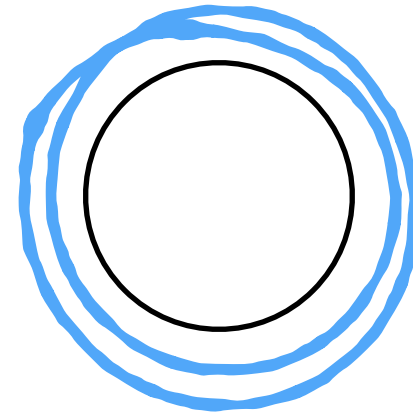
How???



Momentum

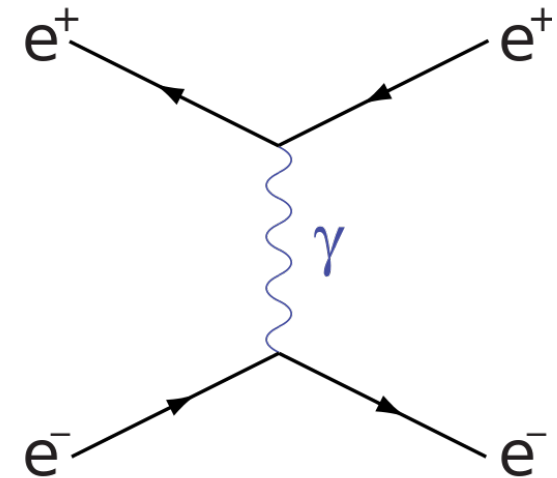
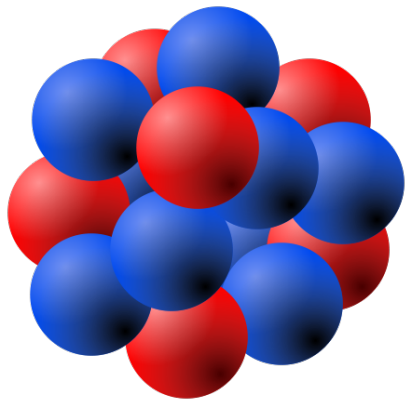
T-duality


$$R \leftrightarrow \frac{\ell_s^2}{R}$$



Winding

S is for Strong/Weak



- Type IIB string theory contains a fundamental string F1 and a string-like, solitonic D1-brane
- S-duality exchanges the two, like in electromagnetic duality
- In fact, EM duality of super-Yang-Mills derives from this!

Conclusion

- Duality = non-trivial equivalence of theories
- Ubiquitous in SUSY Field Theory and Strings
- Useful to study real world phenomena

Thank You