# Non-geometric spacetimes in string theory

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## overview

1. motivation
2. non-commutative and non-associative spacetimes
3. strings in (bosonic) backgrounds
4. from geometric to non-geometric fluxes

## uncertainty relations of spacetime

- non-commutative spacetime $\rightarrow$ *physical* minimal areas

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  - Jacobi identity (infinitesimal version of *associativity* - blackboard):

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  - violation of Jacobi identity $\rightarrow$ higher uncertainty relation, e.g.

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**How do these emerge in string theory?**

## strings in (bosonic) backgrounds

- single string picture:
  - string moving in/coupling to a background spacetime
    generalisation of point particle moving in curved space
  - different background fields, among others:
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    metric $G_{\mu\nu}$, Kalb-Ramond field $B_{\mu\nu}$: (higher) gauge field
- modifications in relation string $\leftrightarrow$ (target) spacetime
  - compact extra dimensions: winding modes of string
  - finite string length
  - not necessarily ordinary manifolds:
    interplay between metric $G$ and other fields
  - dualities, e.g. $T$-duality - $R \leftrightarrow \frac{1}{R}$

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# example 2: from geometric to non-geometric fluxes I

[Shelton, Taylor, Wecht 05]

chain of $T$-dualities:

$$\mathbf{H}_{abc} \quad \xrightarrow{T_1} \quad \mathbf{f}^c{}_{ab} \quad \xrightarrow{T_2} \quad \mathbf{Q}_c{}^{ab} \quad \xrightarrow{T_3} \quad \mathbf{R}^{abc}$$

- start with $\mathbf{H}$-flux on flat 3-torus $T^3$: $\mathbf{H}_{123} = \partial_{[1} B_{23]} = h$ $\mathbb{R}^3$ with $x^i \sim x^i + 1$ + gauge field $\to$ geometric

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- $T$-duality along $x^1$:
  - 'twisted torus': $(x^1, x^2, x^3) \sim (x^1 - hx^2, x^2, x^3 + 1)$
  - similar to a Lie group: vector fields $e_i^a$

$$\partial_{[i}e_{j]}^c + \mathbf{f}^c{}_{ab}e_i^a e_j^b = 0 \rightarrow \text{geometric } \mathbf{f}\text{-flux, here } \mathbf{f}^3{}_{12} = h$$

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    $\to$ 'globally non-geometric'
  - for closed strings in **Q**-flux: $[\hat{x}^a, \hat{x}^b] \ \sim \ \mathbf{Q}_c{}^{ab} w^c$
- (formal) $T$-duality along $x^3$: $\theta^{12} = h\tilde{x}^3$
  - $\tilde{x}^3$ 'winding' coordinate
  - no description in terms of standard coordinates
    $\to$ 'locally non-geometric'

$$\mathbf{R}^{abc} = \tilde{\partial}^{[c}\theta^{ab]} \longrightarrow \text{non-geometric } \mathbf{R}\text{-flux}\ , \text{ here } \mathbf{R}^{123} = h$$

  - for closed strings in **R**-flux: $Jac(\hat{x}^a, \hat{x}^b, \hat{x}^c) \ \sim \ \mathbf{R}^{abc}$

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    $\to$ non-commutative Yang-Mills theories
  - closed strings: non-geometric **Q**- and **R**-flux describe non-commutative resp. non-associative spacetimes.

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  - closed strings: non-geometric **Q**- and **R**-flux describe non-commutative resp. non-associative spacetimes.
- string dualities: different backgrounds, same physics
  new view on spacetime and its mathematical description