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Introduction to AdS/CFT duality

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From? University of Würzburg

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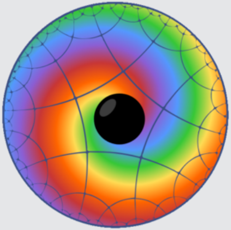

Intuitive motivation

What is so special about **gravity**?
What makes it so different to QFTs?

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What makes it so different to QFTs?

$$\downarrow$$

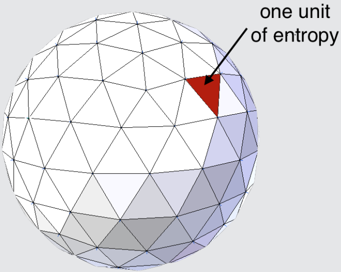
It contains **black holes.**

Intuitive motivation

Black holes have an entropy.

Black hole entropy

$$S_{BH} \propto Area_{Horizon}$$



What does this mean for a general spacetime region A?

Intuitive motivation

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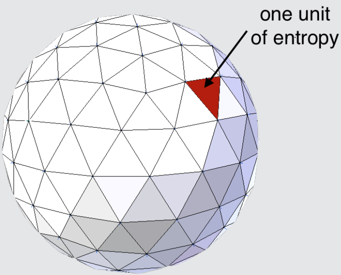


Black hole entropy

$$S_{BH} \propto Area_{Horizon}$$

What does this mean for a general spacetime region A?

$\rightarrow$ statistical physics: # of DoFs $\propto S$

Bekenstein bound

$$\# \text{ of DoFs } \leq Area(\partial A)$$

(Bekenstein 1972, 1981)

Intuitive motivation

In contrast, in a **local** theory the # of DoFs scales as

$$\text{\# of DoFs} \propto \mathit{Volume}(A).$$

Holographic principle

For theory of quantum gravity, the degrees of freedom in a region A can be encoded on its boundary ∂A.

(Hooft, 1993; Susskind 1994)

Taking it one step further

Duality

two theories describe **same physics**

- Both theories have same dynamics.
- one-to-one map between DoFs on both sides

The holographic principle hints to a dual theory living on the boundary.

$\Downarrow$

vastly different theory,
different spacetime dimension

AdS/CFT duality

theory in Anti-de Sitter space

$\updownarrow$

conformal field theory on boundary

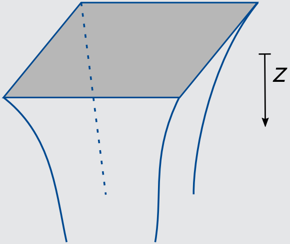
(Maldacena, 1997)

Anti-de Sitter space

- highly symmetric solution to Einstein equations with negative cosmological constant
- constant negative curvature

metric

$$ds^2 \propto \frac{1}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$



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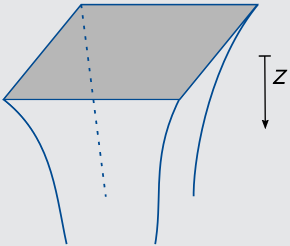

Boundary of AdS

AdS space has a boundary at $z = 0$. The boundary coordinates are t and $\vec{x}$.

Symmetries

Symmetries have to match on both sides.

- Which symmetry corresponds to coordinate transformations?
- Consider a metric of the form

$$ds^2_{AdS} \propto \frac{1}{z^2}\left(dz^2 + \hat{g}_{\mu\nu}(z,t,\vec{x})dx^\mu dx^\nu\right)$$

- Coordinate transformation which preserves this form changes boundary metric

$$\hat{g}_{\mu\nu}(0,t,\vec{x}) \longrightarrow e^{2\sigma(x)} \cdot \hat{g}_{\mu\nu}(0,t,\vec{x})$$

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coordinate transformations in AdS

$$\updownarrow$$

conformal transformation on boundary

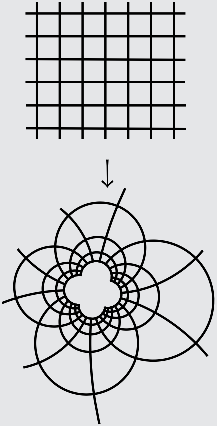
Conformal field theory

- conformal symmetry

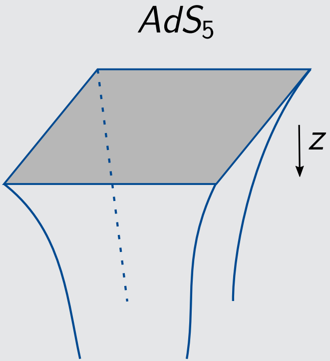
$$g_{\mu\nu}(x) \longrightarrow e^{2\sigma(x)} \cdot g_{\mu\nu}(x)$$

  - extension of Poincare symmetry
  - preserves angles and causality

- Theory has no length-scale.
  - dimensionless coupling constants
  - vanishing beta functions



What do we have to do to obtain a 3 + 1 dimensional CFT?



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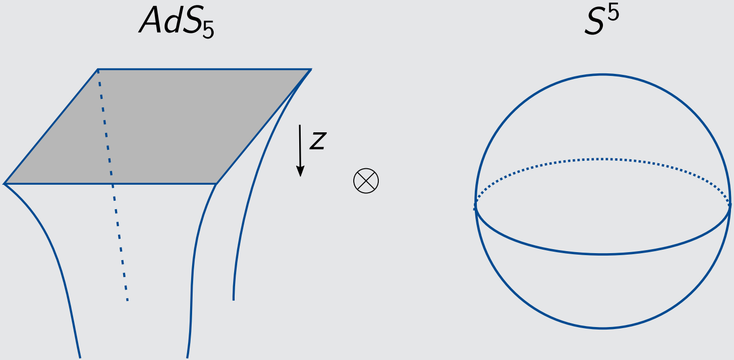
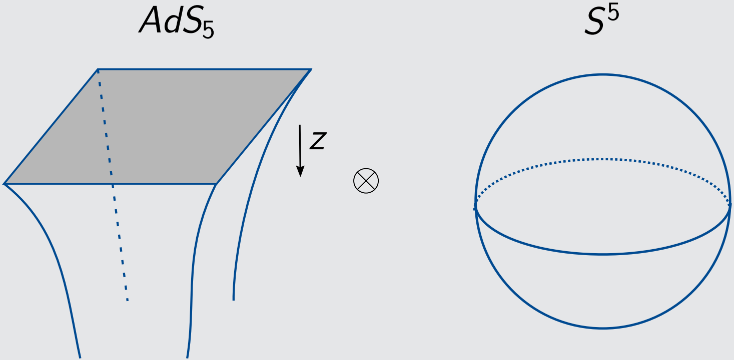


What do we have to do to obtain a $3+1$ dimensional CFT?



compactify $S^5 \longrightarrow$ effective theory on AdS_5

Example for $d = 4$

- supersymmetric Yang-Mills theory (SYM)
- gauge symmetry $SU(N)$

field content	
one	gauge field
four	Weyl fermions
six	(real) scalars

- dimensionless coupling-constant g_{YM}

$$\updownarrow$$

- string theory on $AdS_5 \times S^5$
- additional symmetries:
  $\mathcal{N} = 4$ supersymmety, $SO(6)$ symmetry

AdS/CFT duality

dictionary:

$$
\begin{aligned}
g_{YM}^2 &\propto \text{string coupling} \\
\lambda = g_{YM}^2 N &\propto \text{string length}^{-4}
\end{aligned}
$$

strongest form

$N,\ \lambda$ arbitrary $\longleftrightarrow$ quantum string theory

weak form

$N \to \infty$, λ large $\longleftrightarrow$ point-particle limit, classical field theory: supergravity

AdS/CFT duality

I promised you a one-to-one map:

Field-operator map

CFT operator $\mathcal{O}$ $\Longleftrightarrow$ **AdS field φ** with mass m

$$x \longrightarrow \lambda x$$
$$\mathcal{O}(x) \longrightarrow \lambda^{-\Delta}\mathcal{O}(x)$$

$$m^2 \propto \Delta(\Delta - d)$$

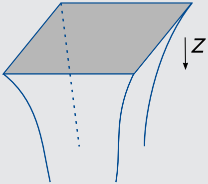
near-boundary expansion:

$$\varphi(z,t,\vec{x}) \sim \underset{\substack{\uparrow \\ \text{source of } \mathcal{O}}}{\varphi_{(0)}}(t,\vec{x}) \cdot z^{d-\Delta} + \underset{\substack{\uparrow \\ \text{expectation value}}}{\langle \mathcal{O}(t,\vec{x}) \rangle} \cdot z^{\Delta} + \cdots$$

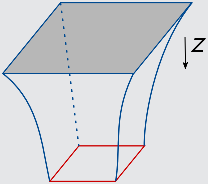
$$Z_{CFT}[\varphi_{(0)}] = Z_{AdS}[\varphi] = \exp(iS_{AdS}[\varphi])$$

Generalisations

- finite temperature $\longleftrightarrow$ black hole



$\Longrightarrow$



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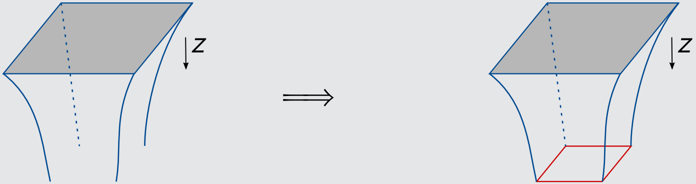


- thermodynamics $\longleftrightarrow$
- temperature
- entropy

- black hole thermodynamics
- Hawking temperature
- Bekenstein entropy

Different approaches

- **Top-down approach**
  - start with string theory in 10 or 11 dimensions
    dimensional reduction on compact space
  - clear statement of the two sides of the duality
  - established holographic dictionary

- **Bottom-up approach**
  - effective AdS Lagrangians with few ingredients to obtain desired features
    e.g. confinement, superconductivity
  - dual field theory not completely known
  - **advantage**: allows to study qualitative behaviour of strongly-coupled field theories with such properties

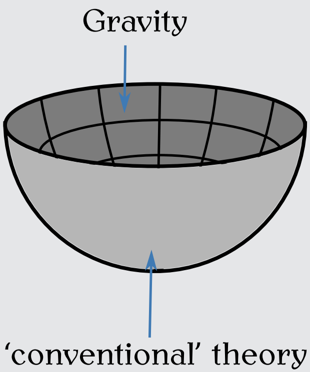
Summary

Holographic principle:

gravity theory in region A

$$\updownarrow$$

encode DoFs on ∂A.



Summary

Holographic principle:

gravity theory in region A

$\updownarrow$

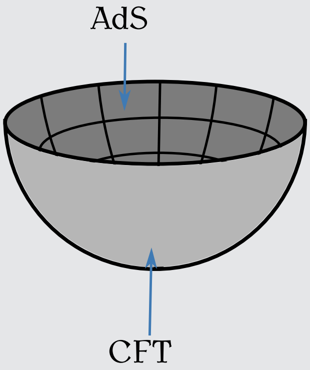
encode DoFs on ∂A.

AdS/CFT:

CFT in d dimensions

$\updownarrow$

theory on AdS in $d+1$ dim.



- one-to-one map between DoFs, same dynamics
- symmetries match
- strong coupled field theories
  $\longrightarrow$ probes interesting new regime

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Thank you for your attention