On couplings to matter in bimetric theory

IMPRS Young Scientist Workshop at Ringberg Castle 2018

by

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September 5, 2018

Contents

Why massive graviton?

Field theoretic perspective

- We have theories for massless and massive particles of different spin s

	massless	massive
spin-0	Klein-Gordon	Klein-Gordon
spin-1	Maxwell	Proca
spin-2	GR	?

Field theoretic perspective

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	massless	massive
spin-0	Klein-Gordon	Klein-Gordon
spin-1	Maxwell	Proca
spin-2	GR	?

- GR is the unique theory (unitary and Lorentz-invariant) for a massless spin-2 field (in 4 dim) [Lovelock '71]

Field theoretic perspective

- GR is a fully non-linear theory of gravity

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  - $d \neq 4$ dimensions
  - higher-order derivatives of the metric tensor
  - non-locality
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  - Add new DoF's: scalars, vectors, tensors
  - Make graviton massive! $\rightarrow$ Gravitational force with finite range

Introduction to bigravity

Linear theory & Boulwere-Deser ghost

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- Lagrangian of lin. gravity + Fierz-Pauli mass term

$$\mathcal{L} = -\frac{1}{4} h_{\alpha\beta} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{m^2}{8} (h_{\mu\nu} h^{\mu\nu} - (1-a) h^2)$$

- Propagates a ghost with mass $m^2 \sim a^{-1}$. Fierz-Pauli tuning: $a = 0$.

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- Propagates a ghost with mass $m^2 \sim a^{-1}$. Fierz-Pauli tuning: $a = 0$.
  [Fierz&Pauli '39]
- BUT: tuning does not persist non-linearly and the ghost will reappear at higher order! [Boulwere&Deser '72]

Bigravity

- Non-linear action for massless & massive graviton [deRham,Gabadadze,Tolley '10; Hassan&Rosen '11]

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  - Symmetric under $g_{\mu\nu} \leftrightarrow f_{\mu\nu}$
  - Potential is symmetric and involves square-root matrix

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- To which metric should matter couple? Physical metric?

Matter couplings

Singly and doubly coupled

$$S_{\mathrm{m}} = \int \mathrm{d}^4 x \sqrt{g}\, \mathcal{L}_{\mathrm{m}}(g; \partial\phi, \phi)$$

- Simplest **choice**: matter couples to one metric (singly-coupled BG)
  - (classically) ghost-free [Hassan&Rosen '11]
  - matter loops do not detune the potential, but generate coupling between f and ϕ. BD ghost reappears above strong coupling scale [deRham et.al. '14, Heisenberg '15]
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- Trivial extension: two independent matter sectors ϕ and χ
- Same matter couples to both metrics: $\phi = \chi$
  - BD ghost at low energies [Yamashita et.al. '14; deRham et.al. '14]
  - A matter field must not have two kinetic terms, but even then matter loops bring back the BD ghost at unacceptable low scales

Effective composite metric

$$S_{\mathrm{m}} = \int \mathrm{d}^4x\sqrt{h}\mathcal{L}_{\mathrm{m}}(h; \partial\phi, \phi)$$

- Matter couples to eff. metric, composed out of g and f `[deRham et.al. '14; Heisenberg '14 & '15]`

$$h_{\mu\nu} = a^2 g_{\mu\nu} + 2ab\, g_{\mu\lambda}(\sqrt{g^{-1}f}\,)^{\lambda}_{\ \nu} + b^2 f_{\mu\nu}$$

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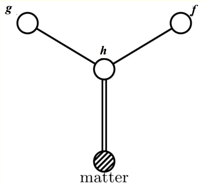
- For computations, switch to trimetric description



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Summary

- The only known non-linear ghost-free theory for a massive graviton involves 2 metric tensors
- More than 1 dynamical metric: Physical metric?
  - *Either* break equivalence principle between both metrics
  - *Or* BD ghost reappers below Planck scale (but above strong coupling scale)
- What's next?
  - Other ghost-free symmetric matter couplings?
  - Study phenomenology, in particular in high energy environments (early universe, BH)