The Idea of Dimensional Reduction

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Introduction

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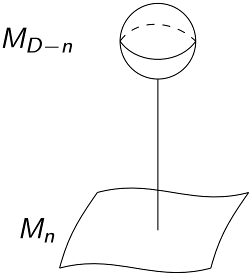
To do so, we take a limit in which the additional dimensions are small and curled up to avoid observation.

The physics (field content, gauge groups,...) of the resulting effective supergravity theory are determined by the geometry of the curled up dimensions. Therefore, we need to understand its structure.

Outline

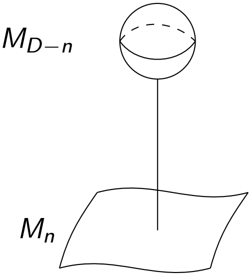
1. Dimensional Reduction
2. Moduli
3. Mirror Symmetry of the Torus

Dimensional Reduction



D-dim. gravity theory coupled to matter

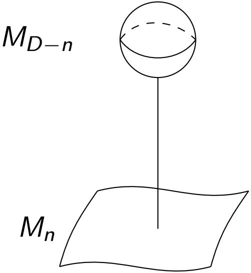
Dimensional Reduction



D-dim. gravity theory coupled to matter

$\Rightarrow$ n-dim. effective theory

Dimensional Reduction



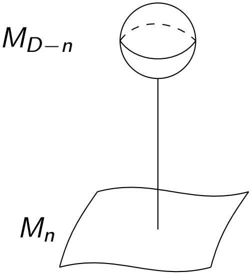
D-dim. gravity theory coupled to matter

$\Rightarrow$ n-dim. effective theory

At every point of the curved spacetime M_n there is a very small deformable internal manifold M_{D-n} whose eigenmodes around a stable ground state correspond to fields on M_n.

$$\text{Mass} \sim \frac{1}{\text{Size } (M_{D-n})}$$

Dimensional Reduction



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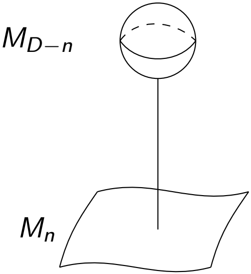

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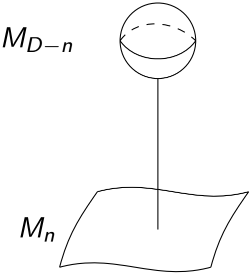

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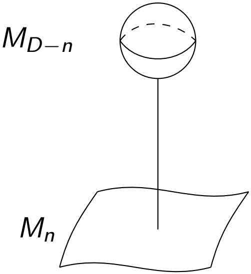

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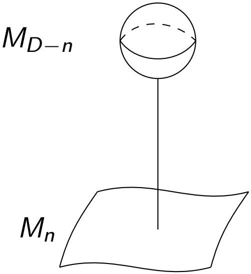

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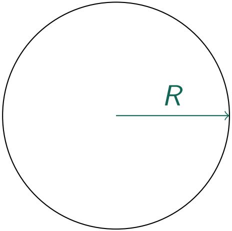
$\Rightarrow$ Study moduli!

Moduli - Circle



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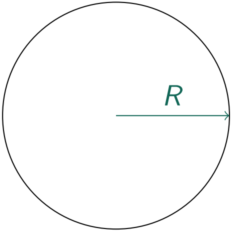
Moduli - Circle



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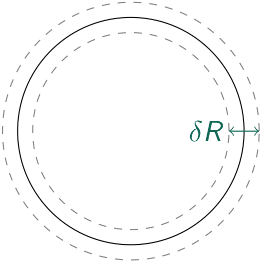
Therefore, only the deformations δR are possible.



Moduli - Torus

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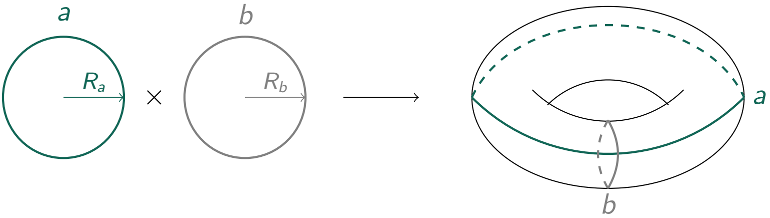

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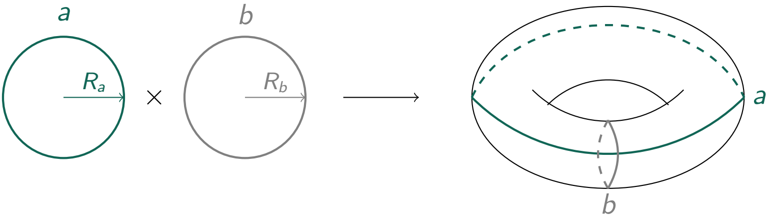

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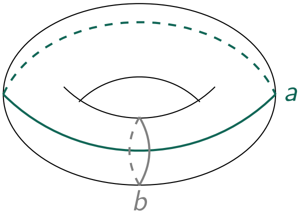
$\Rightarrow$ Two independent eigenmodes to excite!

Moduli - Torus



We choose a special basis for the moduli:

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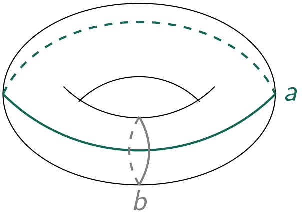

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Volume: $\mathcal{V} = R_a \cdot R_b$

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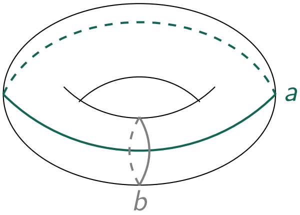


We choose a special basis for the moduli:

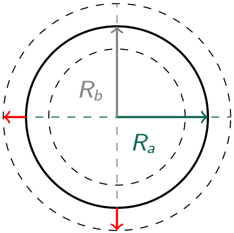
$$\text{Volume:} \quad \mathcal{V} = R_a \cdot R_b$$

and

$$\text{Ratio:} \quad \mathcal{R} = \frac{R_a}{R_b}$$

Moduli - Torus

We have two basic eigenmodes:



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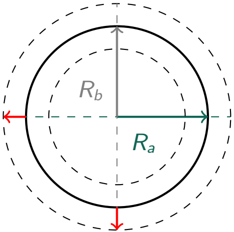
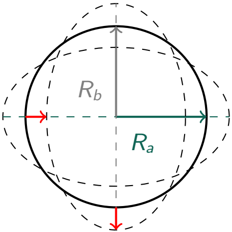


constant ratio: $\mathcal{R} = \frac{R_a}{R_b} = \text{const.}$

(equal phases)

constant volume: $\mathcal{V} = R_a \cdot R_b = \text{const.}$

(opposite phases)



Mirror Symmetry

Consider now the torus $\hat{T}^2$ defined by

$$\hat{R}_b = \frac{1}{R_b}, \quad \hat{R}_a = R_a \quad \Rightarrow \quad \hat{\mathcal{V}} = \hat{R}_a \hat{R}_b = \frac{R_a}{R_b} = \mathcal{R}!$$

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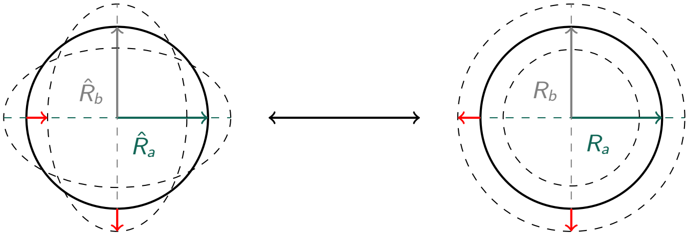
Therefore, we have the correspondence of eigenmodes:



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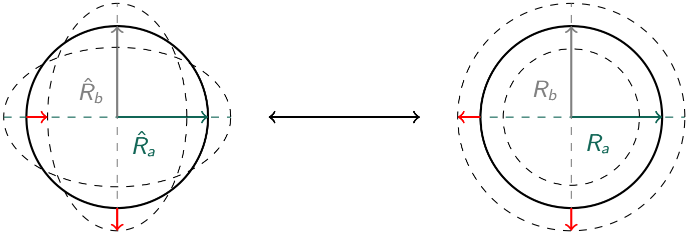

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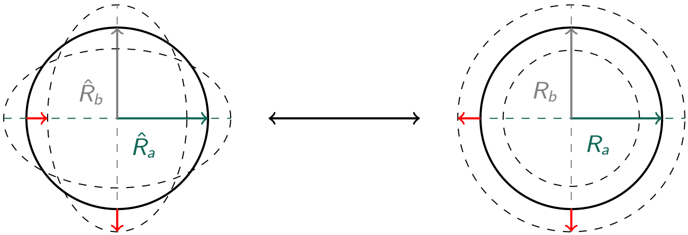

$\hat{\mathcal{V}}$ =const. $\qquad$ $\mathcal{R}$ =const.

Due to this correspondence, on both geometries (stable ground states) we have the same content of massless eigenmodes.

$\Rightarrow$ We have the same **physics** on both configurations!

Mirror Symmetry

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With the same argument, **quantum** effects can be mapped to **classical** effects and are treatable this way.

Thank you for your attention!