Seminar on energy-energy correlations

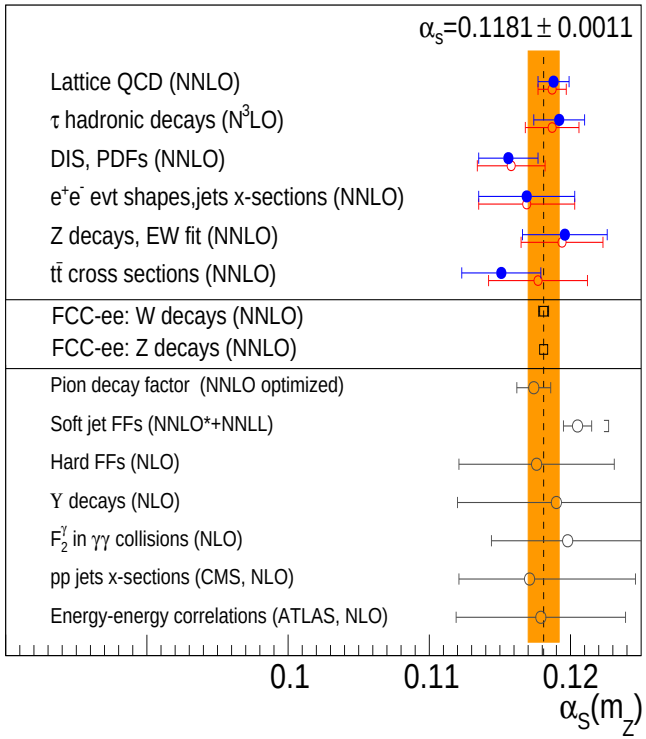
Determination of $\alpha_s(M_Z)$ from energy-energy correlations^{I} and jet rates^{II} in electron positron annihilation with combination of $\mathcal{O}(\alpha_s^3)$ perturbative calculations and nonperturbative corrections from Monte Carlo models

I: Adam Kardos[2], Stefan Kluth[1], Gábor Somogyi[2], Zoltán Tulipánt[2] and Andrii Verbytskyi[1]

II: Andrea Banfi[3], Adam Kardos[2], Pier Francesco Monni[4], Stefan Kluth[1], Gábor Somogyi[2], Zoltán Szőr[5], Zoltán Trocsányi[6], Zoltán Tulipánt[2], Andrii Verbytskyi[1] and Giulia Zanderighi[1]

[1] Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
[2] University of Debrecen
[3] University of Sussex
[4] CERN
[5] Johannes Gutenberg Universität Mainz
[6] Budapestinensis de Rolando Eötvös Nominata

α_s e^+e^-: motivation in the past and in the future



Source: arXiv:1806.06156v1

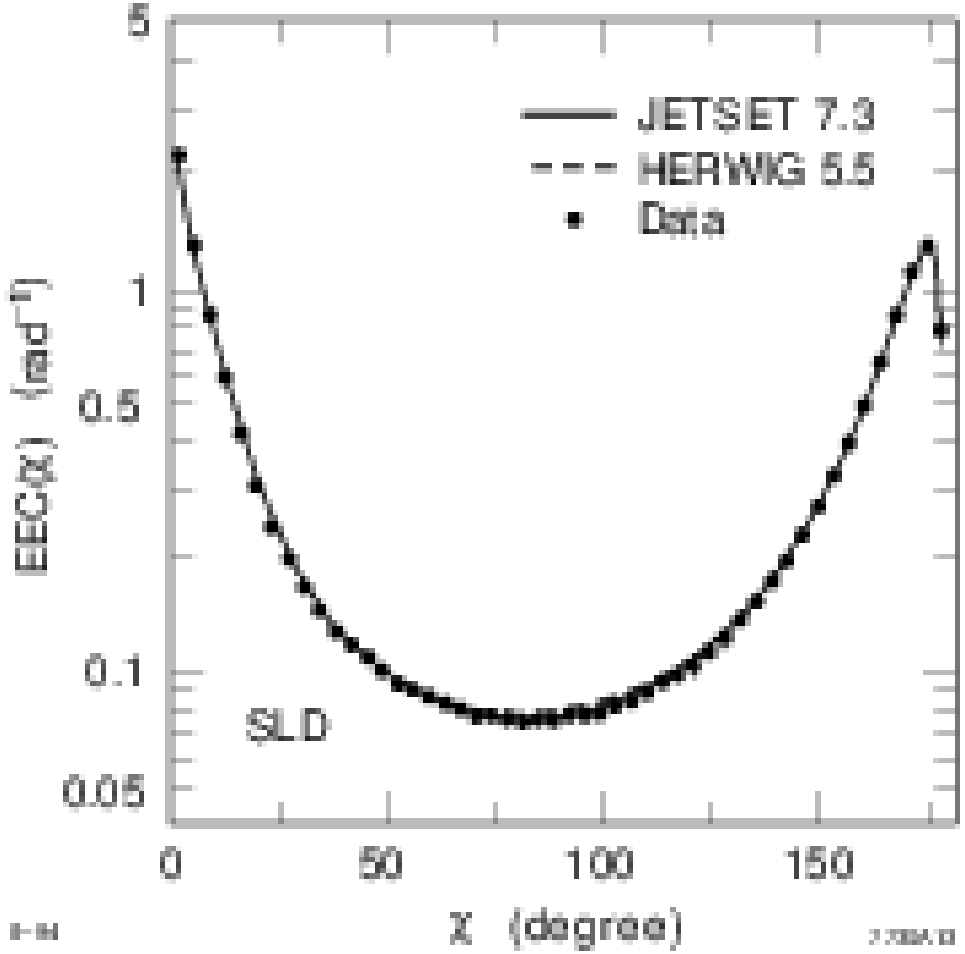
- As of 2019 α_S is known with precision of 1% if calculated from measurements with at least NNLO precision
- However, there is a large spread between measurements
- More measurements is better

+ measurements with new approached/data are important on themselves

PART I: Precise determination of $\alpha_S(M_Z)$ from a global fit of energy-energy correlation to NNLO+NNLL predictions, arXiv:1804.09146, Eur. Phys. J. C **78** (2018) no.6, 498

The energy-energy correlations

$\frac{\mathbf{dEEC}(\chi)}{\mathbf{d}\chi} = \sum_{\mathbf{i}}^{\mathbf{N}} \sum_{\mathbf{j}}^{\mathbf{N}} \frac{\mathbf{E_i E_i}}{\mathbf{E^2_{vis}}} \delta(\cos\chi - \cos\chi_{\mathbf{ij}})$, with $E_{vis} = \sum_i^N E_i$, where E_i is particle energy and χ_{ij} is angle between particles i and j.



- Used multiple times in a **distant** past for α_S extraction
- Inclusive
- Not sensitive to schemes of combinations
- Resummed NNLL predictions became available in **2017**

← **Looks like this[1]**

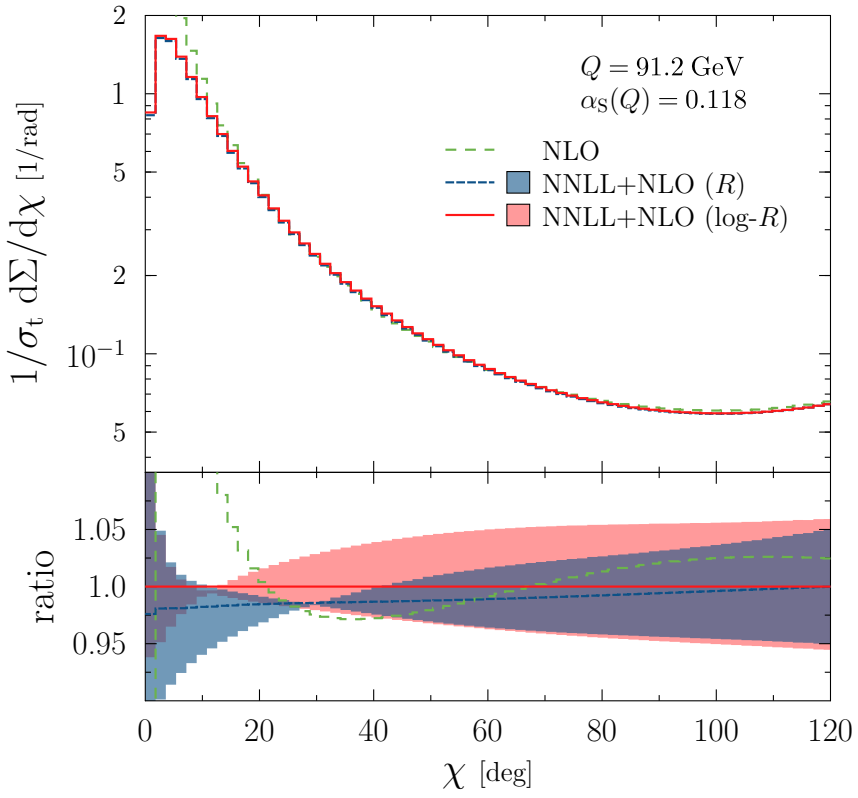
Note: $\chi = 0$ **– selfcorrelations,** $\chi = \pi$ **– correlations with opposite jet.**

Analysis components

- Perturbative and resummed predictions Z. Tulipánt, A. Kardos and G. Somogyi, "Energy-energy correlation in electron-positron annihilation at NNLL + NNLO accuracy," Eur. Phys. J. C **77** (2017) no.11, 749 + some b mass corrections
- Data: LEP, PEP, PETRA, SLC and TRISTAN
- Non-perturbative corrections: NLO MC by Sherpa and Herwig7, analytic hadronisation

Predictions: fixed order, matching, etc.

e^+e^- predictions in NNLO exist for some time, however



$+b$ mass corrections at NLO from ZBB4 program [2].

ColorFulNNLO, V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019 has unique features

- precision
- extendable approach

Resummation and matching have appeared recently:

Z. Tulipánt, A. Kardos and G. Somogyi, "Energy-energy correlation in electron–positron annihilation at NNLL + NNLO accuracy," Eur. Phys. J. C **77** (2017) no.11, 749

Available data

The available data covers wide range of energy: $\sqrt{s} = 14 - 91 GeV$.

Experiment	Data $\sqrt{s}$(average)	MC $\sqrt{s}$	Data events
SLD [1]	91.2(91.2)	91.2	60000
OPAL [3]	91.2(91.2)	91.2	336247
OPAL [4]	91.2(91.2)	91.2	128032
L3 [5]	91.2(91.2)	91.2	169700
DELPHI [6]	91.2(91.2)	91.2	120600
TOPAZ [7]	59.0 – 60.0(59.5)	59.5	540
TOPAZ [7]	52.0 – 55.0(53.3)	53.3	745
TASSO [8]	38.4 – 46.8(43.5)	43.5	6434
TASSO [8]	32.0 – 35.2(34.0)	34.0	52118
PLUTO [9]	34.6(34.6)	34.0	6964
JADE [10]	29.0 – 36.0(34.0)	34.0	12719
CELLO [11]	34.0(34.0)	34.0	2600
MARKII [12]	29.0(29.0)	29.0	5024
MARKII [12]	29.0(29.0)	29.0	13829
MAC [13]	29.0(29.0)	29.0	65000
TASSO [8]	21.0 – 23.0(22.0)	22.0	1913
JADE [10]	22.0(22.0)	22.0	1399
CELLO [11]	22.0(22.0)	22.0	2000
TASSO [8]	12.4 – 14.4(14.0)	14.0	2704
JADE [10]	14.0(14.0)	14.0	2112

Data qualification criteria

- Corrected to charged and neutral final state
- Corrected for ISR
- Full χ range measured
- No overlap with other samples
- Sufficient precision
- Sufficient information on data available

Huge datasets available for combined analysis:
20 datasets from 11 collaborations.

Non-perturbative models

Two approaches are available on the market: analytic and MC based. **We use both.**

Analytic approach

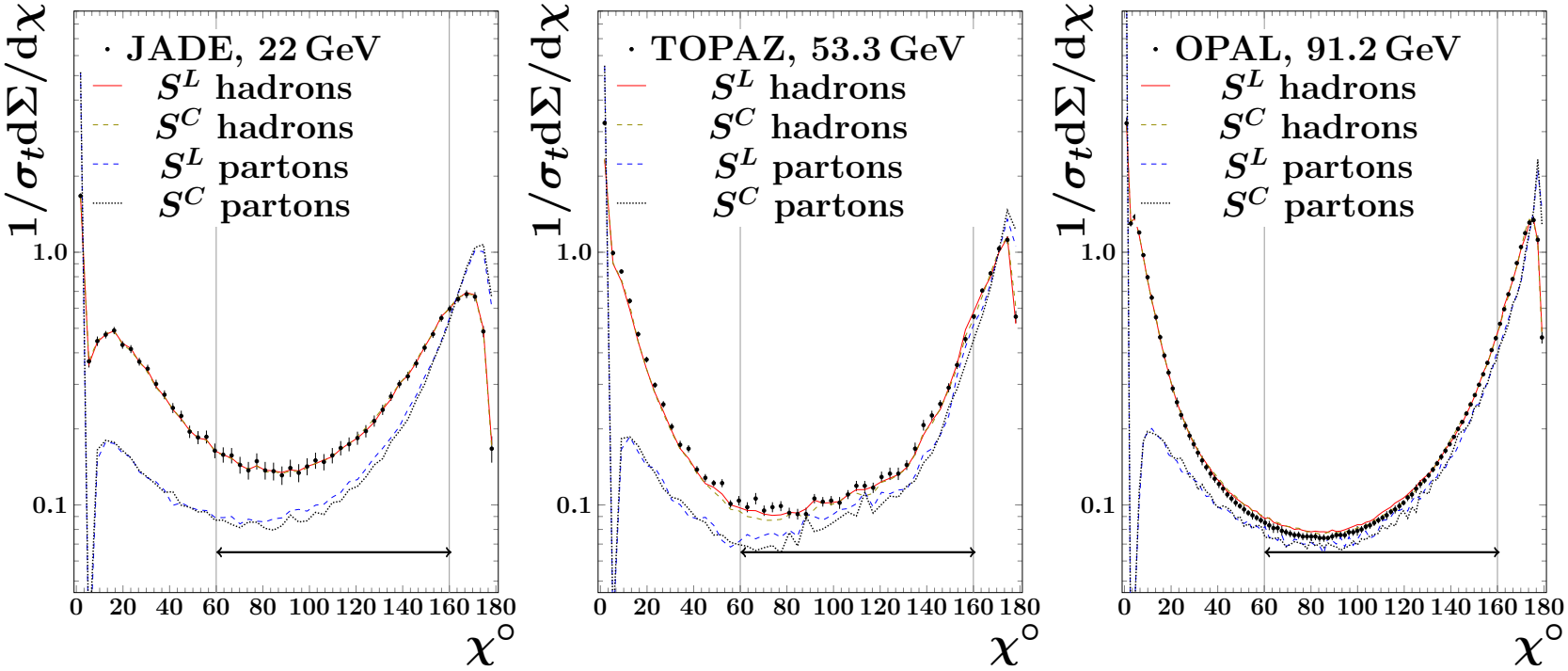
- Calculations with (DMW)

  Y. L. Dokshitzer, G. Marchesini and B. R. Webber, "Nonperturbative effects in the energy energy correlation," JHEP **9907** (1999) 012
- Involves α_S moments at low scales, which are free parameters.
- Note: EEC has a special composition of moments.

MC-based (see backups)

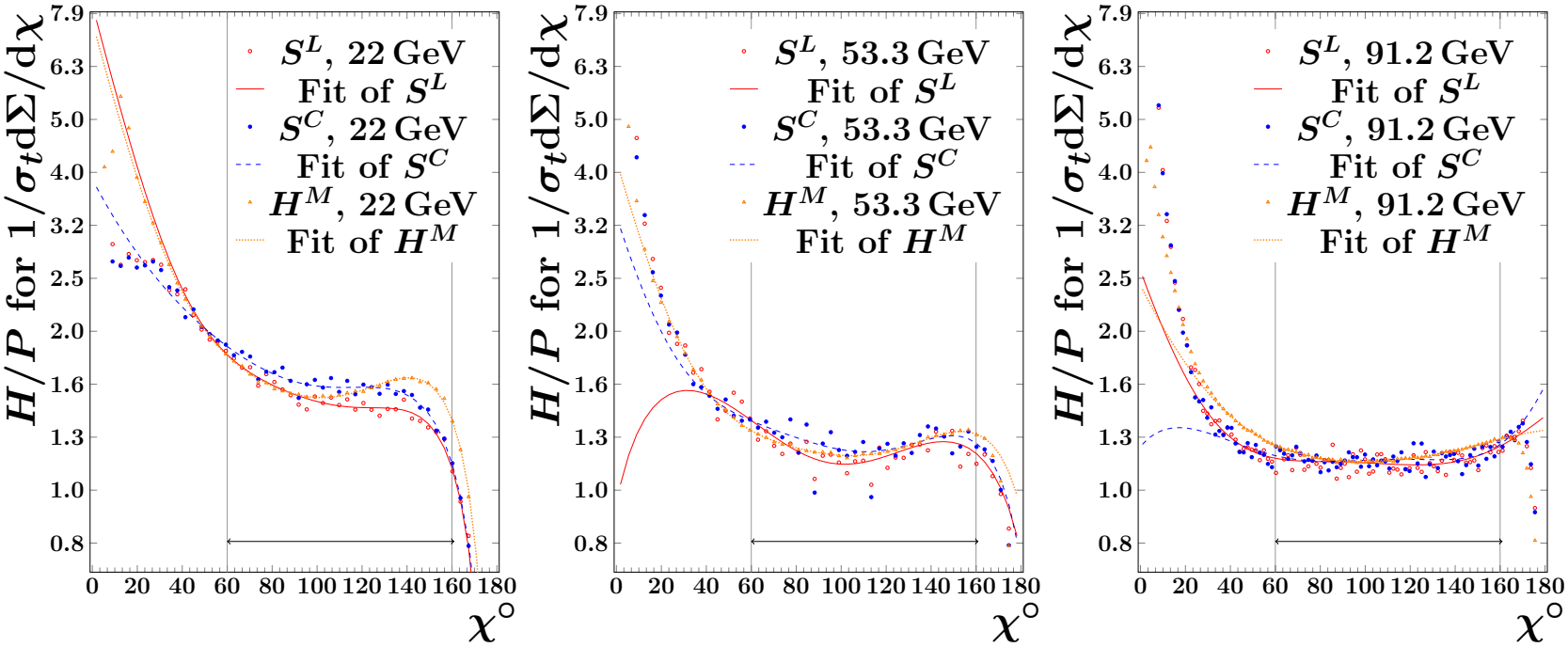
- NLO MC events by particle level generators to extract with point-by-point multiplicative correction factors
- Systematics from multiple hadronisation models
- Simultaneously allows to extract missing correlations of data points

MC based approach: MC distributions



- Good description of data even close to cutoff region.
- More: the samples were reweighted event-by-event to match closely the data: $\log W_{event} = \sum_{bin=1}^{Nbins} k_{bin} EEC_{event}(bin)$.

MC based approach: MC hadronisation



- Hadronisation corrections are ratio of hadron to parton level.
- To avoid binning effects the hadronisation corrections are parametrised with smooth functions. Note: parametrisation is valid only in fit range.

Fits: technique

Fits are performed with MINUIT2 [14] minimising

$$\chi^2 = \sum_{\text{data sets}} \chi^2(\alpha)_{\text{data set}},$$

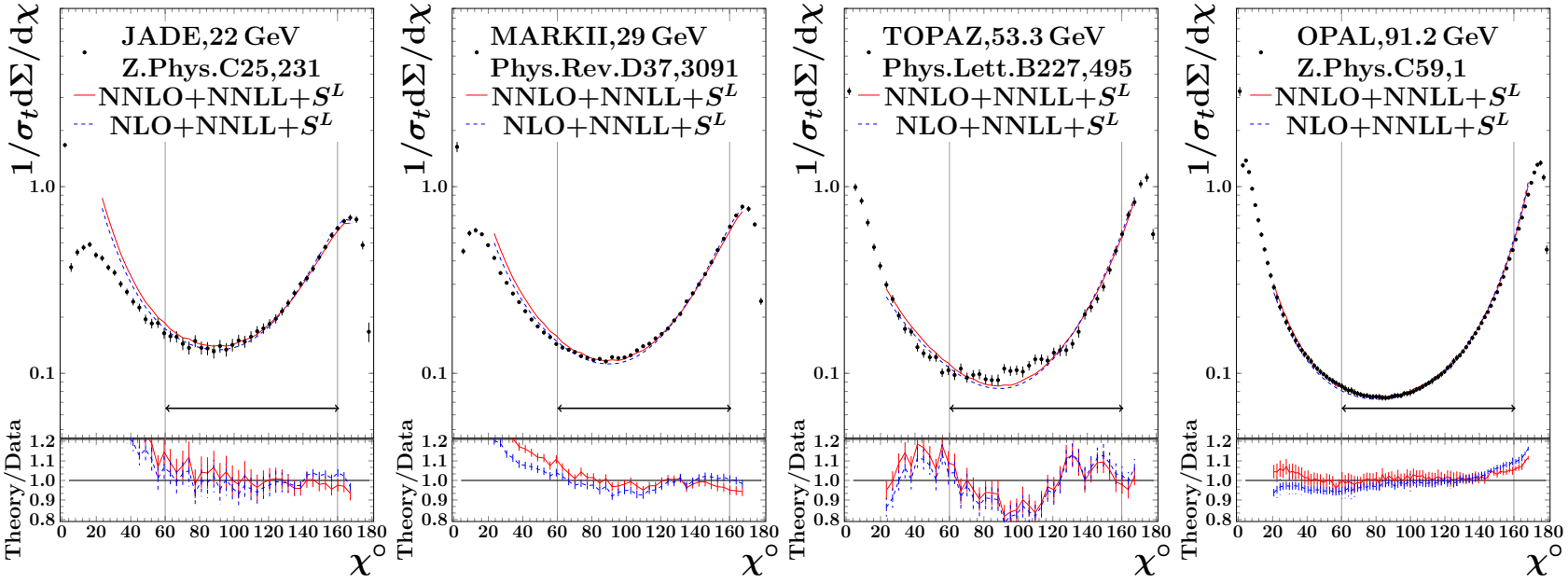
where $\chi^2(\alpha)$ was calculated for each data set as

$$\chi^2(\alpha) = \vec{r}\, V^{-1}\, \vec{r}^T, \qquad \vec{r} \equiv (\vec{D} - \vec{P}(\alpha)), \tag{1}$$

- $\vec{D}$ vector of data points
- $\vec{P}(\alpha)$ vector of fixed order (or resummed) predictions corrected for non-perturbative effects
- V is the covariance matrix for $\vec{D}$

Result for NNLO+NNLL:
$\alpha_S(M_Z) = 0.11750 \pm 0.00018$, $\chi^2/n.d.f = 1022/623 = 1.64$

Fits: distributions

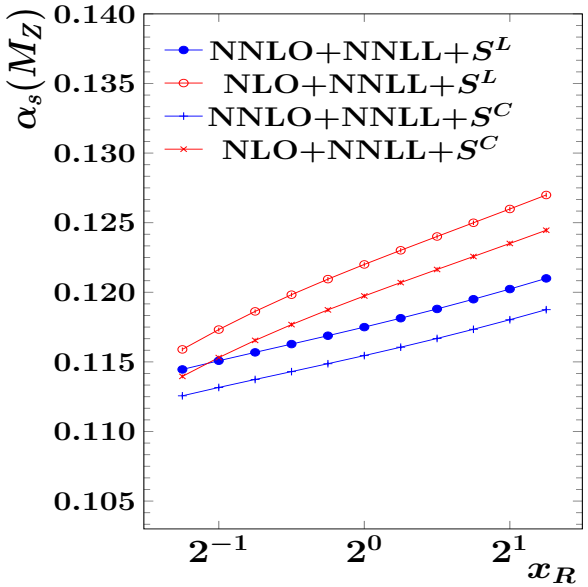


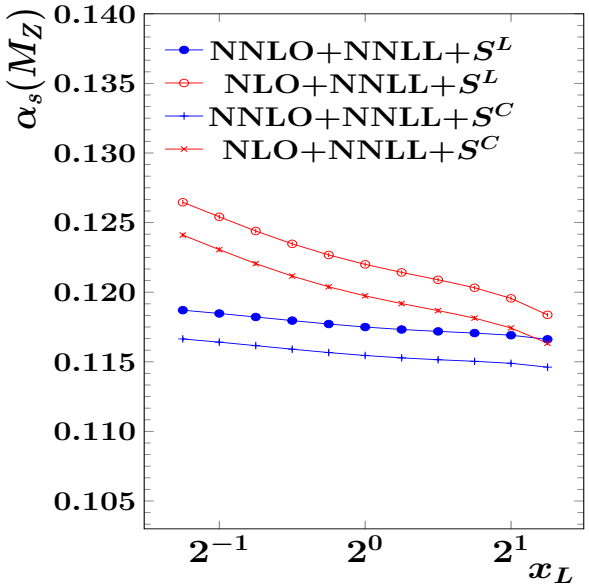
- The fits are done in different ranges.
- Criteria for central result: validity of NNLO, hadronisation corrections and resummation.
- Results are insensitive to $\pm 5^\circ$ changes of fit ranges.

Ranges:

- $117 - 177^\circ$
- $117 - 165^\circ$
- $60 - 165^\circ$
- $60 - 160^\circ$ (central)

Systematics and uncertainties





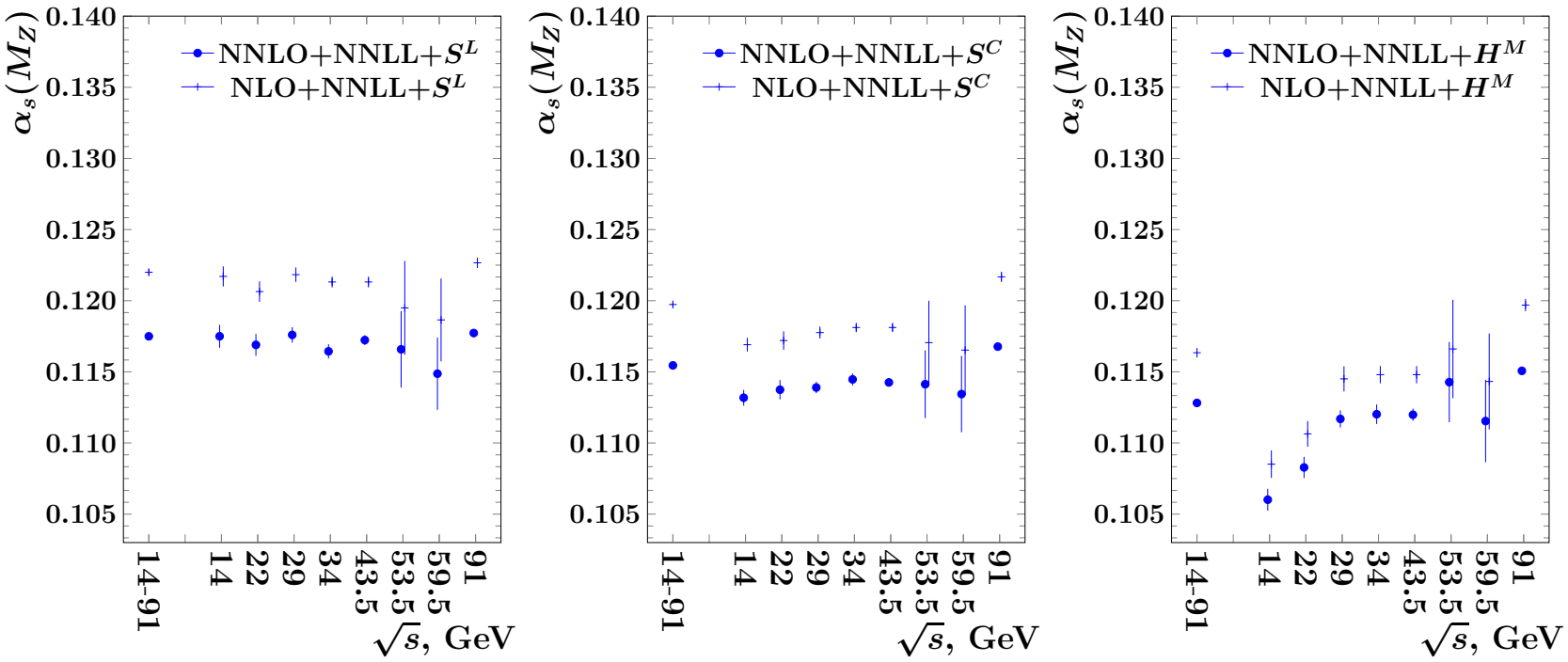
The uncertainties that were estimated:

- Variation of renormalisation scale by $2^{\pm 1}$: (*res.*)
- Variation of resummation scale by $2^{\pm 2}$: (*ren.*)
- Variation of matching power 1 or 2: neglected
- Variation of hadronisation model S^L or S^C: (*hadr.*)
- Fit uncertainty is $\chi^2 + 1$ criterion from MINUIT: (*exp.*)

(Some)Checks

- Analytic hadronisation
- Fit range variation (see backups)
- Power in resummation expressions

- Herwig7 for hadronisation
- Stability across $\sqrt{s}$ (see below)
- Scheme of b mass treatment
- NLO only fits



Note: a lot of work to remove the dependence on $\sqrt{s}$ (artefact of MC).

Conclusions for part I

Extraction of $\alpha_S(M_Z)$ from energy-energy correlations in e^+e^- collisions has been performed with NNLO+NNLL precision **for the first time** using datasets in wide range of centre-of-mass energies. The results are

$\alpha_S(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$

for NLO+NNLL(logR) scheme and

$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$

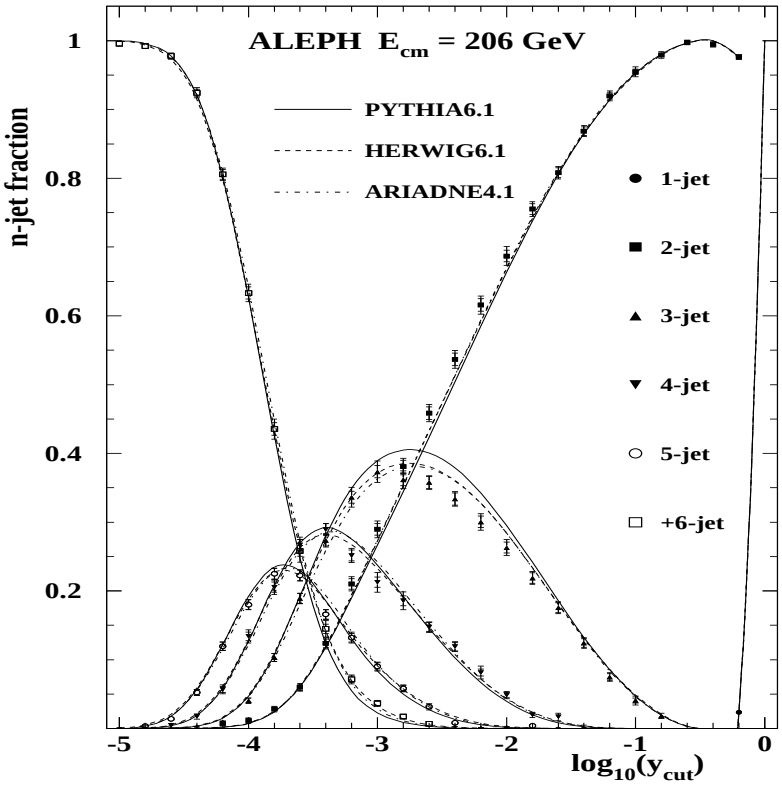
for NNLO+NNLL(logR) scheme.

Note: with better resummation (which almost exists) the renormalization scale dependence will be reduced as well.

PART II: High precision determination of $\alpha_S(M_Z)$ from a global fit of jet rates, arXiv:1902.08158, Submitted to JHEP

Durham jet rates

Durham jet algorithm is a sequential jet algorithm with distance. $d_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$, where E_i is particle energy and θ_{ij} is angle between particles i and j. Jet rates R_n – fraction of n-jet events for $y = d_{min}/E_{vis}^2$.



- R_3 was used multiple times in past for α_S extraction
- R_2 and R_3 can be naturally combined for the first time
- Resumed NNLL predictions became available in for R_2 in **2016**

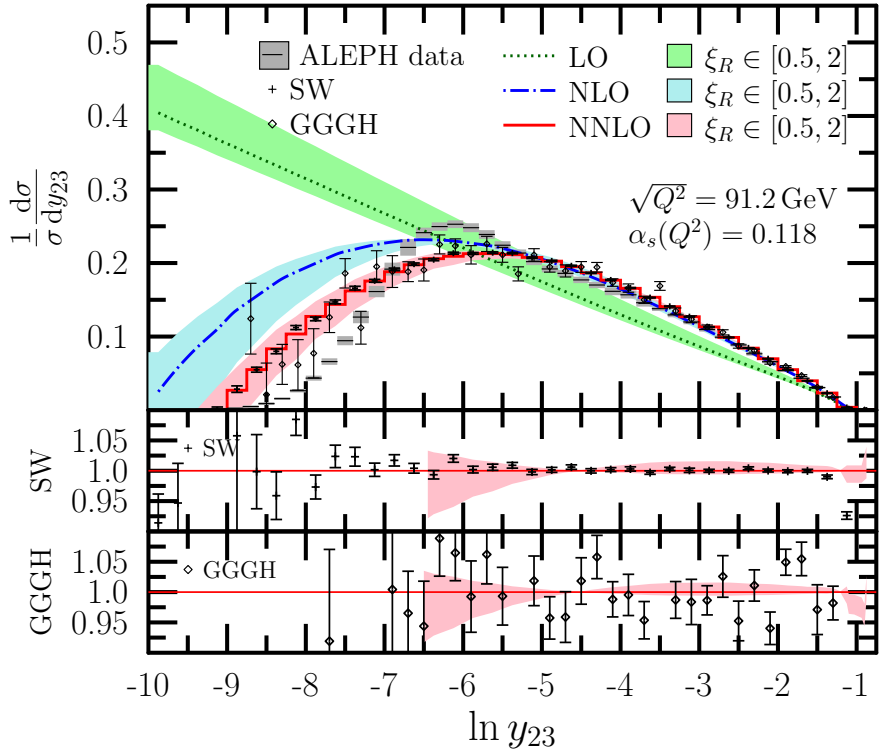
← **Looks like this[15]**

Analysis components

- Perturbative predictions V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019 $+$ some b mass corrections
- Resummation A. Banfi et al., "The two-jet rate in e^+e^- at next-to-next-to-leading-logarithmic order," Phys. Rev. Lett. **117** (2016) no.17, 172001; (S. Catani et al., "New clustering algorithm for multi - jet cross-sections in e^+e^- annihilation," Phys. Lett. B **269** (1991) 432 for tests)
- Data: LEP and PETRA **(YES!)**. New OPAL measurements used to build correlation model for older measurements.
- Non-perturbative corrections: NLO MC by Sherpa and Herwig7 with Lund and cluster hadronisation models, i.e. S^L, S^C, H^L, H^C.

Predictions: fixed order, matching, etc.

e^+e^- predictions in NNLO exist for some time, however



$+b$-mass corrections

Main focus on α_S^3+NNLL for R_2

ColorFulNNLO, V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019:

- precision
- extendable approach

NLL resummation/matching is well known for R_3, for NNLL R_2 have appeared recently: A. Banfi et al., "The two-jet rate in e^+e^- at next-to-next-to-leading-logarithmic order," Phys. Rev. Lett. **117** (2016) no.17, 172001

Available data

The data covers wide range of energy: $\sqrt{s} = 35 - 207 GeV$.

Experiment	Data $\sqrt{s}$(average)	MC $\sqrt{s}$	Data events
OPAL [16]	91.2(91.2)	91.2	1508031
OPAL [16]	189.0(189.0)	189	3300
OPAL [16]	183.0(183.0)	183	1082
OPAL [16]	172.0(172.0)	172	224
OPAL [16]	161.0(161.0)	161	281
OPAL [16]	130.0 – 136.0(133.0)	133	630
L3 [17]	201.5 – 209.1(206.2)	206	4146
L3 [17]	199.2 – 203.8(200.2)	200	2456
L3 [17]	191.4 – 196.0(194.4)	194	2403
L3 [17]	188.4 – 189.9(188.6)	189	4479
L3 [17]	180.8 – 184.2(182.8)	183	1500
L3 [17]	161.2 – 164.7(161.3)	161	424
L3 [17]	135.9 – 140.1(136.1)	136	414
L3 [17]	129.9 – 130.4(130.1)	130	556
JADE [16]	43.4 – 44.3(43.7)	44	4110
JADE [16]	34.5 – 35.5(34.9)	35	29514
ALEPH [15]	91.2(91.2)	91.2	3600000
ALEPH [15]	206.0(206.0)	206	3578
ALEPH [15]	189.0(189.0)	189	3578
ALEPH [15]	183.0(183.0)	183	1319
ALEPH [15]	172.0(172.0)	172	257
ALEPH [15]	161.0(161.0)	161	319
ALEPH [15]	133.0(133.0)	133	806

Data qualification criteria

- Corrected to charged and neutral final state
- Corrected for ISR
- No overlap with other samples
- Sufficient precision
- Sufficient information on data available

Huge datasets available for combined analysis:
20+ datasets from 4 collaborations.

Non-perturbative corrections: MC based

Challenge: simultaneous correction of R_2 and R_3.

Introduce ξ_1, ξ_2, so

$$R_{2,parton} = \cos^2 \xi_1,$$

$$R_{3,parton} = \sin^2 \xi_1 \cos^2 \xi_2$$

and

$$R_{2,hadron} = \cos^2(\xi_1 + \delta\xi_1),$$

$$R_{3,hadron} = \sin^2(\xi_1+\delta\xi_1)\cos^2(\xi_2+\delta\xi_2).$$

Setup: $e^+e^- \to jjjjj$ merged samples with massive b

Differences to EEC:

- OpenLoops [18] instead of GoSam as OLP
- SHERPA2.2.6
- Herwig7.1.4, also 3-jet FS in NLO.
- No reweighting
- Herwig7+Lund (H^L) is taken for central result

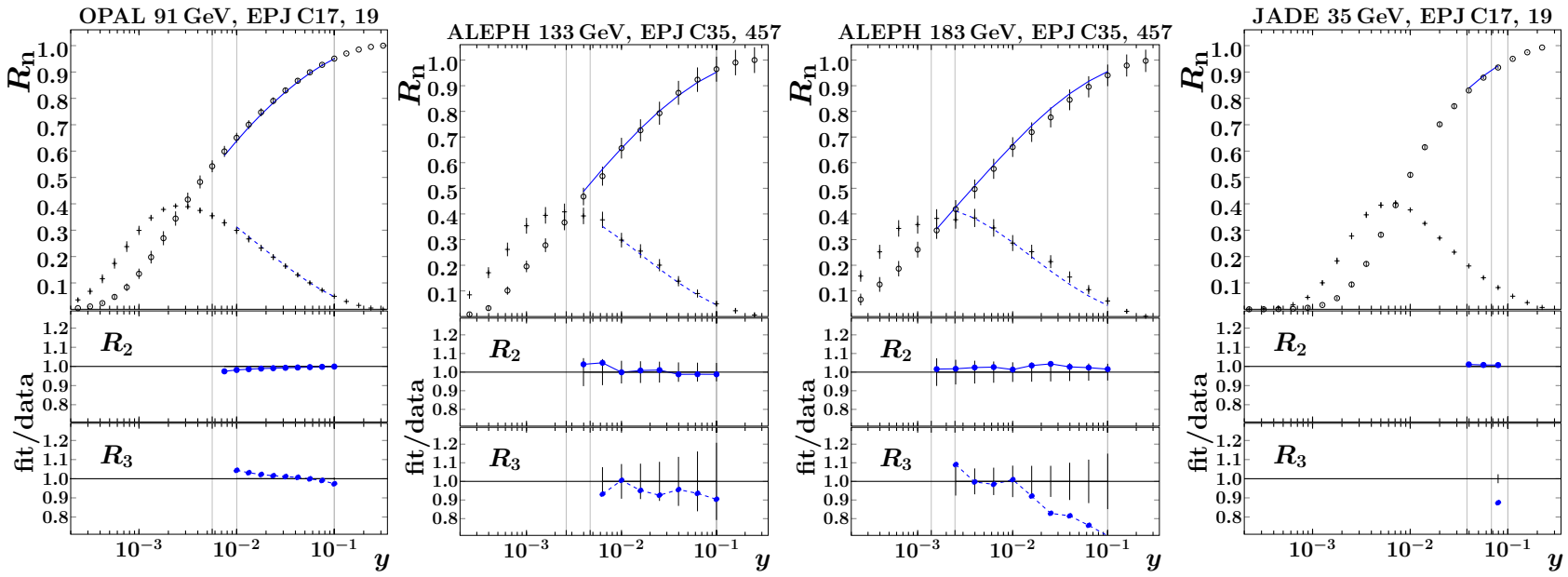
Approach preserves normalisation. $\delta\xi_1(y)$ and $\delta\xi_2(y)$ are corrections to be extracted, see backups.

Fits: technique

Fits are performed with MINUIT2 [14] in same way as for EEC. In addition some checks were performed with modified χ^2 definition.

Result for R_2 fits at N^{3}LO+NNLL:
$\alpha_S(M_Z) = 0.11881 \pm 0.00063$, $\chi^2/n.d.f = 39/150 = 0.26$

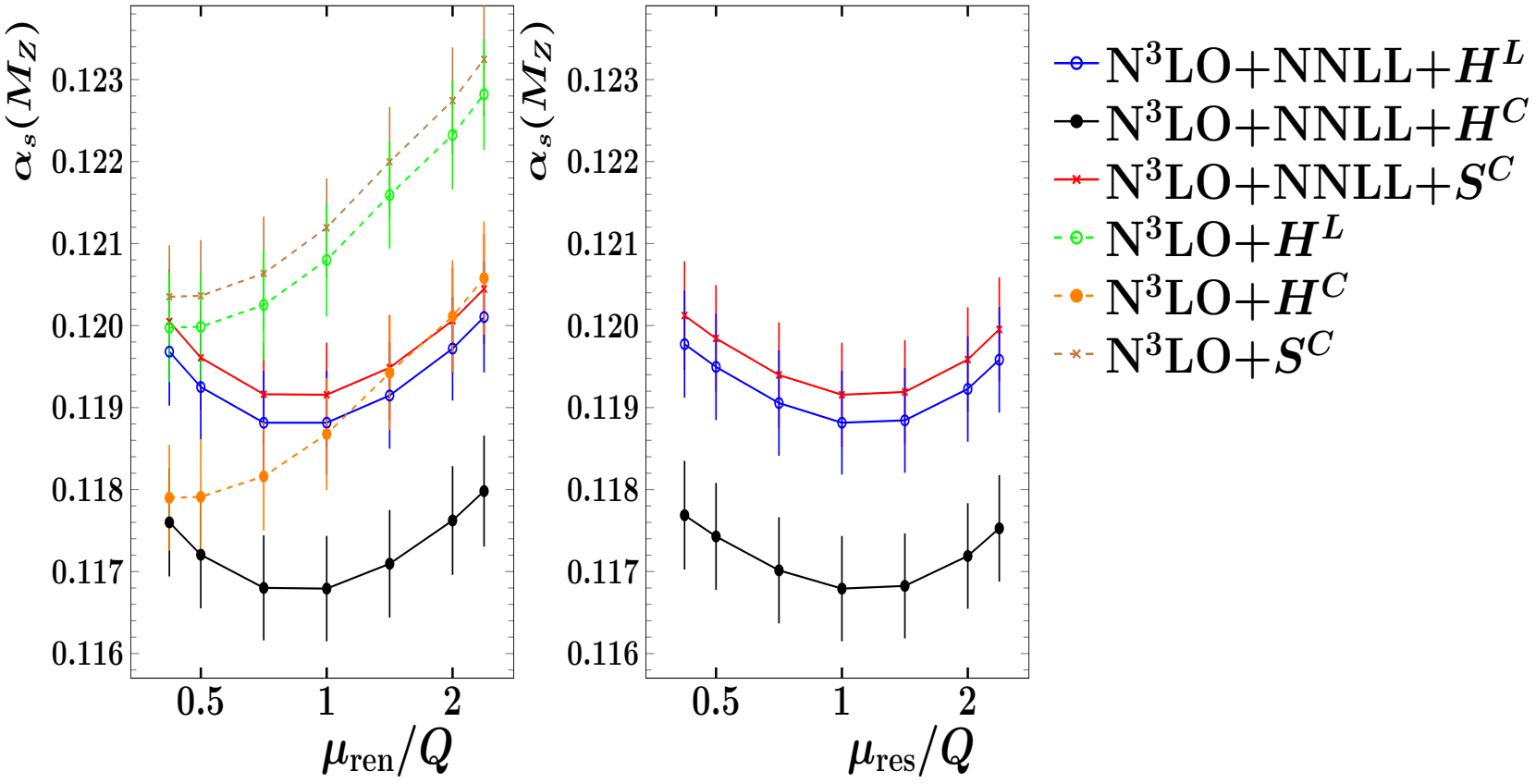
Fits: distributions



Central result and fit range selection

- Validity of N^{3}LO and resummation.
- Validity of reference hadronisation model H^L(Herwig7 with Lund)
- Smallest $\chi^2/ndof$, low sensitivity to fit range.
- Q^2 dependent fit range $[-2.25 + \mathcal{L}, -1]$, $\mathcal{L} = \log \frac{M_Z^2}{Q^2}$
- Separate ranges for R_2 and R_3 (if used).

Systematics and uncertainties

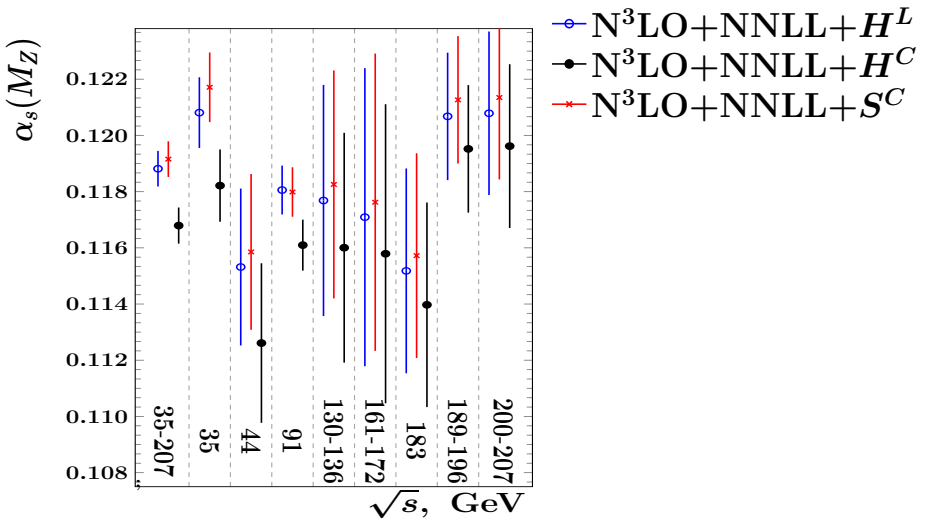


The uncertainties that were estimated:

- Variation of renormalisation scale by $2^{\pm 1}$: (*res.*)
- Variation of resummation scale by $2^{\pm 2}$: (*ren.*)
- Variation of hadronisation model H^C instead of H^L: (*hadr.*)
- Fit uncertainty is $\chi^2 + 1$ criterion from MINUIT: (*exp.*)

(Some) Checks

- Simultaneiuos R_2+R_3 fit (see below)
- Separate R_3 fit
- Variation of χ^2 definition
- Changes of fit ranges
- Multiplicative hadronisation corrections
- Sherpa MC hadronisation S^C
- Stability across $\sqrt{s}$ (see below)
- Exclusion of data $\sqrt{s} < M_Z$



Simultaneiuos R_2+R_3 fit is not more precise, but much more unstable with fit variations, see backups. The result of such fit is:

$\alpha_S(M_Z) = 0.11989 \pm 0.00045(exp.) \pm 0.00098(hadr.) \pm 0.00046(ren.) \pm 0.00017(res.)$

Conclusions for part II

Extraction of $\alpha_S(M_Z)$ from jet rates in e^+e^- collisions has been performed with N^{3}LO+NNLL precision for the first time from R_2 in wide range of centre-of-mass energies.
The obtained value is

$$\alpha_S(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$$

for α_S^3+NNLL(R_2) scheme.

Conclusions and outlook

- The following values of $\alpha_S(M_Z)$ were obtained in analyses:
  - $\alpha_S(M_Z) = 0.11750 \pm 0.00018(\text{exp.}) \pm 0.00102(\text{hadr.}) \pm 0.00257(\text{ren.}) \pm 0.00078(\text{res.})$
  - $\alpha_S(M_Z) = 0.11881 \pm 0.00063(\text{exp.}) \pm 0.00101(\text{hadr.}) \pm 0.00045(\text{ren.}) \pm 0.00034(\text{res.})$
- The presented results are precise, most precise in their subclass.
- The presented results are in good agreement.
- Hadronisation uncertainty would dominate combined result.

- **Combined analyses can bring precise results, but data quality already sets some limits.**
- **Low energy ($\sqrt{s} < M_Z$) data is relevant.**
- **Understanding of non-perturbative effects starts to be more important than precise perturbative calculations. Can be adressed with better MC and/or better observables (see talk by G. Somogyi).**

Final

- The following values of $\alpha_S(M_Z)$ were obtained in analyses:
  - $\alpha_S(M_Z) = 0.11750 \pm 0.00287(\text{comb.})$
  - $\alpha_S(M_Z) = 0.11881 \pm 0.00131(\text{comb.})$

Determination	Data and procedure	Reference
0.1175 ± 0.0025	ALEPH 3-jet rate (NNLO+MChad)	[19]
0.1199 ± 0.0059	JADE 3-jet rate (NNLO+NLL+MChad)	[20]
0.1224 ± 0.0039	ALEPH event shapes (NNLO+NLL+MChad)	[21]
0.1172 ± 0.0051	JADE event shapes (NNLO+NLL+MChad)	[22]
0.1189 ± 0.0041	OPAL event shapes (NNLO+NLL+MChad)	[23]
$0.1164\ ^{+0.0028}_{-0.0026}$	Thrust (NNLO+NLL+anlhad)	[24]
$0.1134\ ^{+0.0031}_{-0.0025}$	Thrust (NNLO+NNLL+anlhad)	[25]
0.1135 ± 0.0011	Thrust (SCET NNLO+N^{3}LL+anlhad)	[26]
0.1123 ± 0.0015	C-parameter (SCET NNLO+N^{3}LL+anlhad)	[27]

Table: Determinations of the strong coupling from jet rates and event shapes in e^+e^- collisions. Source: arXiv:1712.05165v2

Problems: Data

- The old data has no correlations. Some correlation models should be invented. Alternative is to discard most data, see Ref. [19].
- Systematic uncertainties are not split into categories. Virtually impossible to provide single dataset for QCD observables combined out of data of multiple experiments. Would be beneficial, see HERA data combination in Ref. [28].
- 20-50% measurements from LEP/PETRA/SLC are available as **figures** only
- Several observables were measured only for certain energies, e.g. no EEC measurements for $\sqrt{s} > M_Z$
- Low energy data ($\sqrt{s} < M_Z$) is even more important than high-energy data ($\sqrt{s} > M_Z$)

$$\Downarrow$$

- Re-analysis could be an option

Problems: MC

- LHC-era generators and are not tuned well to describe any other data good. LHC data cannot constrain the hadronisation at lower energies. **Poor description of e^+e^- data out of the box**.
- Complexity of codes hides a lot of bugs, e.g. negative energies in `SHERPA` showers. These problems are ignored at LHC, so fixes are done either by authors or interested individuals. Requres a lot of work.
- Complexity of codes hides a lot of technical aspects, so it is virtually impossible to reproduce setp of one generator with another.
- Taking hadronisation into account with multiplication of theory predictions by a coefficient is not sustainable.
- Hadronisation model bias estimation is a two-point systematics – the wors possible option

$$\Downarrow$$

- A lot of work is needed

Problems: QCD Theory

- Even $\mathcal{O}(\alpha_S^3)$ predictions should be supplied at least with some resummation to provide reliable results.
- Resummation of higher orders is still done on case-by-case basis

$$\Downarrow$$

- ARES and similar programs for more universal resummation?

Problems: QCD Theory+

- New techniques can help to bring theory closer to experiment and reduce model (MC) dependence E.g. grooming can be considered, lot of work is needed to get new **published** measurements.
- Not clear if it will work at all for e^+e^-. "There is nothing to groom " – *G. Salam*
- Significant difficulties with theory predictions, e.g. numerically demanding computations.
- Resummation is not available

$$\Downarrow$$

- Better approaches than grooming are needed

Analytic calculations: SYM Theory $\neq$ QCD

- However, what can be done in SYM and QCD?

$$\Downarrow$$

- Triple energy correlations – couple measurements were done, but no data are available (QCD). $\leftarrow$ not useful
- C parameter is directly related to EEC. Multiple measurements and predictions for C (QCD) are available.
  - For one event $C = \frac{3}{2}\int_{-1}^{1} EEC(\theta)\sin\theta^2 d(cos\theta)$
  - Analytic coefficients for moments can be derived, e.g. for QCD $\langle C\rangle^1 = \frac{\alpha}{2\pi}A + (\frac{\alpha}{2\pi})^2 B + \ldots$

$$A_{analytic,me} = -44 + 16\pi^2/3 = 8.63789015,\ B_{analytic,me} \approx 173.0675082000$$

$$A_{CoLoRFulNNLO,prel.} = 8.63759494921,\ B_{CoLoRFulNNLO,prel.} = 173.00969342$$

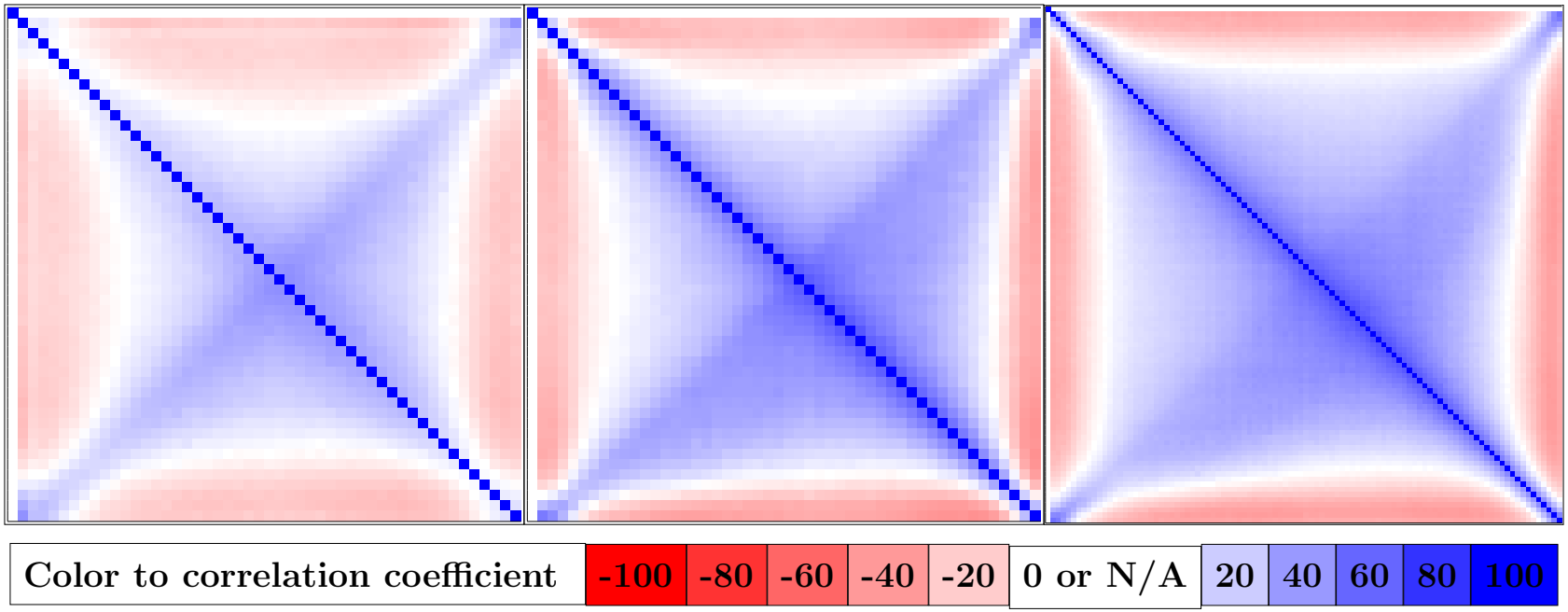
$$A_{EERAD3} = 8.6379,\ B_{EERAD3} = 172.778$$

$$A_{MERCUTIO2} = 8.63780,\ B_{MERCUTIO2} = 172.8$$

  - All moments should be easy to derive. And then ... fully differential distribution from moments?

Backup slides part I: MC based approach: correlations

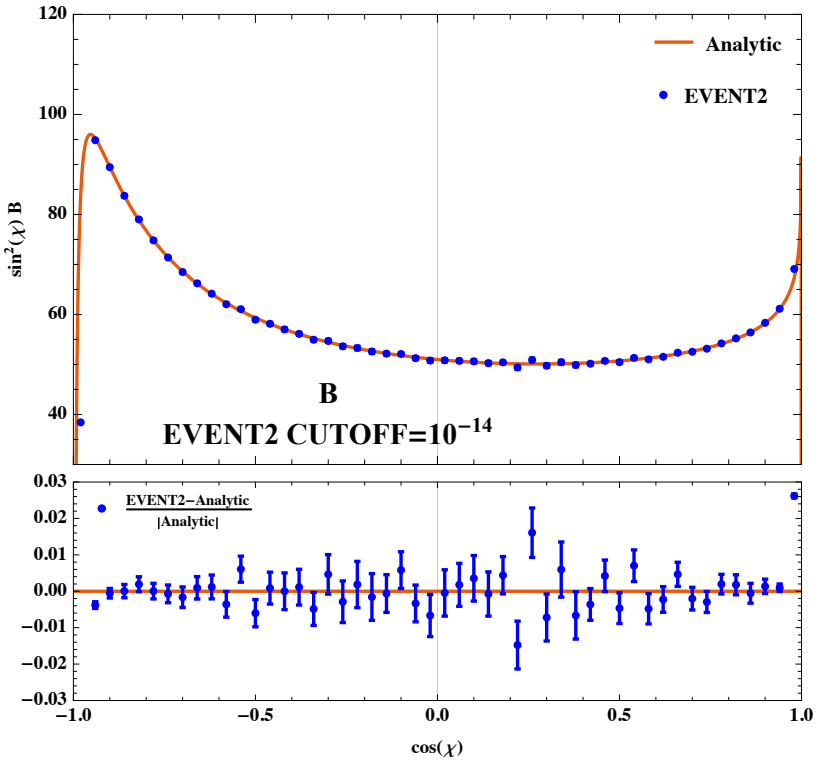
| JADE, $\sqrt{s} = 22 GeV$ | TOPAZ, $\sqrt{s} = 59 GeV$ | OPAL, $\sqrt{s} = 91 GeV$ |



- All measurements are provided without correlations.
- MC samples are used to model correlations between points, see original Fisher papers [29].
- Note the ridge coming from 3−jet events $\chi_x + \chi_y = \pi$

Backup slides part I: Theory work in progress?

- N^3LL resummation under study with recently SCET calculations [30].
- NLO analytic results available [31].

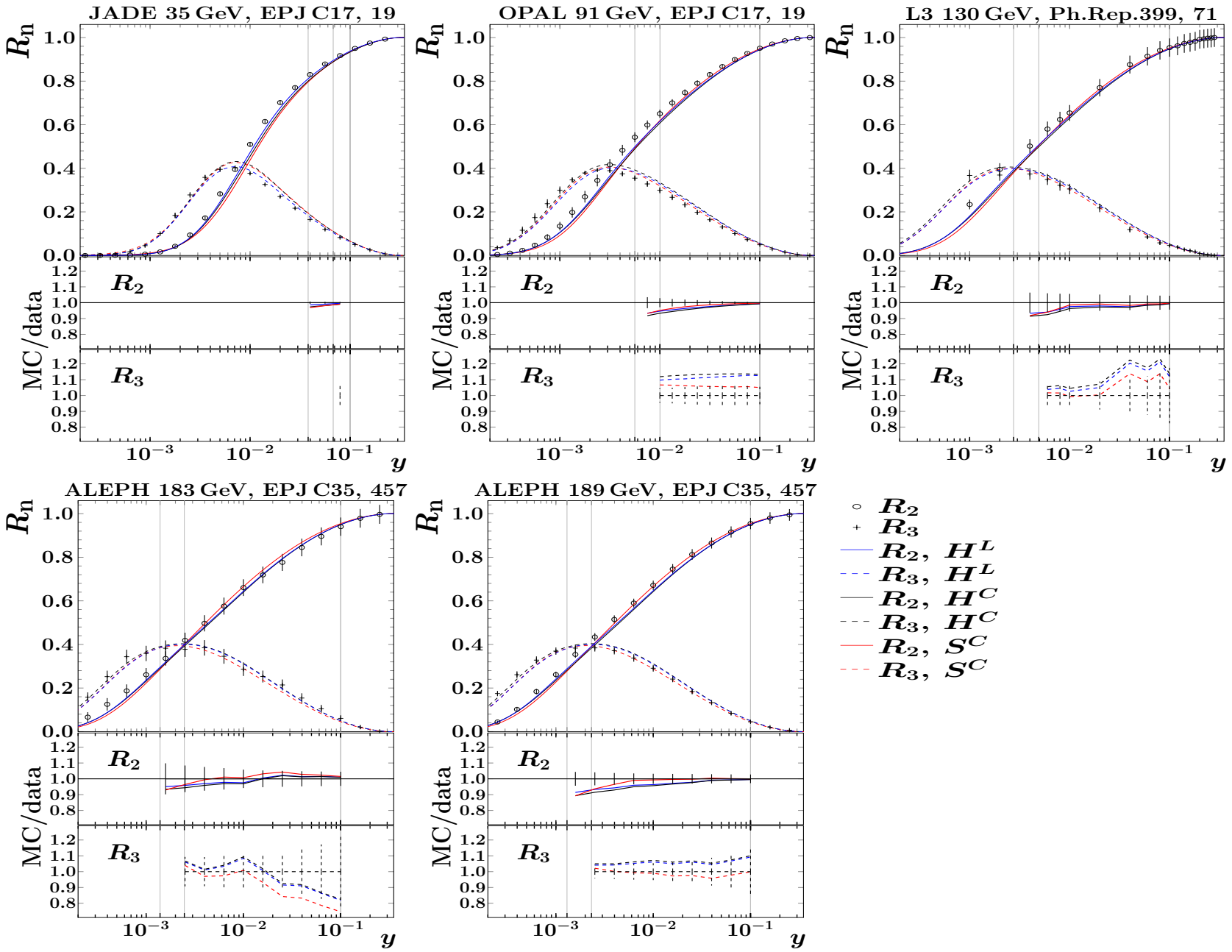


Backup slides part I: fits

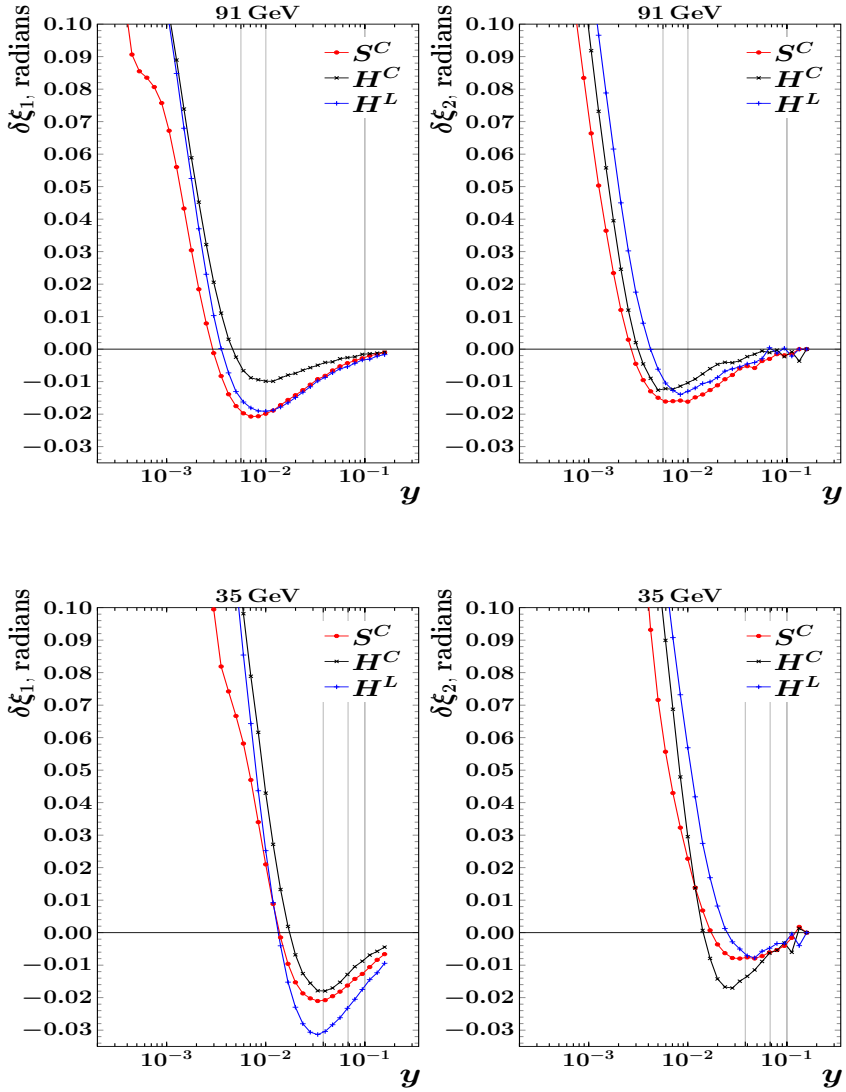
Fit range, $^\circ$ Hadronization	NLO+NNLL $\chi^2/ndof$	NNLO+NNLL $\chi^2/ndof$
$117-165^\circ$ S^L	0.12042 ± 0.00025 $765/298 = 2.57$	0.11760 ± 0.00020 $513/298 = 1.72$
$60-165^\circ$ S^L	0.12134 ± 0.00022 $1720/664 = 2.59$	0.11746 ± 0.00018 $1211/664 = 1.82$
$60-160^\circ$ S^L	0.12200 ± 0.00023 $1417/623 = 2.27$	0.11750 ± 0.00018 $1022/623 = 1.64$
$117-165^\circ$ S^C	0.11796 ± 0.00022 $631/298 = 2.12$	0.11521 ± 0.00017 $395/298 = 1.32$
$60-165^\circ$ S^C	0.11900 ± 0.00021 $1557/664 = 2.34$	0.11530 ± 0.00015 $951/664 = 1.43$
$60-160^\circ$ S^C	0.11973 ± 0.00022 $1321/623 = 2.12$	0.11545 ± 0.00016 $845/623 = 1.36$
$117-165^\circ$ H^M	0.11272 ± 0.00037 $1842/298 = 6.18$	0.11044 ± 0.00029 $1201/298 = 4.03$
$60-165^\circ$ H^M	0.11472 ± 0.00033 $3845/664 = 5.79$	0.11180 ± 0.00023 $2203/664 = 3.32$
$60-160^\circ$ H^M	0.11634 ± 0.00033 $3091/623 = 4.96$	0.11281 ± 0.00023 $1738/623 = 2.79$
$117-165^\circ$ $An.^{DMW}$	0.12154 ± 0.00045 $730/295 = 2.48$	0.11781 ± 0.00037 $558/295 = 1.89$
$60-165^\circ$ $An.^{DMW}$	0.13555 ± 0.00052 $7525/661 = 11.38$	0.12937 ± 0.00039 $4896/661 = 7.41$
$60-160^\circ$ $An.^{DMW}$	0.13606 ± 0.00061 $7364/620 = 11.88$	0.12950 ± 0.00044 $4827/620 = 7.78$

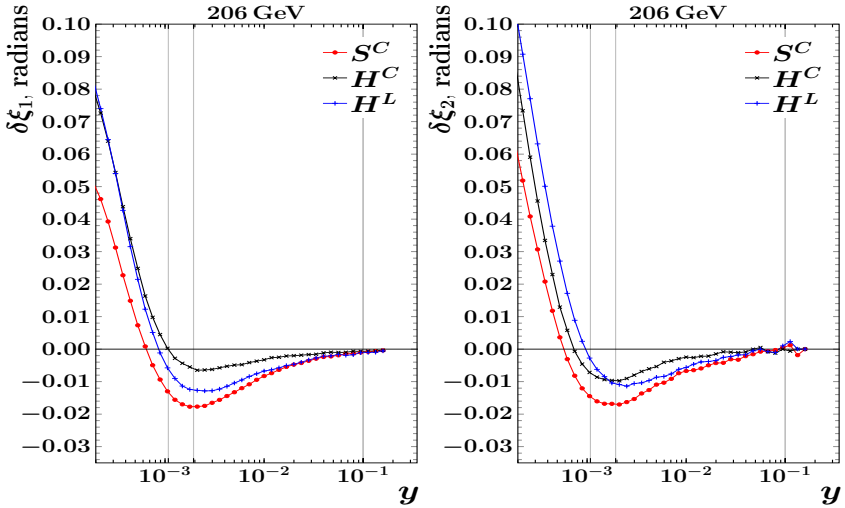
Table: Results of the fits of the matched predictions at NLO+NNLL and NNLO+NNLL accuracy to experimental data. The given uncertainty is fit uncertainty scaled by $\sqrt{\chi^2/ndof}$.

Backup slides part II: Hadron level distributions



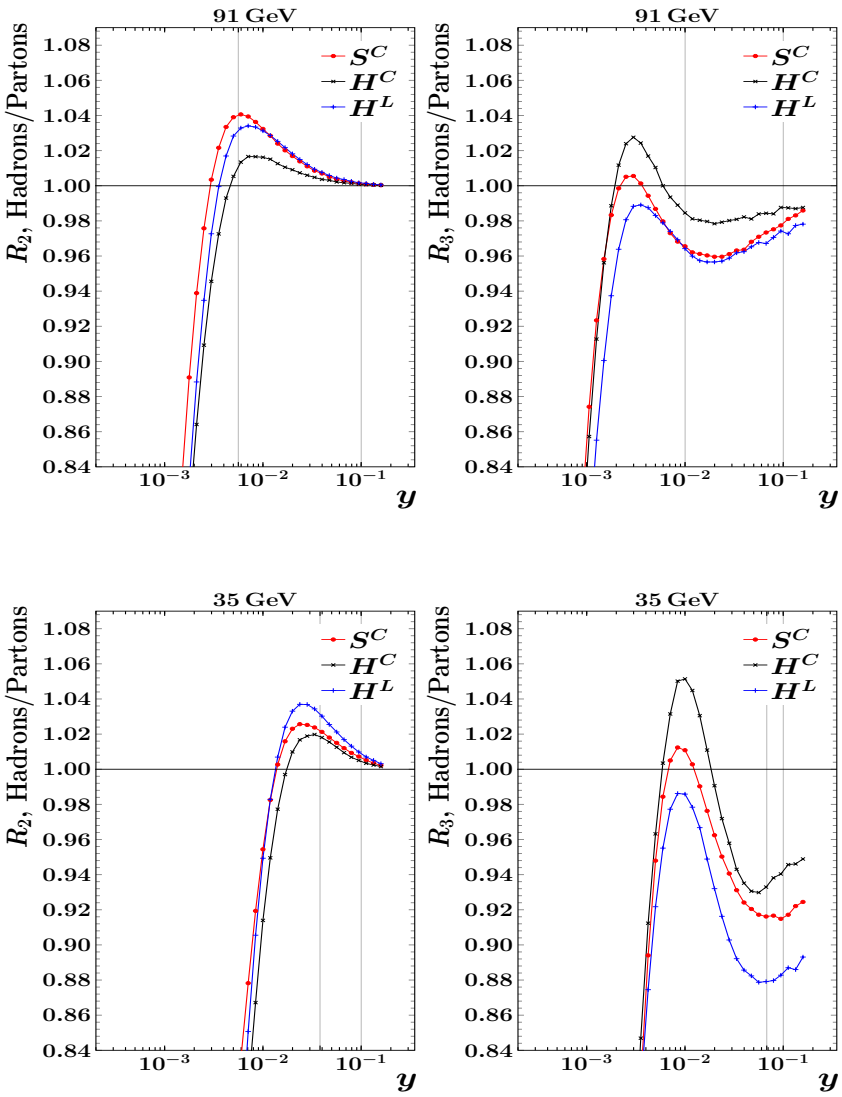
Backup slides part II: Hadronisation corrections

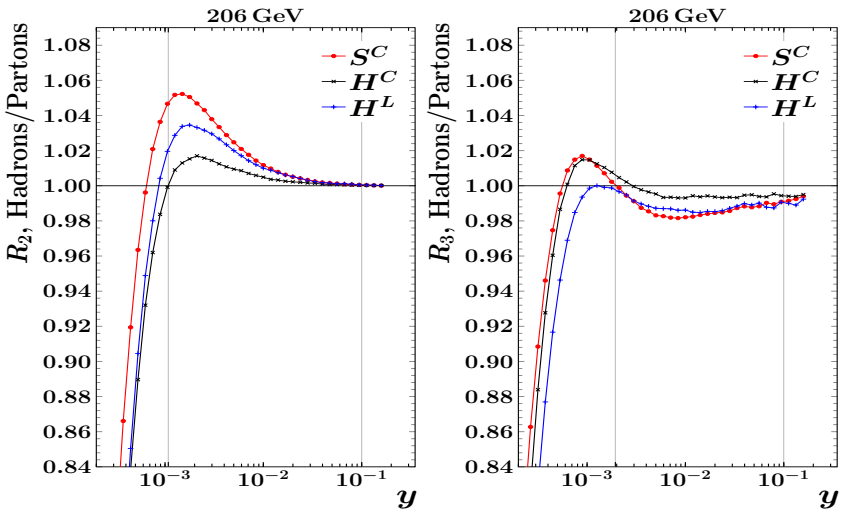




- To avoid binning effects the hadronisation corrections are parametrised with smooth functions.

Backup slides part II: hadron to parton level ratios





- To avoid binning effects the hadronisation corrections are parametrised with smooth functions.

Backup slides part II: R_2 fits

Fit ranges, log y Hadronisation	N^{3}LO $\chi^2/ndof$	N^{3}LO+NNLL $\chi^2/ndof$
$[-1.75+\mathcal{L},-1]$ S^C	0.12121 ± 0.00095 $20/86 = 0.24$	0.11849 ± 0.00092 $20/86 = 0.24$
$[-2+\mathcal{L},-1]$ S^C	0.12114 ± 0.00081 $26/100 = 0.26$	0.11864 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ S^C	0.12119 ± 0.00060 $44/150 = 0.29$	0.11916 ± 0.00063 $44/150 = 0.29$
$[-2.5+\mathcal{L},-1]$ S^C	0.12217 ± 0.00052 $89/180 = 0.50$	0.12075 ± 0.00055 $107/180 = 0.59$
$[-1.75+\mathcal{L},-1]$ H^C	0.11957 ± 0.00098 $22/86 = 0.26$	0.11698 ± 0.00093 $22/86 = 0.25$
$[-2+\mathcal{L},-1]$ H^C	0.11923 ± 0.00079 $29/100 = 0.29$	0.11687 ± 0.00076 $28/100 = 0.28$
$[-2.25+\mathcal{L},-1]$ H^C	0.11868 ± 0.00068 $43/150 = 0.28$	0.11679 ± 0.00064 $40/150 = 0.27$
$[-2.5+\mathcal{L},-1]$ H^C	0.11849 ± 0.00050 $58/180 = 0.32$	0.11723 ± 0.00053 $58/180 = 0.32$
$[-1.75+\mathcal{L},-1]$ H^L	0.12171 ± 0.00109 $21/86 = 0.25$	0.11897 ± 0.00092 $21/86 = 0.24$
$[-2+\mathcal{L},-1]$ H^L	0.12144 ± 0.00078 $28/100 = 0.28$	0.11893 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ H^L	0.12080 ± 0.00069 $43/150 = 0.28$	0.11881 ± 0.00063 $39/150 = 0.26$
$[-2.5+\mathcal{L},-1]$ H^L	0.12024 ± 0.00051 $57/180 = 0.32$	0.11897 ± 0.00053 $52/180 = 0.29$

Table: Fit of $\alpha_s(M_Z)$ from experimental data for R_2 obtained using N^{3}LO and N^{3}LO+NNLL predictions for R_2. The reported uncertainty comes from `MINUIT2`.

Backup slides part II: R_2+R_3 fits

Fit ranges, log y Hadronisation	N^{3}LO, NNLO $\chi^2/ndof$	N^{3}LO+NNLL, NNLO $\chi^2/ndof$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ S^C	0.12195 ± 0.00072 $120/143 = 0.84$	0.12078 ± 0.00066 $140/143 = 0.98$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ S^C	0.12163 ± 0.00061 $153/187 = 0.82$	0.12065 ± 0.00056 $176/187 = 0.94$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ S^C	0.12075 ± 0.00044 $208/251 = 0.83$	0.11994 ± 0.00041 $222/251 = 0.88$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ S^C	0.12143 ± 0.00043 $321/331 = 0.97$	0.12089 ± 0.00044 $336/331 = 1.01$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^C	0.12068 ± 0.00073 $126/143 = 0.88$	0.11956 ± 0.00066 $147/143 = 1.03$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^C	0.12006 ± 0.00061 $163/187 = 0.87$	0.11913 ± 0.00054 $188/187 = 1.01$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^C	0.11869 ± 0.00043 $221/251 = 0.88$	0.11793 ± 0.00043 $238/251 = 0.95$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^C	0.11845 ± 0.00045 $302/331 = 0.91$	0.11799 ± 0.00047 $310/331 = 0.94$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^L	0.12248 ± 0.00068 $121/143 = 0.85$	0.12129 ± 0.00063 $141/143 = 0.99$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^L	0.12211 ± 0.00057 $155/187 = 0.83$	0.12110 ± 0.00053 $180/187 = 0.96$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^L	0.12071 ± 0.00044 $209/251 = 0.83$	0.11989 ± 0.00045 $227/251 = 0.90$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^L	0.12041 ± 0.00044 $266/331 = 0.80$	0.11990 ± 0.00044 $278/331 = 0.84$

Table: Simultaneous fit of $\alpha_s(M_Z)$ from experimental data for R_2 and R_3 obtained using N^{3}LO and N^{3}LO+NNLL predictions for R_2 and NNLO predictions for R_3. The reported uncertainty comes from `MINUIT2`.

Backup slides part II: MC based approach: MC setups

$e^+e^- \to jjjjj$ merged samples with massive b quarks and 2-jet final state in NLO precision.

- **Default setup "S^L": Sherpa2.2.4+ (Comix, Amegic, GoSam ME libraries and OLPs) + Lund (Pythia6) hadronisation**
- Setup for hadronisation systematics: "S^C": Sherpa2.2.4+ (Comix, Amegic, GoSam ME libraries and OLPs) + Ahadic cluster hadronisation
- Setup for cross-check: "H^M": Herwig7.1.1 (Herwig, Madgraph, GoSam ME libraries and OLPs) + Herwig cluster hadronisation

Merging scale was chosen to minimise its size impact on parton level in fit range.

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