

Pulse Shape Simulation for Segmented Detectors



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Outline:

- Motivation
- Electric Field
- Charge Carrier Drift
- Evaluation I + II
- Summary and Outlook

GERDA

Pulse Shape Simulation - Why?

➔ samples of SSE and MSE are needed to understand efficiencies of PSA

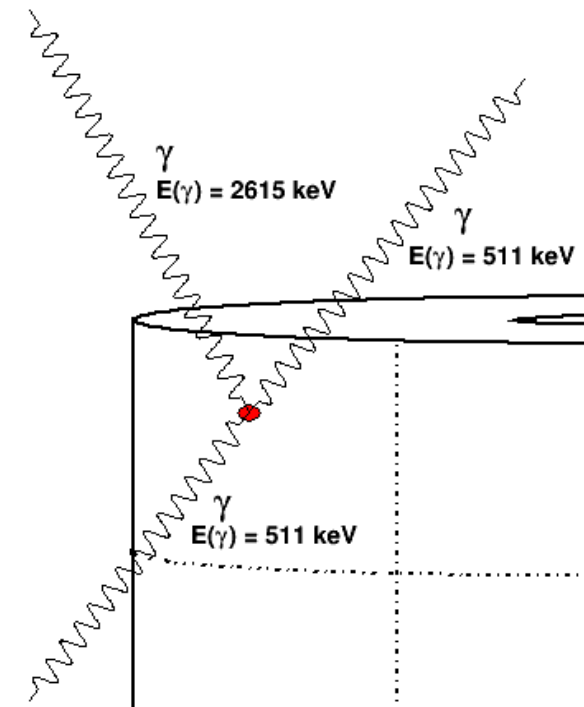
- MSE can be easily extracted from data (MeV photon peaks)
- SSE can be also be extracted from data (Double Escape Peak)

BUT:

- samples are not pure SSE or MSE
- events are not homogeniously distributed throughout the detector

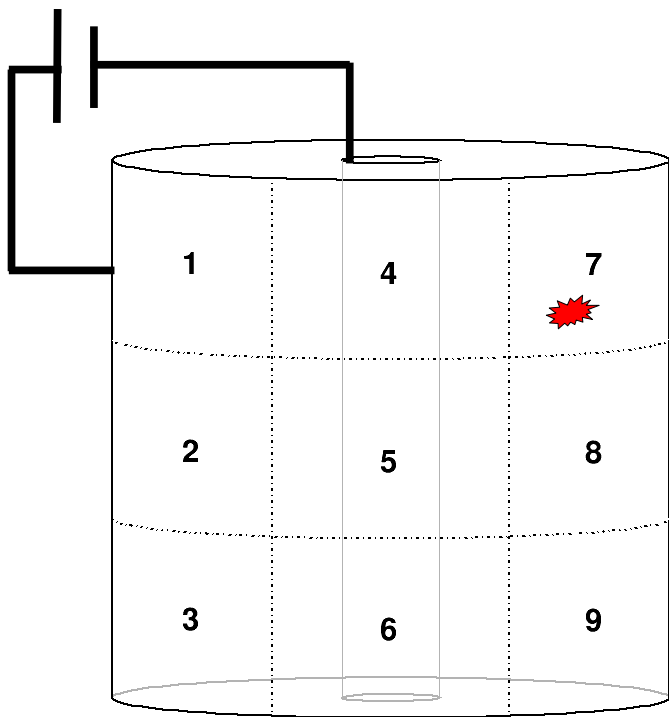
can be overcome

BUT takes long time to record samples



- Data should be supplemented by simulated pulses
 - PSS can give insights into crystal properties
 - helps reconstructing interaction positions

Basic Principle



- **energy deposit** \Rightarrow electrons holes created
- charges **drift** under influence of external **E-Field**
- drifting charges **induce pulses** on electrodes

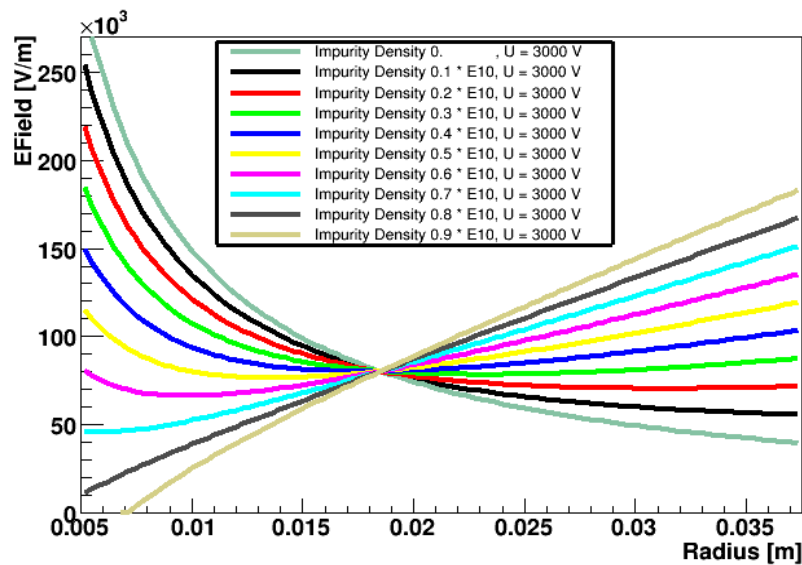
1. simulate energy deposit using MaGe
2. group hits according to position bandwidth and sampling frequency
3. determine number of electron hole pairs

4. **calculate E-Field** inside detector
5. **calculate drift** of charge carriers
 \Rightarrow **calculate induced charges** using **weighting potentials**
6. take into account electronics effect, i.e noise, bandwidth...

Solve Poisson-equation: $\nabla^2 \varphi(\vec{r}) = \frac{1}{(\epsilon_0 \cdot \epsilon_R)} \cdot \rho(\vec{r})$

numerical procedure: **Successive Overrelaxation (SOR)**

agreement
numerical and analytical solution
better than 1%
numerical calculation works



- impurity density ρ dominates the electric field
- ρ changes with radius, height and maybe with azimuthal angle ϕ

Drift:

$$\vec{v}(\vec{r}) = \mu_{e,h} \vec{E}(\vec{r})$$

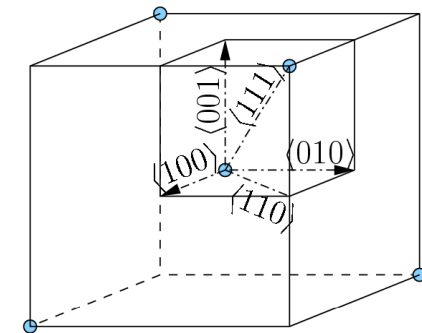
with $\mu_{e,h}$ depends on temperature, electric Field and **structure** of **germanium crystal**

Transverse anisotropy:

- in direction $\langle 100 \rangle$, $\langle 110 \rangle$ and $\langle 111 \rangle$
 $\mu_{e,h}$ **parallel** to E-Field **otherwise not!**

Longitudinal anisotropy:

- along crystal axes $\langle 100 \rangle$, $\langle 110 \rangle$ and $\langle 111 \rangle$
 $\mu_{e,h}$ has different magnitude
- Charge carrier drift in **any** direction can be computed using mobilities along $\langle 100 \rangle$ and $\langle 111 \rangle$ directions
- experimental data along axes exists
 \Rightarrow mobility can extracted along axes



$$v = \frac{\mu_0 E}{\left[1 + \left(\frac{E}{E_0} \right)^\beta \right]^{1/\beta}} - \mu_n E$$

Drift Parameters

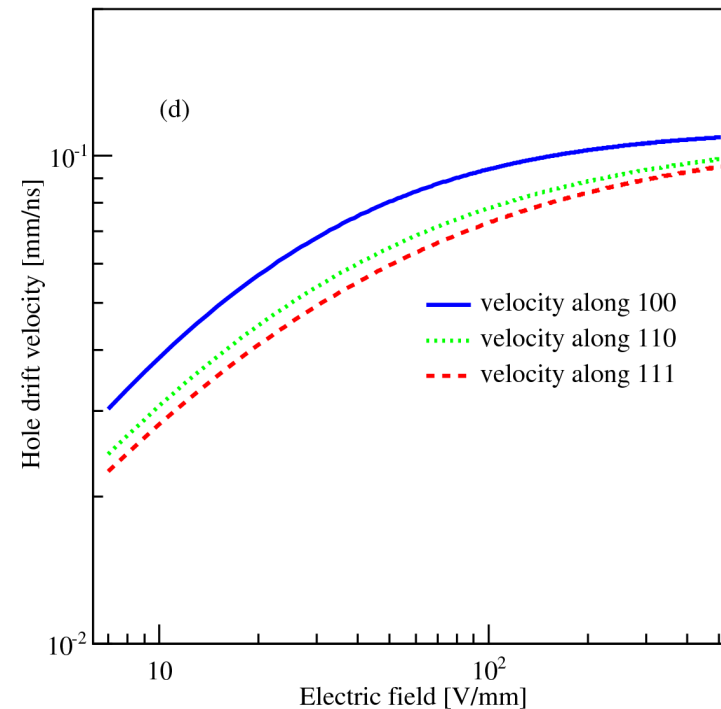
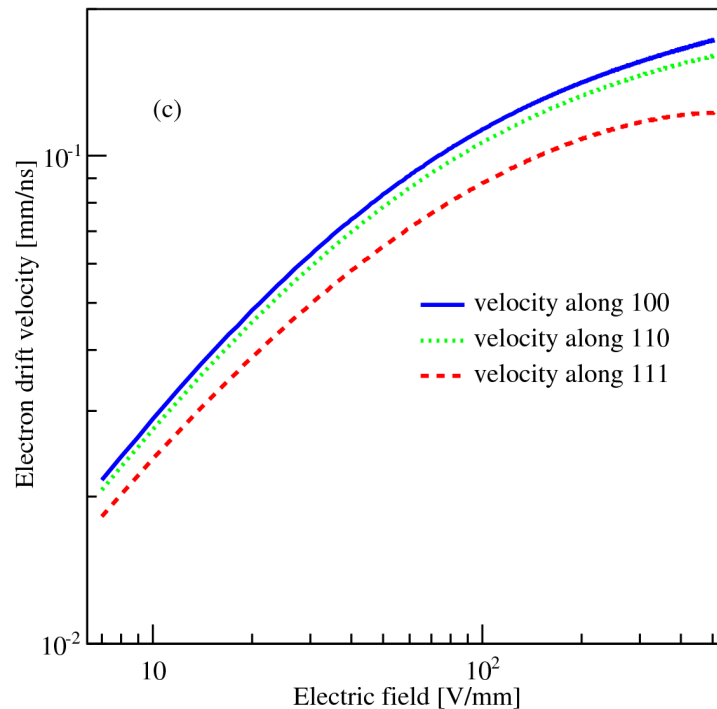
Reference	Carrier	Direction	μ_0 [$\frac{\text{cm}^2}{\text{V}\cdot\text{s}}$]	E_0 [$\frac{\text{V}}{\text{cm}}$]	β	μ_n [$\frac{\text{cm}^2}{\text{V}\cdot\text{s}}$]
Ref. [102]:	Electron	$\langle 111 \rangle$	42420	251	0.87	62
		$\langle 100 \rangle$	40180	493	0.72	589
Ref. [103]:	Hole	$\langle 111 \rangle$	107270	100	0.58	0
		$\langle 100 \rangle$	66333	181	0.744	0
Ref. [104]:	Electron	$\langle 111 \rangle$	38536	538	0.641	510
		$\langle 100 \rangle$	38609	511	0.805	-171
	Hole	$\langle 111 \rangle$	61215	182	0.662	-
		$\langle 100 \rangle$	61824	185	0.942	-

$$v = \frac{\mu_0 E}{\left[1 + \left(\frac{E}{E_0} \right)^\beta \right]^{1/\beta}} - \mu_n E$$

[102]:Mihailescu *et al.* NIM A 447: 350

[103]:Reggiani *et al.*, Phys. Rev. B 16: 2781

[104]:Bruyneel *et al.*, NIM A569:764



Shockley-Ramos Theorem:

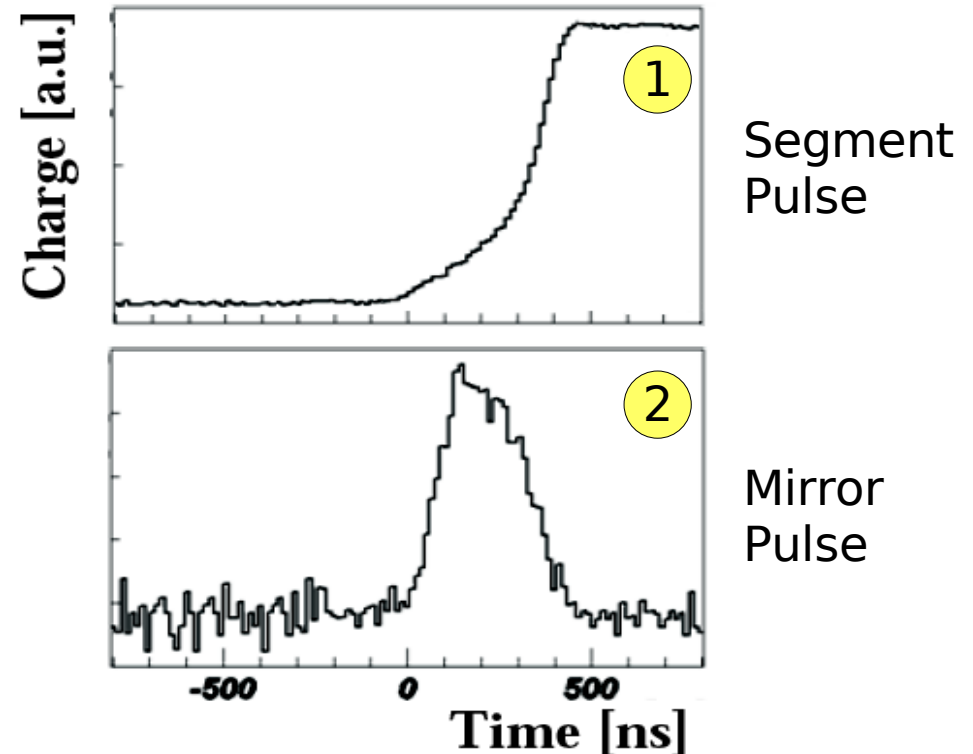
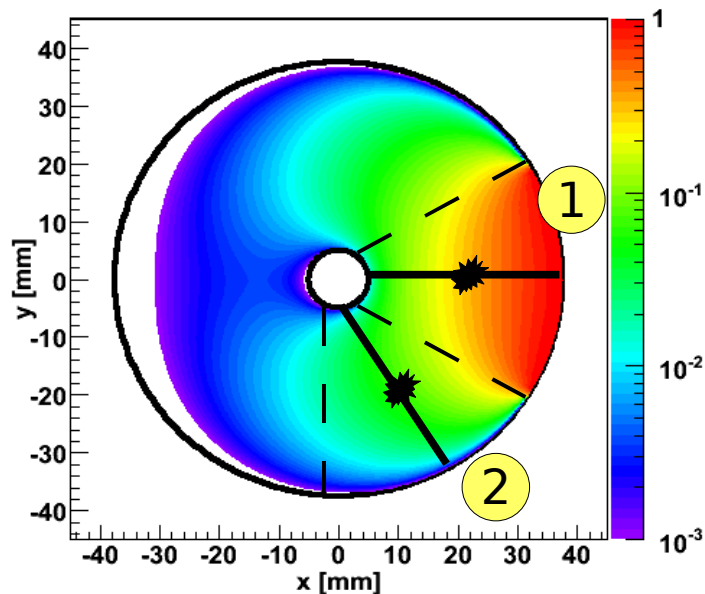
$$Q_{induced}^i(t) = q_e \cdot \phi_W^i(\vec{r}(t)) + q_h \cdot \phi_W^i(\vec{r}'(t))$$

Weighting Potential:

calculated solving Laplace equation
BC WP on electrode 1, otherwise 0

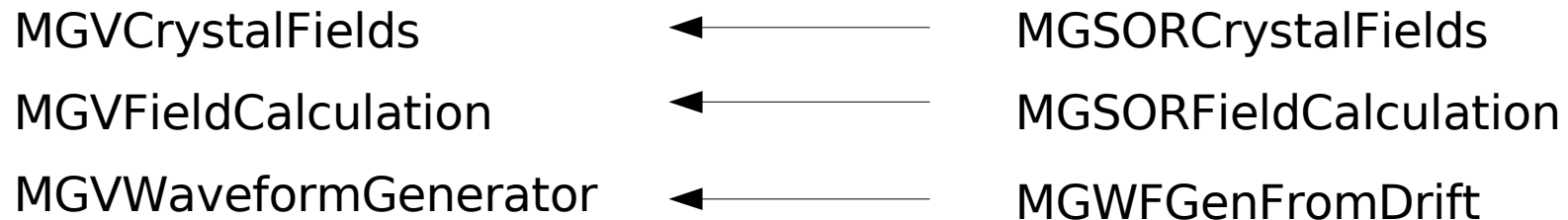
no analytical solution in 3D for 18 segments

Need numerical calculation (SOR)



Connection to MaGe

Virtual base classes:



- MaGe/waveform

- **MGVCrystalFields**

```
//Set and access the MGCrystalData
```

```
inline void SetCrystalData(MGCrystalData* crystal){fCrystalData = crystal;};  
inline const MGCrystalData* GetCrystalData() const {return fCrystalData;};
```

```
//Methods to save, load and draw(?) the electric and weighting fields
```

```
virtual void SaveFields() =0;  
virtual void LoadFields() =0;  
// virtual void DrawFields();
```

```
//Methods to access the electric and weighting field
```

```
virtual CLHEP::Hep3Vector GetEField(CLHEP::Hep3Vector coordinates)const =0;  
virtual CLHEP::Hep3Vector GetWField(CLHEP::Hep3Vector coordinates, size_t segment) const =0;  
virtual double GetWPotential(CLHEP::Hep3Vector coordinates, size_t segment) const =0;
```

```
protected:
```

```
MGCrystalData* fCrystalData;
```


- MaGe/waveform

- **MGVFieldCalculation**

```
MGVFieldCalculation(){};  
MGVFieldCalculation(MGVCrystalFields* crystalFields){fCrystalFields = crystalFields;};  
// virtual ~MGVFieldCalculation();
```

```
//Methods to access MGVCrystalFields  
inline void SetCrystalFields(MGVCrystalFields* crystalFields){fCrystalFields = crystalFields;};  
inline MGVCrystalFields* GetCrystalFields(){return fCrystalFields;};
```

```
//Methods to calculate efield and weighting fields  
virtual void CalculateFields() = 0;
```

protected:

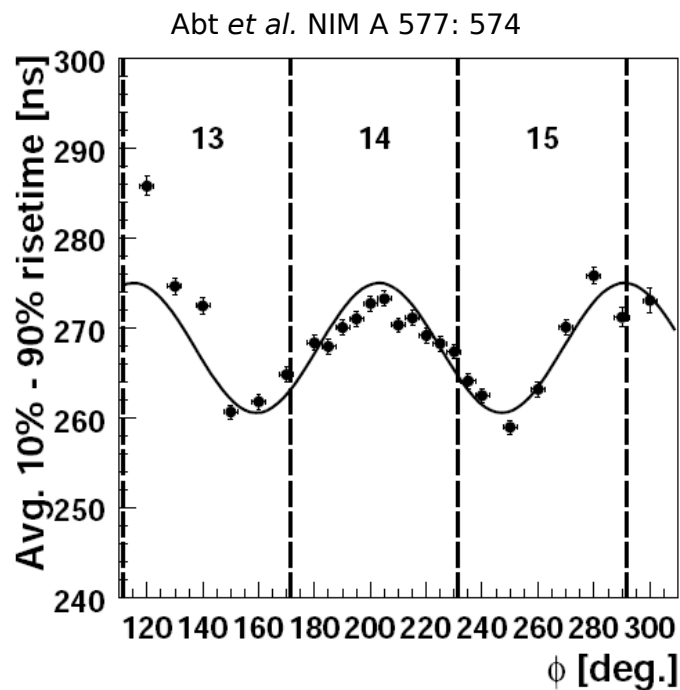
```
MGVCrystalFields* fCrystalFields;
```

Open files, show how it works!

Files can be found in **MaGe/sandbox**

Data:

- 18-fold seg. n-type HPGe detector
- 121.78 keV line from 75kBq collimated ^{152}Eu source
- rise time 10%-90% calculated:

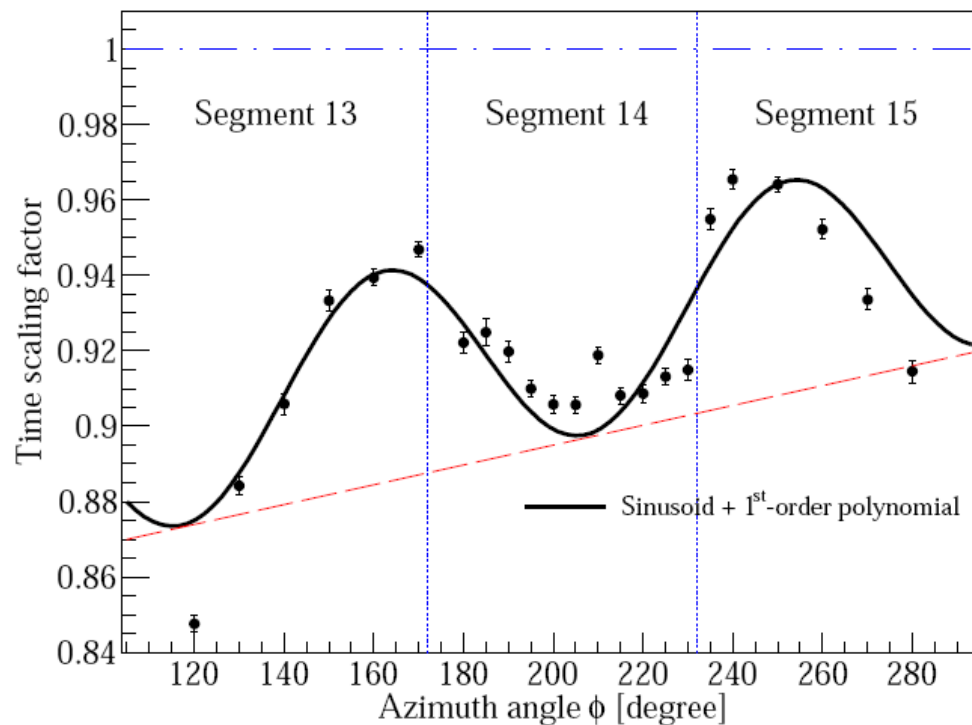


Simulation:

- $\rho_t = 0.70 \cdot 10^{10} \text{ cm}^{-3}$
- $\rho_b = 1.35 \cdot 10^{10} \text{ cm}^{-3}$
- single pulse on $\langle 110 \rangle$ axis at $r=37.5\text{mm}$ (max radius)
- $\langle 110 \rangle$ axis at 290 degree
- added $50 \mu\text{s}$ decay time from preamp
- 37.5 MHz bandwidth of electronics

Evaluation I

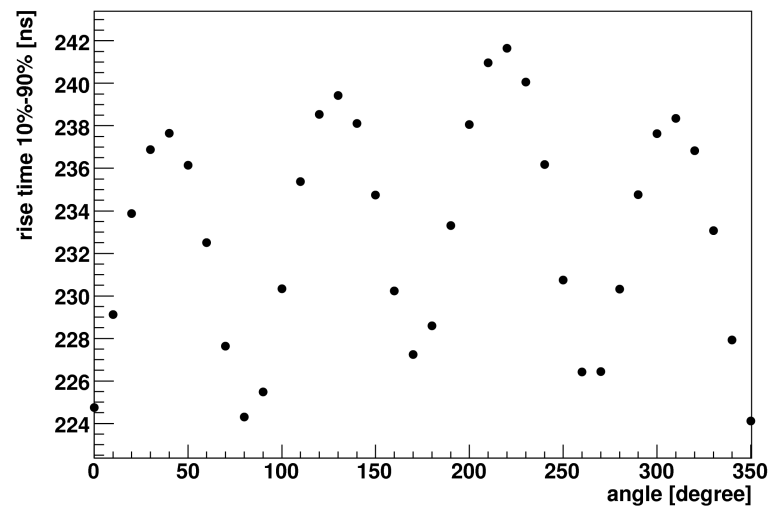
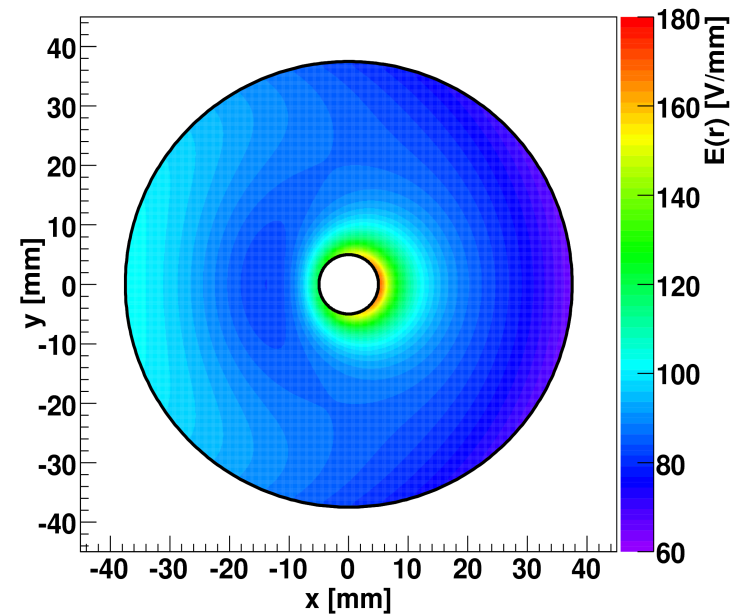
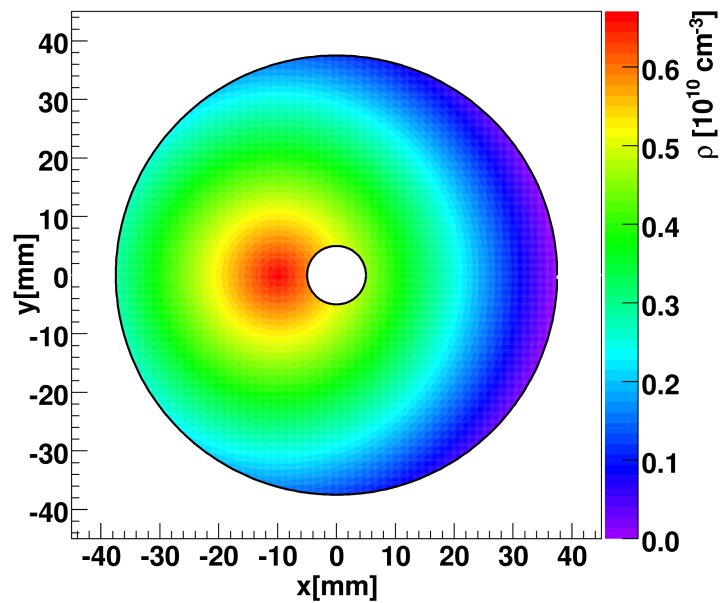
- each data pulse fitted with simulated pulse;
3 parameter: time offset, amplitude scale factor, time scale factor
- pulse with $\chi^2 > 200$ removed; background event
- Time scaling factor histogrammed and fitted with Gaussain



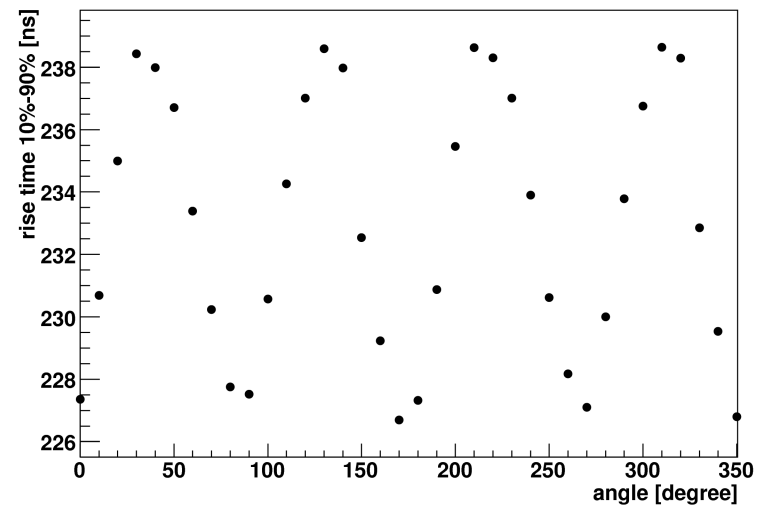
- overall shift of about 10 %

- azimuthal shift about 3%
 - detector effect?
 - temperature effect?

Evaluation I



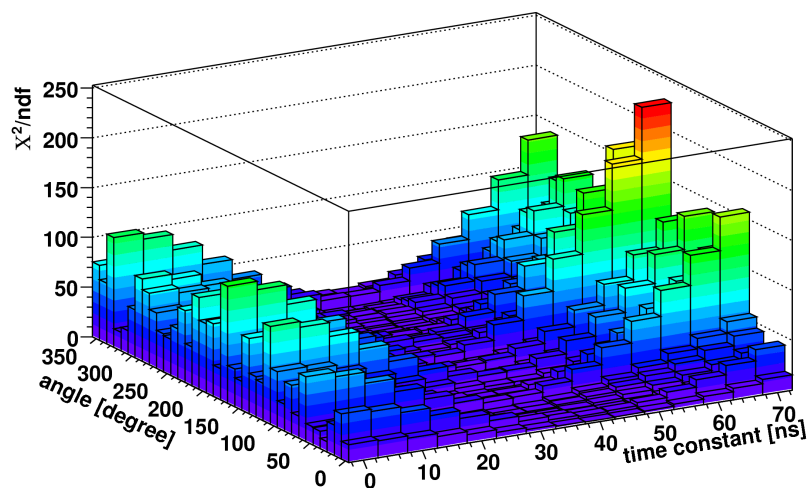
single charge on outside



full simulation 122keV

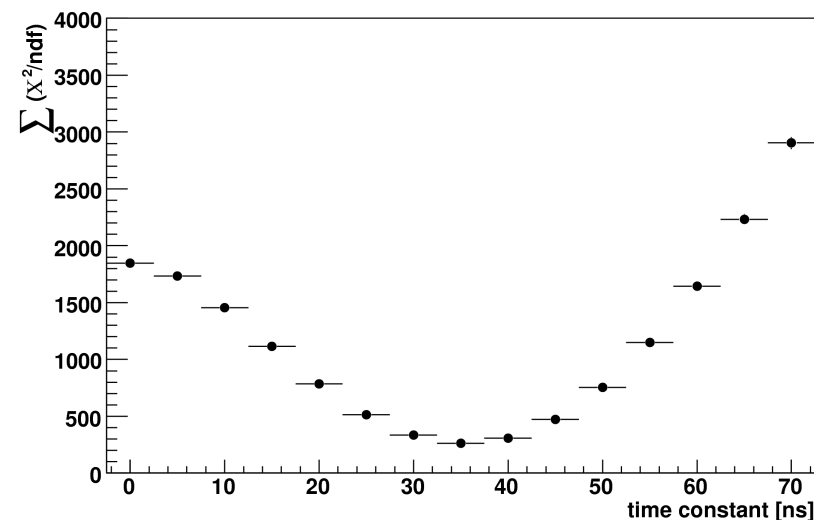
Data:

- 19-fold seg. n-type HPGe detector
- 121.78 keV line from 42kBq collimated ^{152}Eu source
- all pulses added and normalized



Simulation:

- 121.78 keV photons collimated
- all pulses added and normalized
- added 50 μs decay time from preamp
- several bandwidths simulated; χ^2 fit determined best bandwidth



Evaluation II

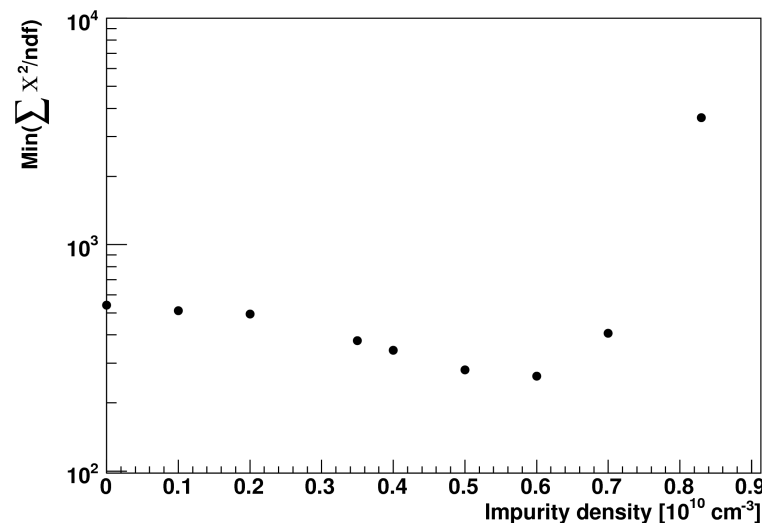
Vary impurity densities ρ :

lowest χ^2 at $\rho = 0.6 \cdot 10^{10} \text{ cm}^{-3}$

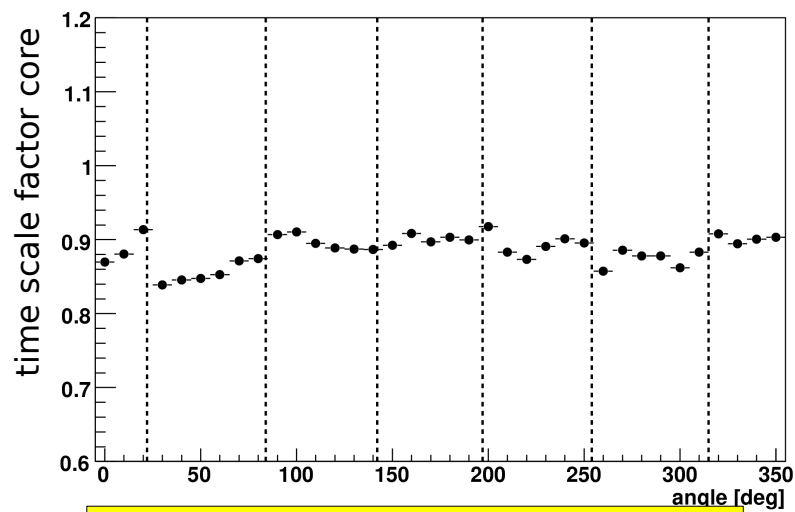
coincides with ρ calculated from depletion voltage 2250V

in the middle of given ρ range for that detector:

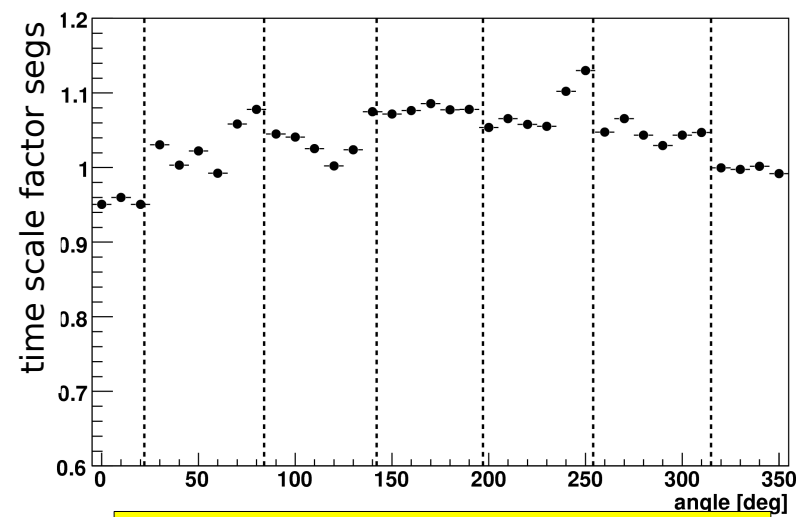
$0.44 \cdot 10^{10} \text{ cm}^{-3} - 1.30 \cdot 10^{10} \text{ cm}^{-3}$



average $\chi^2/\text{ndf} = 6.9$
individuals much better

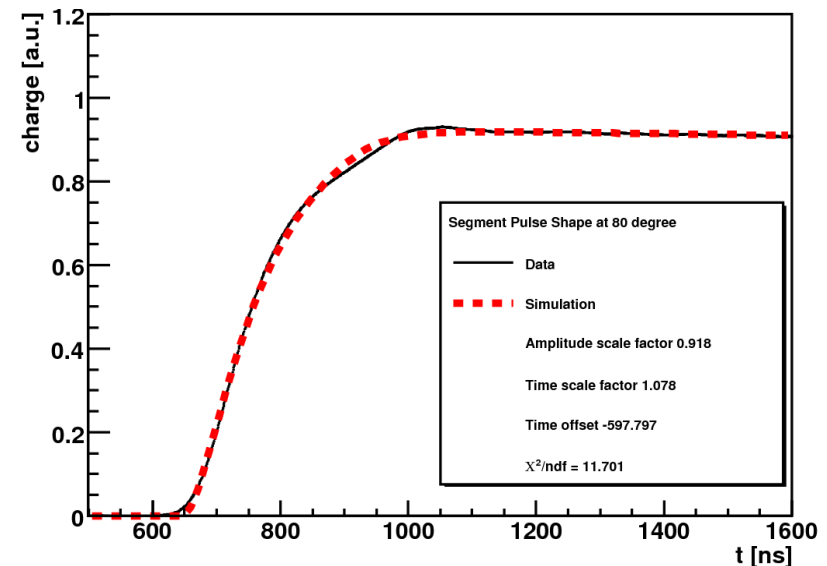
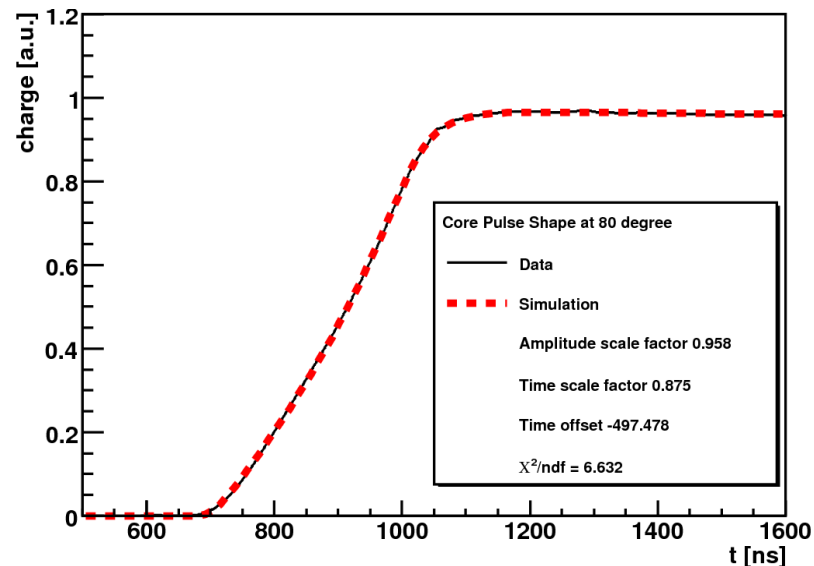
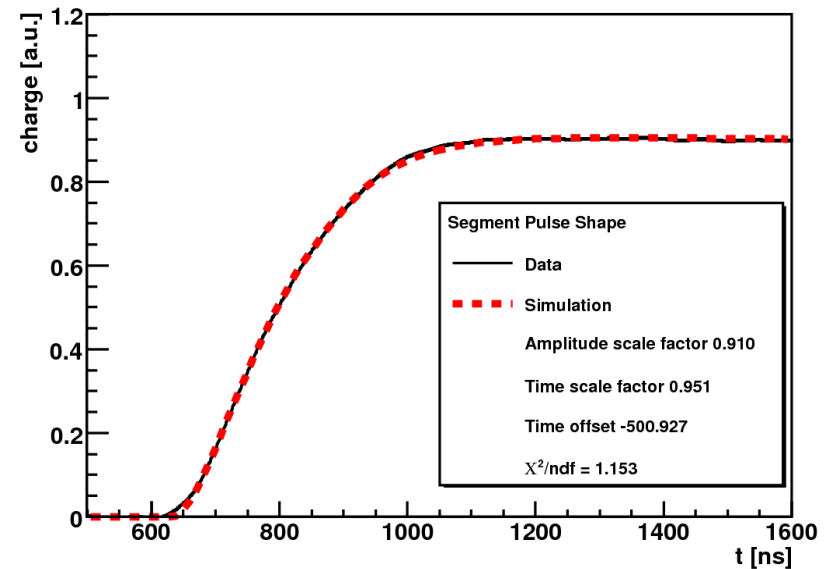
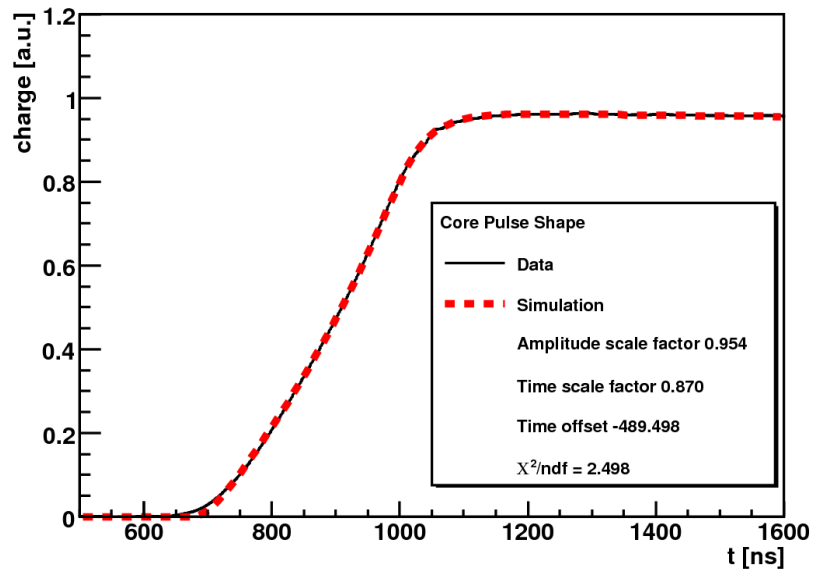


~10% scaling needed



~10% scaling needed

Evaluation II



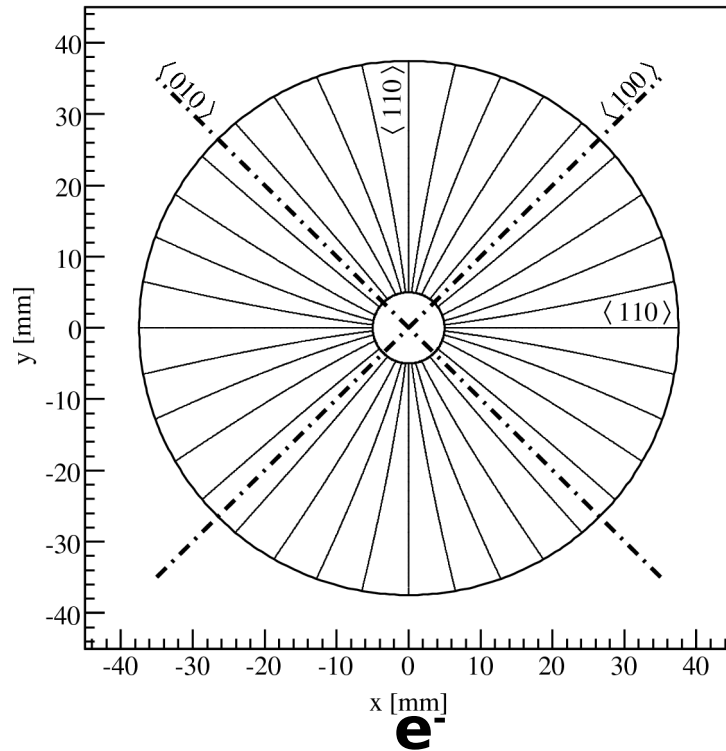
- core shape well matched
- uncertainty not properly estimated?

- shape for some preamps good
- transfer function needed!

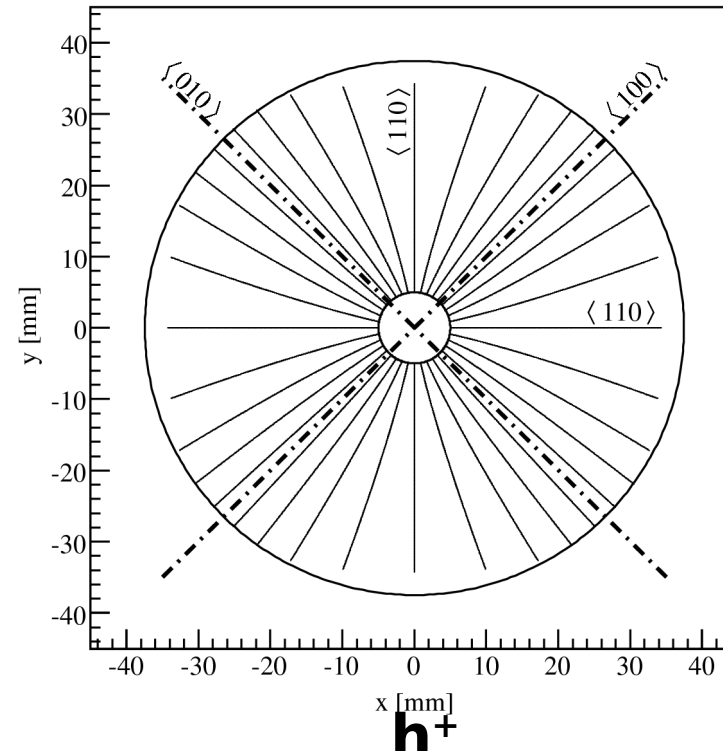
Summary and Outlook

- **PSS needed to fully understand PSA**
- **Numerical simulation** of electric field **better** than **1%**
Weighting potential calculated with **same algorithm**
- Drift implemented taking into account **longitudinal** and **transverse anisotropy**
- Correct impurity density can be extracted
azimuthal change in rise time can hardly be explained by impurities
- **Agreement** of shape **good** after **scaling** of about **10% core**
5% segments
- Replace electronics simulation by measured transfer function

- drift charges in **fixed time interval** Δt
- start position **equally spaced** on outer/inner surface



drift from outside in



drift from inside out

- holes drift slower than electrons
- drift along $\langle 100 \rangle$ axis faster than $\langle 110 \rangle$