Towards NNLO simulations matched to parton showers for processes with final-state jets in GENEVA

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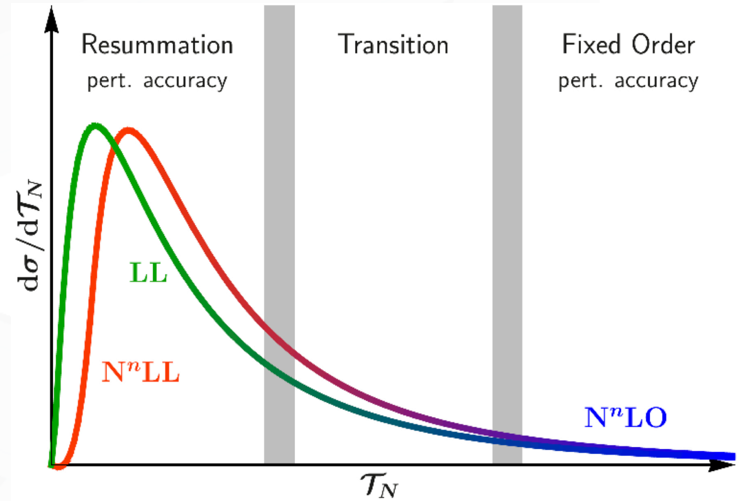
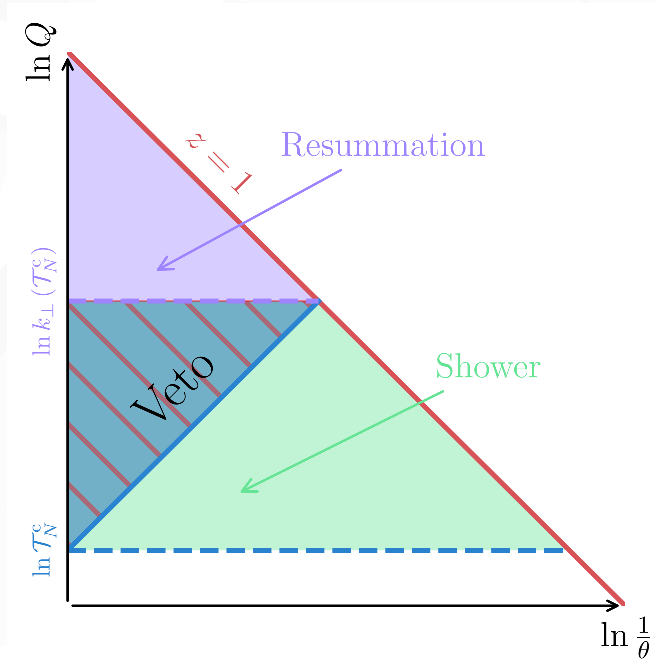
The Geneva method

- Monte Carlo fully-differential event generation at higher-orders (NNLO)
- Resummation plays a key role in the defining the events in a physically sensible way
- Results at partonic level can be further evolved by different shower matching and hadronization models





Resolution parameters for N extra emissions

- The key idea is the introduction of a resolution variable r_N that measure the hardness of the $N+1$-th emission in the Φ_N phase space.

- For color singlet production one can have $r_0 = q_T, p_T^j, k_T$-ness,….

- N-jettiness is a valid resolution variable: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$

- The limit $\tau_N \to 0$ describes a N-jet event where the unresolved emissions are collinear to the final state jets/initial state beams or soft

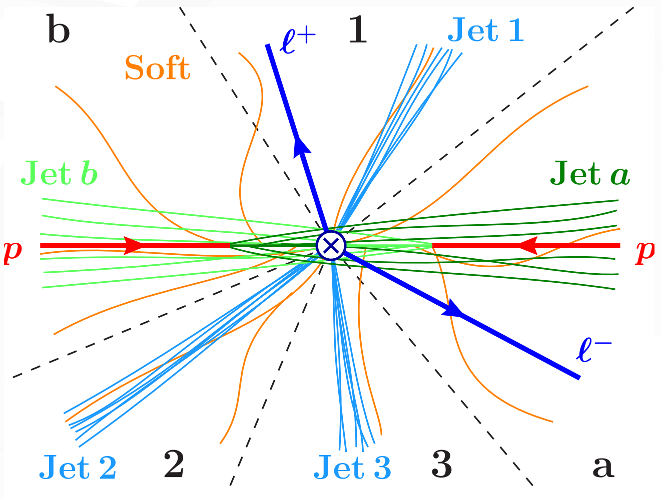
- For color-singlet final states, it reduces to 0-jettiness

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

[Stewart, Tackmann,Waalewijn `09,`10]

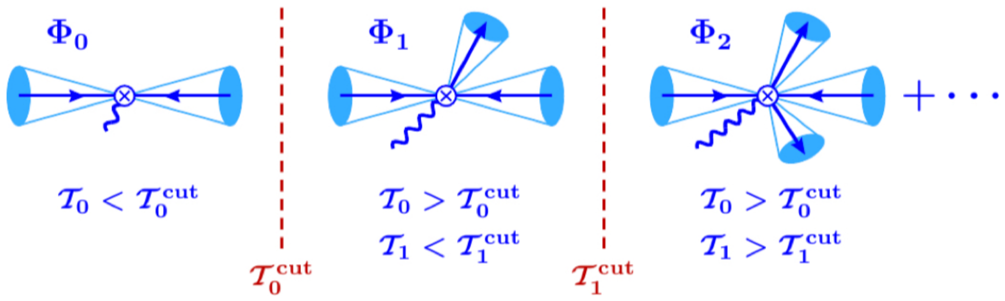
- When an extra jet is present 1-jettiness used for r_1

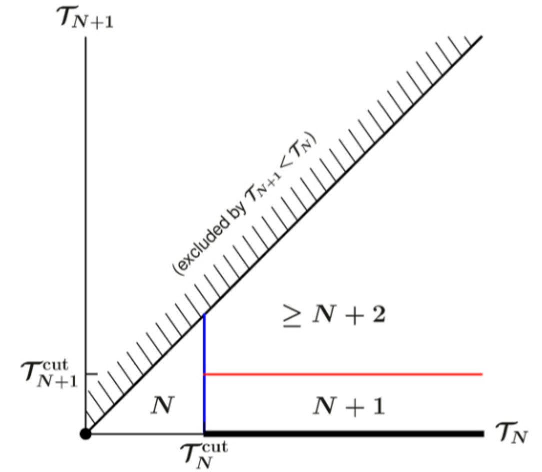
$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$



Resolution parameters for N extra emissions

At NNLO one needs a 0-jet and a 1-jet resolution parameters. Iterating the procedure, the phase space is sliced into jet-bins





Different choices are possible for the resolution parameters, but one always has:

- Emissions below $\mathcal{T}_N^{\rm cut}$ are unresolved (i.e. integrated over) and the kinematic considered is the one of the event before the extra emission(s).
- Emissions above $\mathcal{T}_N^{\rm cut}$ are retained and the kinematics is fully specified.

An M-parton event is considered a N-jet event, $N \leq M$, fully differential in Φ_N

- Price to pay: power corrections in $\mathcal{T}_N^{\rm cut}$ due to PS projection.
- Advantage: vanish for IR-safe observables as $\mathcal{T}_N^{\rm cut} \to 0$

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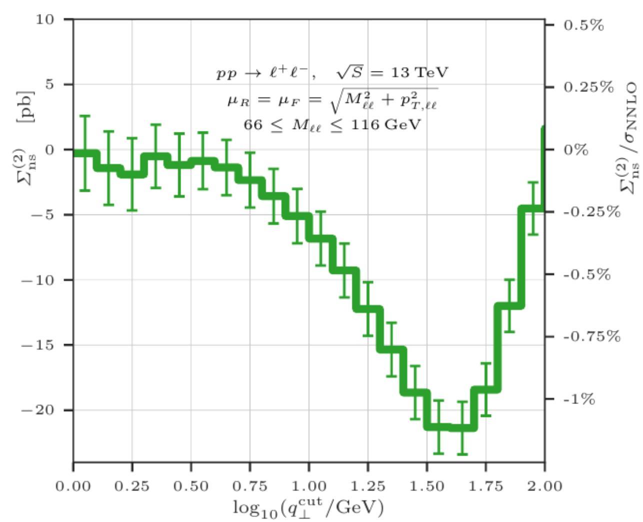
Resummation of resolution parameters

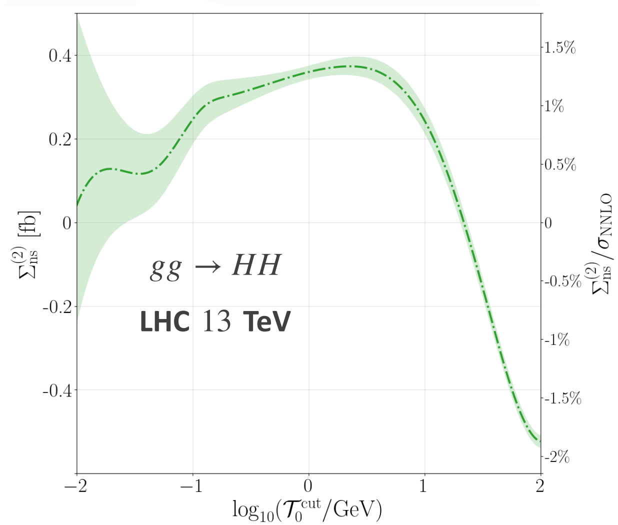
Resumming resolutions parameters not really a new idea, SMCs have been doing it since the '80s with Sudakov factors

Using resummation at higher orders has several benefits: systematically improvable (NLL,NNLL,N3LL,…), lowering theoretical uncertainty at each step. Including primed accuracy captures the exact singular behaviour at $\delta(r_N)$.

The higher the accuracy the lower the cuts can be pushed without risking missing higher logarithms being numerically relevant. The lower the cut the smaller the nonsingular power corrections due to phase-space projections will affect the results differentially.

For NNLO event generation one needs at least NNLL'$_{r_0}$ + NNLO accuracy to control the full α_s^2 singular contributions.





From resummation to event generation

Final GENEVA partonic formulae combine resummation and matching to fixed-order

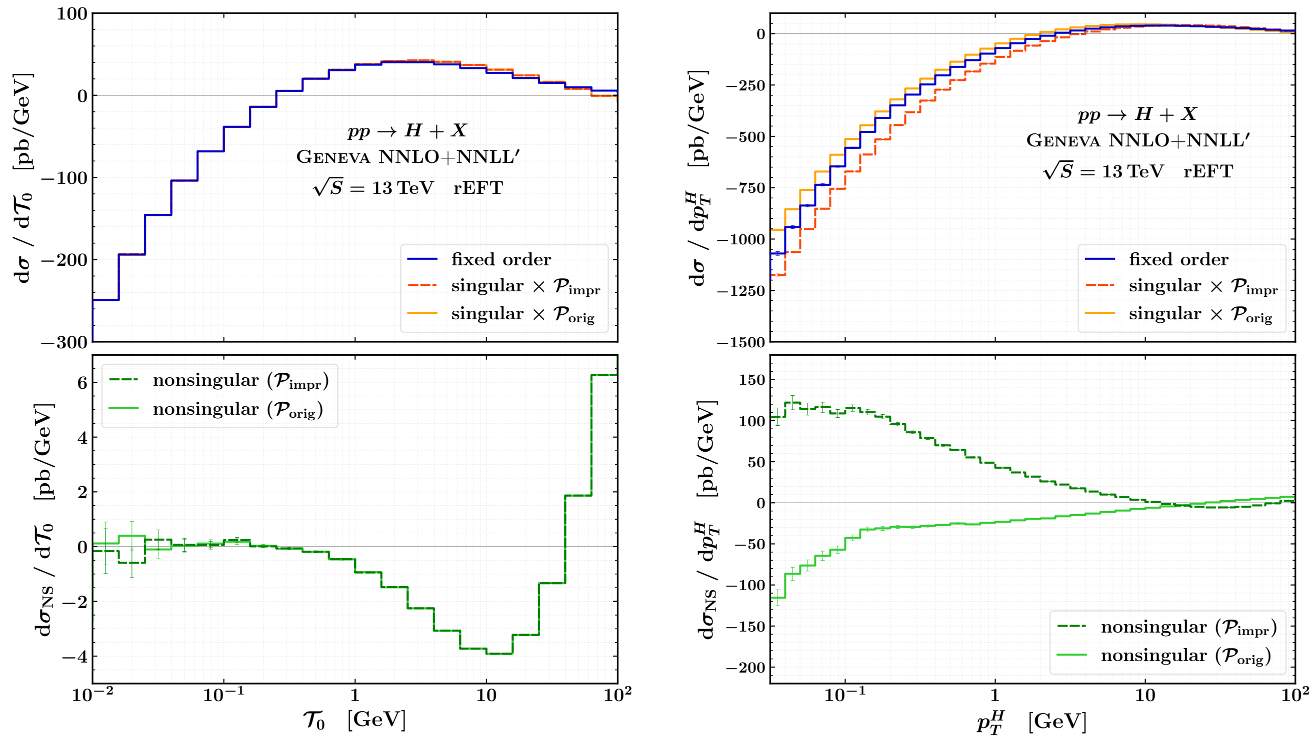
$$\frac{d\sigma^{\mathrm{MC}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma_0^{\mathrm{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}$$

Resummed formulae need to be made more differential via splitting functions, capturing the singular behaviour of different resolution variables as best as they can.

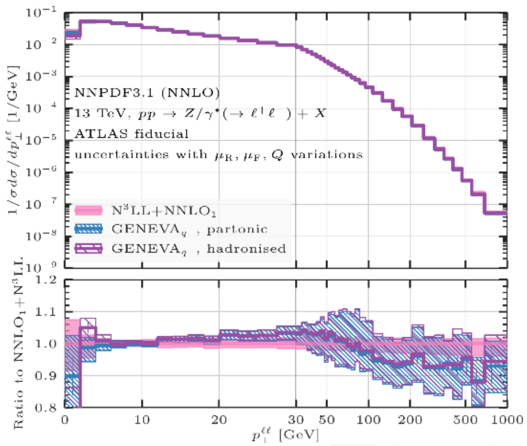
$$\frac{d\sigma^{\mathrm{MC}_1}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1}\, U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_1^{\mathrm{match}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

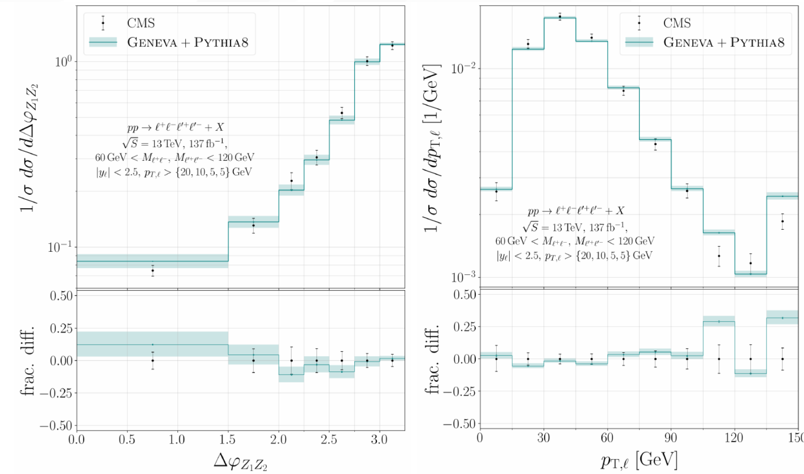
$$\frac{d\sigma^{\mathrm{MC}_{\geq 2}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1}\, U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{d\sigma_{\geq 2}^{\mathrm{match}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

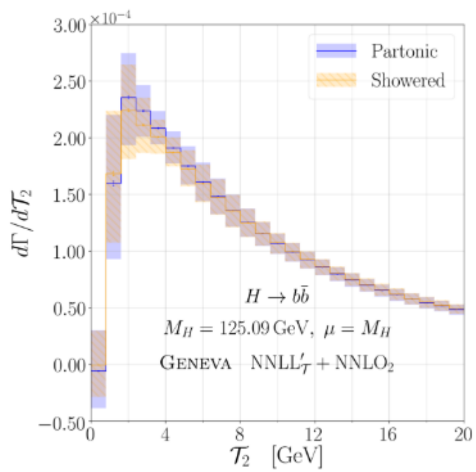


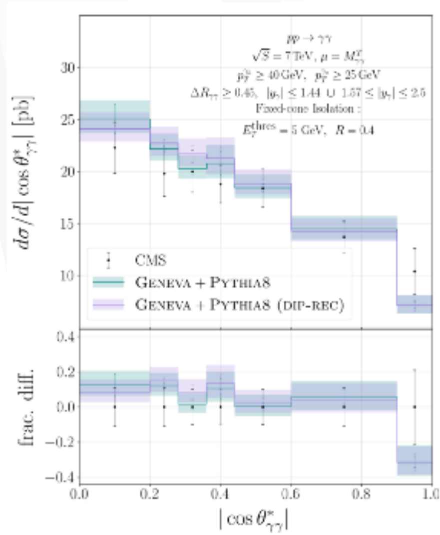
Implemented processes

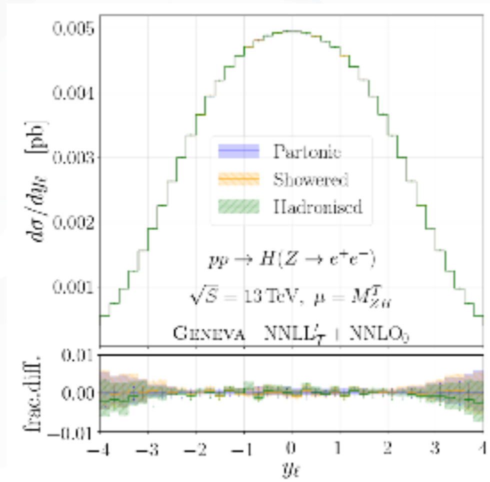
Method has been tested and validated with several color singlet production processes:
DY, ZZ, $W\gamma$, VH, $\gamma\gamma$, ggH, ggHH, Higgs decays using both zero-jettiness and q_T











Using the jet pT as resolution variable

GENEVA recently extended to jet veto resummation in [Gavardi et al. 2308.11577].

Factorization most easily derived for cumulant of the cross-section. SCET II problem.
Numerical derivative to get the spectrum. For hardest-jet we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0}(p_T^{\mathrm{cut}}, \mu, \nu) = \sum_{a,b} H_{ab}(\Phi_0, \mu) B_a(Q, p_T^{\mathrm{cut}}, R, x_a, \mu, \nu) B_b(Q, p_T^{\mathrm{cut}}, R, x_b, \mu, \nu) S_{ab}(p_T^{\mathrm{cut}}, R, \mu, \nu)$$

Two loop Beam and Soft functions recently computed in [Abreu et al. 2207.07037, 2204.02987]

Focus on $W^+W^- \to \mu^+\nu_\mu e^-\bar{\nu}_e$ with jet veto, in 4-flavor scheme to avoid top contaminations.
Massless two-loop hard function taken from `qqVVamp` [Gehrmann et al. 1503.04812]

Interface to SCETlib [Tackmann et al.] allows to perform also resummation also for pT of the second jet at the cumulant level. Refactorization of soft sector into global soft, soft-coll and nonglobal contributions [Cal et al.]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1}(p_T^{\mathrm{cut}}, \mu, \nu) = \sum_{\kappa} H_\kappa(\Phi_1, \mu) B_a(Q, p_T^{\mathrm{cut}}, R, x_a, \mu, \nu)\, B_b(Q, p_T^{\mathrm{cut}}, R, x_b, \mu, \nu) S_\kappa(p_T^{\mathrm{cut}}, y_J, \mu, \nu)$$
$$\times\, \mathcal{S}_j^R(p_T^{\mathrm{cut}} R, \mu)\, J_j(p_T^J R, \mu)\, \mathcal{S}_j^{\mathrm{NG}}\left(\frac{p_T^{\mathrm{cut}}}{p_T^J}\right).$$

[Banfi et al. hep-ph/0206076]

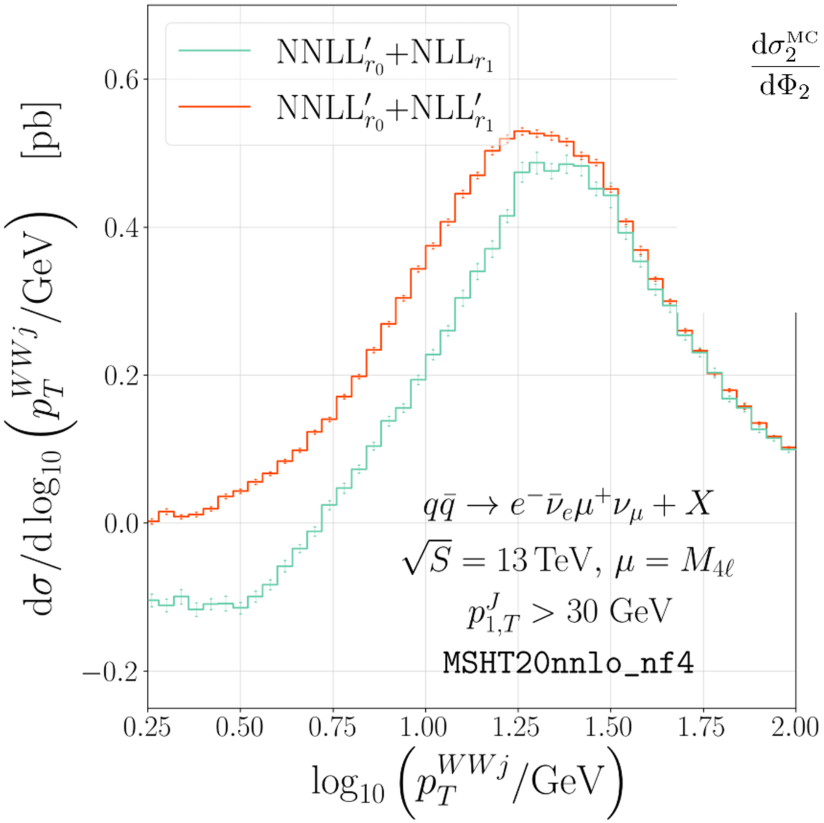
Resumming second jet resolution at NLL’ in GENEVA

Extension of the GENEVA approach to include resummation of r_1^{cut} to NLL’ accuracy

Now truly capturing the correct nonsingular behaviour when approaching the single-jet limit

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_1^{\mathrm{MC}}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) = \Bigg\{ & \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'r_0}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'r_0}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0}\Bigg|_{\mathrm{NLO}_1}\Bigg] \mathcal{P}_{0\to1}(\Phi_1)\, U_1(\Phi_1, r_1^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{NLO}_1}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{\mathrm{NLL}'r_1}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) - \frac{\mathrm{d}\sigma^{\mathrm{NLL}'r_1}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}})\Bigg|_{\mathrm{NLO}_1}\Bigg\}\, \theta(r_0 > r_0^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_1}_{\mathrm{nonproj}}}{\mathrm{d}\Phi_1}\, \theta(r_0 < r_0^{\mathrm{cut}})
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_2^{\mathrm{MC}}}{\mathrm{d}\Phi_2} = \Bigg\{ & \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'r_0}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'r_0}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0}\Bigg|_{\mathrm{NLO}_1}\Bigg] \mathcal{P}_{0\to1}(\Phi_1)\, U_1'(\Phi_1, r_1)\, \mathcal{P}_{1\to2}(\Phi_2) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_2}}{\mathrm{d}\Phi_2} + \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NLL}'r_1}}{\mathrm{d}\Phi_1\,\mathrm{d}r_1} - \frac{\mathrm{d}\sigma^{\mathrm{NLL}'r_1}}{\mathrm{d}\Phi_1\,\mathrm{d}r_1}\Bigg|_{\mathrm{LO}_2}\Bigg] \mathcal{P}_{1\to2}(\Phi_2)\Bigg\}\, \theta(r_1 > r_1^{\mathrm{cut}})\, \theta(r_0 > r_0^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_2}_{\mathrm{nonproj}}}{\mathrm{d}\Phi_2}\, \theta(r_1 < r_1^{\mathrm{cut}})\, \theta(r_0 > r_0^{\mathrm{cut}})\,.
\end{aligned}
$$



NLL’ accuracy of the second jet only maintained in presence of an hard first jet.

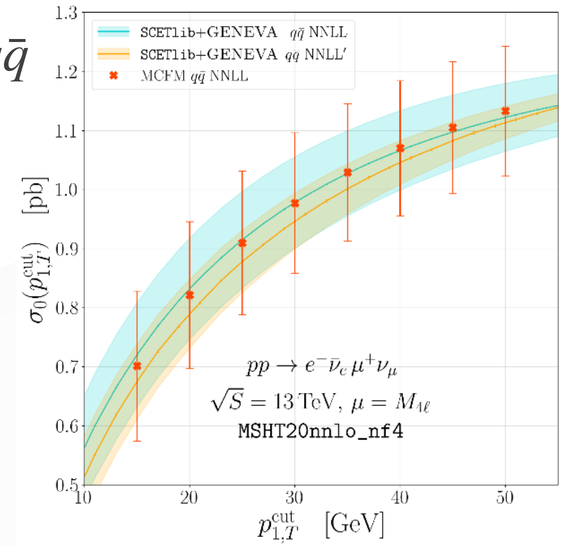
Resummation formula not able to handle the $r_0 \sim r_1 \ll \mu_H$ hierarchy, double resummation required there.

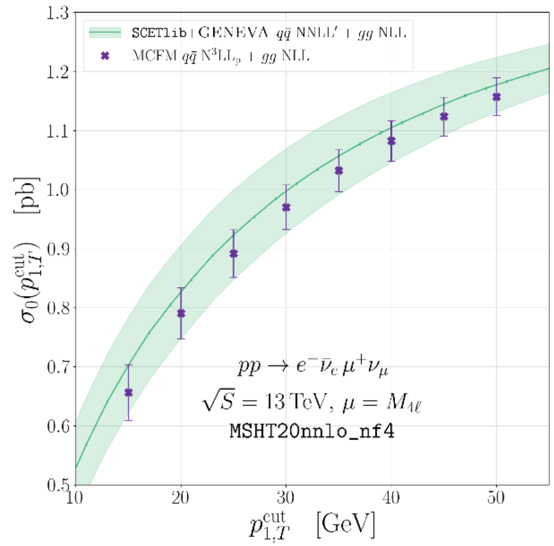
Validation of WW production

We include the resummation of the $q\bar{q}$ channel at NNLL’ and the gg channel at NLL

Jet veto resummation available in MCFM up to partial N3LL accuracy. Different treatment of uncertainties.

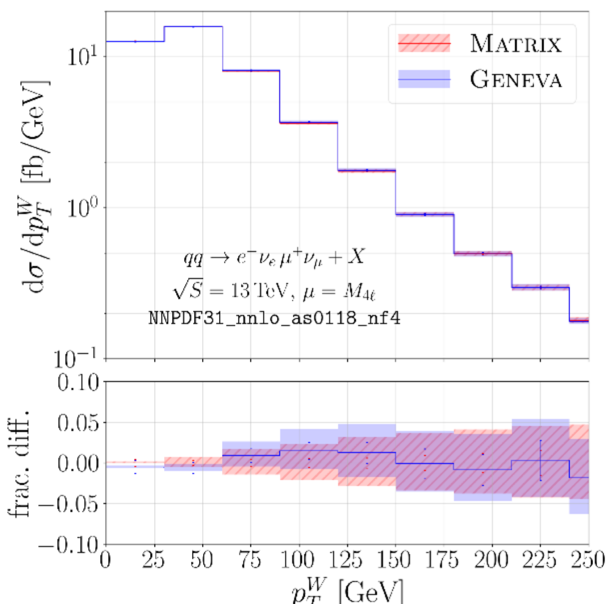
[Campbell et al. 2301.11768]

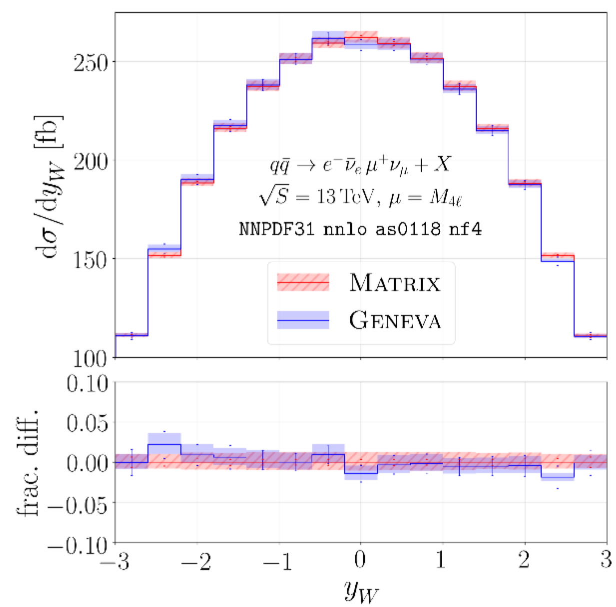




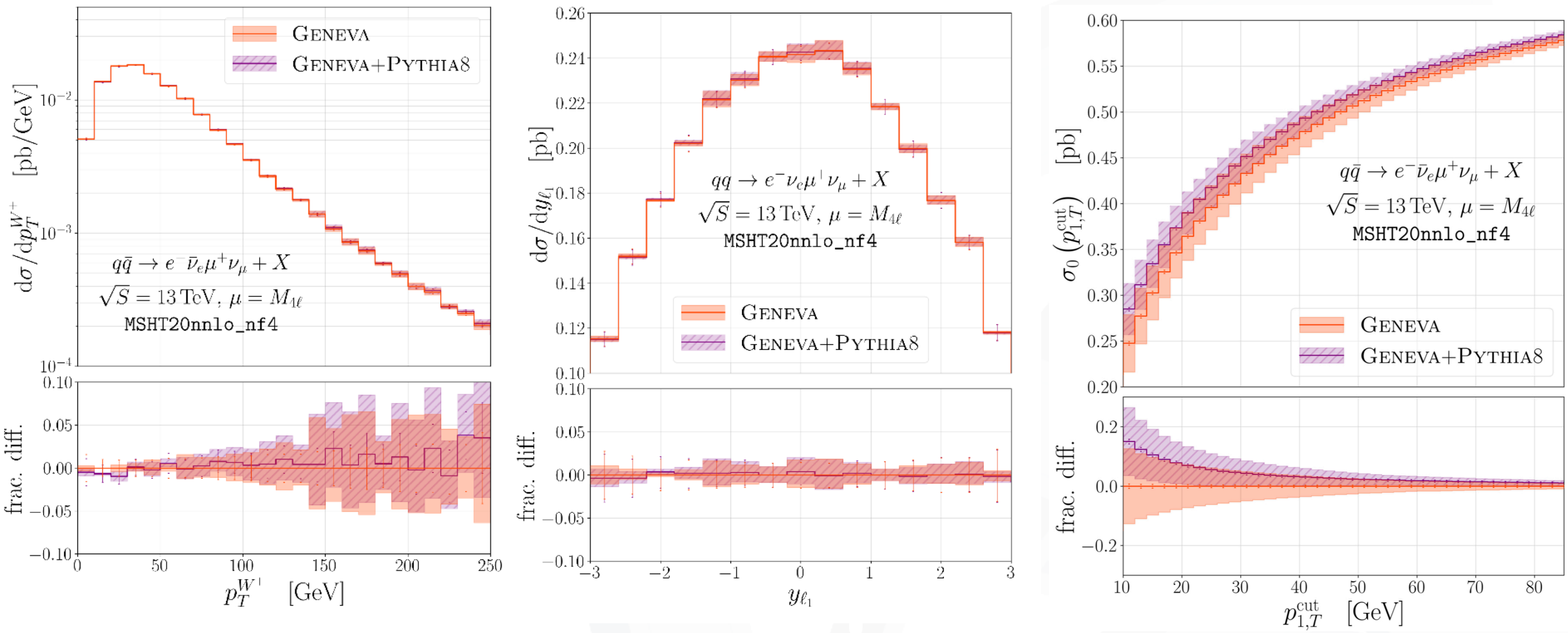
NNLO validation against MATRIX

[Grazzini et al. 1711.06631]





Showering



Inclusive quantities well-preserved by the shower, pT of the hardest jet is extremely sensitive to shower effects and gets mildly shifted. Few percent effect at 30 GeV.

This is entirely due to FSR emissions (the shower splits the hardest jet above pT cut into 2 jets below pT cut). Placing constraints to avoid this preserves pT_{1st} but not physically motivated.

Investigating resummation of different 1-jet resolution variable $\mathcal{T}_1^{k_T}$ (SCET II fact.)

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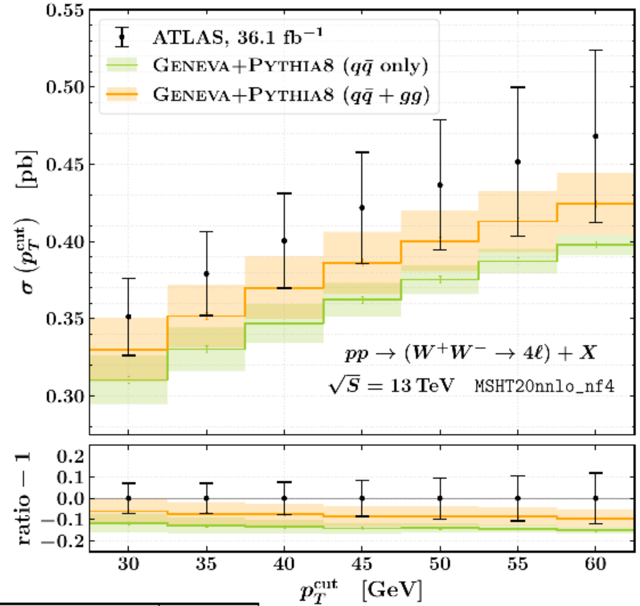
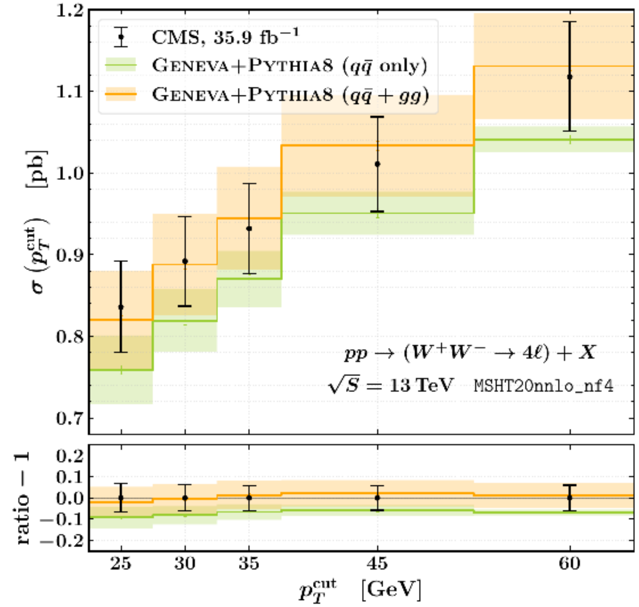
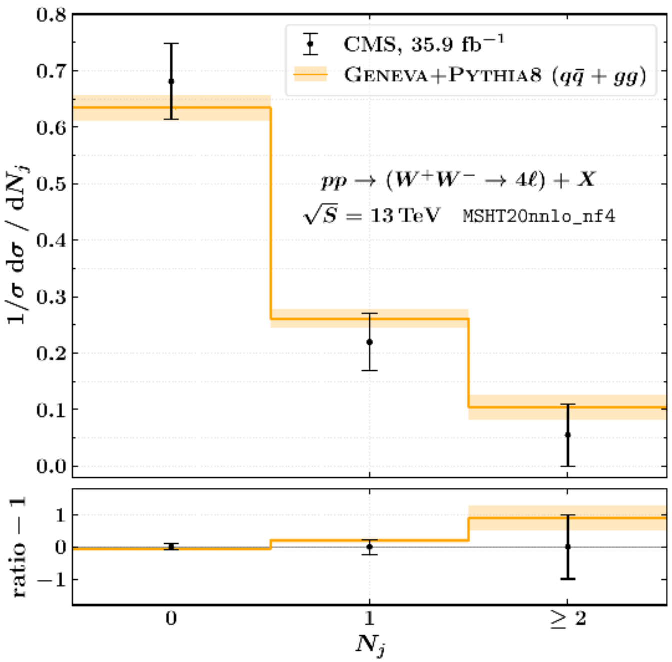
Data comparison

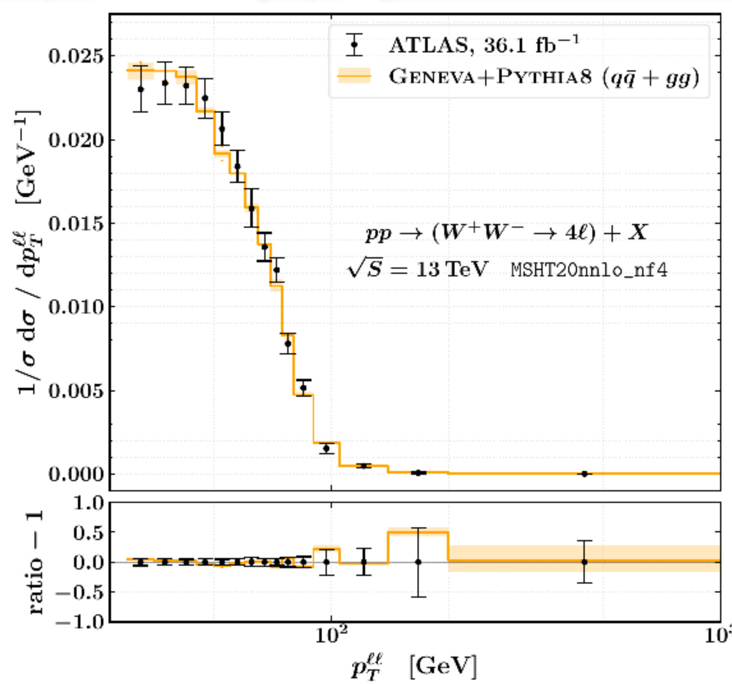
Inclusion of gg channel necessary for agreement with data.

Extension of gg channel to NLO+NLL' ongoing



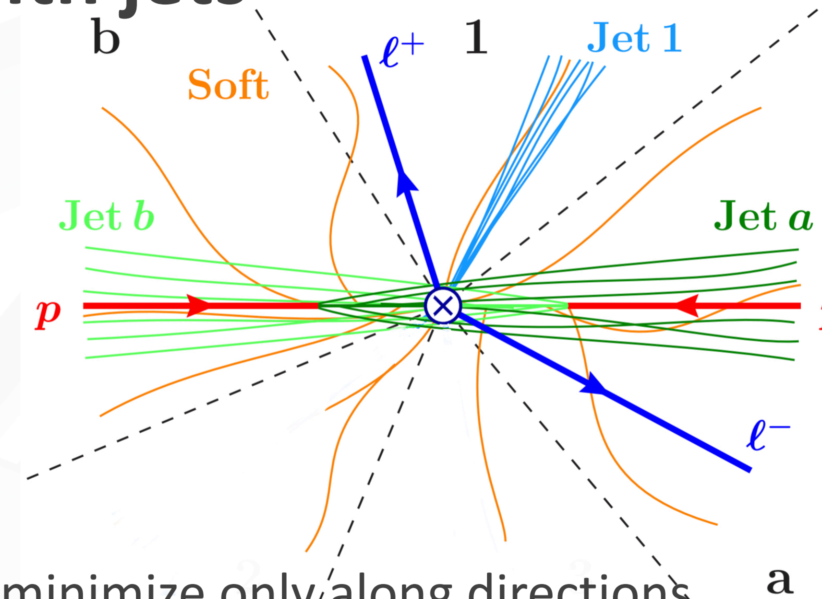






Extension to processes with jets

- Focus of color-singlet plus jet production



$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$

- To remove energy-dependence and minimize only along directions $Q_i = 2E_i$'s must be frame-dependent

$$\hat{\mathcal{T}}_1 = \sum_k \min\{\frac{\hat{n}_a \cdot \hat{p}_k}{\rho_a}, \frac{\hat{n}_b \cdot \hat{p}_k}{\rho_b}, \frac{\hat{n}_J \cdot \hat{p}_k}{\rho_J}\}$$

- The choice of the ρ_i's determines the frame in which the one-jettiness resummation is performed. Possible choices:
  LAB , UB -frame $Y_{Vj} = 0$ and CS-frame $Y_V = 0$

$$\rho_a = e^{\hat{Y}_V}\,, \quad \rho_b = e^{-\hat{Y}_V}\,, \quad \rho_J = \frac{e^{-\hat{Y}_V}(\hat{p}_J)_+ + e^{\hat{Y}_V}(\hat{p}_J)_-}{2\hat{E}_J}$$

- No preliminary jet clustering needed to find hard directions

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Resummation of one-jettiness for Z+jet

Factorization formula in the region $\mathscr{T}_1 \ll Q$ hard scale: $\sqrt{s}, M_{\ell^+\ell^-}, M_{T,\ell^+\ell^-}, \mathscr{T}_0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{\kappa=\{q\bar{q}g, qgq, ggg\}} H_\kappa(\Phi_1) \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J \, B_{\kappa_a}(t_a) B_{\kappa_b}(t_b) J_{\kappa_J}(s_J)$$
$$\times S_\kappa\left(n_{a,b}\cdot n_J, \mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}\right)$$

We left the choice of the frame free, keeping in mind the issues for GENEVA.

It is convenient to transform the soft, beam and jet functions in Laplace space to solve the RG equations, the factorization formula is turn into a product.
The color factorizes in soft and hard functions for 3 colored partons.

$$\mathcal{L}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1}\right] = \sum_\kappa H_\kappa(\Phi_1)\, \tilde{S}_\kappa\left(\ln\frac{\lambda_E^2}{\mu^2}\right) \tilde{B}_{\kappa_a}\left(\ln\frac{Q_a\,\lambda_E}{\mu^2}\right) \tilde{B}_{\kappa_b}\left(\ln\frac{Q_b\,\lambda_E}{\mu^2}\right) \tilde{J}_{\kappa_J}\left(\ln\frac{Q_J\,\lambda_E}{\mu^2}\right)$$

Hard, soft, beam and jet functions

Hard functions known analytically up to 2-loops. [Gehrmann, Tancredi et al. `12, `22]

From NNLL' accuracy include the loop-squared $gg \to Zg$, although numerically very small (per mille)

Beam and jet boundary conditions known up to 3-loop [Mistlberger et al. `20]

[Becher, Bell `10] [Gaunt et al. `14]

We compute the one-loop soft boundary terms as on-the-fly integrals using results in

[Jouttenus et al. `11]

$$S^{\kappa\,(1)}_{\mathcal{T}_1,-1} = 2c^\kappa_s\Big[L^2_{ab} - \frac{\pi^2}{6} + 2(I_{ab,c} + I_{ba,c})\Big] + 2c^\kappa_t\Big[L^2_{ac} - \frac{\pi^2}{6} + 2(I_{ac,b} + I_{ca,b})\Big] + 2c^\kappa_u\Big[L^2_{bc} - \frac{\pi^2}{6} + 2(I_{bc,a} + I_{cb,a})\Big]$$

use the following abbreviation for the finite integrals

$$I_{ij,m} \equiv I_0\Big(\frac{\hat{s}_{jm}}{\hat{s}_{ij}}, \frac{\hat{s}_{im}}{\hat{s}_{ij}}\Big)\ln\frac{\hat{s}_{jm}}{\hat{s}_{ij}} + I_1\Big(\frac{\hat{s}_{jm}}{\hat{s}_{ij}}, \frac{\hat{s}_{im}}{\hat{s}_{ij}}\Big)$$

Also studied for different jet measures in

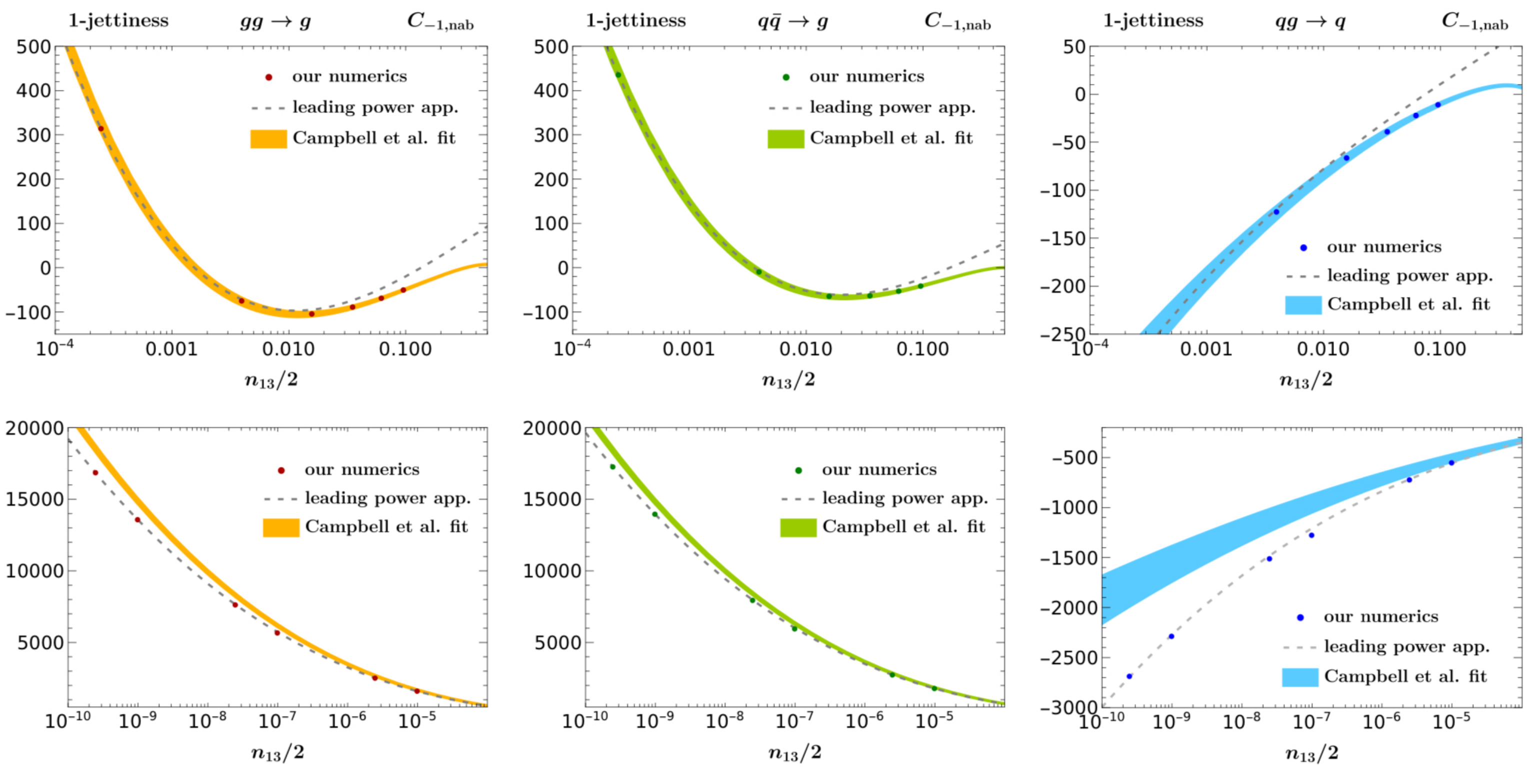
[Bertolini et al. `17]

Hard, soft, beam and jet functions

The 2-loop contribution $S^{\kappa\,(2)}_{\mathcal{T}_1\,-1}$ newly calculated via SoftServe, interfaced to GENEVA in the form of an interpolation grid [Bell, Dehnadi, Morhmann, Rahn `23]

Approach validated comparing to the interpolation used in MCFM.

[Campbell, Ellis, Mondini, Williams `18]



Refined treatment reproduces leading power behavior at extreme angles, important for resummation >= NNLL' and for N3LO singular contribution

Hard evolution

For every channel ($q\bar{q}g$, qgq, ggg,…), ***hard anomalous dimension*** has the form [T. Becher and M. Neubert 1908.11379]

$$\mathbf{\Gamma}_C^\kappa(\mu) = \Gamma_C^\kappa(\mu)\ \mathbf{1} = \left\{\frac{\Gamma_{\text{cusp}}(\alpha_s)}{2}\left[(C_c - C_a - C_b)\ln\frac{\mu^2}{(-s_{ab}-\text{i}0)} + \text{cyclic permutations}\right]\right.$$

4-loops

$$\left. + \gamma_C^a(\alpha_s) + \gamma_C^b(\alpha_s) + \gamma_C^c(\alpha_s) + \frac{C_A^2}{8} f(\alpha_s)(C_a + C_b + C_c)\right\}\mathbf{1}$$

3-loops

$$+ \sum_{(i,j)}\left[-f(\alpha_s)\boldsymbol{\mathcal{T}}_{iijj} + \sum_{R=F,A} g^R(\alpha_s)(3\boldsymbol{\mathcal{D}}^R_{iijj} + 4\boldsymbol{\mathcal{D}}^R_{iiij})\ln\frac{\mu^2}{(-s_{ij}-\text{i}0)}\right] + \mathcal{O}(\alpha_s^5)$$

$f(\alpha_s)$ and $g^R(\alpha_s)$ start at $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^4)$ computed in [Henn, Korchemsky, Mistlberger 1911.10174], [Von Manteuffel, Panzer, Schabinger 2002.04617]. Evaluated these contributions as functions of N_c using the *colour space formalism*

$$\boldsymbol{\mathcal{D}}^R_{ijkl} = d_R^{abcd}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\boldsymbol{T}_l^d \qquad \boldsymbol{\mathcal{T}}_{ijkl} = f^{ade}f^{bce}(\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\boldsymbol{T}_l^d)_+$$

$$d_R^{a_1\ldots a_n} = \text{Tr}_R(\boldsymbol{T}^{a_1}\ldots\boldsymbol{T}^{a_n})_+ \equiv \frac{1}{n!}\sum_\pi \text{Tr}(\boldsymbol{T}_R^{a_{\pi(1)}}\ldots\boldsymbol{T}_R^{a_{\pi(n)}})$$

Using color conservation and symmetry properties of d_R^{abcd}, we found the following relations

$$3(\boldsymbol{\mathcal{D}}^R_{iijj} + \boldsymbol{\mathcal{D}}^R_{jjii}) + 4(\boldsymbol{\mathcal{D}}^R_{iiij} + \boldsymbol{\mathcal{D}}^R_{jjji}) = (D_{kR} - D_{iR} - D_{jR})\ \mathbf{1} \qquad i \neq j \neq k$$

Quartic Casimirs

Similarity to the quadratic case $\quad \boldsymbol{T}_a\cdot\boldsymbol{T}_b = [\boldsymbol{T}_c^2 - \boldsymbol{T}_a^2 - \boldsymbol{T}_b^2]/2$

$$C_4(R_i, R) = \frac{d_{R_i}^{abcd}d_R^{abcd}}{N_{R_i}} \equiv D_{iR}$$

Hard evolution

$$\Gamma_C^\kappa(\mu) = \left[-\bar{c}^\kappa \Gamma_{\rm cusp}(\alpha_s) + \sum_{R=F,A} \bar{c}_4^{\kappa,\,R}\, g^R(\alpha_s)\right] \ln\frac{Q^2}{\mu^2} + \sum_{i=a,b,c} \gamma_C^i(\alpha_s) + f(\alpha_s)\, c_f^\kappa - \bar{c}_L^\kappa \Gamma_{\rm cusp}(\alpha_s) + \sum_{R=F,A} g^R(\alpha_s)\, \bar{c}_{4,\,L}^{\kappa,\,R}$$

$$c_f^\kappa = -\left[\frac{C_A^2}{4}\bar{c}^\kappa + \sum_{i\neq j} \frac{\langle \mathcal{M}|\boldsymbol{T}_{iijj}|\mathcal{M}\rangle}{\langle \mathcal{M}|\mathcal{M}\rangle}\right]$$

$$\bar{c}^\kappa = c_s^\kappa + c_u^\kappa + c_t^\kappa = -(C_a + C_b + C_c)/2 \quad\longleftrightarrow\quad \bar{c}_4^{\kappa,\,R} = D_{aR} + D_{bR} + D_{cR}$$

$$\bar{c}_L^\kappa = c_s^\kappa L_s + c_u^\kappa L_u + c_t^\kappa L_t \qquad \bar{c}_{4,\,L}^{\kappa,\,R} \equiv c_{4,\,s}^{\kappa,\,R} L_s + c_{4,\,u}^{\kappa,\,R} L_u + c_{4,\,t}^{\kappa,\,R} L_t$$

$$c_s^\kappa = \boldsymbol{T}_a \cdot \boldsymbol{T}_b\,, \quad c_u^\kappa = \boldsymbol{T}_b \cdot \boldsymbol{T}_c\,, \quad c_t^\kappa = \boldsymbol{T}_a \cdot \boldsymbol{T}_c \quad\longleftrightarrow\quad c_{4,\,s}^{\kappa,\,R} = D_{aR} + D_{bR} - D_{cR}$$

$$c_{4,\,t}^{\kappa,\,R} = D_{aR} + D_{cR} - D_{bR}$$

$$c_{4,\,u}^{\kappa,\,R} = D_{bR} + D_{cR} - D_{aR}$$

Kinematic dependent logs

$$L_s = \ln\frac{-s_{ab} - i0}{Q^2} = \ln\frac{s_{ab}}{Q^2} - i\pi$$

$$L_u = \ln\frac{s_{bc}}{Q^2} \qquad L_t = \ln\frac{s_{ac}}{Q^2}$$

Beam, Jet and soft evolution

Beam and Jet functions in ***Laplace space:***

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln\tilde{B}_a(\varsigma_B,x,\mu) = -2\Big[C_a\,\Gamma_{\mathrm{cusp}}(\alpha_s) + 2\sum_{R=F,A} D_{aR}\;g^R(\alpha_s)\Big]\ln\Big(\frac{Q_a\varsigma_B}{\mu^2}\Big) + \gamma_B^a(\alpha_s)$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln\tilde{J}_c(\varsigma_J,\mu) = -2\Big[C_c\,\Gamma_{\mathrm{cusp}}(\alpha_s) + 2\sum_{R=F,A} D_{cR}\;g^R(\alpha_s)\Big]\ln\Big(\frac{Q_J\varsigma_J}{\mu^2}\Big) + \gamma_J^c(\alpha_s)$$

The soft functions depend on $\hat{s}_{ij} = \dfrac{2\,q_i\cdot q_j}{Q_iQ_j}$ which are frame dependent

$$\hat{s}_{aJ}^{\mathrm{LAB}} = \frac{n_a\cdot n_J}{2} = \rho_a\rho_J\,\hat{s}_{aJ}^{\mathrm{CS}}$$

Moderately sized $\hat{s}_{aJ}^{\mathrm{CS}}$ may require to evaluate the LAB-frame soft function at very small values of $\hat{s}_{aJ}^{\mathrm{LAB}}$ depending on the boost factor $\rho_a\rho_J$

Soft functions in ***Laplace space:***

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln\tilde{S}^\kappa(\varsigma_S,\mu) = 2\left[-\bar{c}^\kappa\Gamma_{\mathrm{cusp}}(\alpha_s) + \sum_{R=F,A}\bar{c}_4^{\kappa,R}g^R(\alpha_s)\right]\ln\left(\frac{\varsigma_S^2}{\mu^2}\right)$$
$$+\Big[\gamma^\kappa_{S_{N=1}}(\alpha_s) + 2\Gamma_{\mathrm{cusp}}(\alpha_s)\big(c_s^\kappa L_{ab} + c_t^\kappa L_{ac} + c_u^\kappa L_{bc}\big)$$
$$-2\sum_{R=F,A} g^R(\alpha_s)\big(c_{4,s}^{\kappa,R}L_{ab} + c_{4,t}^{\kappa,R}L_{bc} + c_{4,u}^{\kappa,R}L_{bc}\big)\Big]$$

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N3LL resummed formula

Combine the solutions to the RG equations for the hard, soft, beam and jet functions to obtain

$$
\begin{aligned}
\frac{d\sigma^{\mathrm{N^3LL}}}{d\Phi_1 d\mathcal{T}_1} = \sum_{\kappa} \exp\Big\{ & 4(C_a + C_b) K_{\Gamma_{\rm cusp}}(\mu_B, \mu_H) + 4 C_c K_{\Gamma_{\rm cusp}}(\mu_J, \mu_H) - 2(C_a + C_b + C_c) K_{\Gamma_{\rm cusp}}(\mu_S, \mu_H) \\
& - 2 C_c L_J\, \eta_{\Gamma_{\rm cusp}}(\mu_J, \mu_H) - 2(C_a L_B + C_b L'_B) \eta_{\Gamma_{\rm cusp}}(\mu_B, \mu_H) + K_{\gamma_{\rm tot}} \\
& + \left[C_a \ln\left(\frac{Q_a^2 u}{st}\right) + C_b \ln\left(\frac{Q_b^2 t}{su}\right) + C_{\kappa_j} \ln\left(\frac{Q_J^2 s}{tu}\right) + (C_a + C_b + C_c) L_S \right] \eta_{\Gamma_{\rm cusp}}(\mu_S, \mu_H) \\
& + \sum_{R=F,A} \Big[8\,(D_{aR} + D_{bR}) K_{g^R}(\mu_B, \mu_H) + 8\, D_{cR} K_{g^R}(\mu_J, \mu_H) \\
& - 4\,(D_{aR} + D_{bR} + D_{cR}) K_{g^R}(\mu_S, \mu_H) - 4 D_{cR} L_J \eta_{g^R}(\mu_J, \mu_H) - 4\,(D_{aR} L_B + D_{bR} L'_B) \eta_{g^R}(\mu_B, \mu_H) \\
& + 2\left[D_{aR} \ln\left(\frac{Q_a^2 u}{st}\right) + D_{bR} \ln\left(\frac{Q_b^2 t}{su}\right) + D_{cR} \ln\left(\frac{Q_J^2 s}{tu}\right) + (D_{aR} + D_{bR} + D_{cR}) L_S \right] \eta_{g^R}(\mu_S, \mu_H) \Big] \Big\} \\
& \times H_\kappa(\Phi_1, \mu_H) \tilde{S}^\kappa(\partial_{\eta_S} + L_S, \mu_S) \tilde{B}_{\kappa_a}(\partial_{\eta_B} + L_B, x_a, \mu_B) \tilde{B}_{\kappa_b}(\partial_{\eta'_B} + L'_B, x_b, \mu_B)\, \tilde{J}_{\kappa_J}(\partial_{\eta_J} + L_J, \mu_J) \\
& \times \frac{Q^{-\eta_{\rm tot}}}{\mathcal{T}_1^{\,1-\eta_{\rm tot}}} \frac{\eta_{\rm tot}\, e^{-\gamma_E \eta_{\rm tot}}}{\Gamma(1 + \eta_{\rm tot})}
\end{aligned}
$$

Up to NNLL′

where we defined $\eta_{\rm tot} = \eta_B + \eta'_B + \eta_J + 2\eta_S$

$$
L_H = \ln\left(\frac{Q^2}{\mu_H^2}\right), \quad L_B = \ln\left(\frac{Q_a Q}{\mu_B^2}\right), \quad L'_B = \ln\left(\frac{Q_b Q}{\mu_B^2}\right)
$$

$$
L_J = \ln\left(\frac{Q_J Q}{\mu_J^2}\right), \quad L_S = \ln\left(\frac{Q^2}{\mu_S^2}\right)
$$

$$
K_{g^R}(\mu_H, \mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} g^R(\alpha_s) \int_{\alpha_s(\mu_H)}^{\alpha_s} \frac{d\alpha'_s}{\beta[\alpha'_s]}
$$

$$
\eta_{g^R}(\mu_H, \mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} g^R(\alpha_s)
$$

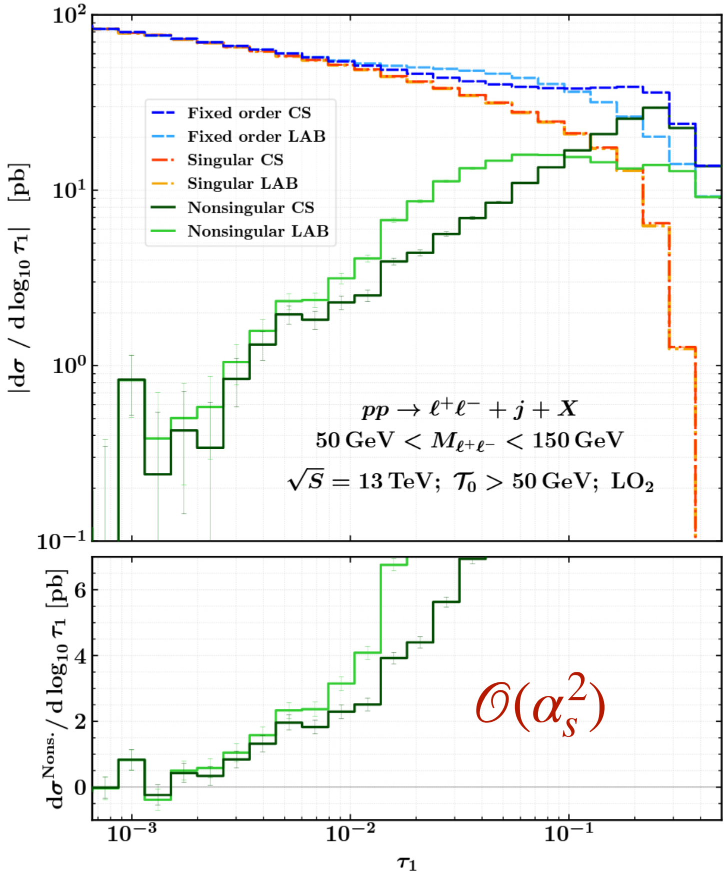
$$
K_f(\mu_H, \mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} f(\alpha_s)
$$

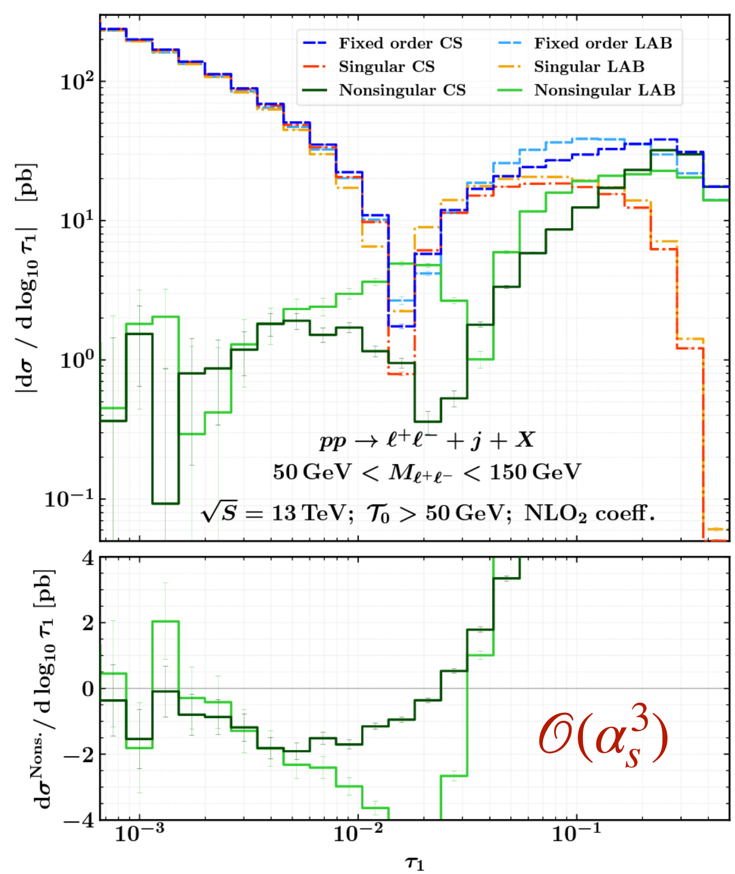
Nonsingular behavior

Dimensionless definition

$$\tau_1 = 2\mathcal{T}_1/\sqrt{M_{\ell^+\ell^-}^2 + q_T^2}$$

- Different $\mathcal{T}_1$ choices have different subleading power corrections
- Investigated for one-jettiness subtraction at LL NLP [Boughezal, Isgro', Petriello `20]





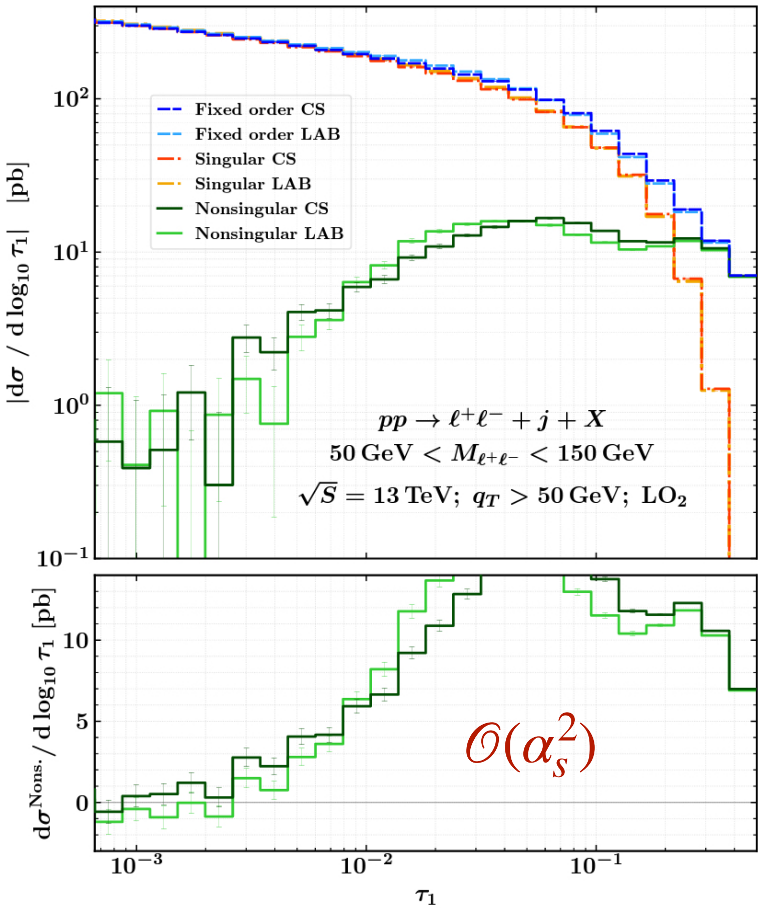
- CS frame better than LAB across different cuts. UB frame delicate for IR safety.

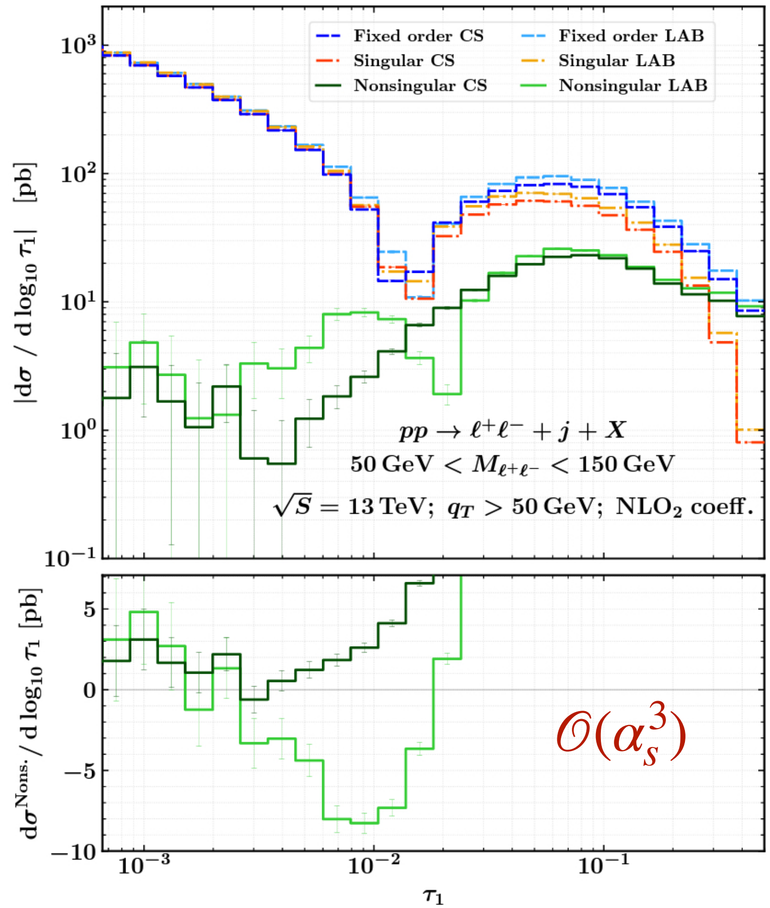
Nonsingular behavior

Dimensionless definition

$$\tau_1 = 2\mathscr{T}_1/\sqrt{M^2_{\ell^+\ell^-} + q_T^2}$$

- Reduced differences when cutting on Z boson trans. momentum q_T





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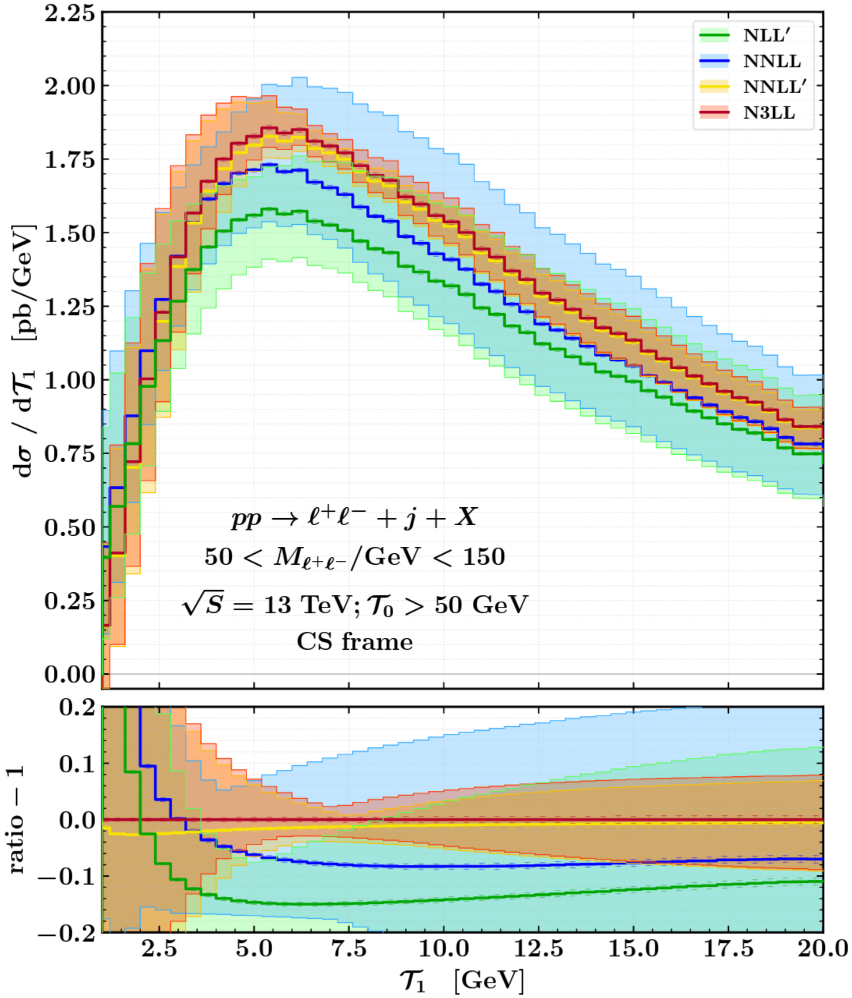
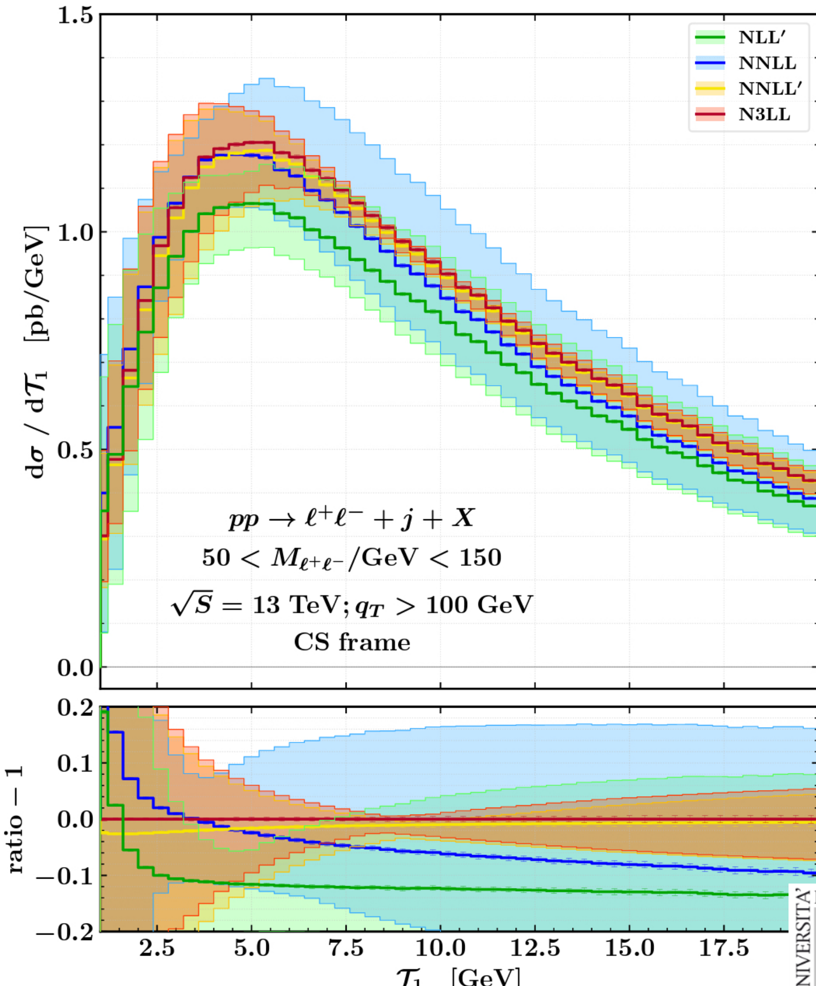
Resummed results

- Summing in quadrature profile scales variations and fixed-order ones
- Nice convergence and reduction of theoretical uncertainties





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Two dimensional profile scales

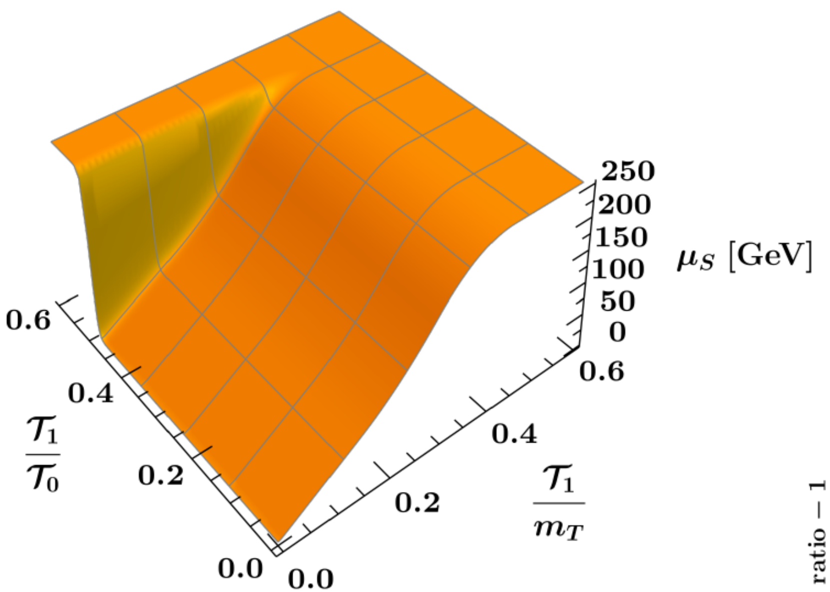
A final state with N particles is subject to the constraint

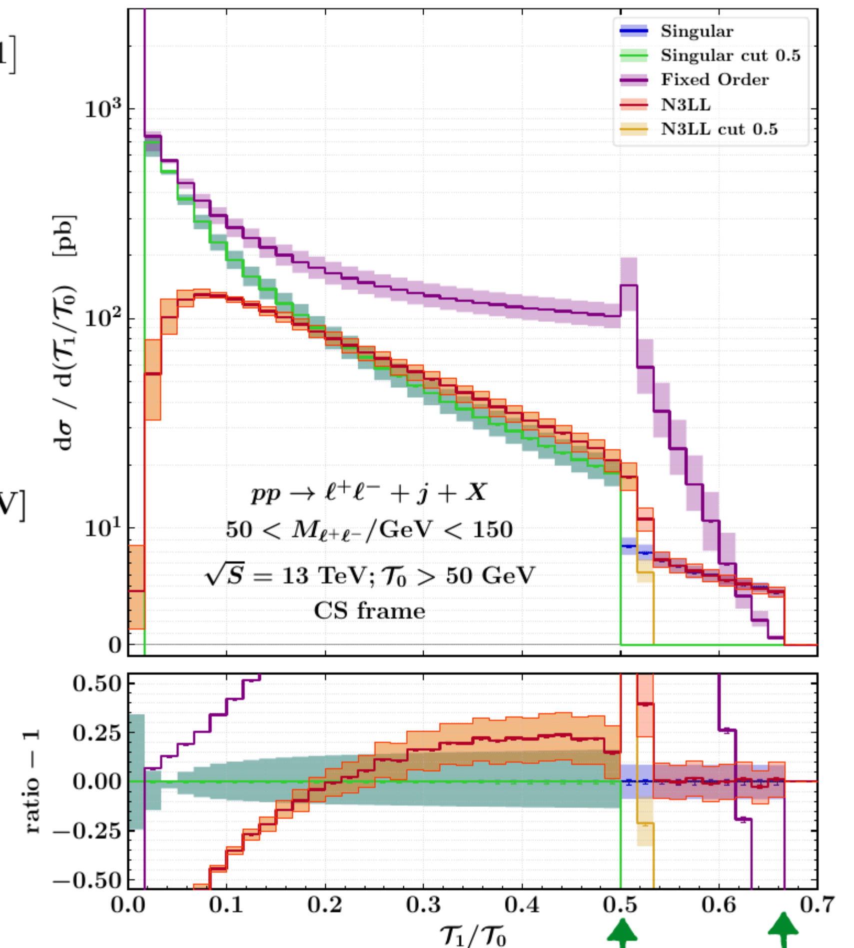
$$\frac{\mathcal{T}_1(\Phi_N)}{\mathcal{T}_0(\Phi_N)} \leq \frac{N-1}{N} = \begin{cases} 1/2\,, & N=2 \\ 2/3\,, & N=3 \end{cases}$$

$$\mu_S(\mathcal{T}_1/\mu_{\rm FO},\mathcal{T}_1/\mathcal{T}_0) = \mu_{\rm FO}\big[\big(f_{\rm run}(\mathcal{T}_1/\mu_{\rm FO})-1\big)s^{(p,k)}(\mathcal{T}_1/\mathcal{T}_0)+1\big]$$

Behaves as smooth Theta function

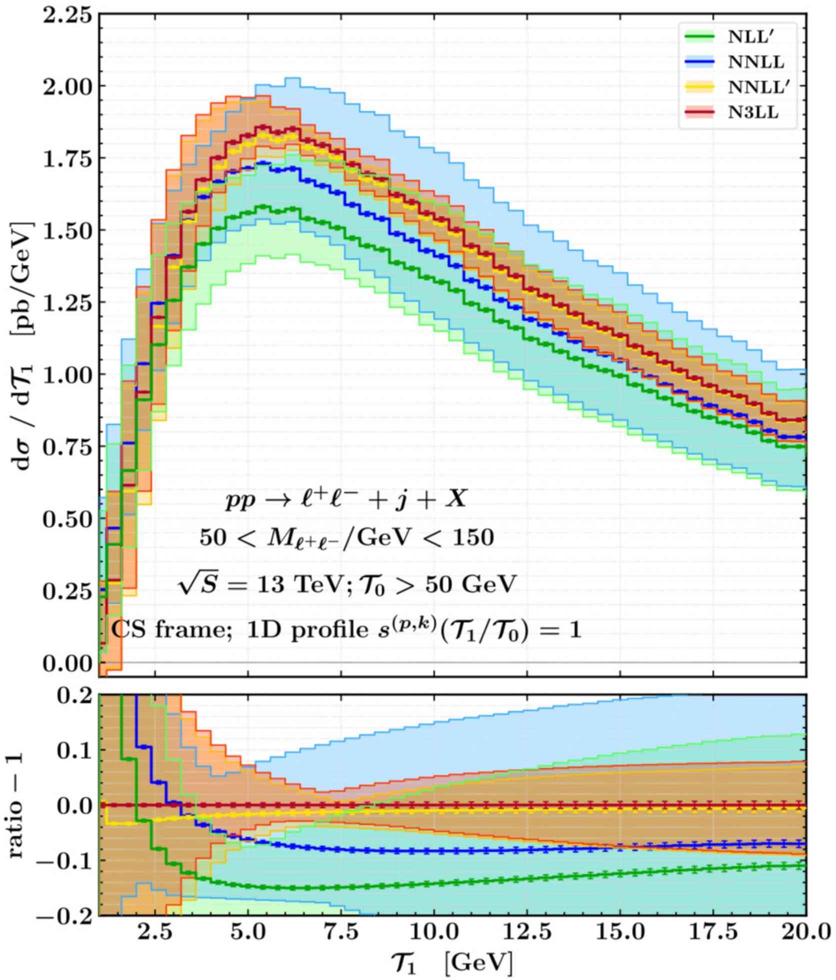
$$s^{(p,k)}(\mathcal{T}_1/\mathcal{T}_0) = \frac{1}{1+e^{pk(\mathcal{T}_1/\mathcal{T}_0-1/p)}}$$

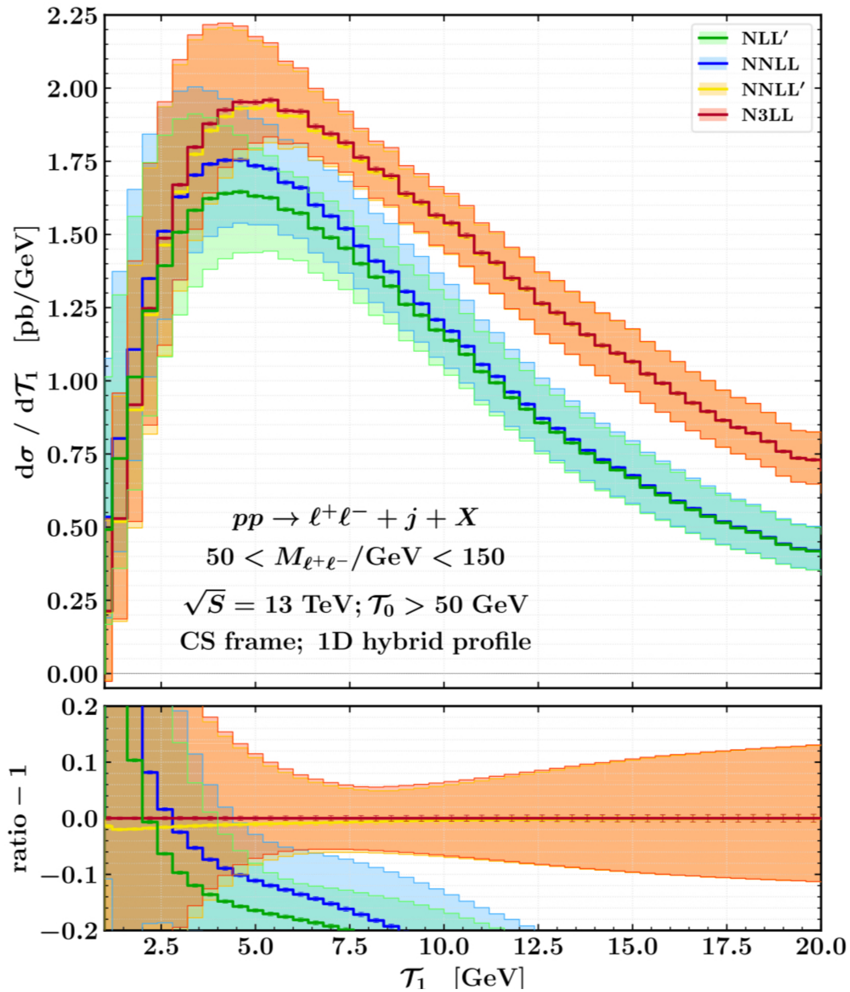




We use $p = 2$ (determines the transition point) and $k = 100$ (slope of the transition)

Alternative profile scales: 1D and hybrid





$$\mu_S(\mathcal{T}_1/\mu_{\mathrm{FO}}, \mathcal{T}_1/\mathcal{T}_0) = \mu_{\mathrm{FO}} f_{\mathrm{run}}(\mathcal{T}_1/\mathcal{T}_0) + \mathcal{T}_1(1 - f_{\mathrm{run}}(\mathcal{T}_1/\mathcal{T}_0))$$

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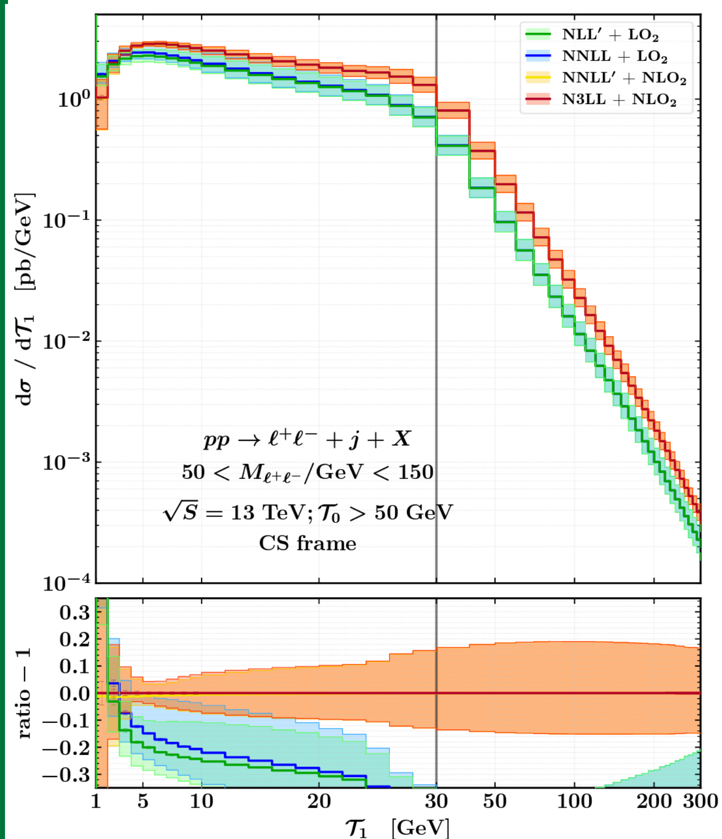
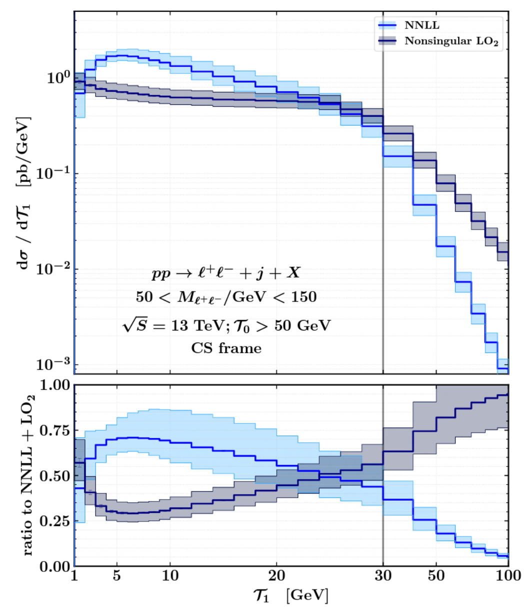
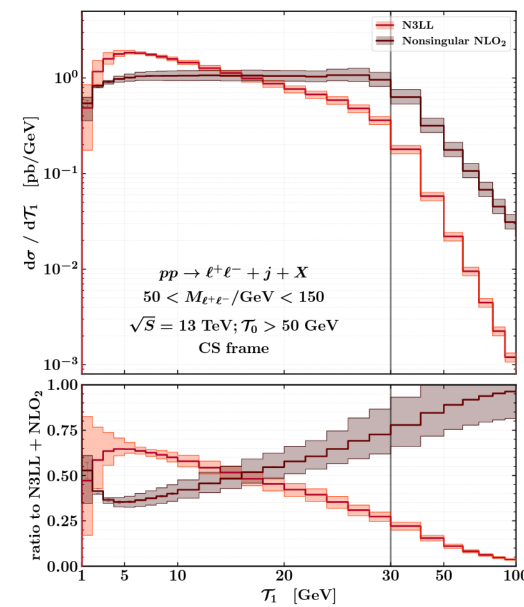
Matched results

$$\frac{d\sigma^{\text{match.}}}{d\Phi_1 d\mathcal{T}_1} = \frac{d\sigma^{\text{res.}}}{d\Phi_1 d\mathcal{T}_1} + \frac{d\sigma^{\text{f.o.}}}{d\Phi_1 d\mathcal{T}_1} - \frac{d\sigma^{\text{res.exp.}}}{d\Phi_1 d\mathcal{T}_1}$$



- $\mathcal{O}(\alpha_s^3)$ gives sizable contribution, important to include it for small values of $\mathcal{T}_0$

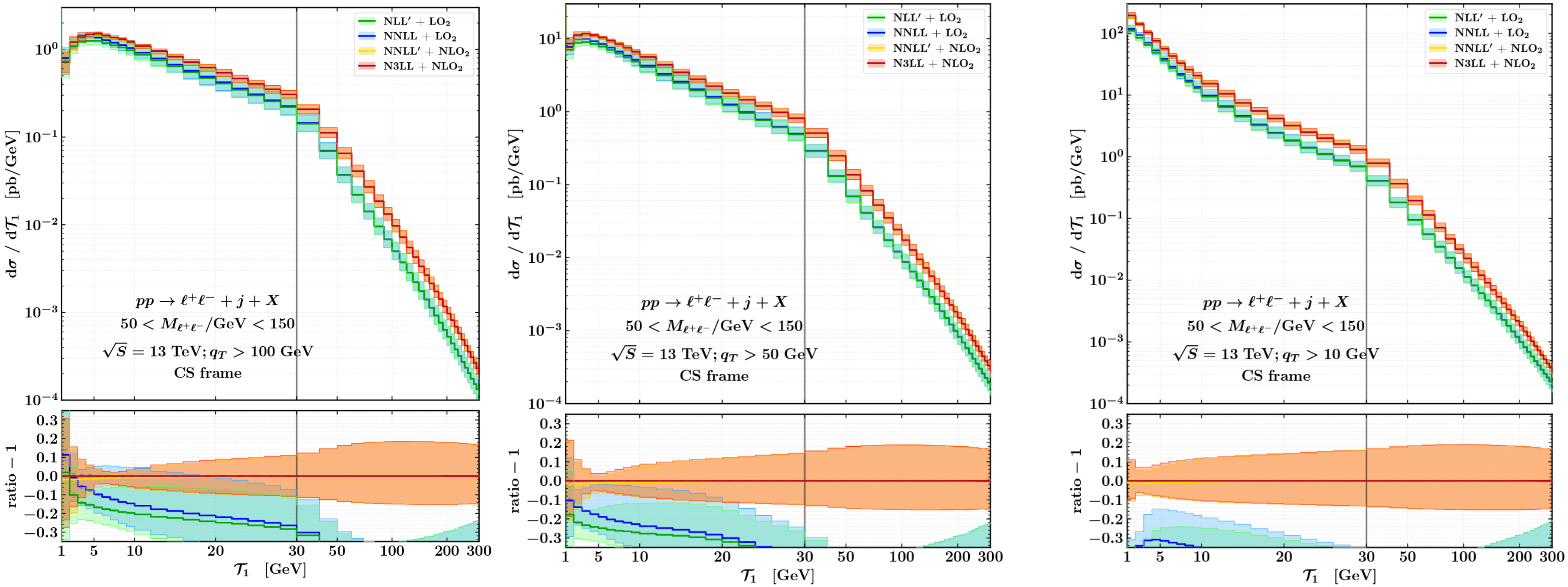




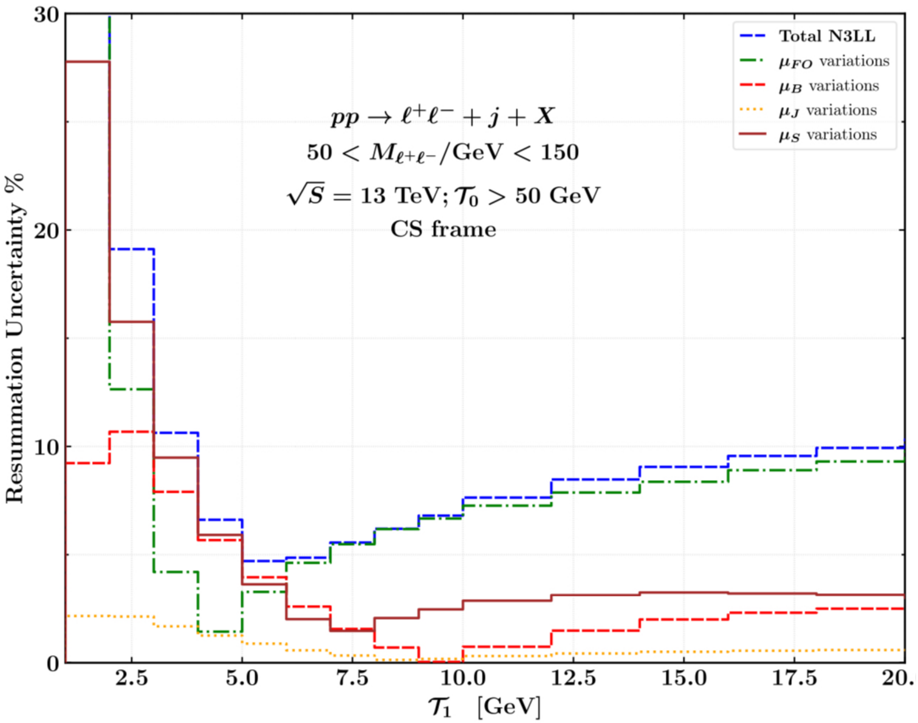
- Nonsingular divergent for $\mathcal{T}_0 \to 0$. Joint $(\mathcal{T}_0, \mathcal{T}_1)$ resummation required to handle both divergencies

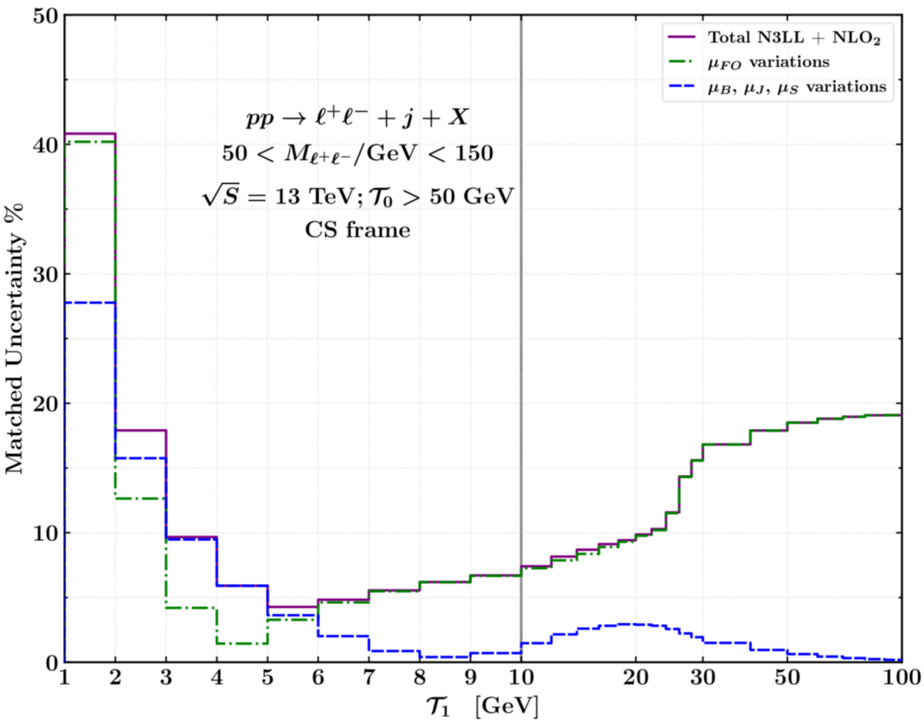
Matched results

- Similarly large nonsingular contribution when cross section define by cut on Z transverse momentum in the limit $q_T \to 0$



Final uncertainty budget





Conclusion and outlook

- The inclusion of state-of-the-art theoretical predictions in SMC generators is mandatory to match the experimental precision and fully exploit the discovery potential of LHC measurements

- GENEVA method allows for interfacing higher-order resummation of resolution variables in event generation with NNLO accuracy and parton showers.

- Several color-singlet processes implements, using different resolution variables: N-jettiness, qT, jet veto…

- Implemented one-jettiness resummation, prerequisite for Vj@NNLO+PS in GENEVA. Studied different $\mathcal{T}_1$ definitions, performed resummation up to N3LL and matched to corresponding fixed-order. Observed nice convergence and reduction of theory unc. in presence of an hard jet.

Thank you for your attention.

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BACKUP

Zero-jettiness factorization for top-quark pairs

Factorization formula derived using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. [SA et al. 2111.03632]

In case of boosted regime $M_{t\bar{t}} \gg m_t$ one would instead need a modified two-jettiness [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int \mathrm{d}t_a\, \mathrm{d}t_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Hard functions (color matrices)

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Zero-jettiness resummation for top pairs

Resummed formula valid up to NNLL' accuracy

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \mathrm{Tr}\Big\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \Big\}$$
$$\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}\,.$$

where

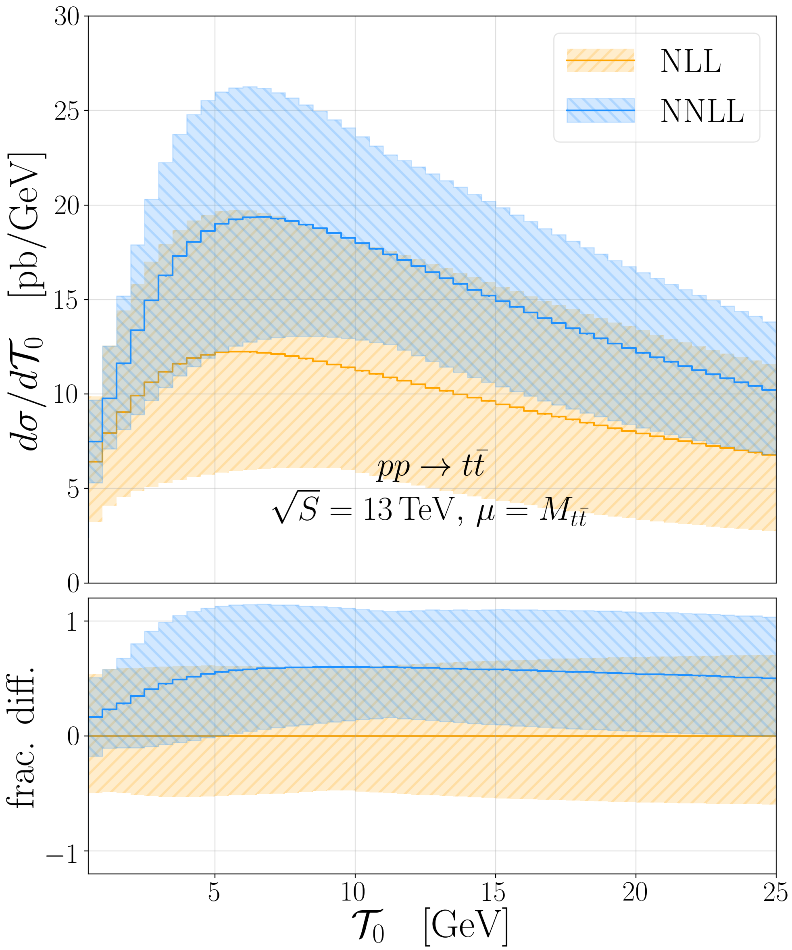
$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) =$$
$$\exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \right]$$

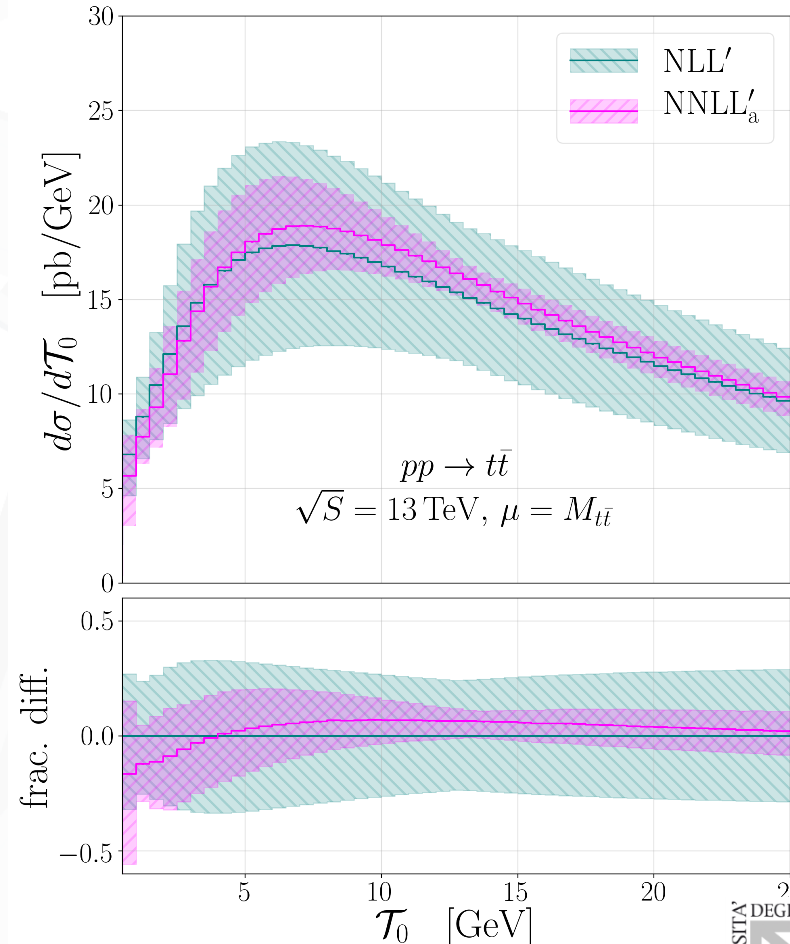
and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

The final accuracy depends on the availability of the perturbative ingredients

Resummed results

NNLL'_a is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales





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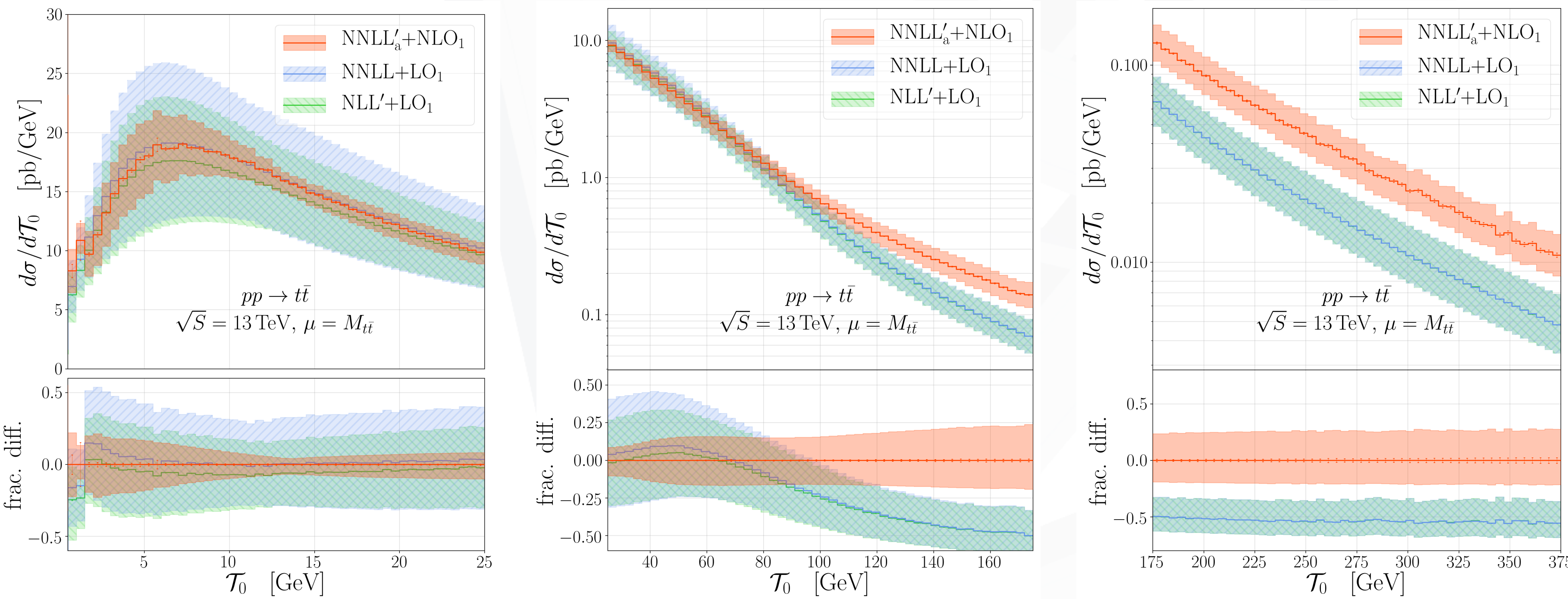
Matched results

Matching to $t\bar{t}+j$ @NLO improves the perturbative accuracy across the whole spectrum

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$

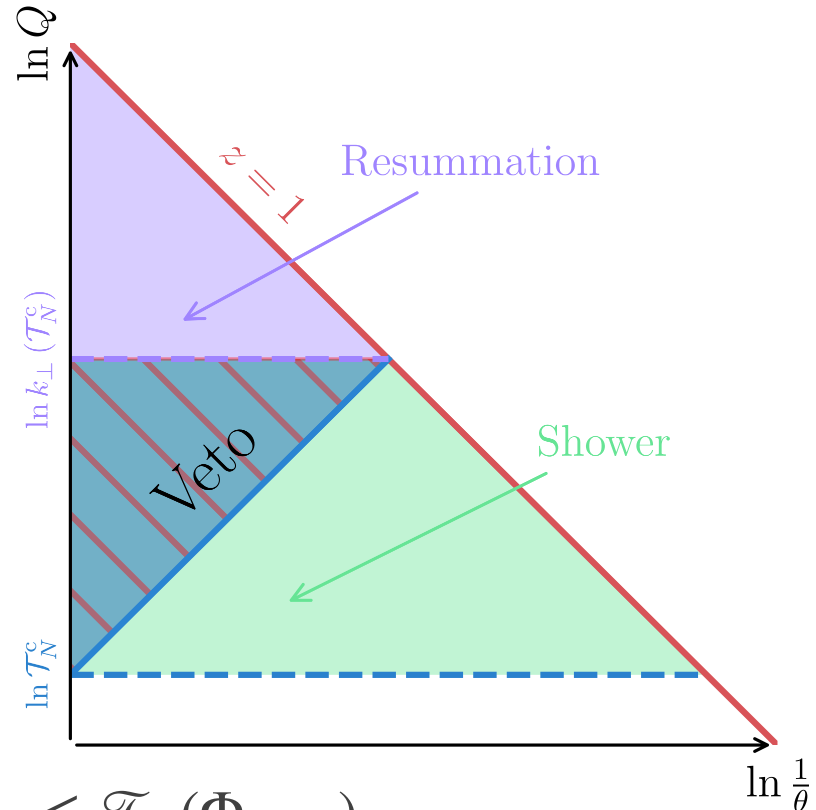


Extension to full NNLL' and to event generation is in progress.

Interface with the parton shower

$\mathcal{T}_N(\Phi_{N+1})$ measures the hardness of the N+1-th emission

- If shower ordered in k_T, start from largest value allowed by N-jettiness
- Let the shower evolve unconstrained.
- At the end veto an event if after $M \geq 1$ shower emissions $\mathcal{T}_N(\Phi_{N+M}) > \mathcal{T}_N(\Phi_N + 1)$ and **retry** the whole shower.



$$\mathcal{T}_{N+M-1}(\Phi_{N+M}) \leq \mathcal{T}_{N+M-2}(\Phi_{N+M}) \leq \ldots \leq \mathcal{T}_N(\Phi_{N+M})$$

Ensures the relevant phase space is correctly covered to avoid spoiling the resummation accuracy for $\mathcal{T}$. Shower accuracy for other observables is more delicate for dipole shower, effects numerically negligible .

0-jet and 1-jet bins are treated differently: starting scale is resolution cutoff.

Method rather independent from shower used: PYTHIA8, DIRE & SHERPA.

Interface with the parton shower

Effect of shower on resolution variables different from what is resummed more marked, albeit shower accuracy is maintained.

GENEVA framework allows this comparison for DY when resumming q_T or $\mathscr{T}_0$

Best approach here would be joint $(\mathscr{T}_0, \vec{q}_T)$ resummation, avoids need of splitting func.