Recent Progress in EW Calculations

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Outline

1. Introduction
2. Automated tools for NLO EW corrections
3. Logarithmic approximation of EW corrections
4. Recent calculations for specific processes
5. Polarised vector bosons
6. Conclusion and outlook
7. Backup

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Relevance of electroweak (EW) corrections

Generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_\mathrm{s}^2)$ $\Rightarrow$ NLO EW $\sim$ NNLO QCD
typical: few per cent (for inclusive observables)

systematic enhancements

- by (soft and/or collinear) photon emission:
  kinematic effects such as radiative tails
  collinear logarithms $\propto \alpha \ln(m_\mu/Q)$ for bare muons
  $\Rightarrow$ huge effects ($> 100\%$) possible (in radiative tails)
- at high energies:
  EW Sudakov logarithms $\propto (\alpha/s_\mathrm{w}^2) \ln^2(M_\mathrm{W}/Q)$ and subleading logs
  $\Rightarrow$ EW corrections of several 10% in high-energy tails of distributions
  or cross sections dominated by high scales

$\Rightarrow$ NLO EW corrections can be sizeable
$\Rightarrow$ must be included in theoretical predictions

automation of (fixed-order) NLO EW corrections basically done

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Automated tools for NLO EW corrections

NLO EW matrix element providers

tool	collaboration	
GoSam	Chiesa et al.	1407.0823
MadGraph5_aMC@NLO	Frixione et al.	1804.10017
NLOX	Honeywell et al.	1812.11925, 2101.01305
OpenLoops	Pozzorini et al.	1907.13071
Recola	Actis et al.	1211.6316, 1605.01090

$2 \to 6$ and simpler processes routinely available.

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Automated tools for NLO EW corrections

State-of-the-art applications:

- $2 \to 6$ processes

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{b}\bar{\mathrm{b}}$ $(\mathrm{t}\bar{\mathrm{t}})$	Denner, Pellen 1607.05571
$\mathrm{pp} \to 4\ell\mathrm{jj}$ (VBS)	Denner et al. 1611.02951, 1708.00268, 1904.00882, 2009.00411, 2107.10688, 2202.10844 Dittmaier et al. 2308.16716
$\mathrm{pp} \to \ell_1^-\bar{\nu}_{\ell_1}\ell_2^+\nu_{\ell_2}\ell_3^+\nu_{\ell_3}$ (WWW)	Schönherr 1806.00307, Dittmaier et al. 1912.04117
$\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\nu_\mu\mathrm{jj}_b$ (tZj)	Denner, Pelliccioli, Schwan 2207.11264

- $2 \to 7$ processes

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{b}\bar{\mathrm{b}}\mathrm{H}$ $(\mathrm{t}\bar{\mathrm{t}}\mathrm{H})$	Denner, Lang, Pellen, Uccirati 1612.07138

- $2 \to 8$ processes

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\tau^+\nu_\tau\mu^-\bar{\nu}_\mu\mathrm{b}\bar{\mathrm{b}}$ $(\mathrm{t}\bar{\mathrm{t}}\mathrm{W})$	Denner, Pelliccioli 2102.03246
$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{b}\bar{\mathrm{b}}\tau^+\tau^-$ $(\mathrm{t}\bar{\mathrm{t}}\mathrm{Z})$	Denner, Lombardi, Pelliccioli 2306.13535
$\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^-\mathrm{b}\bar{\mathrm{b}}\gamma\gamma$ $(\mathrm{t}\bar{\mathrm{t}}\gamma\gamma)$	(narrow-width approximation)
$\to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell\gamma\gamma$	Stremmer, Worek 2403.03796

Full NLO corrections (all orders in $\alpha_\mathrm{s}^{n-m}\alpha^{m+1}$) exist for several processes.

Expansion in multiple couplings

Example: $\mathrm{pp} \to 4\ell\mathrm{jj}$ (vector-boson scattering: $\mathrm{pp} \to \mathrm{VVjj}$)

LO: pure EW diagrams $\mathcal{O}(e^6)$ and diagrams with gluons $\mathcal{O}(e^4 g_\mathrm{s}^2)$

NLO: EW and QCD corrections to both types of diagrams

at level of cross section:

$$\begin{tikzcd}
\textbf{LO} & & \mathcal{O}(\alpha^6) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & & \mathcal{O}(\alpha_\mathrm{s}\alpha^5) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & & \mathcal{O}({\alpha_\mathrm{s}}^2\alpha^4) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & \\
\textbf{NLO} & \mathcal{O}(\alpha^7) & & \mathcal{O}(\alpha_\mathrm{s}\alpha^6) & & \mathcal{O}({\alpha_\mathrm{s}}^2\alpha^5) & & \mathcal{O}({\alpha_\mathrm{s}}^3\alpha^4)
\end{tikzcd}$$

full NLO corrections = all NLO orders

consequences:

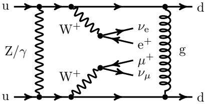
- QCD and EW corrections cannot be separated in general
- QCD corrections to leading LO terms well defined
- consider well-defined orders $\mathcal{O}(\alpha_\mathrm{s}^n \alpha^m)$
- automation must deal with expansion in different couplings

Mixing of virtual EW and QCD corrections

Virtual diagrams mix QCD and EW corrections:

- EW correction to LO QCD amplitude
- QCD correction to LO EW amplitude
- QED and QCD IR singularities

example from VBS



⇒ separation into QCD and EW is not well-defined at NLO

real subtraction terms with both gluons and photons needed

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Origin of leading EW corrections

For energies $Q \lesssim 300\,\mathrm{GeV}$:

- corrections related to the running of the electromagnetic coupling $\alpha(Q)$ $\propto \alpha\log(m_f/Q)$

  $\Rightarrow$ incorporated by suitable choice of renormalisation of α
  - $\alpha(0)$ for external isolated photons
  - $\alpha(M_\mathrm{Z})$ or α_{G_μ} otherwise

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)$$

- corrections originating from soft photons or collinear massless fermion–antifermion or (anti)fermion–photon pairs $\propto \alpha\log(m_f/Q)$
  - YFS resummation (Yennie–Frautschi–Suura)
  - electromagnetic parton showers
- top-mass corrections $\propto \alpha m_\mathrm{t}^2/(M_\mathrm{W}^2 s_\mathrm{w}^2)$

  $\Rightarrow$ (partially) incorporated by using α_{G_μ}

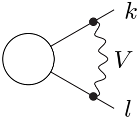
For energies $Q \gtrsim 300\,\mathrm{GeV}$ in addition:

- logarithmic electroweak corrections involving $\alpha\ln(Q/M_\mathrm{W})$ and $\alpha\ln^2(Q/M_\mathrm{W})$

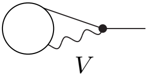
Universality of leading-logarithmic EW corrections

Origin of leading-logarithmic virtual EW corrections

- double logarithms from soft–collinear singular diagrams: $(\alpha/s_{\mathrm{w}}^2)\ln^2(s_{kl}/M_{\mathrm{W}}^2)$
  $\Rightarrow$ angular-dependent logarithms of the form
  $\ln\frac{s_{kl}}{s}\ln\frac{s}{M_{\mathrm{W}}^2}$, $\ln\frac{t}{u}\ln\frac{s}{M_{\mathrm{W}}^2}$



- single logarithms from collinear-singular diagrams and wave-function renormalisation (self-energies): $(\alpha/s_{\mathrm{w}}^2)\ln(Q/M_{\mathrm{W}})$



- single logarithms from coupling renormalisation at scale $M_{\mathrm{W}} \ll \sqrt{s}$
  $\Rightarrow$ running of EW couplings (e, s_{w}, λ, g_{Yukawa}) from M_{W} to $\sqrt{s}$

Leading-logarithmic EW corrections depend only on gauge structure of model, external lines and their polarisations, (and on the running of the couplings).
$\Rightarrow$ Leading-logarithmic EW corrections are universal.

Real emission of EW vector bosons

- separate IR-finite contribution, experimentally identifiable
- can be included as extra LO process if needed

Automation of leading EW logarithms

General results for virtual EW logarithmic corrections to arbitrary non-mass suppressed processes in Sudakov limit, $|s_{kl}| \gg M_{\mathrm{W}}^2$, exist
Denner, Pozzorini hep-ph/0010201

EW virtual corrections in logarithmic approximation implemented in

- ALPGEN (specific processes) Chiesa et al. 1305.6837
- MCFM (specific processes) Campbell et al. 1608.03356
- SHERPA (general processes) Bothmann, Napoletano 2006.14635
- MADGRAPH5_AMC@NLO (general processes) Pagani, Zaro 2110.03714
  Pagani, Vitos, Zaro 2309.00452
- OPENLOOPS (general processes) Lindert, Mai 2312.07927

optionally including some universal subsubleading non-mass-singular terms

Non-logarithmic terms can be consistently included via SCET$_{\mathrm{EW}}$
Chiu, Manohar et al. 1409.1918 and refs. therein.
Non-logarithmic terms are process dependent!
Recent implementation of SCET approach in Monte Carlo integrator based on RECOLA2 for di-boson production Denner, Rode 2402.10503

Features of logarithmic approximation

- Simple formulas, complexity of tree-level calculation
- non-logarithmic terms neglected ⇒ typical accuracy of few percent
- implementations in MadGraph5_aMC@NLO and OpenLoops contain in addition to logarithms of Denner, Pozzorini '00
  - $\mathrm{i}\pi$ terms resulting from $\ln(-s_{kl}/M_\mathrm{W}^2 - \mathrm{i}\varepsilon) = \ln(|s_{kl}|/M_\mathrm{W}^2) - \theta(s_{kl})\mathrm{i}\pi$ in single-logarithmic (obvious) and non-logarithmic terms
  - $\ln^2 \frac{s_{kl}}{s}$ terms resulting from

$$\ln^2 \frac{|s_{kl}|}{M^2} = \ln^2 \frac{s}{M^2} + 2\ln\frac{s}{M^2}\ln\frac{|s_{kl}|}{s} + \ln^2\frac{|s_{kl}|}{s}$$

These terms improve the approximation in many cases,
but are not a result of a consistent expansion.

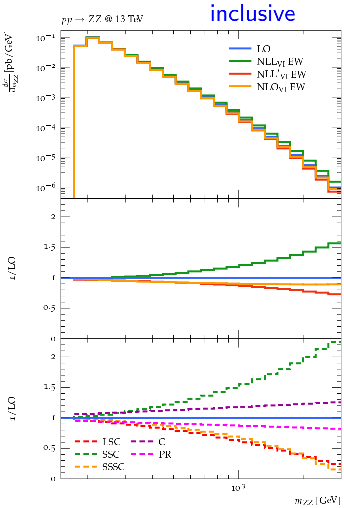
- logarithmic approximation often not useful for inclusive quantities [dominated by small scales, small EW corrections of $\mathcal{O}(\alpha/(s_\mathrm{w}^2\pi)) \sim 1\%$]
- quality needs to be checked case by case
  depends on distribution and phase-space region
- non-logarithmic terms may reach up to 10%
  (e.g. for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}_\mathrm{L}^+\mathrm{W}_\mathrm{L}^-$ for $\sqrt{s} = 3\,\mathrm{TeV}$)

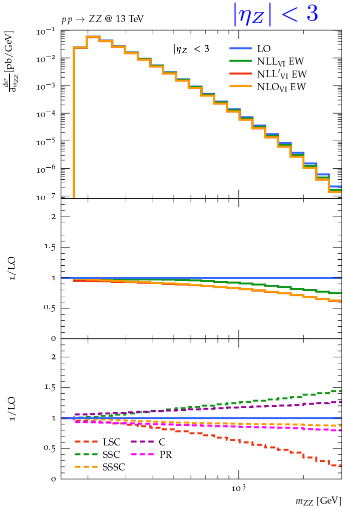
Logarithmic approximation for $\mathrm{pp} \to \mathrm{ZZ}$ (13 TeV)

Virtual corrections with IR poles subtracted via Catani–Seymour I operator

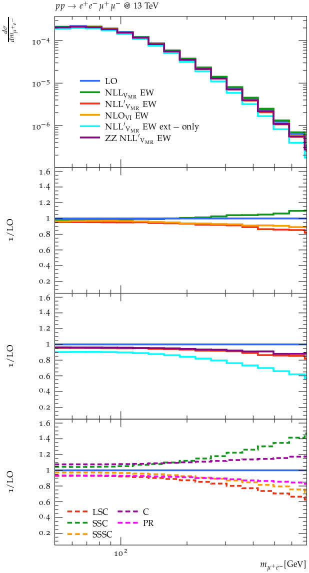
NLL$'_{\mathrm{VI}}$ EW contains squared angular logarithms $\ln^2(t/s)$, NLL$_{\mathrm{VI}}$ EW does not

Lindert, Mai 2312.07927





Logarithmic approximation for $\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^-$



Lindert, Mai 2312.07927

LA for processes with resonances
based on kinematic projectors to include logs for on-shell and off-shell process simultaneously
($w = 10$ scaling factor, $\mu^2 = M^2 - \mathrm{i}M\Gamma$)

$$P(k) = \left| \frac{\mu^2 - w^2 M^2 \Gamma^2}{(k^2 - M^2 + \mathrm{i}wM\Gamma)^2 + \mu^2} \right| = \begin{cases} 1 & \text{if } k^2 \to M^2 \\ 0 & \text{if } k^2 \to \infty \end{cases}$$

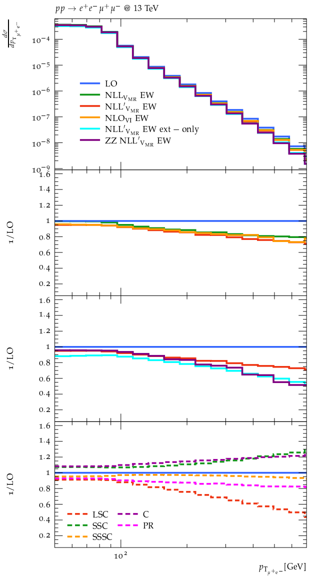
$\mathrm{NLL'}_{\mathrm{V_{MR}}}$EW : logarithms from both full and on-shell process
⇒ describes full NLO EW and on-shell process well

$\mathrm{NLL'}_{\mathrm{V_{MR}}}$EW ext-only : logarithms from external particles of full process
⇒ large deviation

ZZ $\mathrm{NLL'}_{\mathrm{V_{MR}}}$: logarithms from on-shell process

Distribution in $m_{\mu^+\mathrm{e}^-}$:
On-shell logarithms approximate well.

Logarithmic approximation for $\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^-$



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LA for processes with resonances
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$\Rightarrow$ describes full NLO EW and on-shell process well

NLL'$_{\mathrm{V_{MR}}}$EW ext-only : logarithms from external particles of full process
$\Rightarrow$ large deviation

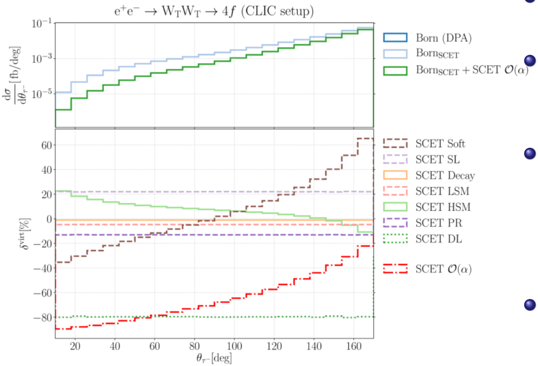
ZZ NLL'$_{\mathrm{V_{MR}}}$: logarithms from on-shell process

Distribution in $p_{\mathrm{T},\mu^+\mathrm{e}^-}$:
On-shell logarithms approximate poorly.

W-pair production at CLIC using SCET$_{\text{EW}}$

$e^+e^- \to W_T^+W_T^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^+$ for $\sqrt{s} = 3\,\text{TeV}$

Distribution in τ production angle for transverse W bosons



Denner, Rode 2402.10503

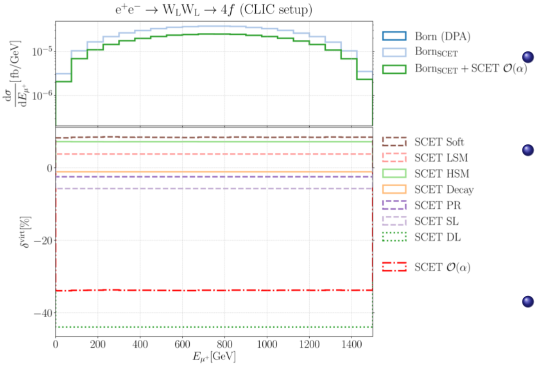
- SCET neglects all power-suppressed corrections $\propto M_W^2/s$
- SCET $\mathcal{O}(\alpha)$ reproduces full $\mathcal{O}(\alpha)$ to better than 0.5%
- $\mathcal{O}(\alpha)$ corrections dominated by double logarithms (DL) and angular-dep. logarithms (Soft)
- Non-logarithmic corrections in high-scale matching (HSM), in low-scale matching (LSM), and in corrections to boson decay (Decay)
- 20% corrections in HSM [contains all(!) $\ln^2(s/t)$ and $\ln(s/t)$ terms]
- -4% corrections in LSM

W-pair production at CLIC using SCET$_{\mathrm{EW}}$

$\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}_\mathrm{L}^+\mathrm{W}_\mathrm{L}^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^+$ for $\sqrt{s} = 3\,\mathrm{TeV}$

Denner, Rode 2402.10503

Distribution in μ energy for longitudinal W bosons

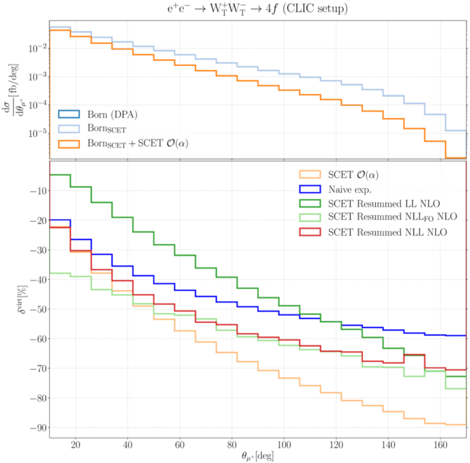


- SCET neglects all power-suppressed corrections $\propto M_\mathrm{W}^2/s$
- SCET $\mathcal{O}(\alpha)$ reproduces full $\mathcal{O}(\alpha)$ to better than 1%
- $\mathcal{O}(\alpha)$ corrections dominated by double logarithms (DL) and angular-dep. logarithms (Soft)
- Non-logarithmic corrections in high-scale matching (HSM), in low-scale matching (LSM), and in corrections to boson decay (Decay)
- 7% constant corrections in HSM [contains all(!) $\ln^2(s/t)$ and $\ln(s/t)$ terms]
- 4% constant corrections in LSM

W-pair production at CLIC using SCET$_{\text{EW}}$

$e^+e^- \to W_L^+W_L^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^+$ for $\sqrt{s} = 3\,\text{TeV}$ Denner, Rode 2402.10503

Distribution in μ production angle for transverse W bosons



- Resummed LL NLO: $\exp(\alpha L^2) + \alpha L + \alpha$
- Resummed NLL$_{\text{FO}}$ NLO: $\exp(\alpha L^2)(1 + \alpha L) + \alpha$
- Resummed NLL NLO: $\exp(\alpha L^2 + \alpha L) + \alpha$
- Naive exp.: $\exp(\delta_{\text{FO}}^{\text{virt}})$
- complete NLL exponentiation important, effects of 20%
- Naive exponentiation deviates by 5–10%

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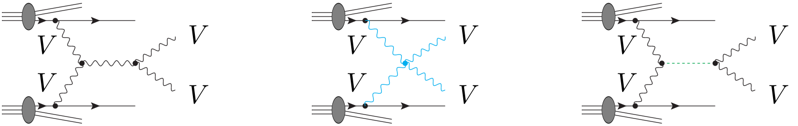
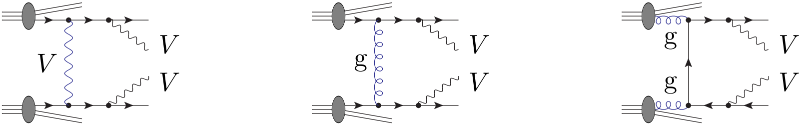
Processes: $\mathrm{pp} \to VV + 2\mathrm{j} \to 4\ell + 2\mathrm{j}$

Vector-boson scattering (VBS) signal



Irreducible background to VBS



- **EW process:** $\mathcal{O}(\alpha^4)$ for stable Vs, $\mathcal{O}(\alpha^6)$ with decays
- **QCD process** $\mathcal{O}(\alpha_\mathrm{s}^2\alpha^2)$ for stable Vs, $\mathcal{O}(\alpha_\mathrm{s}^2\alpha^4)$ with decays
- non-vanishing **interferences** between EW and QCD contributions
  $\mathcal{O}(\alpha_\mathrm{s}\alpha^3)$ for stable Vs, $\mathcal{O}(\alpha_\mathrm{s}\alpha^5)$ with decays
- **gluonic channels** for neutral final states

Corrections to fiducial cross sections

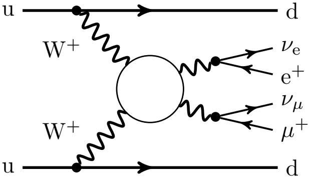
Large NLO EW corrections to VBS processes

process	$\sigma_{\mathrm{LO}}^{\mathcal{O}(\alpha^6)}$ [fb]	$\Delta\sigma_{\mathrm{NLO,EW}}^{\mathcal{O}(\alpha^7)}$ [fb]	δ_{EW} [%]
Biedermann et al. 1708.00268	(Dittmaier et al. 2308.16716)		
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ ($\mathrm{W}^+\mathrm{W}^+$)	$1.4178(2)$	$-0.2169(3)$	-15.3
Denner et al. 1904.0088			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ (ZW^+)	$0.25511(1)$	$-0.04091(2)$	-16.0
Denner et al. 2009.00411			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ (ZZ)	$0.097681(2)$	$-0.015573(5)$	-15.9
Denner et al. 2202.10844			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ ($\mathrm{W}^+\mathrm{W}^-$)	$2.6988(3)$	$-0.307(1)$	-11.4

- EW corrections similar for all processes and rather independent of cuts
  ⇒ intrinsic feature of VBS process
- smaller corrections to $\mathrm{W}^+\mathrm{W}^-$ due to Higgs resonance in fiducial phase space (Higgs contribution about 25%, corresponding EW corrections -6.5%)
- σ^{LO} receives sizeable contributions involving large invariants $s_{ij} \gg M_\mathrm{W}$

Source of large EW corrections

Double-pole approximation (DPA) for outgoing W bosons
effective vector-boson approximation (EVBA) for incoming W bosons



- DPA and EVBA reduce discussion to $V_1V_2 \to V_3V_4$
- DPA accurate for cross section within 1%
- EVBA crude approximation ($\sim 50\%$)
  Kuss, Spiesberger '96, Dittmaier et al. '23
  sufficient to understand dominant effects

high-energy, logarithmic approximation for $V_1V_2 \to V_3V_4$ Denner, Pozzorini '00

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}}\left[1 - \frac{\alpha}{4\pi}4C_{\mathrm{W}}^{\mathrm{EW}}\log^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi}2b_{\mathrm{W}}^{\mathrm{EW}}\log\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right]$$

$$C_{\mathrm{W}}^{\mathrm{EW}} = \frac{2}{s_{\mathrm{w}}^2}, \quad b_{\mathrm{W}}^{\mathrm{EW}} = \frac{19}{6s_{\mathrm{w}}^2} \qquad \text{for transverse W bosons}, \qquad Q \to M_{4\ell}$$

(double EW logs, collinear single EW logs, and single logs from parameter renormalisation included) (angular-dependent logarithms omitted, $\log\frac{t}{u}\log\frac{Q}{M_{\mathrm{W}}}$)

large NLO EW corrections intrinsic feature of VBS

NLO EW corrections in logarithmic approximation

Simple formula for total cross section

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}}\left[1 - \frac{\alpha}{4\pi}4C_{\mathrm{W}}^{\mathrm{EW}}\log^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi}2b_{\mathrm{W}}^{\mathrm{EW}}\log\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right]$$

process	δ_{EW} [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,int}}$ [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,diff}}$ [%]	$\langle M_{4\ell}\rangle$ [GeV]
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$	-16.0	-16.1	-15.0	390
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$	-16.0	-17.5	-16.4	413
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}$	-15.9	-15.8	-14.8	385

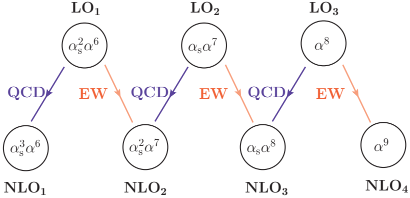
- surprisingly good agreement with complete calculation
- large EW corrections are due to large gauge couplings of vector bosons (C^{EW}) and large scale $Q \sim \langle M_{4\ell}\rangle \sim 400\,\mathrm{GeV}$
- angular-dependent logarithms different for different processes ~ 1–2% owing to cancellations

large NLO EW corrections intrinsic feature of VBS

Associated top-pair and Z-boson production

Process:
$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{b}\bar{\mathrm{b}}\tau^+\tau^-$
$(\mathrm{pp} \to \mathrm{t}\bar{\mathrm{t}}\mathrm{Z})$ $(2 \to 8)$
Denner, Lombardi, Pelliccioli
2306.13535

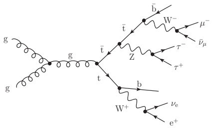
contributions to cross section

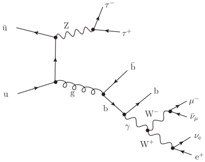


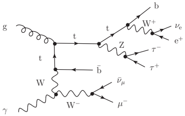
LO:

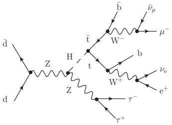
- QCD and EW contributions
- interference of order $\mathcal{O}(\alpha_\mathrm{s}\alpha^7)$ only receives contributions from photon- and bottom-induced channels

Sample diagrams for LO_1 (diags. 1, 2), LO_2 (diag. 3) and to LO_3 (diag. 4)









NLO_1: QCD corrections to LO QCD   Bevilaqua et al. 2203.15688

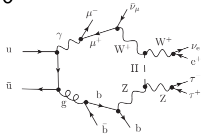
Associated top-pair and Z-boson production

NLO_2: EW corrections to LO_1 and QCD corrections to LO_2

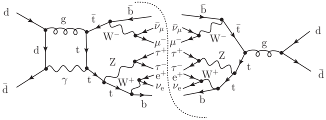
Denner, Lombardi, Pellicciolo 2306.13535

Virtual corrections

- up 10-point functions with maximal rank 6

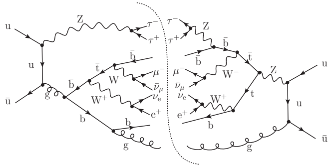


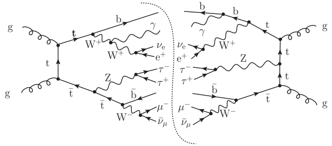
- Classification as QCD or EW corrections ambiguous



Real corrections

- $2 \to 9$ process
- large number of IR-singular regions
- real QCD corrections from gluon and photon radiation





Virtual IR-singularities in $\mathcal{O}(g_s^2 g^8) \times \mathcal{O}(g_s^2 g^6)$ cancelled by both classes of real corr.

Associated top-pair and Z-boson production

NLO_3: QCD corrections to LO_3 (dominant) and EW corrections to LO_2

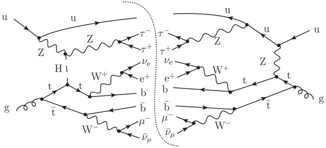
Denner, Lombardi, Pellicciolі 2306.13535

Naively expected to be subleading but comparable to LO_3 and NLO_2

Frederix et al. 1804.10017

not as much enhanced as for $\mathrm{t\bar{t}W}$ production

Dominated by gq channel contribution involving $\mathrm{tZ} \to \mathrm{tZ}$ scattering



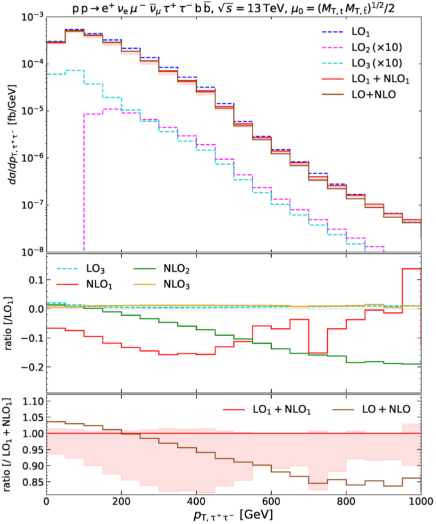
NLO_4: EW corrections to LO_3

Denner, Lombardi, Pellicciolі 2306.13535

0.05% of LO_1 ⇒ negligible

Frederix et al. 1804.10017

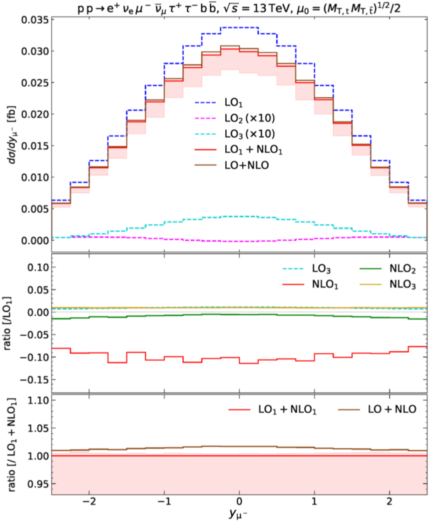
Associated top-pair and Z-boson production



Denner, Lombardi, Pelliccioli 2306.13535

- NLO$_1$ = NLO QCD corrections vary by 15%
- NLO$_2$ = "NLO EW corrections" vary between $+2\%$ and -20%
- NLO$_3$ = $\mathcal{O}(\alpha^2/\alpha_s)$ corrections basically constant at 1%
- corrections beyond NLO$_1$, dominated by EW corrections, strongly distort QCD prediction and exceed QCD scale uncertainty

Associated top-pair and Z-boson production



Denner, Lombardi, Pelliccioli 2306.13535

- y_{μ^-} proxy for $y_{\bar{t}}$
- NLO$_1$ $=$ NLO QCD corrections vary by few % around -10%
- NLO$_2$ $=$ “NLO EW corrections” are negative and below 2% owing to cancellations
- NLO$_3$ $=\mathcal{O}(\alpha^2/\alpha_\mathrm{s})$ corrections basically constant at 1%
- corrections beyond NLO$_1$, including subleading LO contributions stay below 2%

Outline

1. Introduction
2. Automated tools for NLO EW corrections
3. Logarithmic approximation of EW corrections
4. Recent calculations for specific processes
5. **Polarised vector bosons**
6. Conclusion and outlook
7. Backup

Physical interest in polarisation and problems

Observables with polarised massive vector bosons

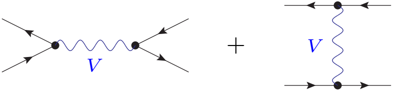
- are important probes of Standard Model gauge and Higgs sectors,
- may provide discrimination power between SM and beyond-SM physics.

Longitudinal polarisation mode of vector bosons is

- a consequence of the EW Symmetry Breaking
- very sensitive to deviations from SM:
  unitarity of cross sections with longitudinally polarised vector bosons realized in SM via cancellation of different contributions.

Challenges and problems

- Unstable massive vector bosons appear only as virtual particles $\Rightarrow$
  - no unique definition of vector-boson polarisations for off-shell bosons
  - diagrams without resonant vector bosons contribute to physical final state

$\mathcal{M} =$ 

- vector bosons are massive $\Rightarrow$
  definition of polarisation depends on reference frame

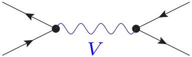
Definition of polarisation based on pole expansion I

Idea: use pole expansion to extract resonant (vector-boson) contributions in gauge-invariant way Ballestrero, Maina, Pellicciolì '17, '19

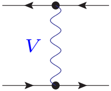
formulation developed by Denner, Pellicciolì '20

- not all diagrams involve required resonances
  resonant diagrams non-resonant diagrams

$$\frac{R(k^2)}{k^2-M^2+\mathrm{i}M\Gamma} =$$



$$N(k^2) =$$



- split full matrix element into resonant part and non-resonant part using pole expansion (gauge-invariant)

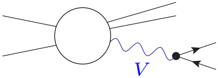
$$\begin{aligned}\mathcal{A} &= \frac{R(k^2)}{k^2-M^2+\mathrm{i}M\Gamma} + N(k^2)\\ &= \frac{R(M^2)}{k^2-M^2+\mathrm{i}M\Gamma} + \frac{R(k^2)-R(M^2)}{k^2-M^2} + N(k^2) = \mathcal{A}_{\mathrm{res}} + \mathcal{A}_{\mathrm{nonres}}\end{aligned}$$

- consider non-resonant part as irreducible background: no resonance
- define polarisation for on-shell residue $R(M^2)$

Definition of polarisation based on pole approximation II

Separate polarisation modes of resonant amplitude

split propagator numerator of resonant particle



$$\mathcal{A}_{\text{res}} = \mathcal{P}_\mu \, \frac{-g^{\mu\nu}}{k^2 - M_{\text{W}}^2 + \text{i}\Gamma_{\text{W}} M_{\text{W}}} \, \mathcal{D}_\nu = \mathcal{P}_\mu \, \frac{\sum_\lambda \varepsilon_\lambda^{\mu\,*}(k)\varepsilon_\lambda^{\nu}(k)}{k^2 - M_{\text{W}}^2 + \text{i}\Gamma_{\text{W}} M_{\text{W}}} \, \mathcal{D}_\nu$$

$$= \sum_{\lambda=\text{L},\pm} \frac{\mathcal{M}_\lambda^{\text{prod}}\,\mathcal{M}_\lambda^{\text{dec}}}{k^2 - M_{\text{W}}^2 + \text{i}\Gamma_{\text{W}} M_{\text{W}}} =: \sum_{\lambda=\text{L},\pm} \mathcal{A}_\lambda \,,$$

$$\left|\mathcal{A}_{\text{res}}\right|^2 = \sum_\lambda \left|\mathcal{A}_\lambda\right|^2 + \sum_{\lambda\neq\lambda'} \mathcal{A}_\lambda^* \, \mathcal{A}_{\lambda'}$$

- incoherent sum $\sum_\lambda \left|\mathcal{A}_\lambda\right|^2$: $\left|\mathcal{A}_\lambda\right|^2 \propto$ “polarised cross sections”,

  “polarisation fractions”: $f_\lambda = \dfrac{\left|\mathcal{A}_\lambda\right|^2}{\sum_\lambda \left|\mathcal{A}_\lambda\right|^2}$
- interferences $\sum_{\lambda\neq\lambda'} \mathcal{A}_\lambda^* \, \mathcal{A}_{\lambda'}$
  vanish for quantities fully inclusive in decay products, but not in general

Method is universally applicable!

Existing results

Fixed-order results at (N)NLO

- results at LO for VBS for ss-WW, WZ, ZZ, os-WW
  Ballestrero, Maina, Pellicciolі '17, '19, '20 [PHANTOM]
- results at NLO QCD for
  - $\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}$ $(\mathrm{W}^+\mathrm{W}^-)$ Denner, Pellicciolі 2006.14867
  - $\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}$ $(\mathrm{W}^+\mathrm{Z})$ Denner, Pellicciolі 2010.07149
  - $\mathrm{pp} \to \mathrm{jj}\ell^+\ell^-$ $(\mathrm{W}^+\mathrm{Z})$ Denner, Haitz, Pellicciolі '22
- results at NLO EW for (diboson production)
  - $\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-$ (ZZ) Denner, Pellicciolі 2107.06579
  - $\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}$ $(\mathrm{W}^+\mathrm{Z})$ Baglio, Dao, Le 2203.01470, 2208.09232
  - $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu$ (WW) Denner, Pellicciolі 2311.16031, Dao, Le 2311.17027
- results at NNLO QCD for
  - $\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}$ $(\mathrm{W}^+\mathrm{W}^-)$ (DPA and NWA) Poncelet, Popescu 2102.13583
  - $\mathrm{pp} \to \ell^\pm\nu_\ell \mathrm{j}$ (Wj) (NWA) Pellen, Poncelet, Popescu 2109.14336

Implementation in Monte Carlo generators

- MADGRAPH5_AMC@NLO: spin-correlated narrow-width approximation (NWA), LO Franzosi, Mattelaer, Ruiz, Shil 1912.01725
- SHERPA: approximate NLO QCD (NWA) Hoppe, Schönherr, Siegert 2310.14803

Example results for polarisation fractions in WW production

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu$ (WW):

state	σ_LO [fb]	$\sigma_\mathrm{NLO\,EW}$ [fb]	$\delta_\mathrm{EW}\,[\%]$	$f_\mathrm{NLO\,EW}\,[\%]$
$\mathrm{b\bar{b}}$ included, $\gamma\mathrm{b}$, $\gamma\mathrm{\bar{b}}$ excluded				
full	$259.02(2)$	$253.95(9)$	-1.96	103.4
unp.	$249.97(2)$	$245.49(2)$	-1.79	100.0
LL	$21.007(2)$	$20.663(2)$	-1.64	8.4
LT	$33.190(3)$	$33.115(3)$	-0.23	13.5
TL	$34.352(5)$	$34.230(5)$	-0.35	13.9
TT	$182.56(2)$	$178.21(3)$	-2.38	72.6
int.	$-21.14(5)$	$-20.6(2)$	-2.45	-8.4

- irreducible background (3.4%) consistent with DPA accuracy
- sizeable interferences (-8.4%) from p_T cuts on charged leptons
- NLO EW corrections differ for various polarised and unpolarised cross sections

Outline

Conclusion

Status of fixed-order EW corrections

- EW corrections automated in several codes
  - EW corrections to $2 \to 5(6)$ processes easily available
  - present frontier $2 \to 7(8)$ processes
- EW corrections typically $\lesssim 5$–10% for inclusive observables
- large EW corrections possible
  - in radiative tails ($> 100\%$)
  - in high-energy tails of distributions [$\mathcal{O}(40\%)$]
  - in fiducial cross sections for specific processes [$\mathcal{O}(20\%)$ for VBS]
- naively suppressed coupling orders may be important due to opening of new kinematic channels (e.g. $\mathrm{tZ/W}$ scattering in $\mathrm{t\bar{t}Z/W}$)
- EW corrections in logarithmic approximation (plus improvements) implemented in automated tools
- methods for EW corrections to processes with polarised vector bosons exist
  - results for VV production available
  - results for VBS within reach

Important topics not mentioned

- matching of EW corrections with parton showers
- PDFs and parton showers including EW effects

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NLO corrections to $\mathrm{t\bar{t}}\gamma$ and $\mathrm{t\bar{t}}\gamma\gamma$ production

Stremmer, Worek 2403.03796
Full NLO corrections to

- $\mathrm{pp} \to \mathrm{t\bar{t}}\gamma \to \mathrm{W^+W^-b\bar{b}}\gamma \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell\mathrm{b\bar{b}}\gamma + X$
- $\mathrm{pp} \to \mathrm{t\bar{t}}\gamma\gamma \to \mathrm{W^+W^-b\bar{b}}\gamma\gamma \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell\mathrm{b\bar{b}}\gamma\gamma + X$

in NWA for top quarks and W bosons at LHC

- full NLO corrections calculated
- Nagy–Soper Bevilacqua et al. 1305.5605 and Catani–Seymour Catani et al. hep-ph/9605323, hep-ph/0201036 subtraction schemes used
  (extended to QED within HELAC-DIPOLES Czakon et al. 0905.0883)
- LO and NLO matrix elements calculated with RECOLA Actis et al. 1605.0190
- bottom- and photon-induced processes included in all subleading contributions.

NLO corrections to $\mathrm{t\bar{t}}\gamma\gamma$ production

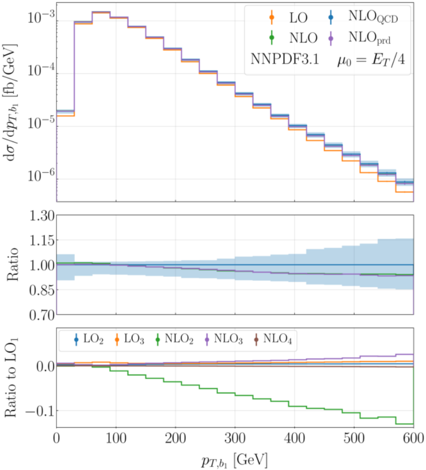
$\mathrm{pp} \to \mathrm{t\bar{t}}\gamma\gamma \to \mathrm{W^+W^-b\bar{b}}\gamma\gamma \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell\mathrm{b\bar{b}}\gamma\gamma + X$
in NWA for top quarks and W bosons at LHC Stremmer, Worek 2403.03796

		σ_i [fb]	Ratio to LO_1
LO_1	$\mathcal{O}(\alpha_s^2\alpha^6)$	$0.15928(3)^{+31.3\%}_{-22.1\%}$	1.00
LO_2	$\mathcal{O}(\alpha_s^1\alpha^7)$	$0.0003798(2)^{+25.8\%}_{-19.2\%}$	$+0.24\%$
LO_3	$\mathcal{O}(\alpha_s^0\alpha^8)$	$0.0010991(2)^{+10.6\%}_{-13.1\%}$	$+0.69\%$
NLO_1	$\mathcal{O}(\alpha_s^3\alpha^6)$	$+0.0110(2)$	$+6.89\%$
NLO_2	$\mathcal{O}(\alpha_s^2\alpha^7)$	$-0.00233(2)$	-1.46%
NLO_3	$\mathcal{O}(\alpha_s^1\alpha^8)$	$+0.000619(1)$	$+0.39\%$
NLO_4	$\mathcal{O}(\alpha_s^0\alpha^9)$	$-0.0000166(2)$	-0.01%
LO		$0.16076(3)^{+30.9\%}_{-21.9\%}$	1.0093
$\mathrm{NLO_{QCD}}$		$0.1703(2)^{+1.9\%}_{-6.2\%}$	1.0690
$\mathrm{NLO_{prd}}$		$0.1694(2)^{+1.7\%}_{-5.9\%}$	1.0637
NLO		$0.1700(2)^{+1.8\%}_{-6.0\%}$	1.0674

- All subleading LO contributions amount to less than 1%.
- NLO_1 corrections dominate, NLO_2 corrections amount to -1.5%.
- Subleading NLO corrections less suppressed than naively expected

NLO corrections to $\mathrm{t\bar{t}}\gamma\gamma$ production

$\mathrm{pp} \to \mathrm{t\bar{t}}\gamma\gamma \to \mathrm{W^+W^-b\bar{b}}\gamma\gamma \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell\mathrm{b\bar{b}}\gamma\gamma + X$
in NWA for top quarks and W bosons at LHC Stremmer, Worek 2403.03796



Distribution in transverse momentum of leading bottom quark

- Corrections beyond NLO$_{\mathrm{QCD}}$ amount to 6% in the tail.
- Approximation NLO$_{\mathrm{prd}}$ that includes the subleading corrections only for $\mathrm{pp} \to \mathrm{t\bar{t}}\gamma\gamma$ reproduces NLO within 1%.
- All subleading LO contributions amount to less than about 1%.
- $\mathrm{NLO_2}$ (EW) corrections reach -13% for large $p_{\mathrm{T,b_1}}$.
- $\mathrm{NLO_3}$ corrections stay below 3%.
- Accidental cancellations between $\mathrm{NLO_2}$ and $\mathrm{NLO_3}$.

Associated top-pair and Z-boson production

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu \mathrm{b}\bar{\mathrm{b}}\tau^+\tau^-$

Denner, Lombardi, Pellicciolli 2306.13535

fiducial cross section (without bottom contributions)

perturbative order	σ_nob [ab]	$\frac{\sigma_\mathrm{nob}}{\sigma_\mathrm{nob,\,LO_1}}$
LO_1	$107.246(5)^{+35.0\%}_{-24.0\%}$	1.0000
LO_2	$0.7522(2)^{+11.1\%}_{-9.0\%}$	$+0.0070$
LO_3	$0.2862(1)^{+3.4\%}_{-3.4\%}$	$+0.0027$
NLO_1	$-11.4(1)$	-0.1072
NLO_2	$-0.89(1)$	-0.0083
NLO_3	$1.126(4)$	$+0.0105$
NLO_4	$-0.0340(9)$	-0.0003
$\mathrm{LO}_1+\mathrm{NLO}_1$	$95.8(1)^{+0.4\%}_{-11.2\%}$	$+0.8933$
LO	$108.285(5)^{+34.7\%}_{-23.8\%}$	$+1.0097$
LO+NLO	$97.0(1)^{+0.5\%}_{-11.2\%}$	$+0.9052$

- LO_1 dominates.
- LO_2 and LO_3 below 1%.
- NLO_1 = NLO QCD corr.: -11%
- NLO_2 = "NLO EW corr.": -0.9%
- $\mathrm{NLO}_3 = \mathcal{O}(\alpha^2/\alpha_\mathrm{s})$ corr. $+1.1\%$.
- NLO_4 negligible.

Example results for polarisation fractions

$pp \to e^+e^-\mu^+\mu^-$ (ZZ):

mode	σ_{LO} [fb]	δ_{QCD}	δ_{EW}	δ_{gg}	σ_{NLO_+} [fb]
full	$11.1143(5)^{+5.6\%}_{-6.8\%}$	$+34.9\%$	-11.0%	$+15.6\%$	$15.505(6)^{+5.7\%}_{-4.4\%}$
unpol.	$11.0214(5)^{+5.6\%}_{-6.8\%}$	$+35.0\%$	-10.9%	$+15.7\%$	$15.416(5)^{+5.7\%}_{-4.4\%}$
Z_LZ_L	$0.64302(5)^{+6.8\%}_{-8.1\%}$	$+35.7\%$	-10.2%	$+14.5\%$	$0.9002(6)^{+5.5\%}_{-4.3\%}$
Z_LZ_T	$1.30468(9)^{+6.5\%}_{-7.7\%}$	$+45.3\%$	-9.9%	$+2.8\%$	$1.8016(9)^{+4.3\%}_{-3.5\%}$
Z_TZ_L	$1.30854(9)^{+6.5\%}_{-7.7\%}$	$+44.3\%$	-9.9%	$+2.8\%$	$1.7933(9)^{+4.3\%}_{-3.4\%}$
Z_TZ_T	$7.6425(3)^{+5.2\%}_{-6.4\%}$	$+31.2\%$	-11.2%	$+20.5\%$	$10.739(4)^{+6.2\%}_{-4.7\%}$

- small irreducible background (0.5%) and interferences (1.2%)
- sizeable QCD and EW corrections
- substantial contribution from
  loop-induced gg fusion for LL and TT
- polarisation fractions roughly conserved
  by NLO corrections owing to cancellations