Department of Theoretical Physics

The path to NNLL accurate parton showers

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Ringberg 2024: 2nd Workshop on Tools for High Precision LHC Simulations

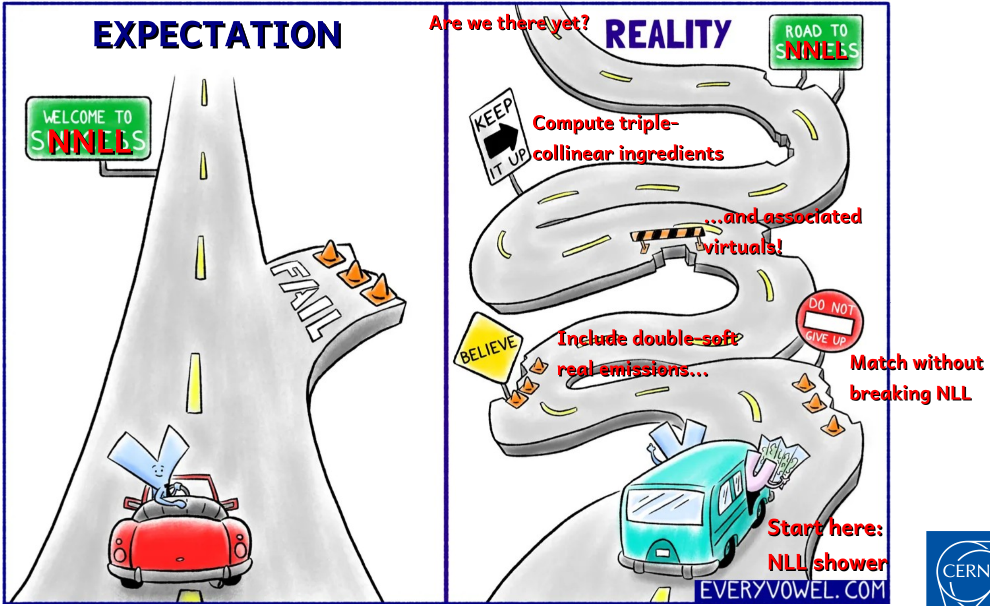
Based on

JHEP 03 (2023) 224 [K. Hamilton, AK, G. P. Salam, L. Scyboz, R. Verheyen]
Phys.Rev.Lett. 131 (2023) [S. Ferrario Ravasio, K. Hamilton, AK, G. P. Salam, L. Scyboz, G. Soyez]
2405.XXXXX [eid. + M. v. Beekveld, M. Dasgupta, B. K. El-Menoufi, J. Helliwell, P. F. Monni, A. Soto-Ontoso]

+

using analytic understanding developed in

JHEP 01 (2019) 083 [A. Banfi, B. K. El-Menoufi, P. F. Monni]
JHEP 12 (2021) [M. Dasgupta, B. K. El-Menoufi]
2307.15734 (accepted in JHEP) [eid. + M. v. Beekveld, J. Helliwell, P. F. Monni]

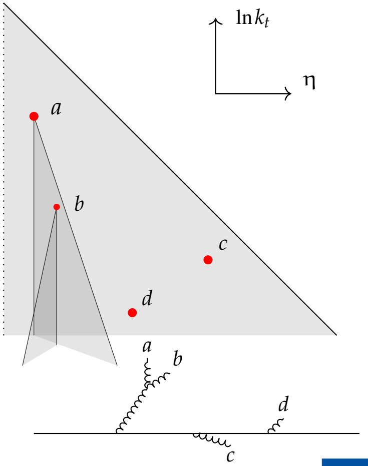

EXPECTATION
WELCOME TO
NNLL
FAIL
Are we there yet?
REALITY
ROAD TO
NNLL
KEEP IT UP
Compute triple-collinear ingredients
...and associated virtuals!
BELIEVE
Include double-soft real emissions...
DO NOT GIVE UP
Match without breaking NLL
Start here: NLL shower
EVERYVOWEL.COM

Start here: NLL shower

- The limited accuracy of parton showers has become one of the main theoretical bottlenecks at the LHC in recent years
- There has been significant progress in improving the hard matrix elements of event generators with NNLO matching and NLO multi-jet merging now state-of-the-art (see next five talks!)
- However, the most widely-used event generators at the LHC, Pythia, Herwig, and Sherpa, are all limited to LL (some exceptions where NLL can be reached, cf. Bewick, Ferrario Ravasio, Richardson, Seymour [1904.11866])
- For this reason, there has been a concerted effort in taking parton showers from LL→NLL
- This has been achieved by several groups including PanScales [1805.09327], [2002.11114], [2011.10054], [2103.16526], [2111.01161], [2205.02237], [2207.09467], [2305.08645], [2312.13275], ALARIC Herren, Höche, Krauss, Reichelt, Schoenherr [2208.06057], [2404.14360], APOLLO Preuss [2403.19452], DEDUCTOR Nagy, Soper [2011.04773], and Forshaw-Holguin-Plätzer [2003.06400]

NLL showers in a nutshell

- A necessary condition for a shower to be NLL is that it correctly describes configurations where all emissions are well-separated in a Lund plane Dasgupta, Dreyer, Hamilton, Monni, Salam [1805.09327]
- A core principle in this picture is that two emissions that are well-separated, should not influence each other (e.g. emission d cannot change the kinematics of c)[a].
- This principle is violated by most standard dipole-showers, due to the way the recoil is distributed after an emission First observed by Andersson, Gustafson, Sjogren '92
- For NLL 2-loop running coupling in the CMW scheme is also required
- For full NLL one also needs to include spin-correlations and subleading colour corrections



[a] Spin-correlations are an exception in this context as they introduce long-range azimuthal correlations at NLL. Collinear spin understood in angular ordered showers for decades due to work of Collins '88 and Knowles '88. Extension to dipole showers studied in Richardson, Webster [1807.01955]. Both collinear and soft spin-correlations are included in PanScales at NLL.

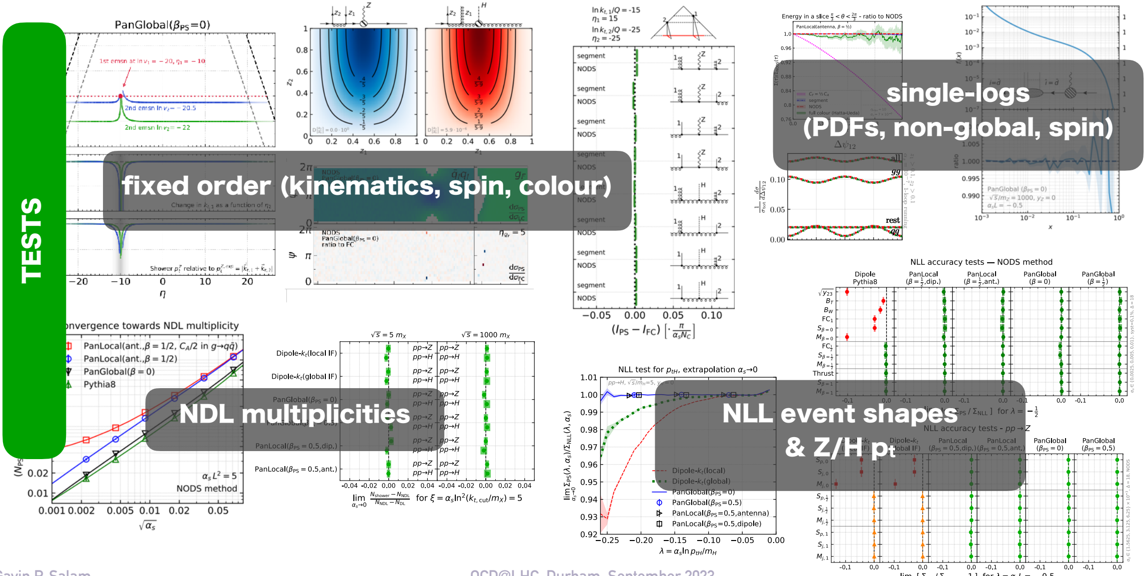
PanLocal	PanGlobal	Colour	Spin
$k_t\sqrt{\theta}$ ordered	k_t or $k_t\sqrt{\theta}$ ordered	nested ordered double soft (NODS)	for correct azimuthal structure in collinear and soft→collinear
Recoil $\perp$: local, $+$: local, $-$: local	**Recoil** $\perp$: global, $+$: local, $-$: local	Designed to ensure LL are full colour (also gets many NLL at full colour)	[Collins-Knowles extended to soft sector]
Dipole partition event CoM	**Dipole partition** event CoM		

e^+e^-: Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]; pp (w/spin+colour): van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]; + pp tests: eid. + Hamilton [2207.09467]; DIS+VBF: van Beekveld, Ferrario Ravasio [2305.08645]

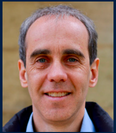
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

AK, Salam, Scyboz, Verheyen [2103.16526], eid. + Hamilton [2111.01161]

a selection of the logarithmic accuracy tests



Oxford


Gavin Salam


Jack Helliwell


Silvia Zanoli

NIKHEF


Melissa van Beekveld

Manchester


Mrinal Dasgupta

UCL


Keith Hamilton

Monash


Basem El-Menoufi


Ludo Scyboz

IPhT


Gregory Soyez

CERN


AK


Silvia Ferrario Ravasio


Pier Monni


Alba Soto-Ontoso

PanScales current members

A project to bring logarithmic understanding and accuracy to parton showers

Analytic structure beyond NLL

Taking an event shape, $\mathcal{O}$, to be less than some value $e^{-|L|}$ we have at NNLL (focusing for now on e^+e^- only)

$$\Sigma(\mathcal{O} < e^{-|L|}) = (1 + \alpha_s C_1 + \ldots) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right] \tag{1}$$

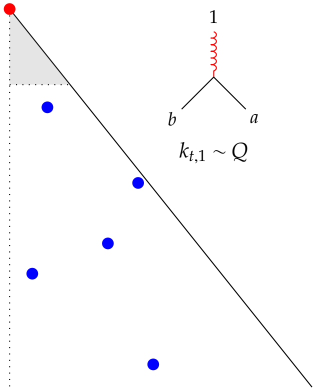
where g_1 accounts for LL terms, g_2 for NLL terms and g_3 and C_1 for NNLL terms[1]. Whereas an analytic resummation in principle retains only the terms that are put in (i.e. g_1 and g_2 at NLL) the shower will instead generate spurious higher order terms

$$\Sigma(\mathcal{O} < e^{-|L|}) = \left(1 + \alpha_s \tilde{C}_1 + \ldots\right) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s \tilde{g}_3(\alpha_s L) + \ldots\right] \tag{2}$$

When thinking about going beyond NLL we need to address two things: 1) what are the necessary analytic ingredients from resummation and 2) how do we compensate the NNLL terms already present in the shower?

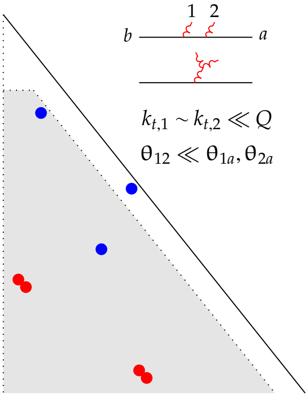
[1] In the language of q_T resummation A_1 is responsible for LL terms, A_2 and B_1 for NLL terms and A_3 and B_2 for NNLL terms (together with the hard coefficient function $C_1(z)$).

Lund plane picture



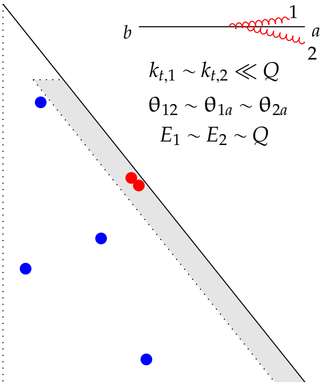
hard matching →

α_S correct for first emission



double-soft →

get any pair of soft commen- -surate energy/angle right

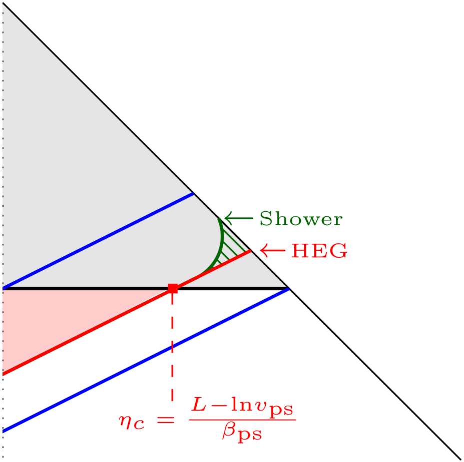


triple-collinear →

account for genuine $2 \to 4$ collinear splittings

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

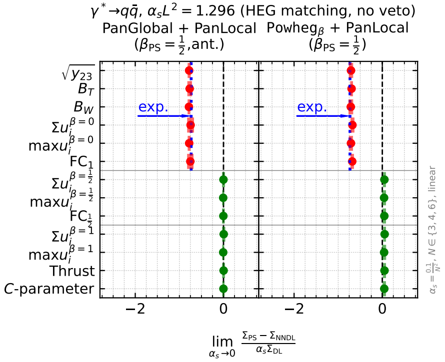
Match without breaking NLL

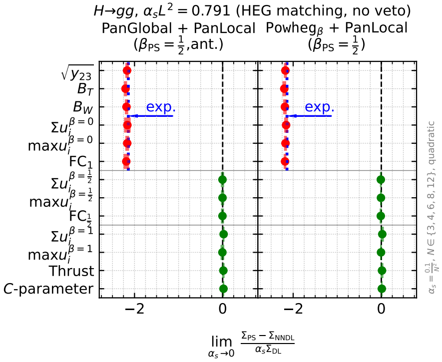


- We have so far explored the two-body decays $\gamma \to q\bar{q}$ and $h \to gg$ @ NLO
- For matching schemes that rely on the shower to generate the first emission (such as MC@NLO, KrkNLO, and MAcNLOPS) the matching works more or less out of the box.
- For POWHEG style matchings (including MiNNLO and GENEVA) log accuracy is more subtle to maintain.
- Main concern related to kinematic mismatch between shower and hardest emission generator (in general they are only guaranteed to agree in the soft-collinear region). This issue has been studied in the past Corke, Sjöstrand [1003.2384] but logarithmic understanding is new.

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

HEG-matching without a veto is not NNDL accurate



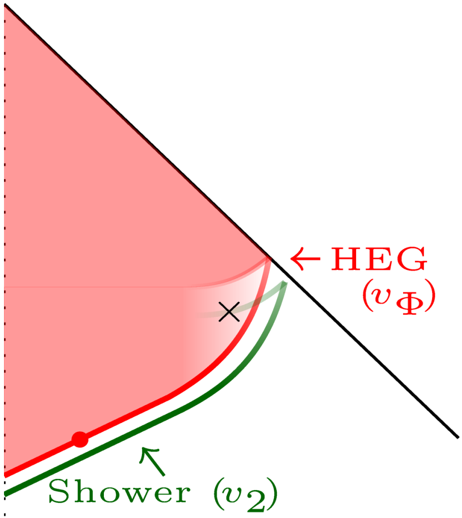


$$\lim_{\alpha_S \to 0} \frac{\Sigma_{PS} - \Sigma_{NNDL}}{\alpha_S \Sigma_{DL}}\Bigg|_{\text{fixed } \alpha_S L^2}$$

Without a veto NLL accurate showers fail our NNDL ($\alpha_S^n L^{2n-2}$) event shape tests. The failure is $\mathcal{O}(1)$, and hence phenomenologically relevant. The dashed blue line indicates the analytically calculated expected value.

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

Further subtleties



- Even when the contours are fully aligned there are issues associated with how dipole showers partition the $g \to gg(q\bar{q})$ splitting function.
- In PanScales we use

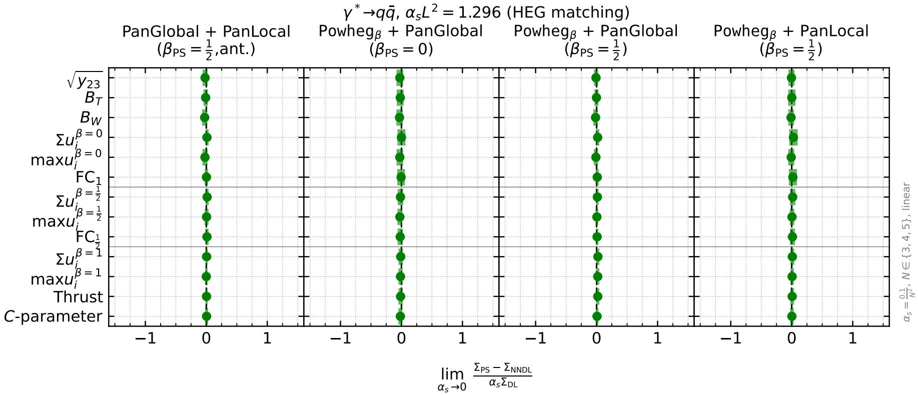
$$\frac{1}{2!} P_{gg}^{\text{asym}}(\zeta) = C_A \left[\frac{1+\zeta^3}{1-\zeta} + (2\zeta - 1) w_{gg} \right],$$

such that $P_{gg}^{\text{asym}}(\zeta) + P_{gg}^{\text{asym}}(1-\zeta) = 2P_{gg}(\zeta)$

- This partitioning takes place to isolate the two soft divergences in the splitting function ($\zeta \to 0$ and $\zeta \to 1$), but there is some freedom in how one handles the non-singular part.
- The HEG needs to partition in exactly the same way. Not clear how easy this is in general, in particular in the soft-large angle region.

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

Proper HEG-matching achieves NNDL accuracy



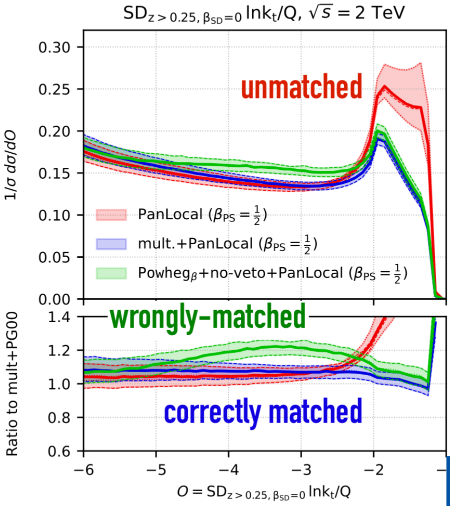
This can be achieved through a standard kinematic veto, as long as shower partioning matches the exact matrix element. A veto however complicates the inclusion of double-soft emissions, since it effectively alters the second emission, complicating the path to further logarithmic enhancement.

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

Phenomenological impact

- Contour mismatch by area $\alpha\Delta$ leads to breaking of NLL and exponentiation
- Correct matching on the other hand augments the shower from NLL to NLL+NNDL for event shapes.
- Impact of NLL breaking terms vary - for SoftDrop they have a big impact due to the single-logarithmic nature of the observable. In particular the breaking manifests as terms with super-leading logs

$$\partial_L \Sigma_{\mathrm{SD}}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L - \bar{\alpha}\Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2}\left(1 - e^{-\bar{\alpha}\Delta}\right)$$



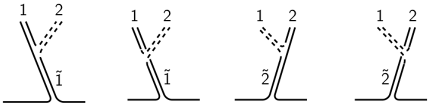
Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

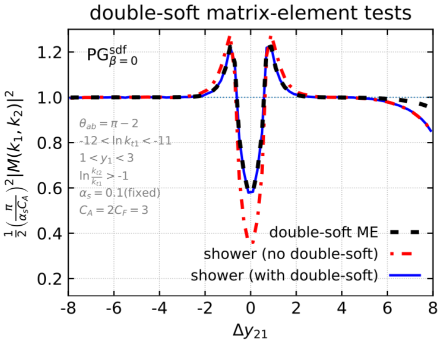
Include double-soft real emissions

- NLO matching is a necessary ingredient for going beyond NLL, but to some extent NLO matching is a solved problem
- Until recently the inclusion of double-soft emissions in an NLL shower was still an open question
- To get them right we must ensure that any pair of soft emissions with commensurate energy and angles should be produced with the correct ME
- Any additional soft radiation off that pair must also come with the correct ME
- In addition must get the single-soft emission rate right at NLO (CMW-scheme)
- This should achieve NNDL accuracy for multiplicities, ie terms $\alpha_S^n L^{2n}$, $\alpha_S^n L^{2n-1}$, $\alpha_S^n L^{2n-2}$
- and next-to-single-log (NSL) accuracy for non-global logarithms, for instance the energy in a rapidity slice, $\alpha_S^n L^n$ and $\alpha_S^n L^{n-1}$ (albeit only at leading-N_C for now)

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

The double-soft ME



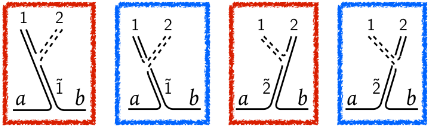


- For now we have focused on PanGlobal
- Any two-emission configuration in a dipole-shower comes with up to four histories (for PanLocal this would in fact be eight)
- We accept any such configuration with the true ME divided by the shower's effective double-soft ME summed over all histories that could have lead to that configuration.

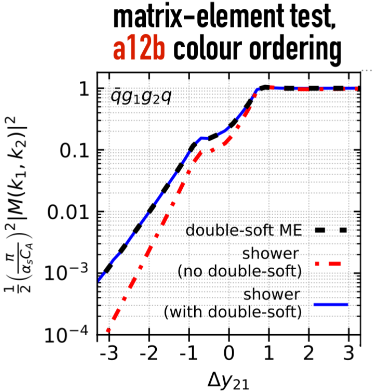
$$P_{\text{accept}} = \frac{|M^2_{\text{DS}}|}{\sum_h |M^2_{\text{shower},h}|}$$

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Correcting the colour-ordering



- We have two distinct colour orderings $a12b$ and $a21b$
- We need to get the relative fractions $F^{(12)}$ and $F^{(21)}$ right in order to ensure that any further emissions are also correct.
- In practice we accept a colour ordering if the shower generates too little of it, and swap them if the shower generates too much (and similarly for $q\bar{q}$ vs gg branchings).



$$P_{\text{swap}} = \frac{F^{(12)}_{\text{shower}} - F^{(12)}_{\text{DS}}}{F^{(12)}_{\text{shower}}}$$

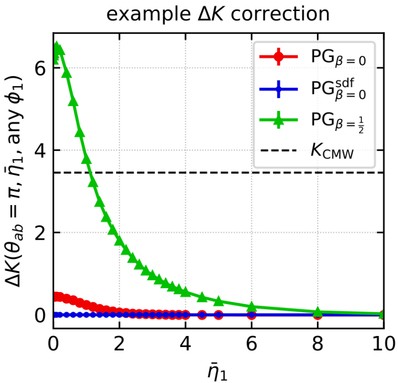
Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

...and associated virtuals!

- The PanScales showers have correct soft emission intensity at NLO in the soft-collinear (sc) region due to the use of the CMW-coupling

$$\alpha_S \to \alpha_S + \alpha_S^2 K_1/2\pi$$

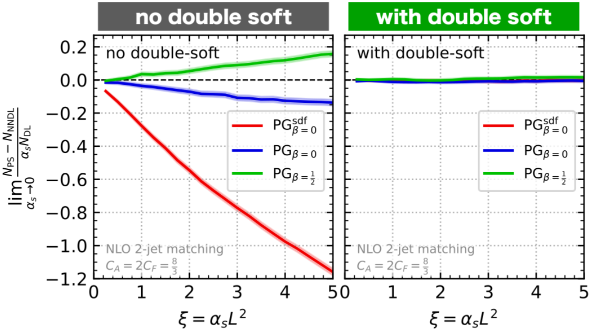
- This in general is not enough the get to soft wide-angle region right and we need to add a ΔK_1 which depends on the rapidity of the single soft emission
- This is related to the fact, that the shower organises its phase space in such a way, that the rapidity of soft pair, y_{12}, does not coincide with the parent rapidity, $y_{\tilde{1}}$.



$$\Delta K_1 = \int d\Phi^{(\mathrm{PS})}_{12/\tilde{1}} |M^{(\mathrm{PS})}_{12/\tilde{1}}|^2 - \int d\Phi^{(\mathrm{PS})}_{12/\tilde{1}_{sc}} |M^{(\mathrm{PS})}_{12/\tilde{1}_{sc}}|^2.$$

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Lund Multiplicities

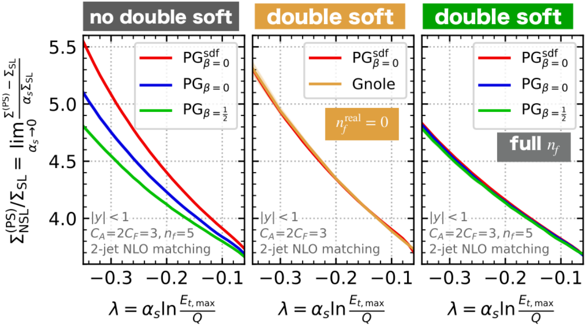


$$\lim_{\alpha_S \to 0} \frac{N_{(\mathrm{PS})} - N_{\mathrm{NNDL}}}{\alpha_S N_{\mathrm{DL}}}\Bigg|_{\text{fixed } \alpha_S L^2}$$

- Reference NNDL ($\alpha_S^n L^{2n-2}$) analytic result from Medves, Soto-Ontoso, Soyez [2205.02861]
- We take $\alpha_S \to 0$ limit to isolate NNDL terms. This is significantly more challenging than at NDL due to presence of $1/\alpha_S$ in denominator.
- Showers without double-soft corrections show clear differences from reference (and each other).
- Adding the double-soft corrections brings NNDL agreement.

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Energy in a slice

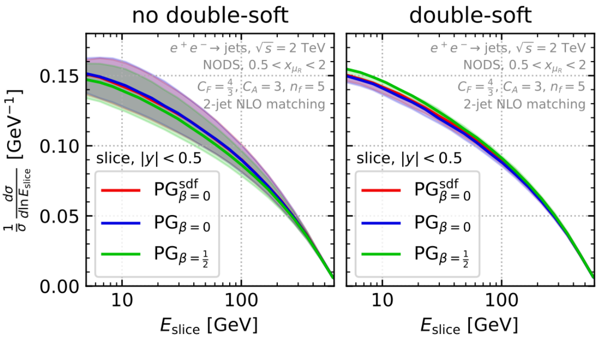


$$\lim_{\alpha_S \to 0} \frac{\Sigma^{(PS)} - \Sigma_{SL}}{\alpha_S}\Bigg|_{\text{fixed } \alpha_S L}$$

- Reference NSL ($\alpha_S^n L^{n-1}$) from Gnole Banfi, Dreyer, Monni [2111.02413] (see also Becher, Schalch, Xu [2307.02283]).
- We did this test semi-blind: only compared to Gnole after we had agreement between the three PanGlobal variants.
- We have NSL agreement with Gnole (using $n_f^{real} = 0$) and agreement between all showers with full-n_f dependence (first calculation of this kind as a by-product!)

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

What about pheno?



- We studied energy flow between two hard (1 TeV) jets as a preliminary pheno case
- The three PanGlobal variants are remarkably close without double-soft corrections, but have large uncertainties
- With double-soft corrections we see a small shift in central values but a significant reduction in uncertainties.

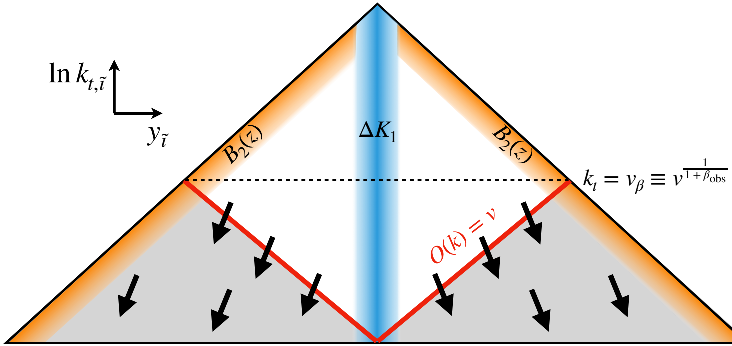
Compute triple-collinear ingredients

- Double-soft corrections are not by themselves enough to reach NNLL accuracy for event shapes. If our showers also had the correct triple-collinear structure (cf. Dasgupta, El-Menoufi [2109.07496], eid. + van Beekveld, Helliwell, Monni [2307.15734], eid. + AK [2402.05170] for work in this direction) we would get them right
- However, it turns out that with the inclusion of real double-soft emissions, only the Sudakov form factor needs to be modified to reach NNLL for event shapes
- Taking

$$\alpha_{\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} \left(K_1 + \Delta K_1(y) + B_2(z) \right) + \frac{\alpha_s^2}{4\pi^2} K_2 \right]$$

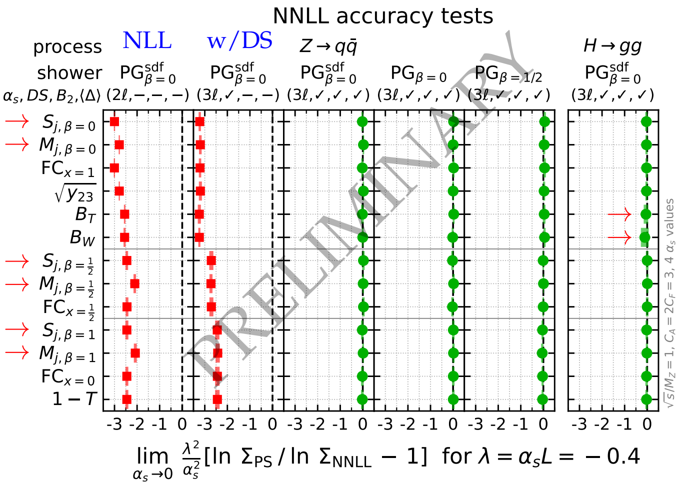
  there are two pieces missing - B_2 which is of triple-collinear origin [2109.07496], [2307.15734] and K_2 (A_3) which is known Banfi, El-Menoufi, Monni [1807.11487], Catani, De Florian, Grazzini [1904.10365]
- However, as discussed above we also have to take into account that our NLL showers generate spurious terms $\tilde{B}_2$ and $\tilde{K}_2$ that effectively have to be compensated. This can be done numerically due to clear connection with shower kinematics

An intuitive picture



Imagine an emission, $\tilde{1}$, sitting anywhere right at the observable boundary (red line). The key observation is that whenever the shower splits $\tilde{1} \to 12$, the kinematic variables $(y_{12}, k_{t,12}, z_{12})$ of the resulting pair, do not agree with that of the parent $(y_{\tilde{1}}, k_{t,\tilde{1}}, z_{\tilde{1}})$. Since the Sudakov was computed assuming conserved kinematics of $\tilde{1}$, and the observable is computed with the actual kinematics of (12), we have generated a mismatch. We correct for this by numerically evaluating the shifts.

Are we there yet?



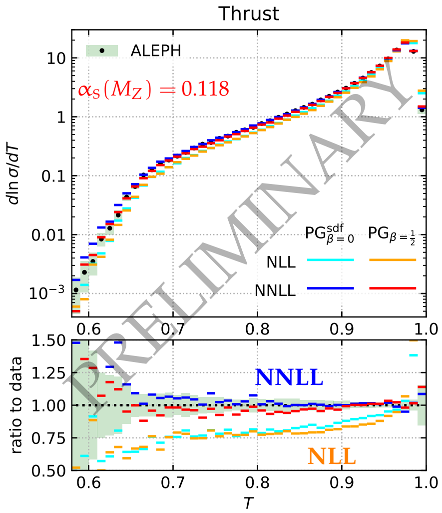
→ New analytic results - see Alba's talk

With no NNLL improvements, the coefficient of NNLL difference is significant, $\mathcal{O}(2-3)$, indicating importance of getting NNLL right

With the inclusion of double-soft, observables with the same β_{obs} align but do still not agree with the analytics

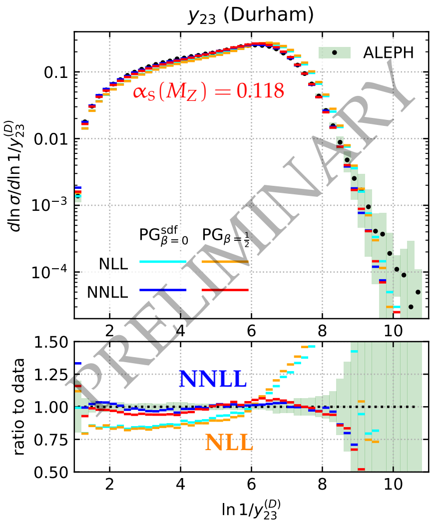
After inclusion of shifts and B_2 and K_2 we have perfect agreement

Not far now...



- Long-standing tension between LEP data and Pythia8 unless using an anomalously large value of $\alpha_S(M_Z) = 0.137$ Skands, Carrazza, Rojo [1404.5630] (also present for PanScales showers)
- Inclusion of NNLL brings large corrections with respect to NLL
- Agreement with data achieved without anomalously large value of α_S
- Beware: no 3j@NLO which is known to be relevant in the hard regions
- Residual uncertainties still need to be worked out

Not far now...



Improved agreement with data across a large range of event shapes

Tuning here still rough

→ We start from the Monash tune (see ref. above) but fix $\alpha_S(M_Z) = 0.118$

For our NLL showers this is the tune we use

For the NNLL showers we tune a number of parameters in the string model semi-automatically

Full tuning exercise still to be done!

Conclusions and outlook

- As the experiments at the LHC record more and more data, it will become increasingly more important to improve on the accuracy of event generators
- NLL accurate showers have now been established by several groups
- First steps towards general NNLL accuracy was taken recently with the inclusion of double-soft corrections in the PanGlobal family of showers
- With these corrections we have reached NNDL accuracy for multiplicity and NSL accuracy for non-global observables
- The next natural step is to get NNLL right for event shapes
- This can be achieved using known ingredients from resummation together with an understanding of how the showers differ from analytic resummation through mainly recoil
- Although still preliminary, we think we have achieved this next step
- The associated NNLL code will be made public in a forthcoming 0.2 release of the PanScales code
- Naturally we now are thinking about how to bring these advances to hadron-collisions
- For full general NNLL the shower needs to also correctly reproduce triple-collinear kinematics (e.g. for fragmentation functions)
- Work in that direction is also ongoing