

MAX-PLANCK-INSTITUT
FÜR PHYSIK



NNLO+PS predictions for bbH production in the four-flavour scheme

Christian Biello

in collaboration with

Aparna Sankar, Marius Wiesemann, Giulia Zanderighi

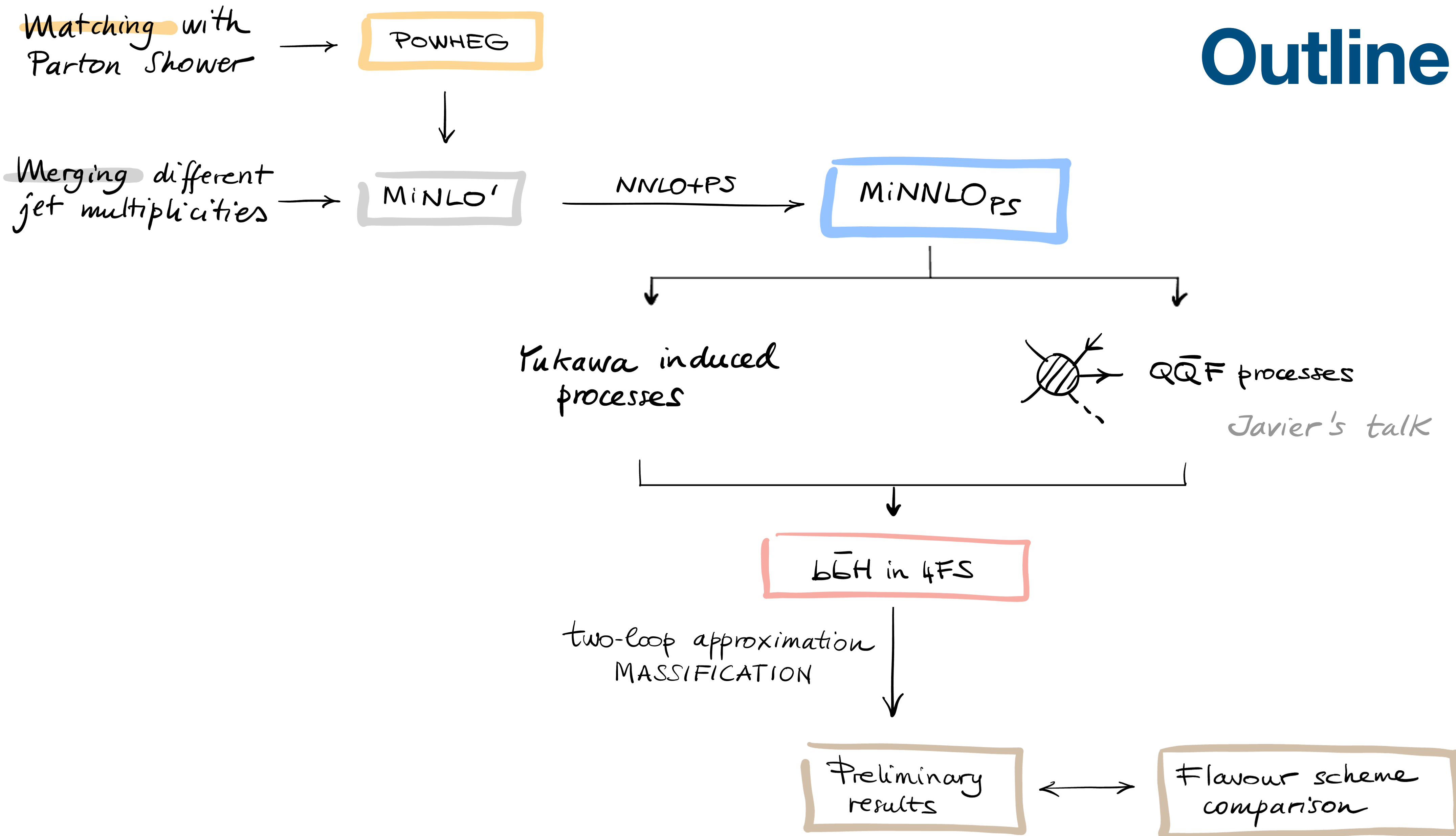
2nd Workshop on Tools for High Precision LHC Simulations

Ringberg Castle

May 9th, 2024



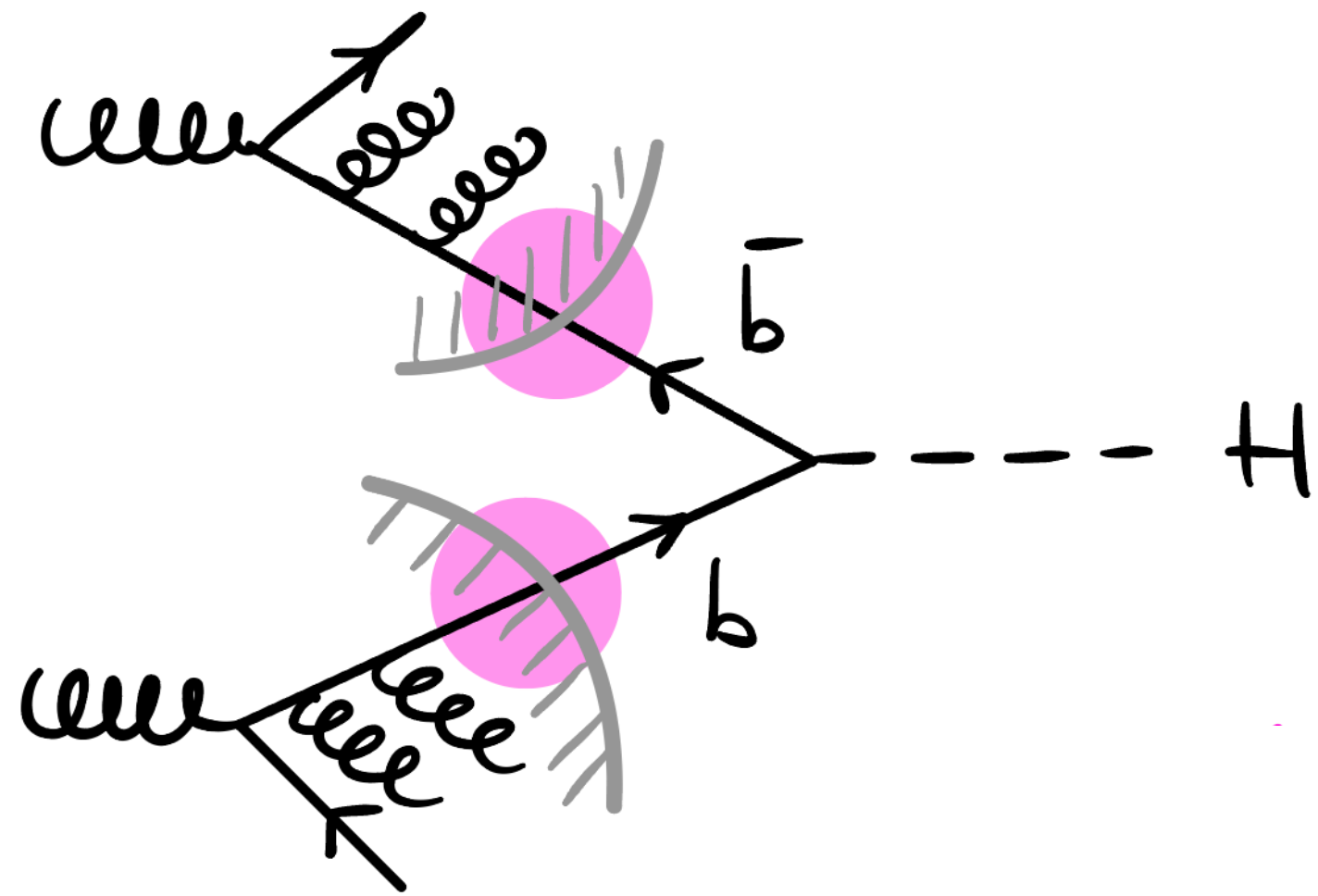
Outline





Matching with PS in bbH: current state of the art

5FS



- Cross-section FO @N³LO

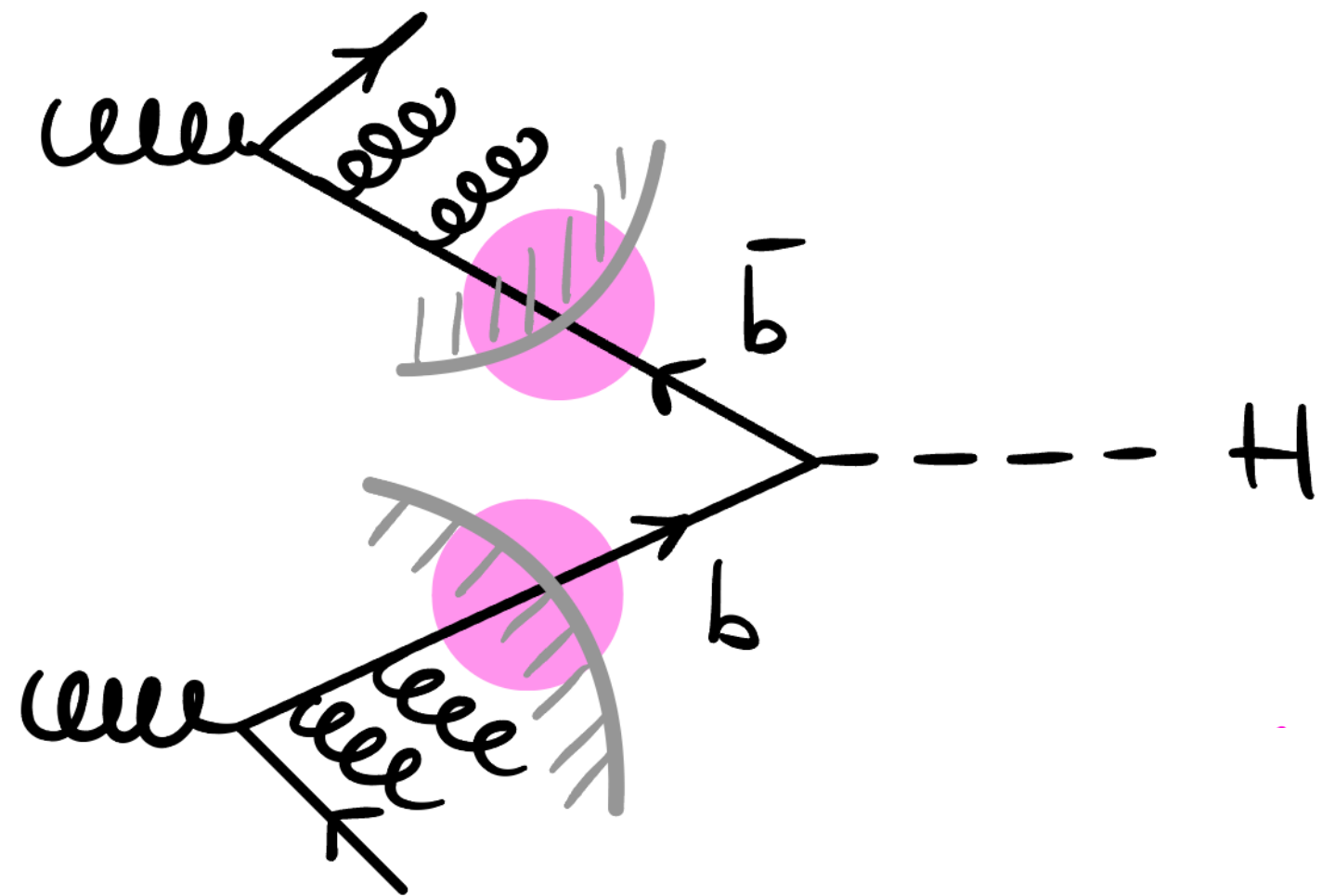
Duhr, Dulat, Mistlberger [1904.09990]

- NNLO_{QCD} + PS **Aparna's talk**



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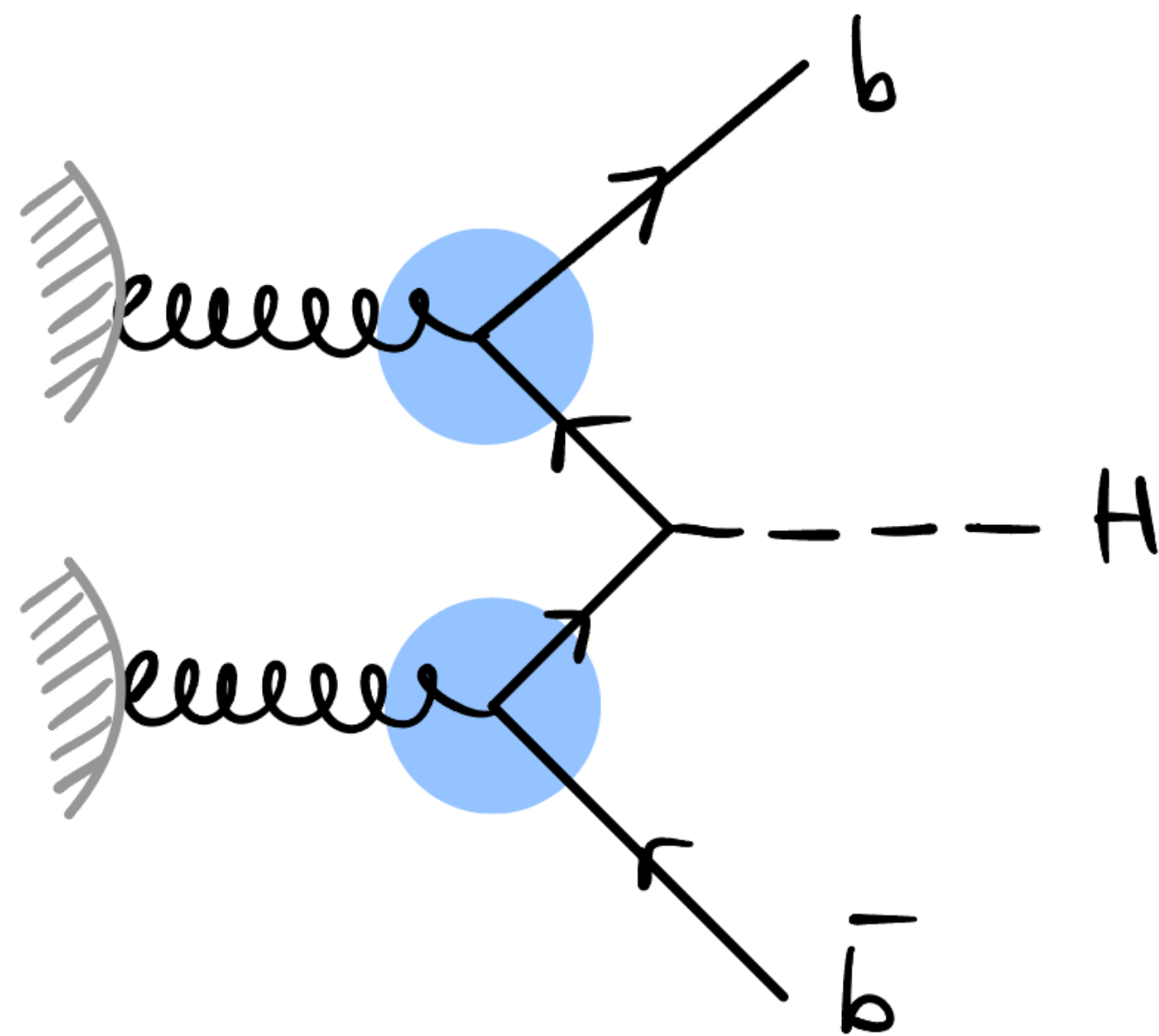


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4FS



- Cross-section FO @NLO

Dittmaier, Krämer, Spira [hep-ph/0309204]

- NLO_{QCD} + PS

Wiesemann, Frederix, Frixione, Hirschi,
Maltoni, Torrielli [1409.5301]

Jäger, Reina, Wackerroth [1509.05843]

- NLO_{QCD} + PS combined with NLO_{EW}

Pagani, Shao, Zaro [2005.10277]

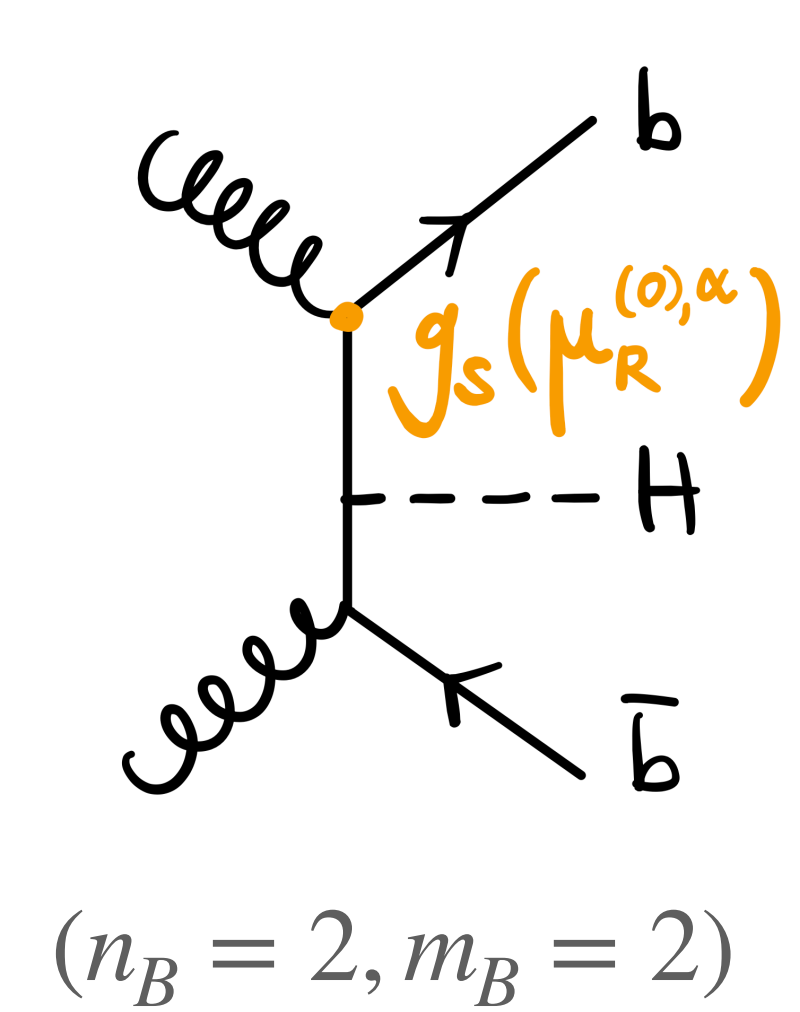
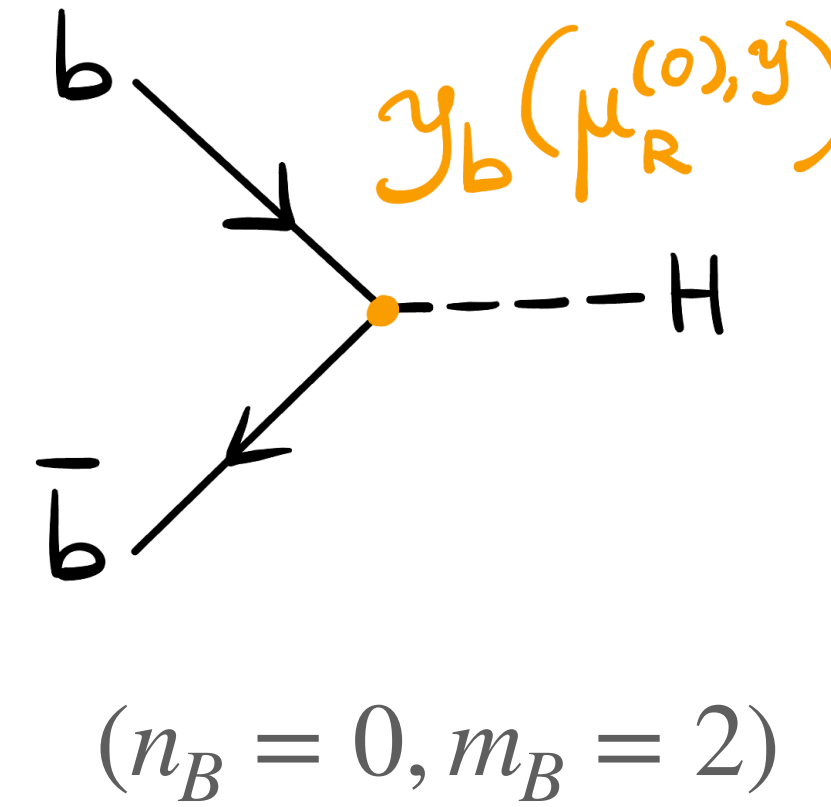
- **NNLO_{QCD} + PS** **This talk**

Yukawa in MiNNLOps

The $\overline{\text{MS}}$ running of this Born couplings

$$\sigma_{LO} \sim \alpha_s^{n_B}(\mu_R^{(0),\alpha}) y^{m_B}(\mu_R^{(0),y})$$

requires some adaptations.



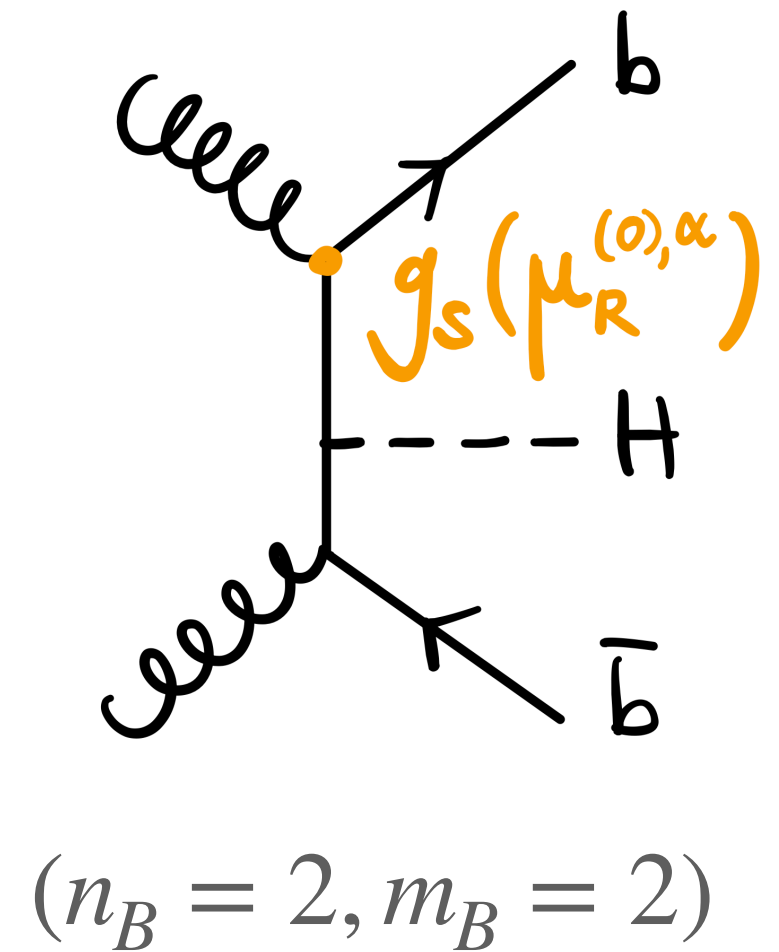
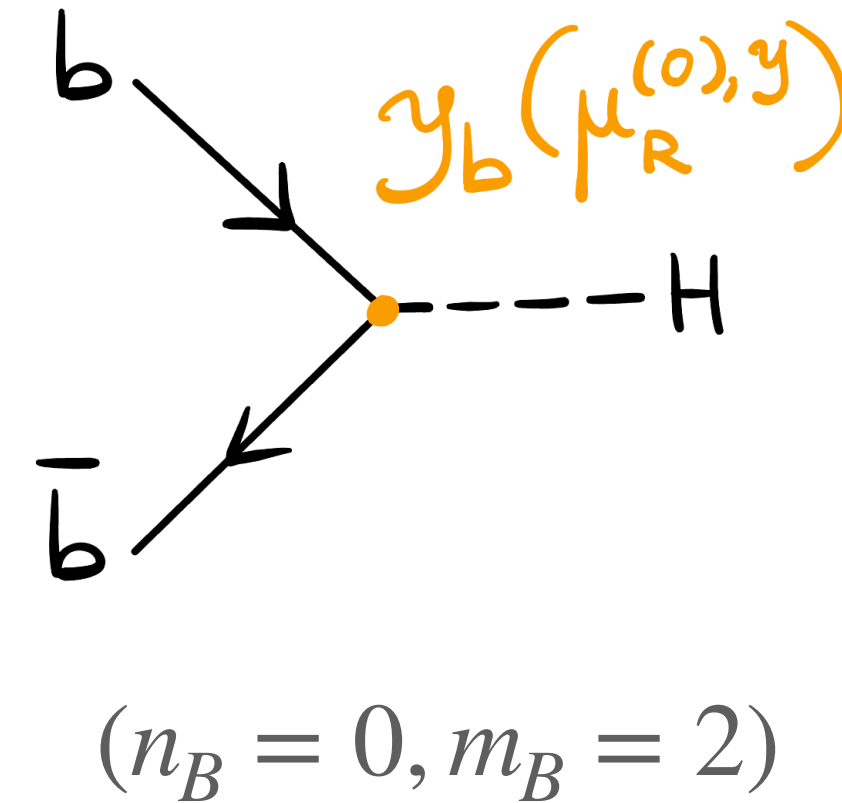


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$$H^{(1)} \supset \text{single } \log(\mu_R^{(0),y})$$

$$H^{(2)} \supset \text{single and double } \log(\mu_R^{(0),y}) \text{ and mixed terms } \sim n_B m_B \log \mu_R^{(0),\alpha} \log \mu_R^{(0),y}$$

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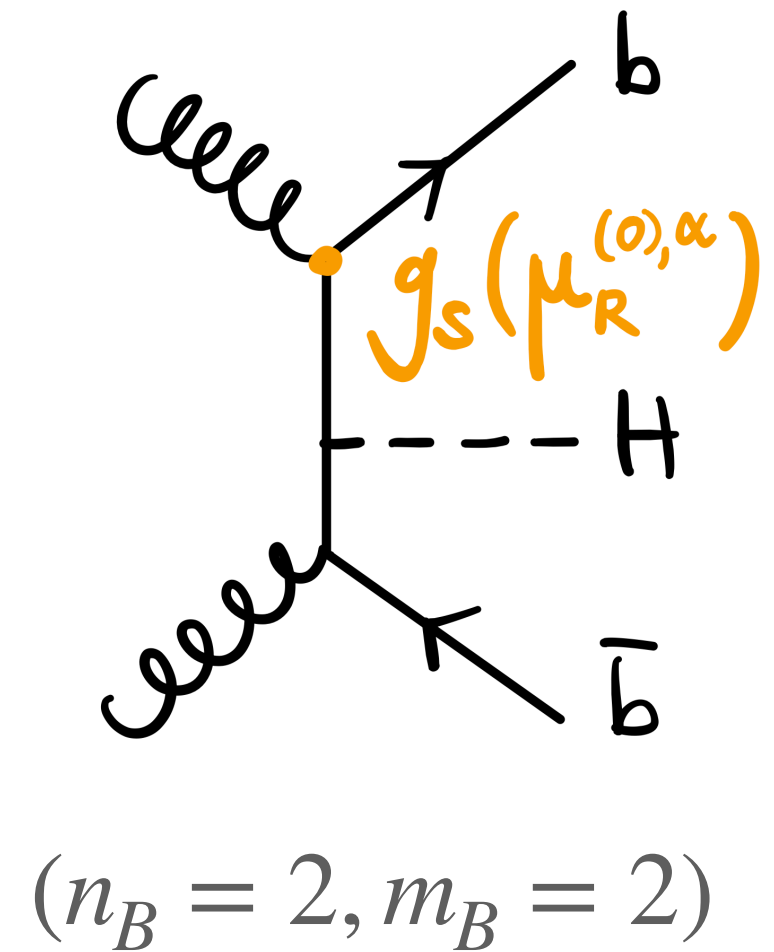
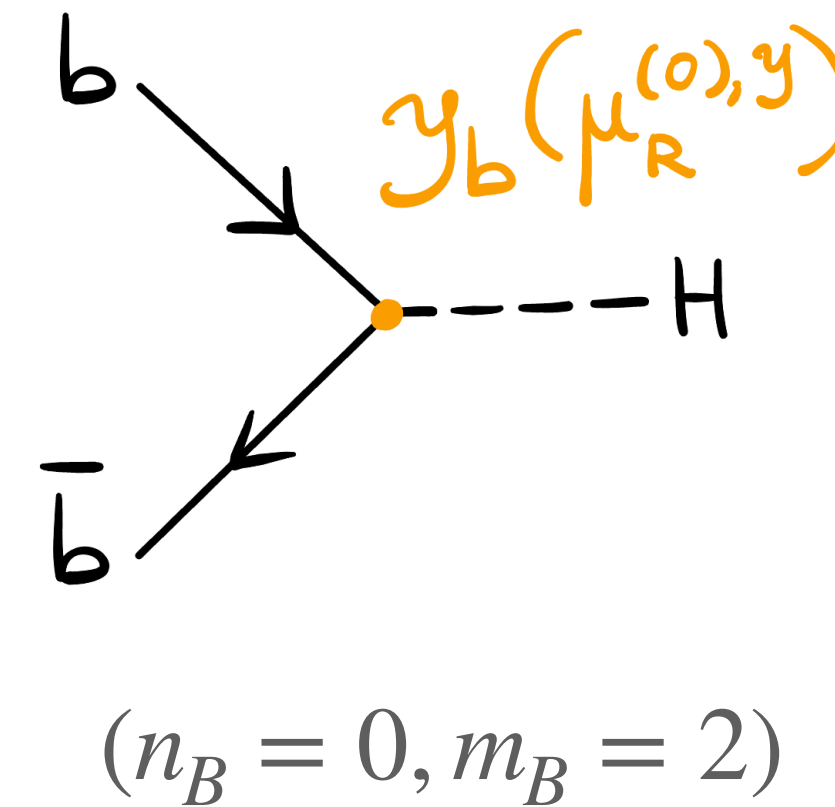
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The Yukawa scale has an interplay with the renormalisation and resummation scale factors

$$\alpha_s(p_T) \rightarrow \alpha_s\left(\frac{K_R}{K_Q} p_T\right) \rightsquigarrow H^{(2)} \supset \log K_R \log \mu_R^{(0),y} \text{ and } \log K_Q \log \mu_R^{(0),y}$$



POWHEG implementation

NLO_{PS} Hbb

For future studies, we provided NLO+PS predictions for the y_b^2 contribution in Hbb.

We checked our code against the public one.

Jäger, Reina, Wackerroth [1509.05843]

We can perform predictions with new settings since we disentangled the Born scales.

	$(\mu_R^{(0),\alpha}, \mu_R^{(0),y})$	NLO _{PS}
default	$(\frac{H_T}{4}, m_H)$	$0.381(2)^{+20.2\%}_{-15.9\%}$ pb
	$(\frac{H_T}{4}, \frac{H_T}{4})$	$0.406(4)^{+16.6\%}_{-14.3\%}$ pb

$$H_T = \sum_{b, \bar{b}, H} \sqrt{m_i^2 + p_{T,i}^2}$$



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NLO_{PS} Hbbj

The starting point of our calculation is the

POWHEG + 1j generator.

We used OpenLoops as amplitude provider (setting $y_t = 0$) and inserted a transverse momentum cut for the Born jet.

MiNLO'



$$\bar{B}(\Phi_{XJ}) = e^{-\tilde{S}(p_T)} \left\{ B (1 - \alpha_s(p_T) \tilde{S}^{(1)}) + V + \int d\phi_{rad} R + \cancel{[D^{(3)}(p_T)]} \times F^{corr} \right\}$$

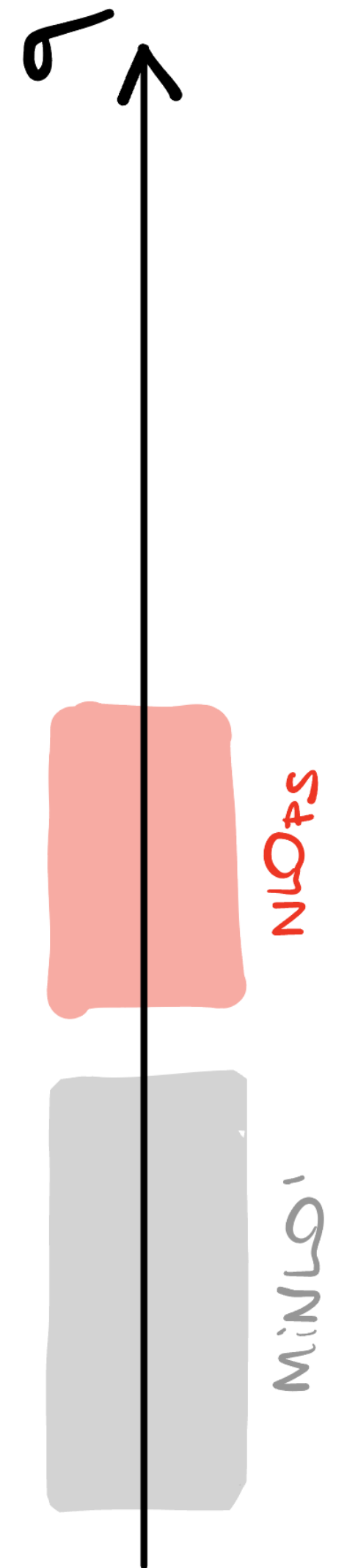
included in the luminosity for MiNNLO_{PS}

In MiNLO' there are no cancellations of the large $\log(m_b)$ in the real (RV, RR) contributions.

We need the **VV contribution** to cancel the quasi-collinear divergences.

Same behaviour observed in $b\bar{b}\ell^+\ell^-$.

Mazzitelli, Sotnikov, Wiesemann [2404.08598]



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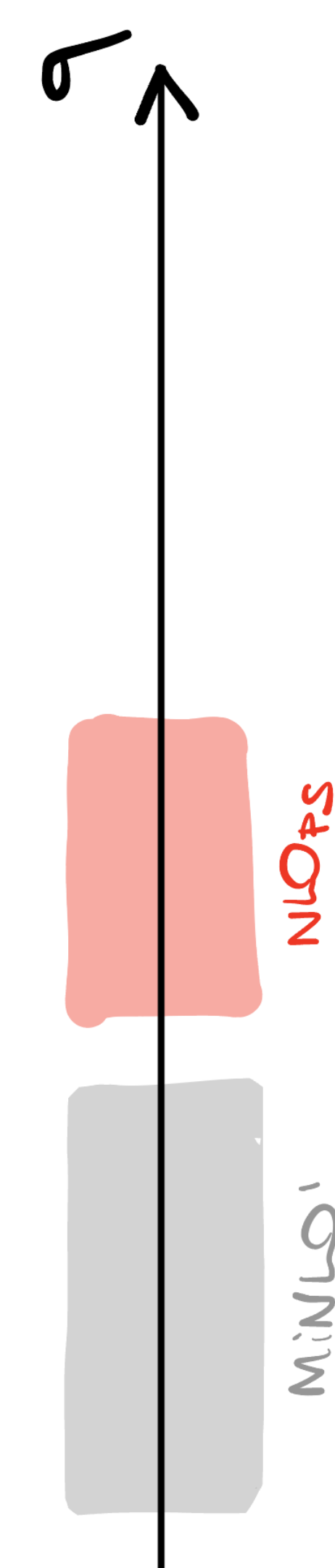
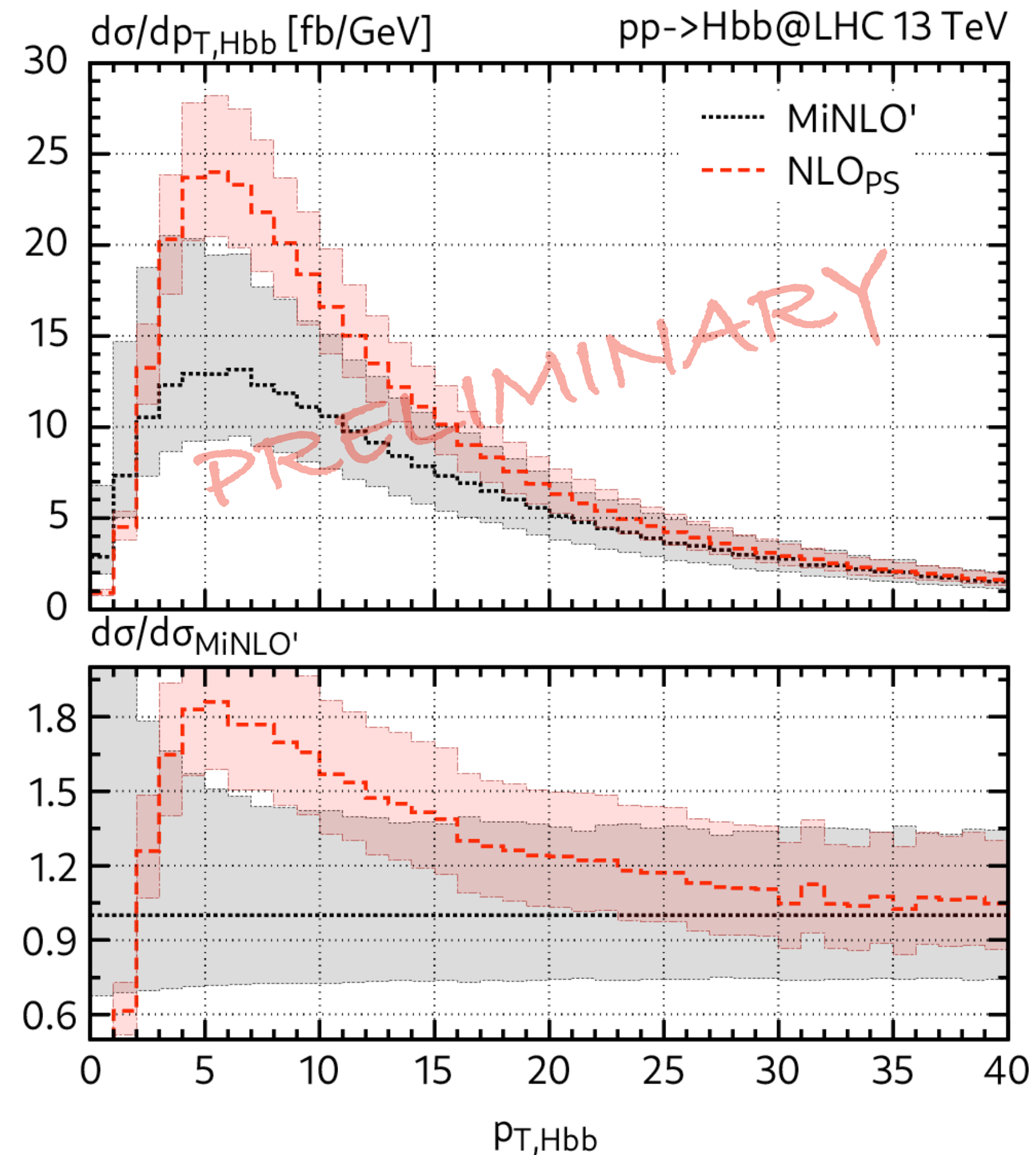
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Two-loop approximation

The double virtual correction for massive bottoms is not known.

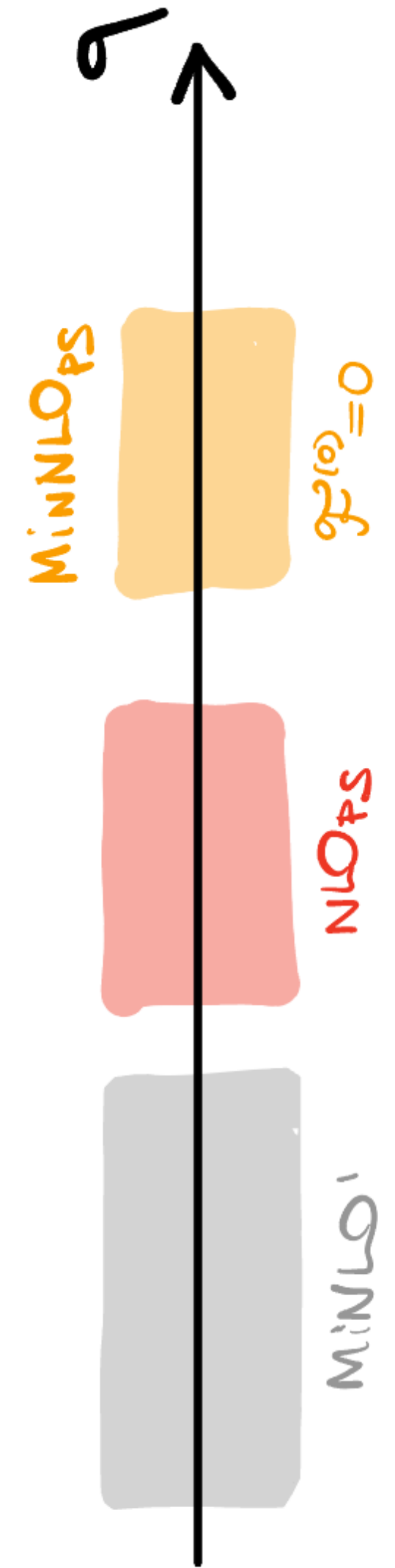
Approximation by retaining all the log-enhanced contributions through the **massification procedure**.

$$|\mathcal{A}^{(2)}\rangle = \log(m_b)\text{-terms} + \text{const.} + \mathcal{O}\left(\frac{m_b}{Q}\right)$$

$$\mathcal{F}^{(2)} |\mathcal{A}_{m_b=0}^{(0)}\rangle + \mathcal{F}^{(1)} |\mathcal{A}_{m_b=0}^{(1)}\rangle + \mathcal{F}^{(0)} |\mathcal{A}_{m_b=0}^{(2)}\rangle$$

MiNNLOPS with only logarithmic contributions in the VV predicts a **total cross-section bigger** than the NLO+PS one.

$(\mu_R^{(0),\alpha}, \mu_R^{(0),y})$	NLO _{PS}	MiNLO'	MiNNLO _{PS} ($\mathcal{F}^{(0)} = 0$)
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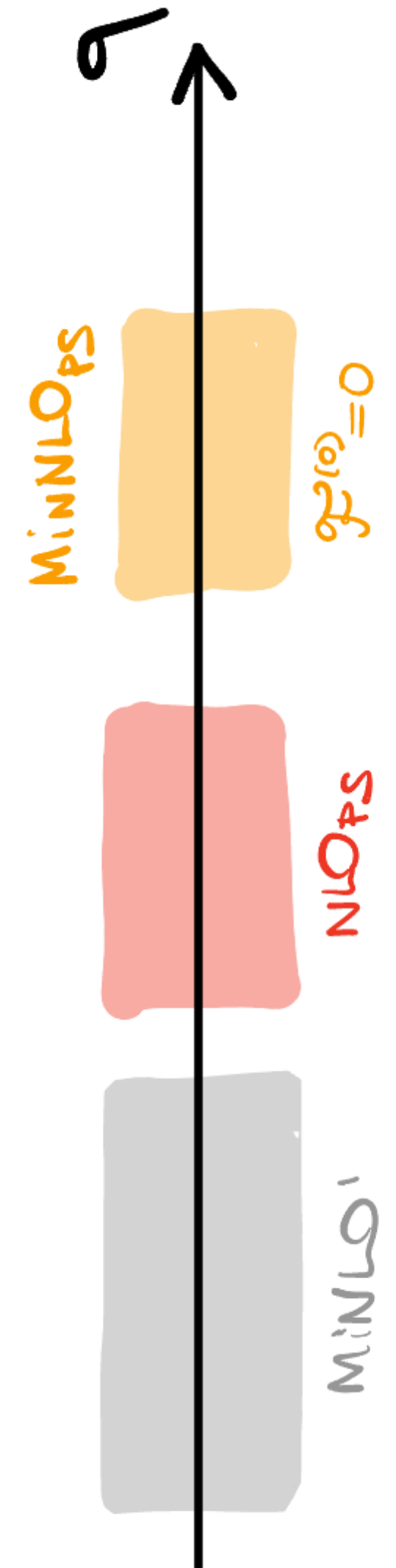
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How to evaluate the massless two-loop?

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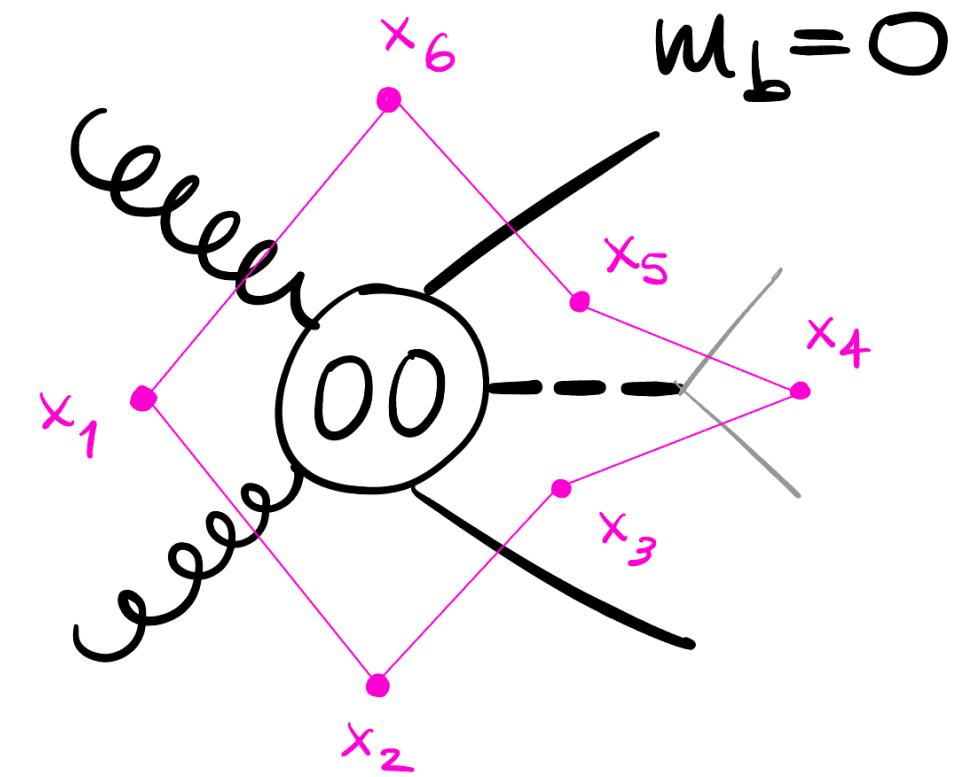
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Massless two-loop library

We used analytic two-loop amplitudes for massless bottoms computed in the leading color approximation.

Badger, Hartanto, Kryś, Zoia [2107.14733]



Momentum
twistor
variables

$$\sum_j \text{coeff}_j(x_i) \cdot \text{MI}_j$$



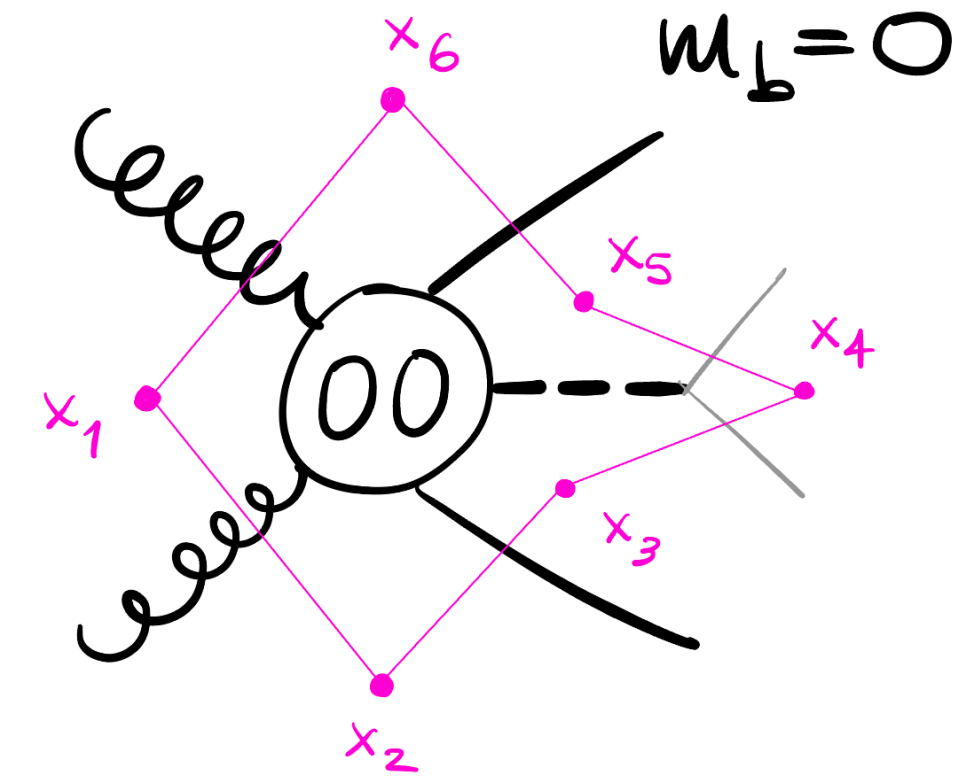
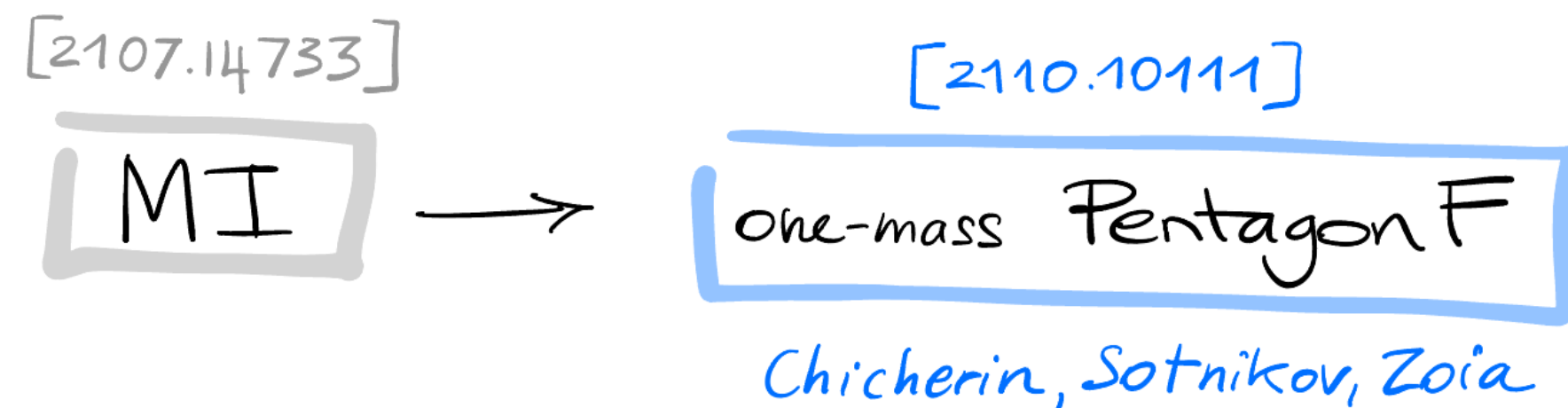


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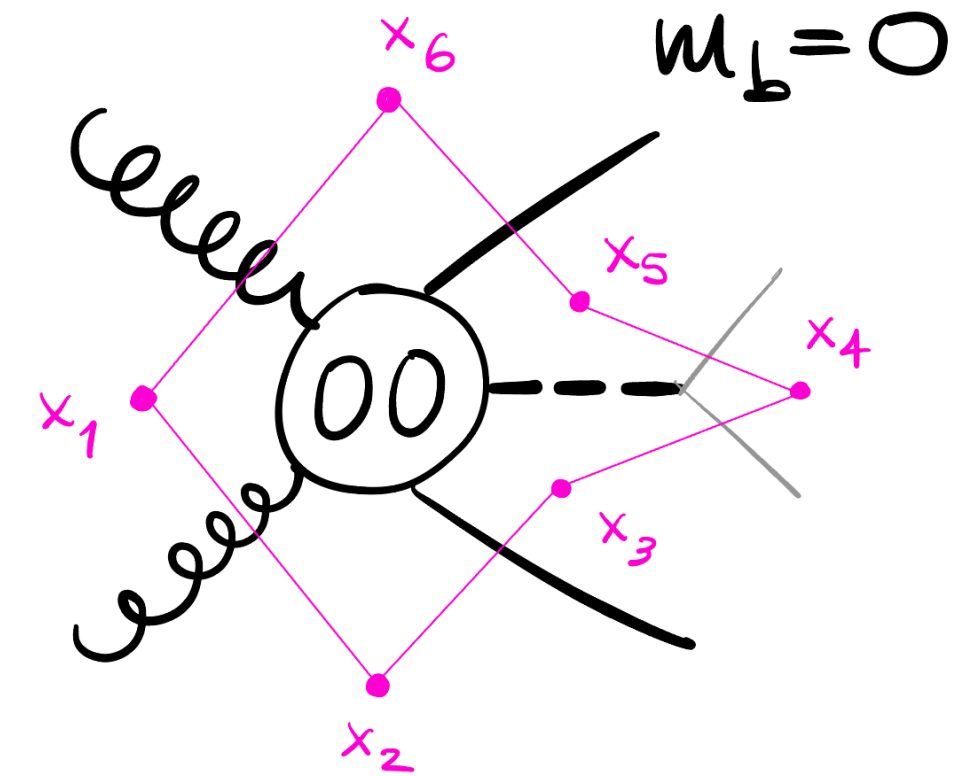
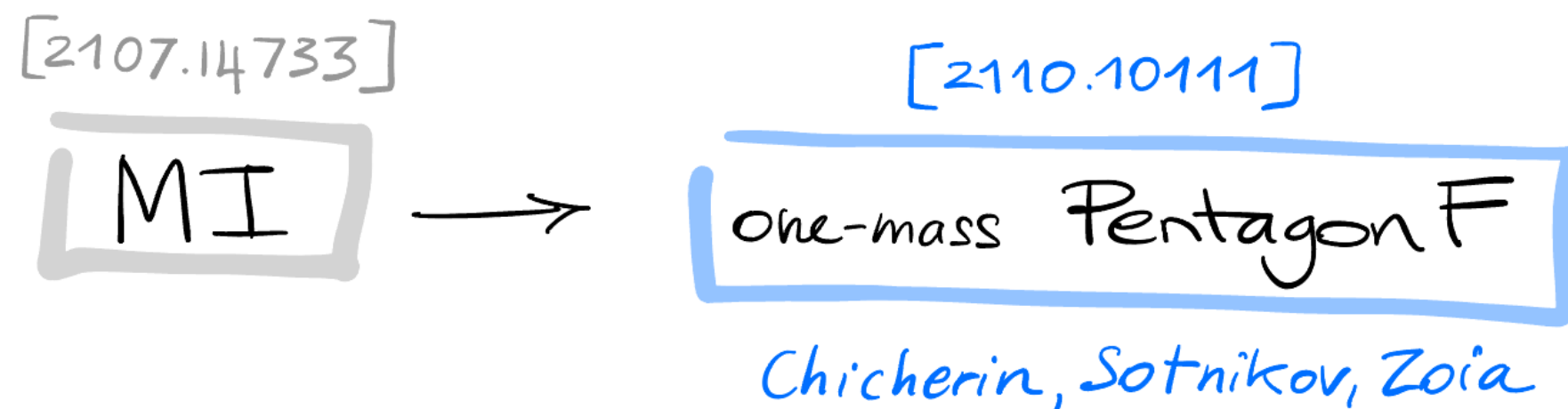


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IR SCHEME in the library

$$\mathcal{A}^{\text{Catani}} = (\mathbb{1} - \mathbb{I}) \mathcal{A}^{\text{UV-ren}} \quad [\text{hep-ph/9802439}]$$

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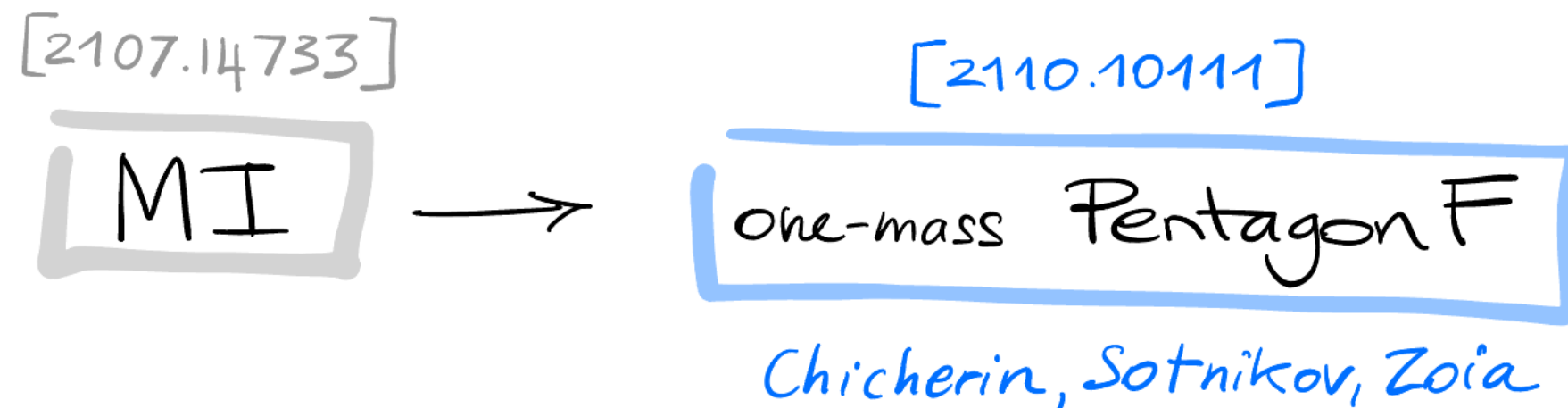


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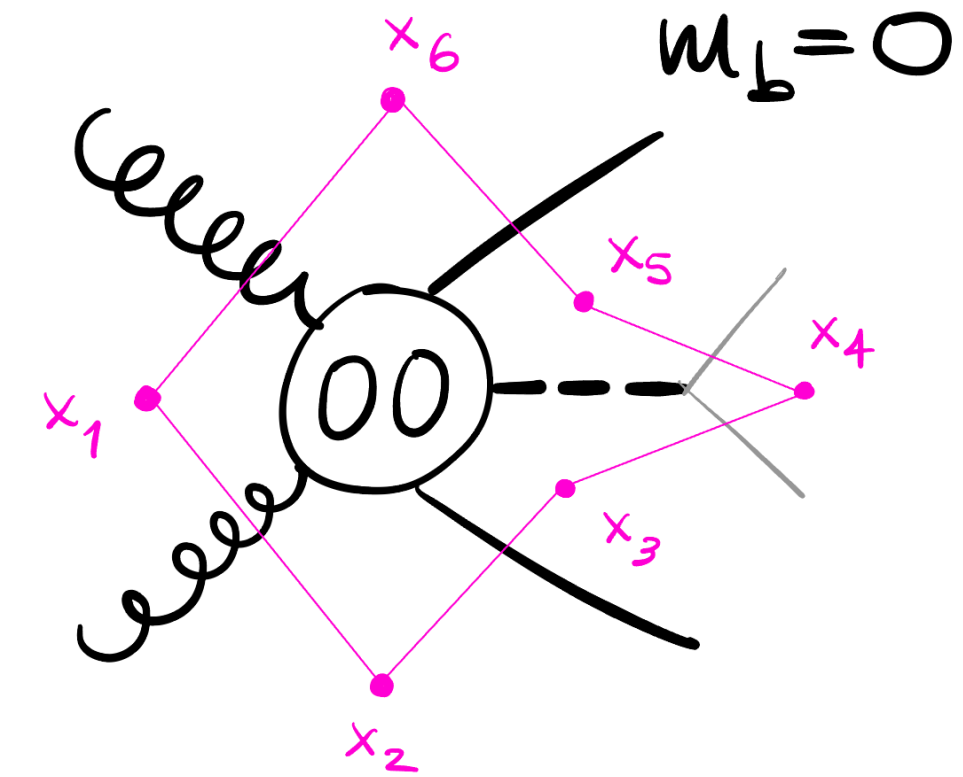
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C++ code interfaced with POWHEG:
~ 3 sec for each PS point in double precision



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Moriond QCD 2024

Checked against the independent Zurich implementation



Original massification

First two-loop massification in Bhabha scattering


Penin [hep-ph/0508127]

Extension for non-abelian theories from factorisation principles

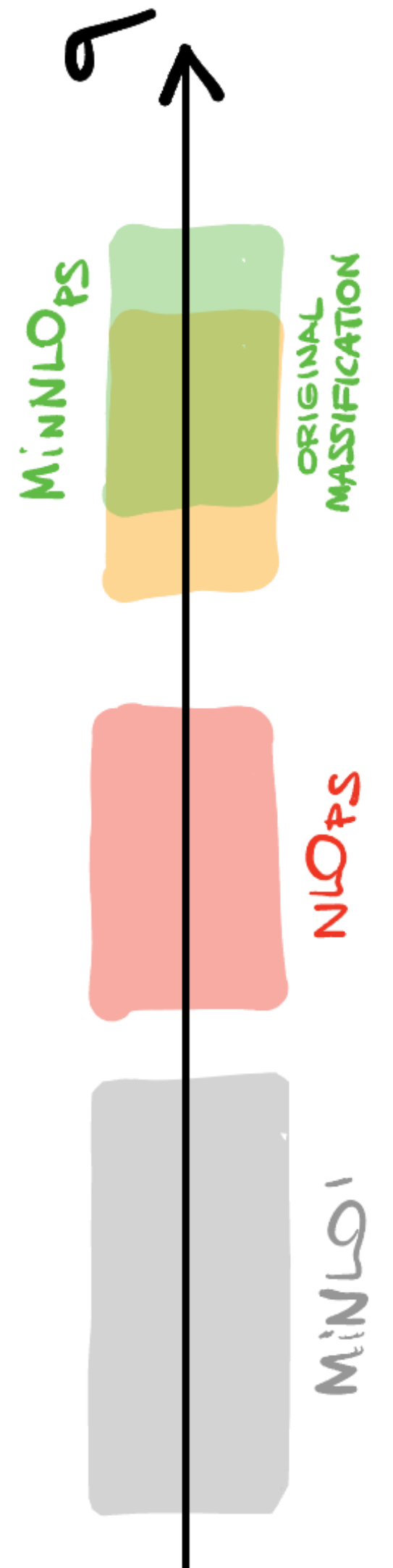
Mitov, Moch [hep-ph/0612149]

$$|\mathcal{A}(m_i, \epsilon)\rangle = \prod_{i = \text{massive external legs}} \left(Z_i \left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \right)^{1/2} |\mathcal{A}(m_i=0, \epsilon)\rangle$$

Universal factors



First check in $q\bar{q} \rightarrow Q\bar{Q}$
Czakon, Mitov, Moch [0705.1975]





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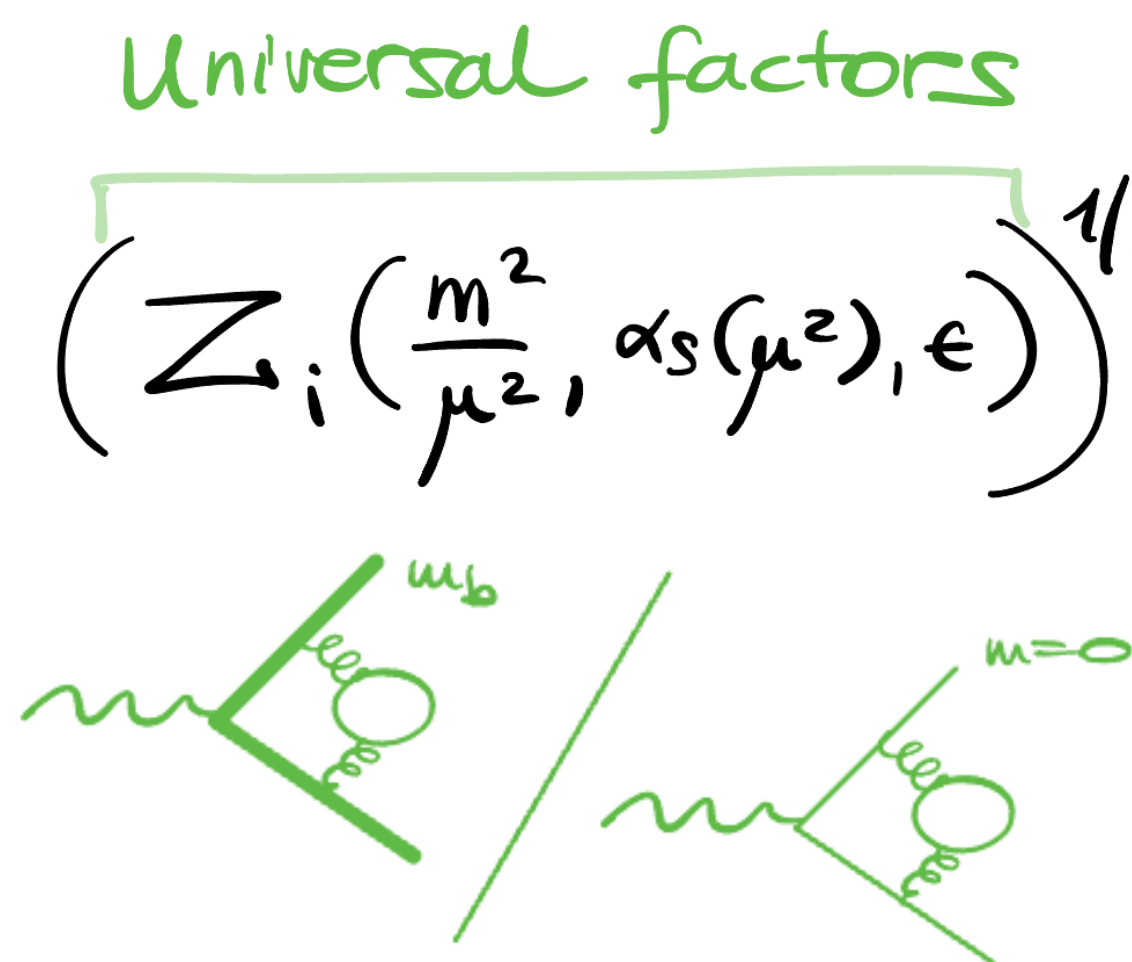
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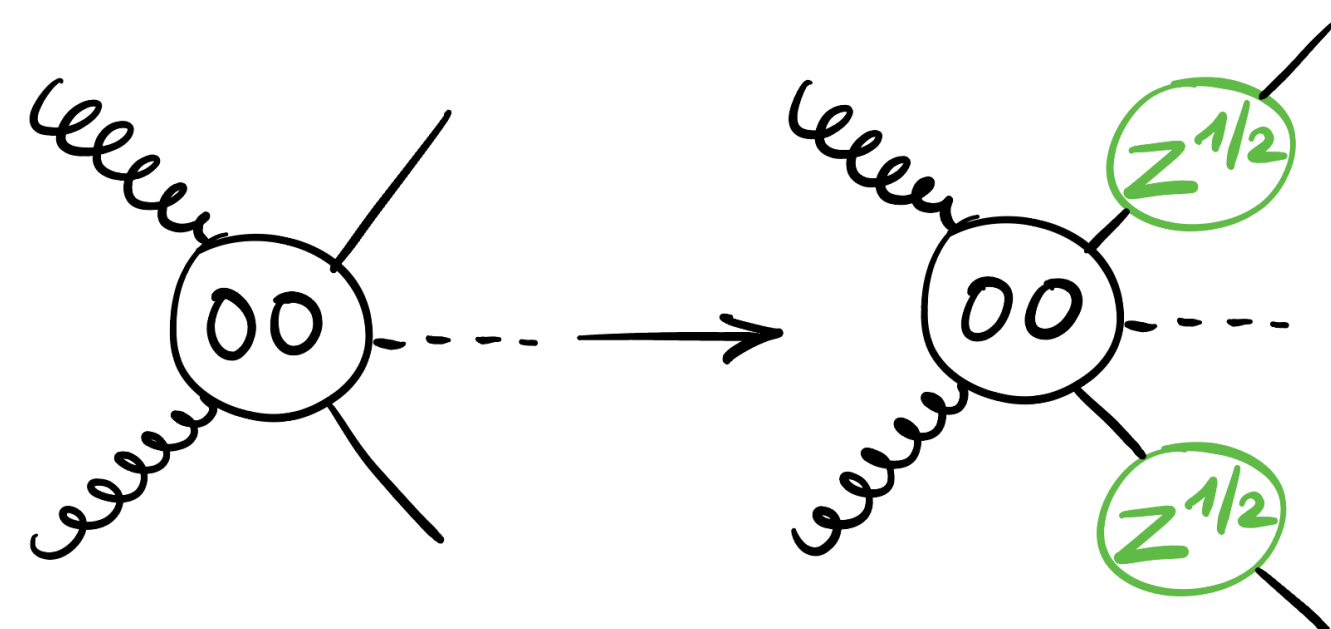
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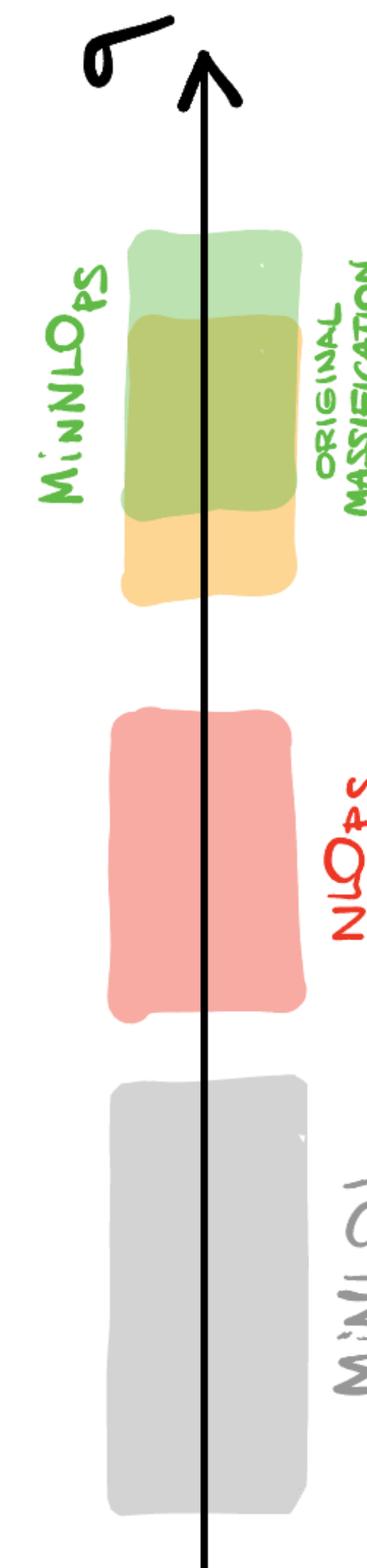
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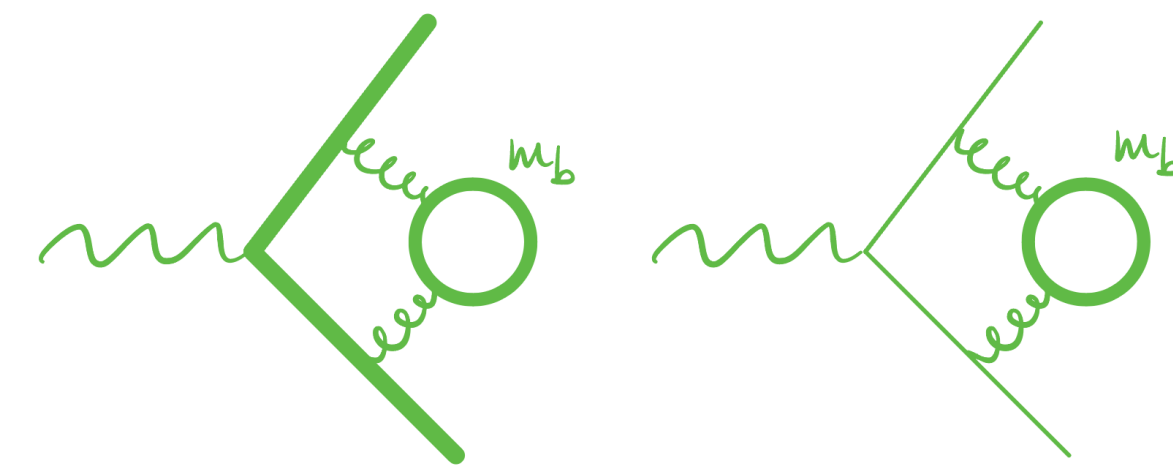
Mapping $\eta : \text{PS}_{m_b} \mapsto \text{PS}_{m=0}$

$\eta_{q\bar{q}}$ preserves the total momentum of $b\bar{b}$

η_{gg} avoids a collinear singularity



Generalised massification



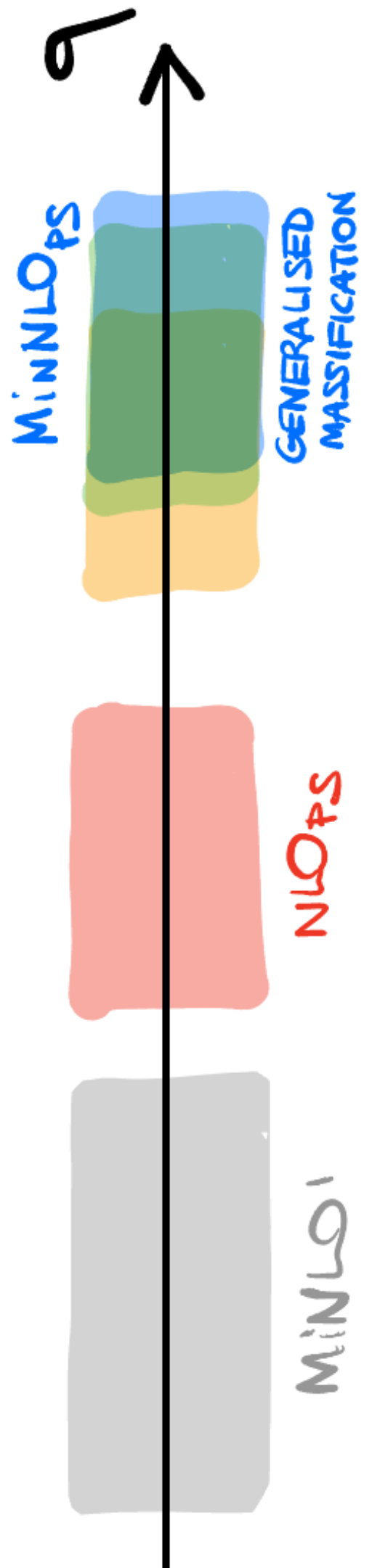
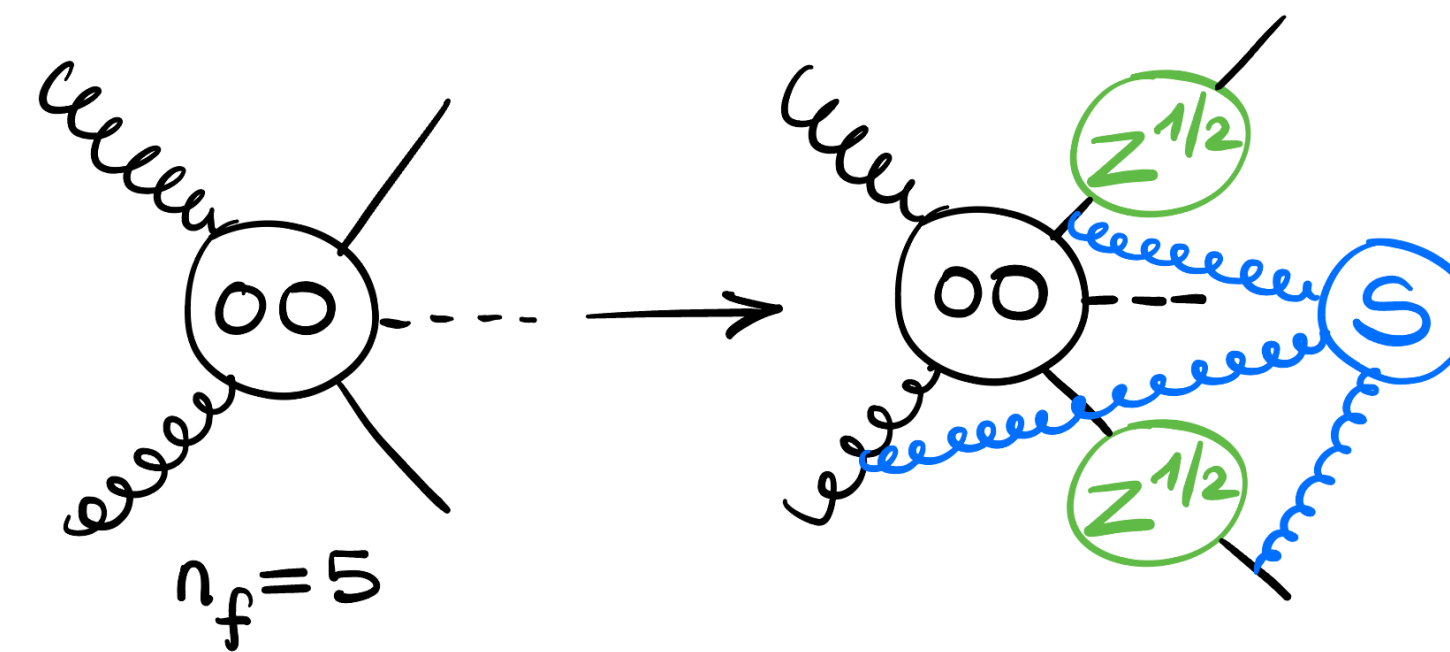
First massification of internal loops in Bhabha using the SCET formalism

Becher, Melnikov [0704.3582]

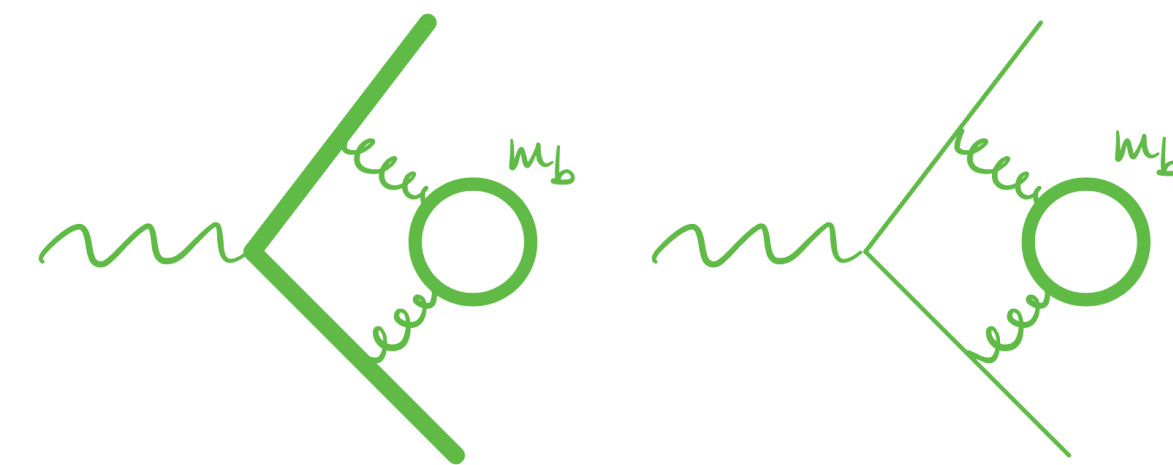
Recent application for QCD amplitudes

Wang, Xia, Yang, Ye [2312.12242]

$$| \mathcal{A}_{\text{massive}} \rangle = \prod_i \underbrace{(Z_i(\{m\}))^{1/2}}_{\text{with massive loop effects}} \underbrace{S(\{m\})}_{\substack{\text{SOFT FUNCTION} \\ O(\alpha_s^2) \text{ effects} \\ T_i \cdot T_j}} | \mathcal{A}_{\text{massless}} \rangle$$



Generalised massification



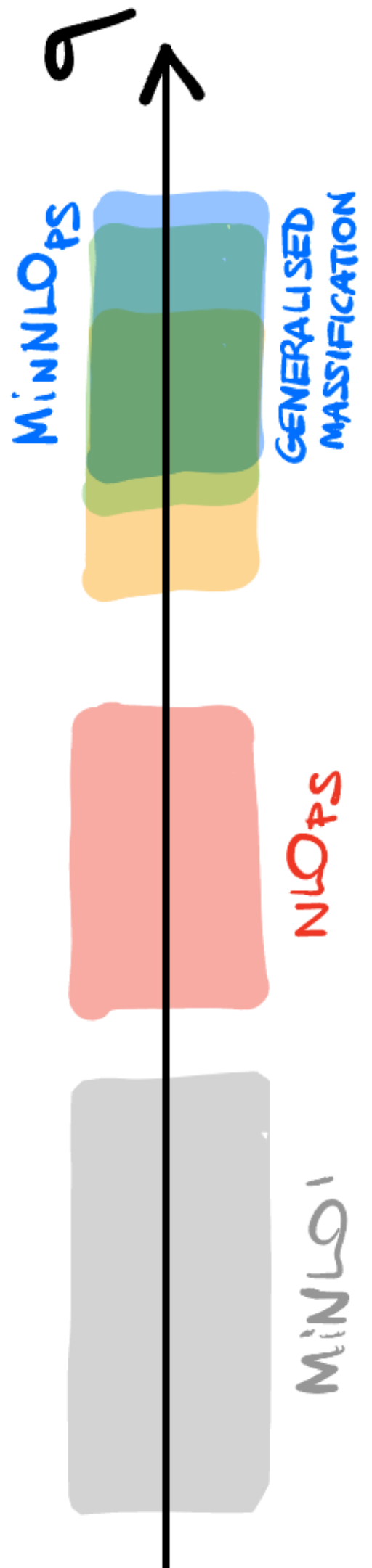
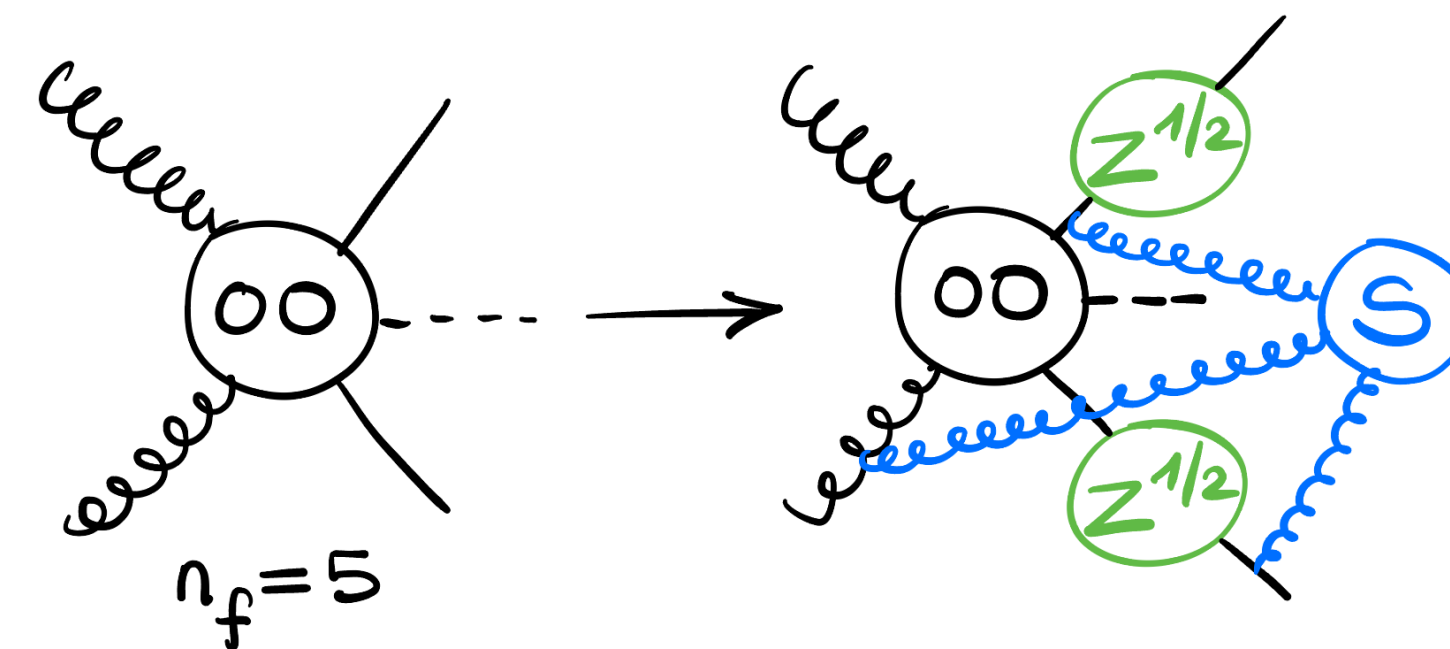
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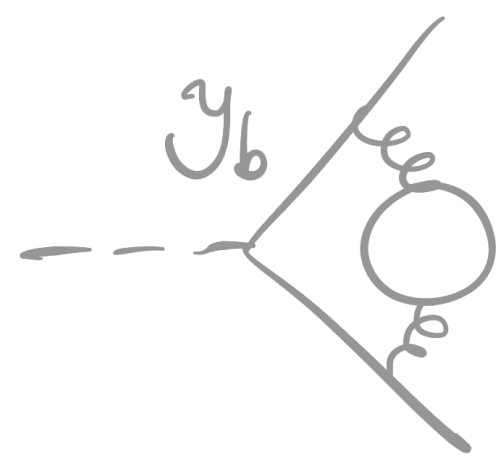
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We applied decoupling relations for α_s and $\overline{\text{MS}}$ Yukawa



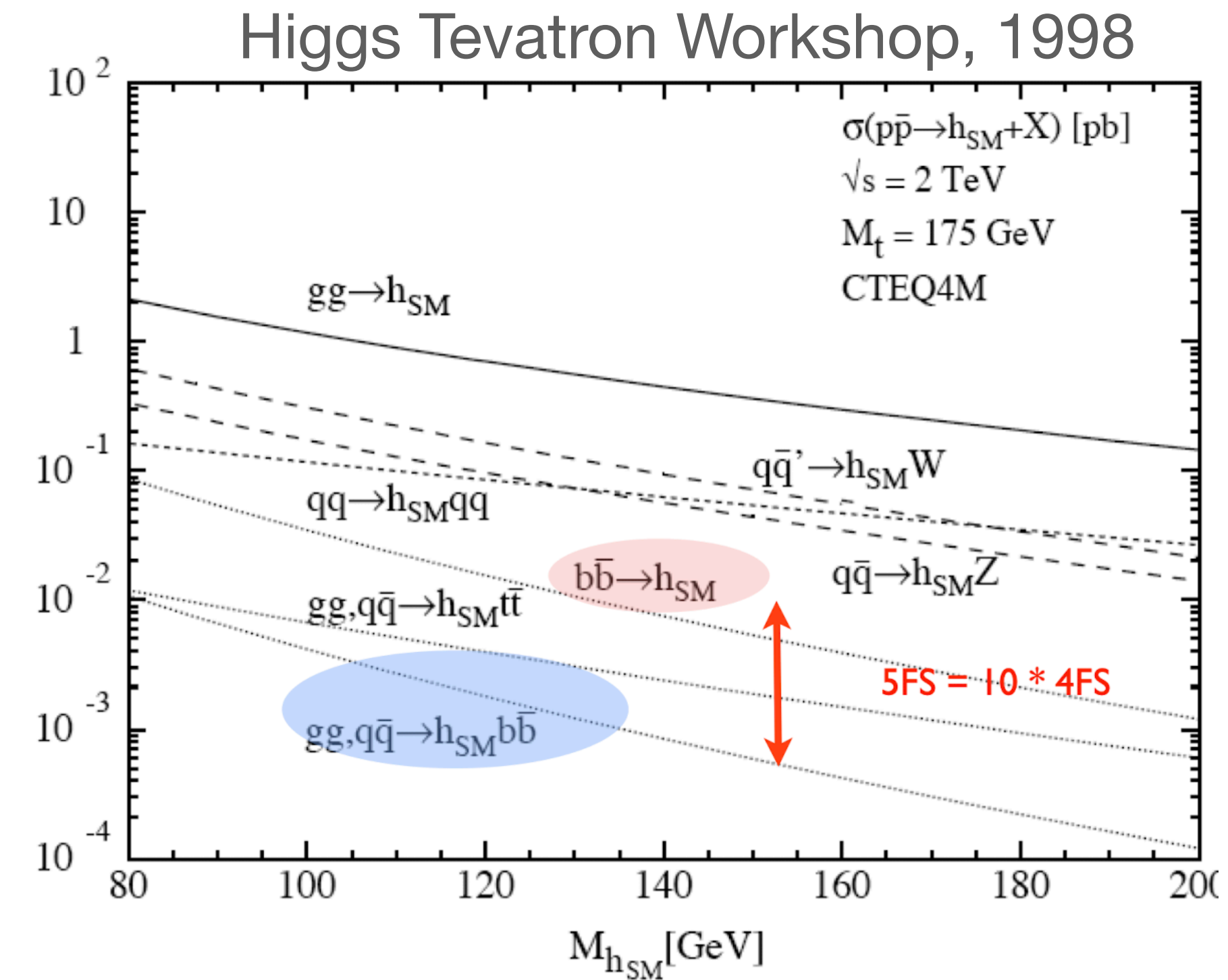
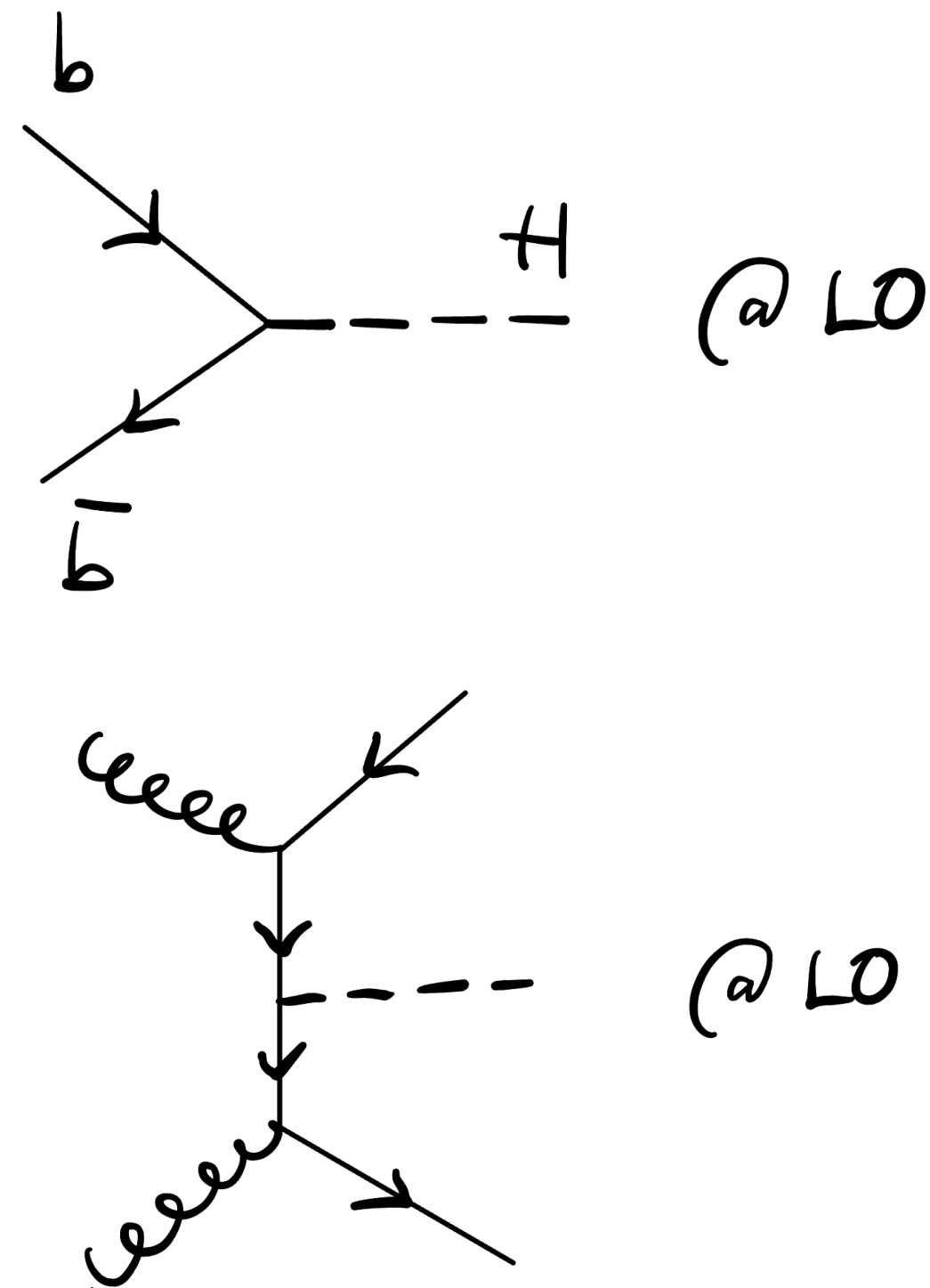
$$y_b^{(n+1)}(\mu) = y_b^{(n)}(\mu) (1 + \alpha_s^2(\mu) \cdot \text{logs})$$

$$\bar{\mathcal{F}}^{(2)} \rightarrow \bar{\mathcal{F}}^{(2)} + \text{logs}$$

Comparison between the flavour schemes



FS comparison: LO

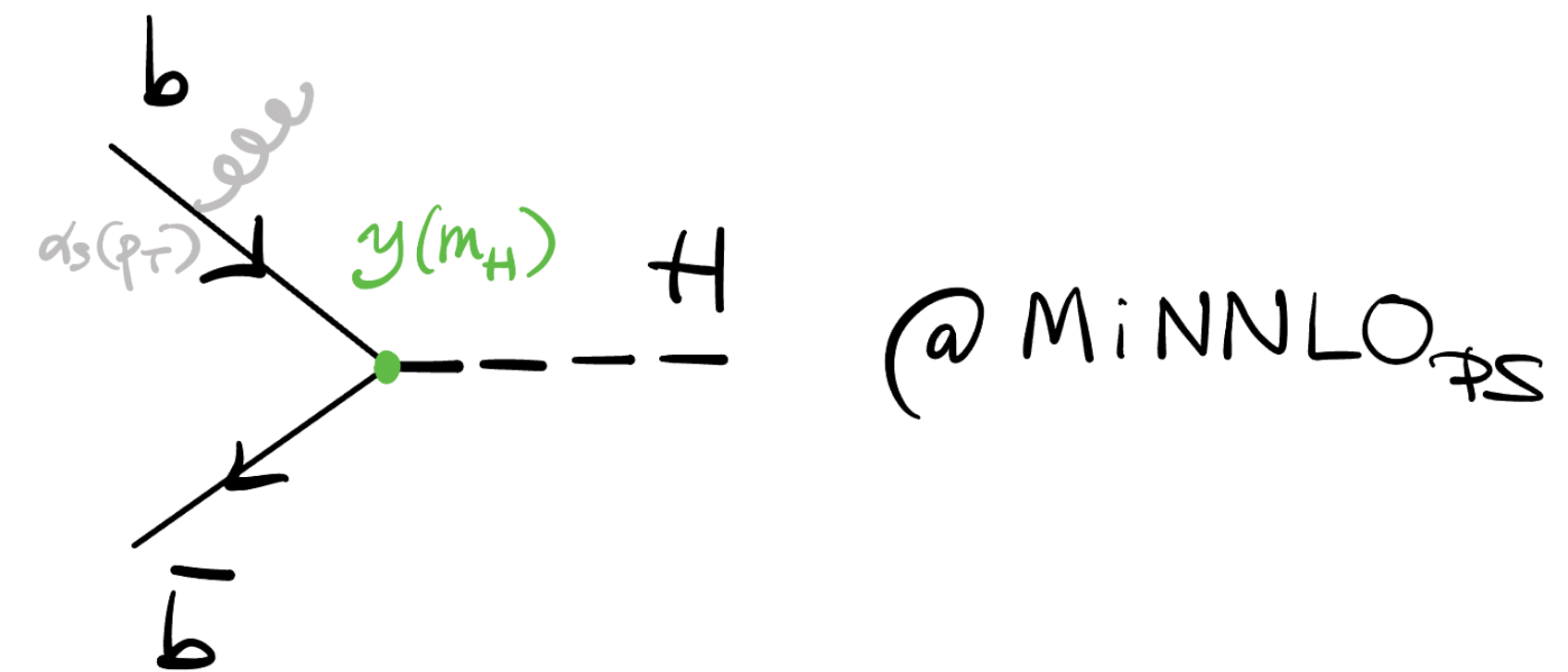
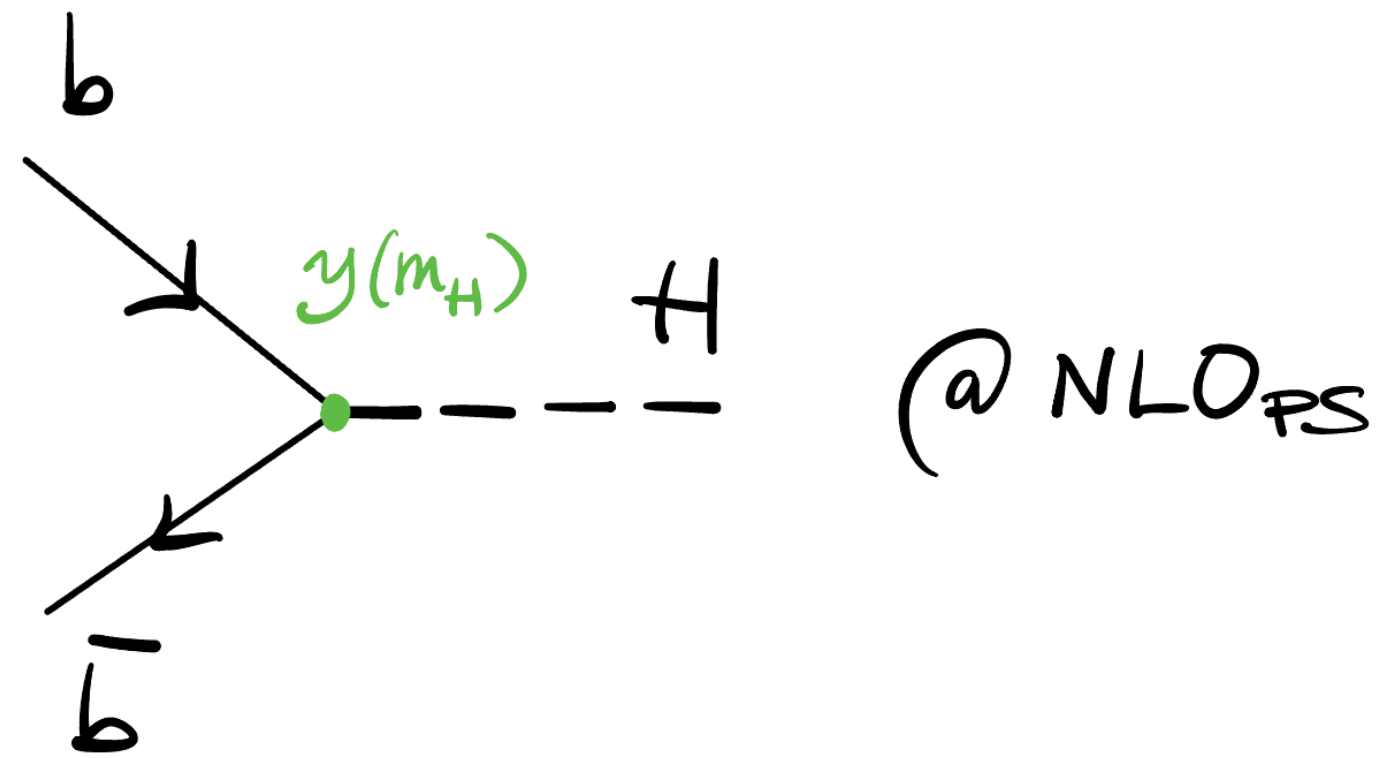


Large differences in the predictions were first observed at leading order: the effect of collinear resummation is extremely large.

Factorisation scales were tuned in order to improve the agreement ($\mu_F^{5FS} = \mu_F^{4FS} / 4$).

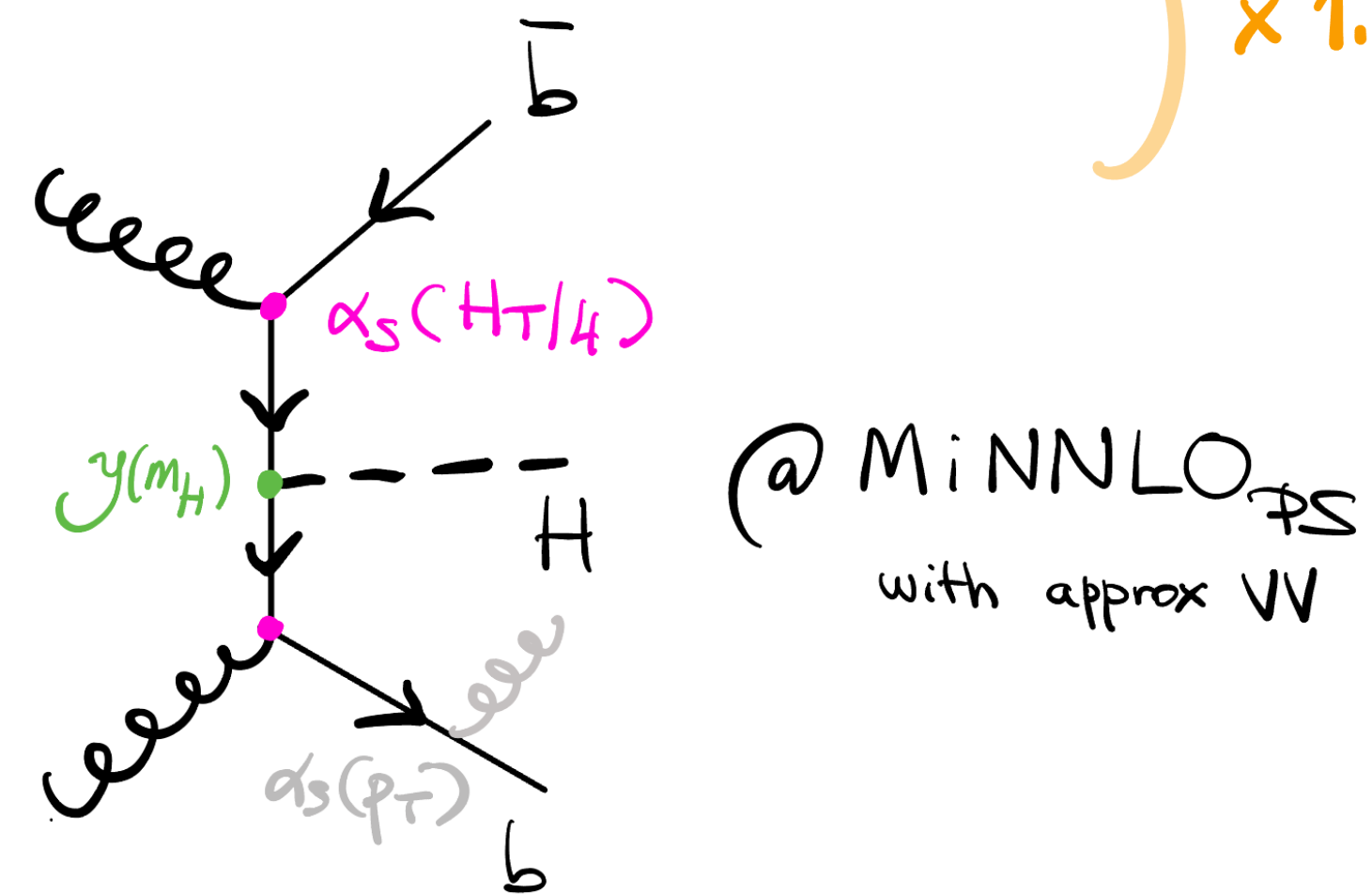
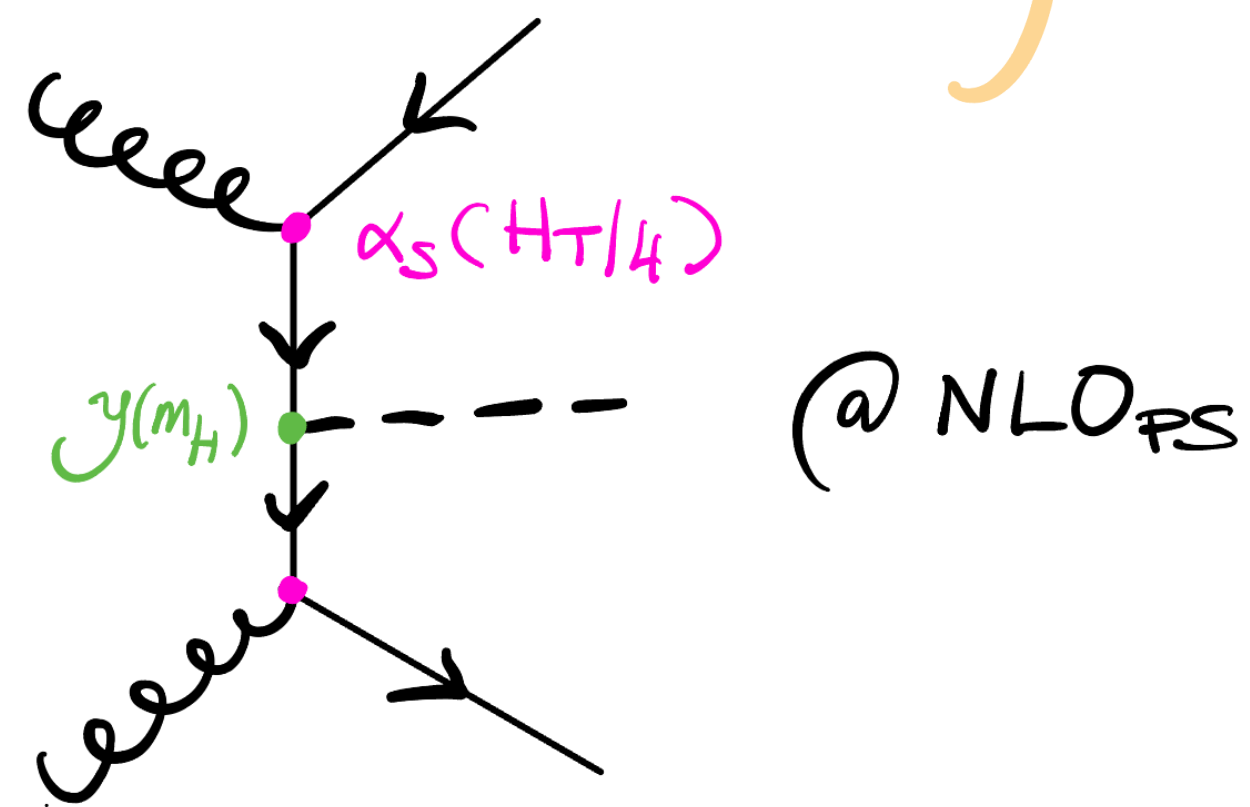


FS comparison: NLO and NNLO



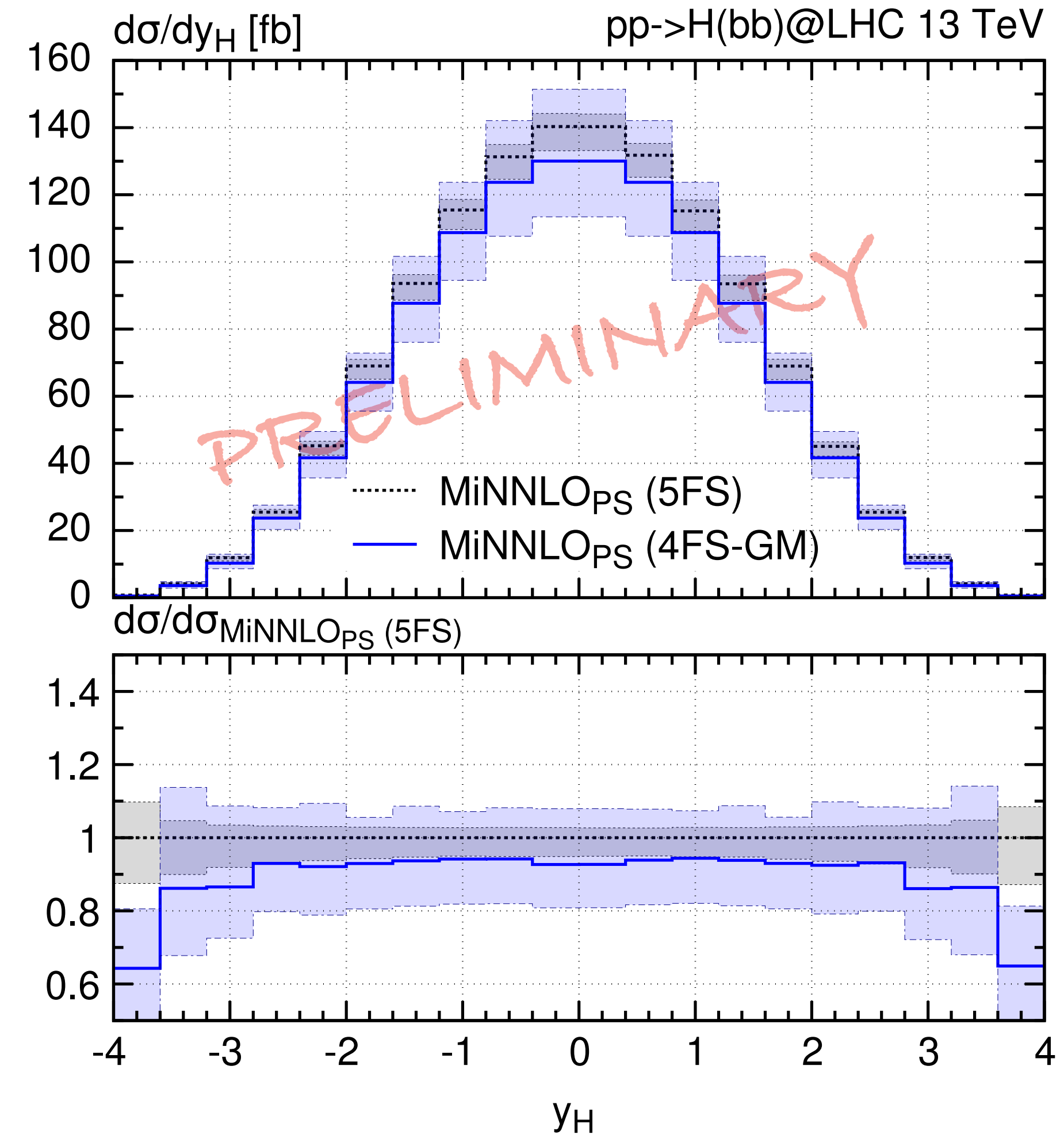
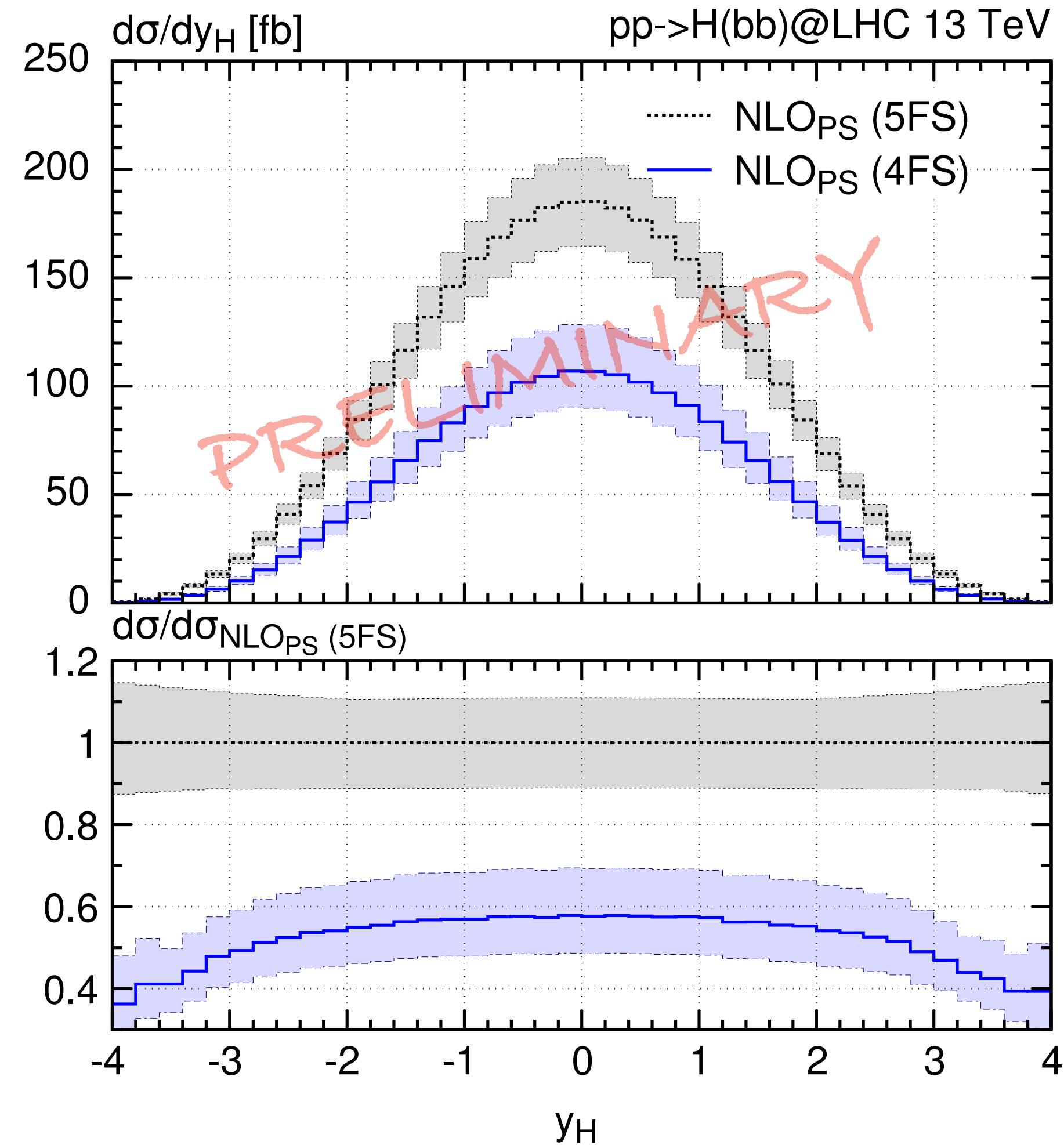
$\times 1.70$

$\times 1.09$



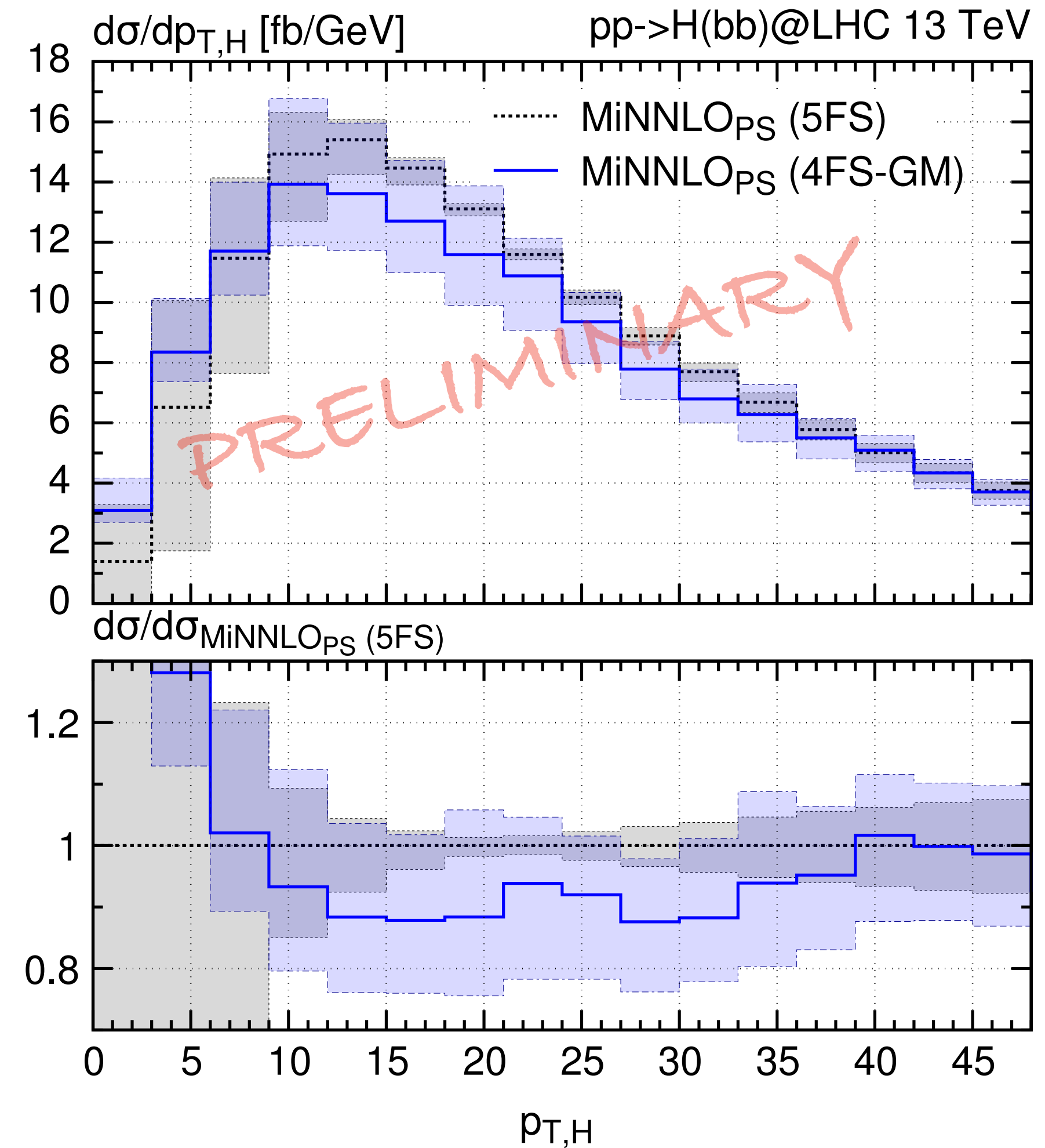
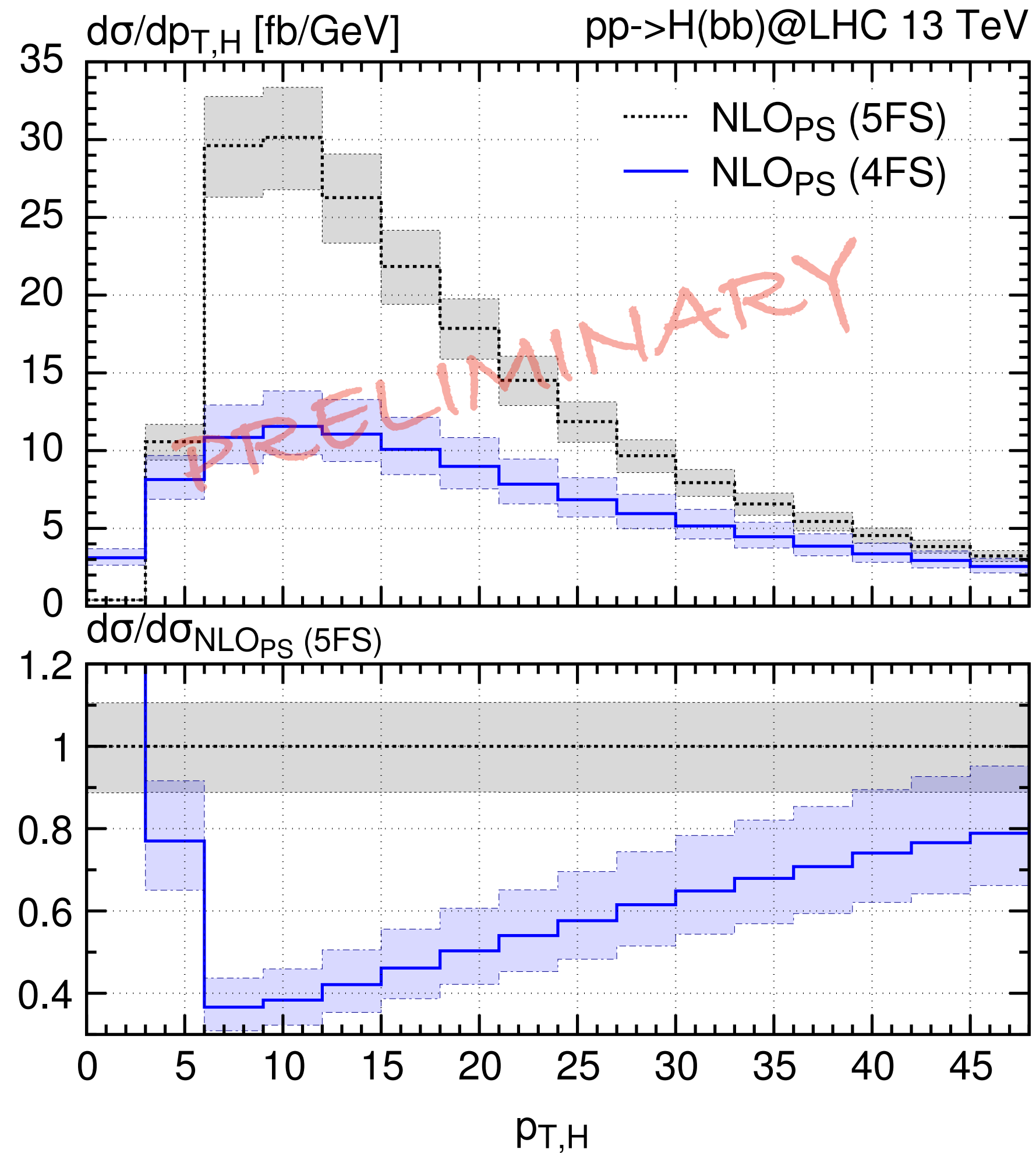


FS comparison: Higgs rapidity





FS comparison: Higgs spectrum





Summary and outlook

- Implementation of the MiNNLOPS method for bbH in 4FS with $\overline{\text{MS}}$ Yukawa
- Approximation of the VV correction using the **massification procedure**
 - For external bottoms
 - For internal bottom loops
- The **theoretical tension between the 4FS and 5FS** predictions significantly decreases at NNLO: they agree within the scale uncertainty
- We can perform a **b-tagging** of the MiNNLOPS events
- **A combination of 4FS and 5FS** results can improve the description of the process in the whole phase space **at differential level**

Thank you for the attention!



Backup slides



Cross-section details

K_R	K_F	MINLO'	MINNLO _{PS} (Orig. Mass.)	MINNLO _{PS} (Gen. Mass.)
1	1	0.277(0)	0.460(7)	0.464(9)
1	2	0.268(8)	0.465(2)	0.470(7)
2	1	0.192(5)	0.403(0)	0.408(1)
2	2	0.195(5)	0.407(0)	0.412(1)
1	$\frac{1}{2}$	0.258(9)	0.457(8)	0.466(0)
$\frac{1}{2}$	1	0.382(7)	0.520(7)	0.527(4)
$\frac{1}{2}$	$\frac{1}{2}$	0.375(3)	0.519(3)	0.525(1)
		$0.277(0)^{+34\%}_{-27\%}$ pb	$0.460(7)^{+13\%}_{-13\%}$ pb	$0.464(9)^{+14\%}_{-13\%}$ pb

NLO+PS (5FS)	NLO+PS (4FS)	MINNLO _{PS} (5FS)	MINNLO _{PS} (4FS-GM)
$0.677(2)^{+11\%}_{-11\%}$ pb	$0.381(0)^{+20\%}_{-16\%}$ pb	$0.509(8)^{+3.0\%}_{-5.0\%}$ pb	$0.469(2)^{+14\%}_{-13\%}$ pb



Massive-massless mapping

We fix the 4-momenta of the incoming partons and the Higgs state k_5 . We want to maintain the invariant mass of the pair

$$m_{QQ} = (k_3 + k_4)^2 = (\tilde{k}_3 + \tilde{k}_4)^2.$$

We introduce the factors

$$\rho_{\pm} = \frac{1 \pm \rho}{2\rho}, \quad \rho = \sqrt{1 - \frac{4m_Q^2}{m_{QQ}^2}} \quad (3)$$

and we define the new momenta as a linear combination of the old ones as follows in the quark-channel,

$$\tilde{k}_3^{\mu} = \rho_+ k_3^{\mu} - \rho_- k_4^{\mu}, \quad (4)$$

$$\tilde{k}_4^{\mu} = \rho_+ k_4^{\mu} - \rho_- k_3^{\mu}. \quad (5)$$

For the gluon channel, we have to avoid the collinear divergence,

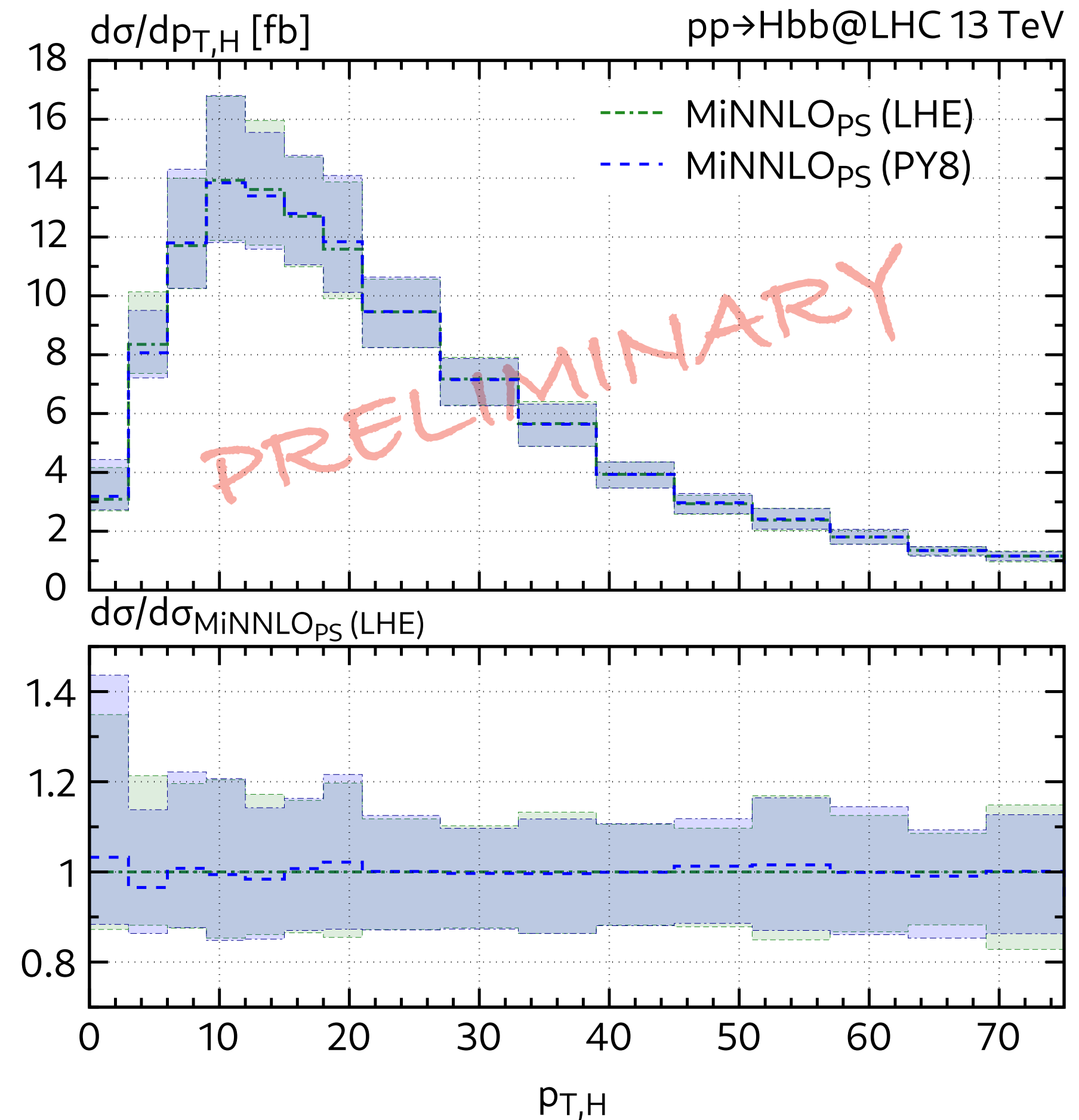
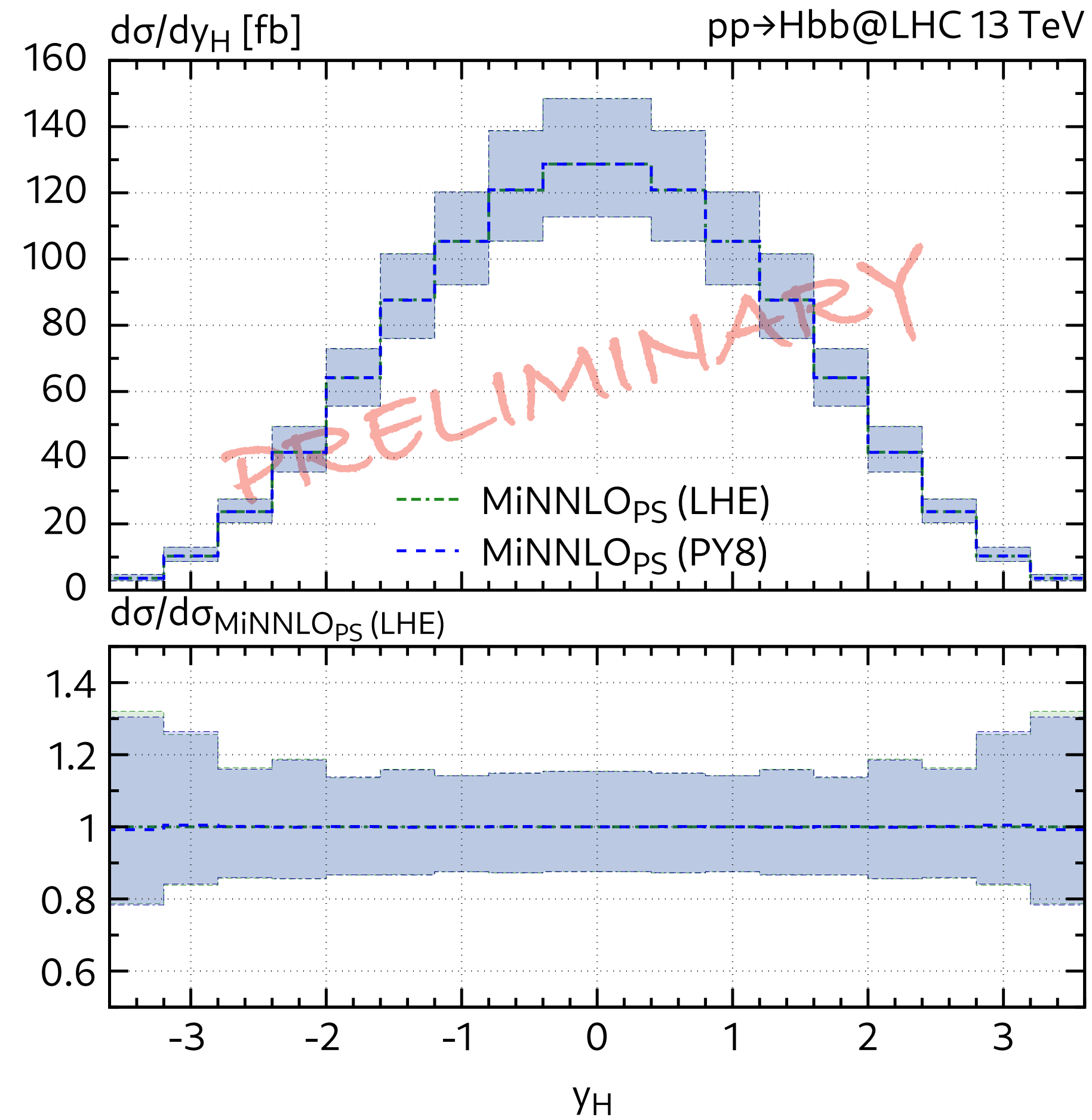
$$\tilde{k}_i^{\mu} = k_i^{\mu} + \left(\sqrt{1 - \frac{m_Q^2 n_x^2}{(p_i \cdot n_i)^2}} - 1 \right) \frac{p_i \cdot n_i}{n_i}, \quad \text{for } i=3,4, \quad (6)$$

where n_i is the transverse component to both k_1 and k_2 . The momentum conservation is restored by performing a Boost such that

$$\tilde{k}_1 + \tilde{k}_2 = k_1 + k_2 - (k_3 + k_4 - \tilde{k}_3 - \tilde{k}_4). \quad (7)$$

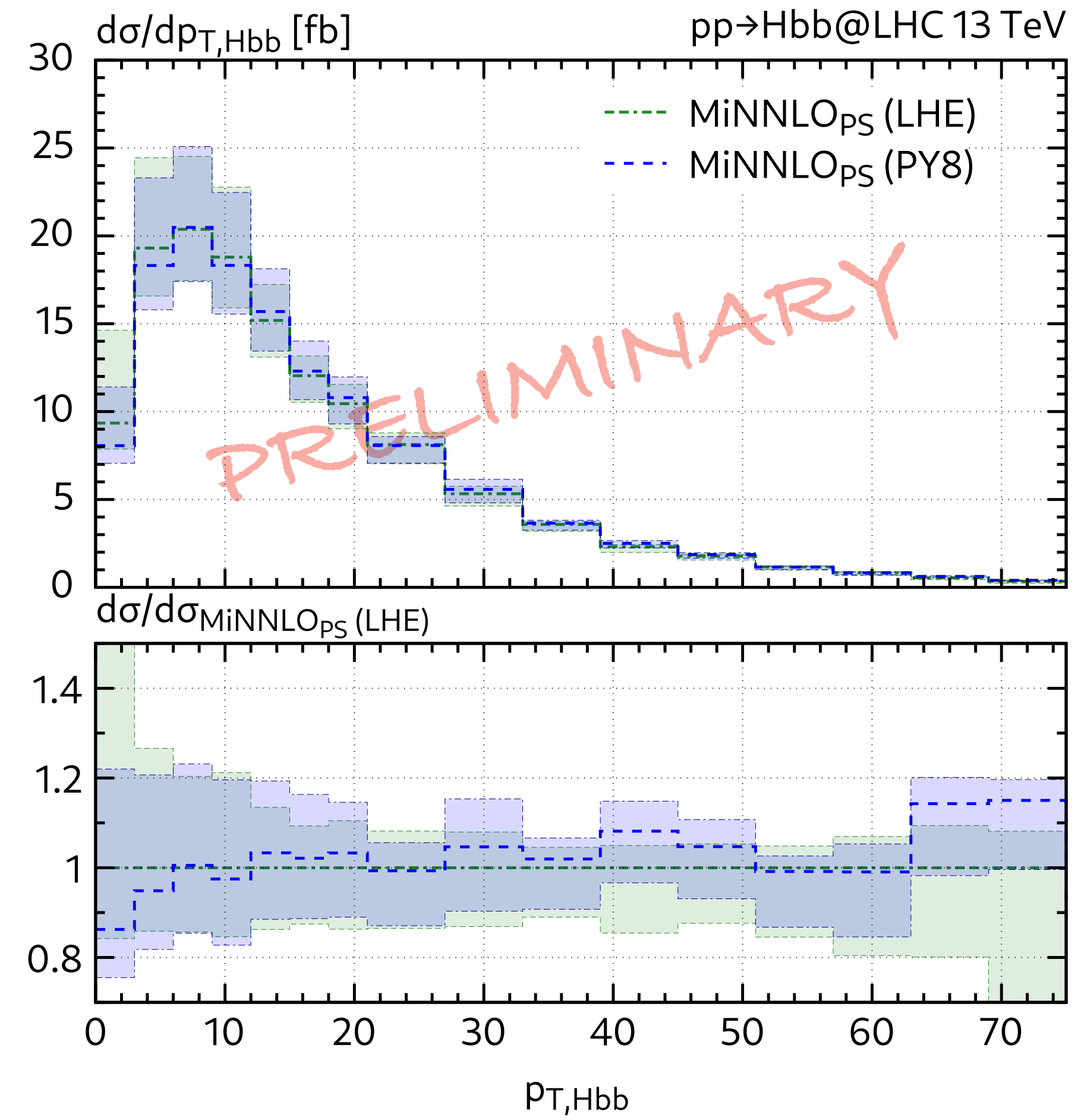
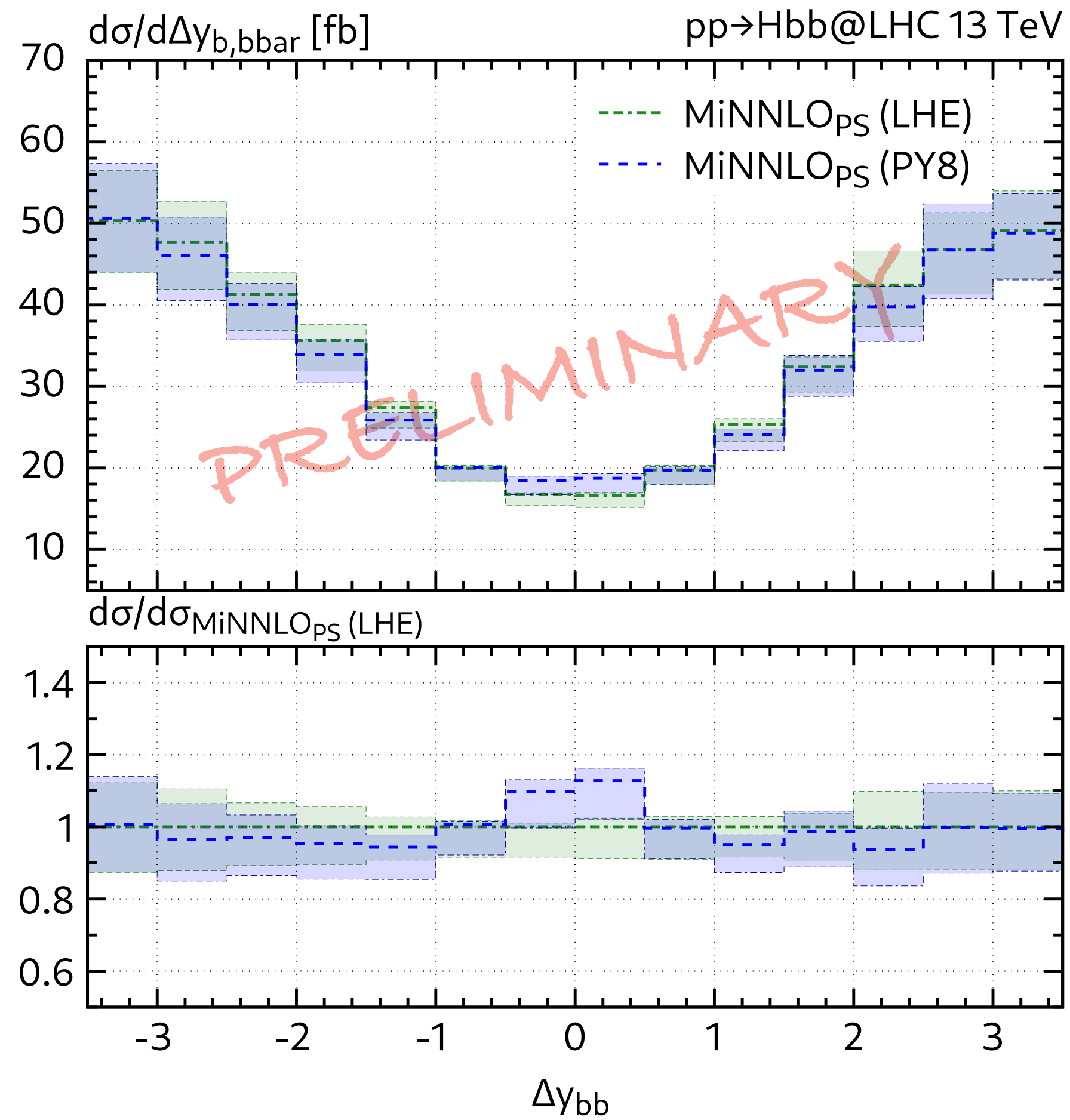


Shower effects in 4FS





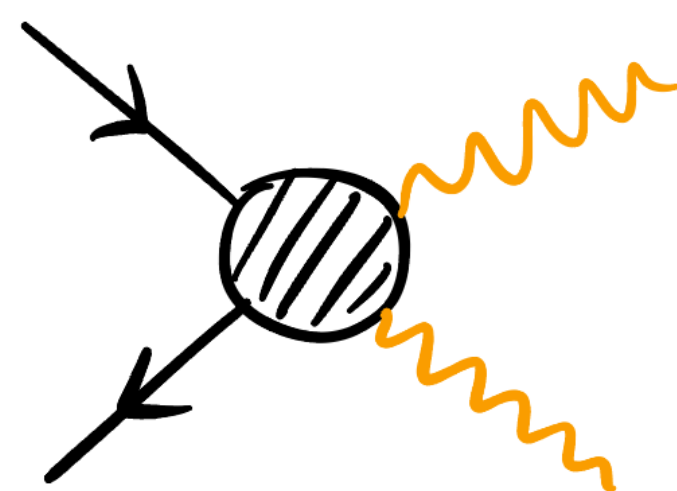
Shower effects in 4FS





Phenomenology with MiNNLOps

DIBOSON
PROCESSES

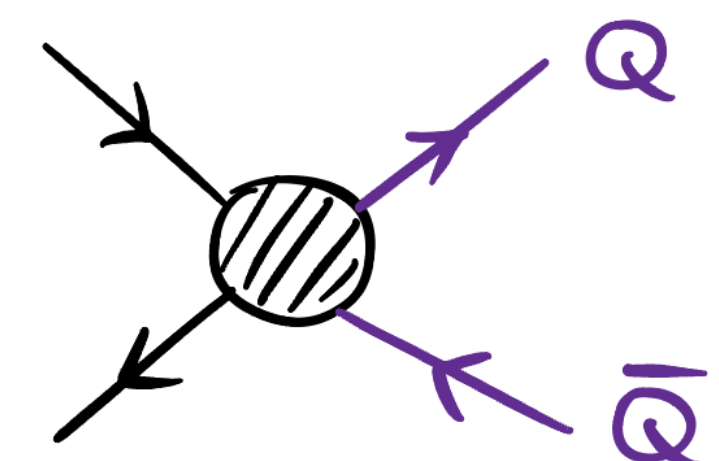


- $Z\gamma$ [2010.10478, 2108.11315]
- WW [2103.12077]
- ZZ [2108.05337]
- $WH/ZH(H \rightarrow b\bar{b})$ [2112.04168]
- $\gamma\gamma$ [2204.12602]
- WZ [2208.12660]
- SMEFT studies [2204.00663, 2311.06107]

$gg \rightarrow H, W/Z$ [1908.06987,
2006.04133,
2402.00596]

5FS $b\bar{b} \rightarrow H$ [2402.04025]

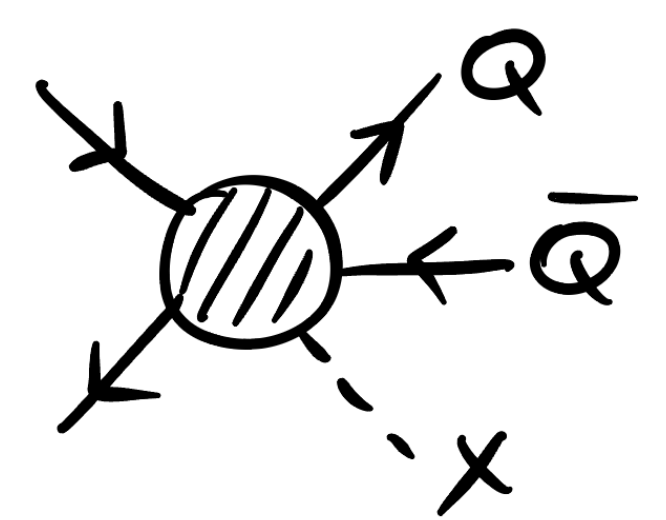
HEAVY-QUARK
PRODUCTION



$t\bar{t}$ [2012.14267, 2112.12135]

$b\bar{b}$ [2302.01645]

$b\bar{b}H$ 4FS

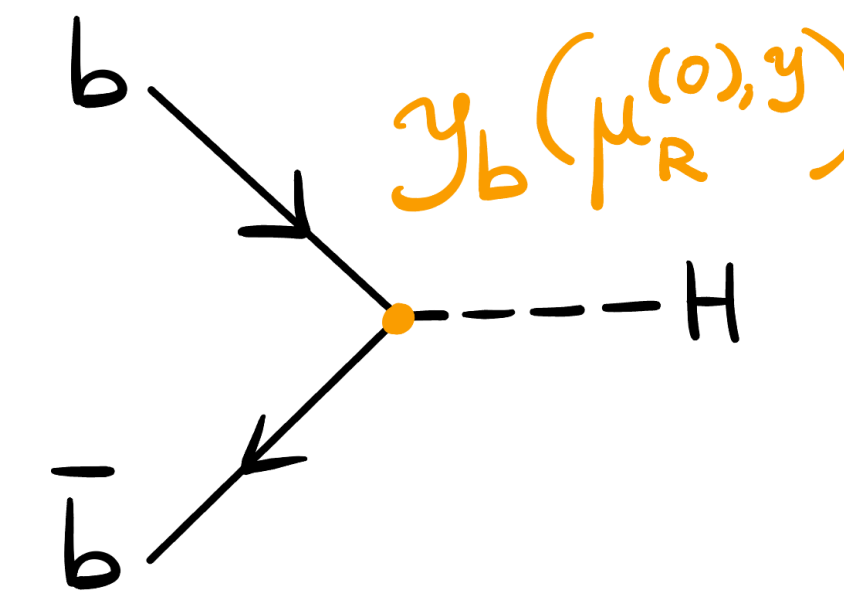




MiNNLOps for Yukawa induced processes

The Yukawa coupling is renormalised in $\overline{\text{MS}}$ scheme.

The running of this Born coupling requires some adaptations of the MiNNLOps method to take account the extra scale dependence.



$$H^{(1,2)} \rightarrow H^{(1,2)} \left(\log \frac{\mu_R^{(0),y}}{m_H} \right)$$

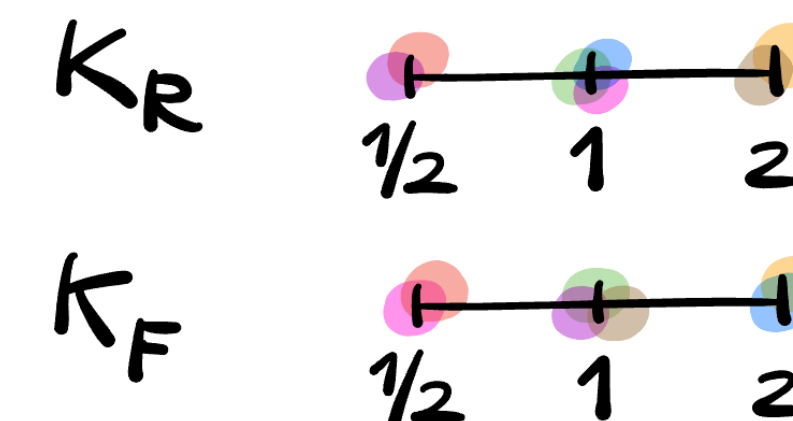
$y_b(m_b = 4.18 \text{ GeV}) \xrightarrow{\text{3-loop running}} y_b(m_H) \rightarrow y_b(K_R m_H)$

$\alpha_s(p_T) \rightarrow \alpha_s(K_R p_T)$

$f_a(p_T) \rightarrow f_a(K_F p_T)$

MiNNLO_{PS}

7-point scale variation





MiNNLOps in a nutshell

$$NLO X_j \longrightarrow NNLO X$$

MiNNLOps is an extension of MiNLO' to achieve NNLO+PS accuracy for inclusive observables.

Monni, Nason, Re, Wiesemann, Zanderighi [1206.3572]

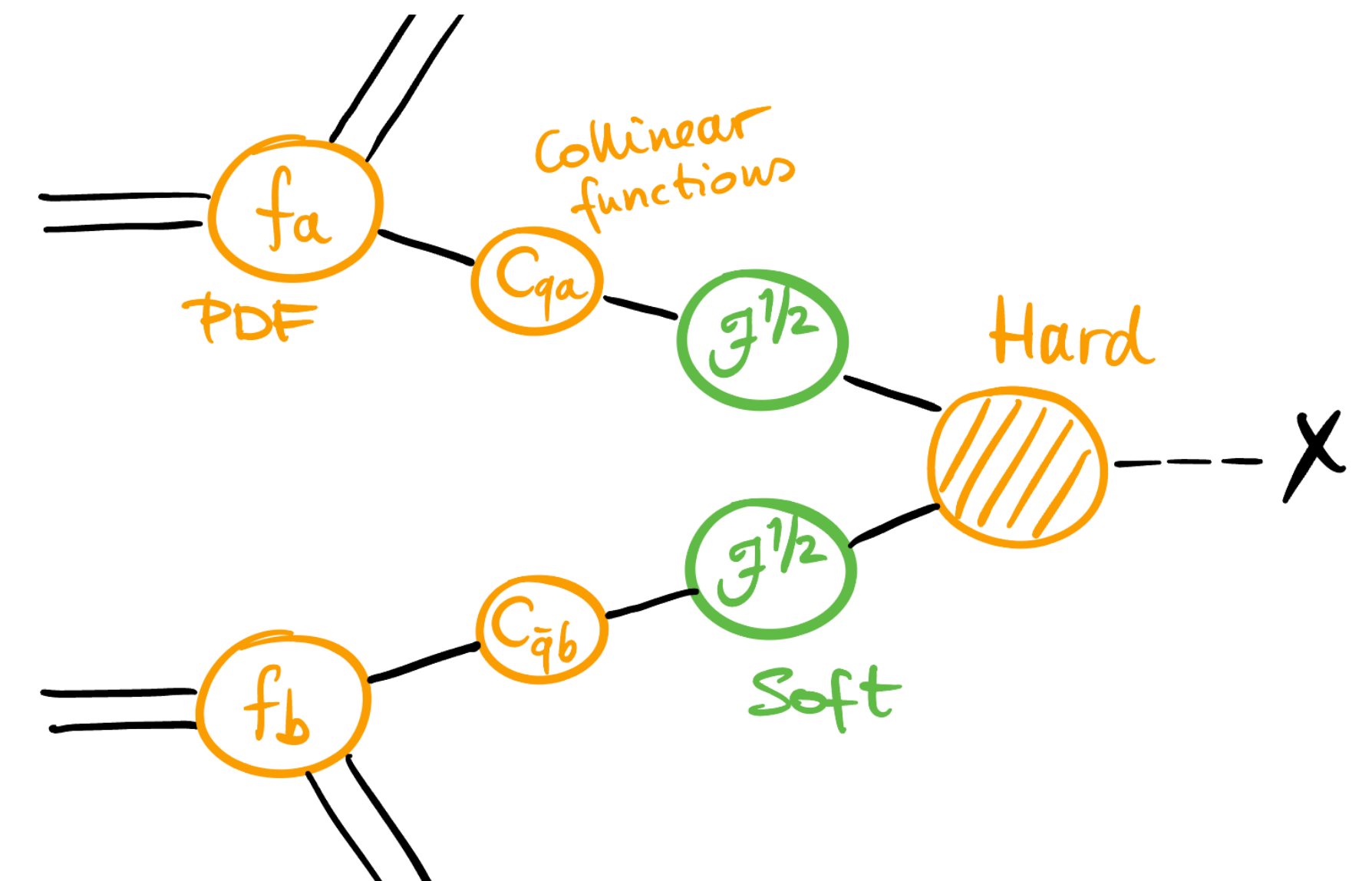
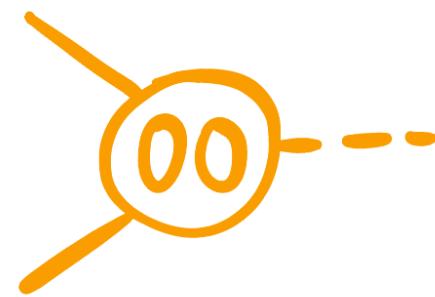
Split the differential inclusive cross-section into the singular and regular part in the small transverse momentum limit: $d\sigma = d\sigma^{sing} + d\sigma^{reg}$.

$$\frac{d\sigma^{sing}}{dp_T d\Phi_X} = \frac{d}{dp_T} \left\{ \mathcal{F}(p_T) \mathcal{L}(p_T) \right\} =: \exp \left[-\tilde{S}(p_T) \right] D(p_T)$$

Sudakov form factor

$$\mathcal{F}(p_T) = \exp \left[-\tilde{S}(p_T) \right]$$

Luminosity: it also contains





MiNNLOps in a nutshell

$$d\sigma = d\sigma^{sing} + d\sigma^{reg}$$

The modified POWHEG function is

$$\bar{B}(\Phi_{XJ}) = e^{-\tilde{S}(p_T)} \left\{ B \left(1 - \alpha_s(p_T) \tilde{S}^{(1)} \right) + V + \int d\phi_{rad} R + \left[D(p_T) - D^{(1)} - D^{(2)} \right] \times F^{corr} \right\}$$

MiNLO' structure

Extra term: it ensures NNLO accuracy.

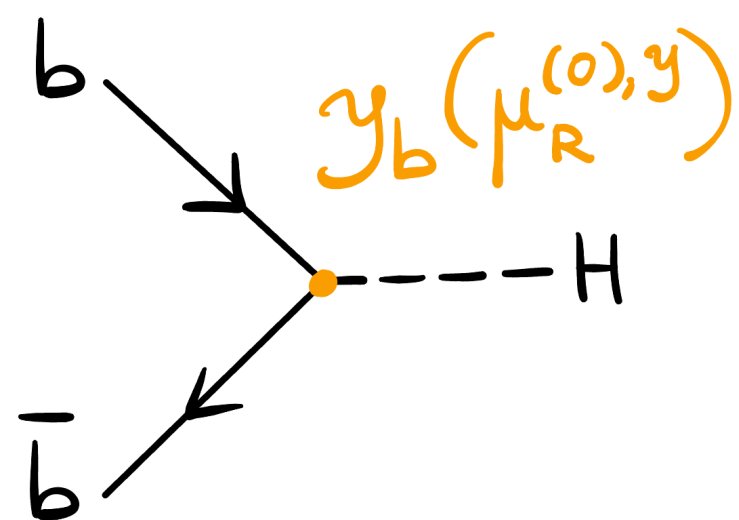
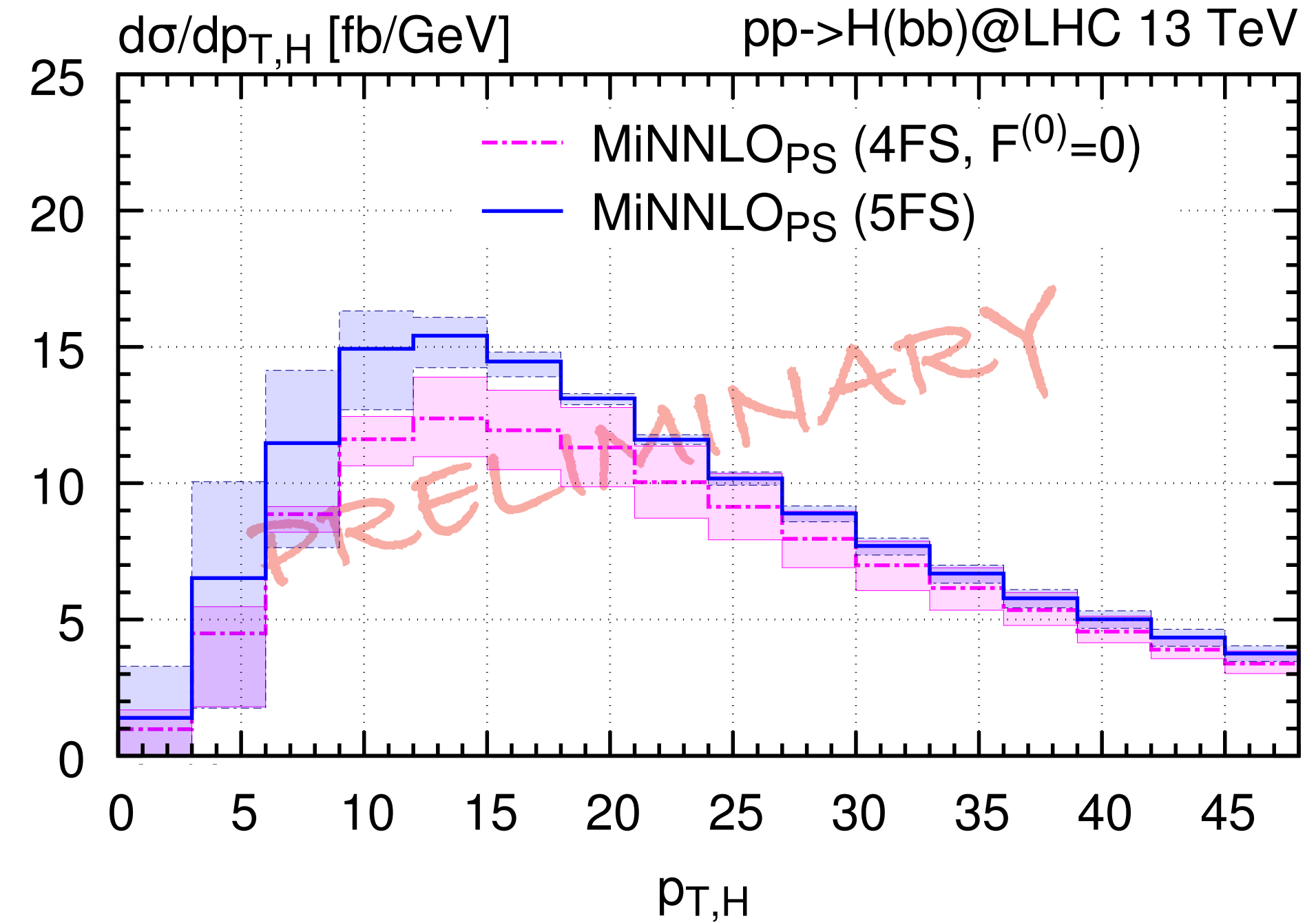
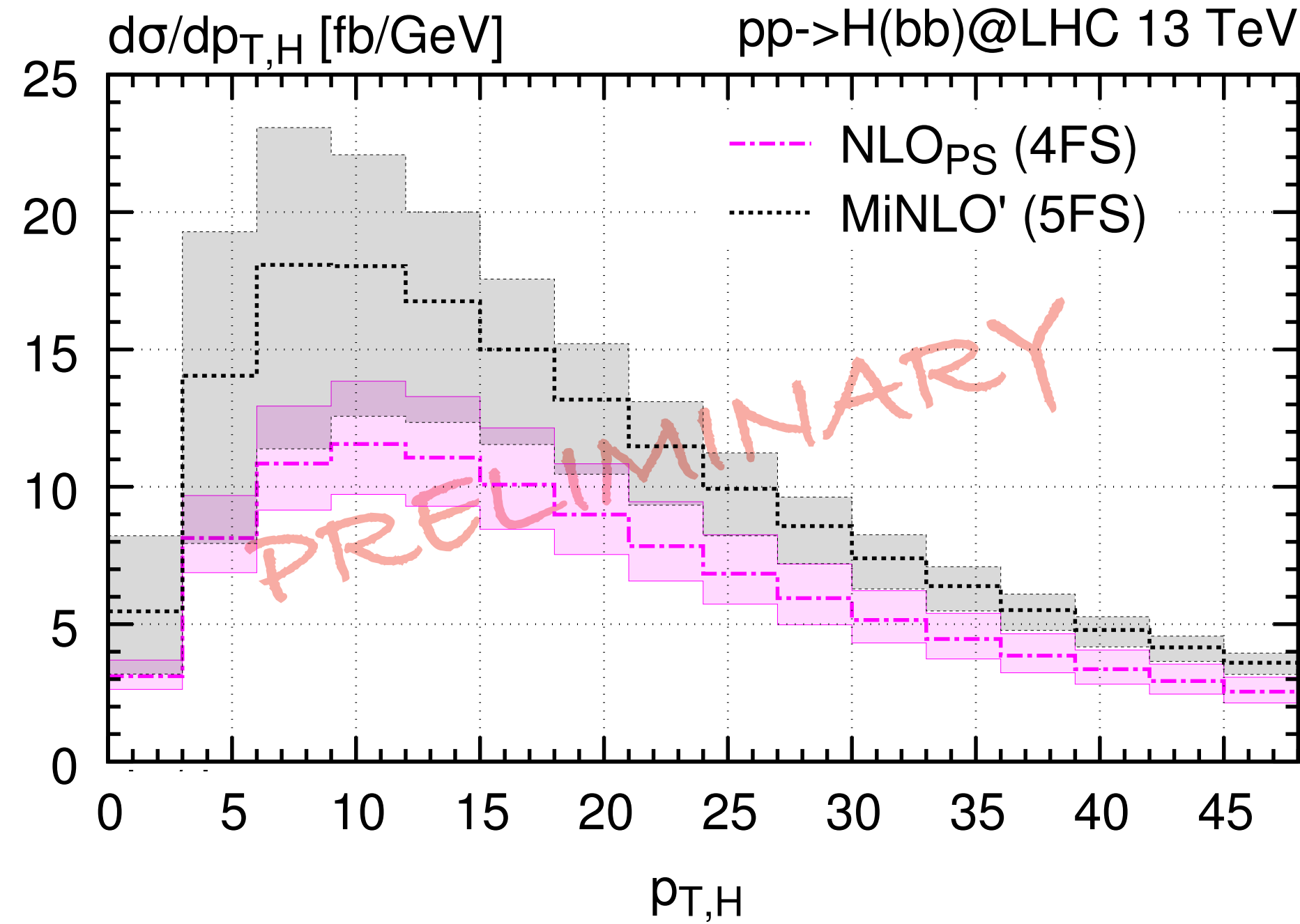
F^{corr} encodes the spreading of the D-terms upon the full Φ_{XJ} .

- In the singular part, the QCD scales must be $\mu_F \sim \mu_R \sim p_T$.
- For the regular part, different scale choices can be performed:
 - the transverse momentum p_T (original choice)
 - the **hard scale** Q (FOatQ=1)

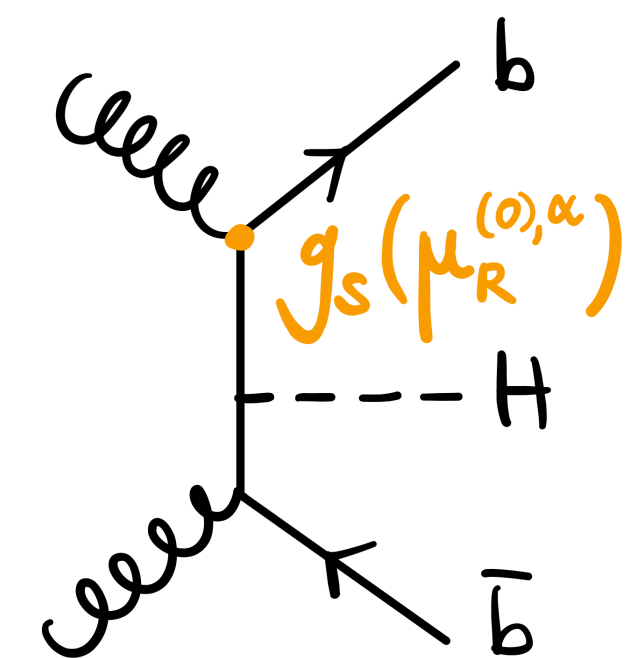
Gavardi, Oleari, Re [2204.12602]



Before the two-loop...



$(\mu_R^{(0),\alpha}, \mu_R^{(0),y})$	NLO _{PS} (5FS)	NLO _{PS} (4FS)	MINNLO _{PS} (5FS)	MINNLO _{PS} (4FS, $\mathcal{F}^{(0)} = 0$)
$(\frac{1}{4}H_T, m_H)$	$0.646(0)^{+10.4\%}_{-10.9\%}$ pb	$0.381(2)^{+20.2\%}_{-15.9\%}$ pb	$0.509(8)^{+2.9\%}_{-5.3\%}$ pb	$0.434(1)^{+6.4\%}_{-10.0\%}$ pb





5FS scale variation

We studied the effects of the correlation between the renormalisation scale factors.

We compare:

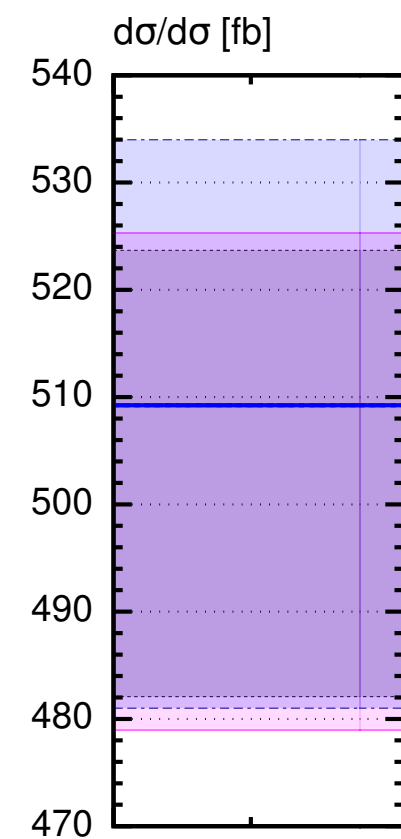
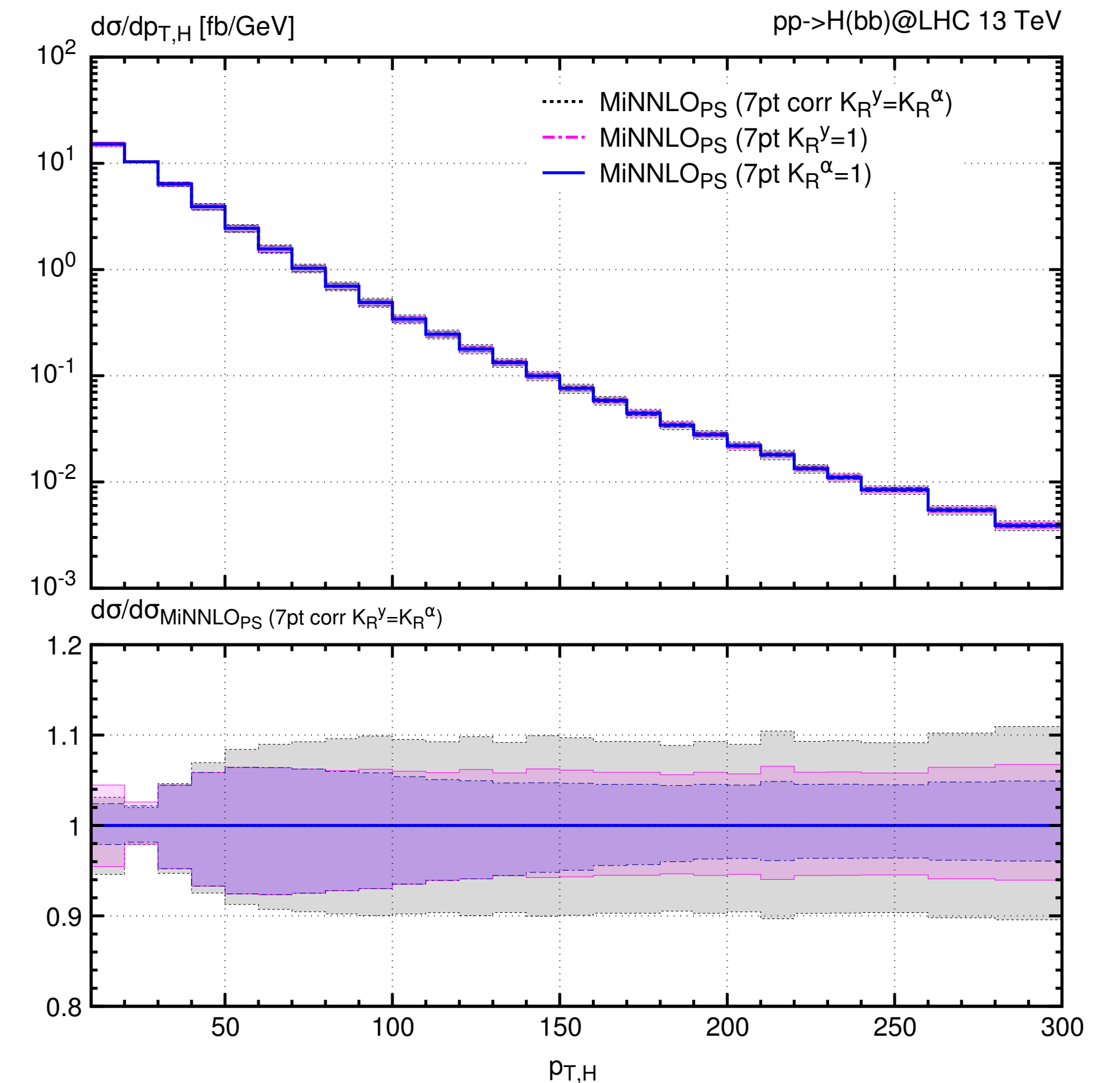
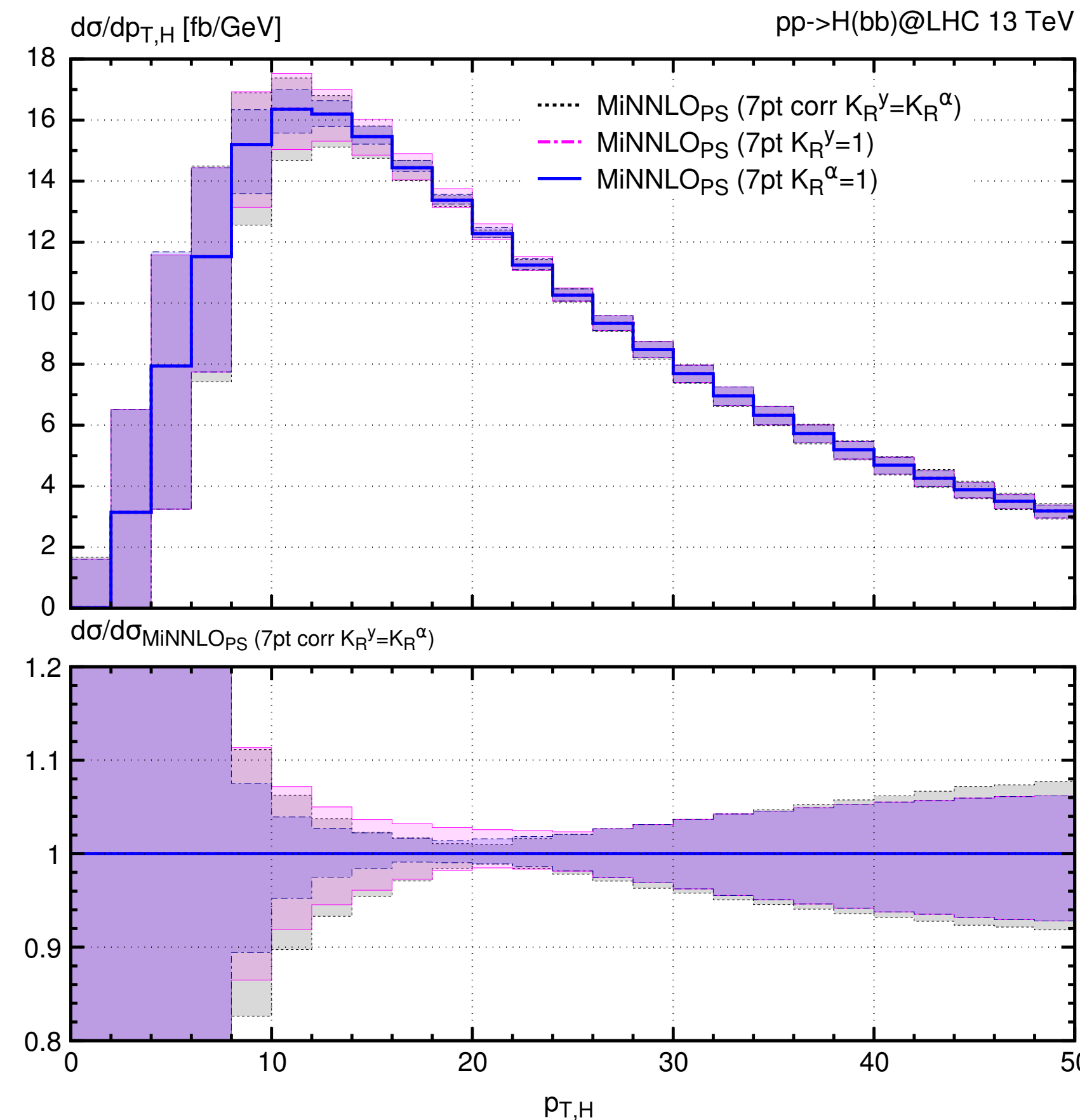
- The standard prediction
- 7pt s.v. for (K_R^α, K_F)
- 7pt s.v. for (K_R^γ, K_F)

$$y_b(m_H) \rightarrow y_b(K_R m_H)$$

$$\alpha_s(p_T) \rightarrow \alpha_s(K_R p_T)$$

$$f_a(p_T) \rightarrow f_a(K_F p_T)$$

MiNNLO_{PS}





5FS scale variation

We compare:

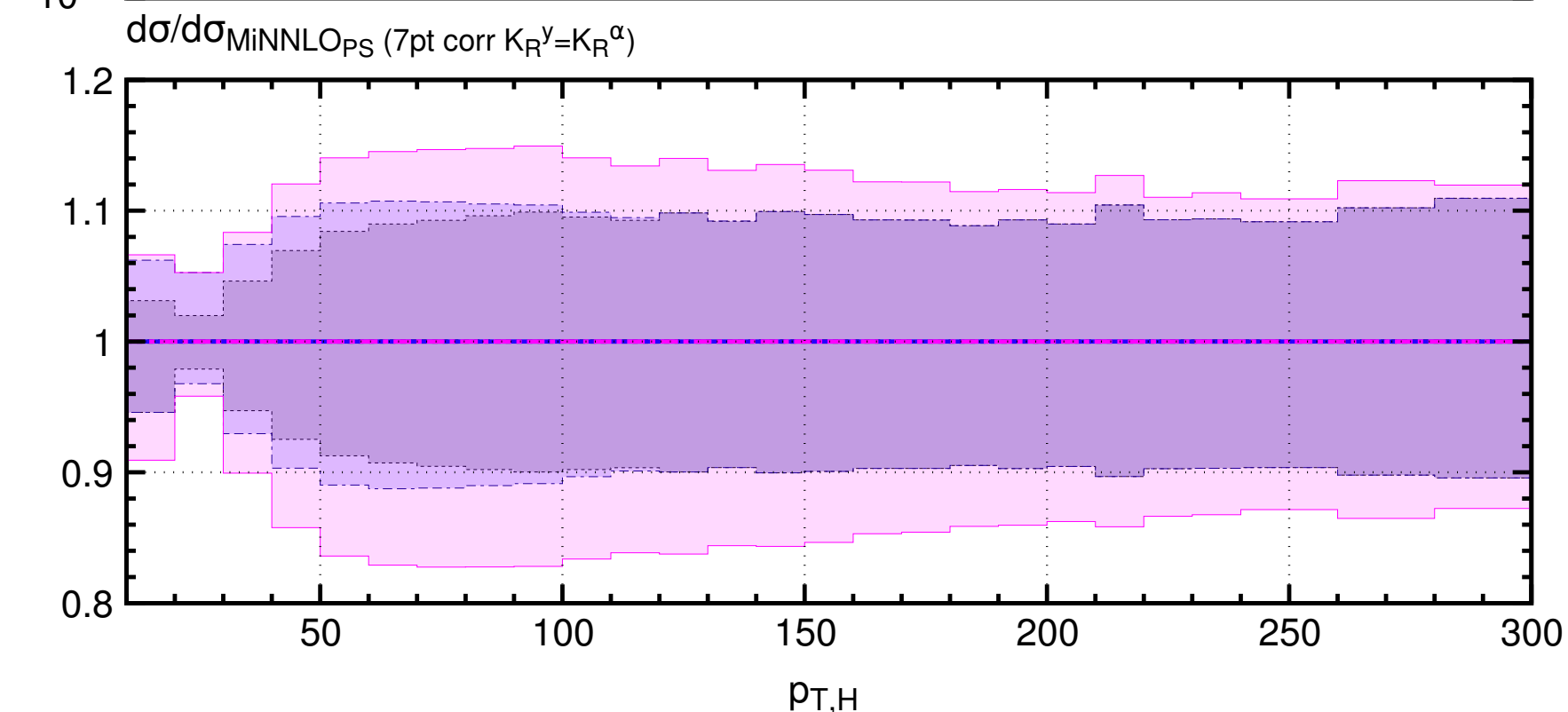
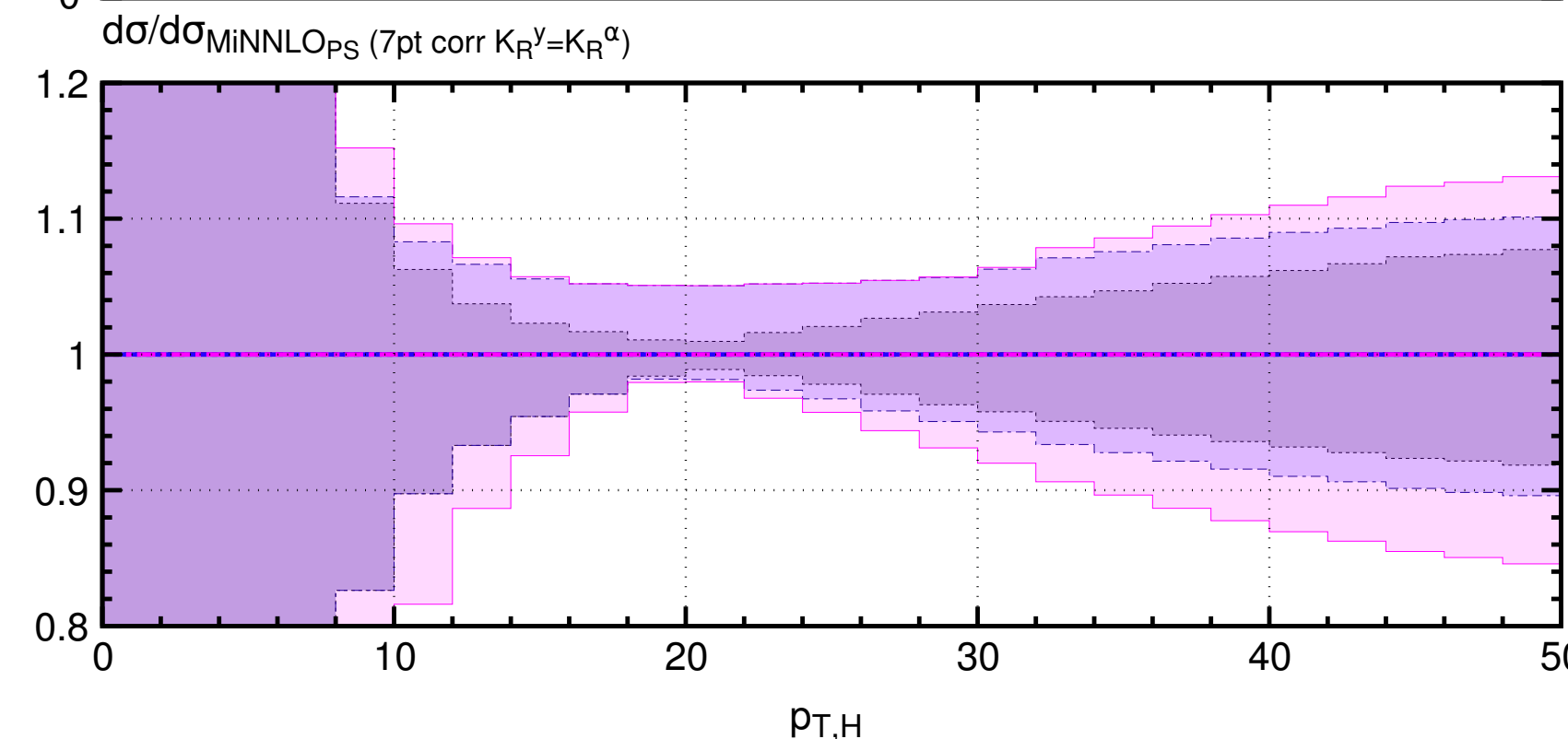
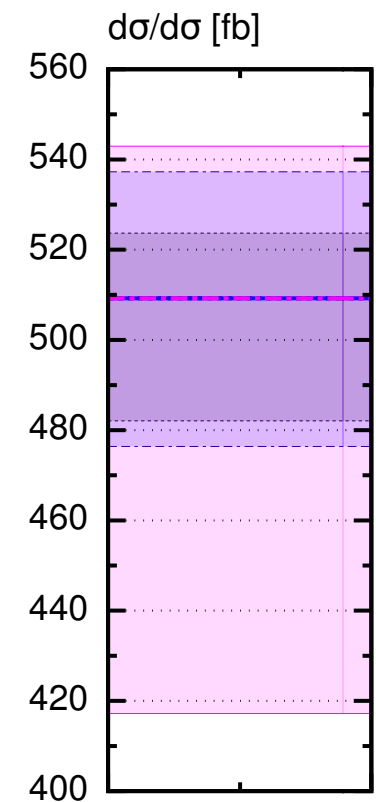
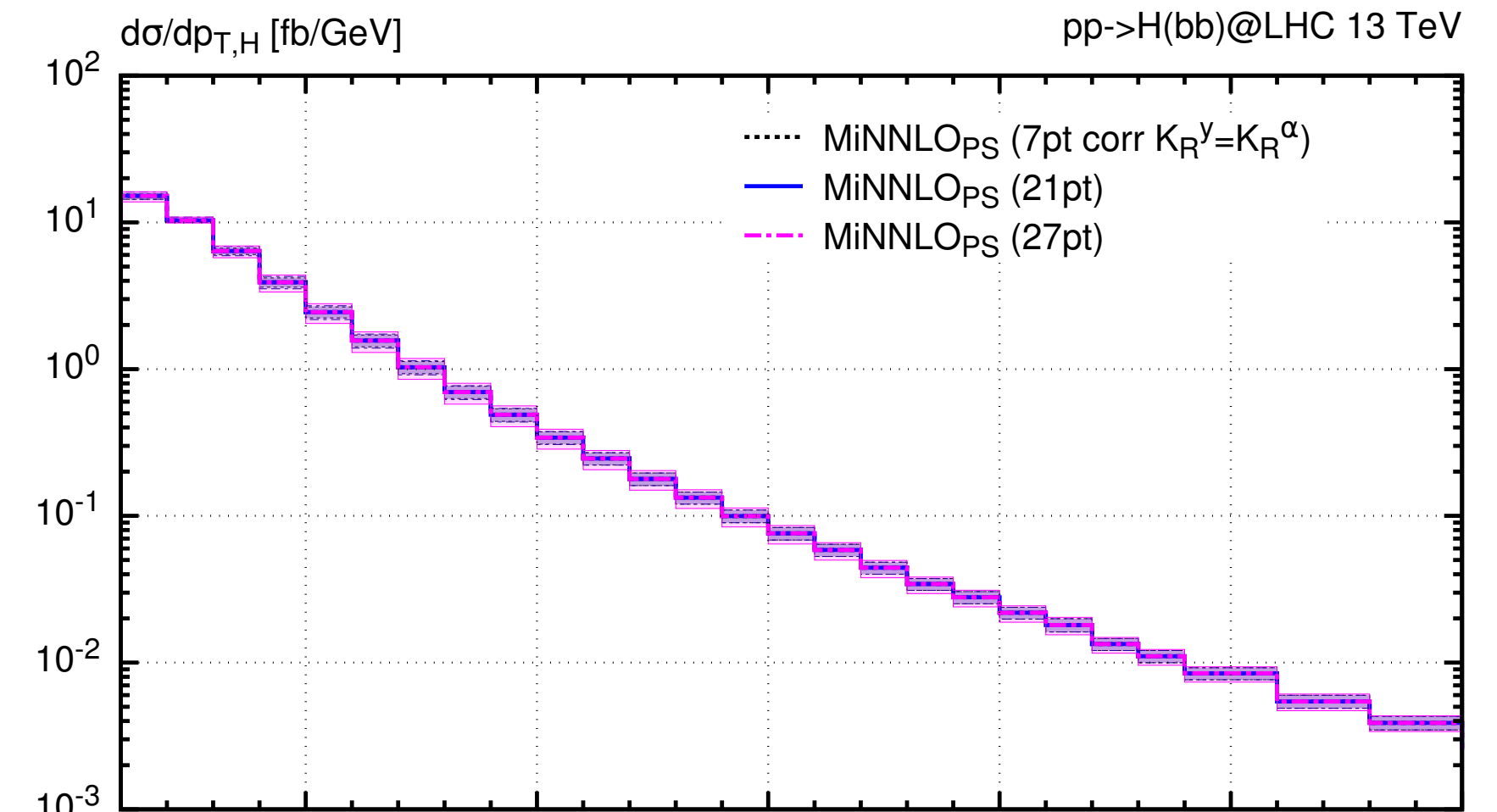
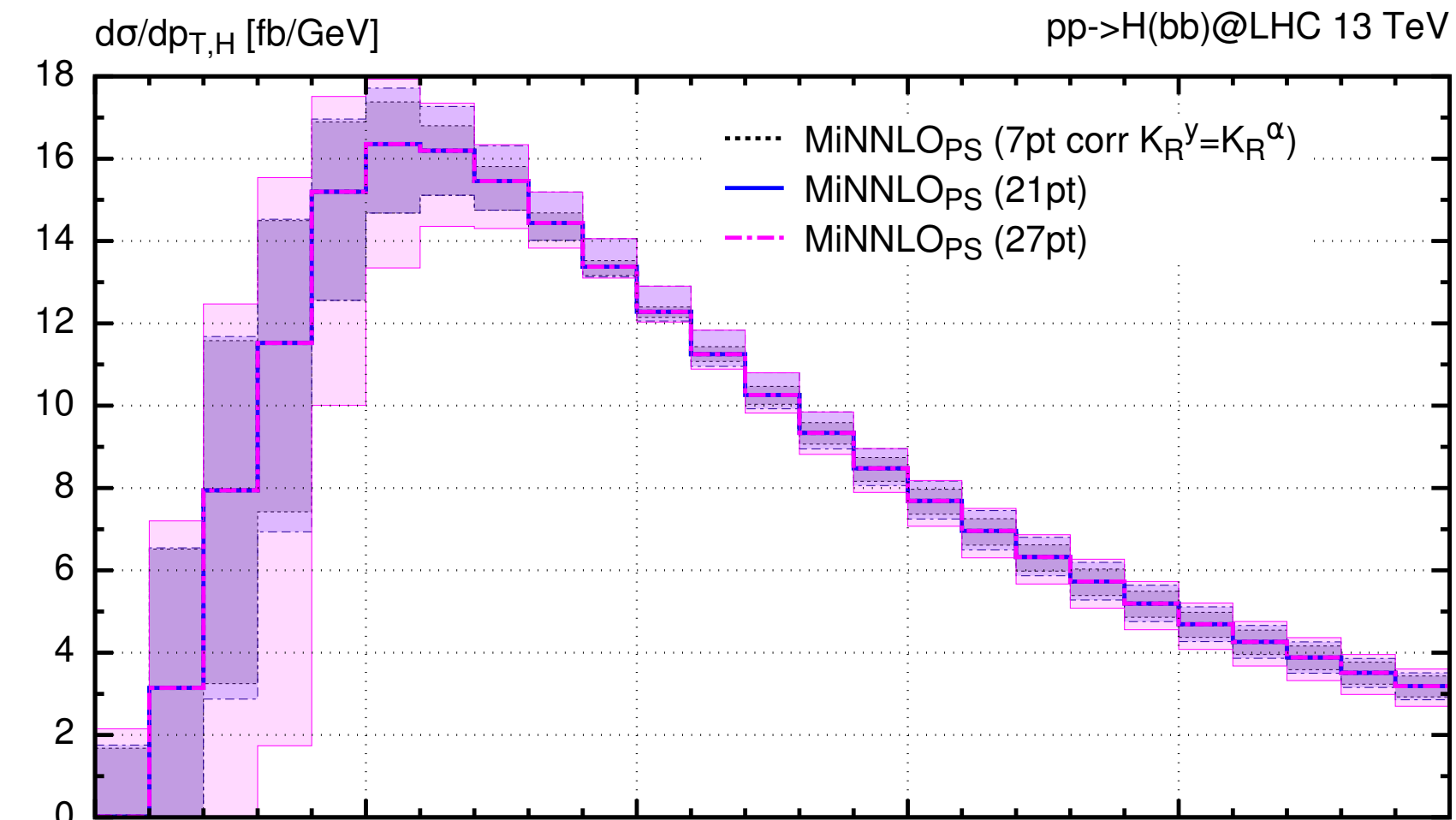
- The standard prediction
- 21pt s.v.: for any value of $K_F = \frac{1}{2}, 1, 2$, we perform a 7pt s.v. for (K_R^y, K_R^α)
- 27pt s.v. (K_R^α, K_R^y, K_F) including $K_i/K_y = 4$.

$$y_b(m_H) \rightarrow y_b(K_R m_H)$$

$$\alpha_s(p_T) \rightarrow \alpha_s(K_R p_T)$$

$$f_a(p_T) \rightarrow f_a(K_F p_T)$$

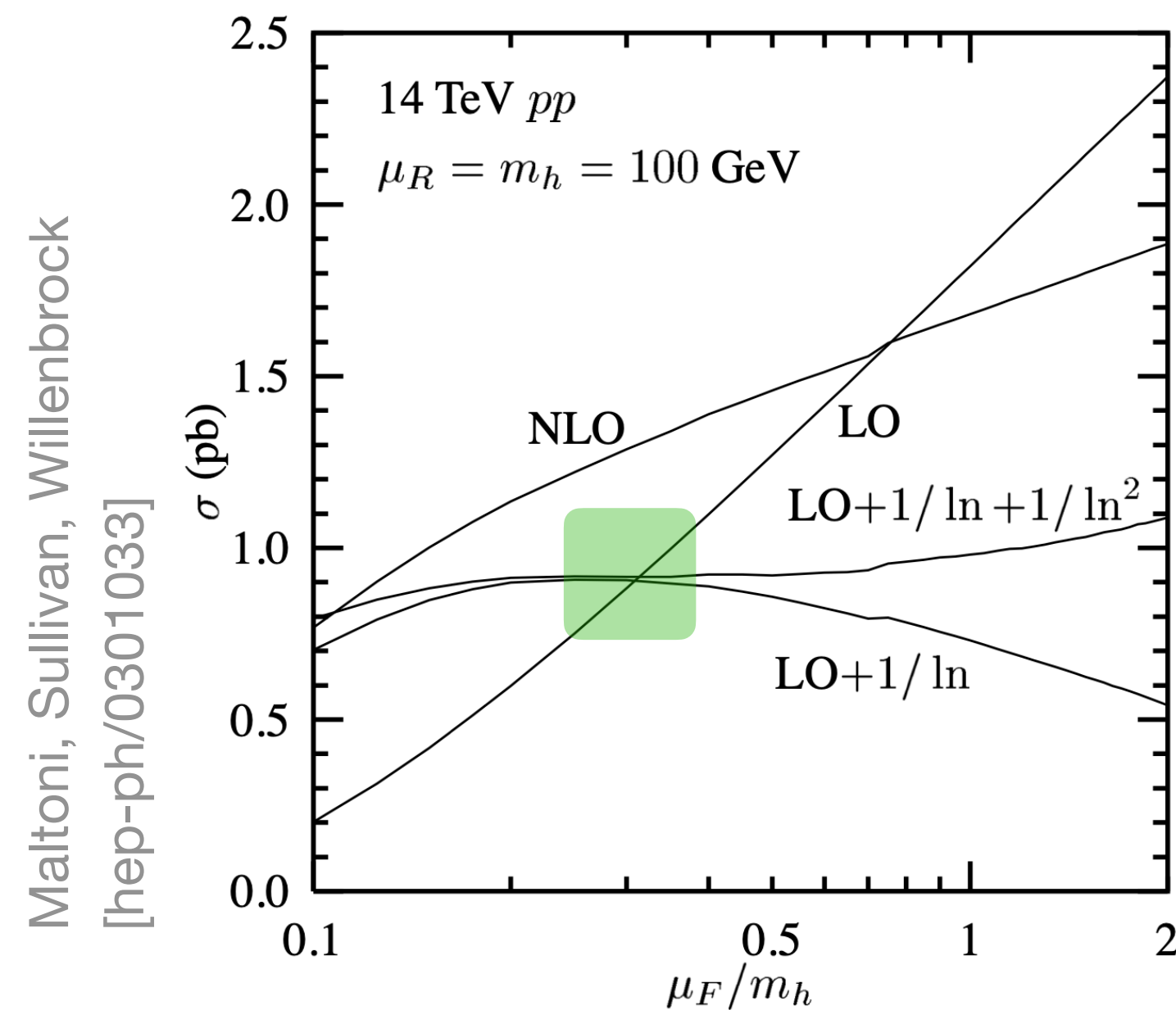
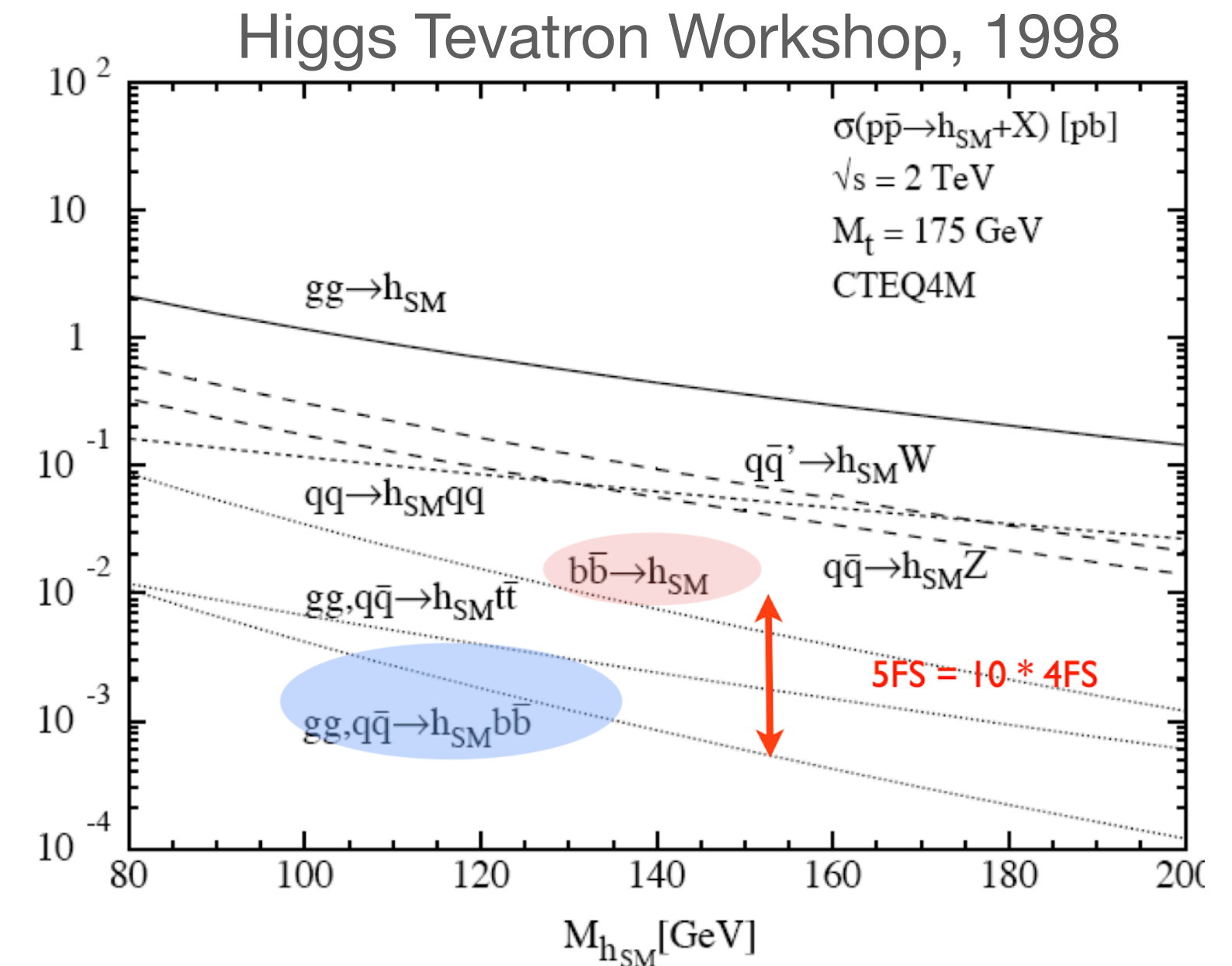
MiNNLO_{PS}





Historical LO comparisons

Large differences in the predictions were first observed at the leading order: the effect of collinear resummation is extremely large.



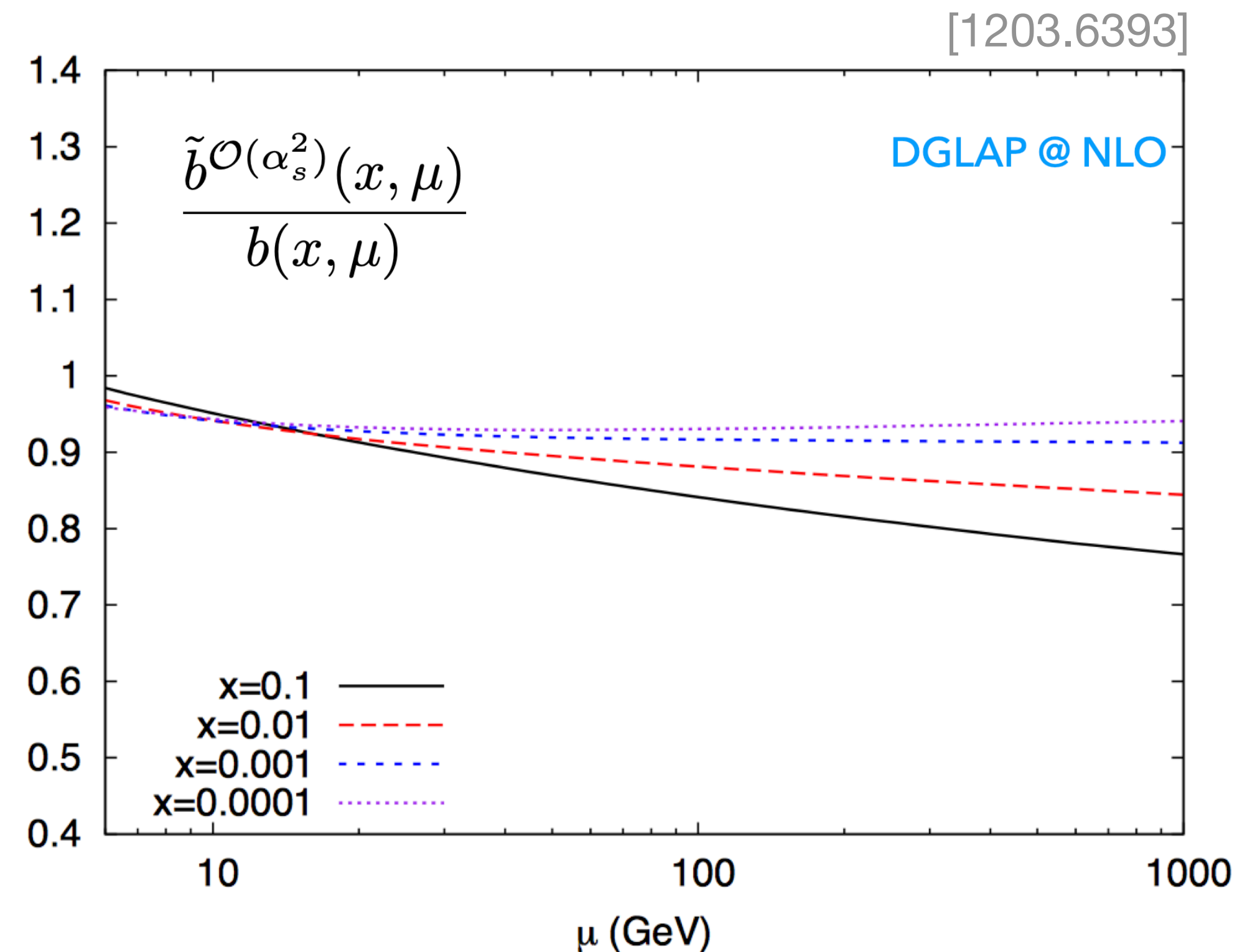
For $\mu_F = m_H/4$, FO computations in the different schemes become compatible, indeed the collinear logs have a small effect. This also improved the convergence of the perturbation series.

The improvement of the compatibility opens the possibility to match together the predictions at least at the inclusive level (Santander matching, FONLL...)



Differences between schemes

Lot of progress in understanding the origin of the differences. The predictions can be merged into a consistent picture by taking into account two main results.



1. At NLO, the resummation effects of collinear logs are important only at high Bjorken- x
2. The possibly large ratios m_H^2/m_b^2 are always accompanied by universal phase space factors f

$$\ln^2 \frac{m_H^2 f}{m_b^2} = \ln^2 \frac{\tilde{\mu}^2}{m_b^2}, \quad \tilde{\mu} < m_H$$

FONLL



- FONLL matches the flavour schemes

$$\sigma^{FONLL} = \sigma^{4FS} + \sigma^{5FS} - \text{double counting.}$$

For a consistent subtraction, we have to express the two cross-sections in terms of the same α_s and PDFs.

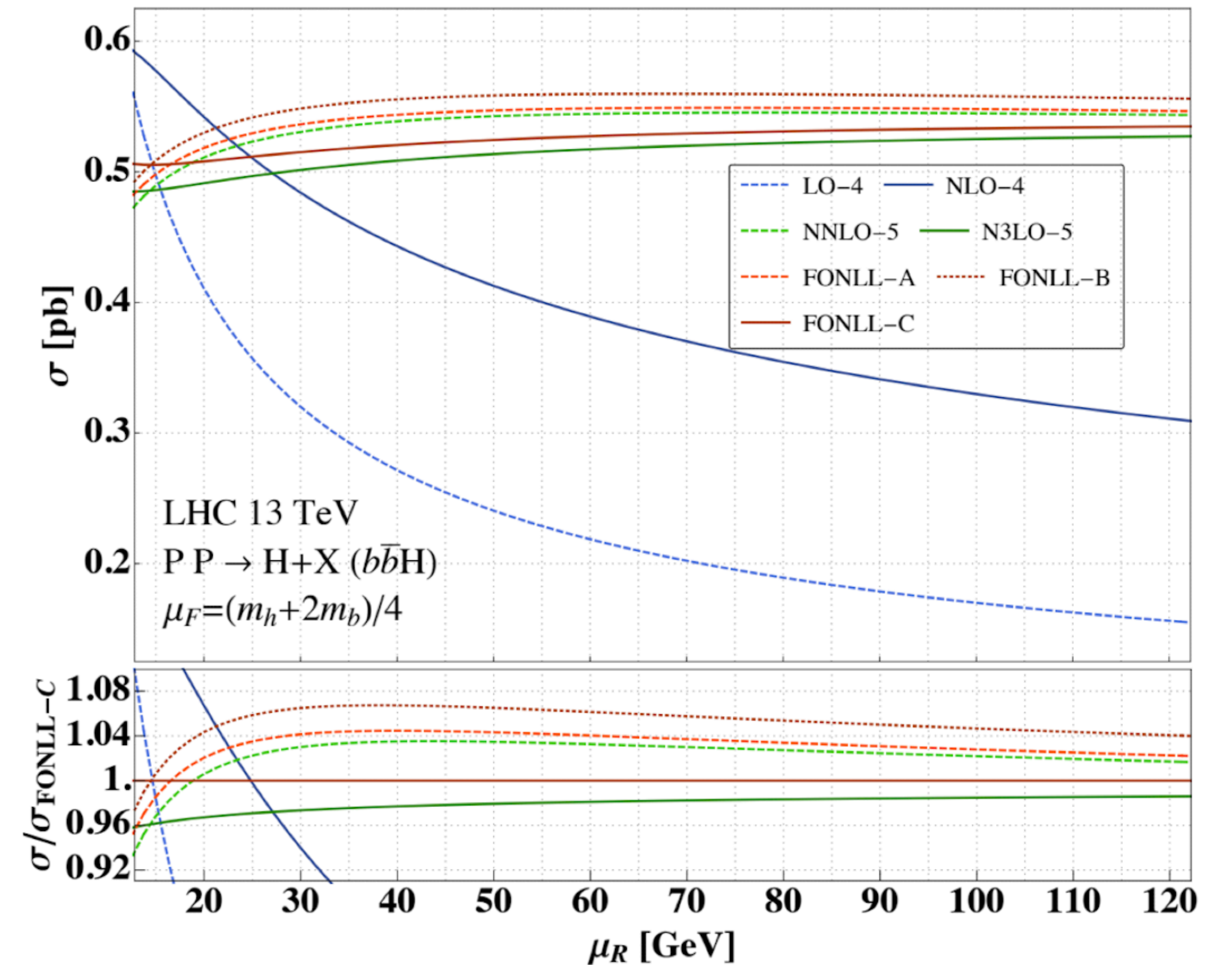
- Currently, the flavour matching for bbH is performed at

$$FONLL_C := N^3LO_{5FS} \oplus NLO_{4FS}.$$

- Differential FONLL applied for Z+b-jet

$$d\sigma^{FONLL} = d\sigma^{5FS} + \left(d\sigma_{m_b}^{4FS} - d\sigma_{m_b \rightarrow 0}^{4FS} \right)$$

Duhr, Dulat, Hirschi, Mistlberger [2004.04752]



[Gauld, Gehrmann-De Ridder,
 Glover, Huss, Majer, 2005.03016]



Exclusive observables

Recent developments in fully differential calculations, for example:

1. Introduce an unphysical scale μ_b in order to switch from 4FS to 5FS in a region where mass effects and collinear logs are not crucial [Bertone, Glazov, Mitov, Papanastasiou, Ubiali, 1711.03355]
2. Massive 5FS at NLO [Krauss, Napoletano, 1712.06832]
3. Differential FONLL applied for Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, Huss, Majer, 2005.03016]

$$d\sigma^{FONLL} = d\sigma^{5FS} + \left(d\sigma_{m_b}^{4FS} - d\sigma_{m_b \rightarrow 0}^{4FS} \right)$$