Realistic effects in angular-coefficient extraction in di-boson events at the LHC

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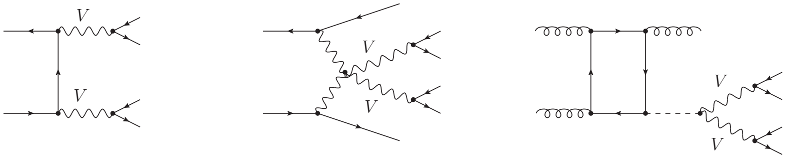
Motivations

LHC luminosities accumulated in Run 2 ($\approx 150\,\text{fb}^{-1}$) and foreseen in next runs ($300\,\text{fb}^{-1}$ in Run 3, and $3000\,\text{fb}^{-1}$ in High-Lumi) at 13/14 TeV CoM energy enable precise measurements of EW processes: multi-boson production.

Polarisation & spin correlations of EW bosons

- are non trivial to extract
- are crucial probes of interplay between gauge and scalar sector in SM
- provide discrimination power between SM and new physics (NP)
- allow to construct quantum observables for entanglement tests



Di-boson: simplest, non-trivial spin correlations with EW bosons

Experimental results

We cannot directly measure the spin state of EW bosons but we can:

- extract spin-sensitive coefficients from angular distributions
- perform fits of LHC data with polarised templates

Run-1 analyses mostly relied on angular-coefficient extraction:

- W+jets [ATLAS 1203.2165, CMS 1104.3829, CMS 2008.04174]
- Z+jets [CMS 1504.03512, ATLAS 1606.00689]
- $t\bar{t}$ [CMS 1605.09047, ATLAS 1612.02577, CMS ATLAS 2005.03799]

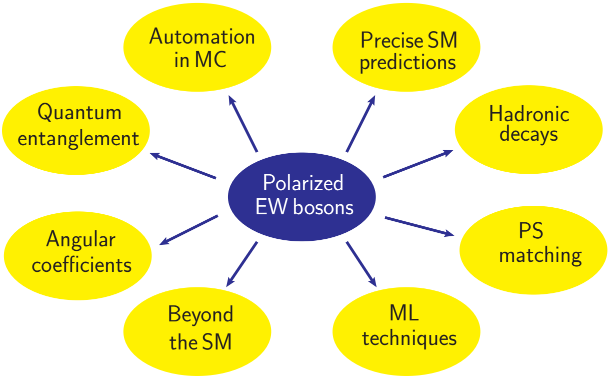
Run-2 analyses mostly rely fits with polarised templates:

- WZ, singly polarised [ATLAS 1902.05759, CMS 2110.11231]
- $W^{\pm}W^{\pm}$ scattering [CMS 2009.09429]
- WZ, doubly polarised [ATLAS 2211.09435, 2402.16365]
- ZZ, doubly longitudinal [ATLAS 2310.04350]

More ongoing analyses and promising sensitivity studies at High-Lumi [CMS-PAS-FTR-18-014, CERN-LPCC-2018-03, Roloff et al. 2108.00324].

What is needed from the theory side?

Proper understanding, precise predictions and new ideas to extract polarisations and spin correlations.



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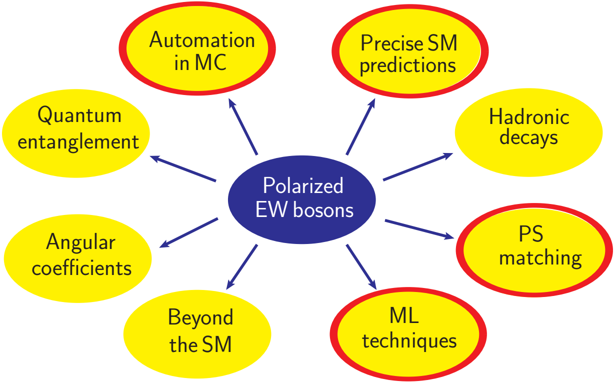

my usual talk

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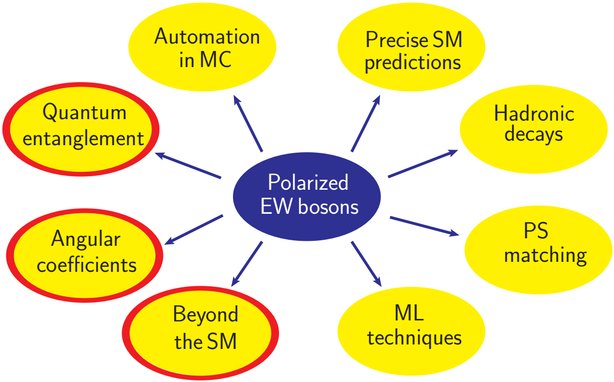
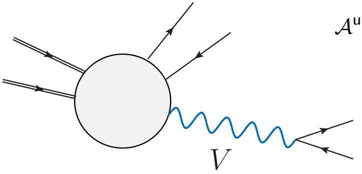


this talk, based on ongoing two works with
Michele Grossi, Alessandro Vicini, Hesham El Faham and Eleni Vryonidou

Amplitude structure

Tree-level structure for single resonant boson (in pole/narrow-width approximation):



$$\begin{aligned}\mathcal{A}^{\rm unpol} &= \mathcal{P}_\mu \,\frac{-g^{\mu\nu}}{k^2-M_V^2+iM_V\Gamma_V}\,\mathcal{D}_\nu \\ &= \mathcal{P}_\mu \,\frac{\sum_{\lambda'}\varepsilon^\mu_{\lambda'}\varepsilon^{*\nu}_{\lambda'}}{k^2-M_V^2+iM_V\Gamma_V}\,\mathcal{D}_\nu \\ &= \sum_{\lambda'}\mathcal{P}_\mu \,\frac{\varepsilon^\mu_{\lambda'}\varepsilon^{*\nu}_{\lambda'}}{k^2-M_V^2+iM_V\Gamma_V}\,\mathcal{D}_\nu = \sum_{\lambda'}\mathcal{A}_{\lambda'}\end{aligned}$$

At the cross section level:

$$|\mathcal{A}^{\rm unpol}|^2 = \underbrace{\sum_{\lambda}|\mathcal{A}_\lambda|^2}_{\text{incoherent sum}} + \underbrace{\sum_{\lambda\neq\lambda'}\mathcal{A}^*_\lambda\mathcal{A}_{\lambda'}}_{\text{interference terms}}$$

Polarisation vectors are defined in a specific Lorenzt frame.

Decay-product angular distributions reflect polarisation state of the decayed V boson [Bern et al. 1103.5445, Stirling et al. 1204.6427, Belyaev et al. 1303.3297].

Angular dependence for one boson

At tree-level, decay of a single resonant boson (θ^*, ϕ^* are ℓ^+ angles in V rest frame, w.r.t. V direction in some Lorentz frame) [Bern et al. 1103.5445, Stirling Vryonidou 1204.6427]:

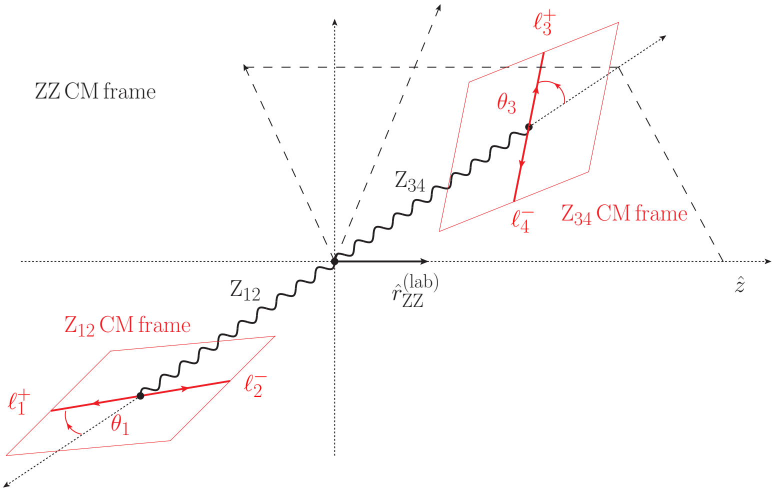
$$\frac{d\sigma}{d\cos\theta^* \, d\phi^* \, dX} = \frac{d\sigma}{dX}\frac{3}{16\pi}\Big[(1+\cos^2\theta^*) + (A_0/2)(1-3\cos^2\theta^*) + A_1 \sin 2\theta^* \, \cos\phi^* + (A_2/2)\sin^2\theta^* \, \cos 2\phi^* + A_3 \sin\theta^* \, \cos\phi^* + A_4\cos\theta^* + A_5 \sin\theta^* \, \sin\phi^* + A_6 \sin 2\theta^* \, \sin\phi^* + A_7 \sin^2\theta^* \, \sin 2\phi^*\Big]$$

$$= \frac{d\sigma}{dX}\left[\frac{1}{4\pi} + \sum_{\ell=1}^{2}\sum_{m=-\ell}^{\ell} \alpha_{\ell m}\, Y_{\ell m}(\theta^*, \phi^*)\right] \qquad (1)$$

8 independent coefficients ($\{A_i\}$ or $\{\alpha_{\ell,m}\}$) extracted through projections [Bern et al. 1103.5445, Stirling Vryonidou 1204.6427, Ballestrero Maina GP 1710.09339, Baglio et al. 1910.13746, Frederix Vitos 2007.08867] or asymmetries [Boudjema Singh 0903.4705] *e.g.*:

$$\int_{-1}^{1} d\cos\theta^* \int_0^{2\pi} d\phi^* \, Y_{\ell m}(\theta^*, \phi^*)\frac{d\sigma}{d\cos\theta^* \, d\phi^* \, dX} = \alpha_{\ell m} = \alpha_{\ell m}(X) \qquad (2)$$

Two bosons: geometric visualisation



Angular dependence for boson pairs

Two resonant bosons (θ_1, ϕ_1, and θ_3, ϕ_3) are ℓ_1^+ and ℓ_3^+ angles in each V rest frame, w.r.t. V direction in some Lorentz frame (typical choice: VV-CM frame):

$$\frac{d\sigma}{\mathrm{d}\cos\theta_1\,\mathrm{d}\phi_1\,\mathrm{d}\cos\theta_3\,\mathrm{d}\phi_3\,\mathrm{d}X} = \frac{d\sigma}{\mathrm{d}X}\Bigg[\frac{1}{(4\pi)^2} + \\ +\frac{1}{4\pi}\sum_{\ell=1}^{2}\sum_{m=-\ell}^{\ell}\alpha^{(1)}_{\ell m}(X)\,Y_{\ell m}(\theta_1,\phi_1) \\ +\frac{1}{4\pi}\sum_{\ell=1}^{2}\sum_{m=-\ell}^{\ell}\alpha^{(3)}_{\ell m}(X)\,Y_{\ell m}(\theta_3,\phi_3) \\ +\sum_{\ell_1=1}^{2}\sum_{\ell_3=1}^{2}\sum_{m_1=-\ell_1}^{\ell_1}\sum_{m_3=-\ell_3}^{\ell_3}\gamma_{\ell_1 m_1 \ell_3 m_3}(X)\,Y_{\ell_1 m_1}(\theta_1,\phi_1)\,Y_{\ell_3 m_3}(\theta_3,\phi_3)\Bigg] \tag{3}$$

80 independent coefficients [Rahaman Singh 2109.09345, Aguilar-Saavedra et al. 2209.13441]], extracted similarly to the single-boson case.

Remark: the two bosons are correlated if $\alpha^{(1)}_{\ell_1,m_1}\alpha^{(3)}_{\ell_3,m_3} \neq \gamma_{\ell_1 m_1 \ell_3 m_3}$.

Angular coefficients and quantum entanglement

Why are these coefficients so relevant?

- constraints for PDFs in single-V production [Gauld et al. 1708.00008, Frederix Vitos 2007.08867, Amoroso et al. 2012.10298, Pellen et al. 2204.12394]
- access to EWSB, sensitivity to NP effects [Han et. al 0911.3656, Brehmer et al. 1404.5951, Ballestrero Maina GP 1710.09339, 1907.04722, Baglio et al. 1810.11034, Buarque-Franzosi et al. 1912.01725, Rahaman Singh 1810.11657, 1911.03111, 2109.09345]
- in VV, direct input for quantum-entanglement observables [Barr 2106.01377, Barr et al. 2204.11063, Aguilar-Saavedra et al. 2209.13441, 2209.14033, Ashby-Pickering et al. 2209.13990, Fabbrichesi et al. 2302.00683, Fabbrichesi et al. 2302.00683, 2304.02403, Morales 2306.17247, Aoude et al. 2307.09675, Fabbri et al. 2307.13783, Bernal et al. 2307.13496, Bernal 2310.10838, Barr et al. 2402.07972, Aguilar-Saavedra 2403.13942] $\rightarrow$ components of the spin-density matrix

The 4ℓ channel: EW production and Higgs decays

Four-charged-lepton is promising channel to measure entanglement and Bell-inequality violation at hadron/lepton colliders [Ashby-Pickering et al. 2209.13990, Aguilar-Saavedra et al. 2209.13441, Fabbrichesi et al. 2302.00683, Aoude et al. 2307.09675]:

- clean signature, no ν reconstruction
- ZZ CM frame precisely reconstructed
- EW production has low sensitivity to NP
- two-boson angular structure not well defined for on-shell Higgs decays
- missing assessment of fiducial cuts and higher-order corrections

Goodness of *entanglement witnesses* (sufficient conditions for system to be entangled) depend on how well we extract spin-density-matrix entries from (LHC) data.

Issues in coefficient extraction

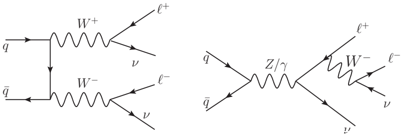
In principle, coefficients extracted directly from data through projections up to $\ell_{1,3} = 2$ of the angular distributions $d\sigma/d\Omega_1 d\Omega_3$

Nice idea, but the analytic structure of $d\sigma/d\Omega_1 d\Omega_3$ in eq. (3)

1. assumes two spin-1 resonances being produced,
2. assumes two-body decays,
3. is not invariant under Lorentz boosts,
4. is not described by $\ell_{1,3} \leq 2$ if selection cuts are applied,
5. is distorted by ν-reconstruction (if applied),
6. is not fully model independent (NP in production and decay)

Resonant production and off-shell effects

Factorised amplitude (production $\otimes$ propagator $\otimes$ decay) not possible for all contributions. *E.g.* diboson (fully leptonic):



Double-resonant and non-double-resonant diagrams at LO. For the latter polarisation cannot be defined: drop them (regarded as non-resonant background).

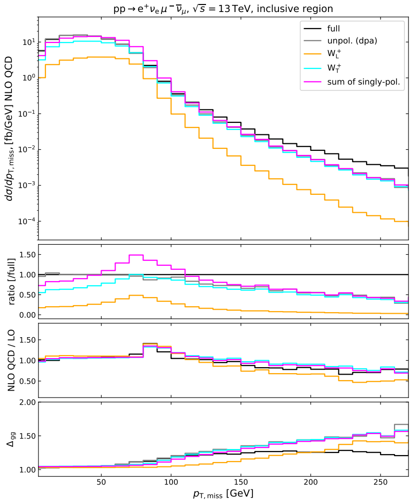
Resonant diagrams treated in gauge-invariant manner:

- DPA: double-pole approximation [Denner et al. 0006307]
- NWA: spin-correlated narrow-width approximation [Artoisenet et al. 1212.3460].

$\rightarrow$ angular expansion allows for a polarisation interpretation (bosons are on-shell)

State-of-art for DPA (un)polarised calc. for VV: (N)NLO QCD [Denner GP 2006.14867, Poncelet Popescu 2102.13583], NLO EW [Denner GP 2107.06579, Le Baglio 2203.01470, Denner Haitz GP 2311.16031, Dao Le 2311.17027], NLO QCD$\times$PS [Hoppe et al. 2310.14803, GP Zanderighi 2311.05220]

Estimating off-shell effects



$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu + X$ @ NLO QCD
fully inclusive [Denner GP 2006.14867]

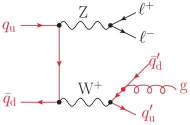
- typically small in single-boson production with jets [Pellen et al. 2109.14336, 2204.12394]
- at the $\lesssim 1\%$ level for Z bosons, owing to physical invariant-mass cuts, *e.g.* $|M_{\ell^+\ell^-} - M_Z| < \Delta$ [Denner GP 2107.06579, Le Baglio 2203.01470]
- larger for W bosons, especially in tails of p_T distributions, where non-resonant diagrams numerically sizeable: up to 50% in W^+W^- for $p_{T,mis} \gtrsim 200$GeV [Denner GP 2006.14867, Poncelet Popescu 2102.13583]

Two-body decays

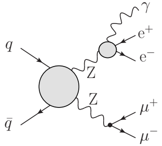
Angular analytic structure ($\ell \leq 2$) only valid for two-body decays of EW bosons.

Higher-rank spherical harmonics appear with three-body decay: QED/QCD radiation off decay products

e.g. QCD corrections to hadronic decays
[Denner Haitz GP 2211.09040]



e.g. EW corrections to leptonic decays
[Denner Haitz GP 2107.06579, Le Baglio 2203.01470,
Denner Haitz GP 2311.16031, Dao Le 2311.17027]



Angles defined after jet clustering/ lepton dressing!

Typically small effects in QED, slightly larger in QCD: three-prong topologies suppressed in both inclusive and boosted fiducial volumes.

If off-shell/non-factorisable effects included: no production $\times$ decay (also at LO).

Lorentz-frame and coordinate-system dependence

Helicity states defined in specific Lorentz frame: same structure, coeff. values change.

For VV:

- modified helicity coord. sys (VV CM) [ATLAS 1902.05759] more natural than lab., reference axis only affects azimuthal coefficients
- frame choice change size of higher-order corrections, even QCD ISR in fully leptonic channel. Table: WZ in fiducial setup [Denner GP 2010.07149].

DPA fractions	LO	NLO QCD	K-factor
VV CM frame			
$W^+_L Z_L$	7.9%	5.7%	1.31
$W^+_L Z_T$	10.6%	15.5%	2.65
$W^+_T Z_L$	9.9%	14.7%	2.68
$W^+_T Z_T$	70.9%	63.5%	1.62
LAB frame			
$W^+_L Z_L$	5.7%	6.0%	1.91
$W^+_L Z_T$	16.5%	17.6%	1.93
$W^+_T Z_L$	22.9%	21.4%	1.69
$W^+_T Z_T$	55.0%	54.8%	1.80

QCD radiation: boost effects on the coefficients

$\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^- + X$ @ fixed order: effect of QCD radiation from ISR on $(\ell = 2, m = 0)$ coefficients in inclusive setup (81GeV$< M_{\ell^+\ell^-} <$101GeV)

calculation	DPA		full off-shell	
		PRELIMINARY		
coefficient	LO	NLO	LO	NLO
$\alpha^{(1)}_{2,0}$	+0.0302(1)	+0.0276(3)	+0.0303(1)	+0.0280(3)
$\alpha^{(3)}_{2,0}$	+0.0304(1)	+0.0274(3)	+0.0299(2)	+0.0277(3)
$\gamma_{2,0,2,0}$	+0.00186(5)	+0.00148(9)	+0.00185(5)	+0.00160(9)

QCD radiation sizeably changes longitudinal components, although angular expansion defined in ZZ CM frame [Grossi GP Vicini 24XX.YYYY]

Remarks: gg-induced [Denner GP 2107.06579, Javurkova et al. 2401.17365] & NNLO corrections (vs NLO) [Poncelet Popescu 2102.13583, Pellen et al. 2204.12394]!

All entanglement observables so far constructed at LO: higher orders unavoidable!

Selection-cut effects

Angular expansion ($\ell \leq 2$ spherical harmonics) valid fully differentially,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*\,\mathrm{d}X}\,.$$

Still valid after integration over independent variables X, if no cuts are applied on decay products,

$$\int_{\mathrm{inc}} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*\,\mathrm{d}X}\mathrm{d}X = \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*}\,.$$

Fiducial cuts on decay products introduce angular modulation f_{cut} which is not a combination of $\ell \leq 2$ spherical harmonics,

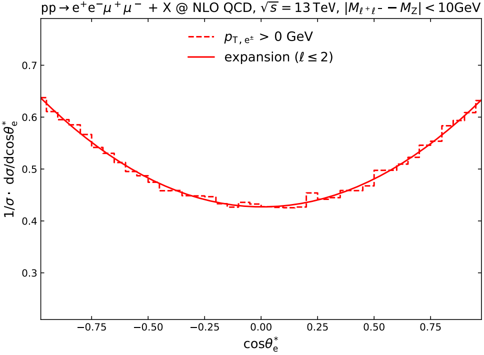
$$\int_{\mathrm{cut}} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*\,\mathrm{d}X}\mathrm{d}X = \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*} f_{\mathrm{cut}}(\theta^*, \phi^*)\,.$$

Therefore

$$\int_{-1}^{1} d\cos\theta^* \int_0^{2\pi} d\phi^*\; Y_{\ell m}(\theta^*, \phi^*)\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*\,\mathrm{d}\phi^*} f_{\mathrm{cut}}(\theta^*, \phi^*) \neq \sigma_{\mathrm{cut}}\alpha_{\ell m}\,.$$

Expansion up to $\ell \leq 2$: ZZ production

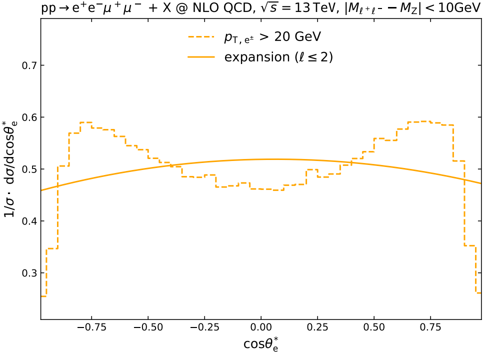
$\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^- + X$: polar decay angle of first Z boson (defined in ZZ CM frame)



fully inclusive

Expansion up to $\ell \leq 2$: ZZ production

$\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^- + X$: polar decay angle of first Z boson (defined in ZZ CM frame)



cut setup

A craft-made extrapolation for $\alpha_{2,0}$ and $\alpha_{2,-2}$ in ZZ

$pp \to e^+e^-\mu^+\mu^- + X$: decay angles defined in ZZ CM frame, in inclusive and ATLAS fid. [ATLAS 2310.04350] setups [Grossi GP Vicini 24XX.YYYY]

$$\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\phi_1\, Y_{\ell,m}(\theta_1,\phi_1)\, \frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_1\, d\phi_1} = \alpha^{(1)}_{\ell,m}$$

setup	$\alpha^{(1)}_{2,0}$	discrepancy	$-\alpha^{(1)}_{2,-2}$	discrepancy
PRELIMINARY		NLO QCD		
inclusive (truth value)	0.0272(4)		0.0167(3)	
extr. from ATLAS fid.	0.0266(6)	−2%	0.0184(4)	+10%
ATLAS fid.	0.0199(4)	−25%	0.0438(9)	+162%
PRELIMINARY		NLO QCD matched to QCD+QED PS + hadr.		
inclusive (truth value)	0.0271(4)		0.0171(3)	
extr. from ATLAS fid.	0.0267(6)	−1%	0.0187(4)	+9%
ATLAS fid.	0.0199(4)	−27%	0.0439(9)	+157%

Basic extrapolation: from unpolarised LHC events (after fid. selection), undo physical Z-boson decays, generate random decays (uniformly distributed in $\cos\theta_1$, and ϕ_1), take ratio to evaluate the geometric acceptance (up to normalisation), fill angular distribution → works at % level for $\alpha_{2,0}$, less stable for $\alpha_{2,-2}$.

Spin correlations at work: $\gamma_{2,0,2,0}$ in ZZ

Projection on $\ell = 2$ sph. harm. [Grossi GP Vicini 24XX.YYYY] related to longitudinal production:

$$\int \mathrm{d}\Omega_1 \int \mathrm{d}\Omega_3 \, Y_{2,0}(\theta_1,\phi_1) Y_{2,0}(\theta_3,\phi_3) \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1 \mathrm{d}\Omega_3} = \gamma_{2,0,2,0}$$

$$f_{\mathrm{L}}^{(i)} = \frac{1}{3}\left(1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(i)}\right), \quad f_{\mathrm{LL}} = \frac{1}{3}\left(1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\gamma_{2,0,2,0}\right)$$

Level of correlation between the two bosons, R_c [ATLAS 2211.09435],

$$R_c = \frac{f_{\mathrm{LL}}}{f_{\mathrm{L}}^{(1)}\, f_{\mathrm{L}}^{(3)}} = \frac{1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\gamma_{2,0,2,0}}{1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\alpha_{2,0}^{(1)}\,\alpha_{2,0}^{(3)}} \tag{4}$$

NLO QCD (DPA) PRELIMINARY	$f_{\mathrm{L}}^{(1)}$	$f_{\mathrm{L}}^{(3)}$	f_{LL}	R_c
inclusive (projection)	0.189(4)	0.191(4)	0.056(2)	1.56(7)
inclusive [GP Zanderighi 2311]	0.185	0.185	0.059	1.73
ATLAS fid. (projection)	0.228(4)	0.214(4)	0.085(3)	1.76(6)
ATLAS fid. [GP Zanderighi 2311]	0.193	0.193	0.059	1.58

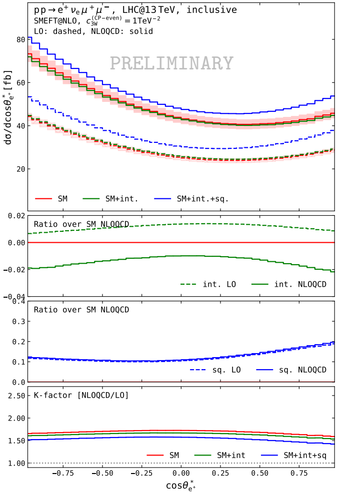
*uncertainties from MC integration, QCD-scale variations smaller

Selection cuts and neutrino reconstruction in WZ

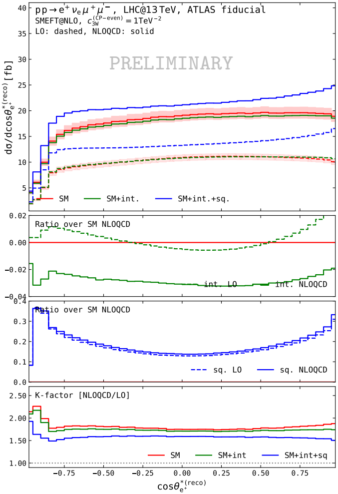
$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^+\mu^- + X$ @ NLO QCD: SM + $\mathcal{O}_{3\mathrm{W}}$ [El Faham GP Vryonidou 24XX.YYYY]

Polar decay angle (e^+ in W^+ rest frame): sensitive to $\alpha^{(1)}_{1,0}$ ($\to A_4$) and $\alpha^{(1)}_{2,0}$ ($\to A_0$)

inclusive



ATLAS fid. [ATLAS:2211.09435] + ν-reco



Extraction of the A_0 coefficients: failure of the projection method

Remark: decay angles defined in WZ CM frame (natural in VV [Denner GP 2010.07149]])

Extraction at NLO QCD in SM/SMEFT through [El Faham GP Vryonidou 24XX.YYYY]

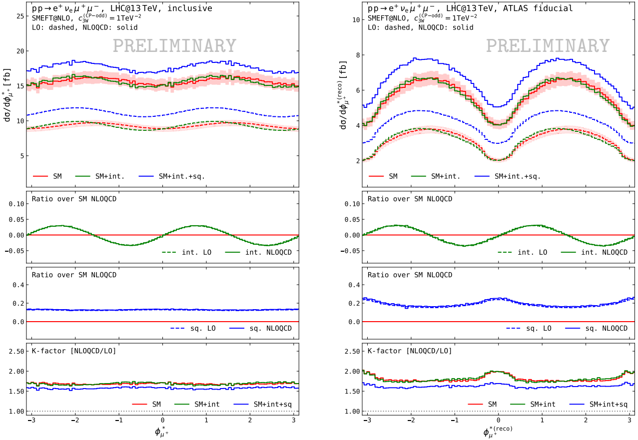
$$4 - 10 \int_{-1}^{1} d\cos\theta^* \int_{0}^{2\pi} d\phi^* \, \cos^2\theta^* \, \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta^* \, d\phi^*} = A_0 = 2f_{\mathrm{L}}$$

setup neutrino reco.	inclusive no	inclusive yes	ATLAS fid. yes
PRELIMINARY		A_0 (W^+ boson)	
SM	0.367(1)	0.515(4)	0.983(3)
SM+int, CP-even	0.378(1)	0.509(4)	0.975(3)
SM+int+sq, CP-even	0.339(1)	0.449(5)	0.861(3)
PRELIMINARY		A_0 (Z boson)	
SM	0.358(1)	0.431(1)	0.839(2)
SM+int, CP-even	0.370(1)	0.440(1)	0.845(3)
SM+int+sq, CP-even	0.332(1)	0.394(2)	0.761(3)

Not physically meaningful if ν-reco. and fiducial cuts are applied: longitudinal component too large, violates perturbative unitarity. Only inclusive results are reliable.

The A_7 coefficients and CP-odd effects

Azimuthal decay angles optimal to probe CP-odd new-physics effects in WZ
SM + $\tilde{O}_{3W}$ @ NLO QCD [El Faham GP Vryonidou 24XX.YYYY]: inc. vs ATLAS fid.



SM-EFT dim-6 interference $\propto \sin 2\phi^*$: $A_7^{(\mathrm{SM})} = -0.002(3)$, $A_7^{(\mathrm{SM+int})} = 0.062(2)$
Mild effects of cuts/reconstruction (relatively to SM).

Conclusions and outlook

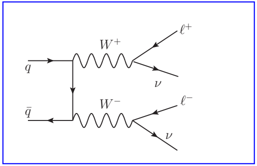
Angular decay structure gives access to spin-density matrix.

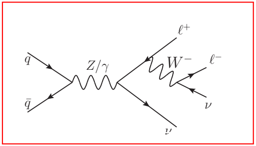
For VV systems, expansion for $\ell \leq 2$ in spherical harmonics has limited validity:

1. off-shell effects, non-two-body decays have a limited impact
2. choose a natural Lorentz frame
3. QCD ISR (at LHC) sizeably change the angular coefficients
4. fiducial cuts disrupt $\ell \leq 2$ expansion, extrapolation needed
5. some angular coefficients enhance sensitivity to aTGC SMEFT effects
6. LO picture may not be enough for entanglement witnesses

Backup

DPA details





$$\mathcal{A}_{\text{full}}(x_1, x_2;\ k_{1\dots4}) = \mathcal{A}_{\text{res}}(x_1, x_2;\ k_{1\dots4}) + \mathcal{A}_{\text{nonres}}(x_1, x_2;\ k_{1\dots4}) \longrightarrow \mathcal{A}_{\text{res}}(x_1, x_2;\ k_{1\dots4})$$

$$\mathcal{A}_{\text{res}}(x_1, x_2;\ k_{1\dots4}) \quad = \quad \mathcal{P}_{\mu\nu}(x_1, x_2;\ k_{12}, k_{34}) = \frac{-\mathrm{i}\, g^{\mu\alpha}}{k_{12}^2 - M_1^2 + \mathrm{i}\Gamma_1 M_1} \frac{-\mathrm{i}\, g^{\nu\beta}}{k_{34}^2 - M_2^2 + \mathrm{i}\Gamma_2 M_2} \mathcal{D}_\alpha(k_1, k_2)\, \mathcal{D}_\beta(k_3, k_4)$$

$$\mathcal{A}_{\text{res}}(x_1, x_2;\ k_{1\dots4}) \quad \xrightarrow{\text{DPA}} \quad \mathcal{A}_{\text{res}}(x_1, x_2;\ \tilde{k}_{1\dots4}) = \mathcal{P}_{\mu\nu}(x_1, x_2;\ \tilde{k}_{12}, \tilde{k}_{34})$$
$$\times \quad \frac{-\mathrm{i}\, g^{\mu\alpha}}{k_{12}^2 - M_1^2 + \mathrm{i}\Gamma_1 M_1} \frac{-\mathrm{i}\, g^{\nu\beta}}{k_{34}^2 - M_2^2 + \mathrm{i}\Gamma_2 M_2} \mathcal{D}_\alpha(\tilde{k}_1, \tilde{k}_2)\, \mathcal{D}_\beta(\tilde{k}_3, \tilde{k}_4)$$

On-shell mapping: $\quad \Phi_4 = \{x_1, x_2; k_{1\dots4}\} \xrightarrow{\text{DPA}} \tilde{\Phi}_4 = \{x_1, x_2; \tilde{k}_{1\dots4}\}$

where $\tilde{k}_{12}^2 = (\tilde{k}_1 + \tilde{k}_2)^2 = M_1^2$ and $\tilde{k}_{34}^2 = (\tilde{k}_3 + \tilde{k}_4)^2 = M_2^2$ ($M_1,\ M_2 =$ masses of the two gauge bosons), and $(k_1 + k_2 + k_3 + k_4)^2 > (M_1 + M_2)^2$.

Polarisation selection: $-g_{\mu\nu} \longrightarrow \varepsilon_\mu^{(\lambda)}(k)\, \varepsilon_\nu^{(\lambda)\,*}(k)\,, \qquad \lambda = \mathrm{L}, +, -$

DPA beyond leading-order: technical details

DPA applied to the subtracted real:

- Only factorisable corrections considered:

$$|\mathcal{A}^{(n+1)}_{\mathrm{ISR}} + \mathcal{A}^{(n+1)}_{\mathrm{FSR}_1} + \mathcal{A}^{(n+1)}_{\mathrm{FSR}_2}|^2 \longrightarrow |\mathcal{A}^{(n+1)}_{\mathrm{ISR}}|^2 + |\mathcal{A}^{(n+1)}_{\mathrm{FSR}_1}|^2 + |\mathcal{A}^{(n+1)}_{\mathrm{FSR}_2}|^2$$

- ISR treated with DPA for two 2-body decays:

$$|\mathcal{A}^{(n+1)}_{\mathrm{ISR}}|^2 \xrightarrow{\mathrm{DPA}(2,2)} |\overline{\mathcal{A}}^{(n+1)}_{\mathrm{ISR}}|^2$$

- $\mathrm{FSR}_{(i)}$ treated with DPA for one 2-body and one 3-body decay:

$$|\mathcal{A}^{(n+1)}_{\mathrm{FSR}_{(\mathrm{i})}}|^2 \xrightarrow{\mathrm{DPA}(3,2)} |\overline{\mathcal{A}}^{(n+1)}_{\mathrm{FSR}_{(\mathrm{i})}}|^2$$

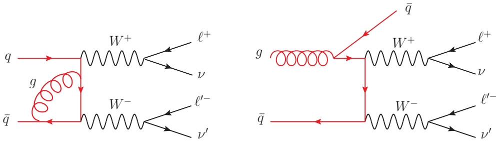
- Subtraction dipoles must be treated consistently: first DPA, second Catani-Seymour (CS) mappings (no commutation for FSR).

- DPAs preserve angles and energy fractions of decay products in resonance CM frame → to avoid mismatch approaching soft and collinear regimes.

- DPA doesn't modify radiation variables: no modification in integrated dipoles.

NLO corrections to the production

- NLO: virtual (V) and real (R) contributions, $V + R$ free of IR singularities;



- subtraction counterterms needed, *e.g.* dipole formalism [Catani, Seymour 9605323]:

$$d\sigma_{\mathrm{nlo}}/d\xi = \int d\phi_n \Big(B + V + \int d\phi_{\mathrm{rad}}\, D\Big)_{d=4} \delta_\xi^{(n)} + \int d\phi_{n+1} \Big(R\, \delta_\xi^{(n+1)} - D\, \delta_\xi^{(n)}\Big)_{d=4}\,; \quad (5)$$

- DPA/NWA usually used for n-body (B, V) $\rightarrow$ also needed for R and D terms;
- separation of polarisations required for all contributions in Eq. 5.

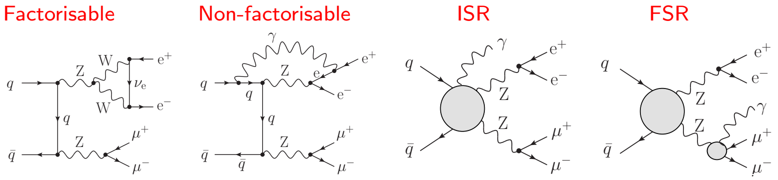
Corrections only affect production of resonance(s) $\longrightarrow$ conceptually straightforward.

N(N)LO QCD corr. with leptonic decays [Denner GP 2006.14867, Poncelet Popescu 2102.13583].

NLO corrections to Z-boson decays

Corrections affect both production and decays of resonance(s).

NLO EW corrections to Z bosons with leptonic decays.



General method has been proposed to separate Z resonant contributions at NLO EW, with leptonic decays [Denner GP 2107.06579].

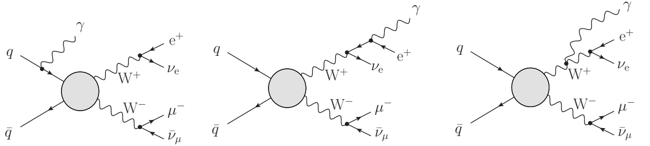
Same structure as NLO QCD corrections to Z/ W bosons with hadronic decays.

Applied to diboson production in semi-leptonic decay channels [Denner Haitz GP 2211.09040].

NLO corrections to W-boson decays

Extended DPA approach to inclusive WZ [Le Baglio 2203.01470, 2208.09232] and $\mathrm{W}^+\mathrm{W}^-$ [Denner Haitz GP 2311.16031, Dao Le 2311.17027] at NLO EW.

Additional feature (compared to Z bosons): photons radiated off boson propagators.



(production only) (decay only) (production & decay)

Even more involved but desirable: NLO corr. to polarised $\mathrm{W}^+\mathrm{W}^+$ scattering.

Precise predictions for diboson @ LHC

- $\rightarrow$ $\mathrm{W}^+(\ell^+\nu_\ell)\,\mathrm{W}^-(\ell'^-\bar{\nu}_{\ell'})$: NLO QCD in the DPA [Denner GP 2006.14867], NNLO QCD in the DPA and NWA [Poncelet Popescu 2102.13583], NLO EW in the DPA [Denner Haitz GP 2311.16031, Dao Le 2311.17027] ;
- $\rightarrow$ $\mathrm{W}^\pm(\ell^\pm\nu_\ell)\,\mathrm{Z}(\ell'^+\ell'^-)$: NLO QCD [Denner GP 2010.07149] and NLO EW [Le Baglio 2203.01470, 2208.09232] in the DPA, nLO QCD in the NWA [Hoppe et al. 2310.14803];
- $\rightarrow$ $\mathrm{Z}(\ell^+\ell^-)\,\mathrm{Z}(\ell'^+\ell'^-)$: NLO EW + QCD in the DPA [Denner GP 2107.06579];
- $\rightarrow$ $\mathrm{W}^\pm(\mathrm{jets})\,\mathrm{Z}(\ell^+\ell^-)$: NLO QCD in the DPA [Denner Haitz GP 2211.09040];
- $\rightarrow$ $V(\ell^+\ell^-)\,V'(\ell'^+\ell'^-)$: NLO QCD in the DPA [GP Zanderighi 2311.05220].

NLO matching to parton shower

Usual assumption: factorisation of PS effects from spin-structure of the multi-boson system $\rightarrow$ not true already with one real emission.

NLO QCD accuracy required.

MC codes simulating intermediate polarised bosons (public or soon-to-be-published):

1. PHANTOM (v1.7): LO, $2 \rightarrow 6$ processes in the DPA, interfaced to PS [Ballestrero Maina GP 1710.09339, 1907.04722, 2007.07133, Maina GP 2105.07972].
2. MG5_AMC@NLO (v2.7): LO, any process in the NWA, multi-jet merging and PS matching, UFO models for BSM/EFT [Buarque-Franzosi et al. 1912.01725], now possible to generate "polarised" Feynman rules [Javurkova et al. 2401.17365].
3. SHERPA: nLO (approx.), any process in the NWA, multi-jet merging and PS matching, UFO models for BSM/EFT [Hoppe et al. 2310.14803]
4. POWHEG-BOX-RES: NLO, diboson processes in the DPA, PS matching [GP Zanderighi 2311.05220].

First efforts towards a public unweighted-event generators capable to treat intermediate polarised bosons beyond LO in SHERPA [Hoppe et al. 2310.14803] and POWHEG-BOX-RES [GP Zanderighi 2311.05220].

Effort needed to incorporate EW effects.

DPA & Powheg-Box-Res

First FKS ($n \to n+1$) mapping, second DPA on-shell mapping:

$$\Phi_{4\ell} = \{x_1, x_2; k_{1\dots4}\} \quad \xrightarrow{\text{FKS}} \quad (\bar{\Phi}_{4\ell}, \Phi_{\rm rad}) = \{\bar{x}_1, \bar{x}_2; \bar{k}_{1\dots4}, k_{\rm rad}\} \xrightarrow{\text{DPA}}$$
$$\xrightarrow{\text{DPA}} \quad (\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi_{\rm rad}) = \{\bar{x}_1, \bar{x}_2; \bar{\tilde{\bar{k}}}_{1\dots4}, k_{\rm rad}\}$$

PowHeg master formula (tailored to DPA):

$$\langle \mathcal{O} \rangle = \int \mathrm{d}\Phi_{4\ell}\, \tilde{\mathrm{B}}(\tilde{\Phi}_{4\ell}) \left[\mathcal{O}(\tilde{\Phi}_{4\ell}) \Delta(t_0) + \int_{t>t_0} \mathrm{d}\Phi_{\rm rad} \mathcal{O}(\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi_{\rm rad}) \frac{\mathrm{R}(\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi_{\rm rad})}{\mathrm{B}(\tilde{\Phi}_{4\ell})} \Delta(t) \right]$$

with NLO-accurate $\tilde{B}$ weight,

$$\tilde{\mathrm{B}}(\tilde{\Phi}_{4\ell}) = \mathrm{B}(\tilde{\Phi}_{4\ell}) + \mathrm{V}_{\rm reg}(\tilde{\Phi}_{4\ell}) + \int \mathrm{d}\Phi_{\rm rad} \left[\mathrm{R}(\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi_{\rm rad}) - \mathrm{CT}(\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi_{\rm rad}) \right]$$

and Sudakov form factor (t = radiation transverse momentum),

$$\Delta(t) = \exp \left[- \int_{t'>t} \mathrm{d}\Phi'_{\rm rad} \frac{\mathrm{R}(\bar{\tilde{\bar{\Phi}}}_{4\ell}, \Phi'_{\rm rad})}{\mathrm{B}(\tilde{\Phi}_{4\ell})} \right]$$