

Top-Bottom Interference Contribution to Fully-Inclusive Higgs Production

Marco Niggetiedt

with M. Czakon, F. Eschment, R. Poncelet, T. Schellenberger and J. Usovitsch

based on

JHEP 02 (2024) 087
[arXiv:2312.09896]

2nd Workshop on Tools for
High Precision LHC Simulations

MAX-PLANCK-INSTITUT
FÜR PHYSIK



Castle Ringberg
May 10th 2024

Gluon fusion

- Gluon fusion is the predominant Higgs boson production mode at the LHC
 - loop induced process

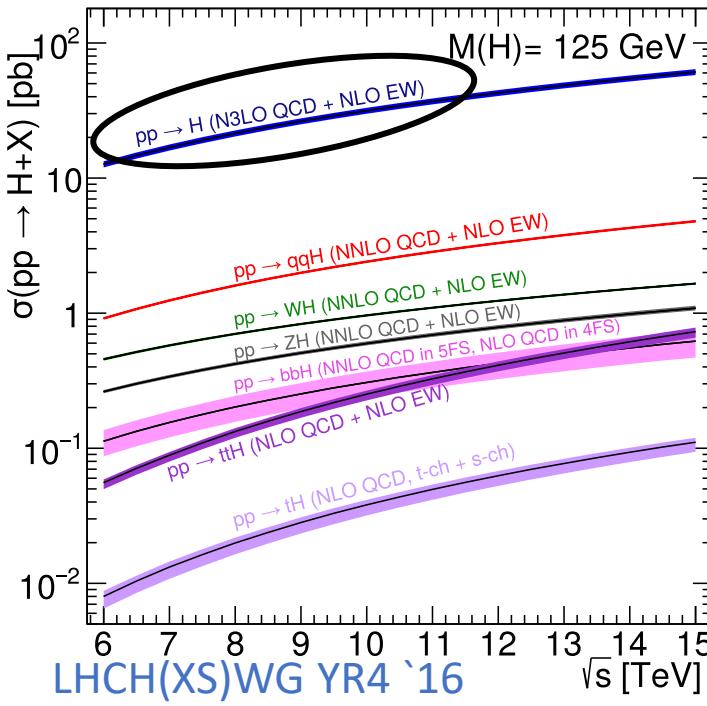
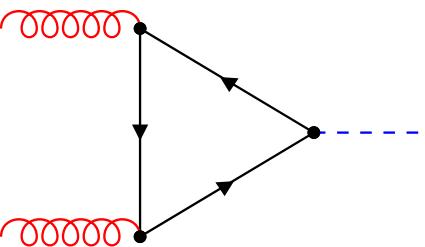
- Higgs boson plays unique role in the SM:
 - Only scalar particle
 - Only particle with Yukawa interactions to fermions

- Predictions for gluon fusion cross section directly impact extraction of Higgs couplings from experimental measurements

- Reducing theory uncertainty is crucial for facilitating high precision measurements of Higgs couplings at the LHC

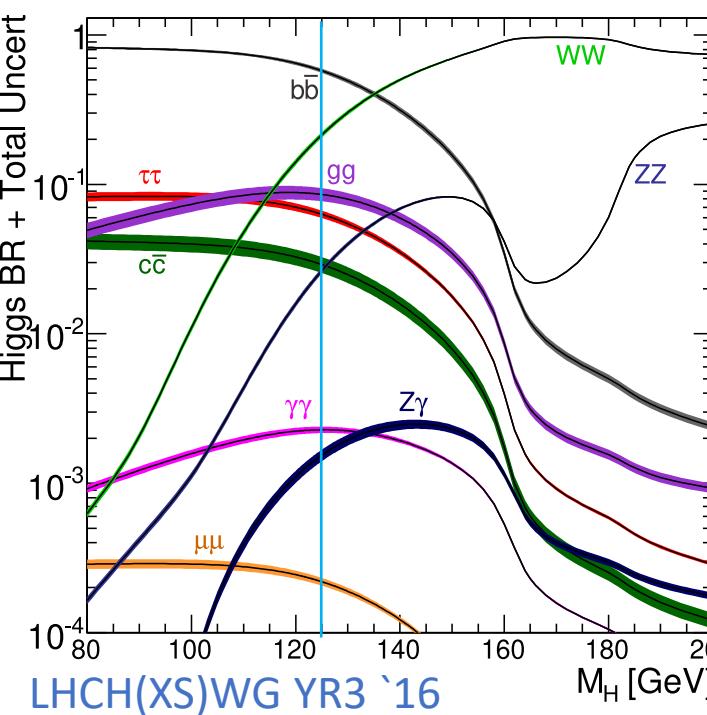
- High luminosity LHC projections anticipate uncertainty $\mathcal{O}(2\%)$ and theory uncertainty to be halved

WG2 Report '19



LHC HIGGS XS WG 2013

LHCH(XS)WG YR4 '16



LHC HIGGS XS WG 2013

LHCH(XS)WG YR3 '16

LHC HIGGS XS WG 2013

M(H) = 125 GeV

Inclusive gluon fusion cross section (YR4 `16)

- Inclusive gluon fusion cross section according to [LHCH\(XS\)WG YR4 `16](#) at the LHC at 13 TeV:

48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)
	- 2.05 pb	(-4.2%)	((t, b, c), exact NLO)
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[LHCH\(XS\)WG YR4 `16](#)

[Anastasiou, Duhr, Dulat, et al. `16](#)

- Sources of uncertainty:

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
$+0.10 \text{ pb}$	$\pm 0.18 \text{ pb}$	$\pm 0.56 \text{ pb}$	$\pm 0.49 \text{ pb}$	$\pm 0.40 \text{ pb}$	$\pm 0.49 \text{ pb}$
-1.15 pb					
$+0.21\%$	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$
-2.37%					

← All sources $\mathcal{O}(1\%)$

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Mistlberger '18

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[Mistlberger '18](#)

[Bonetti, Melnikov, Tancredi '18](#)

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[Becchetti, Bonciani, Del Duca, et al. '21](#)

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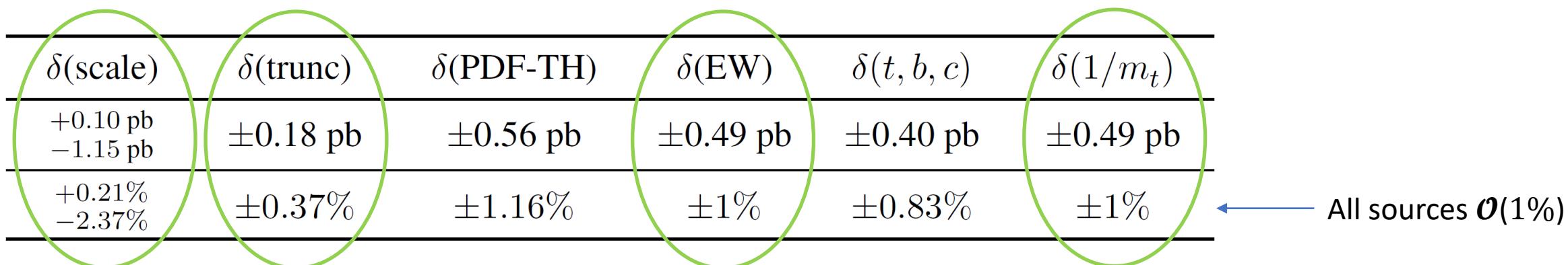
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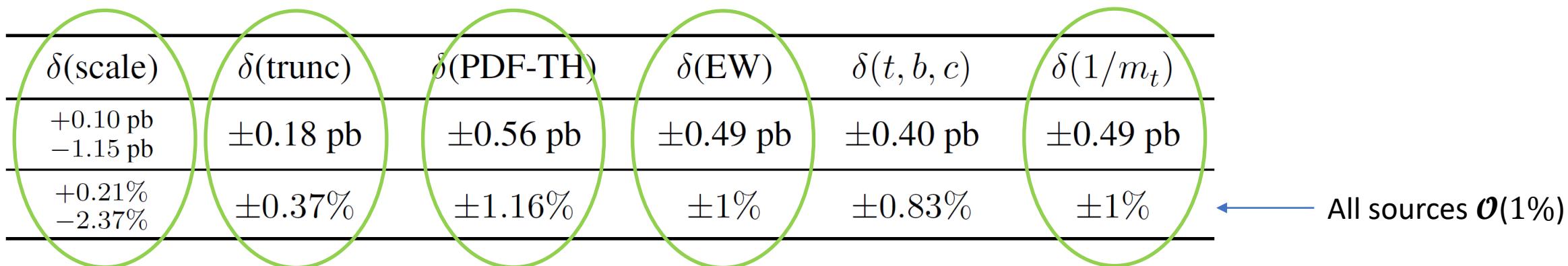
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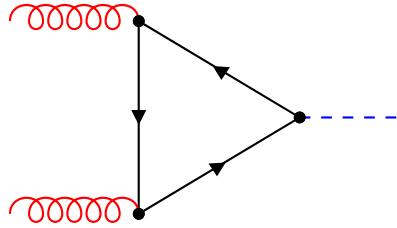
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[Moch, Ruijl, Ueda, et al. '21](#)
[McGowan, Cridge, Harland-Lang et al. '22](#)
[Falcioni, Herzog, Moch, et al. '23/'24](#)
[NNPDF Collaboration '24](#)



Order by order in perturbation theory

- LO contribution exactly known for almost 50 years

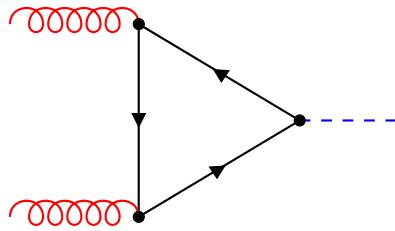


Georgi, Glashow, Machacek, et al. '78

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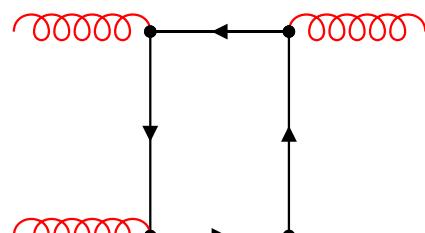
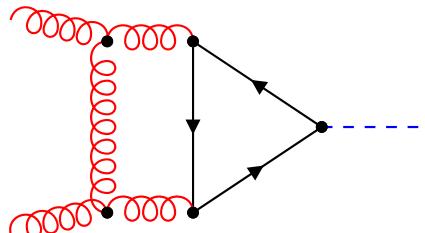
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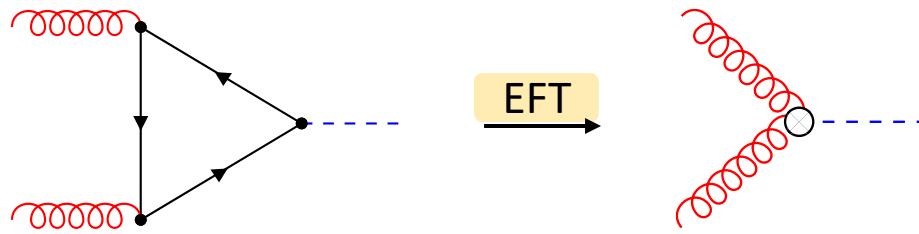
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- NLO contribution exactly known for arbitrary quark masses running in the loop Graudenz, Spira, Zerwas '93



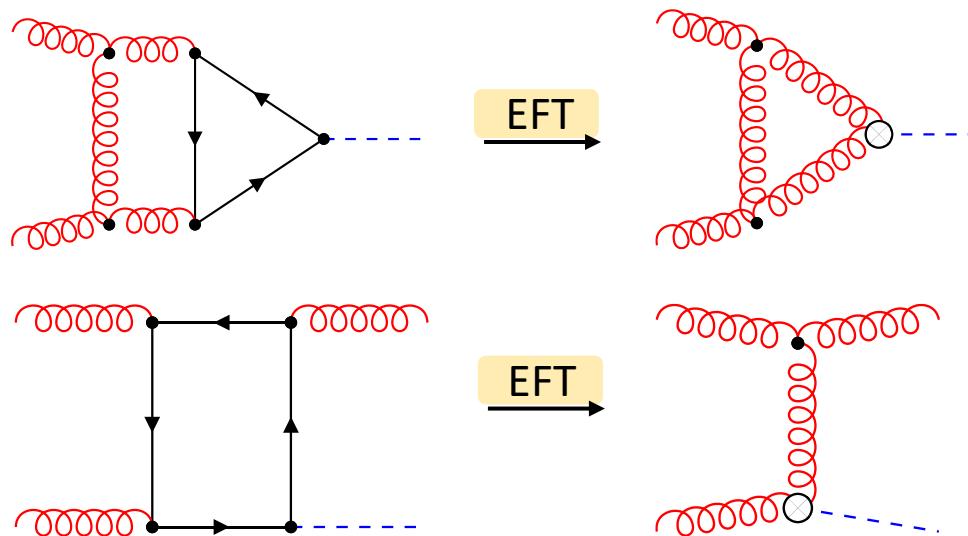
Inclusive cross section in (r)EFT

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Dawson '91
Djouadi, Spira, Zerwas '91

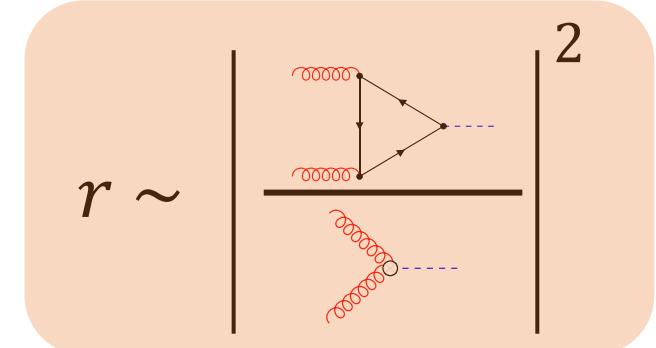
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM},5} - \frac{1}{4} C H G_{\mu\nu}^a G_a^{\mu\nu}$$

\mathcal{L}_{SM}

$m_t \rightarrow \infty$

Chetyrkin, Kniehl, Steinhauser '98
Schröder, Steinhauser '06
Chetyrkin, Kühn, Sturm '06

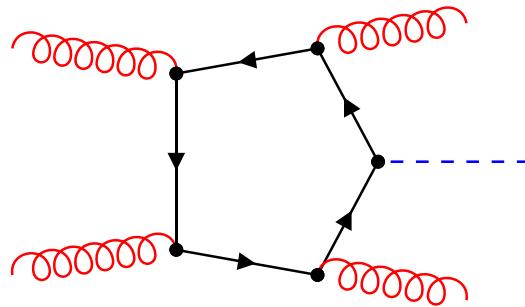
$$\sigma_{\text{HEFT}}^{\text{HO}} = \left(\frac{\sigma^{\text{HO}}}{\sigma^{\text{LO}}} \right)_{M_t \rightarrow \infty} \sigma^{\text{LO}}$$



Computation

Ingredients for exact quark mass effects at NNLO

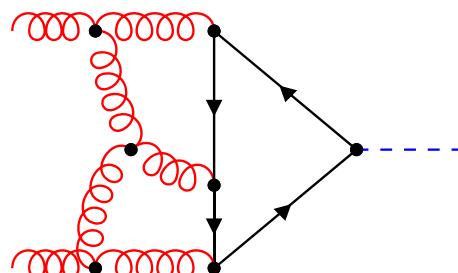
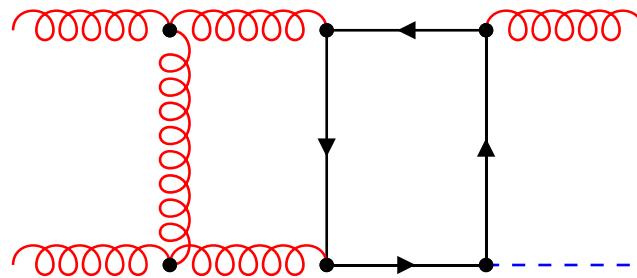
- Beyond NLO:



Analytically: Del Duca, Kilgore, Oleari, et al. '01

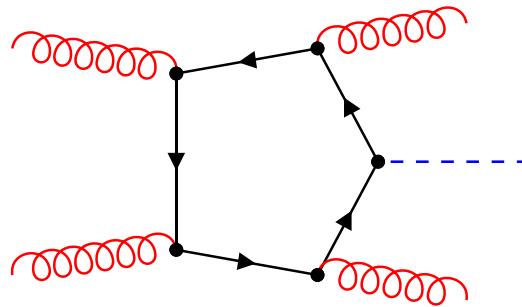
OpenLoops 2: Buccioni, Lang, Lindert, et al. '19

Analytically (more compact and
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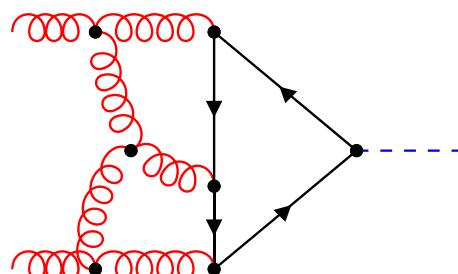
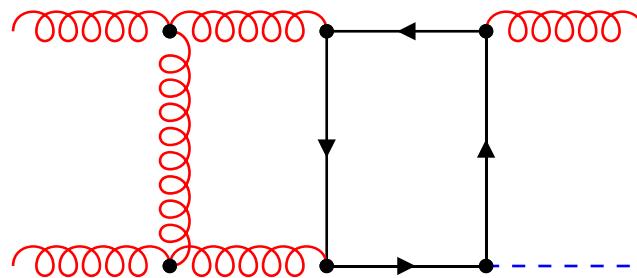


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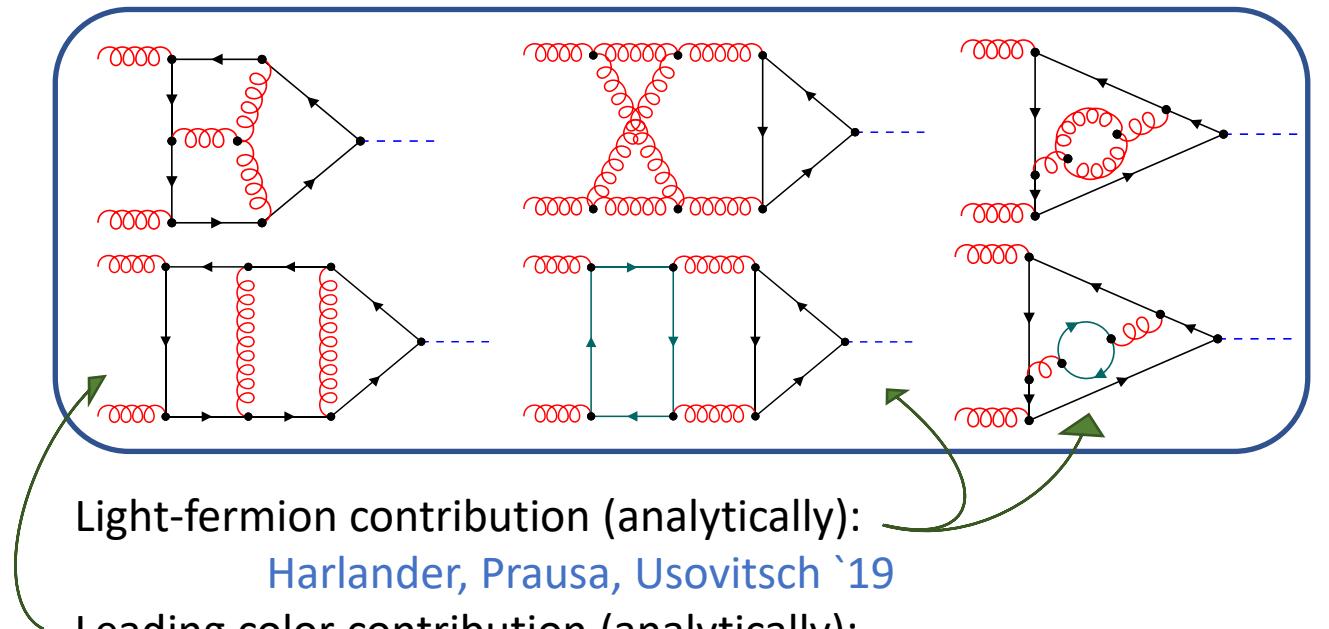
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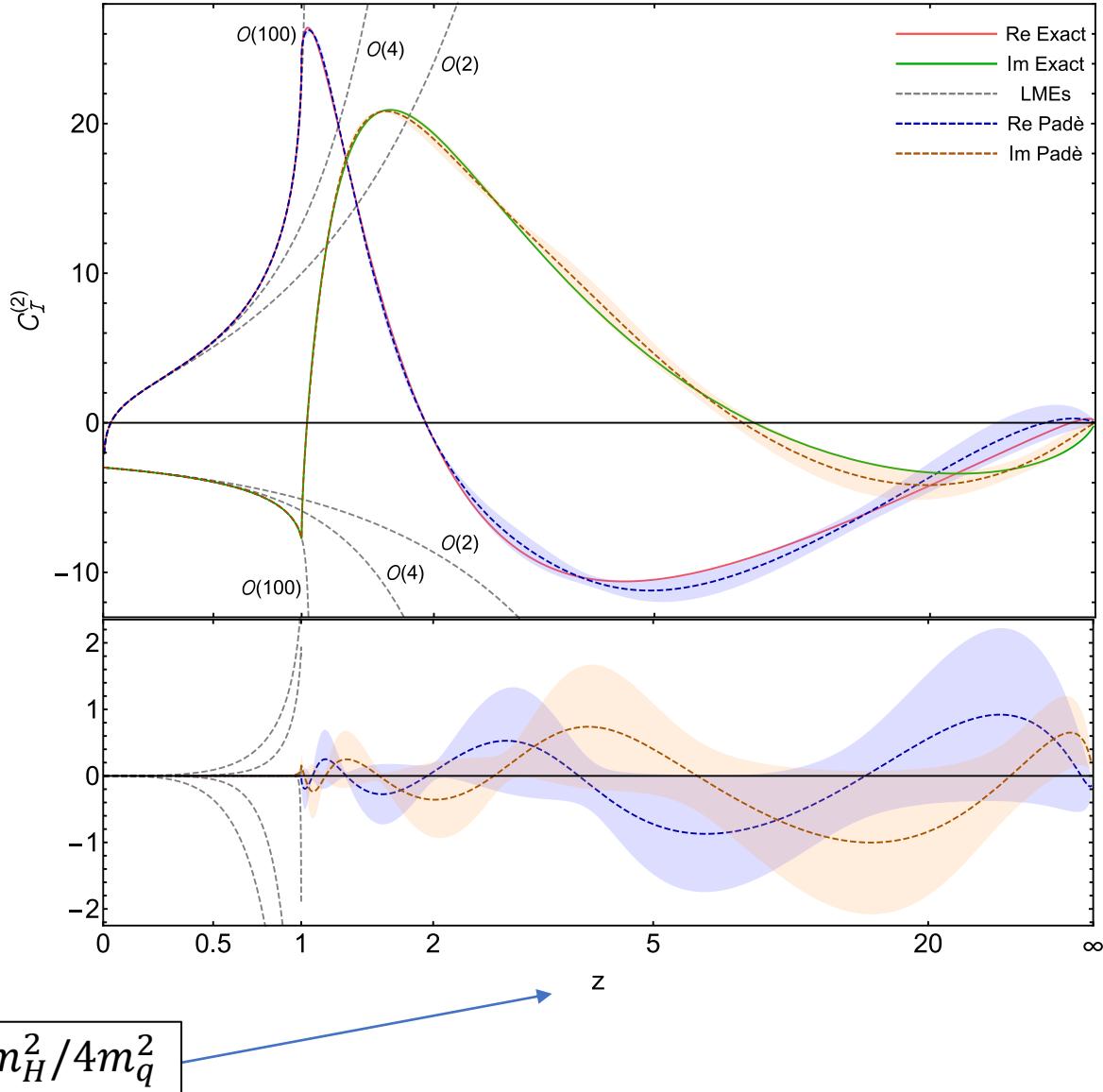
Double-virtual corrections

Double-virtual corrections

One massive flavor

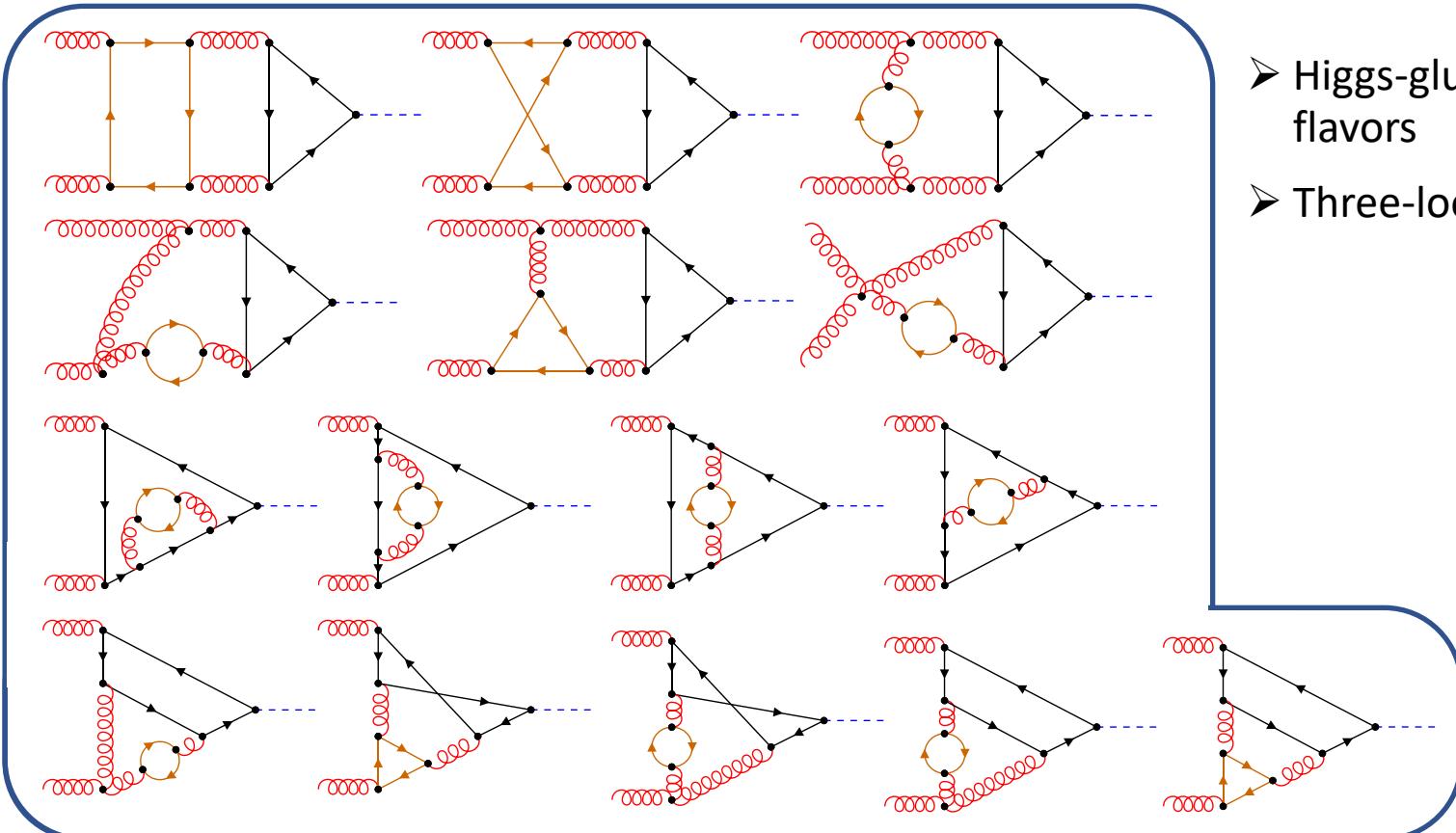


Padé approximation: Davies, Gröber, Maier et al. '19
Numerically exact: Czakon, MN '20



Double-virtual corrections

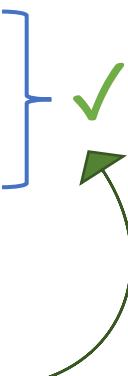
Two massive flavors



- Solution strategy:
 - Compute expansions for $m_b^2 \ll m_H^2 \ll m_t^2$
 - Sample remaining parameter space numerically

- Higgs-gluon form factor in QCD with $n_f = n_t + n_b + n_l$ flavors
- Three-loop coefficient:

$$C^{(2)} = n_t C_t^{(2)} + n_t^2 C_{tt}^{(2)} + n_t n_l C_{tl}^{(2)} \\ + n_b C_b^{(2)} + n_b^2 C_{bb}^{(2)} + n_b n_l C_{bl}^{(2)} \\ + n_t n_b (C_{tb}^{(2)} + C_{bt}^{(2)})$$

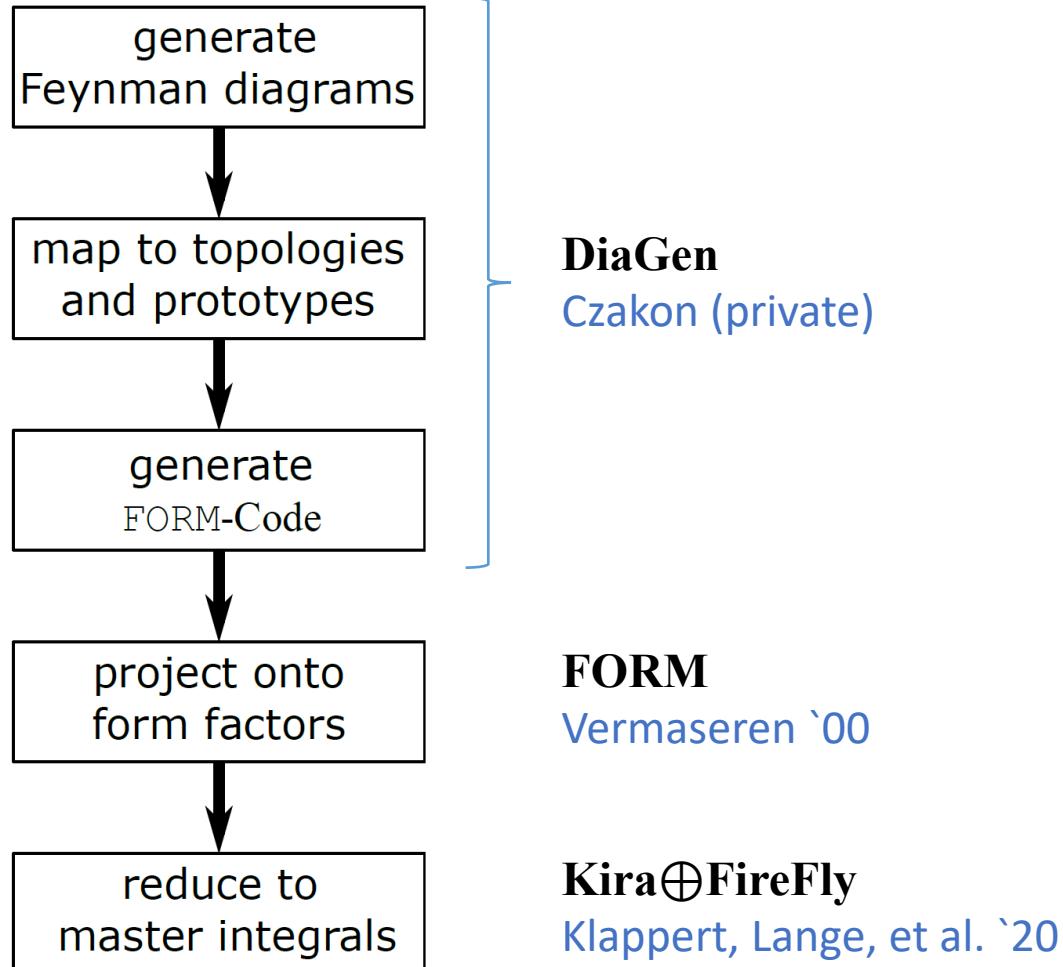


Czakon, MN '20

MN, Usovitsch '23

Note: $\frac{m_H^2}{m_t^2} \approx 0.5$ and $\frac{m_b^2}{m_H^2} \approx 0.0015$

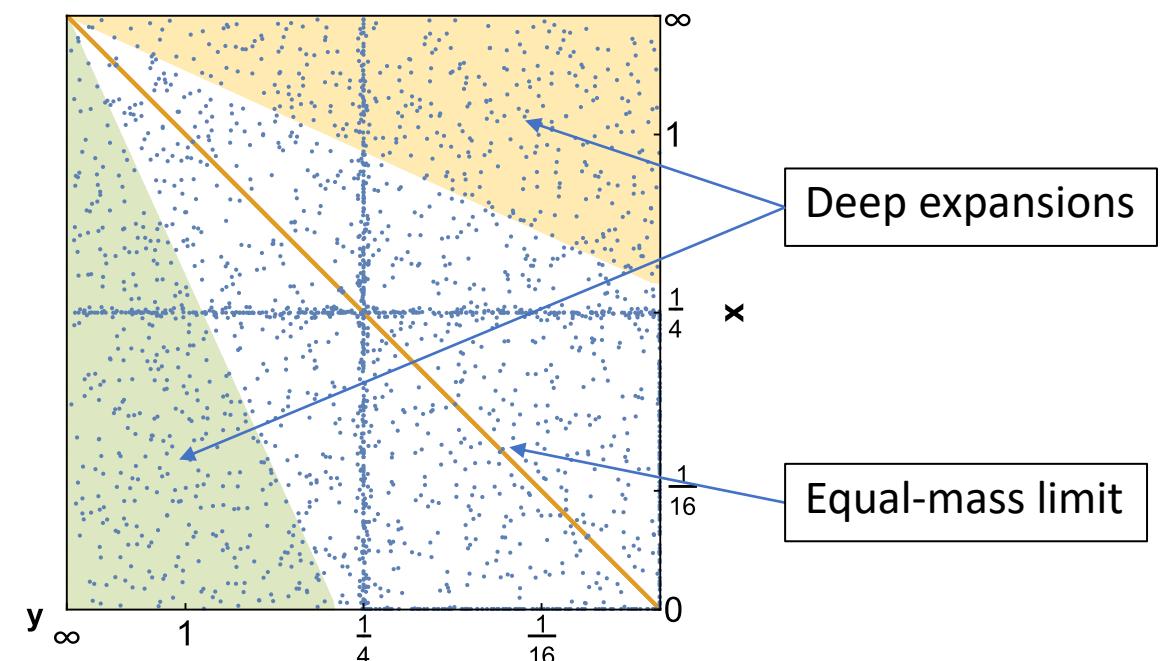
Workflow of the Computation



- Variables: x, y
- Differential operators in $\{x, y\}$ can be expressed in terms of derivatives with respect to $\{m_t^2, m_b^2\}$
- Exploit symmetry: $C_{tb}^{(2)}(x, y) = C_{bt}^{(2)}(y, x)$
- Solve ~ 300 MIs numerically using AMFlow Liu, Ma '22

$$x = m_t^2/m_H^2$$
$$y = m_b^2/m_H^2$$

Kotikov '91
Gehrmann, Remiddi '00



Deep asymptotic expansions

- Employ differential equations for MIs:

$$\frac{dM_i(x, y, \epsilon)}{dx} = A_{ij}(x, y, \epsilon) M_j(x, y, \epsilon) \quad \leftarrow \quad M_i(x, y, \epsilon) = \sum_{l=0}^{\bar{n}_i - \underline{n}_i} \epsilon^{\underline{n}_i + l} I_{\underline{k}_i + l}(x, y)$$

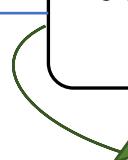
$\frac{dI_k(x, y)}{dx} = B_{kl}(x, y) I_l(x, y)$



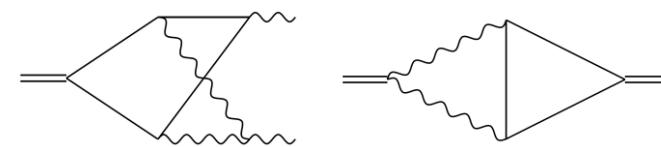
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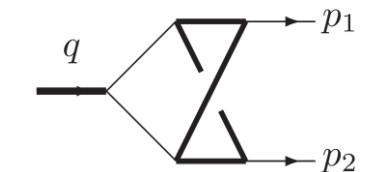
rational functions multiplying boundary condition



 $\{1, i\pi, \pi^2, \zeta_3, \log(y), \text{HPL}(\dots, y), \text{elliptic}\}$



Anastasiou, Beerli, Bucherer '06



von Manteuffel, Tancredi '17

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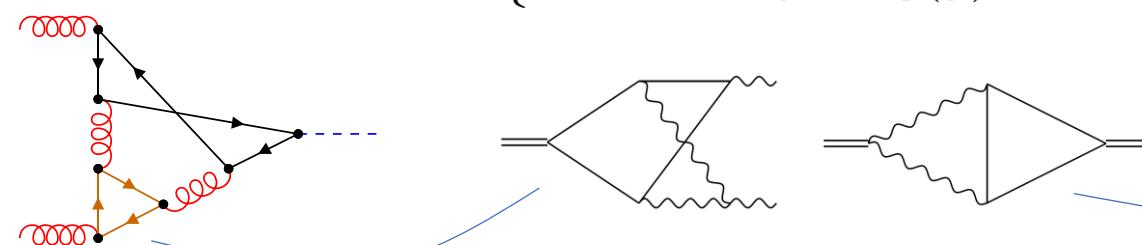
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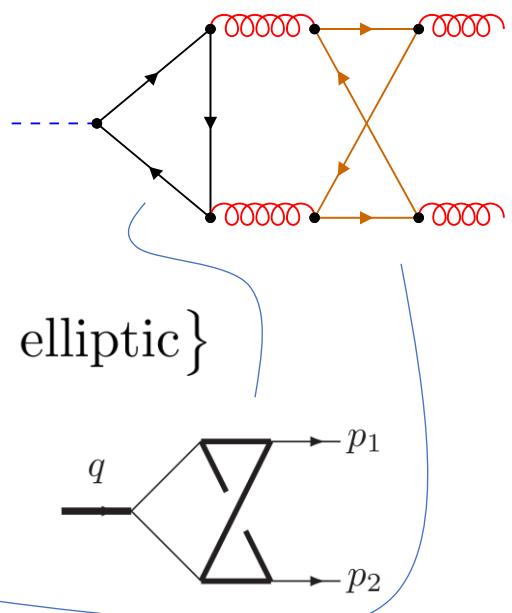
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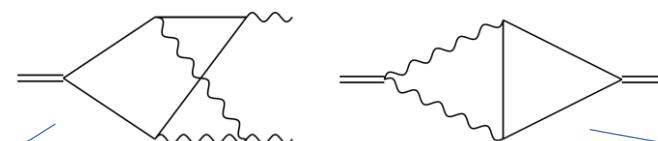
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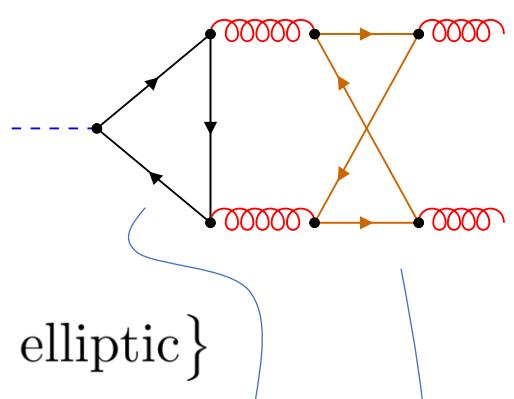
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➤ More efficient: Sample expansion coefficients for different values of y and reconstruct exact form at the amplitude level



Anastasiou, Beerli, Bucherer '06



von Manteuffel, Tancredi '17

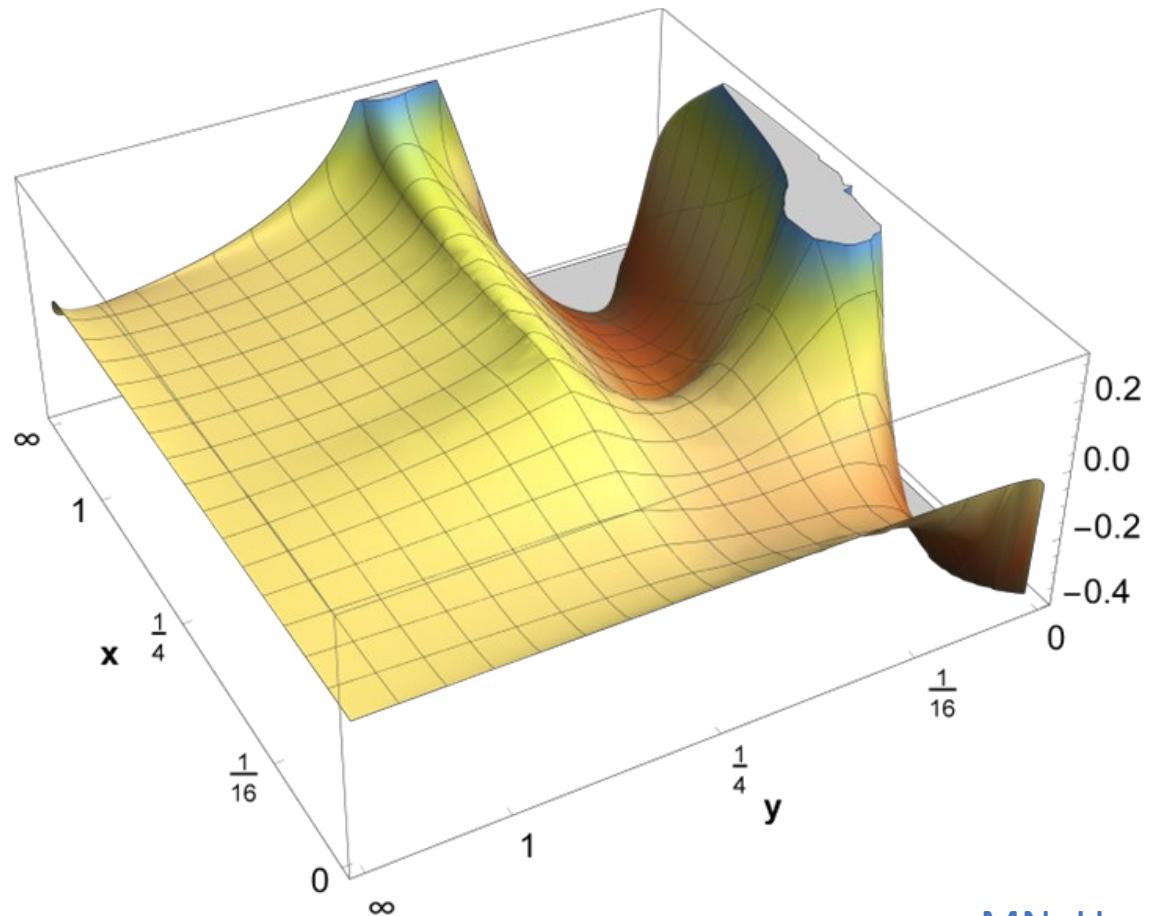
$$x = m_t^2/m_H^2$$

$$y = m_b^2/m_H^2$$

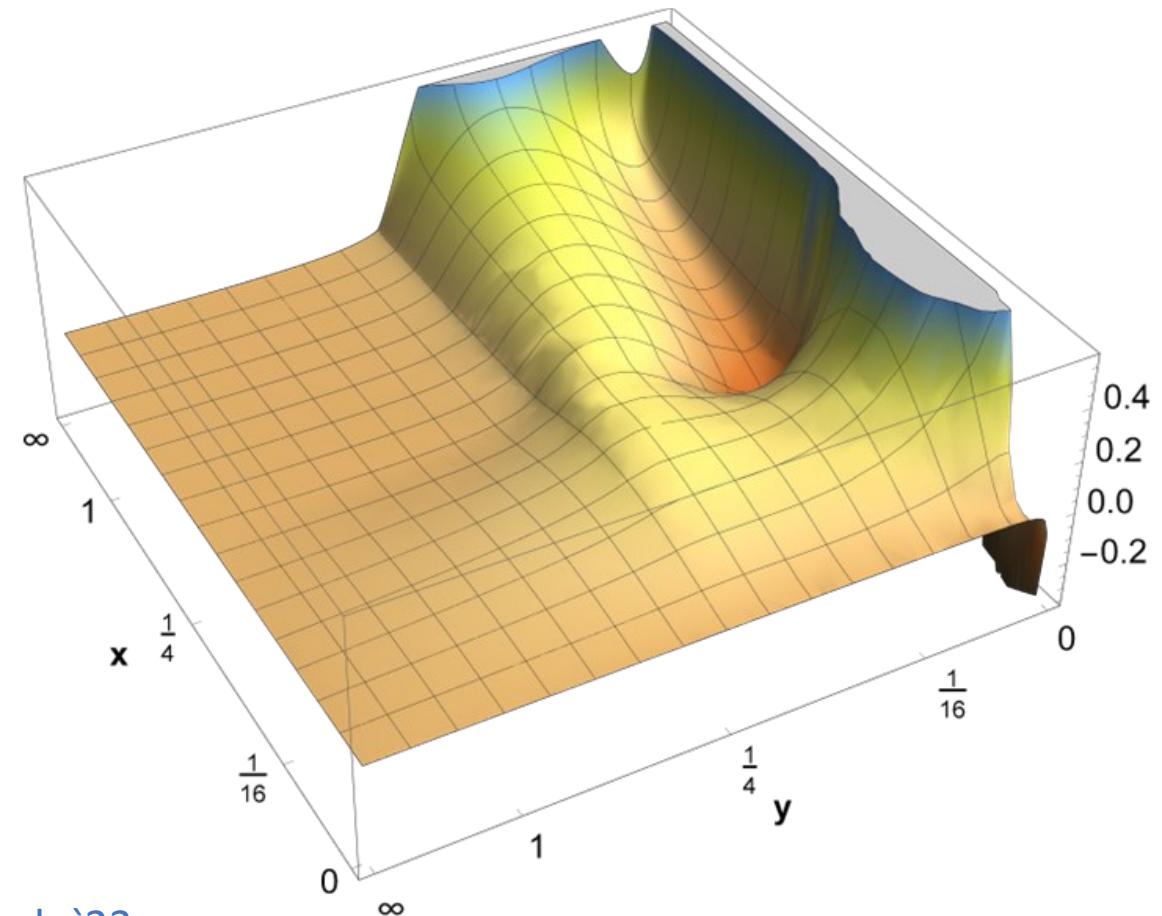
Finite remainder of $C_{tb}^{(2)}$

$$x = m_t^2/m_H^2$$
$$y = m_b^2/m_H^2$$

Re

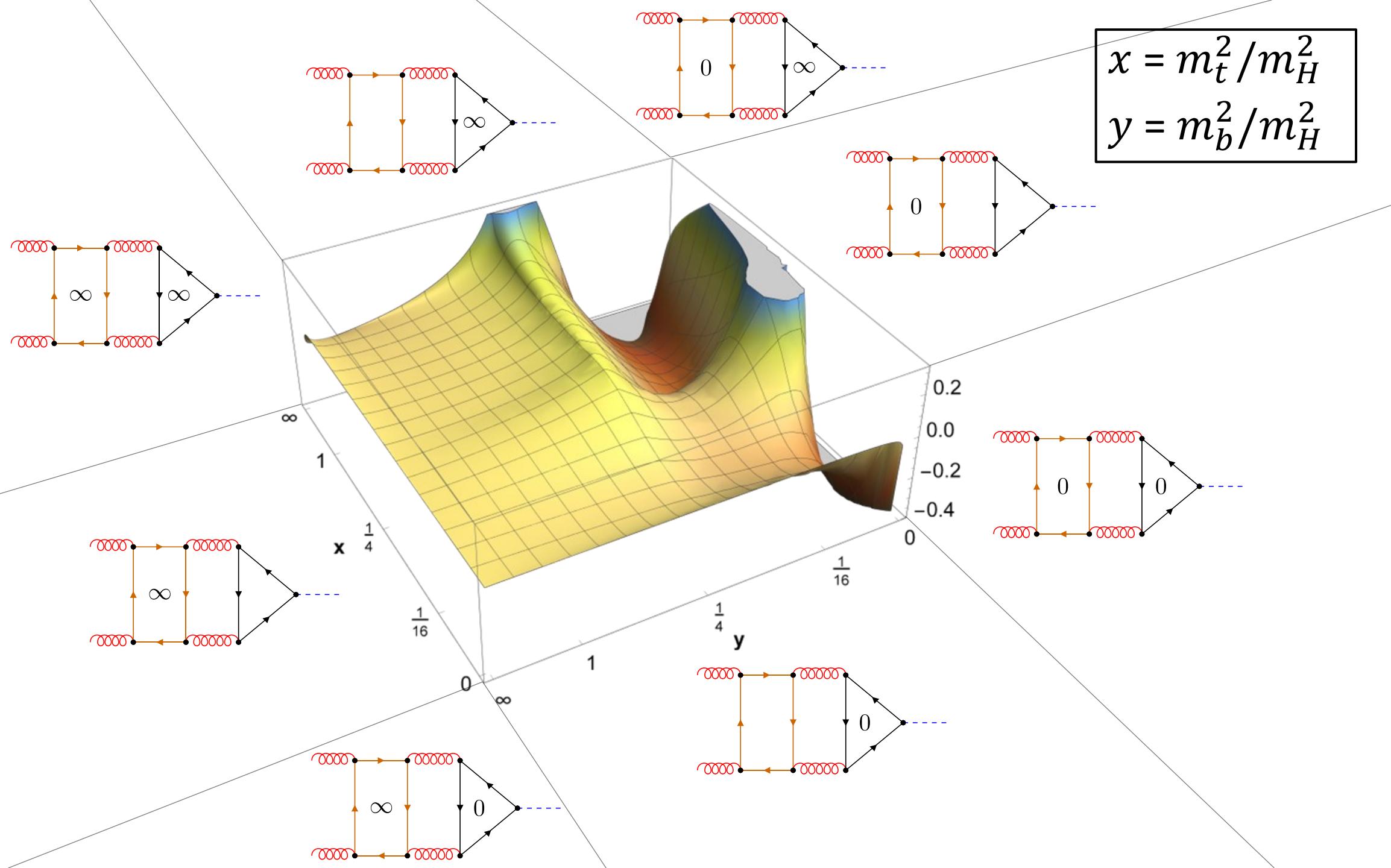


Im



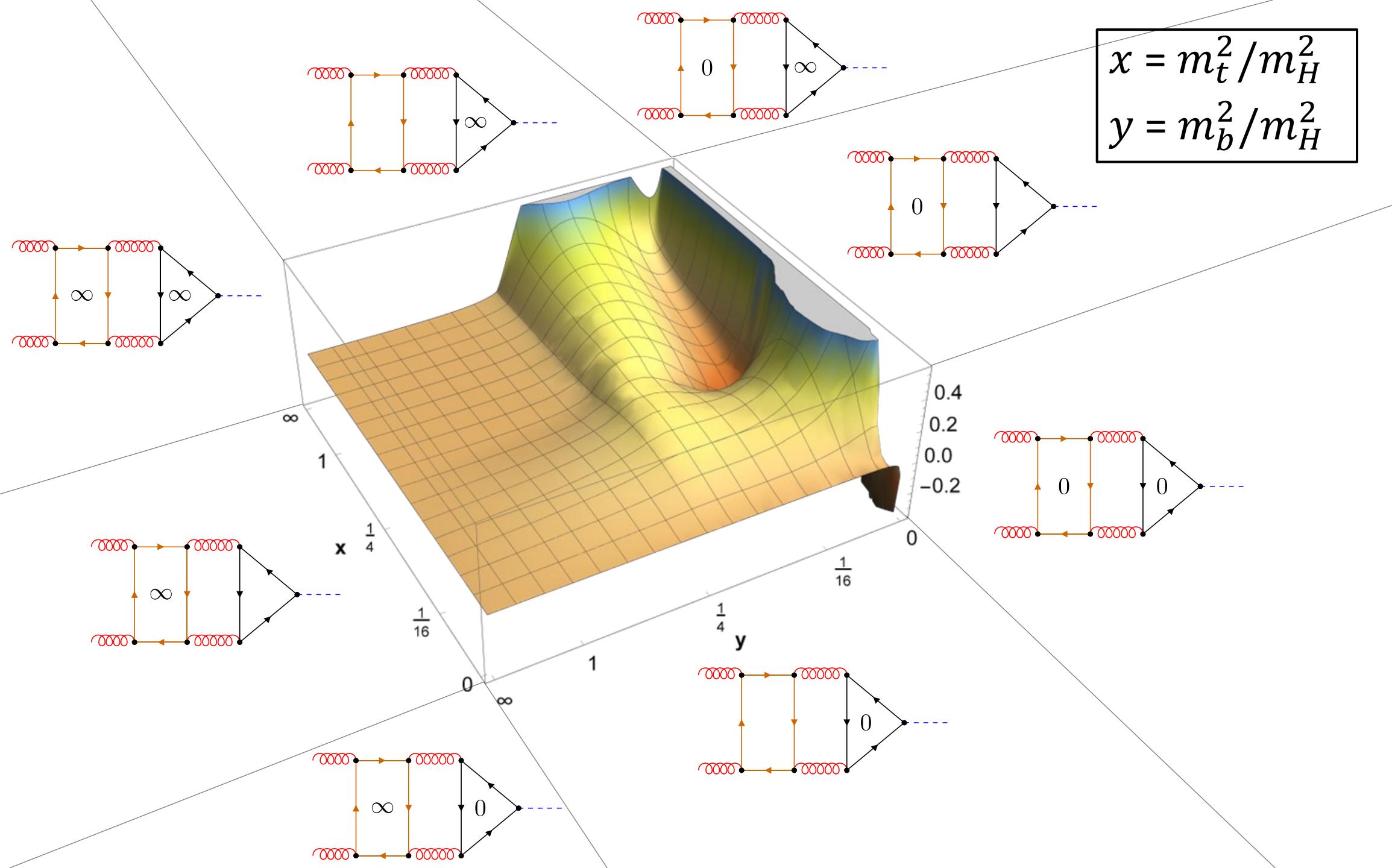
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$$y = m_b^2/m_H^2$$



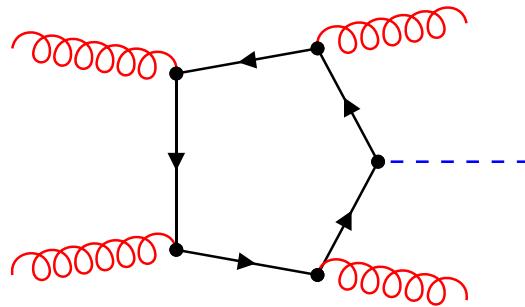
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Ingredients for exact quark mass effects at NNLO

- Beyond NLO:

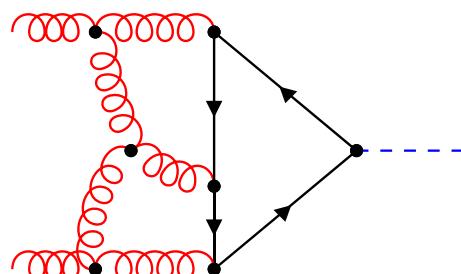
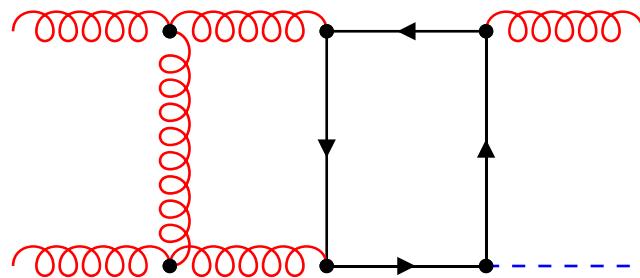


Analytically: Del Duca, Kilgore, Oleari, et al. '01

OpenLoops 2: Buccioni, Lang, Lindert, et al. '19

Analytically (more compact and

implemented in MCFM): Budge, Campbell, De Laurentis, et al. '20

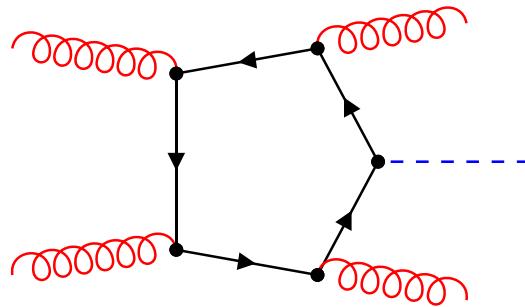


Double-virtual corrections



Ingredients for exact quark mass effects at NNLO

- Beyond NLO:

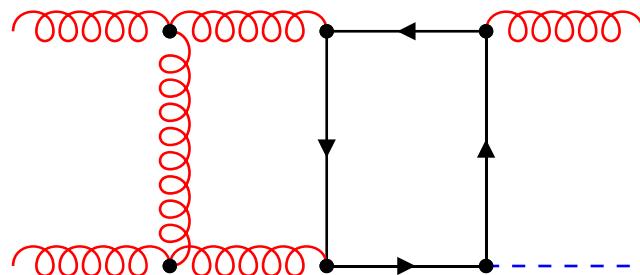


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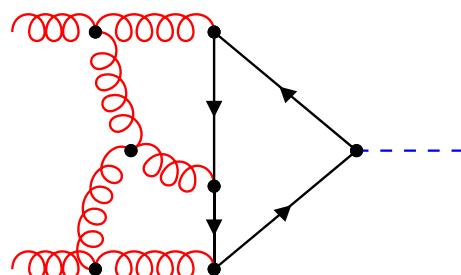
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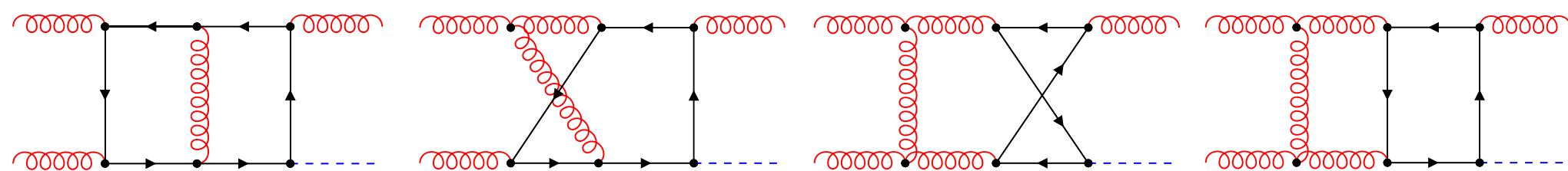
← Real-virtual corrections



← Double-virtual corrections

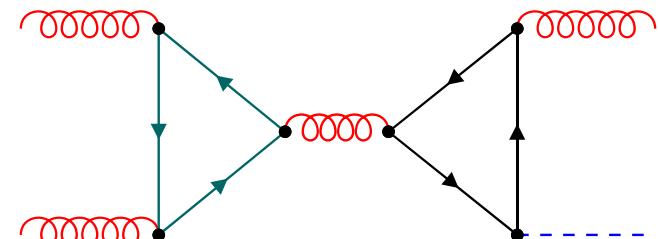
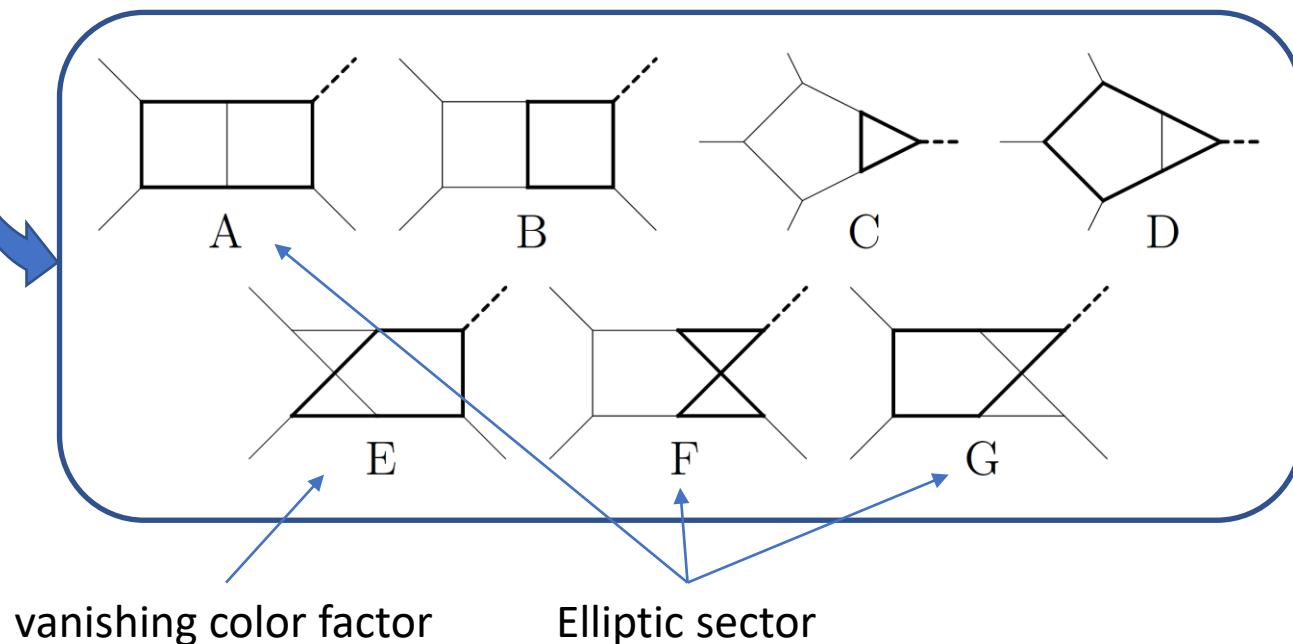


Real-virtual corrections



A,B,C,D: Bonciani, Del Duca, Frellesvig, et al. '16
F: Bonciani, Del Duca, Frellesvig, et al. '19
G: Frellesvig, Hidding, Maestri, et al. '19

Contributions with two closed fermion chains
are always factorizable:



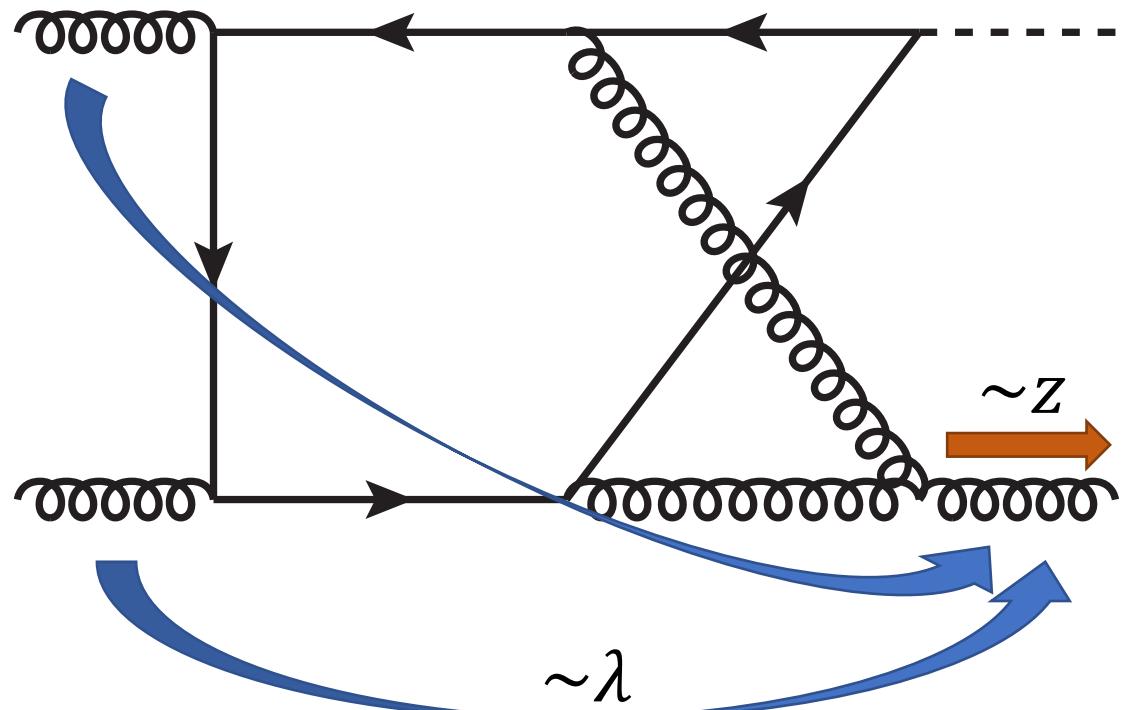
Parametrization

- Variables: $\hat{s}, \hat{t}, \hat{u}, m_H^2, m_q^2$
- Introduce dimensionless variables and fix ratio m_q^2/m_H^2
 - z parametrizes **soft** limit
 - λ parametrizes **collinear** limit

$$\begin{aligned}\hat{t}/\hat{s} &= z \lambda \\ \hat{u}/\hat{s} &= z (1-\lambda)\end{aligned}$$

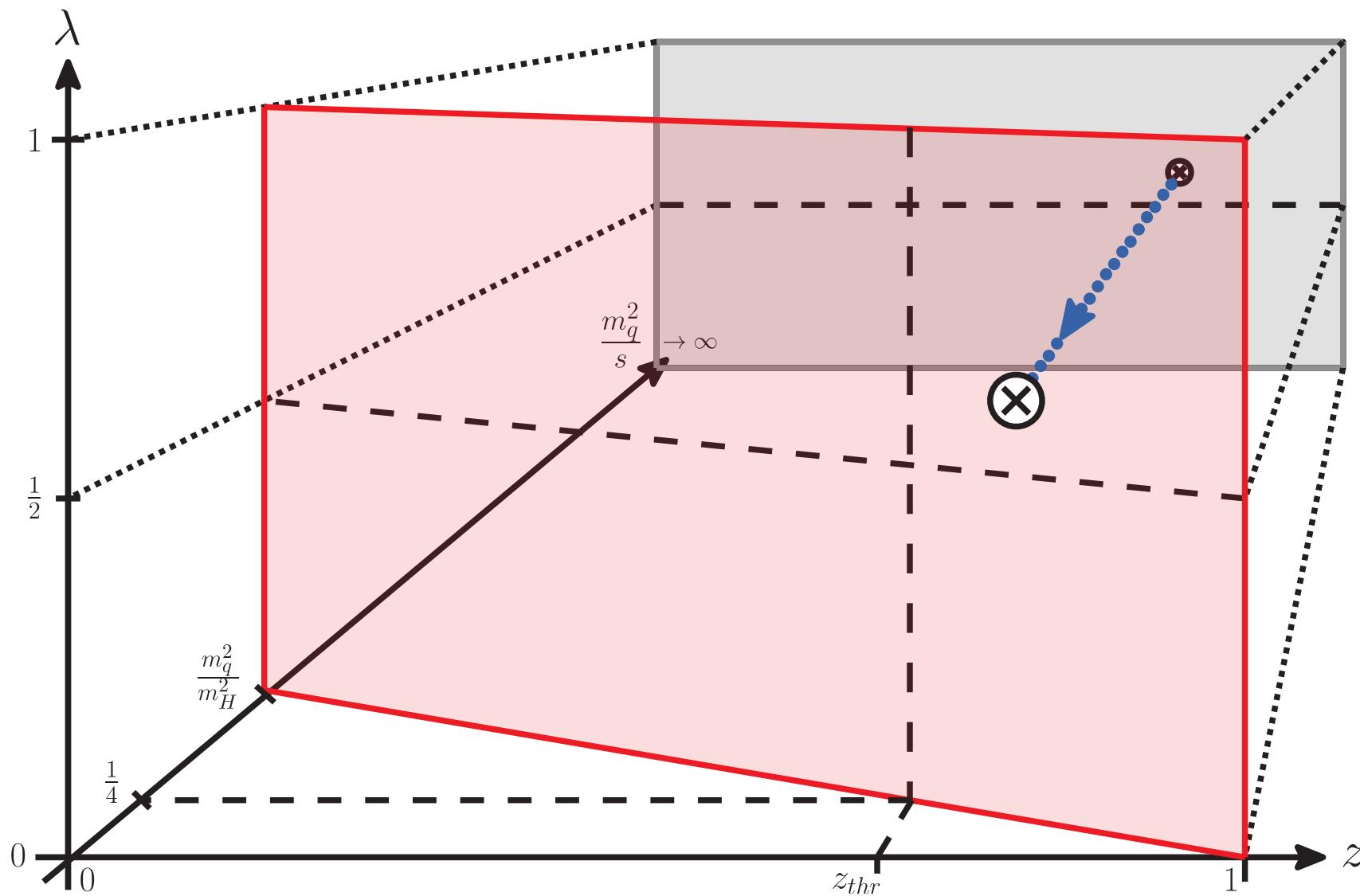


$$\begin{aligned}z &= 1-m_H^2/\hat{s} \\ \lambda &= \hat{t}/(\hat{t}+\hat{u})\end{aligned}$$



$z = 1-m_H^2/\hat{s}$
$\lambda = \hat{t}/(\hat{t}+\hat{u})$
$m_t^2/m_H^2 = 23/12$
$m_b^2/m_H^2 = 1/684$
Range of parameters:
• $\lambda \in (0,1)$
• $z \in (0,1)$

Evolution of differential equations



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2/m_H^2 = 23/12$$

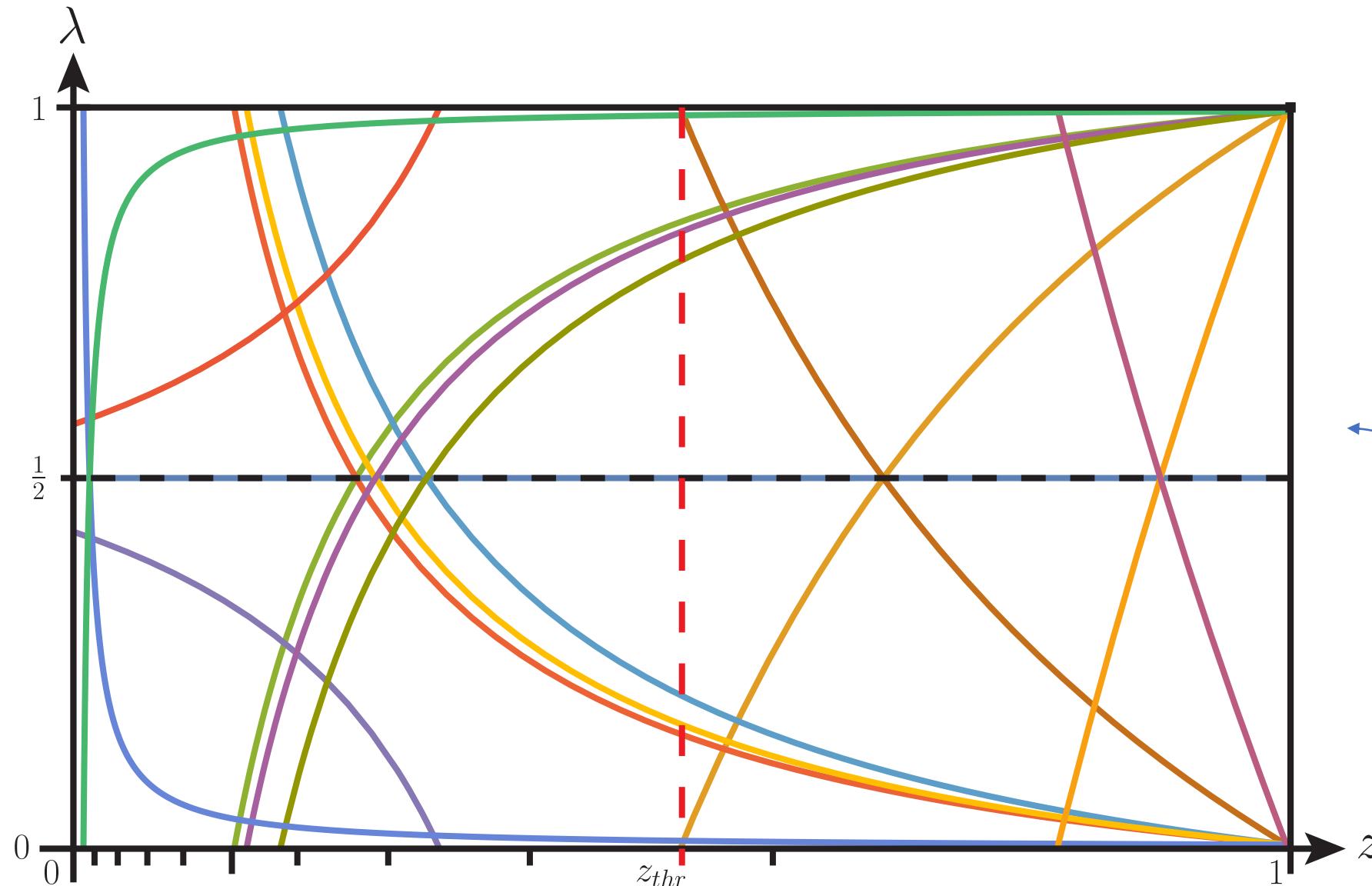
Range of parameters:

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 - $z \in (0,1)$

$$Z_{thr} = 1 - \frac{m_H^2}{4m_q^2}$$

- For $m_t^2/m_H^2 = 23/12$:
 $z_{thr} = 20/23 \approx 0.87$

Evolution in the (z, λ) -plane



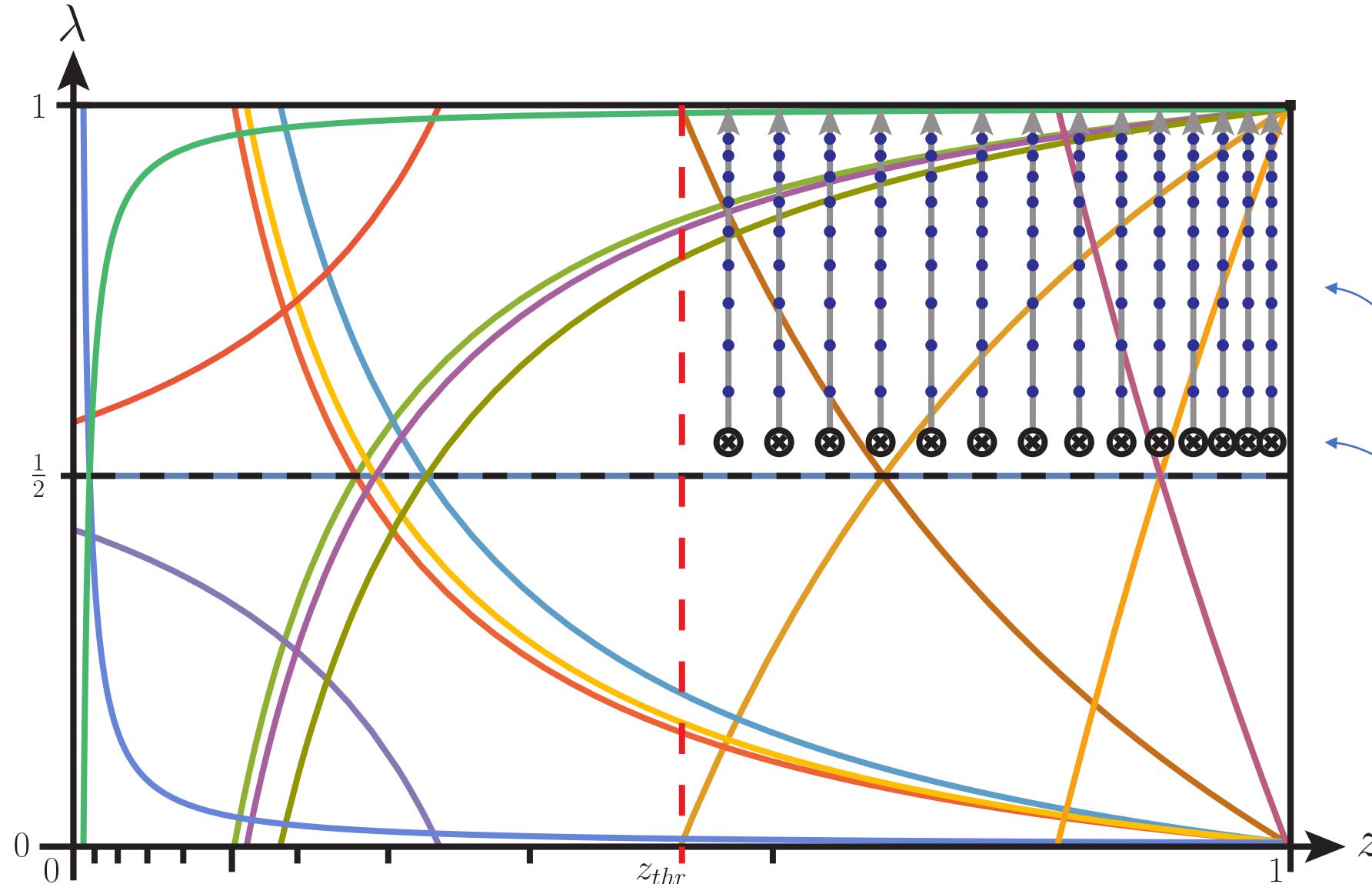
$z = 1 - m_H^2 / \hat{s}$
 $\lambda = \hat{t} / (\hat{t} + \hat{u})$
 $m_t^2 / m_H^2 = 23/12$

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Poles of differential equations in λ

Evolution in the (z, λ) -plane



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$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

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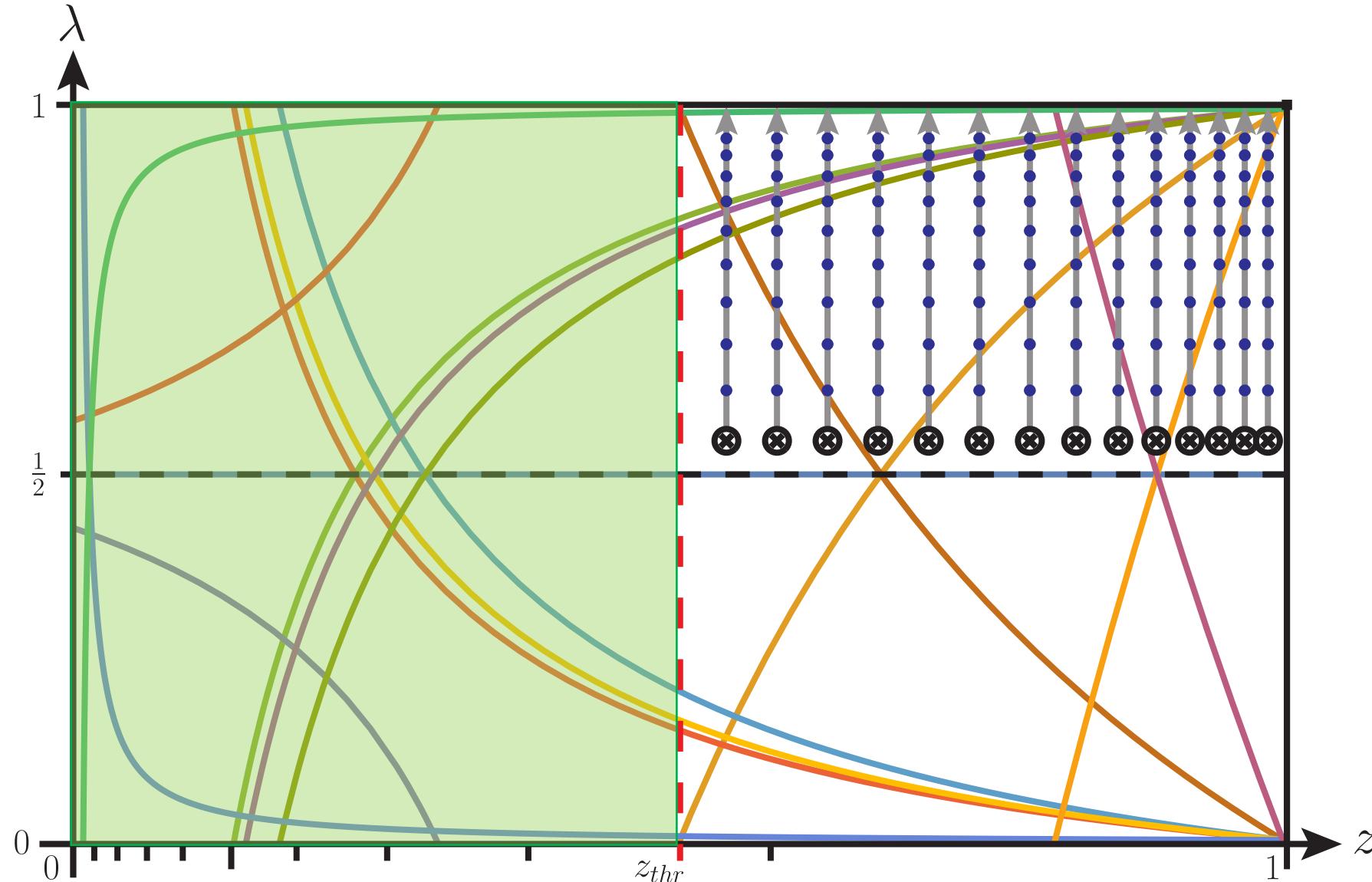
Range of parameters:

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- $z \in (0, 1)$

Collect numerical samples for MI along straight integration contours

Boundaries from numerical integration in the mass

Evolution in the (z, λ) -plane



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 $\lambda = \hat{t} / (\hat{t} + \hat{u})$
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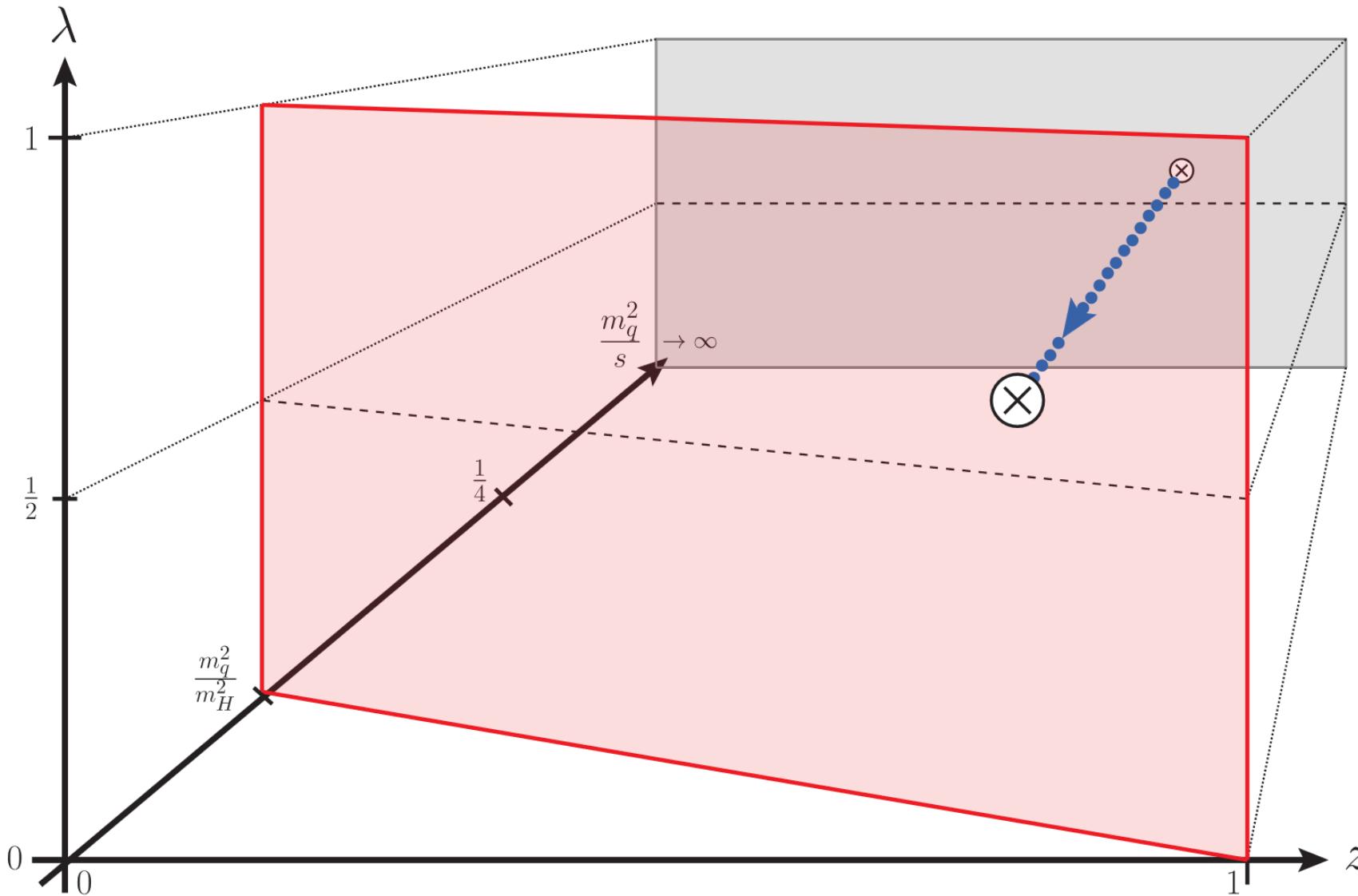
Range of parameters:

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Region below
threshold
covered by LME

LME
 $\mathcal{O}((1/m_q^2)^{40})$

Evolution of differential equations



$$z = 1 - m_H^2/\hat{s}$$

$$\lambda = \hat{t}/(\hat{t} + \hat{u})$$

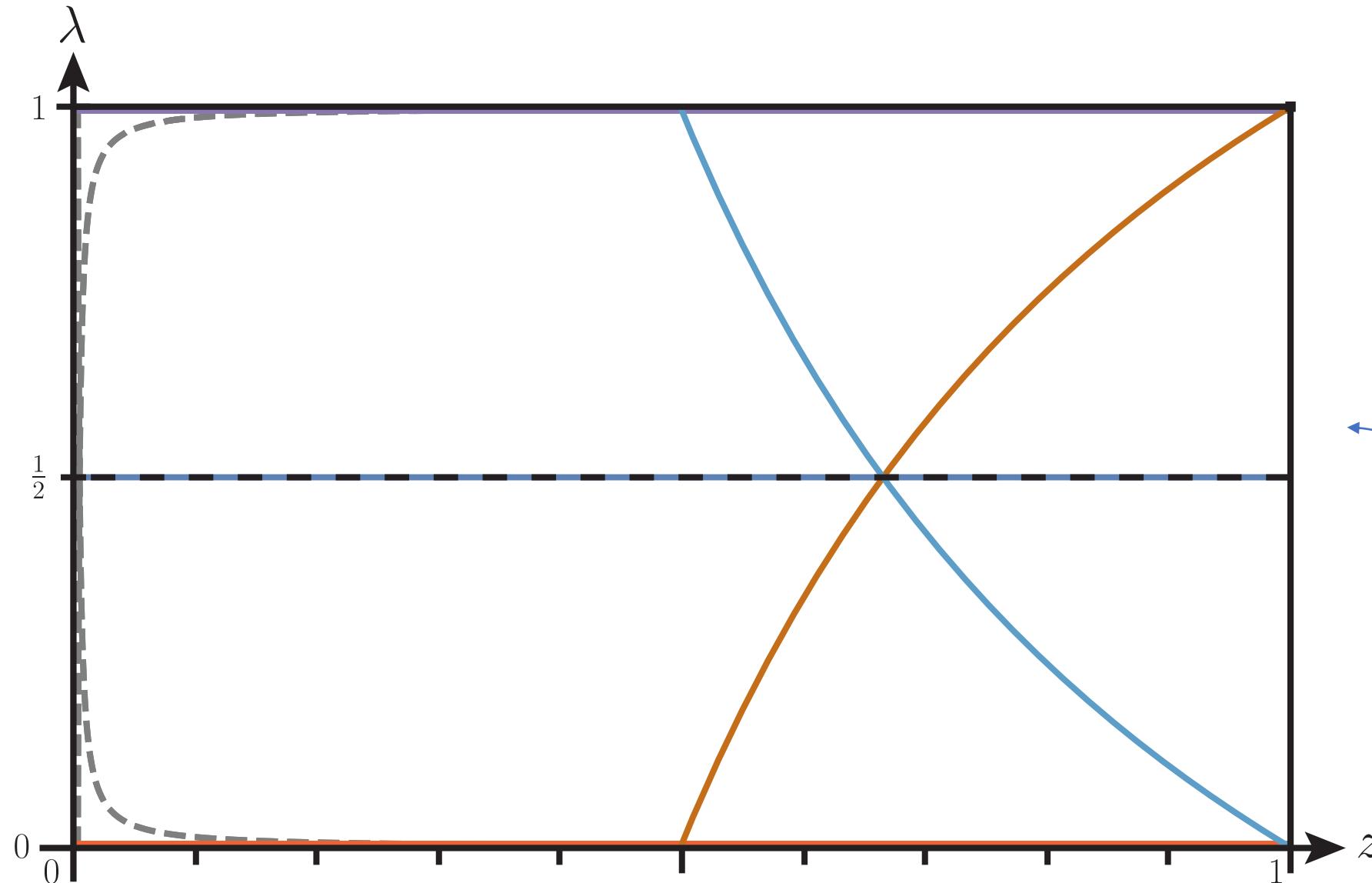
$$m_b^2/m_H^2 = 1/684$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

$$z_{thr} = 1 - \frac{m_H^2}{4m_q^2} < 0$$

Evolution in the (z, λ) -plane



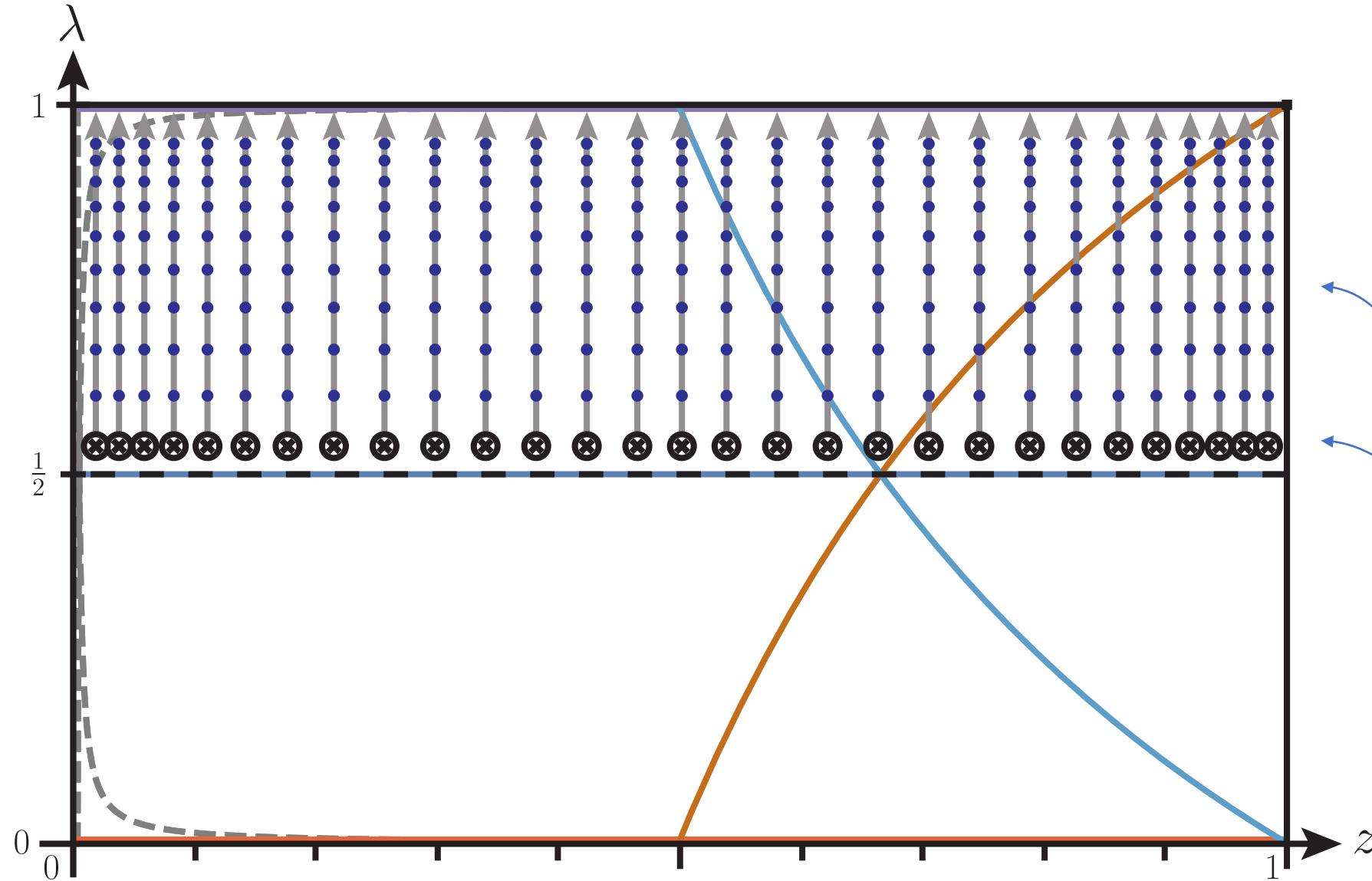
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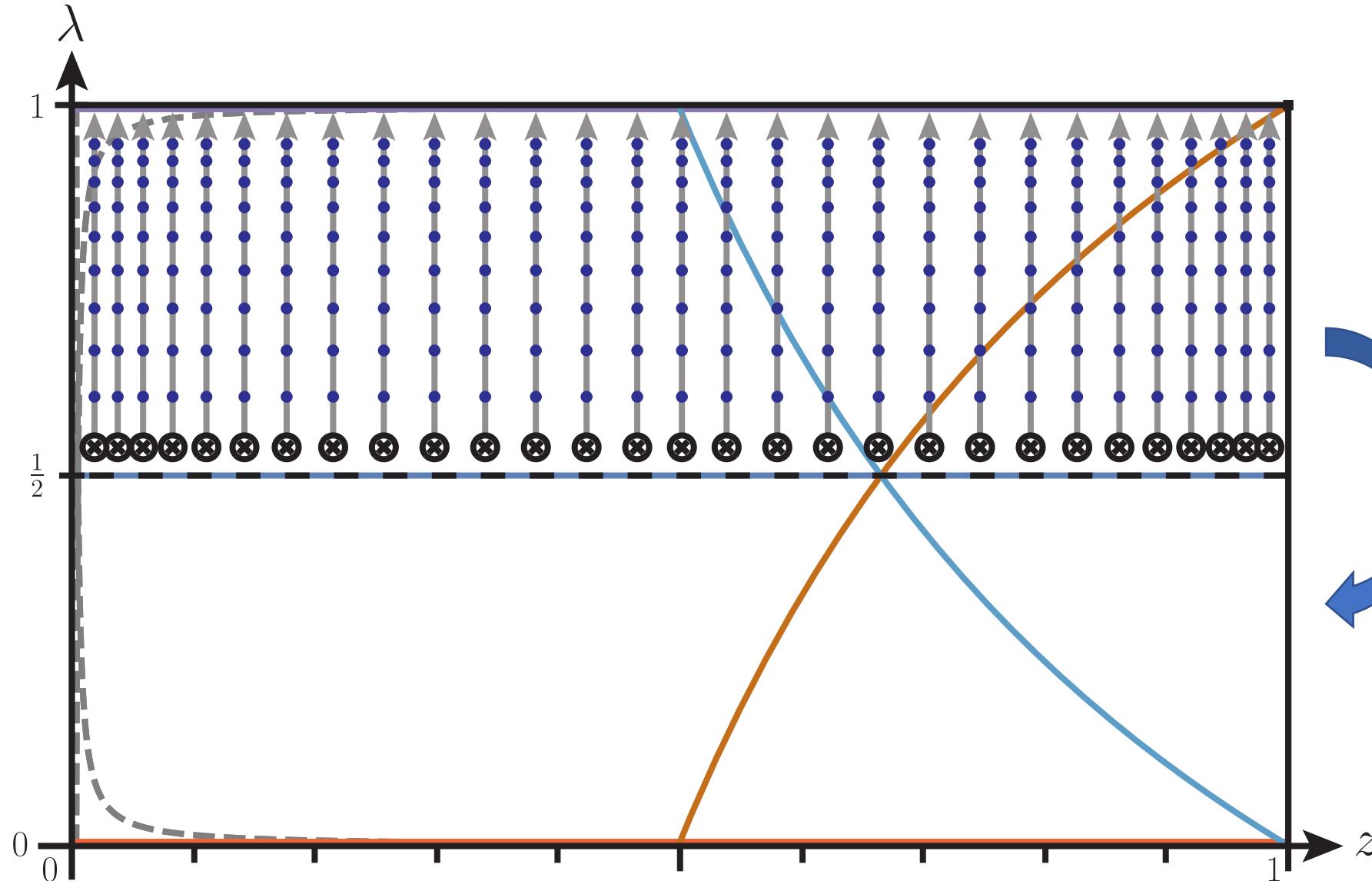
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Range of parameters:

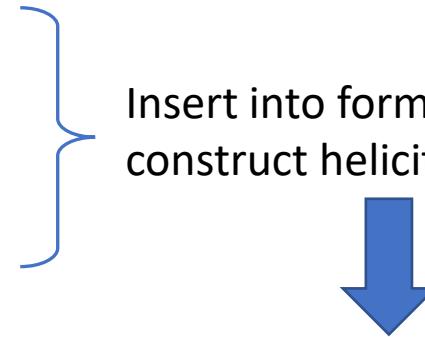
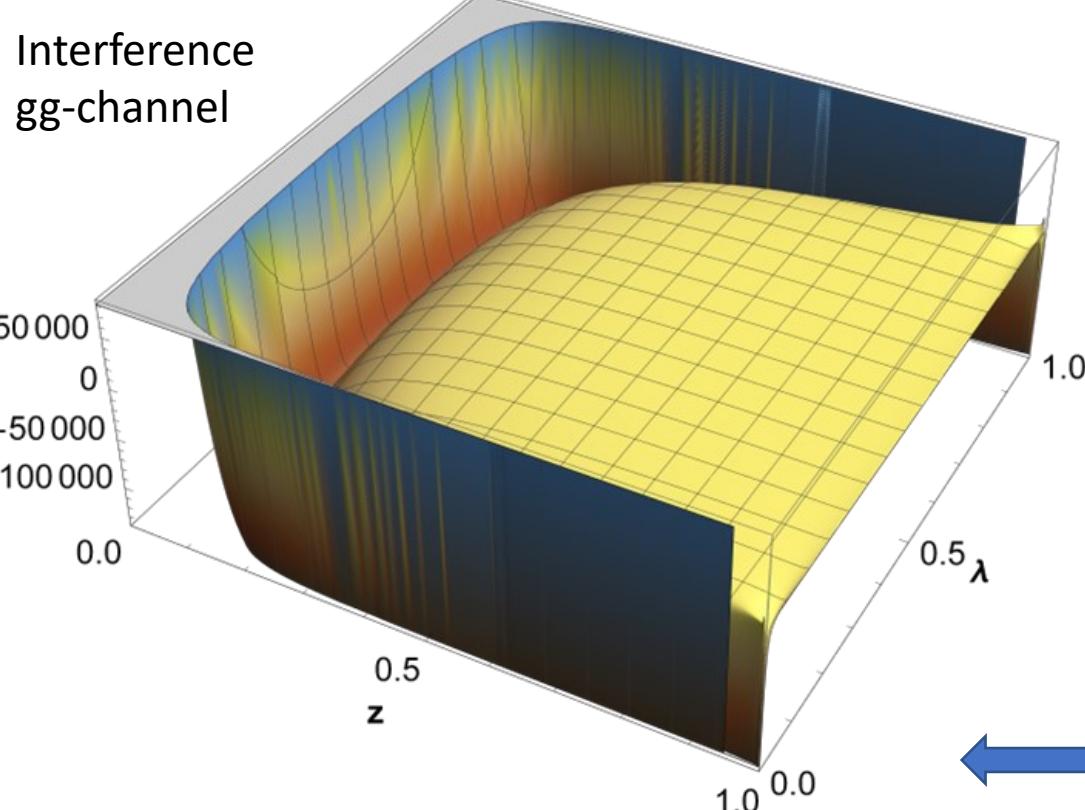
- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Exploit
symmetry of
the problem at
the amplitude
level!

Construction of amplitudes

- Collected 2×10^6 numerical samples for MIs at m_t^2/m_H^2 by evaluation of the LME and numerical evolution above threshold
- Collected 1×10^6 numerical samples for MIs at m_b^2/m_H^2 via numerical evolution in the entire phase space

} Insert into form factors and construct helicity amplitudes

$$\begin{aligned}
 \mathcal{M}_{+++} &= \frac{1}{\sqrt{2}} \frac{s_{12}s_{23}}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \left(F_1 + \frac{s_{13}}{s_{23}} F_2 + \frac{s_{13}}{s_{12}} F_3 + \frac{s_{13}}{2} F_4 \right) \\
 \mathcal{M}_{---} &= \mathcal{M}_{+++} \Big|_{\langle ij \rangle \leftrightarrow [ji]} = -\frac{1}{\sqrt{2}} \frac{s_{12}s_{23}}{[12][23][31]} \left(F_1 + \frac{s_{13}}{s_{23}} F_2 + \frac{s_{13}}{s_{12}} F_3 + \frac{s_{13}}{2} F_4 \right) \\
 \mathcal{M}_{++-} &= \frac{1}{\sqrt{2}} \frac{[12]^3}{[13][23]} \frac{s_{23}}{s_{12}} \left(F_1 + \frac{s_{13}}{2} F_4 \right) & \mathcal{M}_{--+} &= \mathcal{M}_{++-} \Big|_{\langle ij \rangle \leftrightarrow [ji]} \\
 \mathcal{M}_{+-+} &= \frac{1}{\sqrt{2}} \frac{[13]^3}{[12][23]} \frac{s_{12}}{s_{13}} \left(F_2 + \frac{s_{23}}{2} F_4 \right) & \mathcal{M}_{-+-} &= \mathcal{M}_{+-+} \Big|_{\langle ij \rangle \leftrightarrow [ji]} \\
 \mathcal{M}_{-++} &= \frac{1}{\sqrt{2}} \frac{[23]^3}{[12][13]} \frac{s_{13}}{s_{23}} \left(F_3 + \frac{s_{12}}{2} F_4 \right) & \mathcal{M}_{+--} &= \mathcal{M}_{-++} \Big|_{\langle ij \rangle \leftrightarrow [ji]}
 \end{aligned}$$

$$|\mathcal{M}|^2 = \sum_{h \in \text{Hel.}} |\mathcal{M}_h|^2$$



Subtraction of IR divergences for $gg \rightarrow gH$

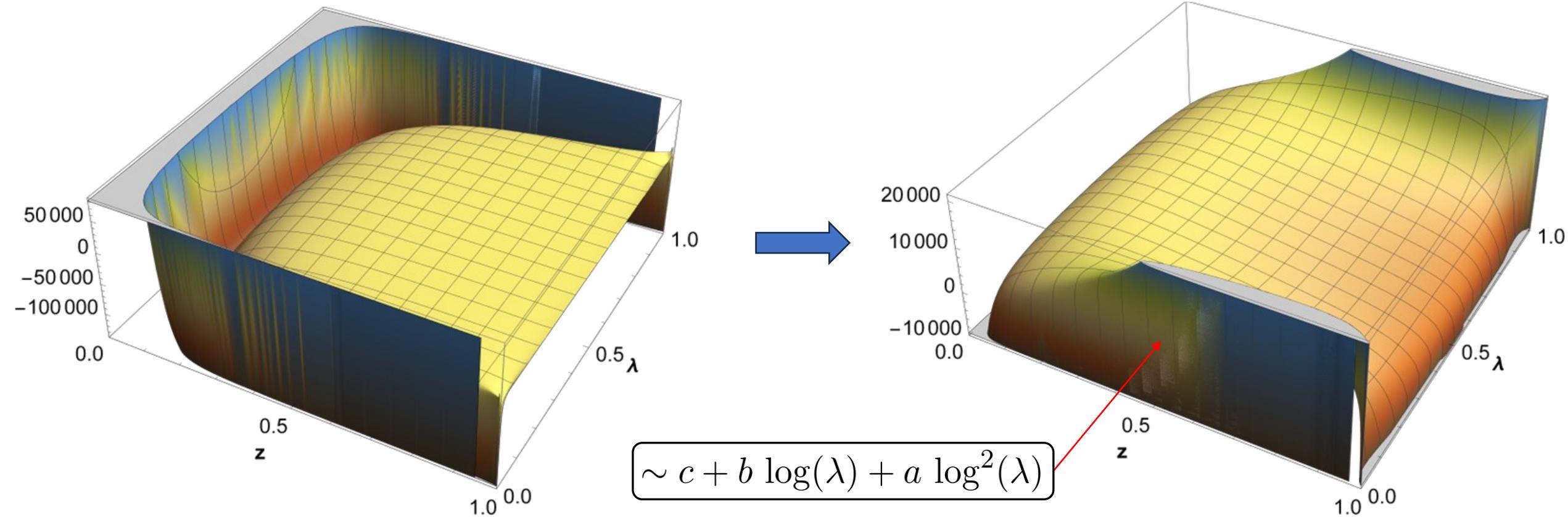
$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$\langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle \Big|_{\text{regulated}} \equiv \langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle - \left[\langle M_{\text{HEFT}}^{(1)} | M_{\text{HEFT}}^{(2)} \rangle + \frac{8\pi\alpha_s}{\hat{t}} \left\langle P_{gg}^{(0)} \left(\frac{\hat{s}}{\hat{s} + \hat{u}} \right) \right\rangle \langle F^{(1)} | (F_{\text{exact}}^{(2)} - F_{\text{HEFT}}^{(2)}) \rangle \right]$$

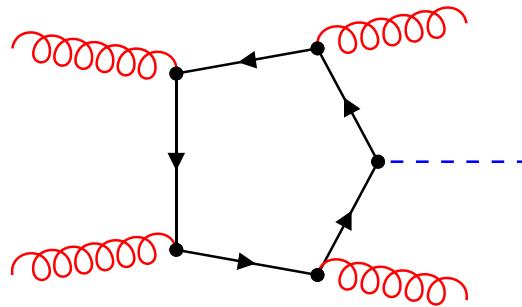
- From unregulated to regulated quantity at $\mu_R = m_H/2$:

$$r = \frac{\sigma_{t \times b}^{\text{LO}}}{\sigma_{\text{HTL}}^{\text{LO}}} \approx -0.129$$



Ingredients for exact quark mass effects at NNLO

- Beyond NLO:

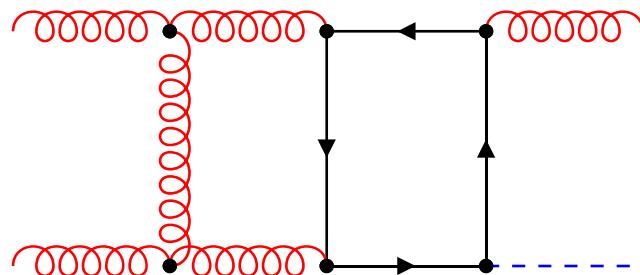


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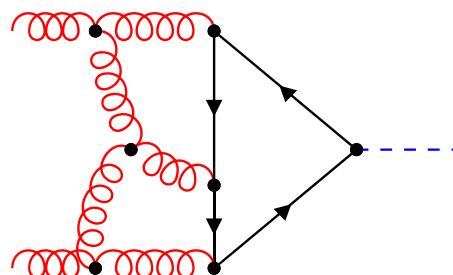
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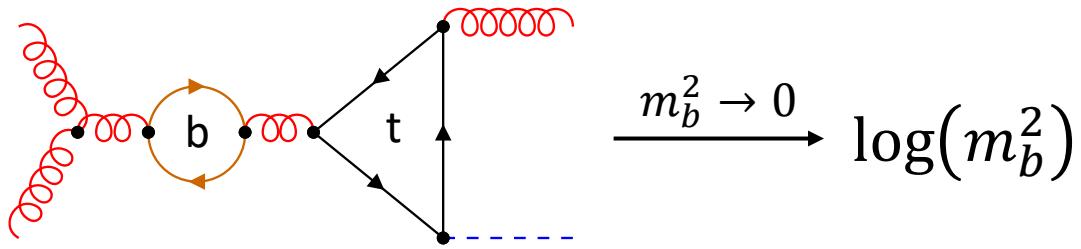
Phase space integration individual contributions performed using **Stripper** framework

Czakon '10

Results

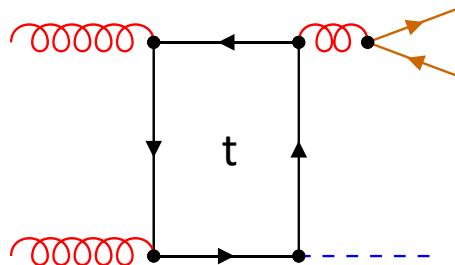
Note on the flavor scheme

- Subsets of diagrams in real-virtual and double virtual contribution give rise logarithmic mass divergences



4-flavor scheme

- Consistent treatment of massive t- and b-quarks
- Exclude b-quark from initial state
- Include massive b-quark splittings in final state



5-flavor scheme

- Treat b-quark as massless particle
- Massive b-quark only present in loops directly attached to the Higgs-boson
- Corresponds to theory with a replica b-quark carrying the mass of a heavy b-quark

Results

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon-fusion at the LHC
 - PDF set: **NNPDF31_nnlo_as_0118** NNPDF Collaboration '17
 - $\mu_R = \mu_F = m_H/2$ (central scale)
 - $m_H = 125$ GeV $\Rightarrow m_t \approx 173.055$ GeV and $m_b \approx 4.779$ GeV (*both* in OS-scheme)
 - HEFT values obtained with **SusHi** Harlander, Liebler, Mantler '16
- Czakon, Eschment, MN, Poncelet,
Schellenberger '23

Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b}/\sigma_{\text{HEFT}}$ [%]
$\sqrt{s} = 8$ TeV				
$\mathcal{O}(\alpha_s^2)$	+7.39	–	-0.895	
LO	$7.39^{+1.98}_{-1.40}$	–	$-0.895^{+0.17}_{-0.24}$	-12
$\mathcal{O}(\alpha_s^3)$	+9.14	-0.0873	-0.268	
NLO	$16.53^{+3.63}_{-2.73}$	$-0.0873^{+0.030}_{-0.052}$	$-1.163^{+0.10}_{-0.08}$	$-7.0^{+1.0}_{-0.8}$
$\mathcal{O}(\alpha_s^4)$	+4.19	$+0.0523(2)$	+0.167(3)	
NNLO	$20.72^{+1.84}_{-2.06}$	$-0.0350(2)^{+0.048}_{-0.013}$	$-0.996(3)^{+0.12}_{-0.05}$	$-4.8^{+0.9}_{-0.8}$
$\sqrt{s} = 13$ TeV				
$\mathcal{O}(\alpha_s^2)$	+16.30	–	-1.975	
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	-12
$\mathcal{O}(\alpha_s^3)$	+21.14	-0.303	-0.446(1)	
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-6.5^{+0.9}_{-0.8}$
$\mathcal{O}(\alpha_s^4)$	+9.72	$+0.147(1)$	+0.434(8)	
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-4.2^{+0.9}_{-0.8}$

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➤ Interference effects much larger than pure top mass effect

Czakon, Eschment, MN, Poncelet,
Schellenberger '23

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- Interference effects much larger than pure top mass effect
- Interference effect at NNLO cancels against NLO

Czakon, Eschment, MN, Poncelet,
Schellenberger '23

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NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-4.2^{+0.9}_{-0.8}$

- Interference effects much larger than pure top mass effect
- Interference effect at NNLO cancels against NLO
- Interference effect at NNLO larger than NLO scale variation (similar in HEFT but less severe)

Czakon, Eschment, MN, Poncelet,
Schellenberger '23

Results

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon-fusion at the LHC
 - PDF set: **NNPDF31_nnlo_as_0118** NNPDF Collaboration '17
 - $\mu_R = \mu_F = m_H/2$ (central scale)
 - $m_H = 125$ GeV $\Rightarrow m_t \approx 173.055$ GeV and $m_b \approx 4.779$ GeV (*both* in OS-scheme)
 - HEFT values obtained with **SusHi** Harlander, Liebler, Mantler '16

Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b}/\sigma_{\text{HEFT}}$ [%]
$\sqrt{s} = 13$ TeV				
$\mathcal{O}(\alpha_s^2)$	+16.30	–	-1.975	
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	-12
$\mathcal{O}(\alpha_s^3)$	+21.14	-0.303	-0.446(1)	
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-6.5^{+0.9}_{-0.8}$
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Czakon, Eschment, MN, Poncelet,
Schellenberger '23

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- Interference NNLO scale variation increases compared to NLO
- Similar effects for different top quark mass ($m_t \approx 170.979$ GeV)

Czakon, Eschment, MN, Poncelet,
Schellenberger '23

Results

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon-fusion at the LHC
 - PDF set: **NNPDF31_nnlo_as_0118** NNPDF Collaboration '17
 - $\mu_R = \mu_F = m_H/2$ (central scale)
 - $m_H = 125$ GeV $\Rightarrow m_t \approx 173.055$ GeV and $m_b \approx 4.779$ GeV (OS-scheme, but Y_b in \overline{MS} with $\bar{m}_b(\bar{m}_b) \approx 4.18$ GeV)
 - HEFT values obtained with **SusHi** Harlander, Liebler, Mantler '16

Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b}(Y_{b,\overline{MS}})$ [pb]
$\sqrt{s} = 13$ TeV				
$\mathcal{O}(\alpha_s^2)$	+16.30	–	-1.975	-1.223
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	$-1.22^{+0.29}_{-0.44}$
$\mathcal{O}(\alpha_s^3)$	+21.14	-0.303	-0.446(1)	-0.623(1)
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-1.85^{+0.26}_{-0.26}$
$\mathcal{O}(\alpha_s^4)$	+9.72	+0.147(1)	+0.434(8)	+0.019(5)
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-1.83(1)^{+0.08}_{-0.03}$

- Interference effects much larger than pure top mass effect
- Interference effect at NNLO cancels against NLO
- Interference effect at NNLO larger than NLO scale variation (similar in HEFT but less severe)
- Interference NNLO scale variation increases compared to NLO
- Similar effects for different top quark mass ($m_t \approx 170.979$ GeV)
- Improved convergence in mixed renormalization scheme compared to OS-scheme

Czakon, Eschment, MN, Poncelet,
Schellenberger '23

Results

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon-fusion at the LHC
 - PDF set: **NNPDF31_nnlo_as_0118** NNPDF Collaboration '17
 - $\mu_R = \mu_F = m_H/2$ (central scale)
 - $m_H = 125$ GeV $\Rightarrow m_t \approx 173.055$ GeV (OS-scheme) and m_b in \overline{MS} -scheme with $\bar{m}_b(\bar{m}_b) \approx 4.18$ GeV
 - HEFT values obtained with **SusHi** Harlander, Liebler, Mantler '16

Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b}(Y_{b, \overline{MS}})$ [pb]	$\sigma_{t \times b}(\bar{m}_b)$ [pb]
$\sqrt{s} = 13$ TeV					
$\mathcal{O}(\alpha_s^2)$	+16.30	–	–1.975	–1.223	–1.118
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	$-1.22^{+0.29}_{-0.44}$	$-1.118^{+0.28}_{-0.43}$
$\mathcal{O}(\alpha_s^3)$	+21.14	–0.303	–0.446(1)	$-0.623(1)$	-0.647
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-1.85^{+0.26}_{-0.26}$	$-1.76^{+0.27}_{-0.28}$
$\mathcal{O}(\alpha_s^4)$	+9.72	+0.147(1)	+0.434(8)	$+0.019(5)$	$+0.02(1)$
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-1.83(1)^{+0.08}_{-0.03}$	$-1.74(2)^{+0.13}_{-0.01}$

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- Interference effect at NNLO cancels against NLO
- Interference effect at NNLO larger than NLO scale variation (similar in HEFT but less severe)
- Interference NNLO scale variation increases compared to NLO
- Similar effects for different top quark mass ($m_t \approx 170.979$ GeV)
- Improved convergence in mixed renormalization scheme compared to OS-scheme
- Similar pattern of corrections for m_b in \overline{MS} -scheme

Czakon, Eschment, MN, Poncelet,
Schellenberger '23 (preliminary)

Summary and outlook

- The top-bottom interference contribution to the total Higgs production cross section was computed with *both* quarks renormalized in the OS-scheme
 - $\mathcal{O}(\alpha_s^4)$ correction at 8 TeV: +0.167 pb
 - $\mathcal{O}(\alpha_s^4)$ correction at 13 TeV: +0.434 pb
- NNLO correction at 13 TeV: $-1.99(1)^{+0.30}_{-0.15}$ pb compatible with previous estimate $-2.18^{+0.20}_{-0.20}$ pb [Anastasiou, Penin '20](#)
- Top-quark and interference contribution not sensitive to small variations of the top-quark mass
- Interference shows signs of poor perturbative convergence
 - Better convergence in \overline{MS} -scheme for the bottom-quark mass or Yukawa coupling only
- Cross checks: at the differential level
 - [Jones, Kerner, Luisoni '18](#)
 - [Caola, Lindert, Melnikov, et al. '18](#)
- Next steps:
 - Complete calculation with quark masses renormalized in \overline{MS} -scheme (compare e.g. [Bonciani, Del Duca, Frellesvig, et al. '22](#))
 - Consistent treatment of massive quarks in 4-flavor scheme
 - Top-charm interference contribution