The Hierarchy Problem and what Neutrinos have to do with it

Philipp Eller, **Manuel Ettengruber**, Alan Zander

Ludwig-Maximilians-Universität München

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1. Introduction
2. Neutrino Oscillations with many Copies of the SM
3. Testing the Model with Neutrino Experiments
4. Conclusion

Introduction: Neutrino Oscillations

For demonstration let us assume a quantum mechanical 2-level system with the stationary states $|\Psi_1\rangle$ and $|\Psi_2\rangle$. The time-evolved states are:

$$|\Psi_i(t)\rangle = e^{-iE_i t}\,|\Psi_i\rangle\,. \tag{1}$$

With the initial state:

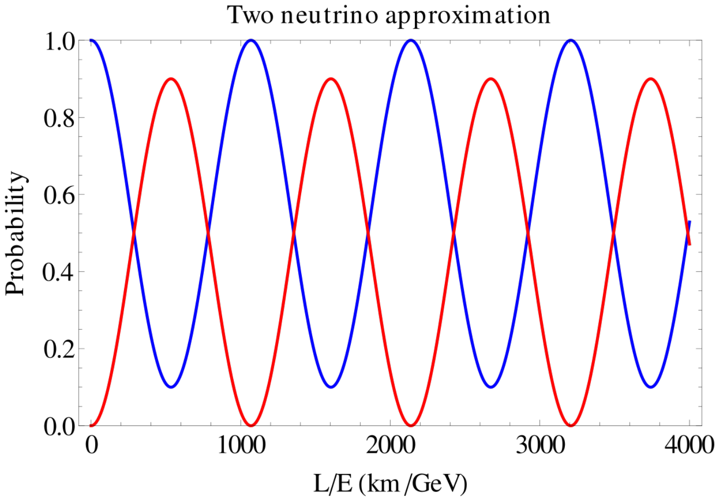
$$|\Psi(0)\rangle = a\,|\Psi_1\rangle + b\,|\Psi_2\rangle\,, \tag{2}$$

We get for the time evolved state:

$$|\Psi(t)\rangle = ae^{-iE_i t}\,|\Psi_1\rangle + be^{-iE_2 t}\,|\Psi_2\rangle\,. \tag{3}$$

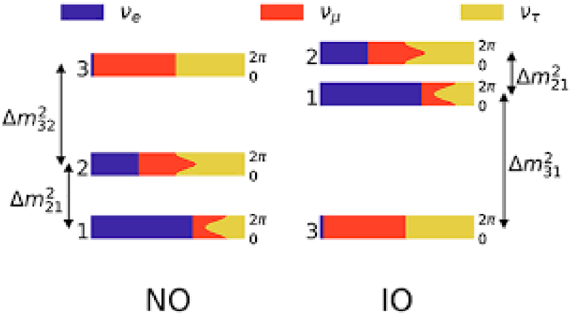
$$\begin{aligned} P_{surv} &= |\,\langle\Psi(0)|\,|\,|\Psi(t)\rangle\,|^2 = ||a|^2 e^{-iE_1 t}\,|\Psi_1\rangle + |b|^2 e^{-iE_2 t}\,|\Psi_2\rangle\,|^2 \\ &= 1 - 4|a|^2|b|^2\sin^2[(E_2 - E_1)t/2]. \end{aligned} \tag{4}$$

Introduction: Neutrino Oscillations

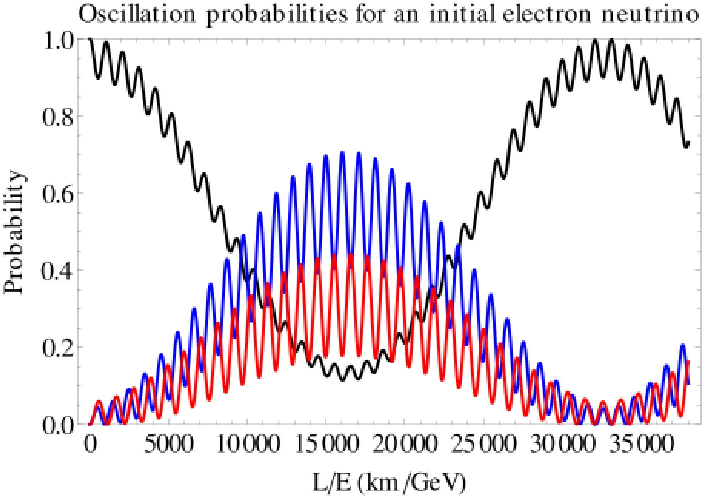


Introduction: Neutrino Oscillations

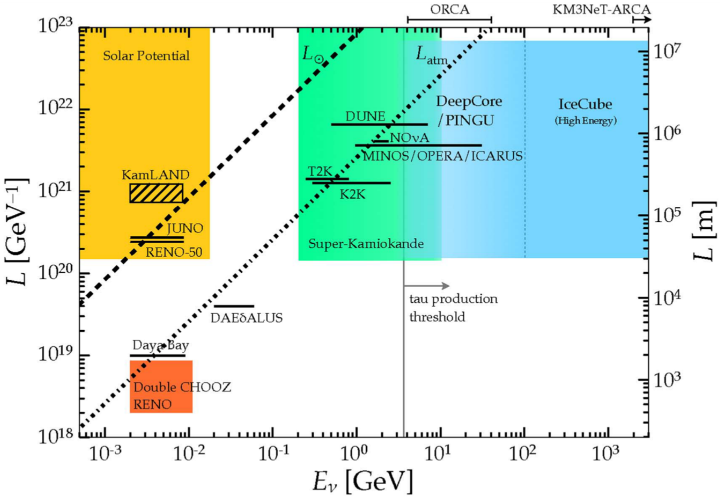
$$|\nu_e\rangle = U_{e1}|m_1\rangle + U_{e2}|m_2\rangle + U_{e3}|m_3\rangle \tag{5}$$



Introduction: Neutrino Oscillations

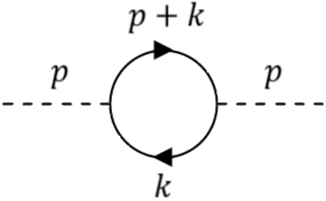


Introduction: Neutrino Oscillations



Introduction: The Hierarchy Problem

$$\mathcal{L} = -\frac{1}{2}\phi(\Box + m^2)\phi + \lambda\phi\bar{\psi}\psi + \bar{\psi}(i\partial - M)\psi, \tag{6}$$



- The evaluation of this diagram looks like the following:

$$i\Sigma_2(p) = (i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{Tr[(p+k+M)(k+M)]}{[(p+k)^2 - M^2 + i\epsilon][k^2 - M^2 + i\epsilon]}. \tag{7}$$

Introduction: The Hierarchy Problem

After regularization this gets:

$$\Sigma_2(p^2) = \frac{3\lambda^2}{4\pi^2}\int_0^1 dx \left([M^2 - p^2 x(1-x)] \ln \frac{M^2 - p^2 x(1-x)}{\Lambda^2} + \Lambda^2 \right), \tag{8}$$

With renormalization the final expression is:

$$\Sigma_2(p^2) = \frac{\lambda^2}{4\pi^2}\left[\frac{(p^2 - m^2)^2}{20M^2} + \mathcal{O}\left(\frac{m^6}{M^4}\right) \right]. \tag{9}$$

There is no Hierarchy Problem in the SM!

Introduction: The Hierarchy Problem

What if the SM is not the final theory of nature? Then there exists a scale of new physics Λ and the renormalization scheme is not working anymore. There is just one known fundamental interaction namely gravity which we expect to live at $\Lambda = M_P$. Then

$$m_P^2 = m^2 - \Sigma(m^2) \approx m^2 - M_P^2. \tag{10}$$

The pole mass of the Higgs-boson is $m_P = 125\mathrm{GeV}$. Then the expression of the bare mass in the Lagrangian is

$$m^2 = (1 + 10^{-34})\Lambda^2. \tag{11}$$

Fine-Tuning + UV-sensitivity = Hierarchy Problem.

Introduction: Many Species Theory

- Motivation: Solution to the Hierarchy Problem, Dark Matter and Neutrino masses
- Through postulating many additional dark sectors one can lower the fundamental scale of gravity M_* via [*Dvali 2007*]:

$$M_* = \frac{M_P}{\sqrt{N}}. \tag{12}$$

  From this equation one can give an upper bound on the number of dark species $N < 10^{32}$.
- The minimal model is that the additional species are charged under dark SM gauge groups.

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Neutrino masses in many species Theories

- In many species theories the neutrino masses get generated by introducing many light states (Infrared solution) instead of few heavy states (UV solution). [*Dvali, Redi 2008*]
- The typical expression for flavor states in such theories looks like [*M.E. 2022*]:

$$|\nu_e\rangle = \sqrt{\frac{N-1}{N}}\left(U_{e1}|m_1\rangle + U_{e2}|m_2\rangle + U_{e3}|m_3\rangle\right) + \frac{1}{\sqrt{N}}\left(U_{e1}|m_1^H\rangle + U_{e2}|m_2^H\rangle + U_{e3}|m_3^H\rangle\right). \quad (13)$$

The masses $m_{1\ldots3}$ are the usual masses of SM neutrinos and the masses $m^H_{1\ldots3}$ are with them related via $m_i^H = \mu m_i$. The massfactor μ can range from 1 to 100 depending on the exact geometry in the species space.

Neutrino Oscillations in many species Theories

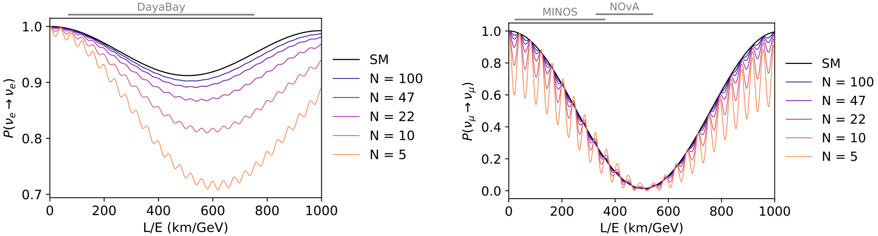
- The two parameters of interest are the number of active species N and the massfactor μ that determines the mass splitting.
- The resulting survival probability of such a composition of the neutrino flavor eigenstate can be calculated via:

$$P(\nu_\mu \to \nu_\mu) = \left(\frac{N-1}{N}\right)^2 \sum_{i=1}^{3}\sum_{j=1}^{3} |U_{\mu i}|^2 |U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)L}{2E}} +$$

$$\frac{N-1}{N^2}\sum_{i=1}^{3}\sum_{j=4}^{6} |U_{\mu i}|^2 |U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)L}{2E}} + \frac{N-1}{N^2}\sum_{i=4}^{6}\sum_{j=1}^{3} |U_{\mu i}|^2 |U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)L}{2E}}$$

$$+ \frac{1}{N^2}\sum_{i=4}^{6}\sum_{j=4}^{6} |U_{\mu i}|^2 |U_{\mu j}|^2 e^{\frac{i(m_i^2-m_j^2)L}{2E}}. \quad (14)$$

Neutrino Oscillations in many species Theories



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The parameters of interest

- The attempt is to make a combined neutrino fit with several different neutrino oscillation experiments to give a first bound on the parameters N and μ.
- Different type of neutrino experiments (accelerator, reactor, atmospheric,...) can probe different scopes of the masssplitting.
- Attempt of a global Neutrino Fit with the free parameters: θ_{12}, θ_{13}, θ_{23}, Δm^2_{12}, Δm^2_{13}, δ_{CP},m_{lightest}, N, μ.

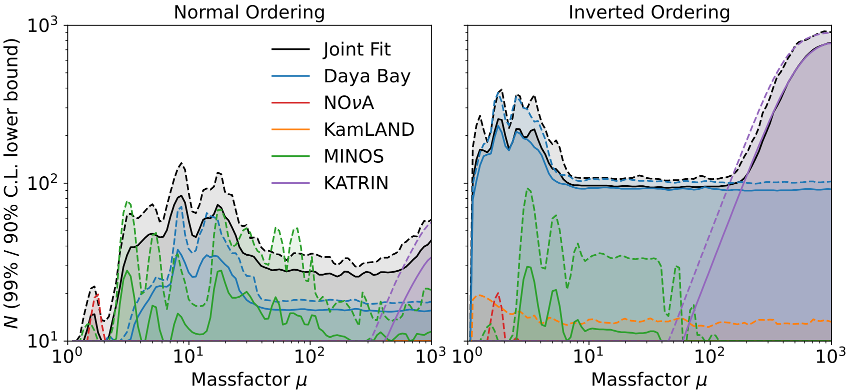
Data Analysis

We combine public data of different neutrino experiments

$$\mathcal{L}_{\mathrm{comb}} = \mathcal{L}_{\mathrm{KATRIN}} \times \mathcal{L}_{\mathrm{MINOS}} \times \mathcal{L}_{\mathrm{KamLAND}} \times \mathcal{L}_{\mathrm{DayaBay}} \times \mathcal{L}_{\mathrm{NO}\nu\mathrm{A}}. \tag{15}$$

We have performed a frequentist analysis by using a likelihood ratio test statistic.

Results


Normal Ordering
Inverted Ordering
Joint Fit
Daya Bay
NOνA
KamLAND
MINOS
KATRIN
N (99% / 90% C.L. lower bound)
10^1
10^2
10^3
10^0
10^1
10^2
10^3
Massfactor μ
Massfactor μ

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Conclusion

- Neutrino experiments are suitable candidates to test the number of active species and the mass splitting with the additional neutrino states.
- The first combined fit of several neutrino experiments are able to give a lower bound up to $N \geq 200$ (depending on μ) for IO and $N \geq 30$ for NO.