Light Quark Mediated Higgs + Jet Production at NNLO and beyond

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Topics discussed

- Introduction/warmup
  - *perturbative vs power corrections*
  - *light quark mediated* $gg \to H$ *through NLL*
- Light quark mediated Higgs+Jet production
  - *factorization structure of* $gg \to Hg$
  - *all-order amplitudes at LL*
  - $pp \to H + j + X$ *cross section*

Based on

K. Melnikov, A.A. Penin, *JHEP* ***05****, 172 (2016)*

T. Liu, A.A. Penin, *Phys.Rev.Lett.* ***119****, 262001 (2017)*

C. Anastasiou, A.A. Penin, *JHEP* ***07****, 195 (2020)*

T. Liu, A.A. Penin, A. Rechman, *JHEP* ***04****, 031 (2024)*

The advent of power corrections

- Power *vs* perturbative corrections
  - $\boxed{\Lambda_X^2/Q^2 \quad \text{vs} \quad \alpha_X^n}$
  - *e.g.* $\Lambda_X = m_b, \; Q = m_H, \; \alpha_X = \alpha_s$ ⇨ $n = 3$
- Logarithmically enhanced power corrections
  - *phenomenologically relevant*
  - *intriguing from QFT point of view*
  - ➥ *weird RG structure, magic relations, etc.*
- Recent stream of the NLP results for
  - *mass, angle, soft momentum, threshold, jetiness, ...*

Higgs production at the LHC

- Total cross section at 13 GeV $\boxed{\sigma_{pp\to H+X} = 48.68\ \textit{pb}}$

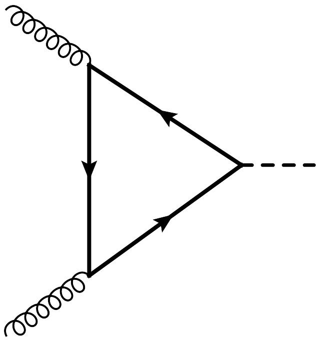
- Dominant theory uncertainties

  - *scale choice* $\begin{matrix}+0.10\\-1.15\end{matrix}$ *pb*
  - *PDF N^{3}LO* ± 0.56 *pb*
  - $m_t < \infty$ *NNLO* ± 0.49 *pb*
  - $m_b > 0$ *NNLO+* ± 0.40 *pb*

Anastasiou et al. JHEP **1605**, 058 (2016)

Bottom quark mass effect

- Leading contribution



$$\propto \alpha_s \ln^2(m_H^2/m_b^2)\,\frac{m_b^2}{m_H^2}$$

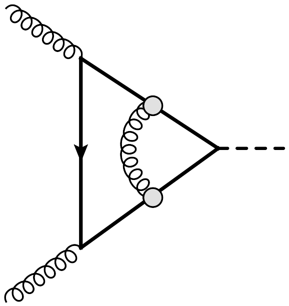
➥ *large logs at subleading power*

- *effective expansion parameter* $\boxed{\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s}$

➥ *resummation is mandatory*

$gg \to H$ amplitude at LL

- Non-Sudakov logs T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



- Factorization formula $\boxed{\mathcal{M}^{b}_{gg\to H} = Z_g^{2\,LL}\, g(z)\, \mathcal{M}^{b\,(0)}_{gg\to H}}$
  - *gluon Sudakov factor* $Z_g^{2\,LL} = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{Q^{2\varepsilon}}\right]$
  - *non-Sudakov double logarithms*

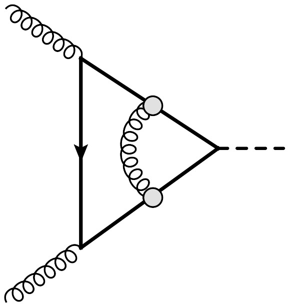
$$\boxed{g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta\, e^{2z\eta\xi} = {}_2F_2\,(1,1;3/2,2;z/2)}$$

  - *double-log variable* $z = \ (C_A - C_F)\ x,\ \ x = \frac{\alpha_s}{4\pi}L^2,\ \ L = \ln(m_H^2/m_q^2)$

eikonal color nonconservation

$gg \to H$ amplitude at LL

- Non-Sudakov logs T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



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➥ *Magic #1: same function for NLP QED scattering in Regge limit*

$gg \to H$ amplitude at NLL

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \underbrace{\left(\frac{\alpha_s(m_H)}{\alpha_s(m_q)}\right)^{\gamma_m^{(1)}/\beta_0}}_{\text{Yukawa RG factor}} \underbrace{Z_g^{2\,NLL}}_{\text{gluon Sudakov form factor}} \underbrace{\left[-\frac{3}{2}\frac{m_q^2}{m_H^2}L^2\,\mathcal{M}^{t\,(0)}_{gg\to H}\right]}_{\text{LO amplitude}}$$

$$C_q = \left[g(z) + \frac{\alpha_s L}{4\pi}\left(2\gamma_q^{(1)} g_\gamma(z) - \beta_0 g_\beta(z)\right)\right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$c_1 = \frac{z}{6} + C_F\frac{\alpha_s L}{4\pi}\,, \qquad c_2 = \frac{z^2}{45} + \frac{z}{5}\frac{\alpha_s L}{4\pi}\left[\frac{3}{2}C_F - \beta_0\left(\frac{5}{6}\frac{L_\mu}{L} - \frac{1}{3}\right)\right],$$

$$c_3 = \frac{z^3}{420} + \frac{z^2}{5}\frac{\alpha_s L}{4\pi}\left[\frac{5}{21}C_F - \beta_0\left(\frac{2}{9}\frac{L_\mu}{L} - \frac{2}{21}\right)\right], \quad \ldots$$

$$L = \ln(m_H^2/m_q^2),\; L_\mu = \ln(m_H^2/\mu^2)$$

- *main NLL effects:*

$$\alpha_s(\mu) \Rightarrow \alpha_s\!\left(m_H\left(\frac{m_q}{m_H}\right)^{2/5}\right)$$

$$m_q^2(\mu) \Rightarrow m_q(m_q)\,m_q(m_H)$$

Higgs production

- Top-bottom interference through NNLO
  - *NLL threshold*

$$\boxed{\delta\sigma^{\rm NNLO}_{pp\to H+X} = -2.18 \pm 0.20 \; pb}$$

C. Anastasiou, A.A. Penin, JHEP ***07****, 195 (2020)*

  - *full result*

$$\boxed{\delta\sigma^{\rm NNLO}_{pp\to H+X} = -1.99^{+0.30}_{-0.15} \; pb}$$

M. Czakon, F. Eschment, M. Niggetiedt, R. Poncelet, T. Schellenberger, JHEP ***07****, 195 (2020)*

- Convergence of the logarithmic expansion

	LO	NLO	NNLO	N^{3}LO
$\delta\sigma^{\rm LL}_{pp\to H+X}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma^{\rm NLL}_{pp\to H+X}$	-1.420	-2.048	-2.183	-2.204
$\delta\sigma_{pp\to H+X}$	-1.023	-2.000	-1.990	

Higgs production

- Top-bottom interference through NNLO
  - *NLL threshold*

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    *C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)*
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    *M. Czakon, F. Eschment, M. Niggetiedt, R. Poncelet, T. Schellenberger, JHEP **07**, 195 (2020)*

- *Why does threshold approximation work so good?*
  - *in principle it should not since the soft emission with $p_\perp \sim m_b$ is not suppressed by PDFs and resolves the bottom loop*

  ➥ *the reason is revealed in the rest of the talk*

Higgs + Jet production

- Top-bottom interference to NLO

  J. M. Lindert, K. Melnikov, L. Tancredi and C. Wever, Phys. Rev. Lett. 118, 252002 (2017)

  R. Bonciani, V. Del Duca, H. Frellesvig, M. Hidding, V. Hirschi, F. Moriello, G. Salvatori, G. Somogyi and F. Tramontano, Phys. Lett. B 843, 137995 (2023)

  X. Chen, A. Huss, S. P. Jones, M. Kerner, J. N. Lang, J. M. Lindert and H. Zhang, JHEP 03, 096 (2022)

  Higher orders?

  Factorization, log structure?

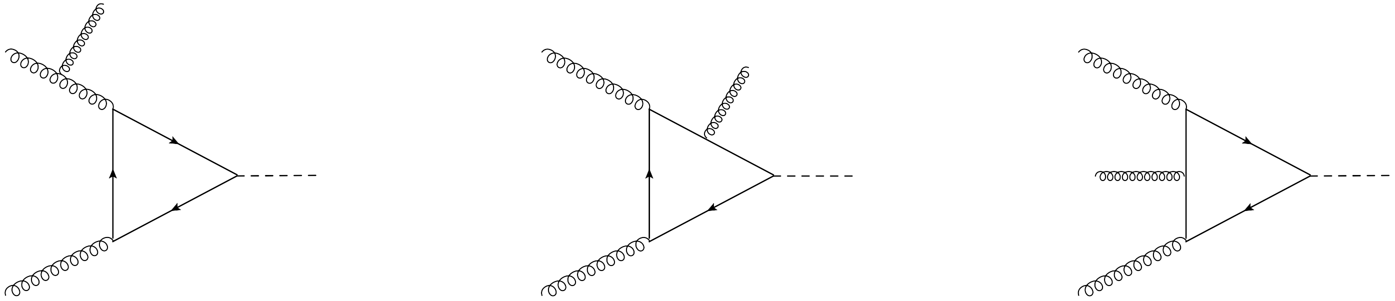
- Brute force calculation of Abelian double logs

  K. Melnikov, A.A. Penin, JHEP ***05****, 172 (2016)*

➥ *full QCD?*

Higgs + Jet production

- Kinematics of $g(p_1) + g(p_2) \to g(p_3) + H(p_H)$
  - $m_q^2 \ll p_\perp^2 \ll s, m_H^2$
  - $s \approx m_H^2$
- One-loop amplitudes



  - *single color structure*
  - *two helicity structures*

Higgs + Jet production

- Amplitudes $M_{++\pm} \propto e^{\frac{\alpha_s}{2\pi} I} \sum_q A^{(q)}_{++\pm}$

- Form factors
  - *large mass expansion* $A^{(t)}_{++\pm} = C_t \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n \tilde{A}^{(n)}_{++\pm} + \mathcal{O}(m_t^{-2})$
  - *small mass expansion* $A^{(q)}_{++\pm} = \frac{m_q^2}{s} \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n A^{(n+1)}_{++\pm} + \mathcal{O}(m_q^4)$

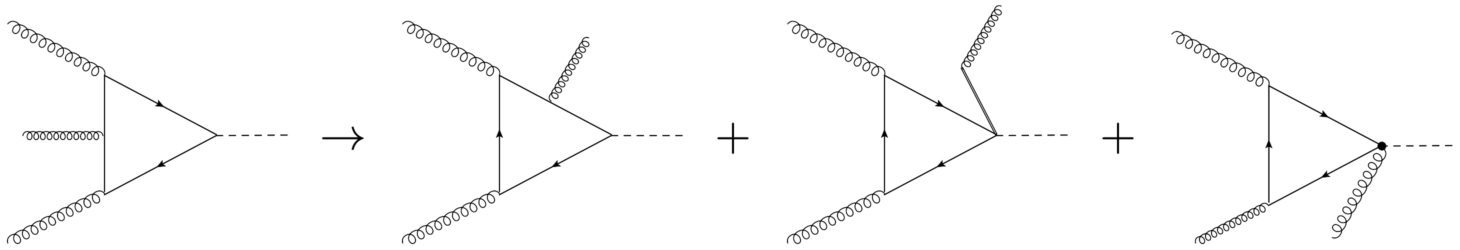
- One-loop Sudakov exponent
  - *physical* $I^{(1)}_{\rm ph} = -\frac{C_A}{2\epsilon^2}\left[2\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-tu}{s\mu^2}\right)^{-\epsilon}\right]$

➥ *accommodate all the double logs of* $p_\perp$ *and rapidity in* $m_q \to \infty$ *limit*

  - *symmetric* $I^{(1)}_{\rm sym} = -\frac{C_A}{2\epsilon^2}\left[\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-t}{\mu^2}\right)^{-\epsilon} + \left(\frac{-u}{\mu^2}\right)^{-\epsilon}\right]$

Factorization

- Jet emission off the soft quark line

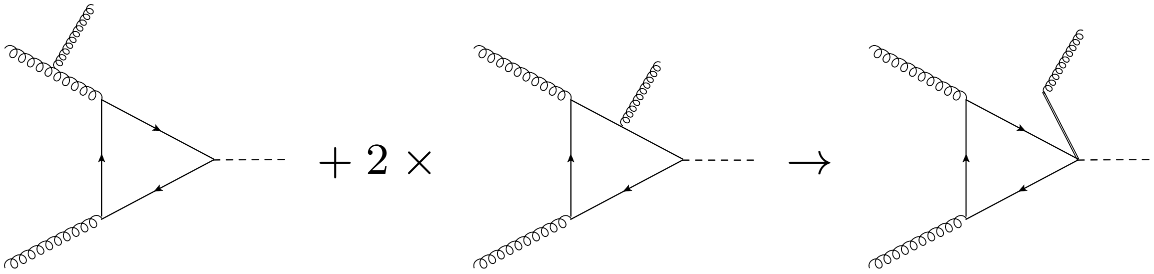


soft dipole

symmetric

  - *soft dipole with* $l_\perp^2 < p_\perp^2$
  - *new symmetric structure* $G^a_{\mu\nu}G^b_{\nu\lambda}G^c_{\lambda\mu}f^{abc}$ *with* $l_\perp^2 < p_\perp^2$

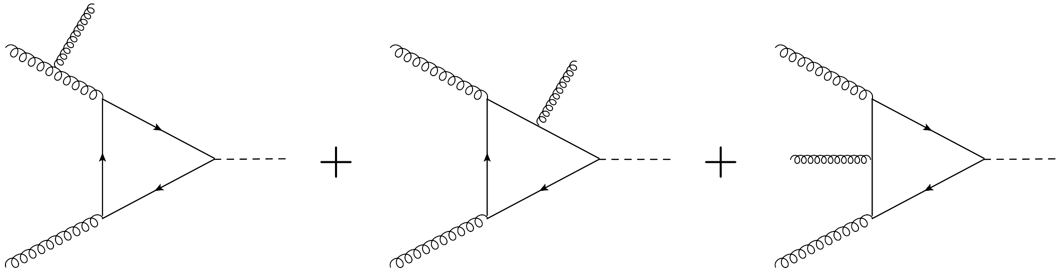
- Jet emission off the eikonal line



eikonal dipole

Factorization

- Effective theory decomposition



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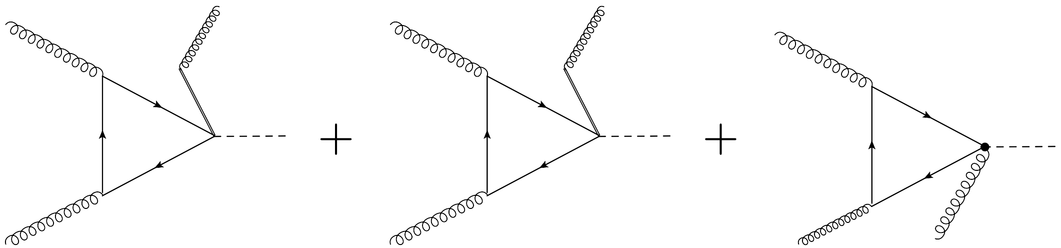


eikonal dipole *soft dipole* *symmetric*

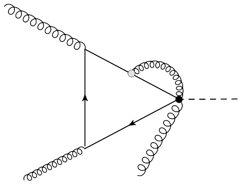
Effective theory decomposition

- Eikonal dipole
  - *standard dipole structure with* $A_{+++} = -A_{++-}$
  - *jet completely factors out from the quark loop*
- Soft dipole
  - *standard dipole structure with* $A_{+++} = -A_{++-}$
  - *quark loop momentum cutoff* $l_\perp^2 < p_\perp^2$
- Symmetric
  - *symmetric tensor structure with* $A_{+++} = 0$
  - *quark loop momentum cutoff* $l_\perp^2 < p_\perp^2$

Double logs

- Soft emission from the factorized gluon line
  - *color is conserved along gluon line*
  - ➥ *only Sudakov double logs*

- Problem reduces to
  - *eikonal dipole:* $g(p_1)g(p_2)H$ *form factor*
  - *soft dipole:* $g(p_1)g(p_2)H$ *form factor with* $p_\perp$*-dependent UV cutoff*
  - *symmetric:*
    - *unresolved vertex:* $g(p_3)g(p_{1,2})H$ *form factor with* $p_\perp$*-dependent UV cutoff*
    - *resolved vertex: additional Sudakov-Wilson contribution*
    - ➥ *soft jet factors out*



Double logs

- Eikonal dipole

$$\left[A^{\rm LL}_{++\pm}\right]_{\rm e.d.}=\pm 2L^2\int_0^1{\rm d}\eta\int_0^{1-\eta}{\rm d}\xi\, e^{2z\eta\xi}=\pm L^2 g(z)$$

- Soft dipole

$$\left[A^{\rm LL}_{++\pm}\right]_{\rm s.d.}=\mp L^2\int_{\tau_t-\tau}^{\tau_t}{\rm d}\eta\int_{1-\tau_t}^{1-\eta}{\rm d}\xi\, e^{2z\eta\xi}$$

  - *transverse momentum variable* $\tau=\ln(p_\perp^2/m_q^2)/L$

- Symmetric

$$\left[A^{\rm LL}_{++-}\right]_{\rm sym.}=-L^2\left[\int_{\tau_t-\tau}^{\tau_t}{\rm d}\eta\int_{1-\tau_t}^{1-\eta}{\rm d}\xi\, e^{2z(\eta-\tau_t+\tau)(\xi-1+\tau_t)}e^{2z(\tau_t-\tau)\xi}+(\tau_t\to\tau_u)\right]$$

Sudakov-Wilson factor

All-order amplitudes

- Leading logarithmic result

$$A^{LL}_{+++} = L^2\left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta))}\right)\right]$$

$$A^{LL}_{++-} = -A^{LL}_{+++} - L^2\left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\, e^{z((1-\tau)^2-\zeta^2)/2}\left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1\right) + (\zeta \to -\zeta)\right]$$

  - *rapidity variable* $\zeta = \ln(t/u)/L$

- Two loops
  - *after infrared matching agrees with the explicit calculation*

K. Melnikov, L. Tancredi, C. Wever, JHEP 1611, 104 (2016)

All-order amplitudes

- ## Leading logarithmic result

$$A^{LL}_{+++} = L^2\left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta)}\right)\right]$$

$$A^{LL}_{++-} = -A^{LL}_{+++} - L^2\left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\, e^{z((1-\tau)^2-\zeta^2)/2}\left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1\right) + (\zeta \to -\zeta)\right]$$

  - *rapidity variable* $\zeta = \ln(t/u)/L$

- ## Boundary values

  - *low transverse momentum* $p_\perp \sim m_q$*:* $A^{LL}_{++\pm} = \pm L^2 g(z)$
  - *high transverse momentum, central rapidity* $p_\perp \sim m_H\,,\;\; t \sim u$*:*

$$A^{LL}_{+++} = \frac{1}{2}L^2 g(z), \quad A^{LL}_{++-} = -\frac{3}{2}L^2 g(z)$$

Higgs + Jet production

- Partonic cross section near threshold

$$d\sigma^{tb}_{gg\to Hg+X} = -\frac{3m_q^2}{m_H^2}L^2 C_t\, C_q(\tau,\zeta)\, d\tilde{\sigma}^{\rm eff}_{gg\to Hg+X}$$

heavy top effective theory cross section

  - *light quark coefficient*

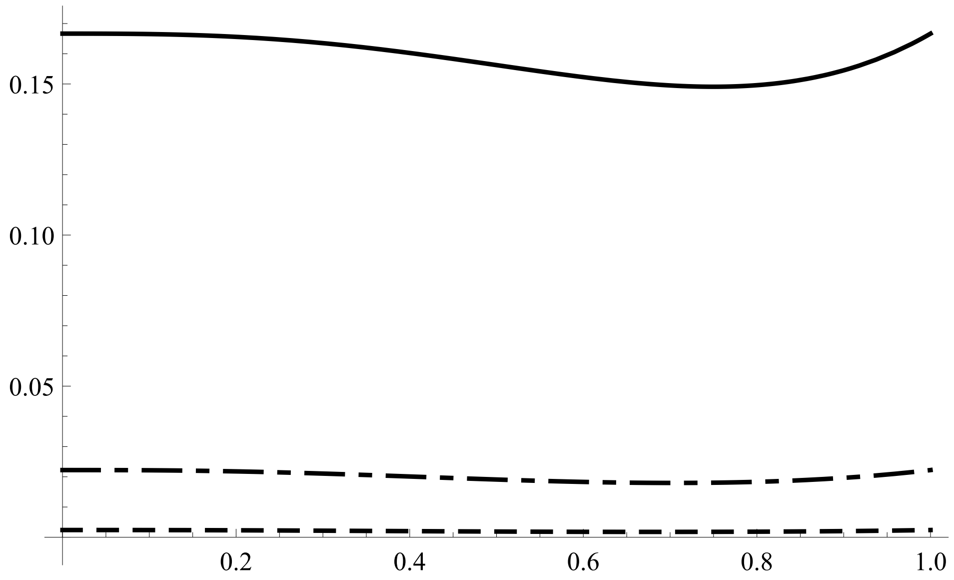
$$C_q(\tau,\zeta) = \frac{A_{+++}-A_{++-}}{2L^2} = 1 + \frac{z}{6}\left(1-\tau^3+\tau^4\right)$$

$$+z^2\left[\frac{1}{45} - \frac{\tau^3}{12} + \frac{\tau^4}{6} - \frac{7\tau^5}{60} + \frac{\tau^6}{30} + \frac{\zeta^2}{12}(\tau^3-\tau^4)\right] + \ldots$$

  - *Magic # 2:* $\boxed{C_q(\tau,\zeta) \approx C_q(0,0) = C_q}$

Jet factorization

- dependence of the $z^{1,2,3}$ coefficients on $m_q < p_\perp < m_H$



➥ a jet with $m_q < p_\perp < m_H$ factors out as a soft jet with $p_\perp \ll m_q$

Higgs + Jet production

- Bottom quark contribution to hadronic cross section

$$d\sigma^{tb}_{pp\to Hj+X} = \left[\frac{C_b}{C_t}\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}\right]\left(\frac{d\sigma^{tb}_{pp\to Hj+X}}{d\sigma^{tt}_{pp\to Hj+X}}\right)^{\rm LO} d\sigma^{tt}_{pp\to Hj+X}\,.$$

  - *numerically observed in NLO*

    J. M. Lindert, K. Melnikov, L. Tancredi and C. Wever, Phys. Rev. Lett. 118, 252002 (2017)

  - *explains the precision of the NLO total threshold cross section*

    C. Anastasiou, A.A. Penin, JHEP ***07****, 195 (2020)*

  - C_b *is known to NLL*

Higgs + Jet production

Bottom quark contribution to hadronic cross section

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- $d\sigma^{tt}_{pp\to Hj+X}$ *is known to NNLO*

  X. Chen, T. Gehrmann, E. W. N. Glover and M. Jaquier, Phys. Lett. B 740, 147 (2015)

  R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze, Phys.Rev.Lett. 115, 082003 (2015)

  R. Boughezal, C. Focke, W. Giele, X. Liu and F. Petriello, Phys. Lett. B 748, 5 (2015)

  F. Caola, K. Melnikov and M. Schulze, Phys. Rev. D 92, 074032 (2015)

  X. Chen, J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover and M. Jaquier, JHEP 10, 066 (2016)

➥ *get top-bottom interference through NNLO*

Higgs + Jet production

- Bottom quark contribution to hadronic cross section

$$d\sigma^{tb}_{pp\to Hj+X} = \left[\frac{C_b}{C_t}\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}\right]\left(\frac{d\sigma^{tb}_{pp\to Hj+X}}{d\sigma^{tt}_{pp\to Hj+X}}\right)^{\rm LO} d\sigma^{tt}_{pp\to Hj+X}\,.$$

  - *higher multiplicities* ⇨ *softer emissions* ⇨ *better factorization*

↪ *complete the existing* $p_\perp$*-resummations*

A. Banfi, P. F. Monni and G. Zanderighi, JHEP 01, 097 (2014)

F. Caola, J. M. Lindert, K. Melnikov, P. F. Monni, L. Tancredi and C. Wever, JHEP 09, 035 (2018)

Summary

Jets with $p_\perp \ll m_H$ *decouple from the light quark loop*

- *explains some mystique phenomena*
- *provides access to higher orders*