Double parton scattering in QCD

Jonathan Gaunt

TUM/MPP Seminar, 10/07/24

OUTLINE

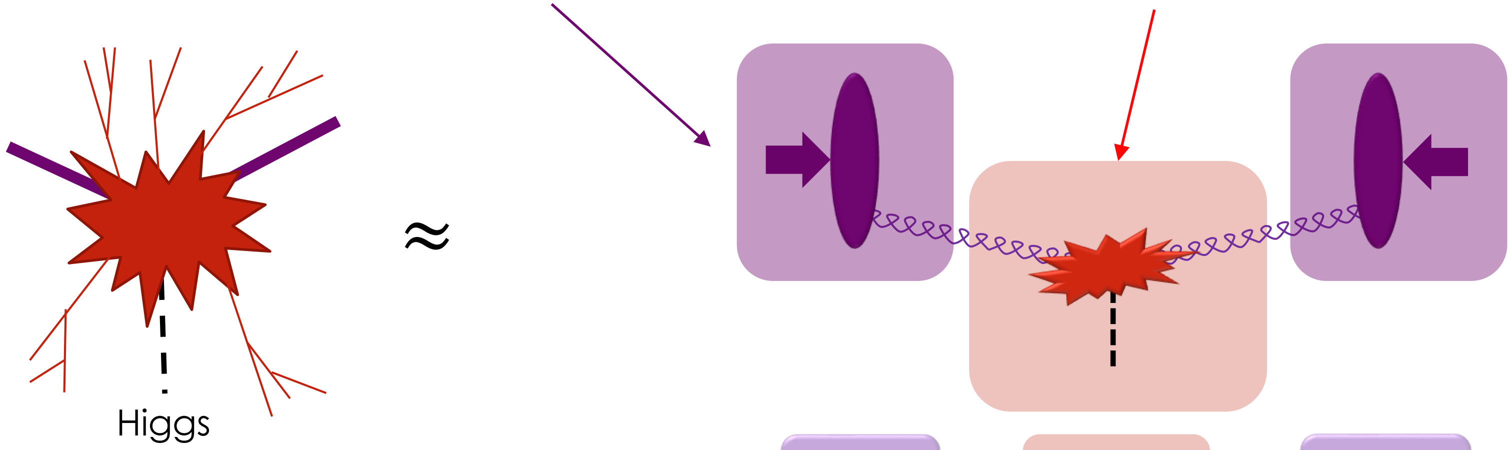
- What is double parton scattering (DPS)?
- Why double scattering is important and interesting, with reference to specific processes and experimental measurements.
- Crudest phenomenogical approach to DPS: ‘the pocket formula’. Extension of the pocket formula to arbitrarily many scatters: ‘eikonal model for multiple scattering’. Some basic improvements on this model.
- Full pQCD framework for DPS, including perturbative correlations. Parton shower implementation of this approach. Effects on DPS cross sections from perturbative and other correlations.

LHC FACTORISATION FORMULA

Standard framework for computing $pp \rightarrow$ some hard final state, say a Higgs boson, assumes this is produced via a single parton-parton collision (SPS):

Long distance proton structure: gluon in proton

Short distance scattering:
gluon + gluon → Higgs

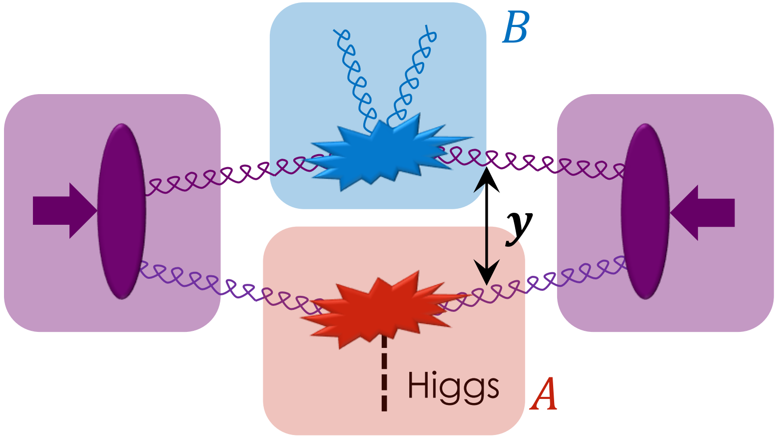


$$\sigma_{pp\rightarrow H} = f_g(x) \otimes \hat{\sigma}_{gg\rightarrow H} \otimes f_g(x')$$

Parton density (PDF)

DOUBLE PARTON SCATTERING

But proton is composite! If the final state can be divided into two hard subsets A & B, this can also be produced via double parton scattering (DPS):



From parton model analysis (no QCD radiation):

$$\sigma_{DPS}^{(A,B)} = \int \underbrace{F_{ik}(x_1, x_2, \boldsymbol{y})} \otimes \hat{\sigma}_{ij}^{A} \hat{\sigma}_{kl}^{B} \otimes \underbrace{F_{jl}(x'_1, x'_2, \boldsymbol{y})} \, d^2\boldsymbol{y}$$

Double parton density (DPD)

Paver, Treleani, Nuovo Cim. A70 (1982) 215.
Mekhfi, Phys. Rev. D32 (1985) 2371.
Blok, Dokshitzer, Frankfurt, Strikman, Phys.Rev. D83 (2011) 071501
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

POWER COUNTING

What is the rough power behaviour of these mechanisms?

$$\sigma_{SPS}^{(A,B)} = f_i(x) \otimes \hat{\sigma}_{ij\to AB} \otimes \ f_j(x')$$

$$^{1}/_{Q^2}$$

$$\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y}) \ \otimes \ \ \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B} \ \ \otimes \ \ F_{jl}({x'}_1, {x'}_2, \boldsymbol{y}) \ \ d^2\boldsymbol{y}$$

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$$\Lambda_{\mathrm{QCD}}^2 \qquad ^1/_{Q^2}\;^1/_{Q^2} \qquad \Lambda_{\mathrm{QCD}}^2 \qquad ^1/_{\Lambda_{\mathrm{QCD}}^2}$$

POWER COUNTING

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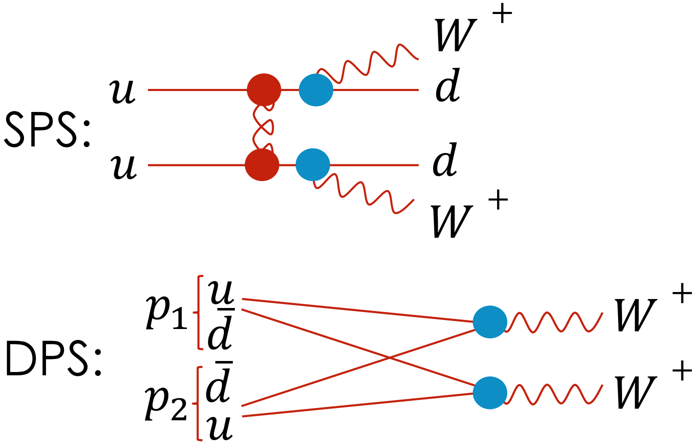
$$\sigma_{DPS}^{(A,B)} = \int \underset{\Lambda_{\mathrm{QCD}}^2}{F_{ik}(x_1, x_2, \boldsymbol{y})} \otimes\ \underset{{}^{1}/_{Q^2}\,{}^{1}/_{Q^2}}{\hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B}}\ \otimes\ \underset{\Lambda_{\mathrm{QCD}}^2}{F_{jl}(x'_1, x'_2, \boldsymbol{y})}\ \ \underset{{}^{1}/_{\Lambda_{\mathrm{QCD}}^2}}{d^2\boldsymbol{y}}$$

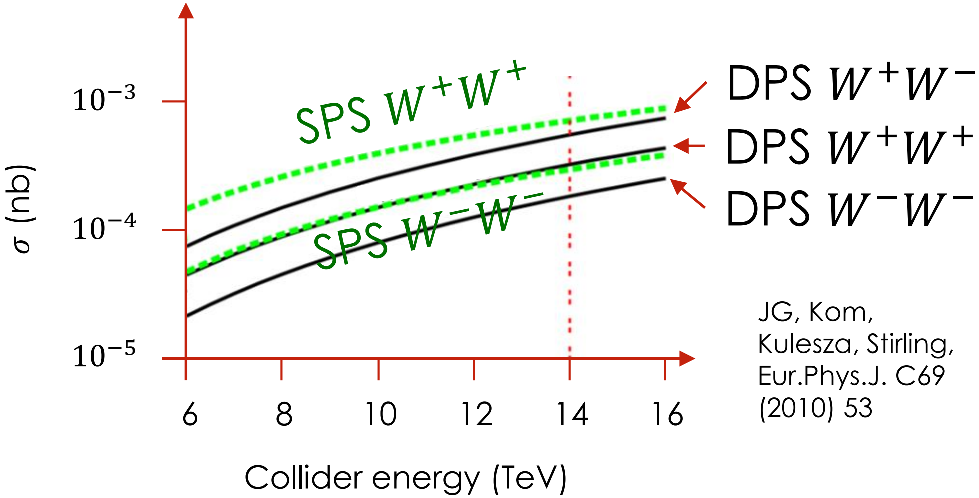
$\Rightarrow {}^{\sigma_{DPS}^{(A,B)}}\big/_{\sigma_{SPS}^{(A)}} \approx {}^{\Lambda_{\mathrm{QCD}}^2}\big/_{Q^2}$, DPS is formally power suppressed at the level of the total cross section! Why then should we care about DPS?

WHY STUDY DPS?

(1) DPS can be a significant background to processes suppressed by small/multiple coupling constants.

'Classic' SM example: same-sign WW production.





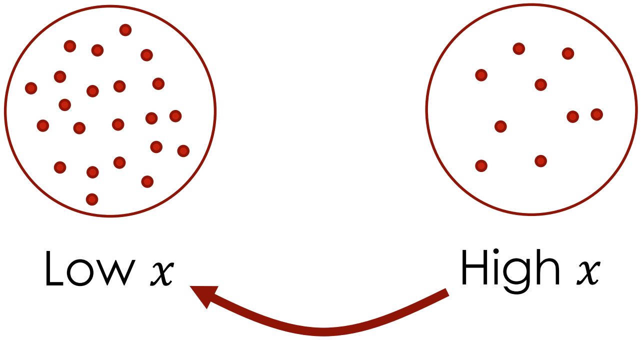
JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53

N.B. same-sign dilepton production an important channel for various new physics searches (doubly charged Higgs, SUSY,…)

WHY STUDY DPS?

(2) DPS grows faster than SPS as collider energy grows.

For a process with given scale, an increase in collider energy means a decrease in x



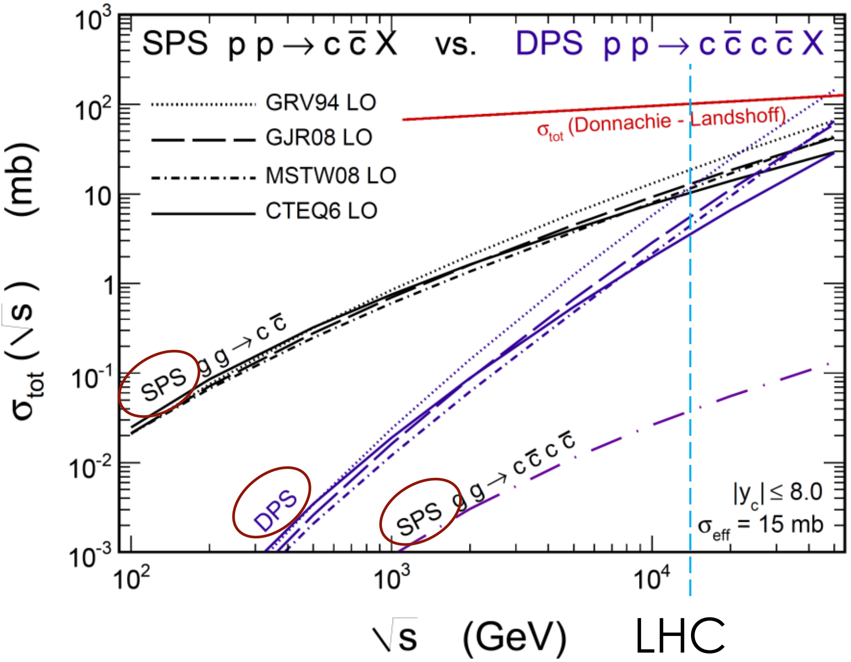
Low x

High x

DPS probability increases

Growth particularly strong for low-scale processes

Łuszczak, Maciuła, Szczurek, Phys. Rev. D79, 094034 (2012)



DPS particularly important for processes involving charm and bottom quarks. ‘10% of all “hard” events have an additional charm pair’ v. *Belyaev, MPI@LHC 2017*

WHY STUDY DPS?

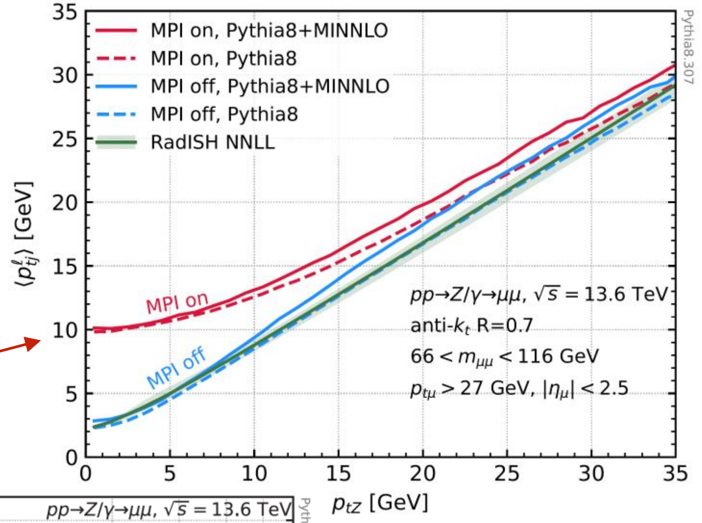
(3) DPS populates phase space in a different way to SPS. Can compete with SPS in certain regions.

E.g. small $p_{T,A}$ and/or $p_{T,B}$

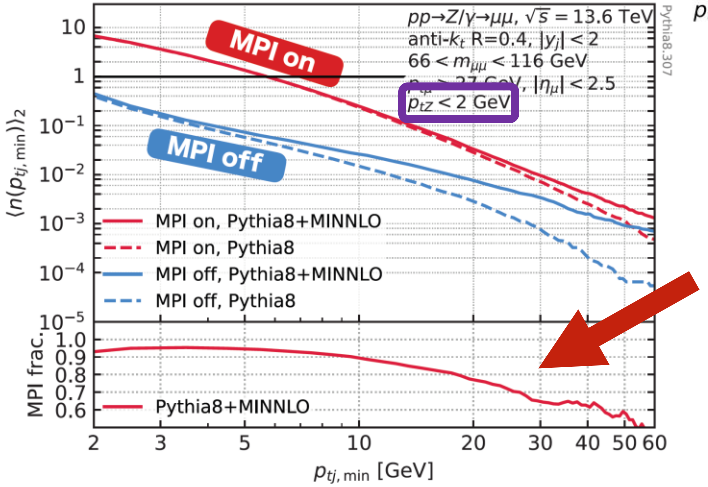
Recent study of Z @ small p_T

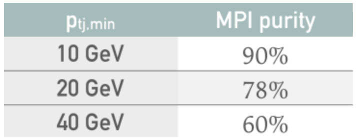
Andersen, Monni, Rottoli, Salam, Soto-Ontoso, PRL 132 (2024) 4, 041901

Look at $\langle p_T \rangle$ of leading jet. At small p_{TZ}, dominated by MPI! (mainly DPS)



By imposing $p_{TZ} < p_{TZ,cut}$ and $p_{Tj} > p_{Tj,cut}$, one can achieve a very high MPI purity!

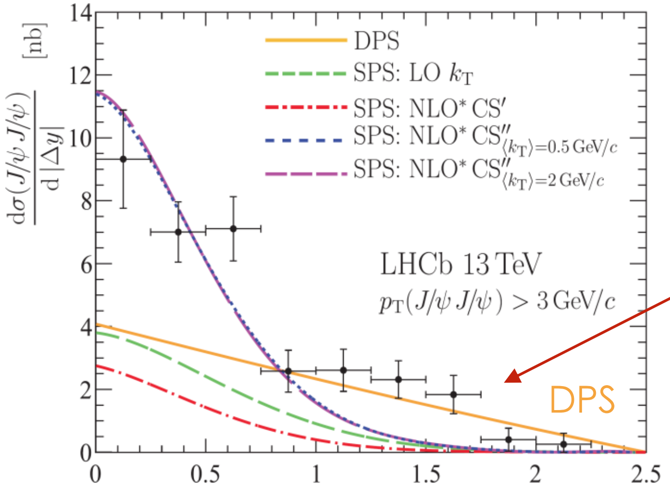




$p_{tj,min}$	MPI purity
10 GeV	90%
20 GeV	78%
40 GeV	60%

WHY STUDY DPS?

Another example: large rapidity separation of A&B



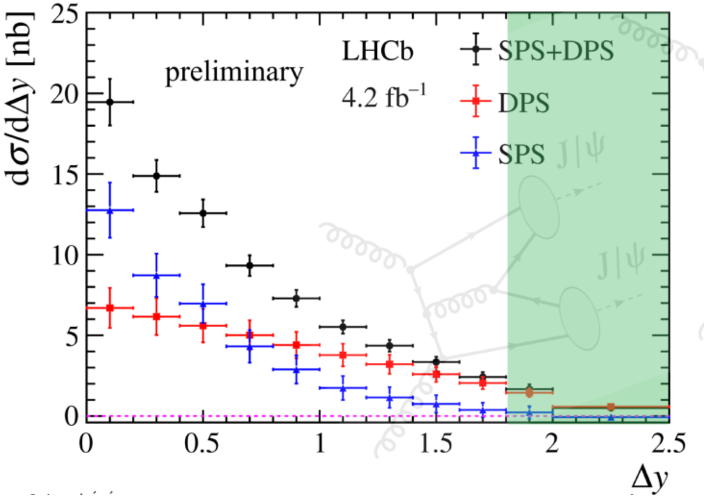
LHCb collaboration,
JHEP 06, 047, (2017)

$$\Delta y = |y_A - y_B|$$

LHCb study of J/ψ pair production .

Need DPS contribution at large Δy to explain data!

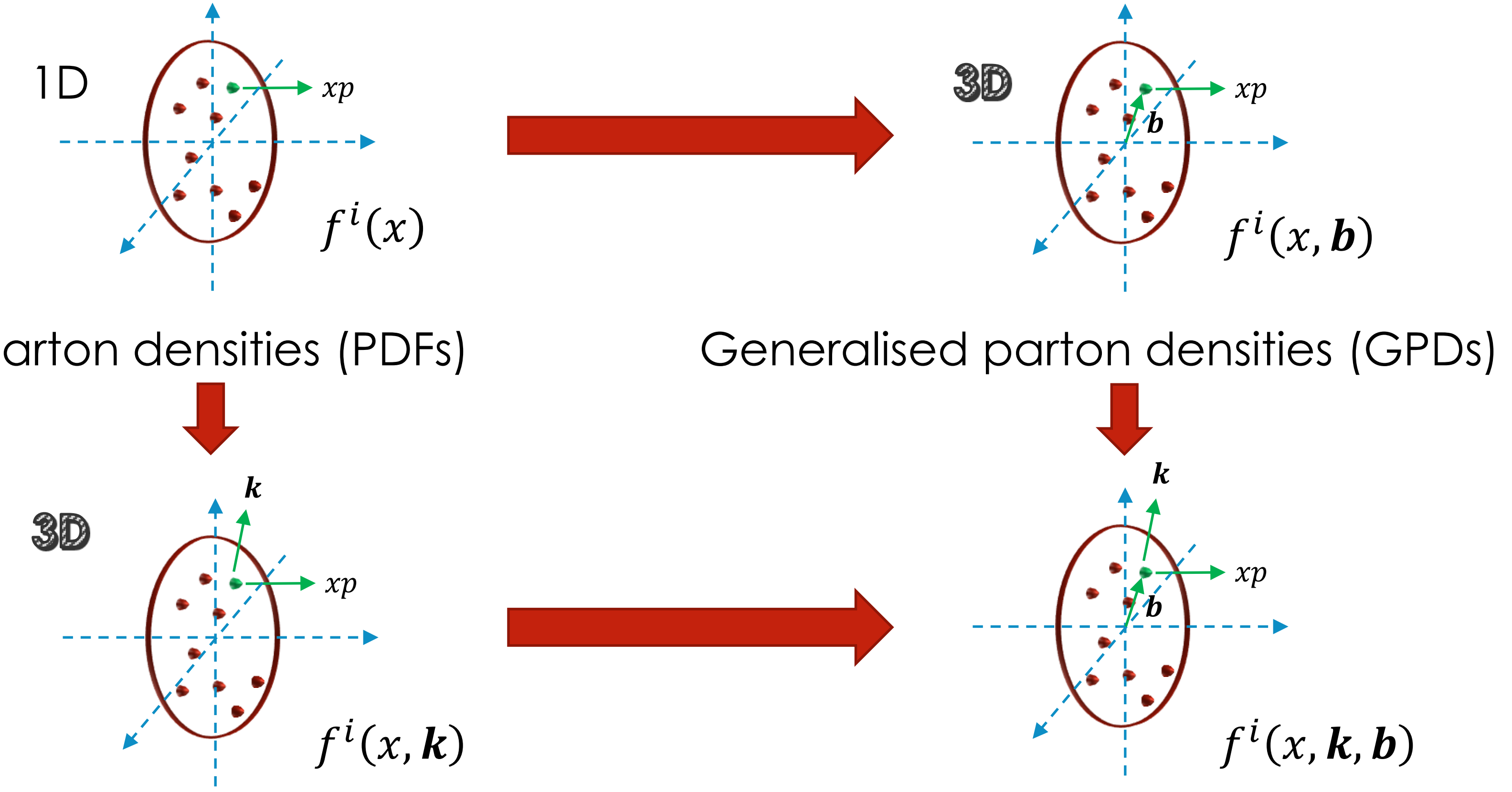
Updated results from LHCb for 2023 – similar picture
(see talk by S. Leontsinis at QCD@LHC 2023)



WHY STUDY DPS?

(4) DPS gives us new information on hadron structure.

From current measurements, one-particle picture of proton:



Parton densities (PDFs)

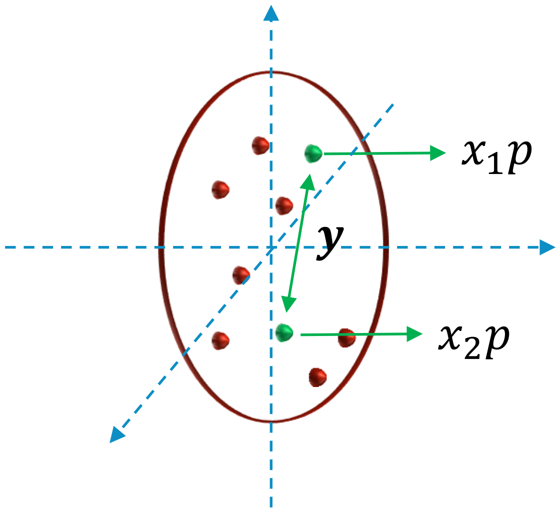
Generalised parton densities (GPDs)

Transverse momentum densities (TMDs)

Generalised transverse momentum dependent densities (GTMDs)

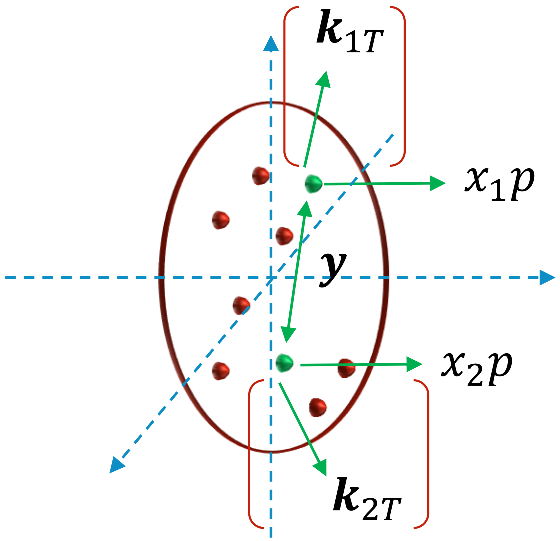
WHY STUDY DPS?

Double parton scattering gives us information, for the first time, on correlation between partons!



Double parton distributions (DPDs)

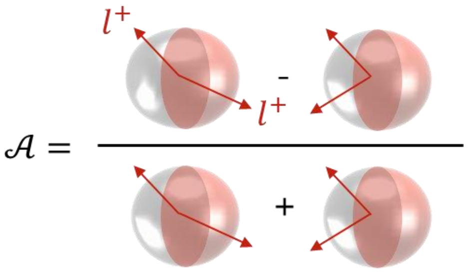
3D⊗3D



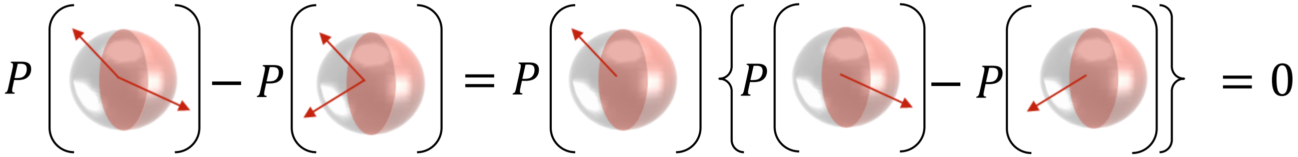
Double parton transverse momentum distributions (DTMDs)

MEASURING CORRELATIONS

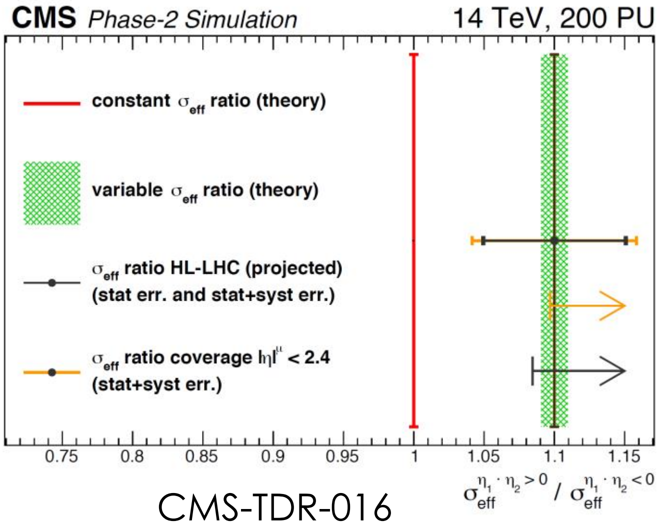
One observable to measure in detail the correlations: $\mathcal{A}$ in $W^{\pm}W^{\pm} \rightarrow l^{\pm}l^{\pm}\nu\nu$



If no correlations:



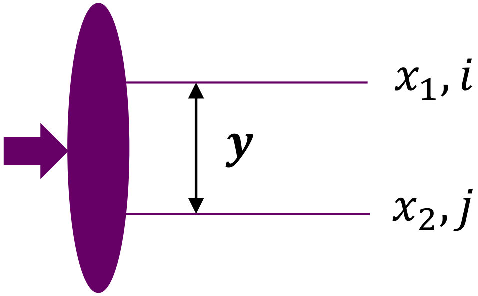
$\mathcal{A} \neq 0$ implies correlations! $\mathcal{A}$ values of $\simeq 0.1$ are measurable at hi-lumi LHC



CMS-TDR-016

DPS ‘POCKET FORMULA’

DPD $F_{ik}(x_1, x_2, \boldsymbol{y})$ is a complex object!



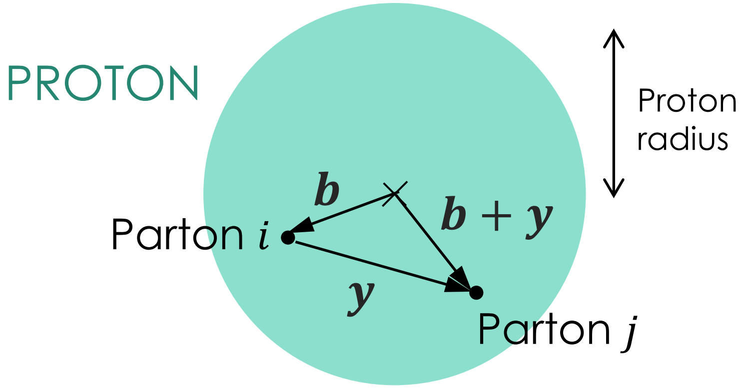
Historically several approximations, for rough estimates of DPS.

(1) Ignore correlations between partons

$$F^{ij}(x_1, x_2, \boldsymbol{y}) \rightarrow \int d^2\boldsymbol{b}\, f^i(x_1, \boldsymbol{b})\; f^j(x_2, \boldsymbol{b}+\boldsymbol{y})$$

GPD

(@ zero skewness)



DPS ‘POCKET FORMULA’

(2) Assume GPD can be written as $f^i(x_1, \boldsymbol{b}) = f^i(x_1) G(\boldsymbol{b})$

Then $F^{ij}(x_1, x_2, \boldsymbol{y}) = f^i(x_1)\, f^j(x_2) \int d^2\boldsymbol{b}\, G(\boldsymbol{b})\, G(\boldsymbol{b} + \boldsymbol{y})$

Inserting into $\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y}) \otimes \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B} \otimes F_{jl}(x'_1, x'_2, \boldsymbol{y})\ d^2\boldsymbol{y}\ \ldots$

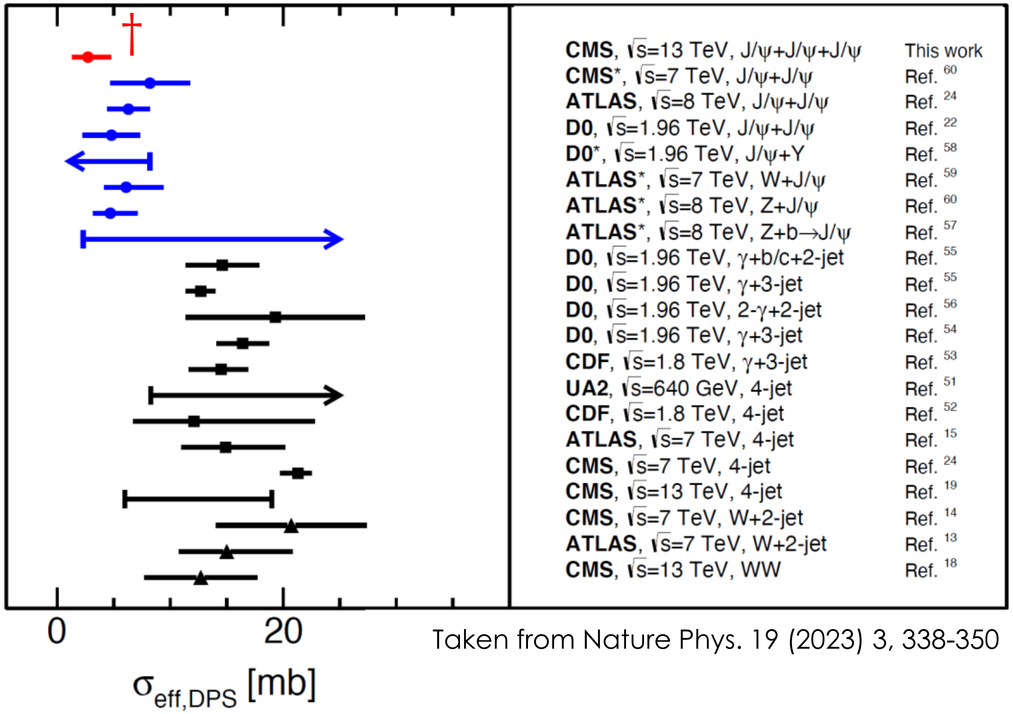
$$\longrightarrow \qquad \boxed{\sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\, \sigma_S^{(B)}}{\sigma_{\text{eff}}}}$$

“DPS pocket formula”

Most pheno estimates of DPS use this!

THE SIZE OF σ_{eff}

If pocket formula picture is the full story, the ratio $\sigma_S^{(A)}\sigma_S^{(B)}/\sigma_D^{(A,B)}$ extracted from various DPS measurements should be universal and roughly the proton transverse area ~ 60 mb.



Taken from Nature Phys. 19 (2023) 3, 338-350

$\sigma_{eff,DPS} \ll 60\text{mb}!$

σ_{eff} with quarkonium $< \sigma_{eff}$ with high-p_T jets/EW bosons

†: Measurement in triple J/ψ. Process receives contributions from triple parton scattering (TPS)!

CMS, Nature Phys. 19 (2023) 3, 338-350

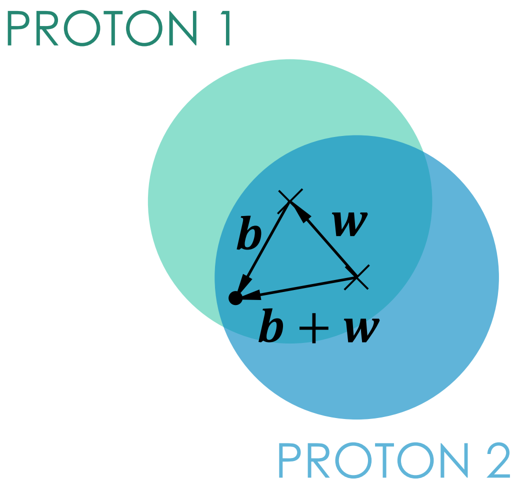
EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Can rewrite pocket formula cross section:

$$\sigma_{\mathrm{D}} = \int \frac{1}{2!}\left(\underbrace{\int f(x_1)f(\bar{x}_1)\hat{\sigma}(x_1,\bar{x}_1)G(\boldsymbol{b})G(\boldsymbol{b}+\boldsymbol{w})d^2\boldsymbol{b}}\right)^2 d^2\boldsymbol{w}$$

(For identical particles)

$$= \int \frac{1}{2!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^2 d^2\boldsymbol{w}$$



EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Generalise to N scatters:

$$\sigma_{\mathrm{N}} = \int \underbrace{\frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N}_{\text{INCLUSIVE N-PARTON SCATTERING PROBABILITY}} d^2\boldsymbol{w}$$

EIKONAL MODEL FOR MULTIPLE INTERACTIONS

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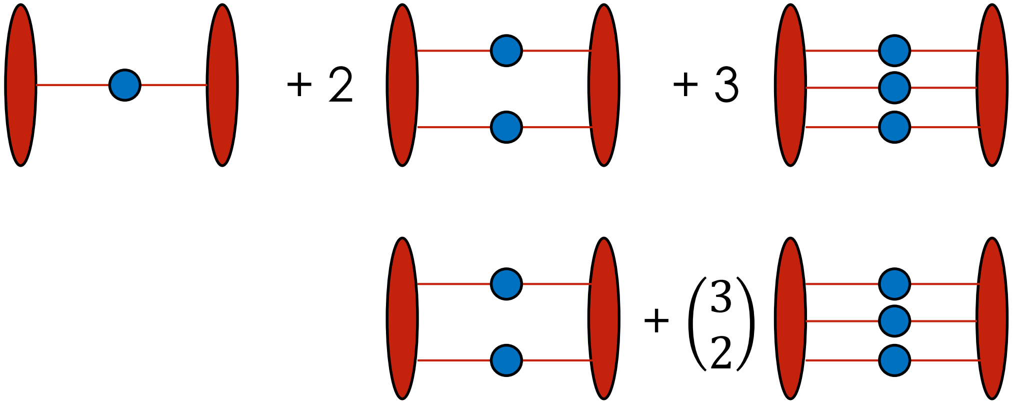


SPS = + 2 + 3 + …

DPS = + $\binom{3}{2}$ + …

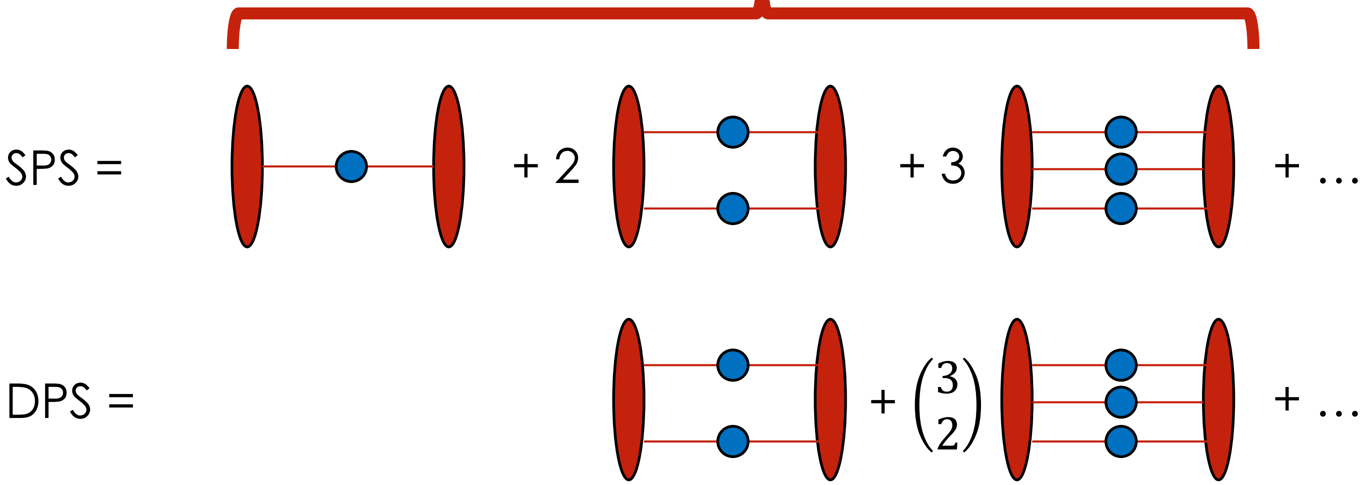
EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Generalise to N scatters:

EXCLUSIVE M-PARTON SCATTERING PROBABILITY

$$\sigma_{\mathrm{N}} = \int \underbrace{\frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N}_{\text{INCLUSIVE N-PARTON SCATTERING PROBABILITY}} d^2\boldsymbol{w} = \int \sum_{M \geq N} \binom{M}{N} P_M(\boldsymbol{w})\, d^2\boldsymbol{w} \qquad P_M(\boldsymbol{w}) = \frac{\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^M}{M!} e^{-\sigma_S \mathcal{G}(\boldsymbol{w})}$$

Poisson distribution



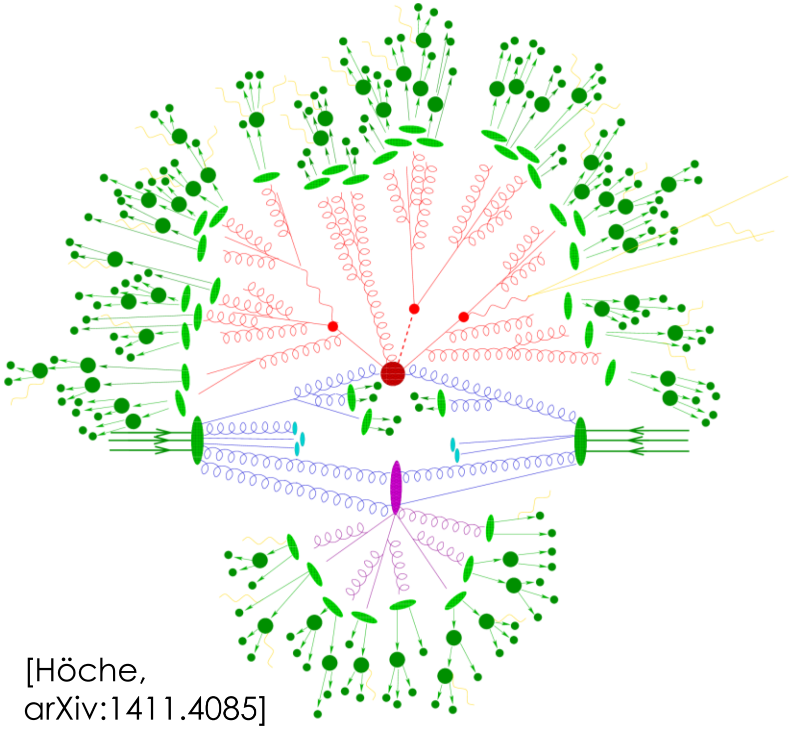
Seymour, Siodmok, arXiv:1308.6749
Calucci, Treleani, Phys.Rev. D79 (2009) 034002, Phys.Rev. D79 (2009) 074013

EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Generalise to N scatters:

$$\sigma_{\mathrm{N}} = \int \frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N d^2\boldsymbol{w} = \int \sum_{M \geq N} \binom{M}{N} P_M(\boldsymbol{w})\, d^2\boldsymbol{w} \qquad P_M(\boldsymbol{w}) = \frac{\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^M}{M!} e^{-\sigma_S \mathcal{G}(\boldsymbol{w})}$$

Poisson distribution



[Höche, arXiv:1411.4085]

This eikonal model is the basis of the multiple interactions models in Monte Carlo event generators!

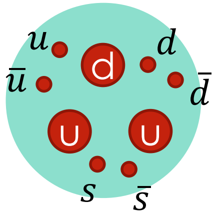
Herwig model ≈ eikonal model.

Butterworth, Forshaw, Seymour, Z.Phys. C72 (1996) 637
Borozan, Seymour, JHEP 0209 (2002) 015
Bahr, Gieseke, Seymour, JHEP 0807 (2008) 076

MULTIPLE SCATTERING IN PYTHIA

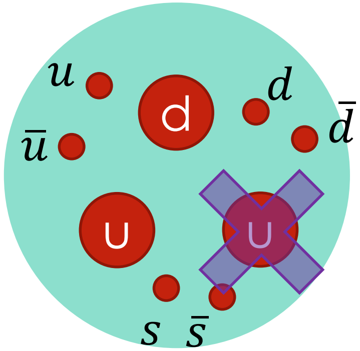
Pythia model has some improvements to this picture.

Sjöstrand, van Zijl, Phys.Rev. D36 (1987) 2019,
Sjöstrand, Skands, JHEP 0403 (2004) 053
Eur.Phys.J. C39 (2005) 129-154



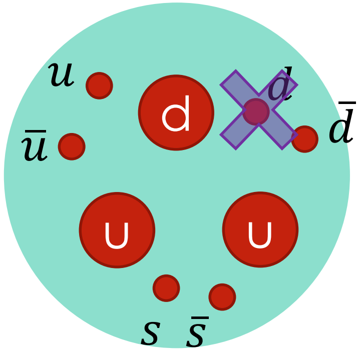
Start at hardest interaction and work 'backwards'. Start with normal PDFs: $\int f^{u_v}(x)\mathrm{d}x = 2\,, \int f^{d_v}(x)\mathrm{d}x = 1, \sum_i \int f^i(x)\; x\; \mathrm{d}x = 1$

Interaction 1 involves valence u parton with momentum z



Adjust PDFs for remaining interactions: Total momentum $1 - z$, number of u valence = 1.

Interaction 1 involves sea d parton with momentum z



Adjust PDFs for remaining interactions: Total momentum $1 - z$, add to $\bar{d}$ distribution 'companion quark distribution'

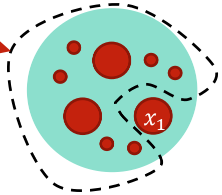
PYTHIA MPDFS: SUM RULES

Can formally state these valence number and momentum conservation constraints in **sum rules**.

E.g. momentum sum rule for equal scale DPDs:

$$\sum_j \int d^2\boldsymbol{y}\, dx_2\, x_2\, \underset{\text{DPD}}{F^{ij}(x_1, x_2, \boldsymbol{y})} = (1 - x_1)\, \underset{\text{PDF}}{f^i(x_1)}$$

$$\sum_i x_i = 1 - x_1$$



JG, Stirling, JHEP 03 (2010) 005
Blok et al., Eur.Phys.J.C 74 (2014) 2926
Diehl, Plößl, Schäfer, Eur.Phys.J.C 79 (2019) 3, 253

How well does Pythia satisfy these sum rules? Issue: no hardness ordering for equal scale DPS, Pythia chooses 'first' randomly.

Symmetrised DPDs satisfy sum rules reasonably, though large deviations in places

x_1	Momentum sum rule ($j_1 = u$). Should = 1.	$\bar{u}u$ number sum rule. Should = 3.
10^{-6}	0.979	2.961
10^{-3}	0.980	3.351
10^{-1}	1.014	3.491
0.2	1.047	3.580
0.4	1.133	3.858
0.8	1.679	7.048

Naively symmetrised Pythia DPDs

Fedkevych, JG, JHEP 02 (2023) 090

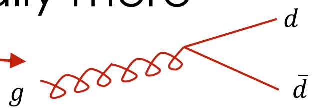
AN IMPROVED MODEL FOR MPDFS

Can one design a model of equal-scale multi-parton PDFs that is symmetric and satisfies sum rules better?

Ongoing work with Oleh Fedkevych, Seonagh Smith

"Minimal" adjustments to Pythia picture:

- Order scatters in x rather than Q + smooth transitions
- Improve "companion quark mechanism" so that it is naturally more symmetric & follows expectations from QCD splitting
- Add a (weak) damping factor at small x fractions



Resultant DPDs satisfy sum rules well!

x_1	$j_1 = u$ MSR. Should = 1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.965	3.072
10^{-3}	0.960	3.035
0.1	1.019	2.902
0.2	1.020	2.904
0.4	1.006	2.953
0.8	1.001	2.995

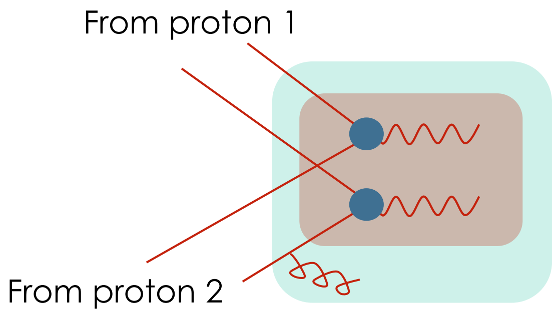
PRELIMINARY

Now checking triple parton distributions + some phenomenology…

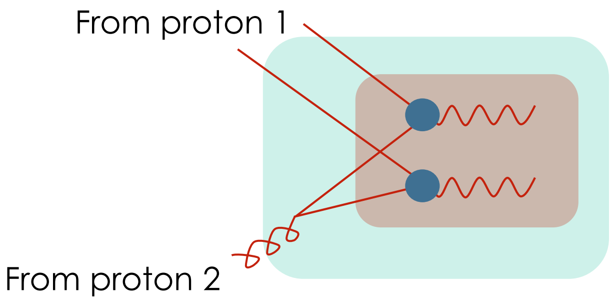
QCD EVOLUTION EFFECTS IN DPS

How do we treat DPS properly in pQCD?

Going 'backwards' from the hard process, what can happen to the two partons?



Emission from single leg. Familiar from single scattering.



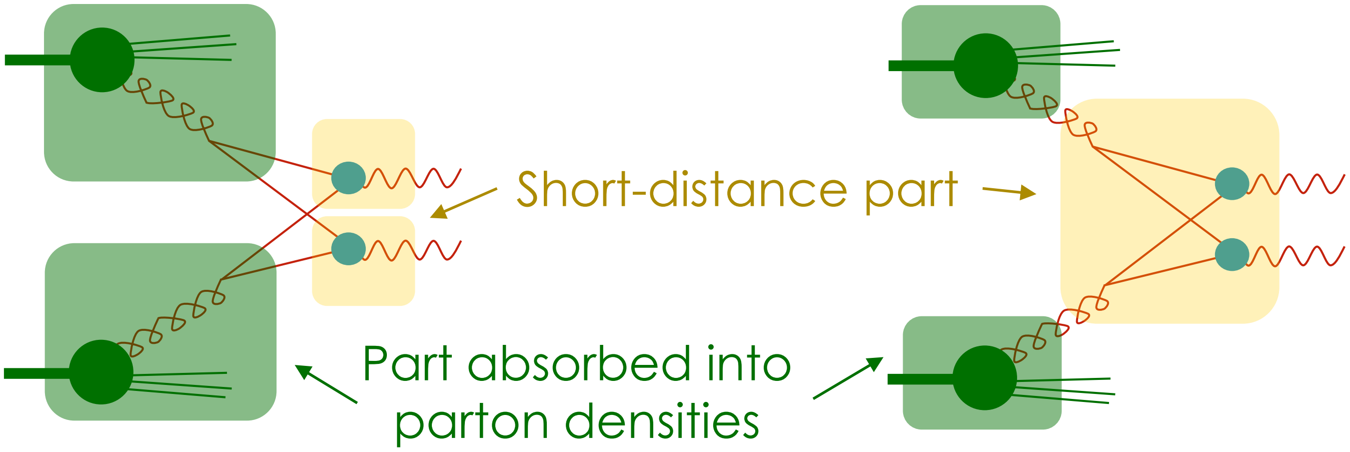
'1→2 splitting'. New effect!

SPS-DPS DOUBLE COUNTING

Problem: if we have a splitting in both protons, process can be thought of either as a contribution to DPS or as a loop correction to SPS:

DPS picture:

SPS picture:

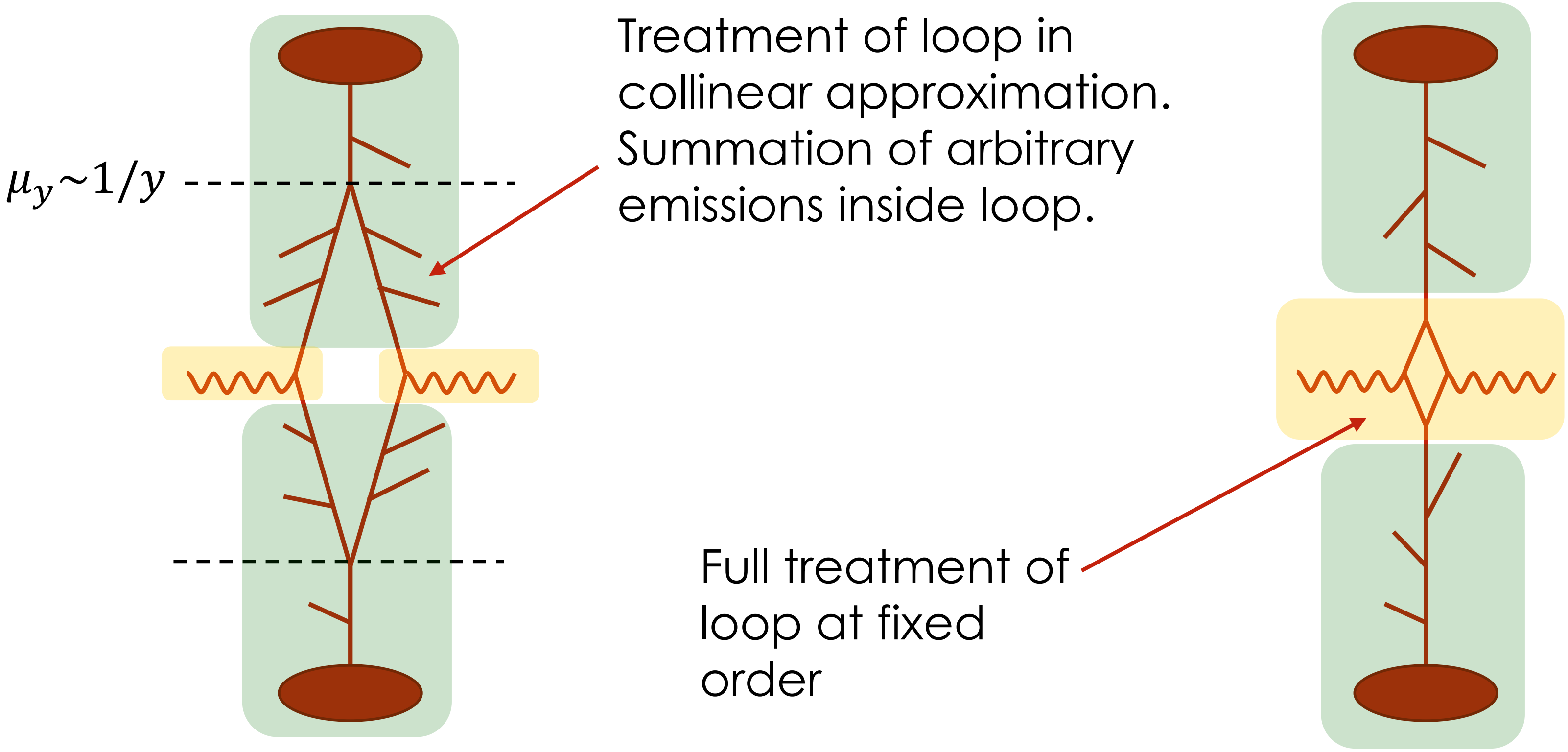


Short-distance part

Part absorbed into parton densities

Double counting issue if splitting is included in a naïve way.

SPS-DPS DOUBLE COUNTING



DPS description

SPS description

Treatment of loop in collinear approximation. Summation of arbitrary emissions inside loop.

Full treatment of loop at fixed order

Most appropriate at large y

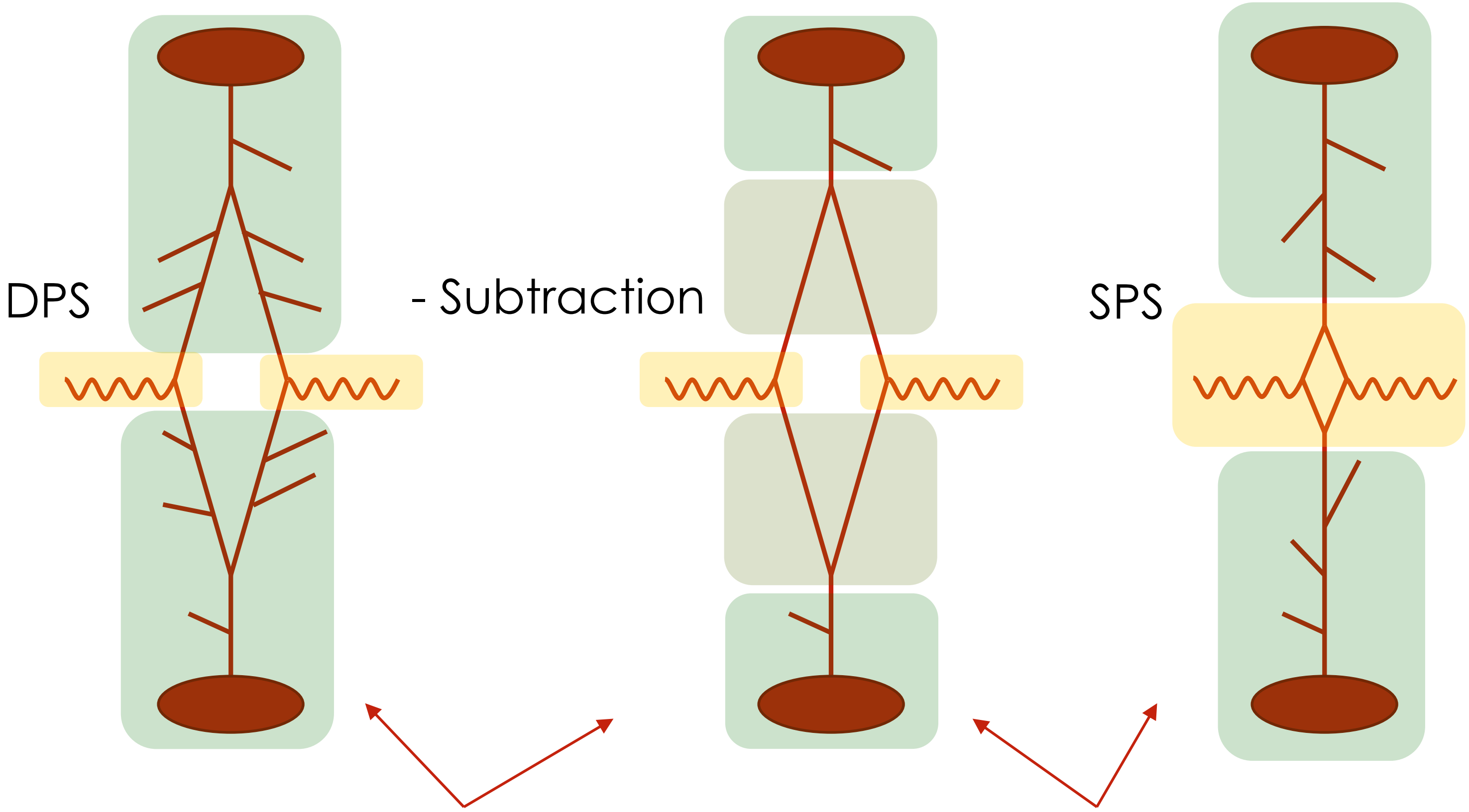
Most appropriate at small y

Want smooth transition from one description to the other as y varies

DPS + SPS WITHOUT DOUBLE COUNTING

Achieve by taking away a subtraction term from the sum of SPS + DPS:

Diehl, JG, Schönwald JHEP 1706 (2017) 083

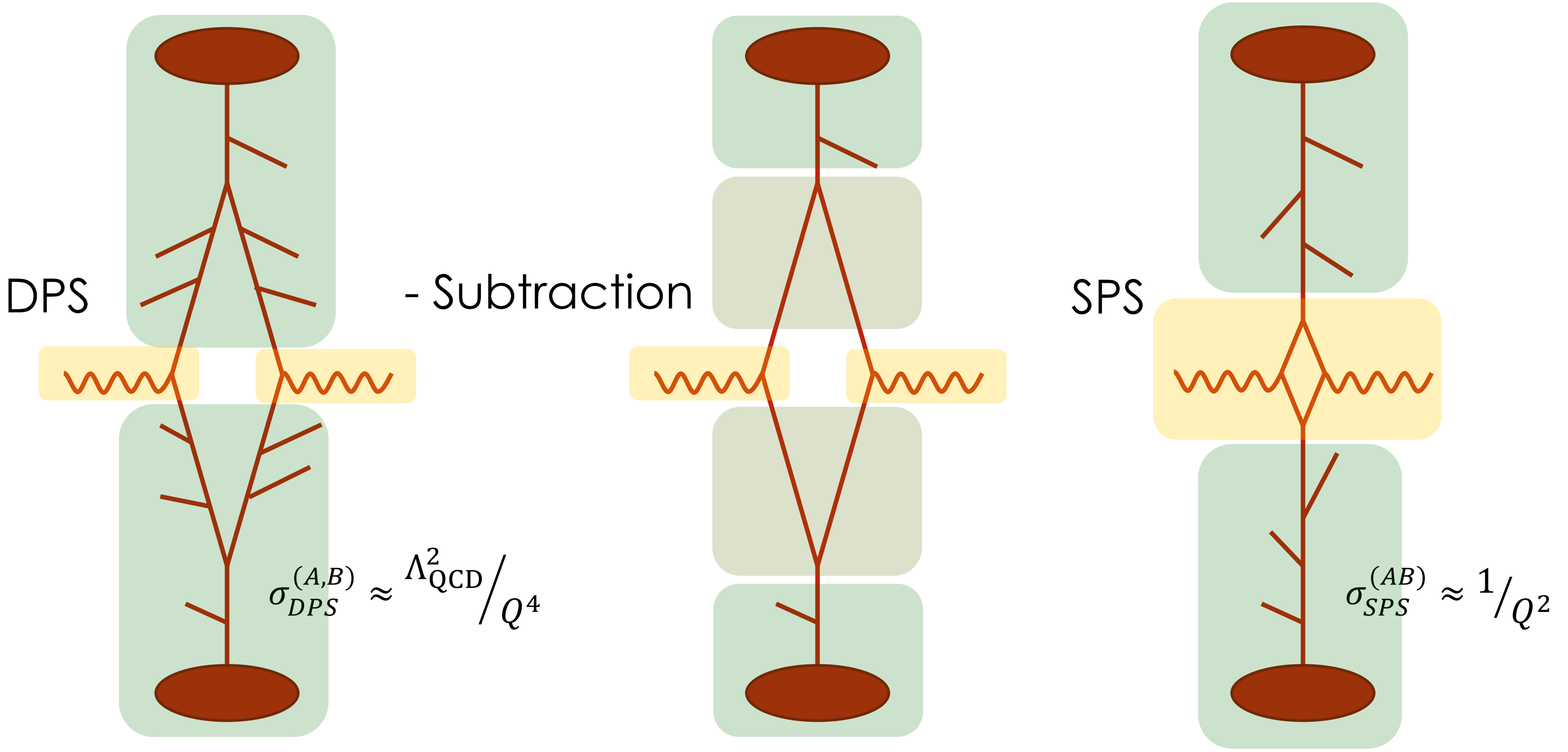


DPS - Subtraction SPS

At small $y{\sim}1/Q$, not much evolution space for DPS to emit inside loop. DPS ~ subtraction and we are left with SPS.

At large y, collinear approximation to loop works well. Subtraction ~ SPS and we are left with DPS.

WHICH REGION IS DOMINANT?



DPS

- Subtraction

SPS

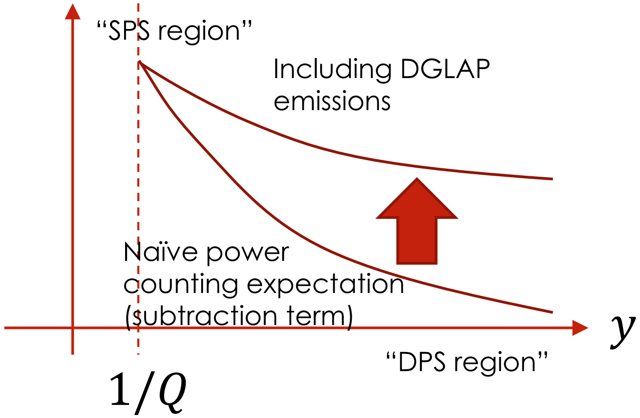
$\sigma_{DPS}^{(A,B)} \approx {\Lambda_{\mathrm{QCD}}^2}/{Q^4}$

$\sigma_{SPS}^{(AB)} \approx {1}/{Q^2}$

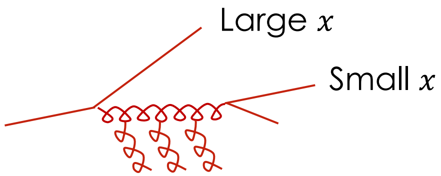
On power counting grounds, expect small y SPS region to be dominant - then DPS ≪ SPS

WHICH REGION IS DOMINANT?

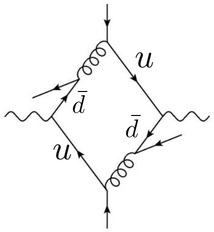
However there are various scenarios where it becomes preferable to have additional emissions in the loop, which compensates naïve power suppression:



- Small x: small x logs prefer earlier 1→2 splitting, 2 legs to emit rather than 1!
- DY at large rapidity separation – preferable to produce one high x & one low x quark via



- Processes where leading order SPS loop is absent, like same-sign WW



Here overlap with SPS is less important, or even numerically irrelevant. Can determine this by looking at y profile of DPS contribution.

PHENO TOOLS FOR DPS

DPS theory developments have been rapid in recent years.
Development of phenomenological tools has lagged behind.

Many experimental extractions of DPS use theoretical predictions of DPS shapes in multiple distributions ('templates').

Typically provided by Monte Carlo event generators.

11 variables in same-sign WW:

$$p_T^{l_1}, p_T^{l_2}, p_T^{miss}, \eta_1\eta_2, |\eta_1 + \eta_2|, m_{T(l_1,p_T^{miss})}, m_{T(l_1,l_2)}, \left|\Delta\phi_{(l_1,l_2)}\right|, \left|\Delta\phi_{(l_2,p_T^{miss})}\right|, \left|\Delta\phi_{(ll,l_2)}\right|, m_{T2}^{ll}$$

CMS, PRL 131 (2023) 091803

Would be very useful to have a Monte Carlo event generator for DPS that includes latest theory developments!

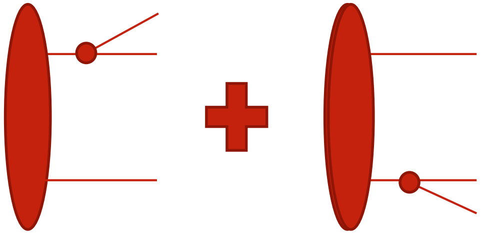
A DPS PARTON SHOWER

Motivated a parton shower implementation of full QCD framework for DPS: dShower. Cabouat, JG, Ostrolenk, JHEP 1911 (2019) 061

Brief summary of algorithm:

- Select x_i of initiating partons and separation y using full DPS formula. Involves use of some DPD set, can be specified by the user.

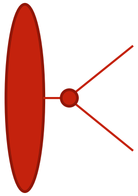
- Backward evolution from hard process with emissions from two legs. Angular ordered shower, as in Herwig.



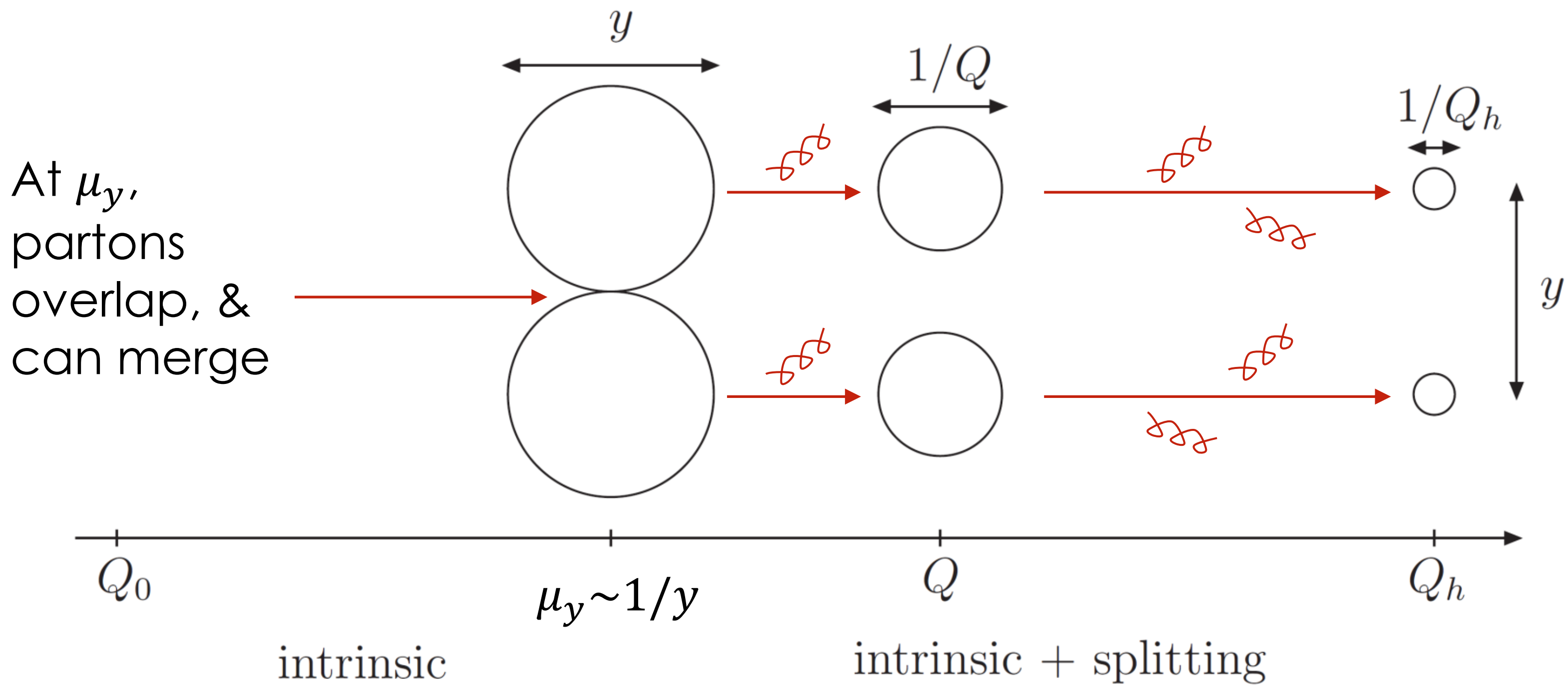
- Shower evolution ‘guided’ by DPDs. Correlations encoded by these DPDs are fed into the shower

A DPS PARTON SHOWER

- Allow possibility of 2→1 'mergings' in backward evolution at appropriate scale.



Intuitive picture:


At μ_y, partons overlap, & can merge

DSHOWER: COMBINING SPS AND DPS

We also developed an algorithm for combining SPS and DPS in the shower without double counting.

Cabouat, JG, JHEP 10 (2020) 012

Need ‘fully differential’ formulation of subtraction formalism:

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \overbrace{\boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma^{SPS}_{A+B}}{dO}\right.}^{\text{Usual SPS shower}} - \left.\frac{d\sigma^{sub}_{(A,B)}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma^{DPS}_{(A,B)}}{dO\, d^2\boldsymbol{y}}$$

Observable

Single parton shower

Double parton shower

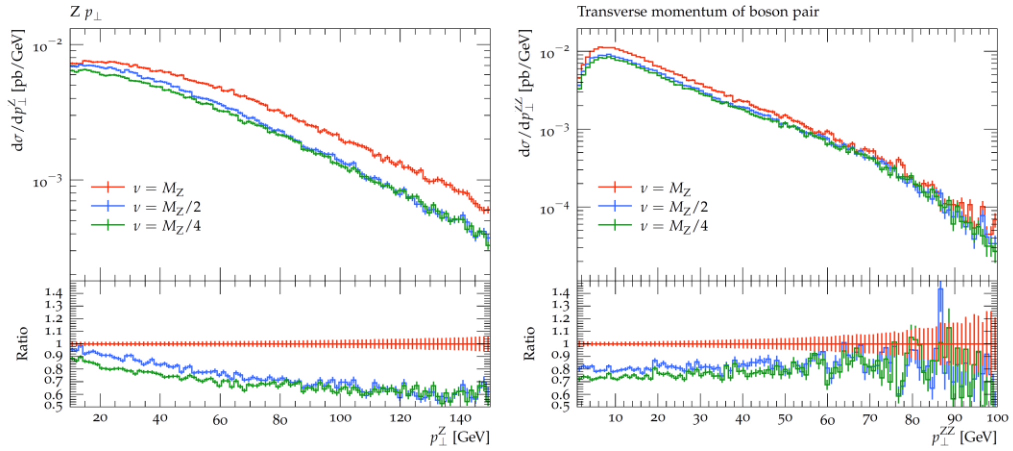
Hard cross section in this term is DPS shower expanded to $\mathcal{O}(\alpha_s^2)$, keeping only merging terms in each proton, integrated over y

[Inspired by methods to match shower with NLO calculations: Frixione, Webber, JHEP 06 (2002) 029, Frixione, Nason, Oleari, JHEP 11 (2007) 070, Nason, JHEP 11 (2004) 040,…]

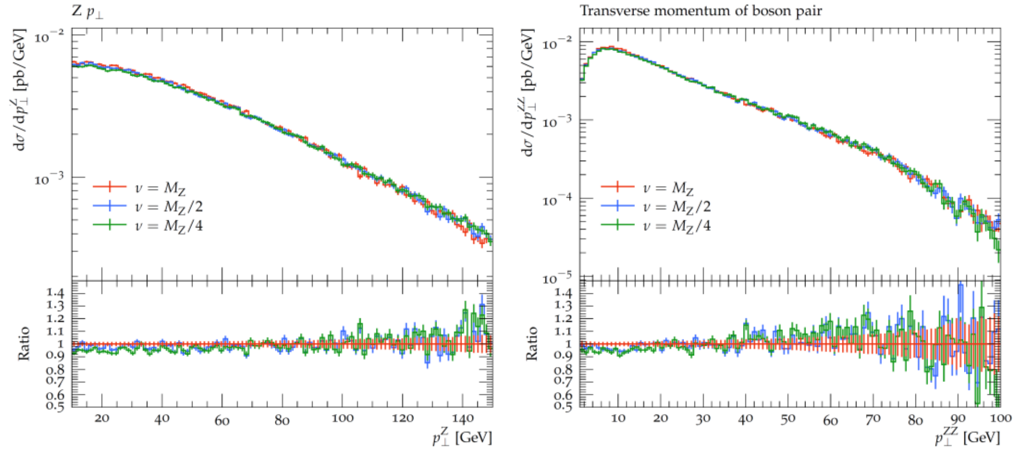
VALIDATION

Validation for ZZ production. DPS & subtraction terms contain a cut-off in y at b_0/ν, ν is (unphysical) scale that demarcates SPS from DPS. Total cross section shouldn't depend on ν.

No subtraction:



Subtraction included:

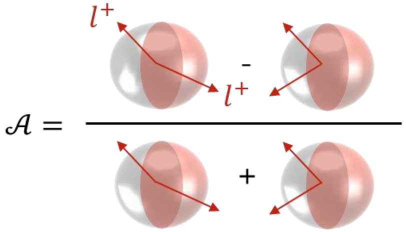


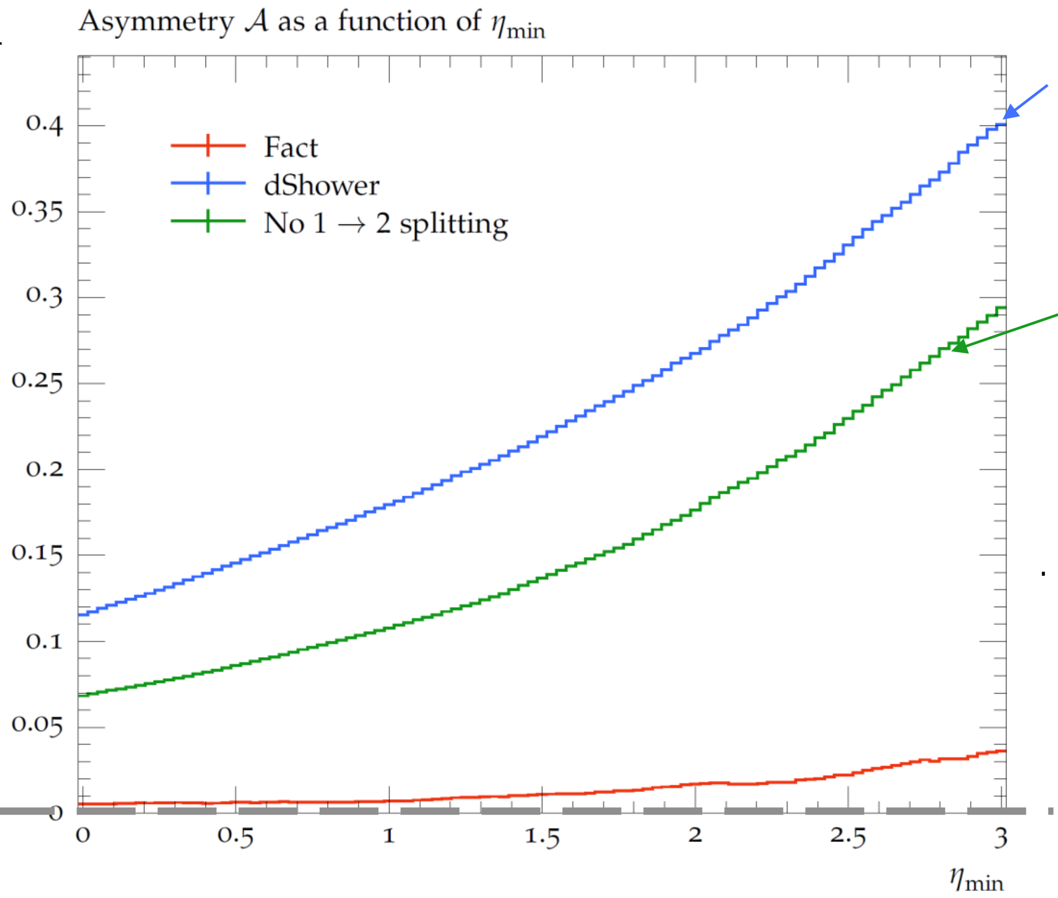
EFFECTS OF CORRELATIONS

dShower predictions take account of correlations from 1→2 splitting and also valence number and momentum constraints. These effects lie beyond the pocket formula.

Can we see the imprint of these in DPS predictions?

WW ASYMMETRY





Includes 1→2 splittings
+ valence number effects

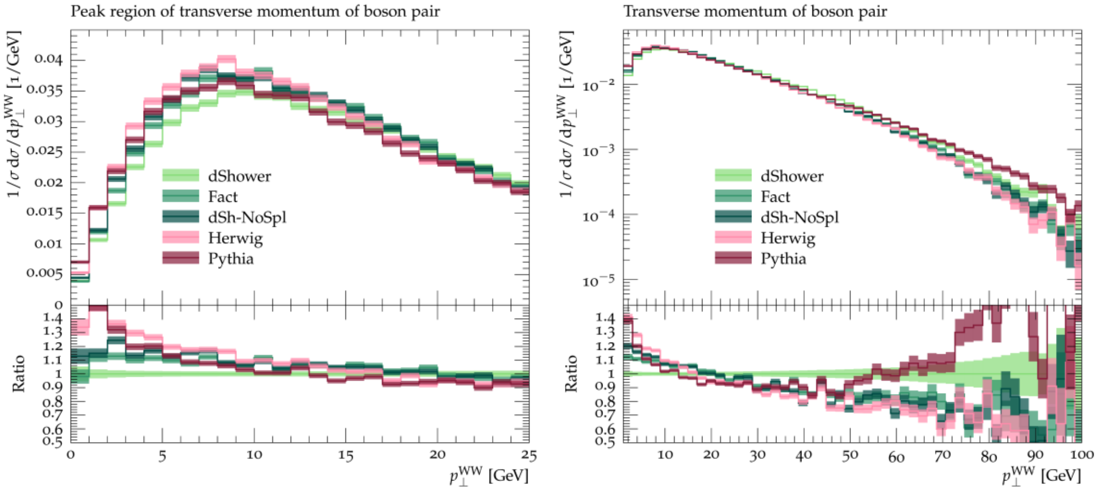
Simple valence number effects

No parton-parton correlations

Cabouat, JG, JHEP 10 (2020) 012

WW TRANSVERSE MOMENTUM

WW p_T spectrum: dShower result skewed more towards larger p_T


Peak region of transverse momentum of boson pair

Transverse momentum of boson pair

(a)

(b)

Explanation: larger qg distributions when including 1→2 splitting effects, leads to greater chance of $\tilde{q}g \rightarrow \tilde{q}q + \bar{q}$ and finite p_T of the $\tilde{q}q$ system.

Cabouat, JG, JHEP 10 (2020) 012

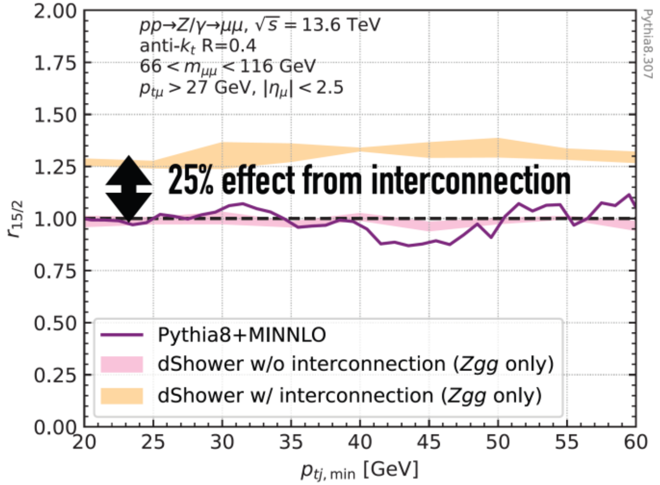
Z + JETS

In Z+jets study of Andersen et al., looked at MPI jet rate when two different cuts on Z p_T were imposed

$$r_{15/2} = \frac{\langle n(p_{tj,\min})\rangle_{15}^{\text{pure-MPI}}}{\langle n(p_{tj,\min})\rangle_{2}^{\text{pure-MPI}}}$$

$p_{TZ} < 15$ GeV

$p_{TZ} < 2$ GeV

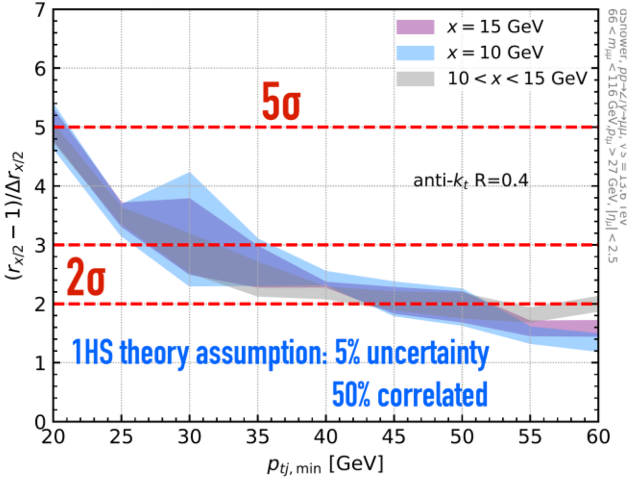


If two scatters are uncorrelated, $r_{15/2} \sim 1$. 1→2 splittings induce $r_{15/2} \sim 1.25$!

Can we measure this experimentally?

- Reasonable assumptions lead to at least 2σ significance → exclusion of pocket formula.
- Significance increases as accuracy of SPS prediction goes up – motivates Z+2j NNLO matched predictions.

significance of signal
of perturbative interconnection

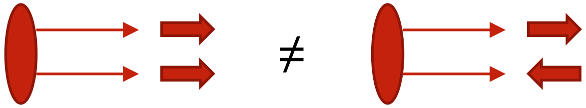


Andersen, Monni, Rottoli, Salam, Soto-Ontoso, PRL 132 (2024) 4, 041901

SPIN CORRELATIONS

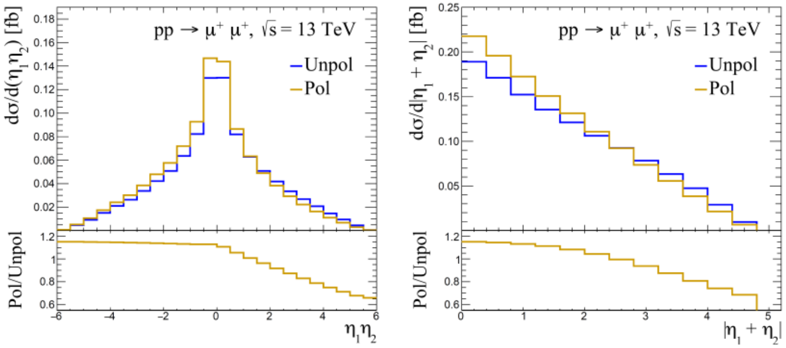
Other types of correlation possible in DPS – e.g. spin correlations

Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009



Spin correlations should be large at high x, but become less significant at smaller x

Spin polarisation effects may have a measurable effect in same-sign WW [Cotogno, Kasemets, Myska, Phys.Rev. D100 (2019) 1, 011503, JHEP 10 (2020) 214]



Few percent effect on lepton pseudorapidity asymmetry, in scenario where 'initial' spin correlations are maximised.

$\lvert\eta_i\rvert$	> 0	> 0.6	> 1.2
A	0.07	0.11	0.16
σ [fb]	0.51	0.29	0.13

SUMMARY

- DPS can compete with SPS for certain processes ($W^{\pm}W^{\pm}$, processes involving charm) and in certain kinematic regions. Relative importance grows with $\sqrt{s}$, and reveals new info on proton structure.

- Simplest approach: neglect correlations → ‘pocket formula’. Models of general MPI in event generators based on this. Pythia: improvements beyond this to account for number & momentum effects, but not perfect – construction of an improved model ongoing.

- Full QCD framework for DPS now developed, including proper effect of perturbative pair generation (“$1 \rightarrow 2$ splittings”). Implemented into parton shower event generator dShower.

- $1 \rightarrow 2$ splittings and/or number & momentum effects (and spin correlations!) can have an appreciable effect on DPS processes at the LHC – examples in same-sign WW and Z + jets.

BACKUP SLIDES

DPD OPERATOR DEFINITION

$$F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B) \propto \int \mathrm{d}y^- \mathrm{d}z_i^- \, e^{i x_i p^+ z_i^-} \, \langle p | \mathcal{O}_i\big(y + \tfrac{1}{2} z_1, y - \tfrac{1}{2} z_1\big) \, \mathcal{O}_j\big(\tfrac{1}{2} z_2, -\tfrac{1}{2} z_2\big) | p \rangle \Big|_{y^+=0,\, z_i^+=0,\, \boldsymbol{z}_i=\boldsymbol{0}},$$

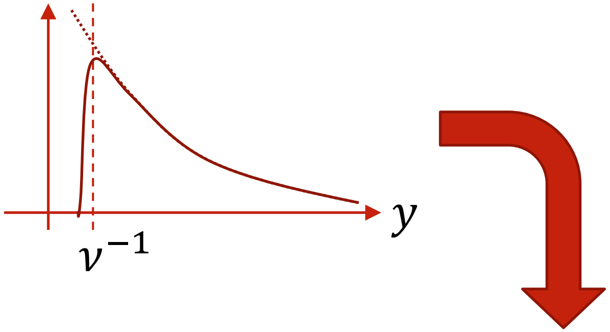
PDF: $f_i(x, \mu) \propto \int \mathrm{d}z^- e^{i x p^+ z^-} \, \langle p | \mathcal{O}_i\big(\tfrac{1}{2} z, -\tfrac{1}{2} z\big) | p \rangle \Big|_{\boldsymbol{z}=\boldsymbol{0},\, z^+=0}$

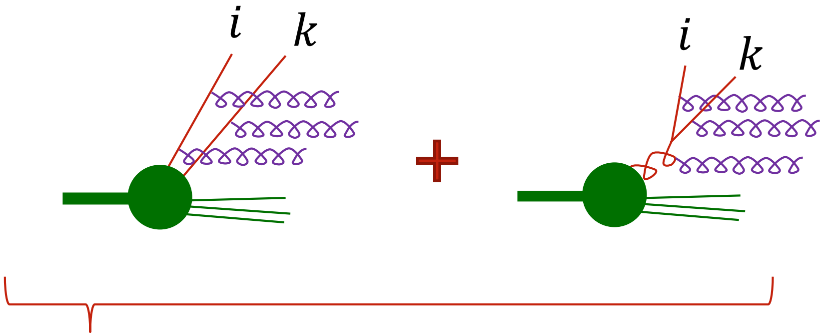
COMBINING SPS AND DPS WITHOUT DOUBLE COUNTING

DPS WITHOUT DOUBLE COUNTING

I focus on SPS & 1v1 DPS overlap. Removal of overlap between 2v1 DPS & 3 particle collision is similar.

<u>Step 1:</u> insert cut-off function into DPS cross section formula





$$\sigma_{DPS}^{(A,B)} = \int dx_i dx'_i d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B)\, F_{jl}(x'_1, x'_2, \boldsymbol{y}, \mu_A, \mu_B)$$

$$\times\, \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B}$$

Choose $\nu \sim Q$ in practice.

Removed divergence. Double counting up to scale ν.

DPS WITHOUT DOUBLE COUNTING

Step 2: For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

σ_{sub}: DPS cross section with DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

$$F_{ij}(x_1, x_2, y, \mu^2) \to \frac{1}{\pi y^2} \frac{f_k(x_1 + x_2, \mu^2)}{x_1 + x_2} \frac{\alpha_s(\mu^2)}{2\pi} P_{k \to ij}\left(\frac{x_1}{x_1 + x_2}\right)$$

General subtraction philosophy used in many QCD calculations (proofs of factorisation, SCET, NLO + PS matching…)

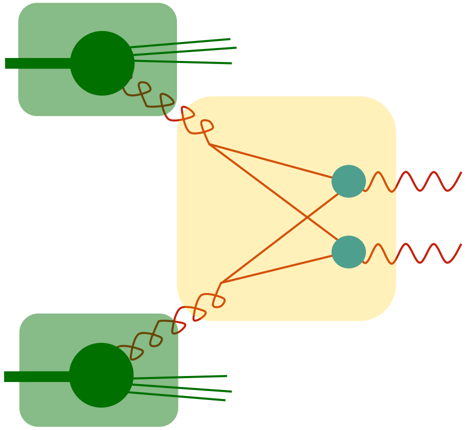
HOW THE SUBTRACTION WORKS

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

For small $\boldsymbol{y}$ (of order $1/Q$) the dominant contribution to σ_{DPS} comes from the (fixed order) perturbative expression $\Rightarrow \sigma_{DPS} \approx \sigma_{sub}$

& $\sigma_{tot} \approx \sigma_{SPS}$ ✓

Dependence on ν cancels order-by-order between σ_{DPS} & σ_{sub}



For large $\boldsymbol{y}$ (much larger than $1/Q$) the dominant contribution to σ_{SPS} is the region of the 'double splitting' loop where DPS approximations are valid

$\Rightarrow \sigma_{SPS} \approx \sigma_{sub}$

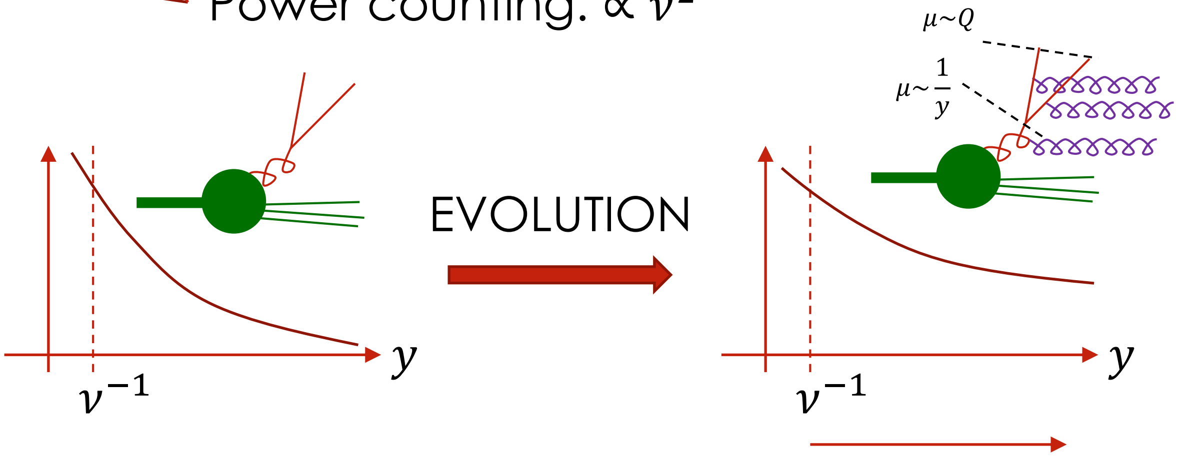
& $\sigma_{tot} \approx \sigma_{DPS}$ ✓

CUTOFF DEPENDENCE

Important: σ_{DPS} is not really 'meaningful' on its own. Can only measure $\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$

Power counting: $\propto \nu^2$

IN CERTAIN CASES:

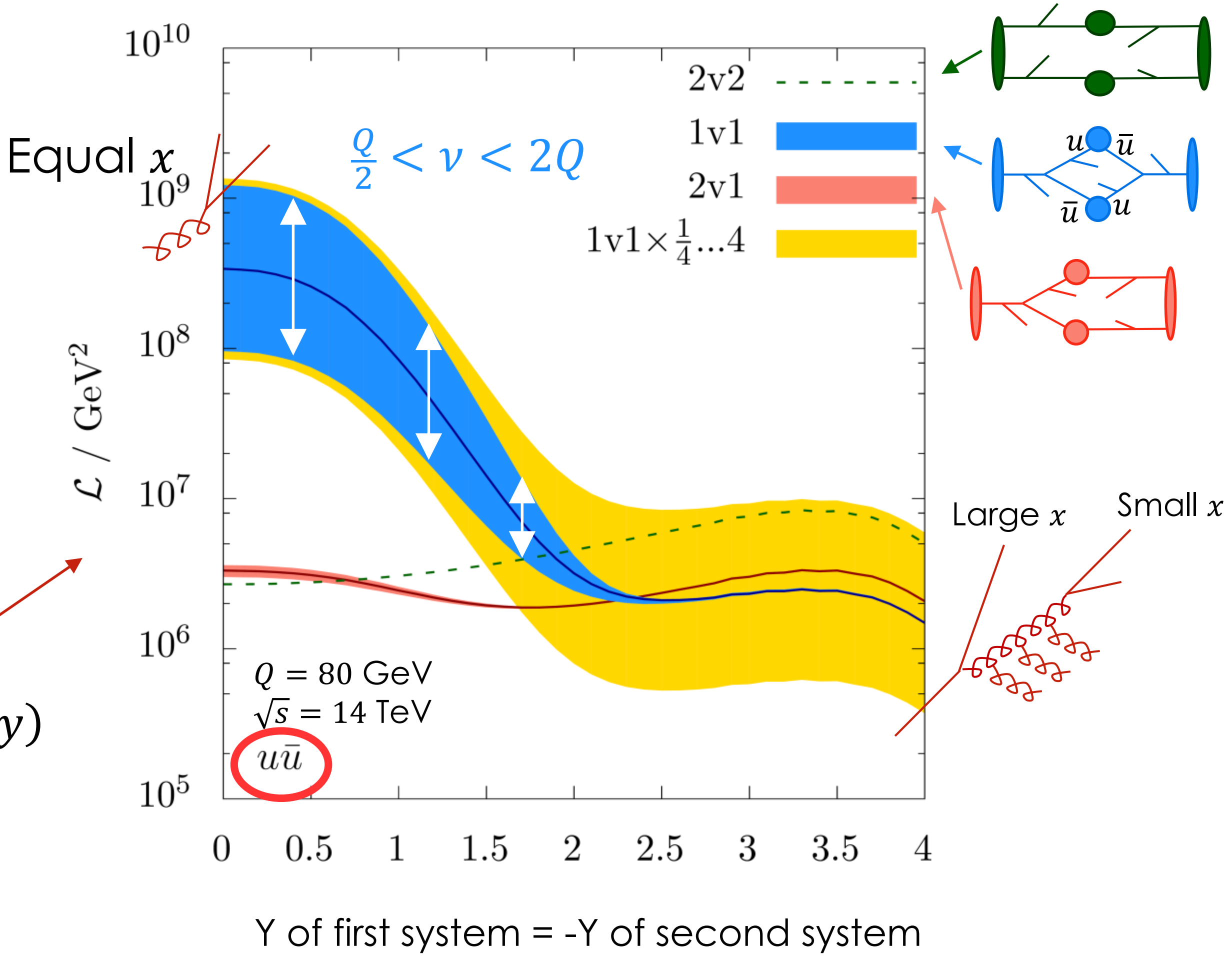


Increasing evolution space between $\mu \sim \frac{1}{y}$ & $\mu \sim \nu \sim Q$

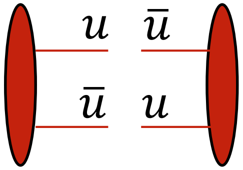
Bulk of σ_{DPS} shifts to large $\boldsymbol{y}$ where DPS approximations are valid. Small $\boldsymbol{y}$ is less important → reduced ν dependence, σ_{sub} and two-loop σ_{SPS} less important.

REDUCED CUTOFF DEPENDENCE

Example: two systems widely separated in rapidity.



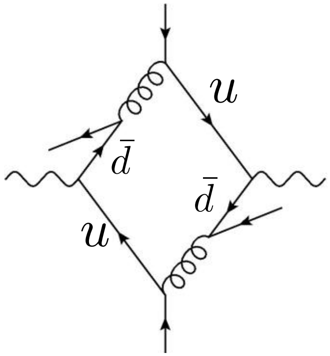
$$\mathcal{L} = \int \Phi(\nu y)^2 F_{u\bar{u}}(y) F_{\bar{u}u}(y)$$

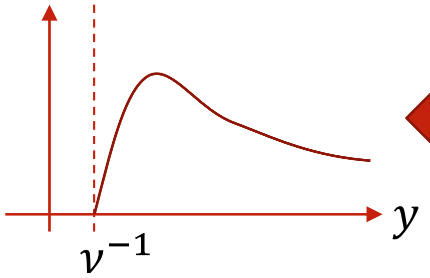


REDUCED CUTOFF DEPENDENCE

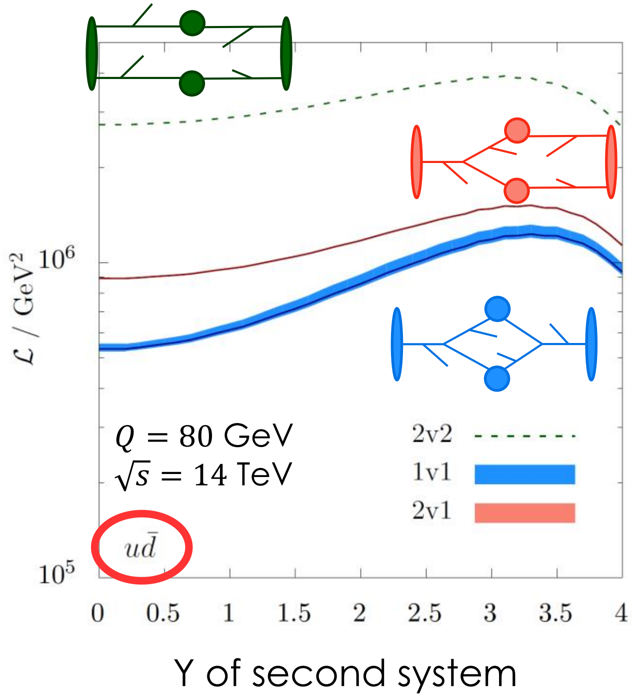
Another example where overlap considerations are less important: processes with no two-loop box contribution

E.g. Same-sign WW production





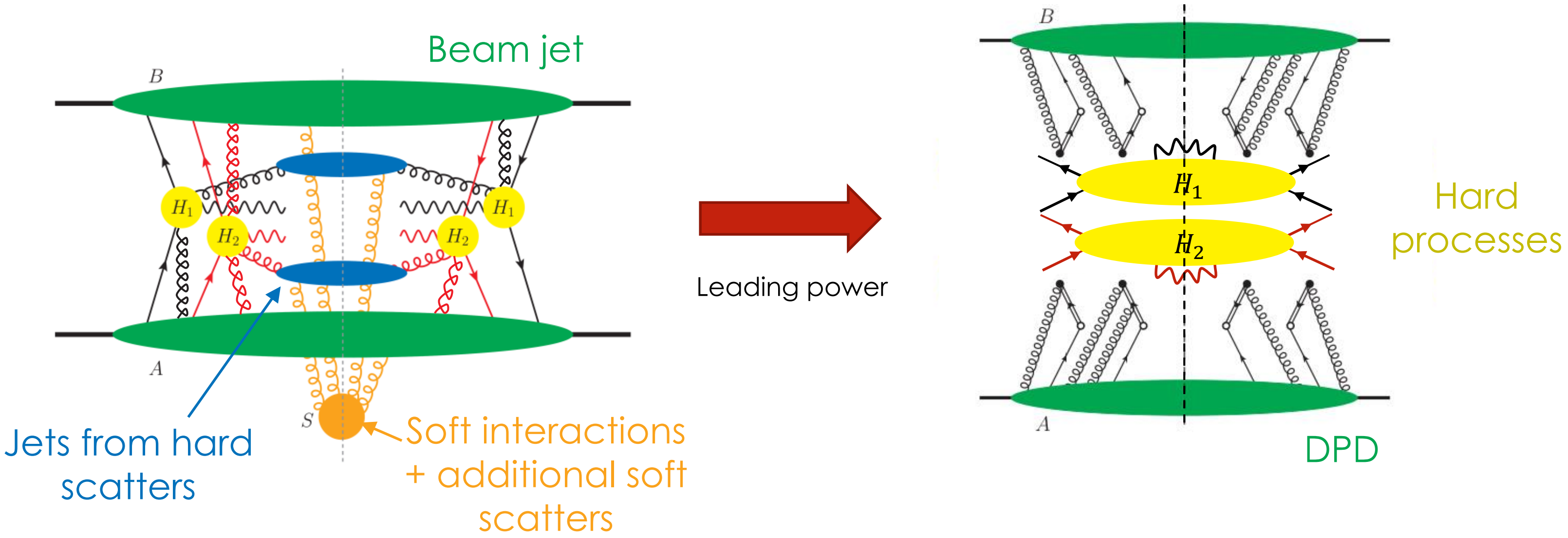
Splitting DPD profile



FACTORISATION IN DPS

FACTORISATION IN DPS

To prove factorisation for DPS inclusive cross section, need to show:



Beam jet

Jets from hard scatters

Soft interactions + additional soft scatters

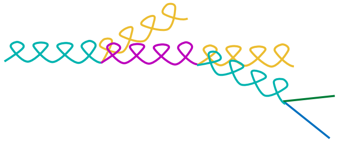
Leading power

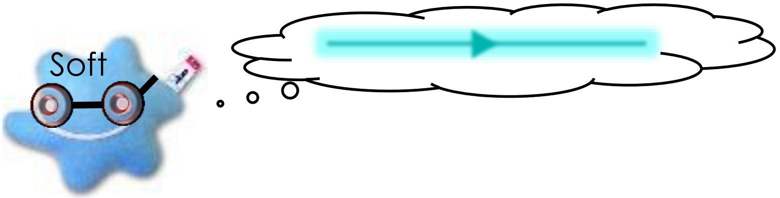
Hard processes

DPD

Key step: need to separate off all soft connections entangling beam and final state jets.

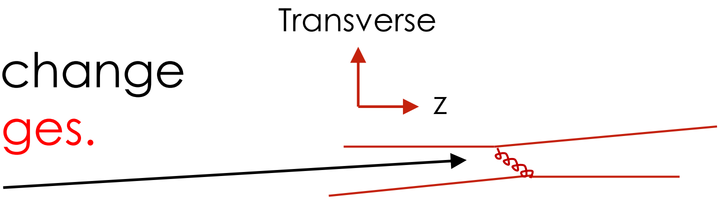
For ‘normal’ soft exchanges, this can be achieved via Ward identities:





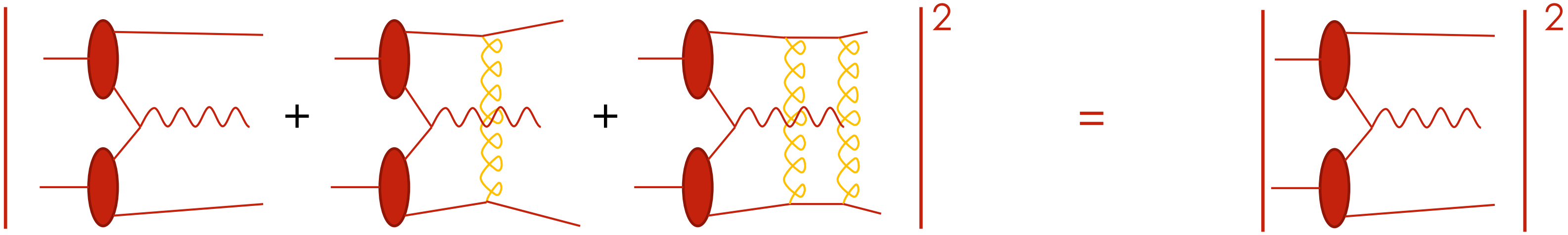
FACTORISATION: SOFT EXCHANGES

However, there is a particular type of soft exchange for which this doesn't work: Glauber exchanges. Soft particles mediating forward scattering.



Treatment of Glauber exchanges is the trickiest part of a factorisation proof!

Single scattering production of colour singlet V: Collins, Soper, Sterman showed that effect of Glauber exchanges cancels if we measure only properties of V, and sum over everything else!



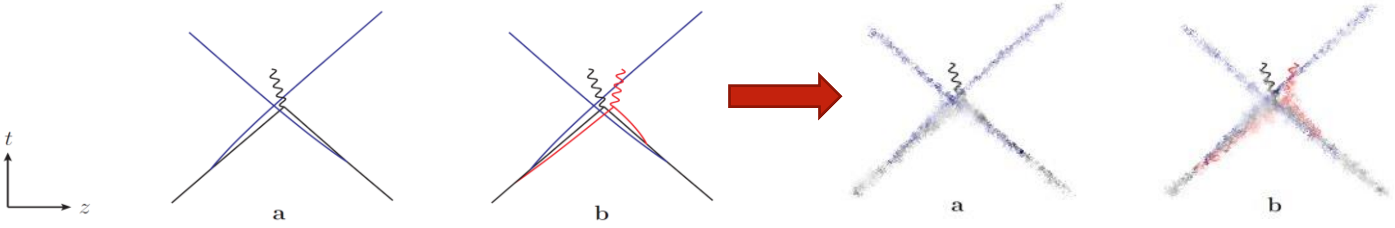
If one starts measuring properties of radiation accompanying V (e.g. global event shape variables), this argument breaks down!

JG, JHEP 1407 (2014) 110
Zeng, JHEP 1510 (2015) 189
Schwartz, Yan, Zhu, Phys.Rev. D97 (2018) no.9, 096017

GLAUBER CANCELLATION IN DPS

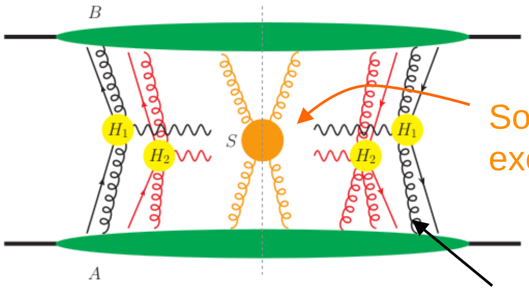
In JHEP 1601 (2016) 076 (Diehl, JG, Schäfer, Ostermeier, Plößl) we adapted the methodology of Collins, Soper, Sterman to show that Glauber exchanges also cancel for DPS production of two colourless systems.

Full proof is very technical, but can get some insight as to why it works by looking at spacetime pictures of single and double scattering:



Other important steps towards factorisation proof made in Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089 Vladimirov, JHEP 1804 (2018) 045, Diehl, Nagar, arXiv:1812.09509.

FACTORISATION IN DPS

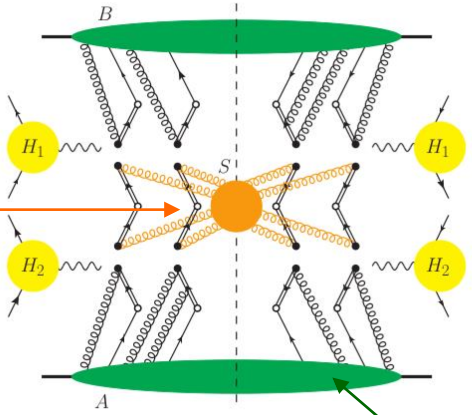


Soft and Glauber exchanges

Extra (unphysically polarised) gluon connections to hard

Initial picture

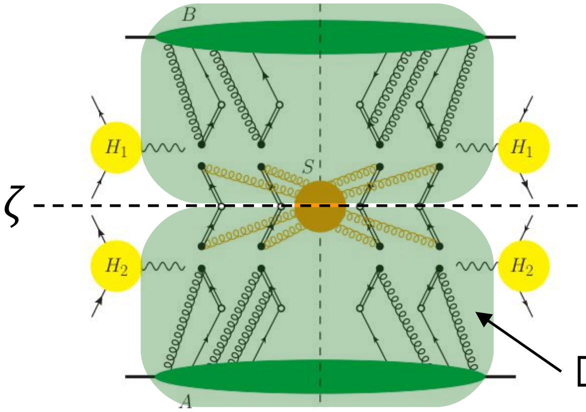
Diehl, JG, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, JHEP 1904 (2019) 124.



Soft factor

Collinear factor

Vladimirov, JHEP 1804 (2018) 045



DPDs

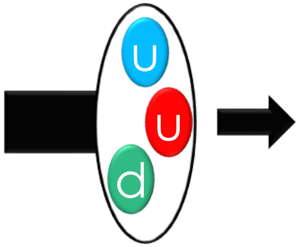
$$\sigma \sim F \otimes F \otimes \hat{\sigma} \otimes \hat{\sigma}$$

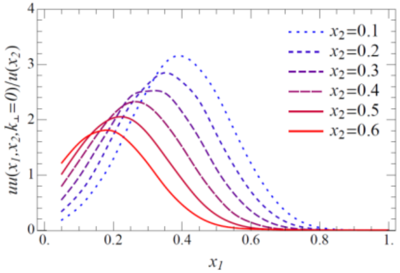
Proven, at least for double Drell-Yan production!

NONPERTURBATIVE DPD CALCULATIONS

NONPERTURBATIVE DPDS

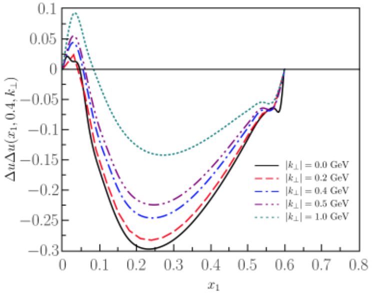
Model calculations:





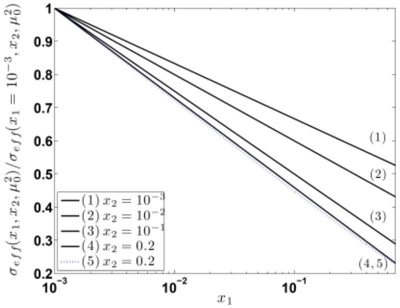
Bag model

[Phys. Rev. D 87, 034009 (2013), Manohar, Waalewijn, Chang]



Light-front CQM

[Rinaldi, Scopetta, Traini, Vento, JHEP 12 (2014) 028]



AdS/QCD

[Traini, Rinaldi, Scopetta, Vento, Phys. Lett. B 768 (2017) 270-273]

General message: factorisation of DPD into separate x_1, x_2, $\boldsymbol{y}$ pieces fails strongly at high x_i, low μ_i where these models are relevant.

Momentum and number sum rules:

[JG, Stirling, JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, Eur.Phys.J. C79 (2019) no.3, 253]
Construction of DPDs to satisfy rules in e.g. JG, Stirling, JHEP 1003 (2010) 005, Golec-Biernat et al. Phys.Lett. B750 (2015) 559-564, Diehl, JG, Lang, Plößl, Schafer, to appear

$$\sum_{j2} \int_0^{1-x_1} \mathrm{d}x_2 \, x_2 \, F^{j_1 j_2}(x_1, x_2; \mu) = (1 - x_1) f^{j_1}(x_1; \mu)$$

$$\int_0^{1-x_1} \mathrm{d}x_2 \, F^{j_1 j_{2,v}}(x_1, x_2; \mu) = \left(N_{j_{2,v}} + \delta_{j_1, \overline{j_2}} - \delta_{j_1, j_2}\right) f^{j_1}(x_1; \mu)$$

$$F(x_1, x_2; \mu) = \int d^2\boldsymbol{y} \Phi(\mu y) F(x_1, x_2, \boldsymbol{y}; \mu) + \mathcal{O}(\alpha_s)$$

NONPERTURBATIVE DPDS

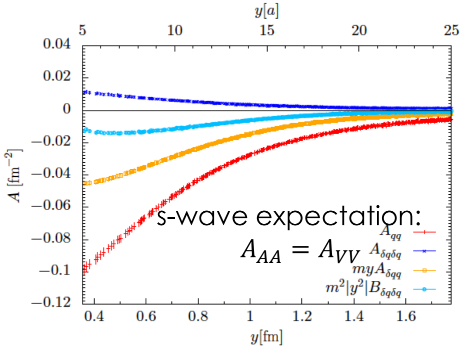
Of course, best theory input would be from lattice calculations!

Ongoing programme to compute DPD Mellin moments. Results so far only for the pion, but calculation with proton is WIP.

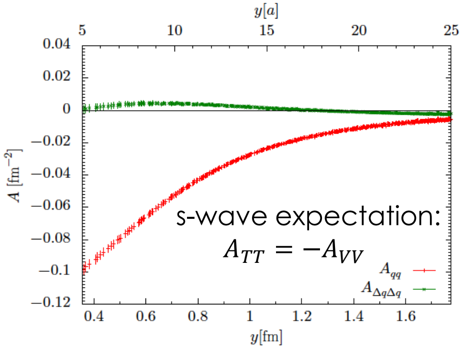
Bali, Castagnini, Diehl, JG, Gläßle, Schäfer, Zimmermann

Test of classical s-wave picture of the pion:

$$-A_{VV} \sim u^+d^+ + u^-d^- + u^+d^- + u^-d^+$$
$$+A_{AA} \sim u^+d^+ + u^-d^- - u^+d^- - u^-d^+$$
$$-A_{TT} \sim u^{\vec{s}}d^{\vec{s}} + u^{-\vec{s}}d^{-\vec{s}} - u^{\vec{s}}d^{-\vec{s}} - u^{-\vec{s}}d^{\vec{s}}$$

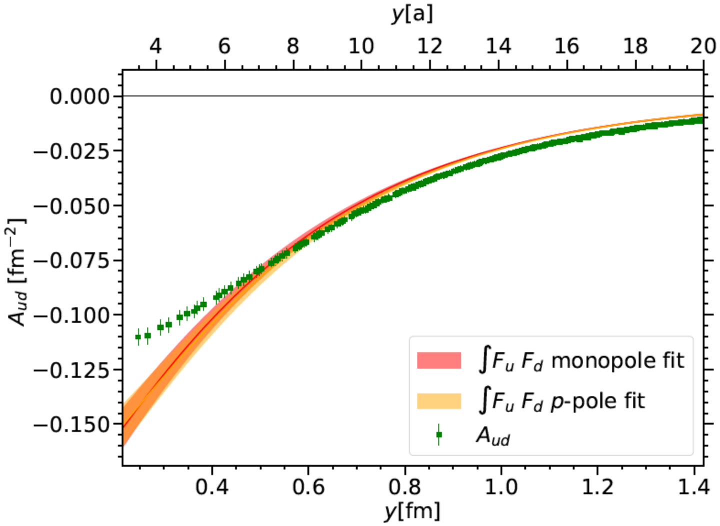


s-wave expectation:
$A_{AA} = A_{VV}$



s-wave expectation:
$A_{TT} = -A_{VV}$

Factorisation test:



arXiv:2006.14826

LATTICE DPDS – SOME DETAILS

$$\underbrace{F(x_1, x_2, \boldsymbol{y})} \propto \int \mathrm{d}y^- \mathrm{d}z_i^- e^{ix_i p^+ z_i^-} \langle p | \mathcal{O}\left(y + \tfrac{1}{2}z_1, y - \tfrac{1}{2}z_1\right) \mathcal{O}\left(\tfrac{1}{2}z_2, -\tfrac{1}{2}z_2\right) | p \rangle \Big|_{y^+=0,\, z_i^+=0,\, \boldsymbol{z}_i=\boldsymbol{0}}$$

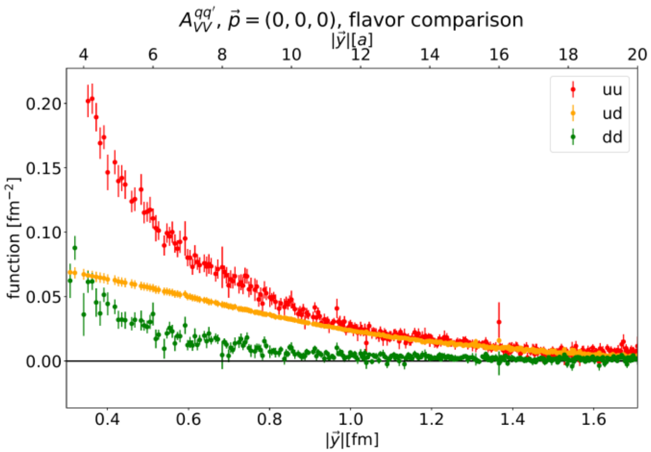
$$\int \mathrm{d}x_1 \mathrm{d}x_2\, F(x_1, x_2, \boldsymbol{y}) \propto \int \mathrm{d}y^- \langle p | \mathcal{O}(y) \mathcal{O}(0) | p \rangle \Big|_{y^+=0}$$

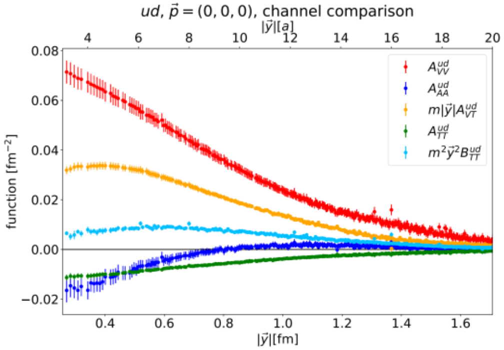
$$\propto \int d(p \cdot y)\, \underbrace{\langle \mathcal{O}\mathcal{O} \rangle \left(p \cdot y, y^2\right)} \Big|_{y^2=-\boldsymbol{y}^2}$$

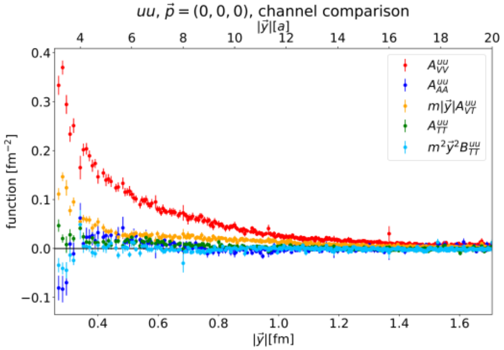
Can compute in Euclidean region on lattice. Implies:

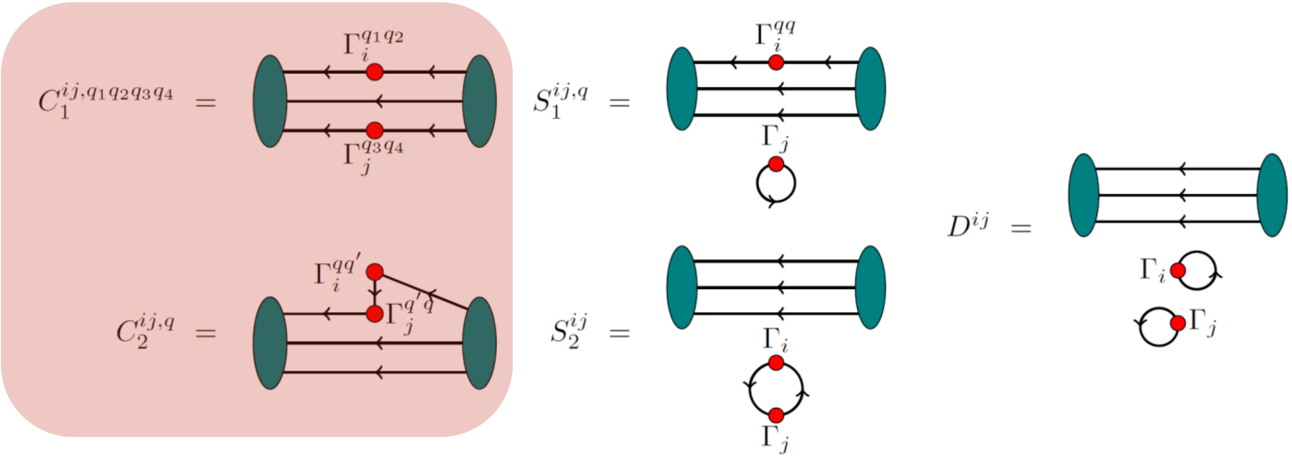
$$\frac{(p \cdot y)^2}{-y^2} = \frac{(\vec{p} \cdot \vec{y})^2}{\vec{y}^2} \leq \vec{p}^2$$

LATTICE DPDS – SOME DETAILS

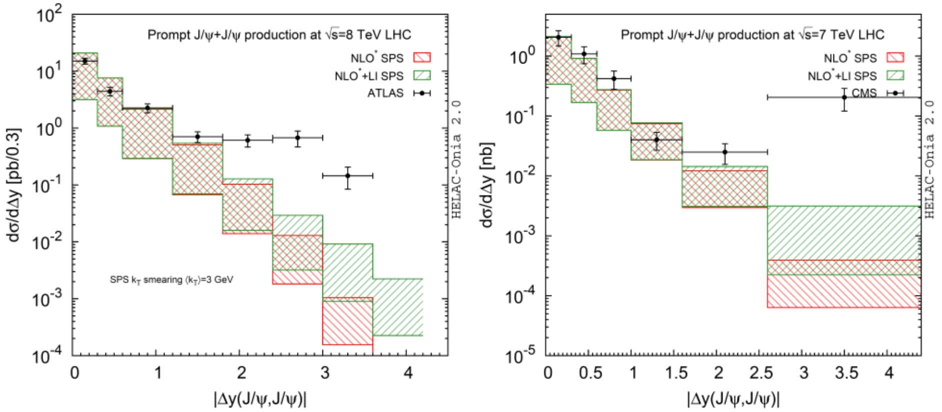

$A_{VV}^{qq'}$, $\vec{p} = (0,0,0)$, flavor comparison


ud, $\vec{p} = (0,0,0)$, channel comparison

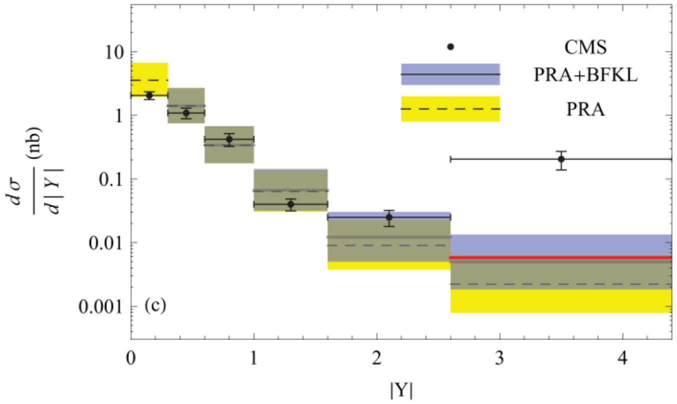

uu, $\vec{p} = (0,0,0)$, channel comparison



STATE-OF-THE-ART DOUBLE J/Ψ SPS



Lansberg, Shao, Yamanaka, Zhang
arXiv:1906.10049



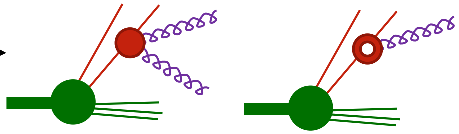
He, Kniehl, Nefedov, Saleev
Phys.Rev.Lett. 123 (2019) no.16, 162002

NEXT-TO-LEADING ORDER

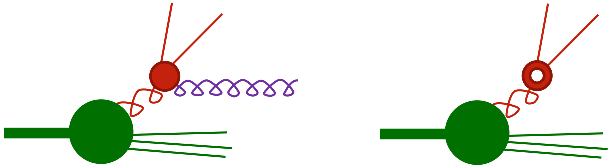
NLO CORRECTIONS TO DPS

DGS framework opens the way for the first NLO computations of DPS. What is needed for these computations?

- NLO corrections to partonic cross sections: already known for many processes from SPS calculations ✓
- NLO 'usual' splitting functions - needed for evolution of $\boldsymbol{F(y)}$: already known since the 80s ✓ Curci, Furmanski, Petronzio, Nucl. Phys. B175, 27 (1980), Furmanski, Petronzio, Phys. Lett. 97B, 437 (1980),…

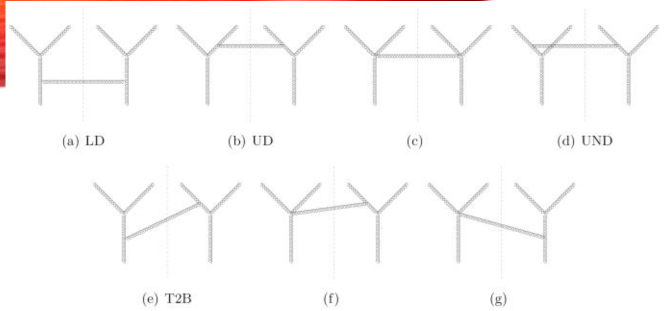


- NLO corrections to the splitting - recently computed! ✓



Diehl, JG, Plößl, Schäfer, SciPost Phys. 7 (2019) 2, 017

NLO: METHOD



Compute graph expressions (`FORM, FeynCalc`). Integrate over minus components using contours.

[Kuipers, Ueda, Vermaseren, Vollinga, Comput. Phys. Commun. 184 (2013) 1453-1467]
[Shtabovenko, Mertig, Orellana, Comput. Phys. Commun. 207 (2016) 432-444]

$$D_1 = \frac{(\boldsymbol{k}_1+\boldsymbol{\Delta})^2}{x_1} + \frac{(\boldsymbol{k}_2-\boldsymbol{\Delta})^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3} \qquad D_2 = \frac{\boldsymbol{k}_1^2}{x_1} + \frac{\boldsymbol{k}_2^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3}$$

$$D_3 = (\boldsymbol{k}_1+\boldsymbol{\Delta})^2 \qquad D_4 = \boldsymbol{k}_2^2 \qquad \tilde{D}_4 = \boldsymbol{k}_1^2 \qquad \tilde{D}_5 = (\boldsymbol{k}_1+\boldsymbol{k}_2)^2$$

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}} \qquad I_2(a_1,a_2,a_3,a_4,a_5) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1...3} D_i^{a_i} \prod_{i=4...5} \tilde{D}_i^{a_i}}$$

$$I_1(1,1,0,0), I_1(0,1,1,0), I_1(1,1,1,0),$$
$$I_1(1,0,1,1), I_1(1,1,1,1), I_1(2,1,1,1)$$

$$I_2(0,1,1,0,1), I_2(1,1,1,1,0)$$

Integration-by-parts reduction to master integrals (`LiteRed`)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

$$\begin{bmatrix} \frac{\partial I_1(1,1,0,0)}{\partial x_1} \\ \frac{\partial I_1(0,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,0,1,1)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,1)}{\partial x_1} \\ \frac{\partial I_1(2,1,1,1)}{\partial x_1} \end{bmatrix} = \begin{bmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacksquare & 0 & 0 & 0 \\ 0 & \blacklozenge & 0 & \blacksquare & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \end{bmatrix} \begin{bmatrix} I_1(1,1,0,0) \\ I_1(0,1,1,0) \\ I_1(1,1,1,0) \\ I_1(1,0,1,1) \\ I_1(1,1,1,1) \\ I_1(2,1,1,1) \end{bmatrix}$$

Construct differential equations in x_1 and solve (`Fuchsia`)

[Gituliar, Magerya, Comput. Phys. Commun. 219 (2017) 329-338]

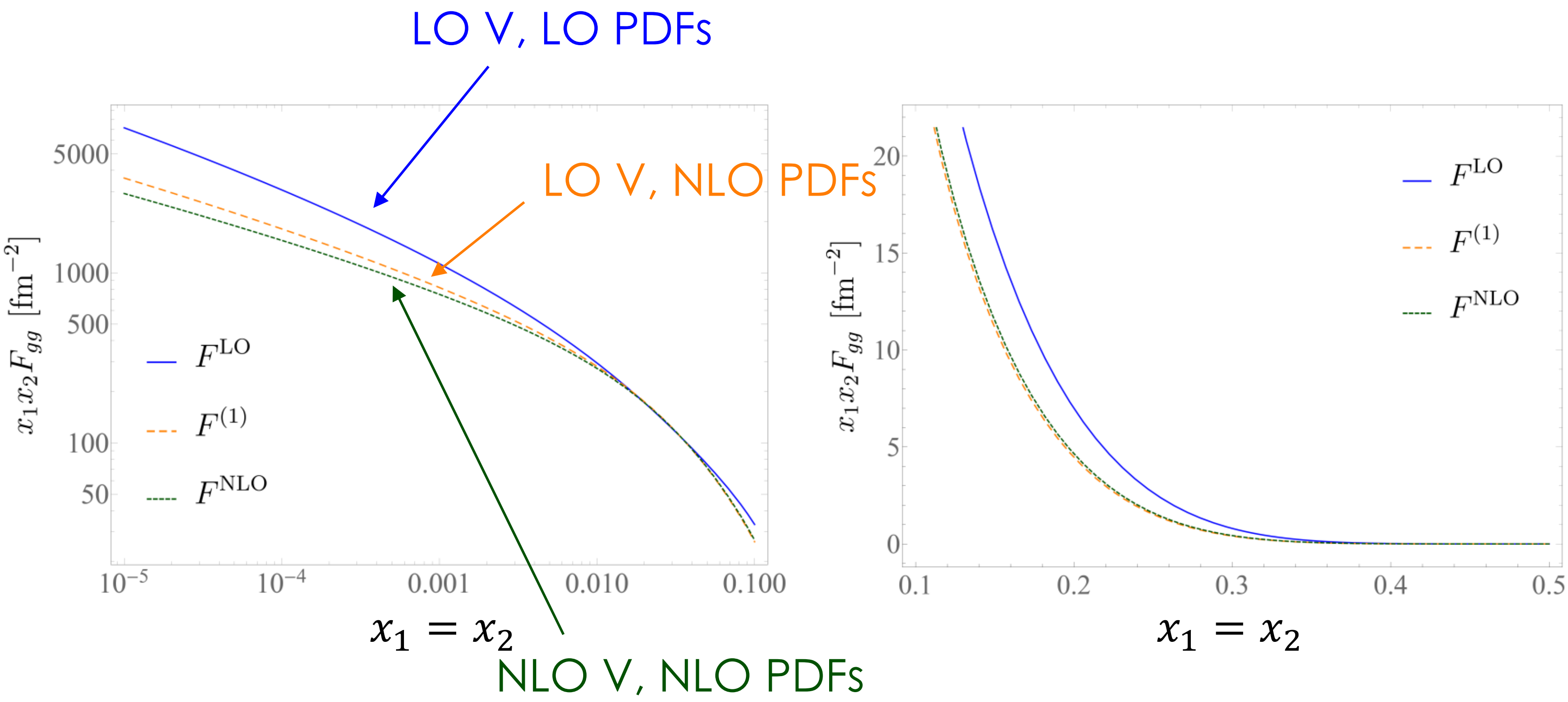
$$I_1(0,1,1,0) \to \pi^{3-2\epsilon} x_3^{1-\epsilon} (x_1 x_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi\epsilon]\Gamma[1-3\epsilon]}$$

Computation of $x_3 \to 0$ limit of master integrals using method of regions (boundary conditions)

Results for bare graphs!

NLO: SOME NUMERICS

Scale $10\ \mathrm{GeV}$, splitting contribution only, no evolution after splitting

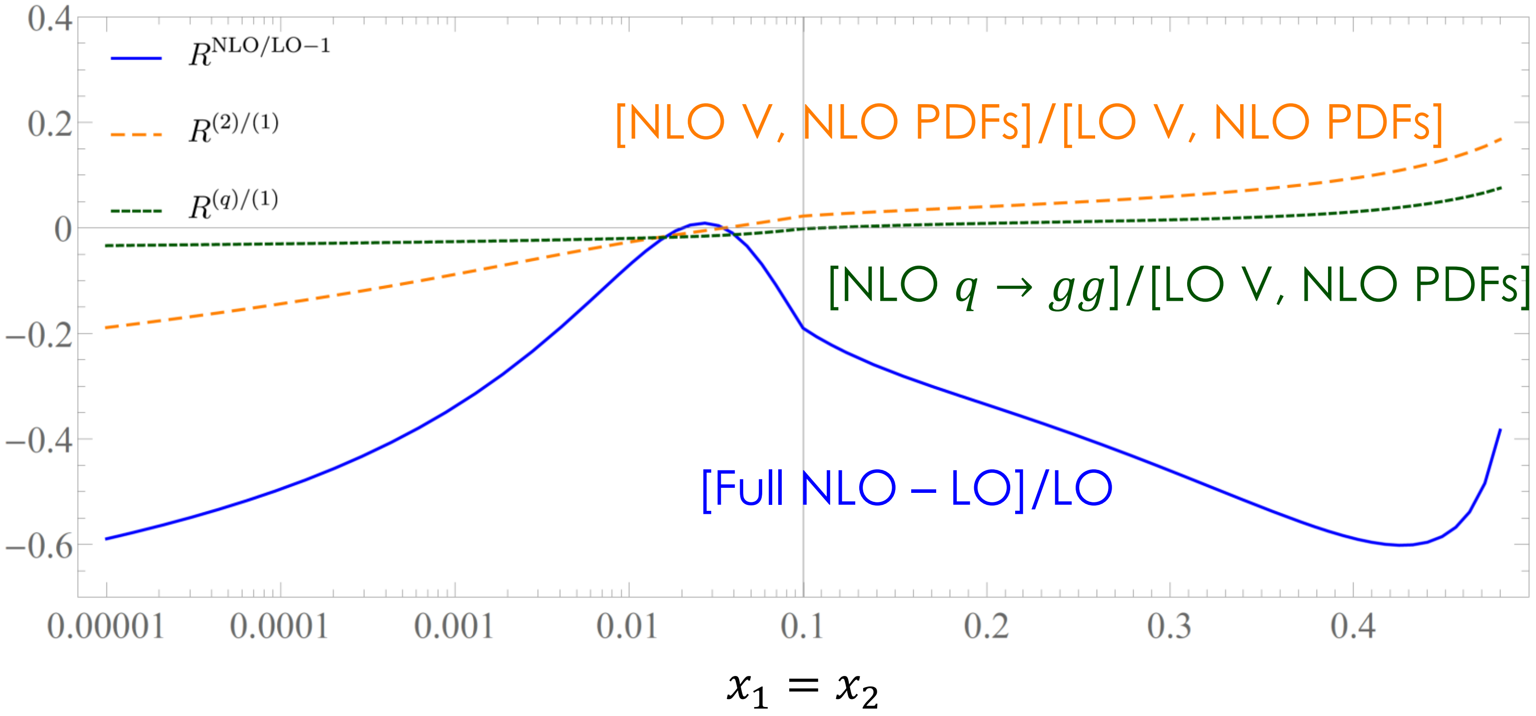


LO V, LO PDFs

LO V, NLO PDFs

NLO V, NLO PDFs

NLO: SOME NUMERICS



[NLO V, NLO PDFs]/[LO V, NLO PDFs]

[NLO $q \to gg$]/[LO V, NLO PDFs]

[Full NLO – LO]/LO

TRANSVERSE MOMENTUM IN DPS

TRANSVERSE MOMENTUM IN DPS

Small $\boldsymbol{q}_i$ region particularly important for DPS – DPS & SPS same power

Parton model analysis: $$\frac{d\sigma^{(A,B)}}{d^2\boldsymbol{q_1} d^2\boldsymbol{q_2}} \sim \int d^2\boldsymbol{y} d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_1\cdot\boldsymbol{q}_1 - i\boldsymbol{z}_2\cdot\boldsymbol{q}_2} \underbrace{F(\boldsymbol{z_1},\boldsymbol{z_2},\boldsymbol{y})}_{\text{DTMDs}}\; F(\boldsymbol{z_1},\boldsymbol{z_2},\boldsymbol{y})$$

Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

QCD treatment of transverse momentum in DPS (including DGS-style double counting subtraction) developed in Buffing, Diehl, Kasemets JHEP 1801 (2018) 044. DPS cross section in QCD:

$$\frac{d\sigma_{\text{DPS}}}{dx_1 dx_2 d\overline{x}_1 d\overline{x}_2 d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} = \frac{1}{C}$$
$$\cdot \sum_{a_1,a_2,b_1,b_2} \widehat{\sigma}_{a_1 b_1}(Q_1,\mu_1)\,\widehat{\sigma}_{a_2 b_2}(Q_2,\mu_2)$$
$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\frac{d^2\boldsymbol{z}_2}{(2\pi)^2} d^2\boldsymbol{y}$$
$$\cdot\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2} W_{a_1a_2b_1b_2}(\overline{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu),$$

Cut-off functions

$$W_{a_1a_2b_1b_2}(\overline{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu) = \Phi(\nu\boldsymbol{y}_+)\,\Phi(\nu\boldsymbol{y}_-)$$
$$\times \sum_R \eta_{a_1a_2}(R)\, {}^{R}F_{b_1b_2}(\overline{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \overline{\zeta})$$
$$\cdot\, {}^{R}F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta).$$

Dependence on ren. scales μ_i AND a rapidity scale ζ.

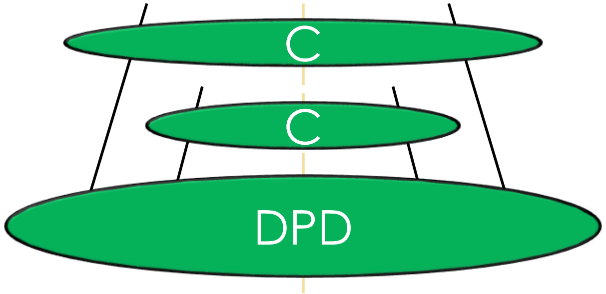
Evolution of DTMDS in all of these scales known at one loop.

TRANSVERSE MOMENTUM IN DPS

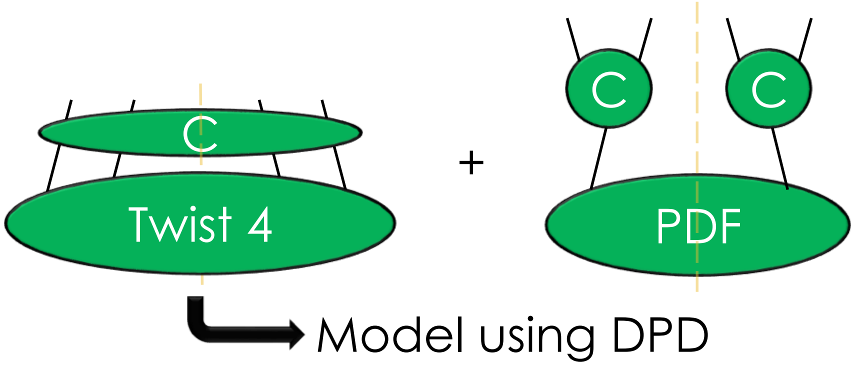
Still need some ‘initial’ expressions for the DTMDs. Function of many arguments $(x_i, \boldsymbol{y}, \boldsymbol{z}_i)$. Hopeless?

For perturbative $|\boldsymbol{q}_i| \gg \Lambda$ can expand DTMDs in terms of collinear quantities:

Large $\boldsymbol{y} \sim 1/\Lambda$:



Small $\boldsymbol{y} \sim 1/q_T \sim |\boldsymbol{z_i}|$:



Model using DPD

So then, need only DPDs and PDFs: very good prospects for phenomenology at perturbative $|\boldsymbol{q}_i|$!

Brief overview of transverse momentum in DPS given in JG, Kasemets, Advances in High Energy Physics, 2019, 3797394

DSHOWER

DSHOWER ALGORITHM

(1) Select x_i of initiating partons and y using DPS formula:

$$\sigma^{\mathrm{DPS}}_{(A,B)}(s) = \frac{1}{1+\delta_{AB}} \sum_{i,j,k,l} \int \mathrm{d}\tau_A\, \mathrm{d}Y_A\, \mathrm{d}\hat{t}_A\, \mathrm{d}\tau_B\, \mathrm{d}Y_B\, \mathrm{d}\hat{t}_B\, \frac{\mathrm{d}\hat{\sigma}_{ij\to A}}{\mathrm{d}\hat{t}_A}\, \frac{\mathrm{d}\hat{\sigma}_{kl\to B}}{\mathrm{d}\hat{t}_B} \times \int 2\pi\, y\, \mathrm{d}y\, \Phi^2(y\nu)\, F_{ik}(x_1, x_3, \boldsymbol{y}, \mu^2)\, F_{jl}(x_2, x_4, \boldsymbol{y}, \mu^2)$$

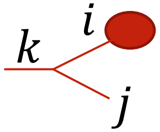
DPDs

Cut-off of DPS for y values $\lesssim 1/\nu \sim 1/Q$

DSHOWER ALGORITHM

(2) Generate QCD emissions, going backwards from hard process

In shower must select an evolution variable. We make the same choice as Herwig:



For ISR:

$$Q^2 = \tilde{q}_{ISR}^2 = -\frac{\left(p_i^2 - m_i^2\right)}{(1-z)} \approx E_k^2\theta_j^2 \longleftarrow \text{Angular ordering}$$

Probability that partons ij survive from Q_h to Q, and then at Q there is an emission from one leg:

$$\mathrm{d}\mathcal{P}_{ij}^{\mathrm{ISR}} = \underset{\substack{\uparrow \\ \text{Emission probability}}}{\mathrm{d}\mathcal{P}_{ij}} \exp \underbrace{\left(-\int_{Q^2}^{Q_h^2} \mathrm{d}\mathcal{P}_{ij}\right)}_{\text{‘Sudakov factor’}}$$

Emission from leg 1

$$\mathrm{d}\mathcal{P}_{ij} = \frac{\mathrm{d}Q^2}{Q^2}\left(\sum_{i'}\int_{x_1}^{1-x_2}\frac{\mathrm{d}x_1'}{x_1'}\frac{\alpha_s(p_\perp^2)}{2\pi}P_{i'\to i}\left(\frac{x_1}{x_1'}\right)\frac{F_{i'j}(x_1', x_2, \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)} + \sum_{j'}\int_{x_2}^{1-x_1}\frac{\mathrm{d}x_2'}{x_2'}\frac{\alpha_s(p_\perp^2)}{2\pi}P_{j'\to j}\left(\frac{x_2}{x_2'}\right)\frac{F_{ij'}(x_1, x_2', \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)}\right)$$

Emission from leg 2

Use ‘competing veto algorithm’ to decide which leg emits

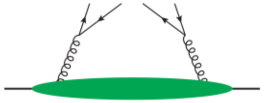
DSHOWER ALGORITHM

(3) At scale $\mu_y \sim 1/y$, decide whether to merge partons i and j. Merging is done with a probability given by:

$$p_{Mrg} = F_{ij}^{spl}(x_1, x_2, y, \mu_y^2)/F_{ij}^{tot}(x_1, x_2, y, \mu_y^2)$$

Total DPD

$$F_{ij}^{spl}(x_1, x_2, y, \mu_y^2) = \frac{1}{\pi y^2}\frac{f_k(x_1+x_2, \mu_y^2)}{x_1+x_2}\frac{\alpha_s(\mu_y^2)}{2\pi}P_{k\to ij}\left(\frac{x_1}{x_1+x_2}\right)\times\text{large } \boldsymbol{y} \text{ suppression}$$

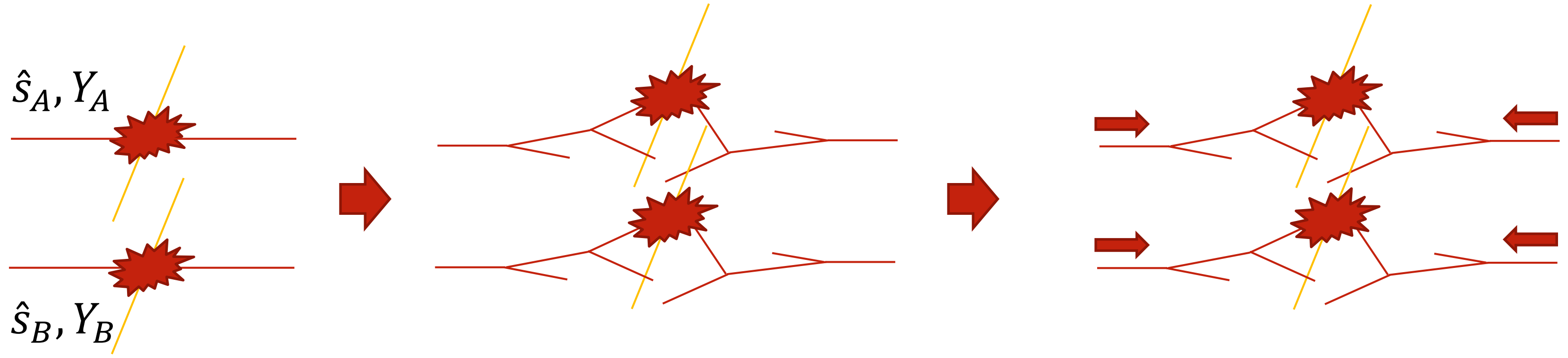


If no merging: continue with two parton branching algorithm from (2), using only ‘intrinsic’ DPDs.

If merging: shower single parton a la Herwig.

KINEMATICS: NO MERGING

No merging:



Generate hard process using DPS σ

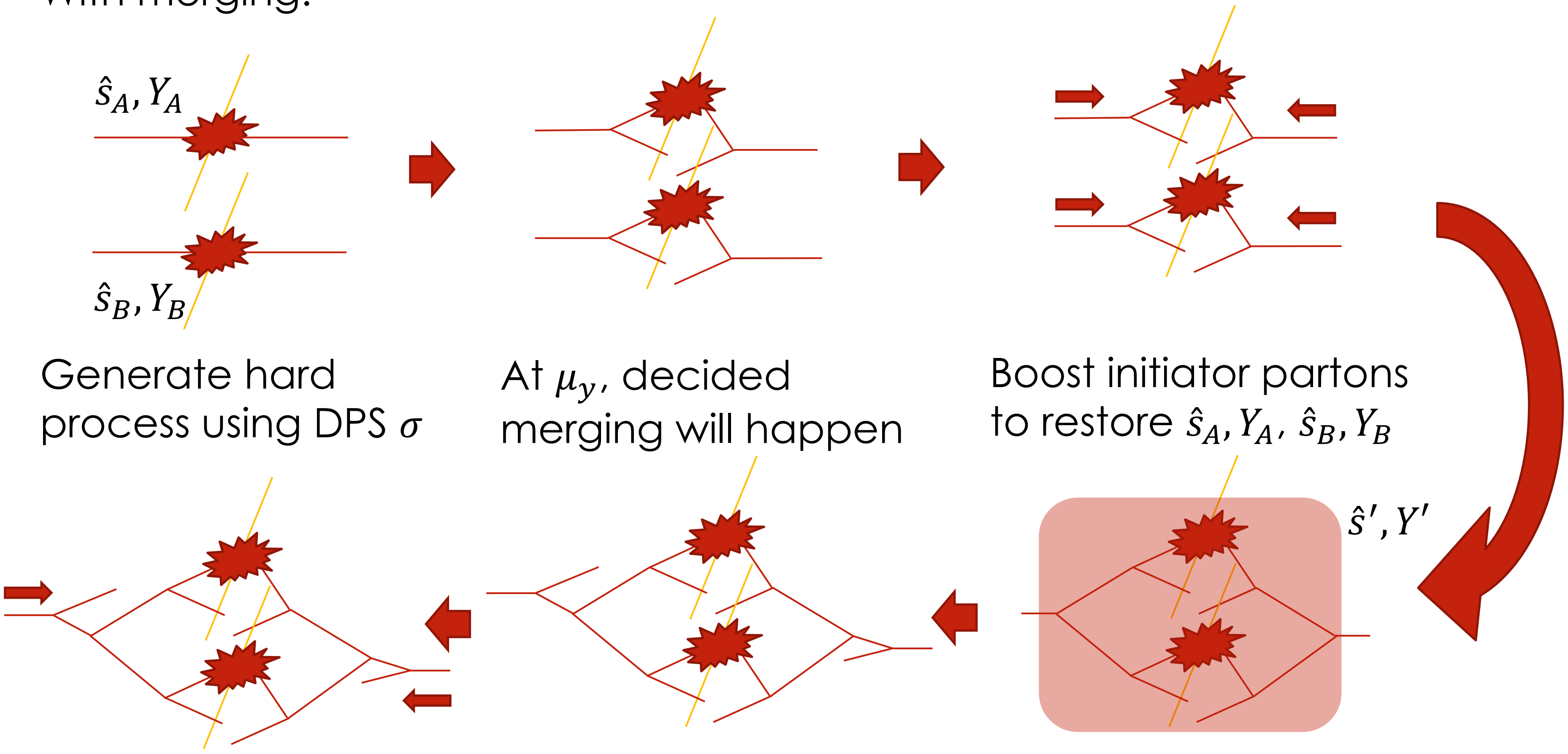
Add shower, kinematics of hard processes altered

Boost initiator partons to restore $\hat{s}_A, Y_A, \hat{s}_B, Y_B$

Works as 4 variables (boosts) and 4 constraints! What about if there is a merging? 2/3 initiator partons → overconstrained system!

KINEMATICS: MERGING

With merging:



Generate hard process using DPS σ

At μ_y, decided merging will happen

Boost initiator partons to restore $\hat{s}_A, Y_A$, $\hat{s}_B, Y_B$

Merge (zero p_T, or $p_T \sim \mu_y$). Define new hard system.

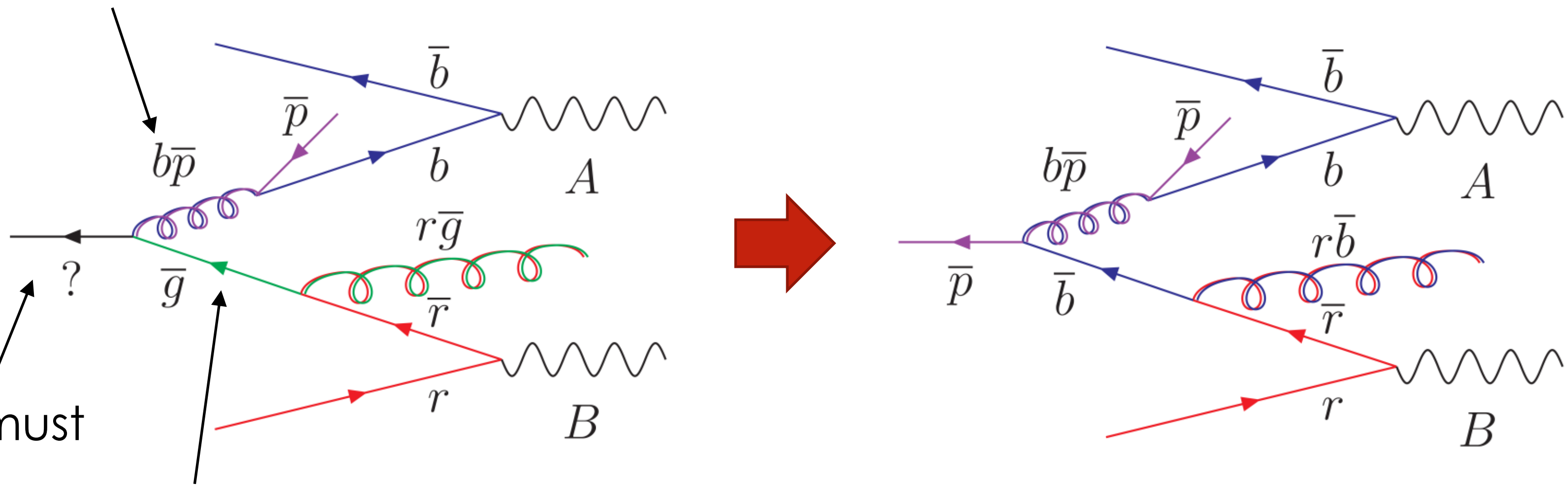
Continue shower

Boost initiator partons to restore $\hat{s}', Y'$

COLOUR WITH MERGING

Shower uses large N_c approximation. Each new emission → new colour. Independent showers before merging.

Mergings require some colour reshuffling. We impose minimal colour disruption.



Change $b \to g$?

This must be $\bar{p}$

Change $\bar{g} \to \bar{b}$?

Not so important for parton-level simulation, but could be important when we add hadronisation

COMBINING DPS AND SPS IN THE SHOWER

IMPLEMENTATION

How do we implement this in practice?

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \underbrace{\boldsymbol{S}_1(t_1)\otimes\left[\frac{d\sigma^{SPS}_{A+B}}{dO} - \frac{d\sigma^{sub}_{(A,B)}}{dO}\right]}_{\text{SPS-type events (‘type 1’)}} + \underbrace{\int d^2\boldsymbol{y}\,\boldsymbol{S}_2(t_2)\otimes\frac{d\sigma^{DPS}_{(A,B)}}{dO\,d^2\boldsymbol{y}}}_{\text{DPS-type events (‘type 2’)}}$$

Phase space for two pieces is different.
Consider e.g. on-shell diboson production (ZZ)

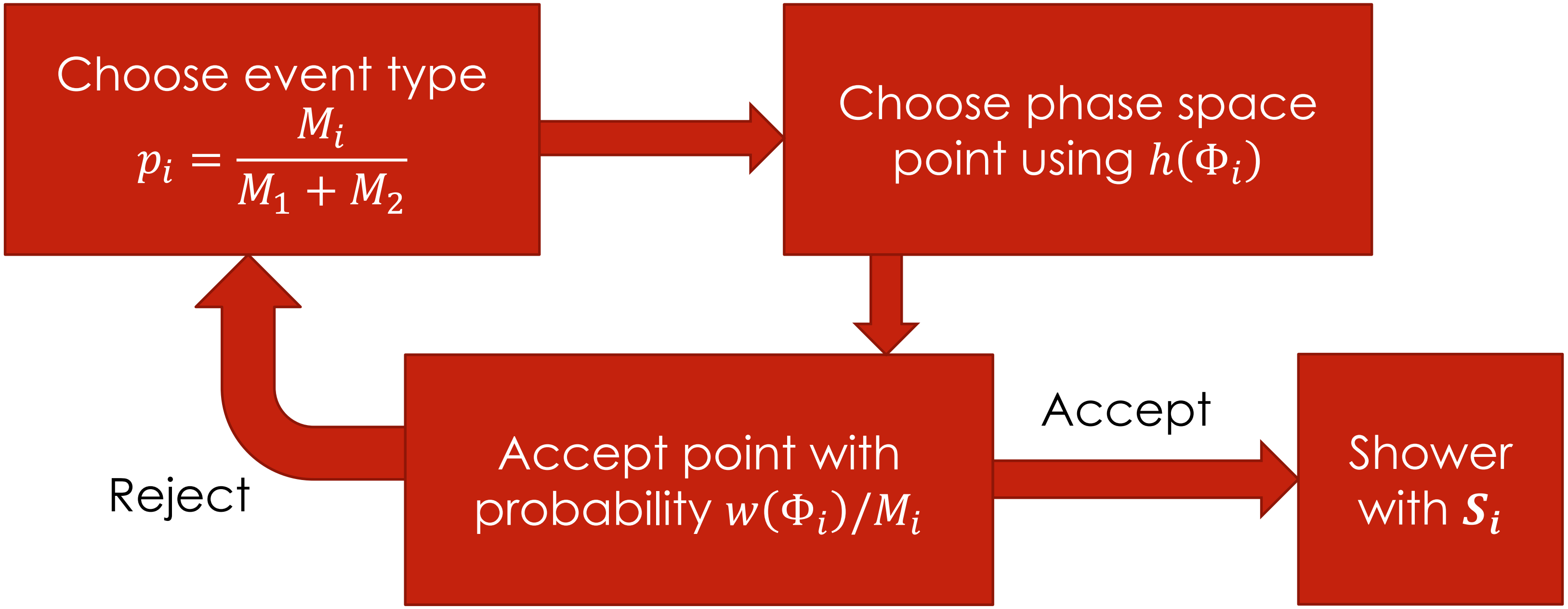
$$\Phi_1 = \{Y_1, Y_2, p_T\} \qquad\qquad \Phi_2 = \{Y_1, Y_2, y\}$$

IMPLEMENTATION

For each event type, define weight: $w(\Phi_i) = \frac{1}{h(\Phi_i)} \frac{d\sigma_i}{d\Phi_i}$ Dimension = $[\sigma]$

$$\int h(\Phi_i) d\Phi_i = 1$$

$$M_i = \max_{\Phi_i} [w(\Phi_i)]$$



Choose event type $p_i = \frac{M_i}{M_1 + M_2}$

Choose phase space point using $h(\Phi_i)$

Accept point with probability $w(\Phi_i)/M_i$

Reject

Accept

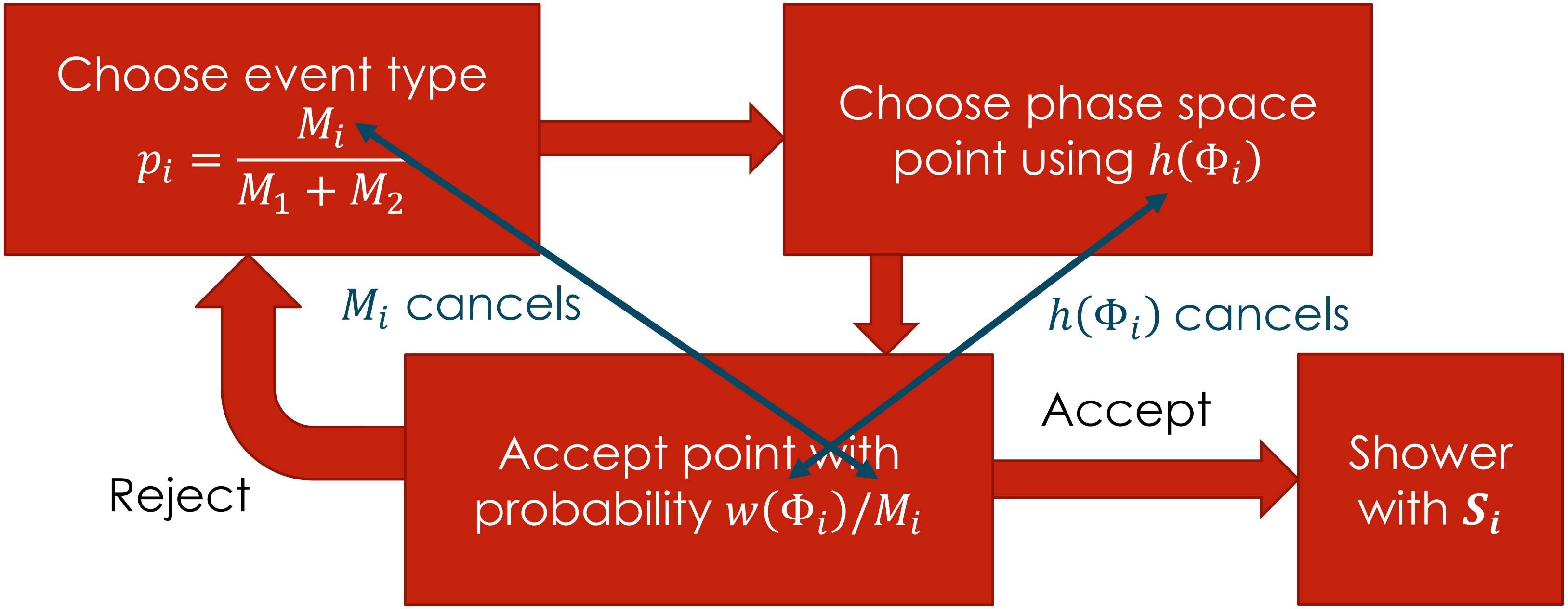
Shower with $\boldsymbol{S_i}$

IMPLEMENTATION

For each event type, define weight: $w(\Phi_i) = \frac{1}{h(\Phi_i)} \frac{d\sigma_i}{d\Phi_i}$ Dimension = $[\sigma]$

$$M_i = \max_{\Phi_i}[w(\Phi_i)]$$

$$\int h(\Phi_i) d\Phi_i = 1$$



Choose event type $p_i = \frac{M_i}{M_1 + M_2}$

Choose phase space point using $h(\Phi_i)$

Accept point with probability $w(\Phi_i)/M_i$

Shower with $\boldsymbol{S}_i$

Accept

Reject

M_i cancels

$h(\Phi_i)$ cancels

THE SUBTRACTION: LARGE & SMALL Y

$$\frac{d\sigma_{A+B}^{tot}}{dO} = \boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma_{A+B}^{SPS}}{dO} - \frac{d\sigma_{(A,B)}^{sub}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma_{(A,B)}^{DPS}}{dO\, d^2\boldsymbol{y}}$$

If sub kinematics correctly reproduces double splitting kinematics of DPS term → DPS & sub cancel at small y, give $d\sigma_{A+B}^{SPS}/dO$

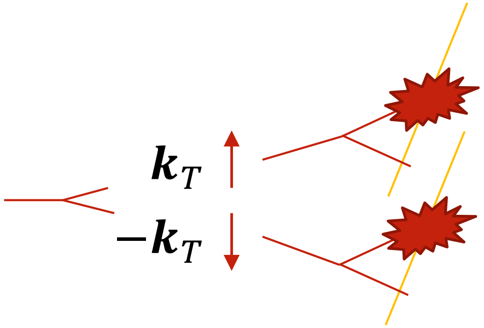
Want sub and SPS loop-induced term to cancel at large y (also differential in O). But we don't have SPS differential in y.

One thing we can look at is p_T of Z bosons – small p_T behaviour dominated by large y!

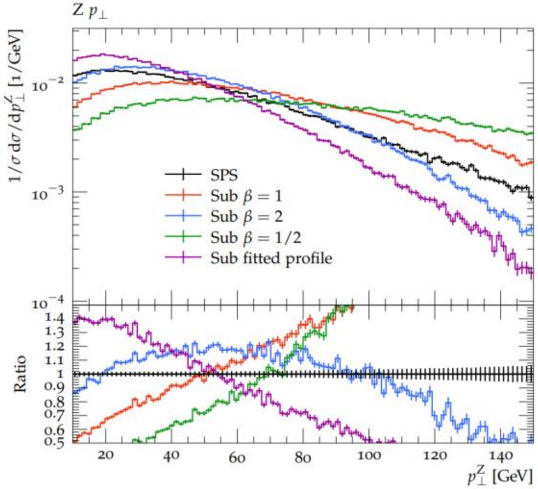
JG, Stirling,
JHEP 06 (2011) 048

THE SUBTRACTION: LARGE & SMALL Y

Want sub and SPS to coincide as closely as possible at small p_T - constrains splitting p_T kinematics in sub & DPS terms.



$\boldsymbol{k}_T$ distributed according to $g(\boldsymbol{k}_T, y)$



Options: (a) Gaussian $g(\boldsymbol{k}_T, y)$:

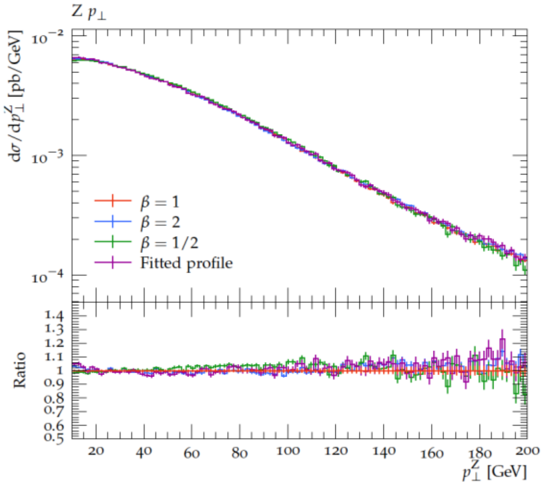
$$g(\boldsymbol{k}_T, y) = \frac{\beta}{\pi} y^2 exp(-\beta y^2 k_T^2)$$

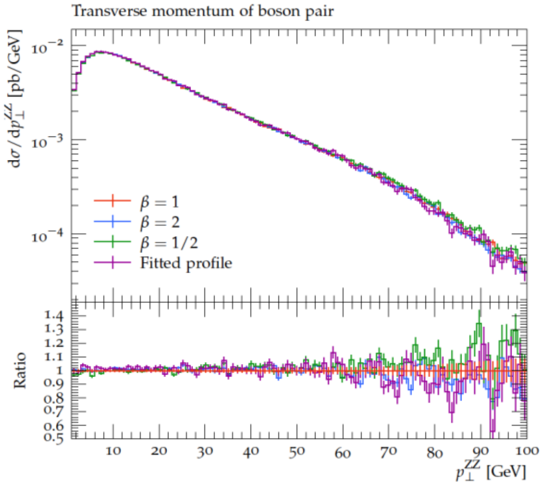
(b) 'Decreasing Gaussian' (more realistic)

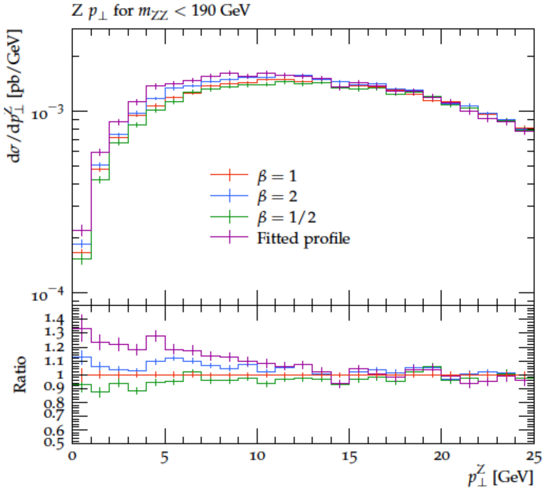
$$g(\boldsymbol{k}_T, y) = \frac{1}{\pi\sqrt{2}} \frac{y}{k_T} exp\left(-\frac{\pi}{2} y^2 k_T^2\right)$$

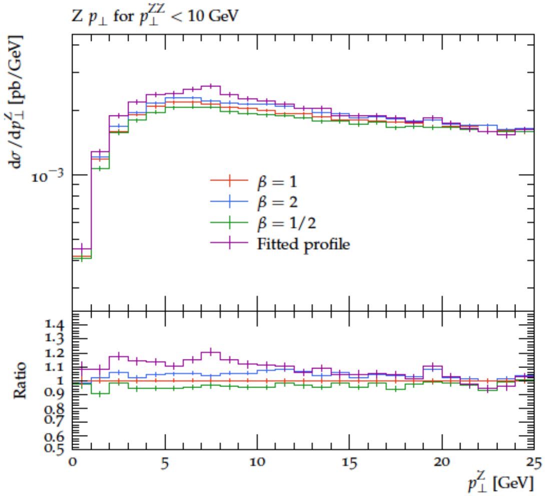
DIFFERENT PROFILES

Many distributions: ~ no difference







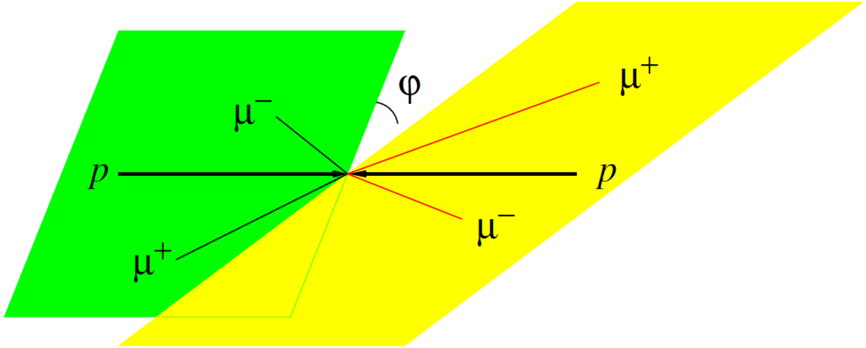


Can see some small differences focussing on region where p_Ts of both bosons are small

CORRELATIONS

CORRELATIONS

Partons in DPS can also be correlated in spin & colour.



Can have interesting effects beyond a change in rate – e.g. transverse spin correlations can cause φ distribution to have a non-flat shape.

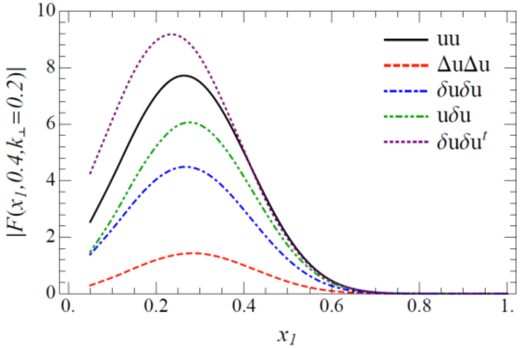
Framework for incorporating these correlations is known.

Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

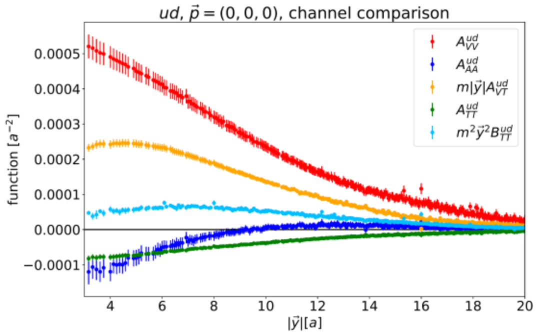
How important are these effects?

SPIN CORRELATIONS

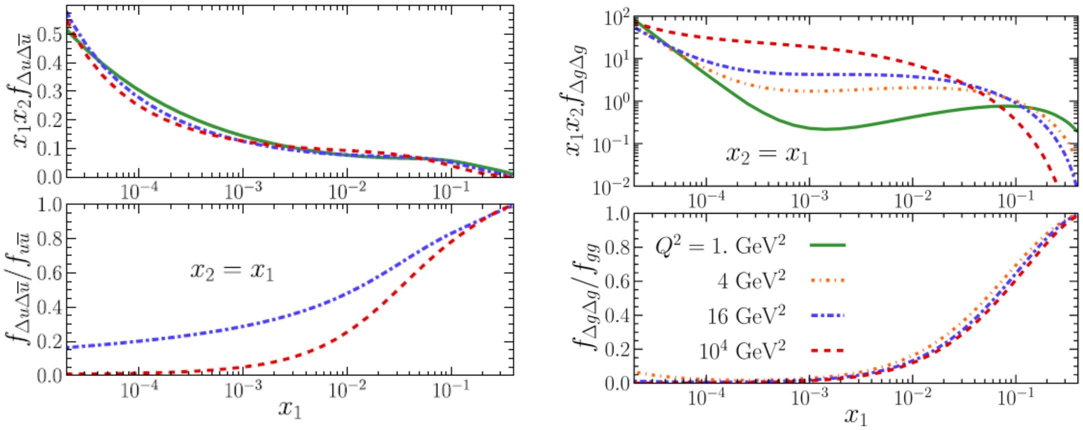
Model and lattice results indicate spin correlations large at larger x and low scale.



Chang, Manohar, Waalewijn,
Phys.Rev. D87 (2013) no.3, 034009



C. Zimmermann, talks at
LATTICE2019, MPI@LHC 2019



Evolution tends to wash out the correlations. Slowest at high x, and for quark channels.

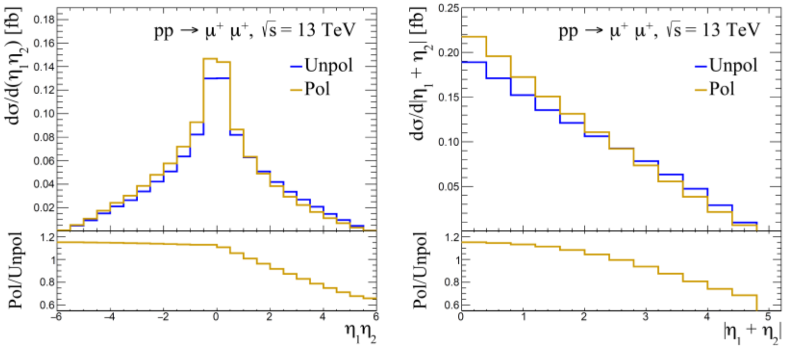
Diehl, Kasemets, Keane, JHEP 1405 (2014) 118

SPIN CORRELATIONS IN $W^{\pm}W^{\pm}$

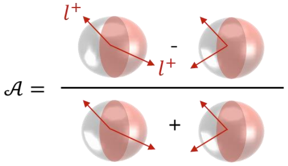
Recently identified that spin polarisation effects may have a measurable effect in same-sign WW [Cotogno, Kasemets, Myska, Phys.Rev. D100 (2019) 1, 011503]

Good process in terms of spin polarisation:

- involves quarks.
- Ws couple only to left-handed quarks



Input at 1 GeV for polarised DPD chosen to yield maximum possible effect



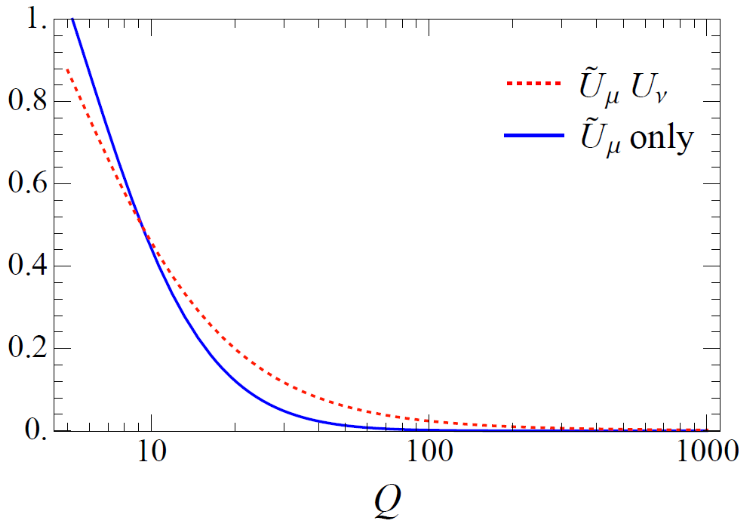
$\|\eta_i\|$	> 0	> 0.6	> 1.2
A	0.07	0.11	0.16
σ [fb]	0.51	0.29	0.13

Few percent effect on lepton pseudorapidity asymmetry

COLOUR CORRELATIONS

Colour correlations are strongly suppressed at high scales

[Technically: Sudakov suppression due to movement of colour between amplitude & conjugate by distance $\boldsymbol{y}$.]



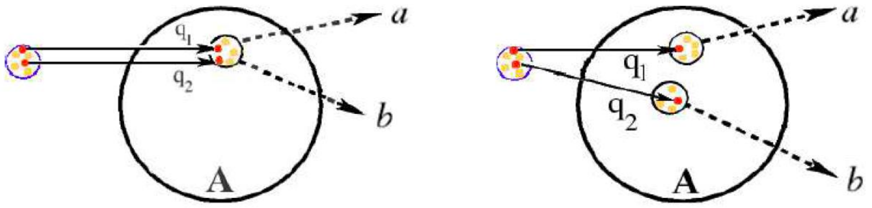
First estimate: negligible at 100 GeV, but could be relevant at moderate scales ~10 GeV.

Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

DPS IN HEAVY ION COLLISIONS

DPS IN pA COLLISIONS

For pA, **two** possible contributions to DPS:



Nuclear thickness: $T(\boldsymbol{B}) = \int \rho(z,\boldsymbol{B})\,\mathrm{d}z$

Assume this is ~ constant over size of one nucleon. Ignore nuclear matter effects.

Strikman, Treleani, Phys.Rev.Lett. 88 (2002) 031801

$$\sigma_{pA,\,\mathrm{I}}^{\mathrm{DPS}} = \frac{m}{2}\int F(x_1,x_2,\boldsymbol{y})F(x_1',x_2',\boldsymbol{y})\hat{\sigma}_a\hat{\sigma}_b dx_i dx_i'\, d^2\boldsymbol{y} \int d^2\boldsymbol{B}\, T(\boldsymbol{B}) = A\sigma_{pp}^{\mathrm{DPS}}$$

Probes L + T correlations in the same way as pp DPS

$$\sigma_{pA,\,\mathrm{II}}^{\mathrm{DPS}} = \frac{m}{2}\frac{A-1}{A}\int f(x_1')f(x_2')\left[\int F(x_1,x_2,\boldsymbol{y})\, d^2\boldsymbol{y}\right]\hat{\sigma}_a\hat{\sigma}_b dx_i dx_i' \int d^2\boldsymbol{B}\, T^2(\boldsymbol{B})$$

Probes longitudinal correlations of **one DPD only**

II contribution in pA probes DPDs in a different way to pp DPS.

DPS IN pA COLLISIONS

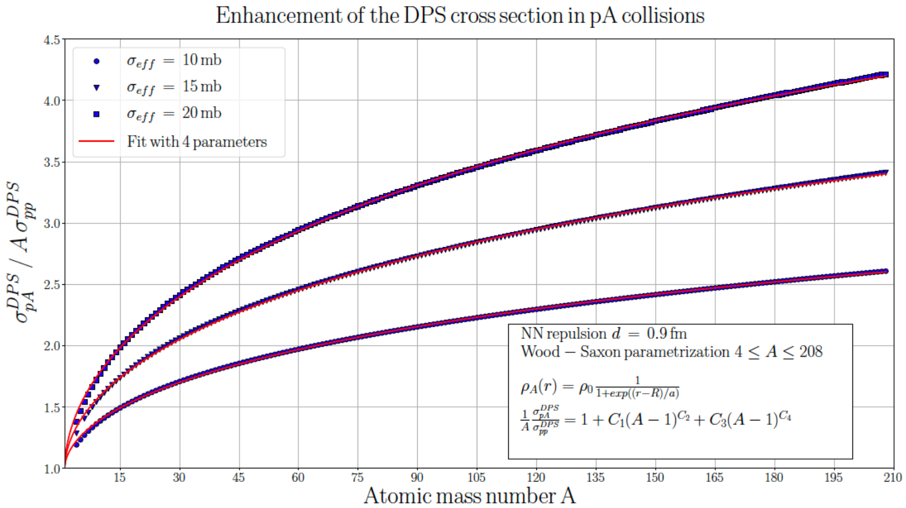
Common simplified ansatz (neglect correlations): $F(x_1, x_2, \boldsymbol{y}) \to f(x_1)\, f(x_2)\, G(\boldsymbol{y})$

Then: $\sigma_{\mathrm{I}}^{\mathrm{DPS}} = A \frac{m}{2} \frac{\sigma_a \sigma_b}{\sigma_{eff}} = A \sigma_{pp}^{\mathrm{DPS}}$ $\sigma^{\mathrm{SPS}} = A \sigma_{pp}^{\mathrm{SPS}}$

$$\sigma_{\mathrm{II}}^{\mathrm{DPS}} = \frac{m}{2} \frac{A-1}{A} \sigma_a \sigma_b \int d^2\boldsymbol{B}\, T^2(\boldsymbol{B})$$

If nucleus is sphere of constant density, $\frac{\sigma_{\mathrm{II}}^{\mathrm{DPS}}}{\sigma^{\mathrm{SPS}}} \propto A^{\frac{1}{3}}$. **Relative importance of DPS grows with A in pA.**

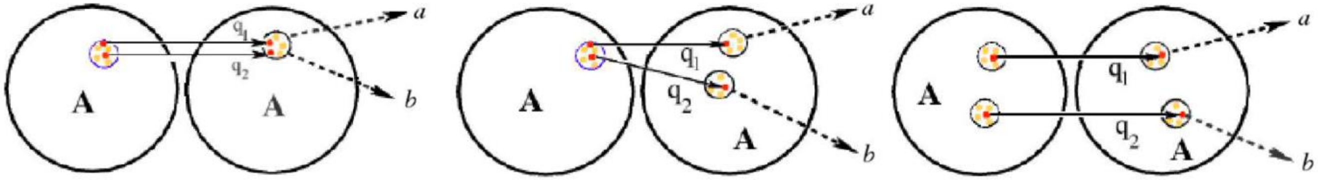
$\frac{\sigma_{\mathrm{II}}^{\mathrm{DPS}}}{\sigma_{\mathrm{I}}^{\mathrm{DPS}}} \sim 2$ at large A, **two contributions comparable**.



Fedkevych, Lonnblad, Phys.Rev.D 102 (2020) 1, 014029

DPS IN AA COLLISIONS

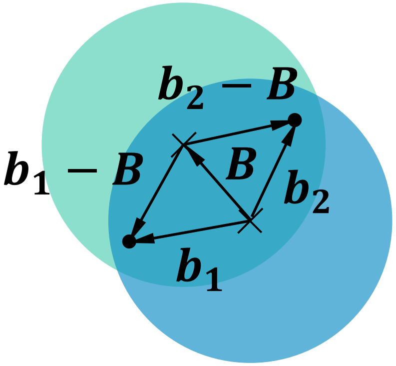
For AA collisions, three contributions to DPS:



$$\sigma_{AA,\mathrm{I}}^{\mathrm{DPS}} = A^2 \sigma_{pp}^{\mathrm{DPS}} \qquad \sigma_{\mathrm{AA,\,II}}^{\mathrm{DPS}} = 2A\sigma_{pA}^{\mathrm{DPS}}$$

$$\sigma_{\mathrm{AA,\,III}}^{\mathrm{DPS}} = \frac{m}{2}\left(\frac{A-1}{A}\right)^2 \sigma_a \sigma_b$$

$$\times \int T(\boldsymbol{b_1})\, T(\boldsymbol{b_2}) T(\boldsymbol{b_1} - \boldsymbol{B}) T(\boldsymbol{b_2} - \boldsymbol{B}) d^2\boldsymbol{b_1} d^2\boldsymbol{b_2} d^2\boldsymbol{B}$$



This contribution corresponds to **double nucleon-nucleon scattering** – doesn't probe parton-parton correlations.

d'Enterria, Snigirev, *Phys.Lett.B* 727 (2013) 157-162, *Adv.Ser.Direct.High Energy Phys.* 29 (2018) 159-187

DPS IN AA COLLISIONS

Relative size of three contributions? Rough estimate using hard sphere nucleus & large A:

$$\sigma_{AA}^{DPS} \approx A^2\sigma_{pp}^{\mathrm{DPS}}\left[\overset{\boxed{\mathrm{I}}}{1}+\overset{\boxed{\mathrm{II}}}{\frac{2}{\pi}A^{1/3}}+\overset{\boxed{\mathrm{III}}}{\frac{1}{2\pi}A^{4/3}}\right]$$

Term III grows much faster than the other two, dominates other two for reasonably large A:

$A = 40$ (Ca): I: II: III = 1: 2.3: 23 87% is term III
$A = 208$ (Pb): I: II: III = 1: 4: 200 97.5% is term III

d'Enterria, Snigirev, *Phys.Lett.B* 727 (2013) 157-162, *Adv.Ser.Direct.High Energy Phys.* 29 (2018) 159-187

In AA collisions, DPS is dominated by double nucleon-nucleon scattering