

# One-loop electroweak Sudakov logarithms: automation and applications

Marco Zaro

Joint Max-Planck & TUM seminar  
17/07/2024

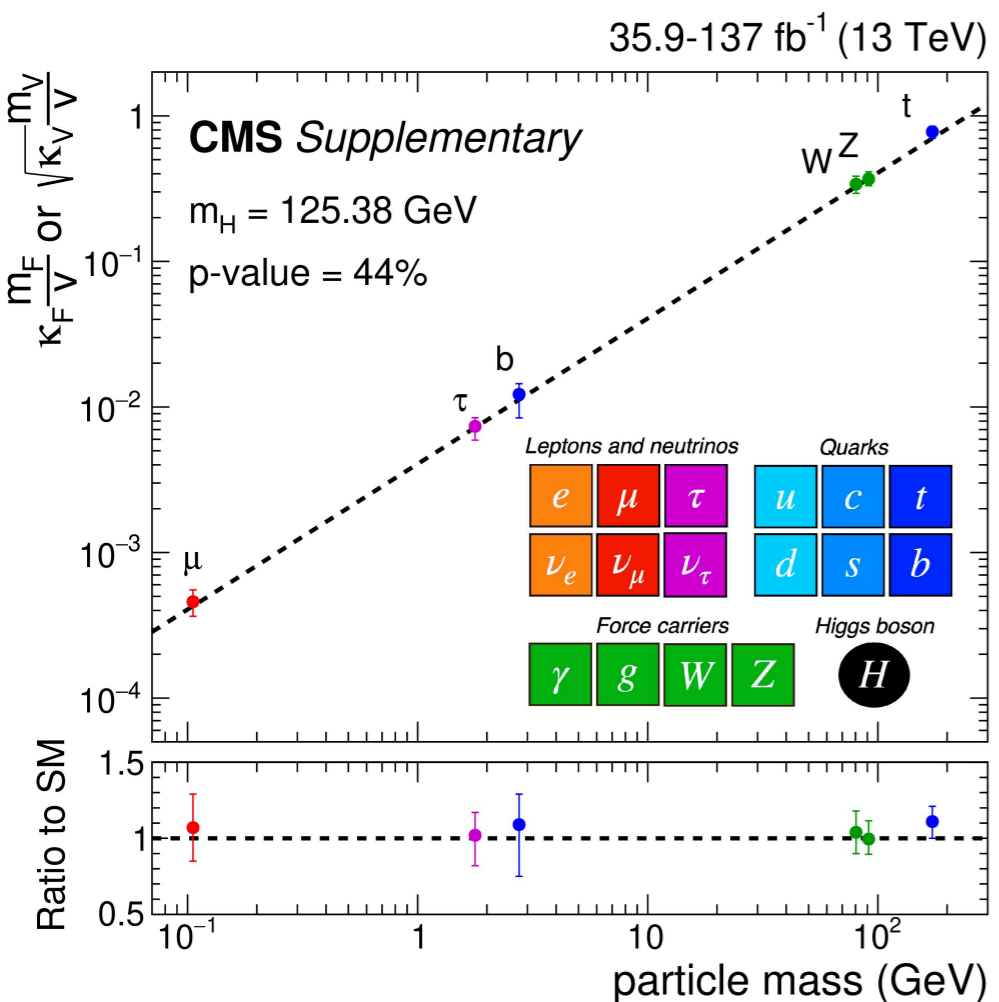


MAX-PLANCK-INSTITUT  
FÜR PHYSIK



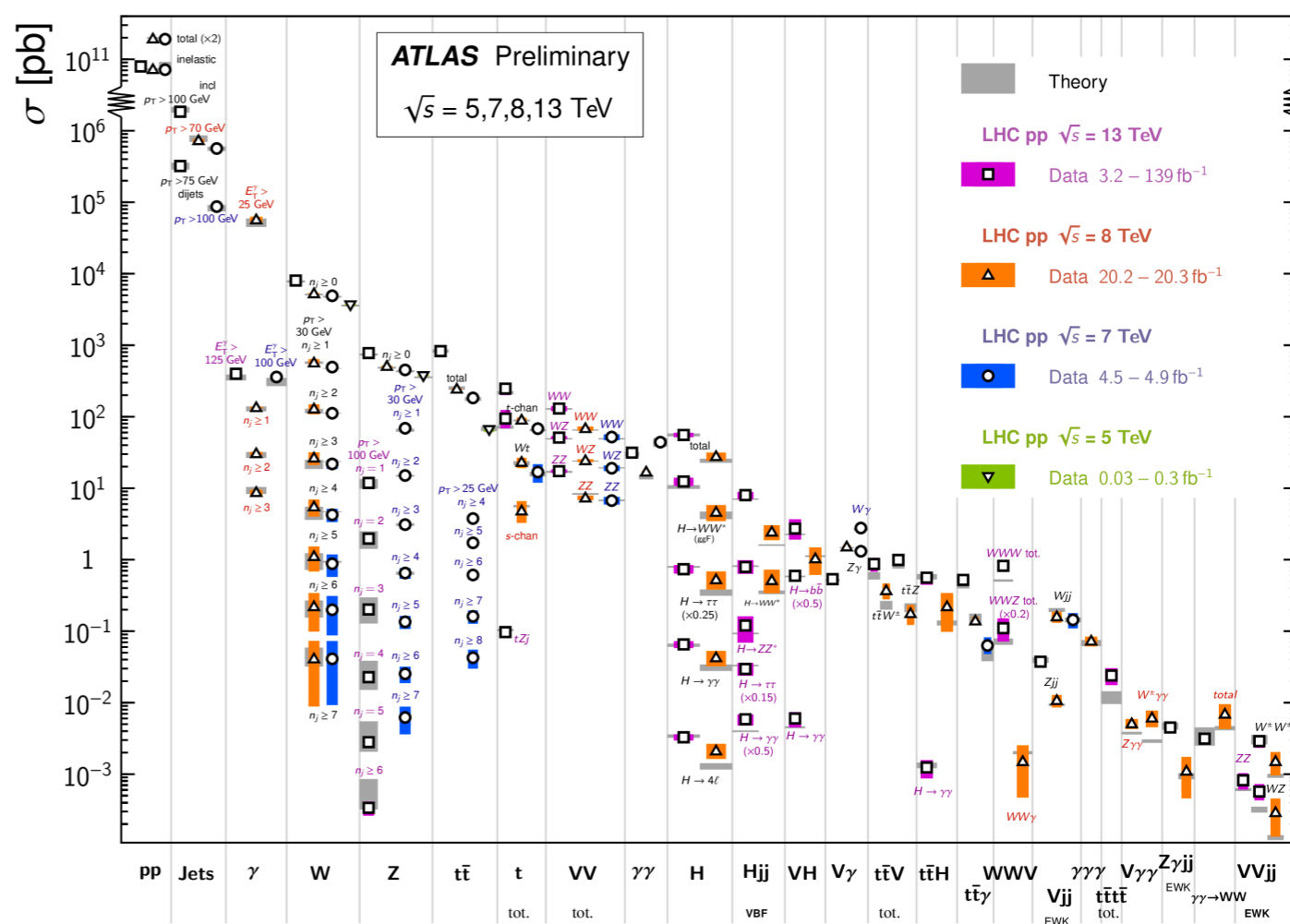


# LHC is running (again)!



## Standard Model Production Cross Section Measurements

Status: February 2022



# A beautiful theory

- The Standard Model encloses our current knowledge of fundamental interactions
- It is a complete theory, and successfully explain phenomena over a vast range of scales (from low-energy QED to the largest energy scales we can probe)

**Electron g-2** ↓ 10<sup>-12</sup>

$$g/2 = 1.001\ 159\ 652\ 180\ 73\ (28) \quad [0.28 \text{ ppt}] \quad (\text{measured})$$

$$g(\alpha)/2 = 1.001\ 159\ 652\ 177\ 60\ (520) \quad [5.2 \text{ ppt}] \quad (\text{predicted}).$$



- Its Lagrangian can be cast in a very compact form

Gauge kinetic term

→

Matter kinetic term

→

Matter-Higgs interaction

→

Higgs kinetic term+potential

→



# New Physics?

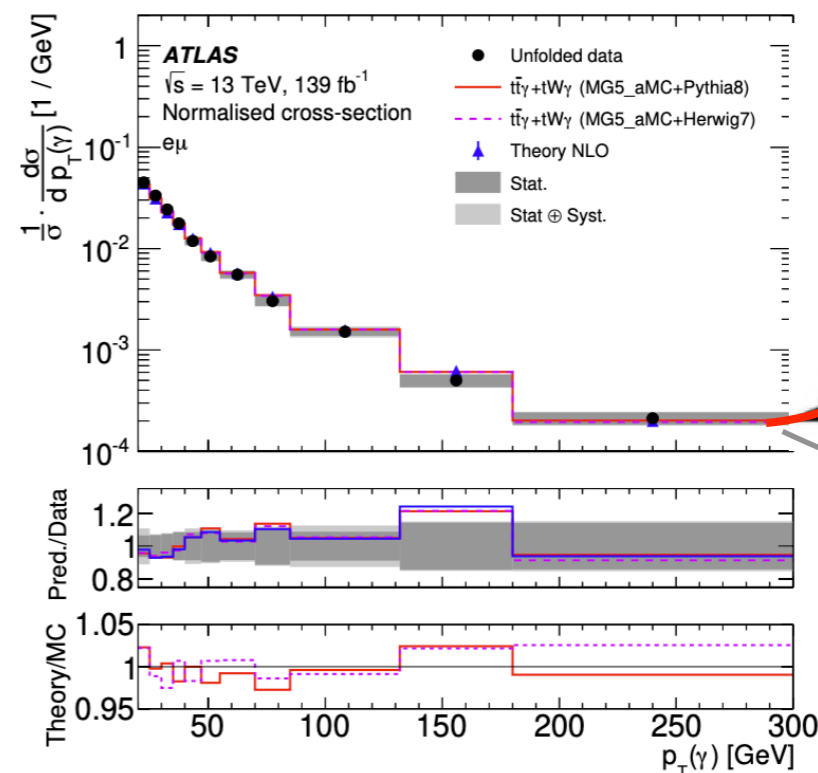
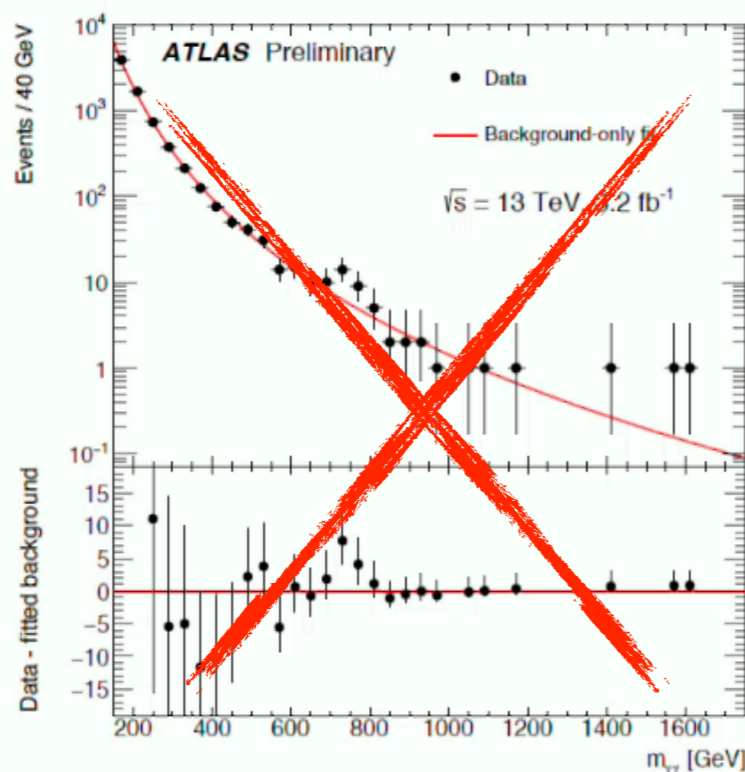
# New Physics?

- We (still) believe that new physics must exist
  - What is Dark Matter made of?
  - Where is all the anti-matter in the universe?
  - Why do particles have such unnatural masses?
  - ...



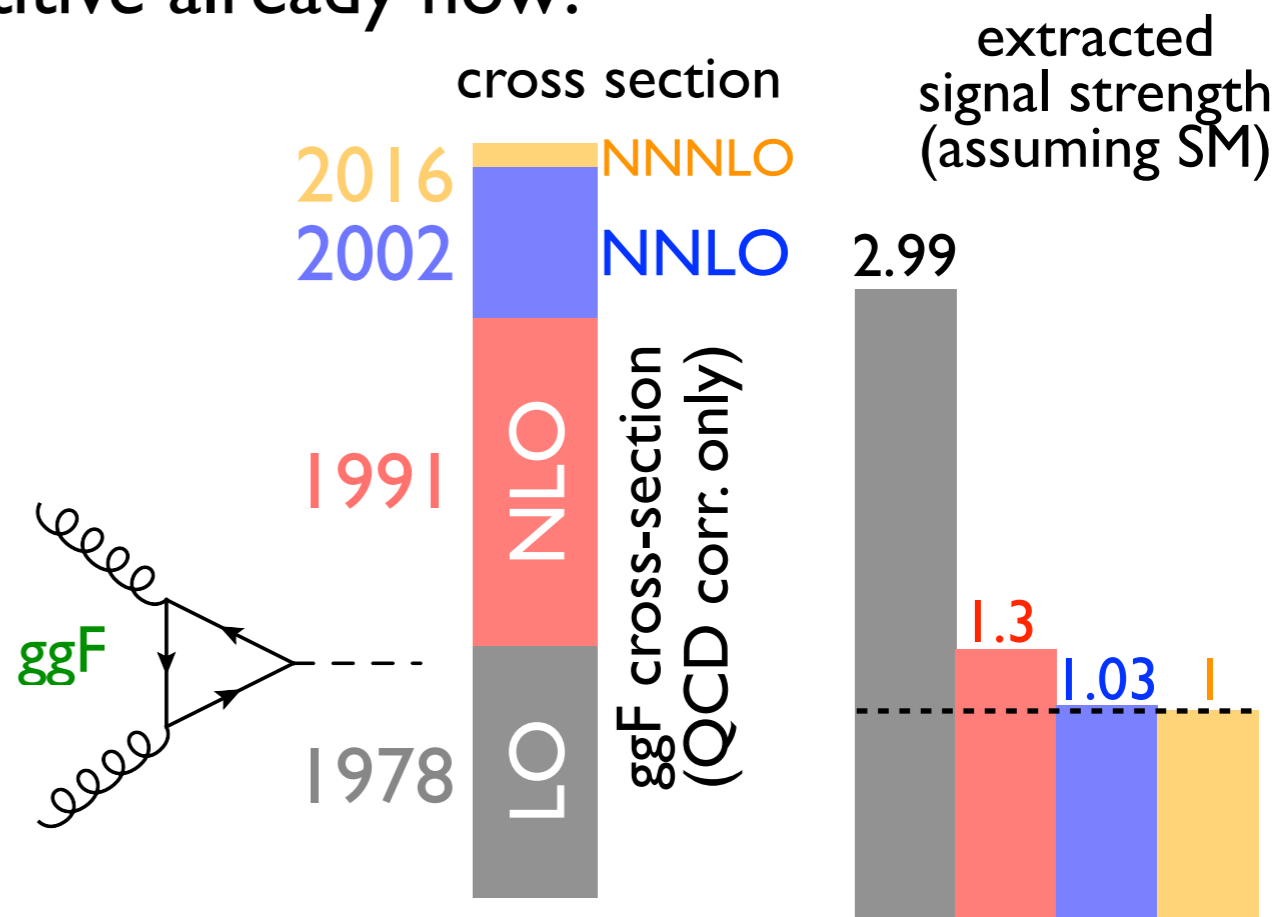
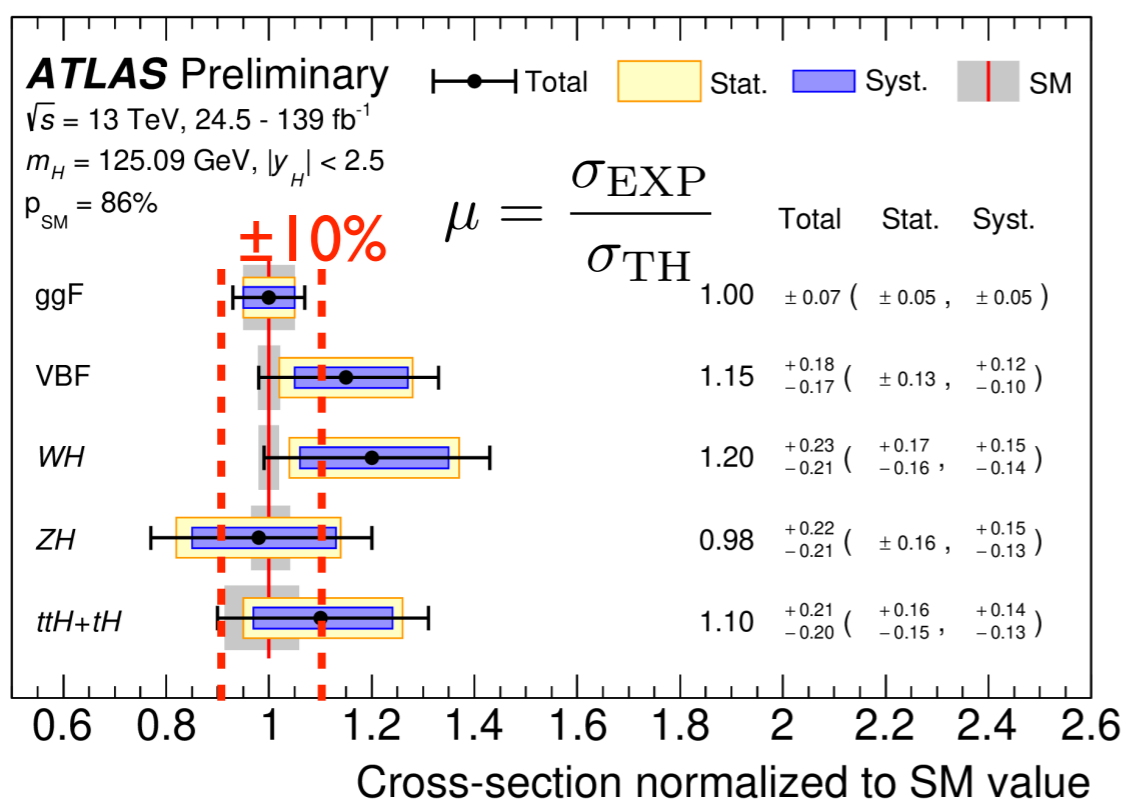
# New Physics?

- We (still) believe that new physics must exist
  - What is Dark Matter made of?
  - Where is all the anti-matter in the universe?
  - Why do particles have such unnatural masses?
  - ...
- New physics must be hiding very well!
  - Change of paradigm: from bump hunting to precision measurements



# Precision for measurements

- Our ability to make measurements and discoveries is limited by the goodness of our theory predictions
- Higgs physics gives a clear example: the dominant production channel receives large perturbative corrections
- Without the inclusion of higher orders, ggF measured rate would be 3\*SM
- Exp. measurements are very competitive already now!





# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)$$



# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

Probability of finding a parton  
into the proton

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)$$

Parton distribution functions:  
must be fit to data, process  
independent

# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)$$

Probability of finding a parton into the proton

Probability that two partons scatter into a given final state

Parton distribution functions: must be fit to data, process independent

Partonic cross section: can be computed in perturbation theory, process dependent

# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)$$

Probability of finding a parton into the proton
Probability that two partons scatter into a given final state

Parton distribution functions: must be fit to data, process independent
Partonic cross section: can be computed in perturbation theory, process dependent

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \alpha_s \hat{\sigma}_{ab \rightarrow X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ab \rightarrow X}^{(2)} + \alpha_s^3 \hat{\sigma}_{ab \rightarrow X}^{(3)} + \dots$$

# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 \underbrace{f_a(x_1) f_b(x_2)}_{\substack{\text{Probability of finding a parton} \\ \text{into the proton}}} \underbrace{\hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)}_{\substack{\text{Probability that two partons} \\ \text{scatter into a given final state}}}$$

Parton distribution functions: must be fit to data, process independent
Partonic cross section: can be computed in perturbation theory, process dependent

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \alpha_s \hat{\sigma}_{ab \rightarrow X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ab \rightarrow X}^{(2)} + \alpha_s^3 \hat{\sigma}_{ab \rightarrow X}^{(3)} + \dots$$

strong coupling,  $\sim 0.1$

# How do we do precision calculations?

- We cannot solve exactly the SM Lagrangian: use perturbation theory
- QCD factorisation theorem

$$\sigma_{pp \rightarrow X}(s) = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_{ab \rightarrow X}(\hat{s} = x_1 x_2 s)$$

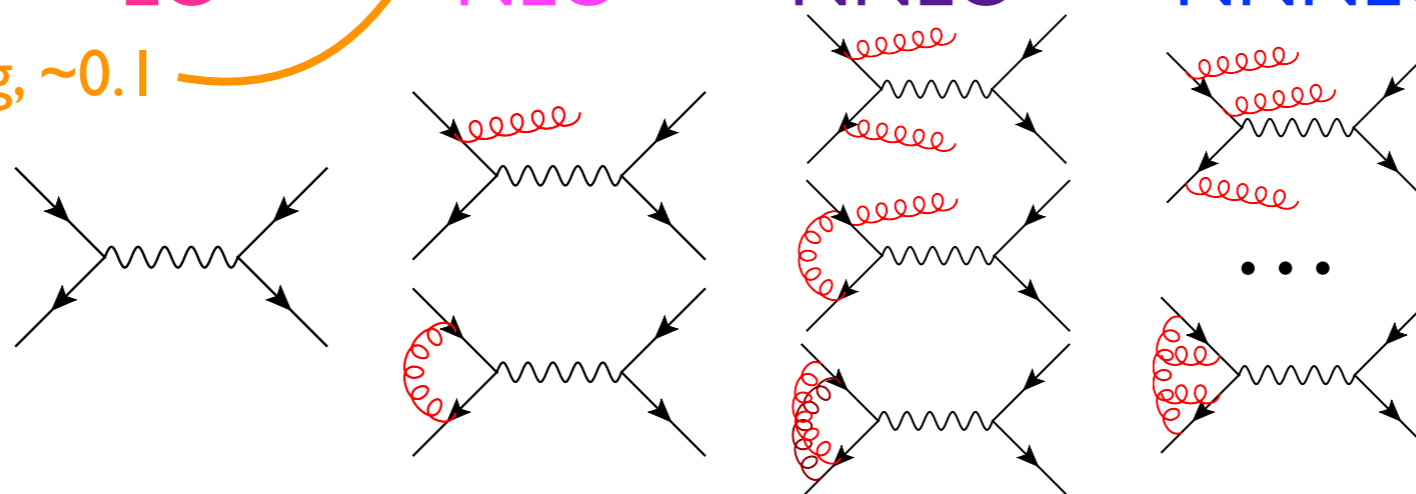
Probability of finding a parton into the proton      Probability that two partons scatter into a given final state

Parton distribution functions: must be fit to data, process independent      Partonic cross section: can be computed in perturbation theory, process dependent

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \alpha_s \hat{\sigma}_{ab \rightarrow X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ab \rightarrow X}^{(2)} + \alpha_s^3 \hat{\sigma}_{ab \rightarrow X}^{(3)} + \dots$$

LO      NLO      NNLO      NNNLO

strong coupling,  $\sim 0.1$



- Going higher orders, the complexity of the computation explodes

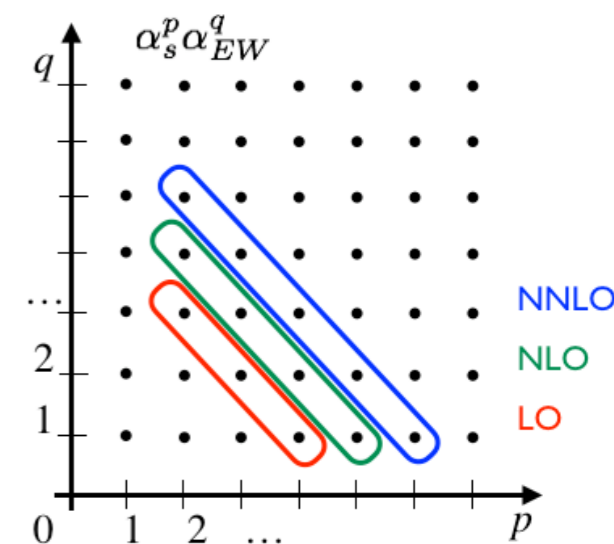


# SM $\neq$ QCD

- So far, we considered only QCD effects
- In the SM also electroweak effects must be accounted for  
→ Multi-coupling expansion
- Since  $\alpha \approx \alpha_s^2$ , EW effects cannot be neglected for precision
  - EW effects grow at large energies: Sudakov enhancement
  - Luckily, NLO EW corrections have been automated in the last years  
see e.g.: Kallweit et al, 1412.5157 (Sherpa+OpenLoops), Biedermann et al, 1704.05783 (Sherpa+Recola+Collier), Frederix, Frixione, Hirschi, Pagani, Shao, MZ, 1804.10017 (MG5\_aMC)
  - Relevance of EW corrections also beyond SM and LHC:
    - Can be  $O(1)$  at high-energy lepton colliders (specially muons)
    - Sizeable effects in Dark-Matter searches, e.g. indirect-detection with heavy DM particles see e.g. Ciafaloni et al, 1104.2996 ..., Cavasonza et al, 1409.8226

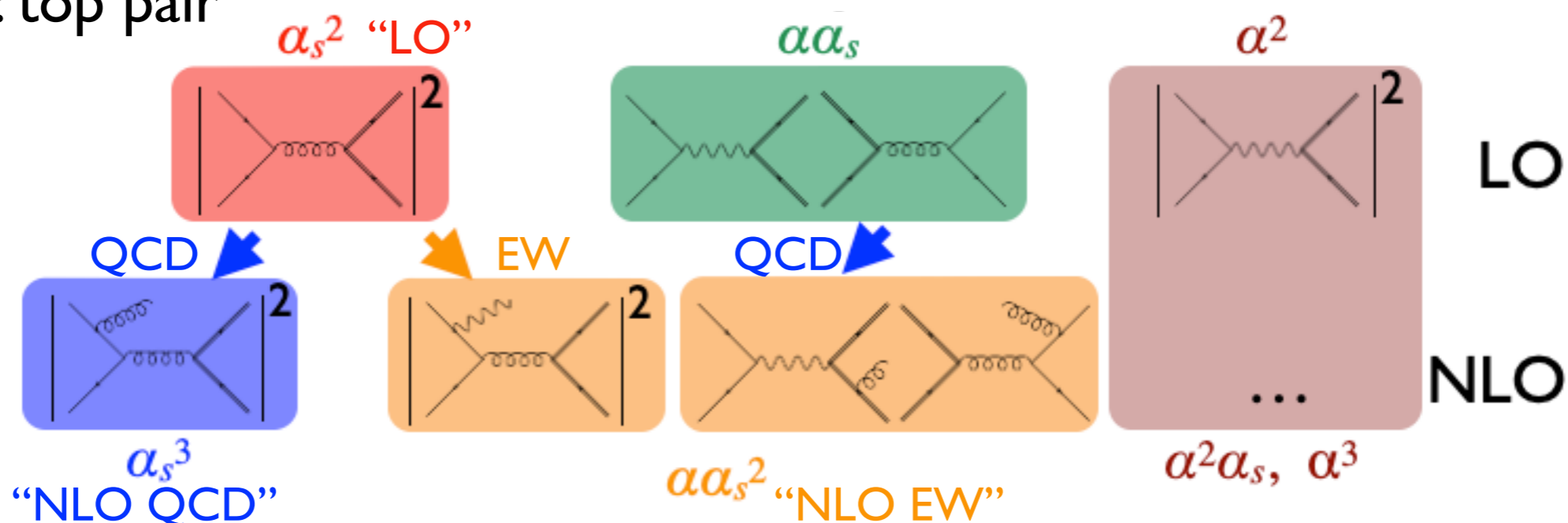
# SM $\neq$ QCD

- So far, we considered only QCD effects
- In the SM also electroweak effects must be accounted for  
→ Multi-coupling expansion
- Since  $\alpha \approx \alpha_s^2$ , EW effects cannot be neglected for precision
  - EW effects grow at large energies: Sudakov enhancement
  - Luckily, NLO EW corrections have been automated in the last years  
see e.g.: Kallweit et al, 1412.5157 (Sherpa+OpenLoops), Biedermann et al, 1704.05783 (Sherpa+Recola+Collier), Frederix, Frixione, Hirschi, Pagani, Shao, MZ, 1804.10017 (MG5\_aMC)
  - Relevance of EW corrections also beyond SM and LHC:
    - Can be  $O(1)$  at high-energy lepton colliders (specially muons)
    - Sizeable effects in Dark-Matter searches, e.g. indirect-detection with heavy DM particles see e.g. Ciafaloni et al, 1104.2996 ..., Cavasonza et al, 1409.8226



# EW corrections vs EW effects

- A general process has several contributions at LO, NLO, ...
- Example: top pair



- The **LO** is often identified with the contribution with most  $\alpha_s$
- At NLO the first two contributions are identified with the **NLO QCD** and **NLO EW** corrections
- This structures induces mixed QCD-EW effects at NLO:  

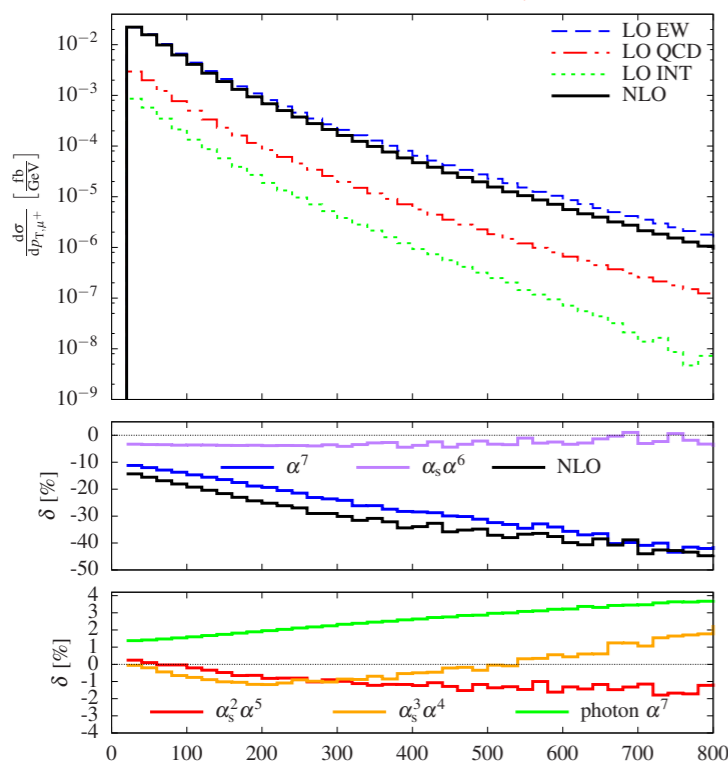
$$\text{NLO}_i = \text{LO}_{i-1} \otimes \text{EW} + \text{LO}_i \otimes \text{QCD}$$



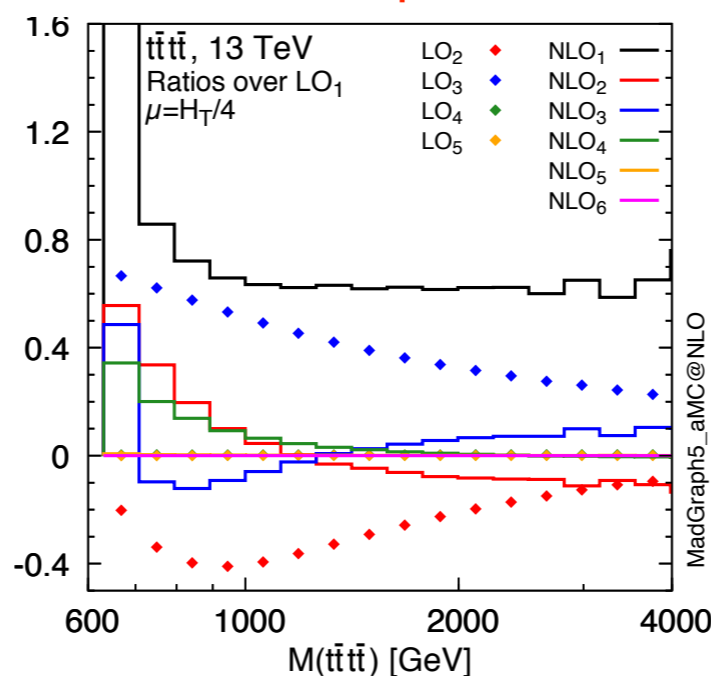
# Large EW corrections

- Despite the naive estimate  $\alpha \sim \alpha_s^2$ , there are cases when EW corrections comparable to NLO QCD or larger. It happens when:
  - Large scales are probed (VBS)
  - Power counting is altered (4 top:  $y_t$  vs  $\alpha$ )
  - New production mechanisms, different than those at the “dominant” LO, enter (ttW, bbH)

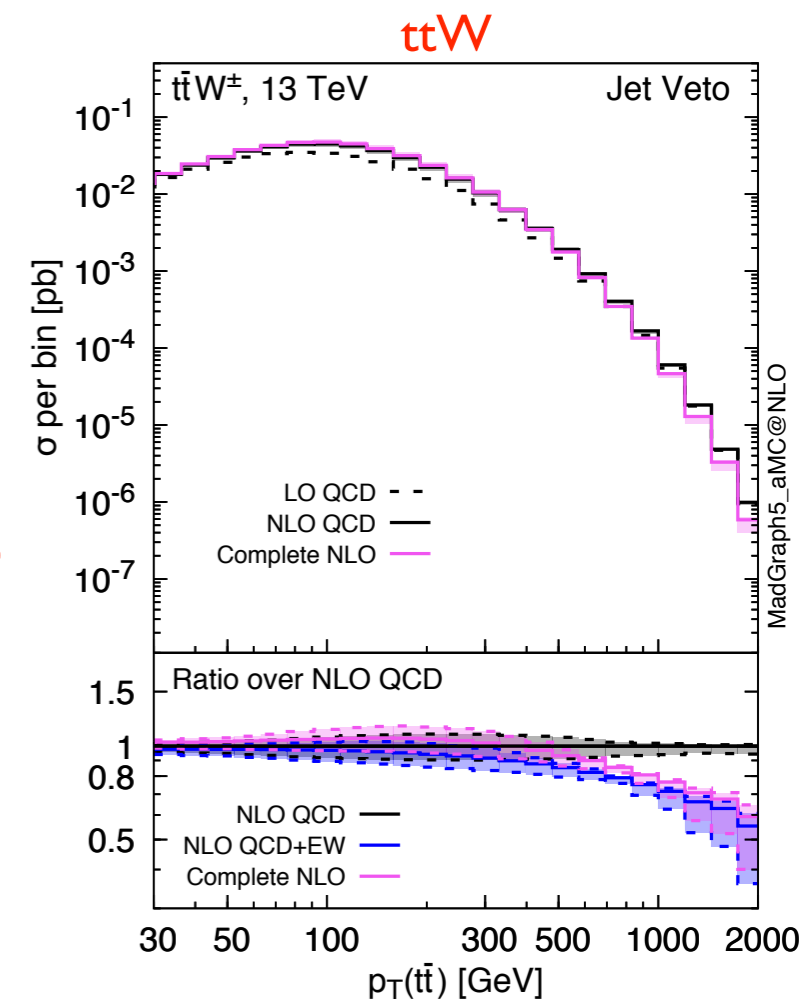
VBS: Biedermann et al, I708.00268



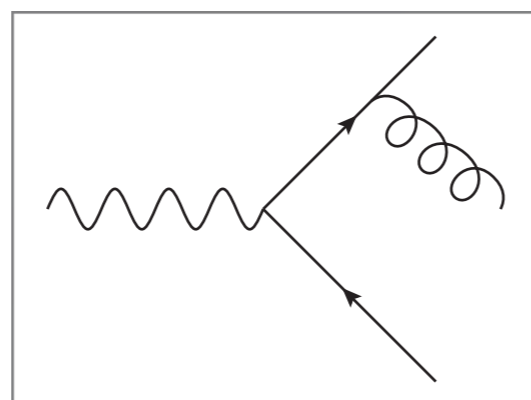
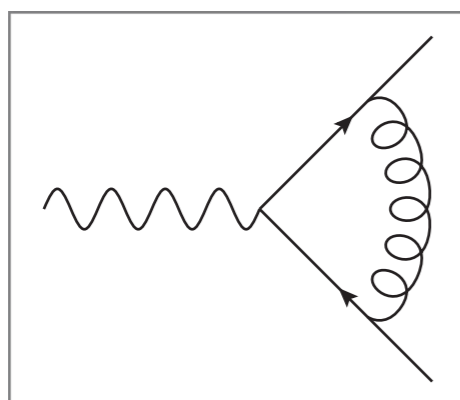
4 top



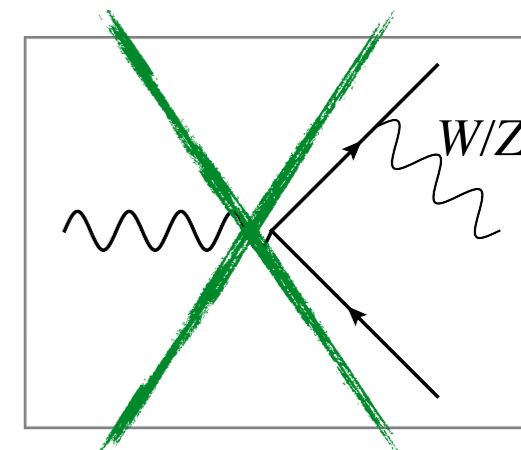
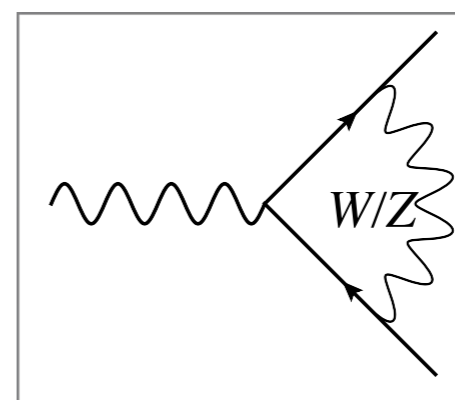
Frederix, Pagani, MZ I711.02116



# EW Sudakov Logarithms



**QCD**



**EW**

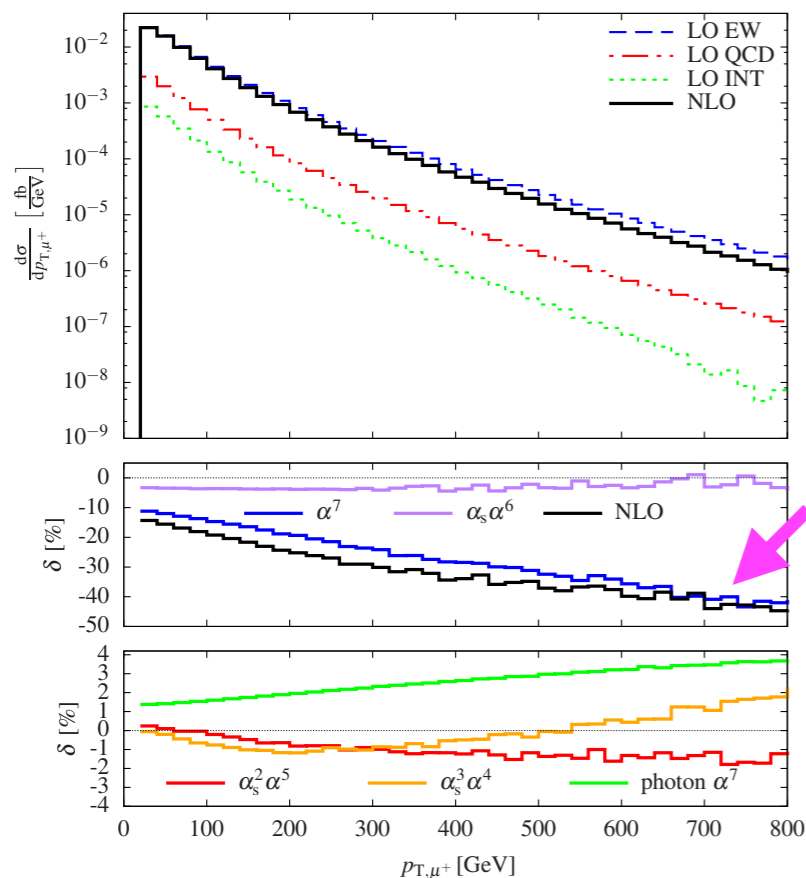
- Since gluons are massless, one must include virtual and real radiation in NLO **QCD** computations
- The W/Z/Higgs boson masses make **EW** corrections finite. No need to include heavy-boson radiation (distinguishable, in principle)
- Even if we included HBR, cancelation will only be partial [Manohar et al, 1409.1918](#)
- However, at high energies, the would-be IR divergence appears via logs:

$$\alpha^k \log^p \frac{s}{m_W^2}, \quad p \leq 2k$$

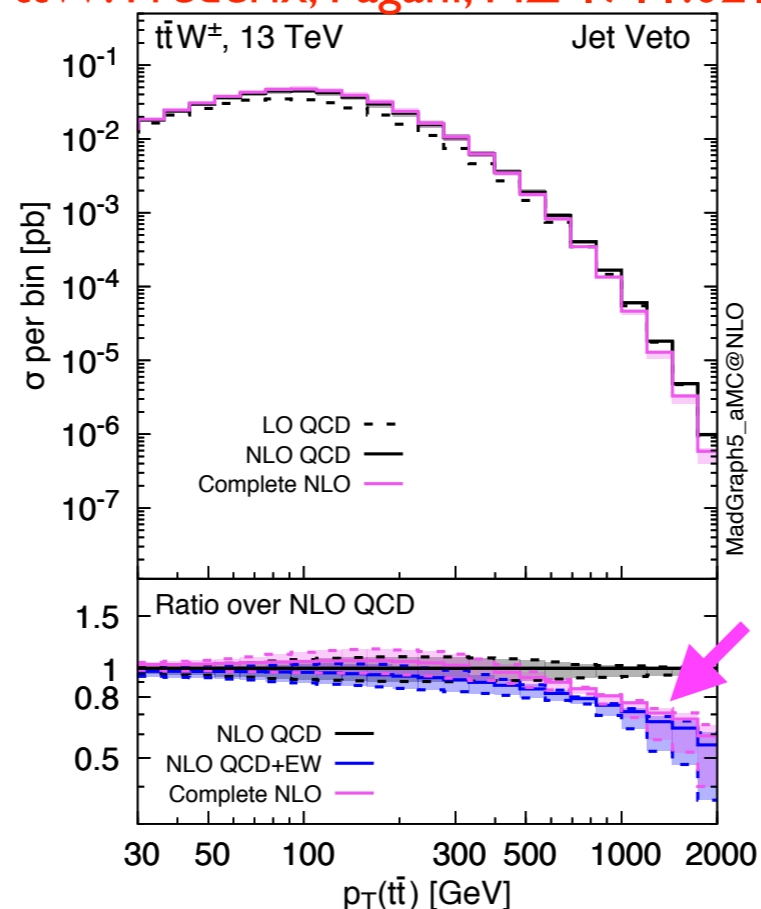
- EWSL are universal, enhance the cross section at high-energy, and can be resummed [Denner, Rode, 2402.10503](#)

# Universality of EWSL

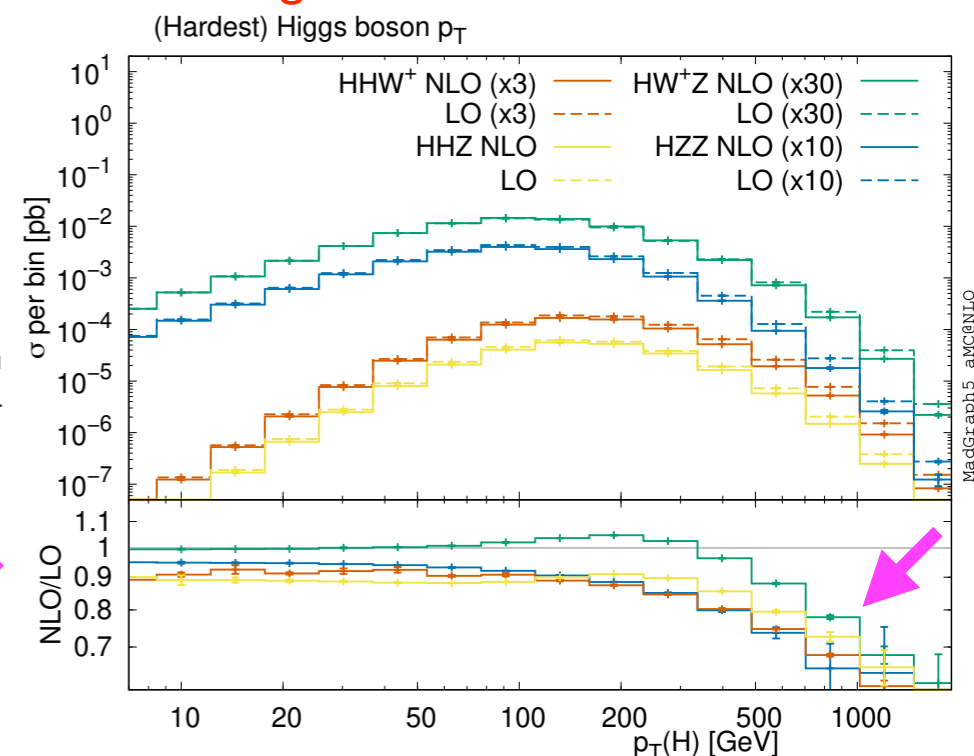
VBS: Biedermann et al, I708.00268



ttW: Frederix, Pagani, MZ I711.02116



Frederix, Frixione, Hirschi, Pagani, Shao, MZ I711.02116



- EWSL enhancement is a feature of many scattering processes
- Still, other effects of different origin can appear in the same kinematic regime: photon PDFs, quasi-collinear enhancements, etc...

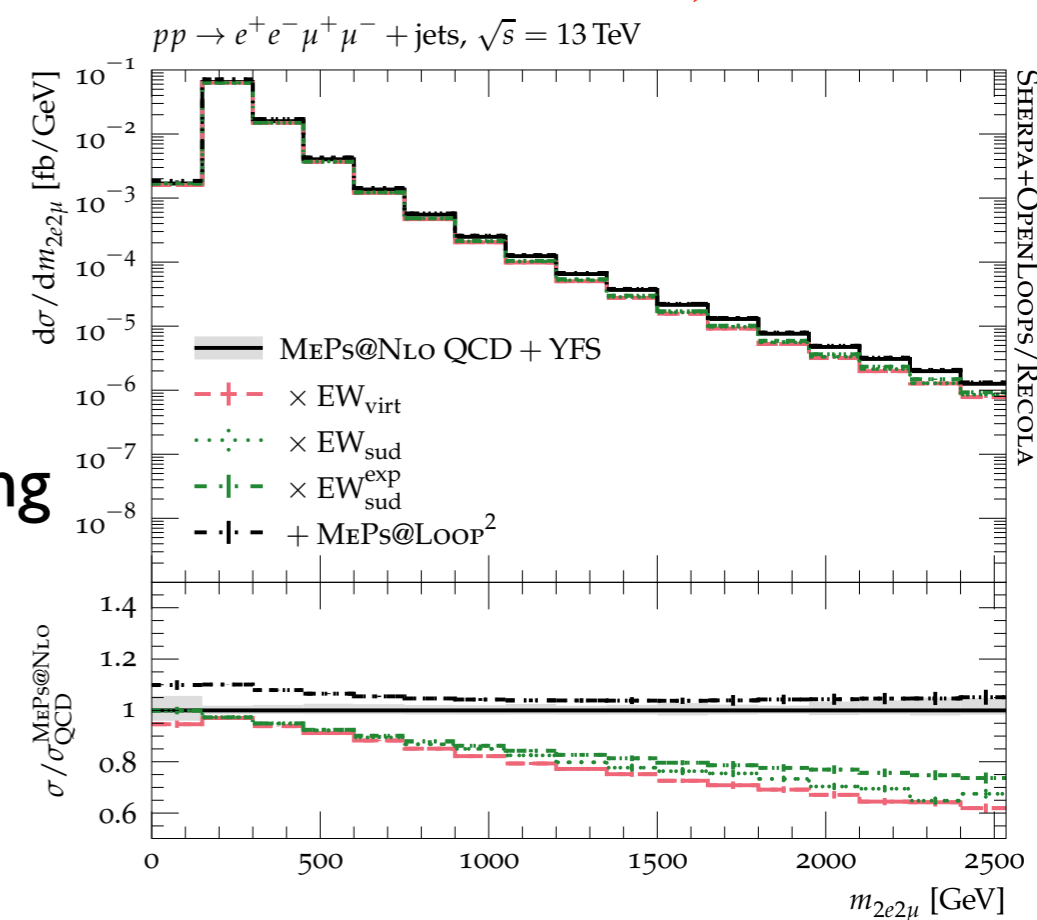
# Don't we have the exact EW corrections?

- Despite the fact we have the exact EW corrections, EWSL have re-gained attention in the recent years:

see e.g. the automation in Sherpa, Bothmann et al, 2006.14635  
or OpenLoops, Lindert et al, 2312.07927

- Can be resummed, providing NLO+NLL EW accuracy
- Much faster and more stable than exact NLO EW corrections
- ★ Only Born-like kinematics: PS merging/matching simplified
- ★ Universal: can be computed also for BSM theories, where UV renormalisation is very complex or even impossible

Bothmann et al, 2111.13453



★ = In this talk



# EWSL by Denner and Pozzorini

hep-ph/0010201 & hep-ph/0104127

- In their seminal works, D&P derived the structure of EWSL for one-loop matrix elements, *where at least one helicity configuration is not mass-suppressed*

- All invariants must satisfy the constraint  $|r_{ij}| \simeq s \gg M_W^2$

- SM is chiral  $\rightarrow$  EWSL must be computed helicity-by-helicity

- Use GBET for longitudinal polarisations

- EWSL decomposed as sum of 4 terms

$$\delta = \delta^{\text{LSC}} + \delta^{\text{SSC}} + \delta^{\text{C}} + \delta^{\text{PR}}$$

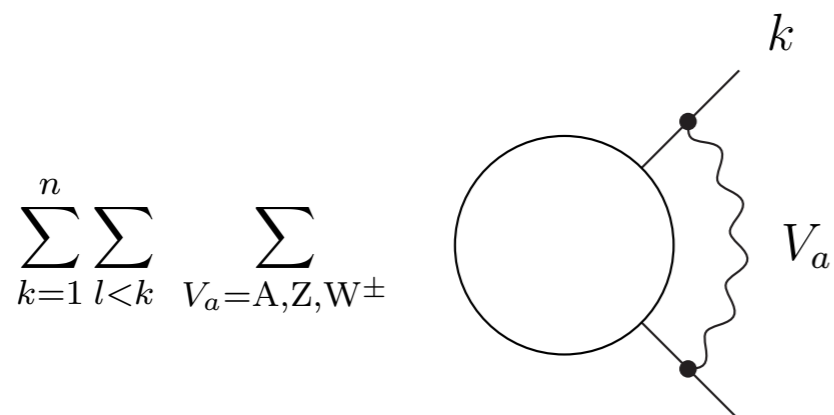
Subleading soft-collinear    Parameter remormalisation

Leading soft-collinear

Collinear single logs

- Photon and fermion masses to regulate IR divergences
- Analytic control of expressions (for simple processes): ability to single out only the dominant terms in the results

# A glance at the anatomy of EWSL



$$L(a, b) = \log^2 \left( \frac{a}{b} \right)$$

$$l(a, b) = \log \left( \frac{a}{b} \right)$$

$$\simeq C_0(p_k, p_l, M_V, M_k, M_l) \simeq \log^2 \left( \frac{-(p_k + p_l)^2 - i\epsilon}{M_V - i\epsilon} \right)$$

$$= L(|r_{kl}|, M_V) - 2i\pi \theta(r_{kl}) l(|r_{kl}|, M_V) \quad *$$

$$= \underbrace{L(s, M_V)}_{\text{LSC}} + 2l(s, M_V) \left[ \log \left( \frac{|r_{kl}|}{s} \right) - i\pi \theta(r_{kl}) \right] + \underbrace{L(|r_{kl}|, s) - 2i\pi \theta(r_{kl}) l(|r_{kl}|, s)}_{\text{SSC}}$$

Imaginary part and extra angular dependence ( $\Delta^{s \rightarrow r_{kl}}$ )

- Soft-collinear terms originate from vector-boson exchange between external legs (in the eikonal approx.)
  - In the strict high-energy approximation (as in D&P), terms with  $s/r_{kl}$  are neglected. Their inclusion can improve angular dependence
  - The imaginary part was not considered in D&P
- Other terms originate from purely-collinear configurations and field renormalisation

\* $M_V$  is further expressed in terms of  $M_W$ ; constant terms are dropped



# Implementation of EWSL in MG5\_aMC

Pagani, MZ arXiv:2110.03714



# Implementation of EWSL in MG5\_aMC

Pagani, MZ arXiv:2110.03714

- Builds on the work by D&P, with some variations:
  - Automate the computation of EWSL for any process, in a fully-numerical framework: MG5\_aMC Alwall, ..., MZ, 1405.0301 & Frixione, ..., MZ, 1804.10017
  - Translate expressions using the modern language of Dim.Reg.
  - Include a missing imaginary part in D&P (relevant for  $2 \rightarrow n$ ,  $n \geq 3$ )
  - Provide results for the squared amplitude, including the tree-loop interference, both due to EW and QCD effects
  - Improve angular dependence by retaining explicit  $r_{kl}$  dependence
  - Obtain approximations for physical cross sections (Virtual+Reals), with the possibility that photons are clustered with charged particles



# The automation of EWSL

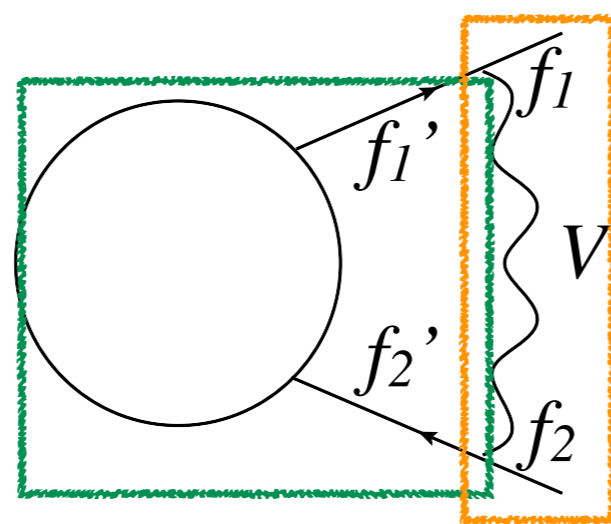
- Use MG5\_aMC to generate all the needed matrix elements:
  - Born ME's  $B_i$ , including those where  $V^{0,\pm} \rightarrow G^{0,\pm}$
  - Isospin-linked Borns  $B_{i,j}$ , and their interferences with  $B_i$
  - Since external particles differ, momenta may need reshuffling to satisfy on-shell relations
  - Keep track of all terms needed for each EWSL contribution, helicity by helicity
  - Compute  $\delta^{\text{PR}}$  with numerical derivatives

$$\delta^{\text{PR}} \mathcal{M} = \frac{\delta \mathcal{M}_0}{\delta e} \delta e + \frac{\delta \mathcal{M}_0}{\delta c_w} \delta c_w + \frac{\delta \mathcal{M}_0}{\delta h_t} \delta h_t + \frac{\delta \mathcal{M}_0}{\delta h_H} \delta h_H^{\text{eff}} \Big|_{\mu^2=s}$$

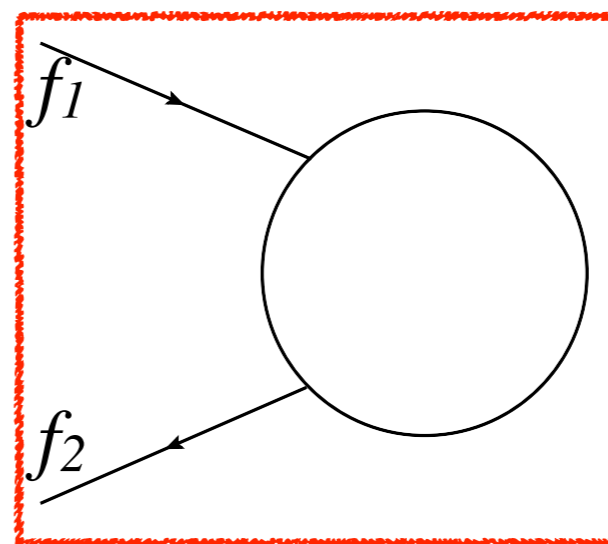
- No special Feynman rule needed by the model  $\rightarrow$  easy to extend BSM
- Formula adapted for both  $G_\mu$  and  $\alpha(M_Z)$  scheme

# Isospin-linked borns

- EWSL originate from loops where EW vector bosons attach to one or two external legs
- This can change e.g. the flavour of a given fermion line ( $u \rightarrow d, l \rightarrow \nu, \dots$ )
- In this case, the Born matrix element is interfered with an ‘isospin-linked’ term



Born amp with  $f_i \rightarrow f_i'$

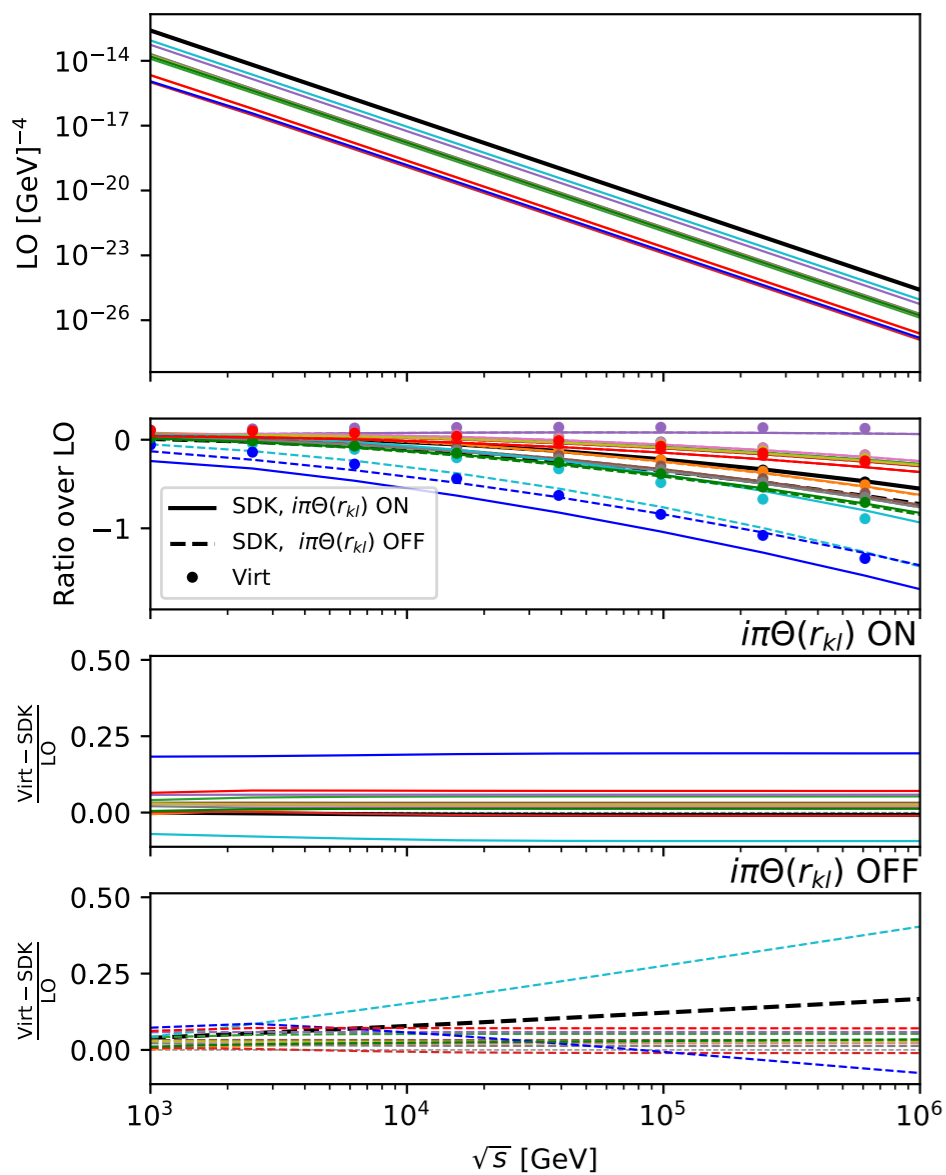


(Original) Born amp.

Logarithmic kernel, couplings...

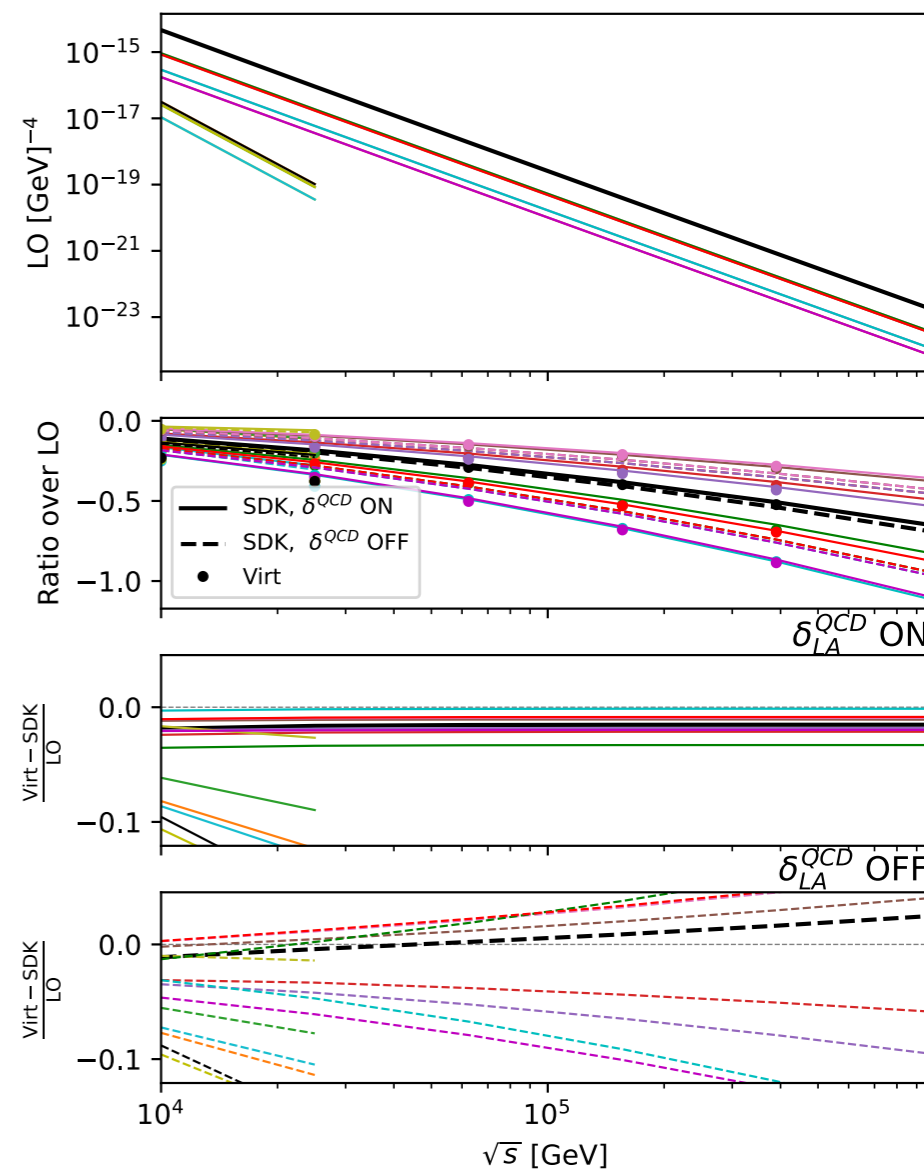
# Validation: Approximated virtual amplitudes

$e^+e^- \rightarrow e^-e^+\mu^-\mu^+$  LO  $O(\alpha^4)$



Im. part ON/OFF

$u\bar{u} \rightarrow h\bar{t}t\bar{g}$  LO  $O(\alpha^1\alpha_s^3)$



QCD contr. ON/OFF

LO amplitude,  
for each leading helicity

Loop or Sudakov / LO

(Loop-Sudakov)/LO  
must be a constant if logs are  
correct  
(checked with a fit)

# From amplitudes to cross sections

- D&P approximate the contribution of virtual diagrams to the cross section
- Real emissions will partly compensate it, in particular the QED part

$$\delta_{i'_k i_k}^{\text{LSC}}(k) = -\frac{1}{2} \left[ \underbrace{Q_k^2 + (I_Z^2)_k + (I_W^2)_k}_{\text{QED part}} C_{i'_k i_k}^{\text{ew}}(k) L(s) - 2(I^Z(k))_{i'_k i_k}^2 \log \frac{M_Z^2}{M_W^2} l(s) + \delta_{i'_k i_k} Q_k^2 L^{\text{em}}(s, \lambda^2, m_k^2) \right]$$

↓

- We introduce a purely-weak Sudakov approximation: QED effects are removed everywhere, except for PR renormalisation
- This assumes that photons are always clustered with charged particles (also massive ones!)
- Other approaches drop the IR-divergent  $^{\text{em}}$  terms. However:
  - This way QED is removed only up to  $M_W$
  - But QED effects appear also elsewhere (SSC, Collinear)
- How does this compare with exact NLO corrections?

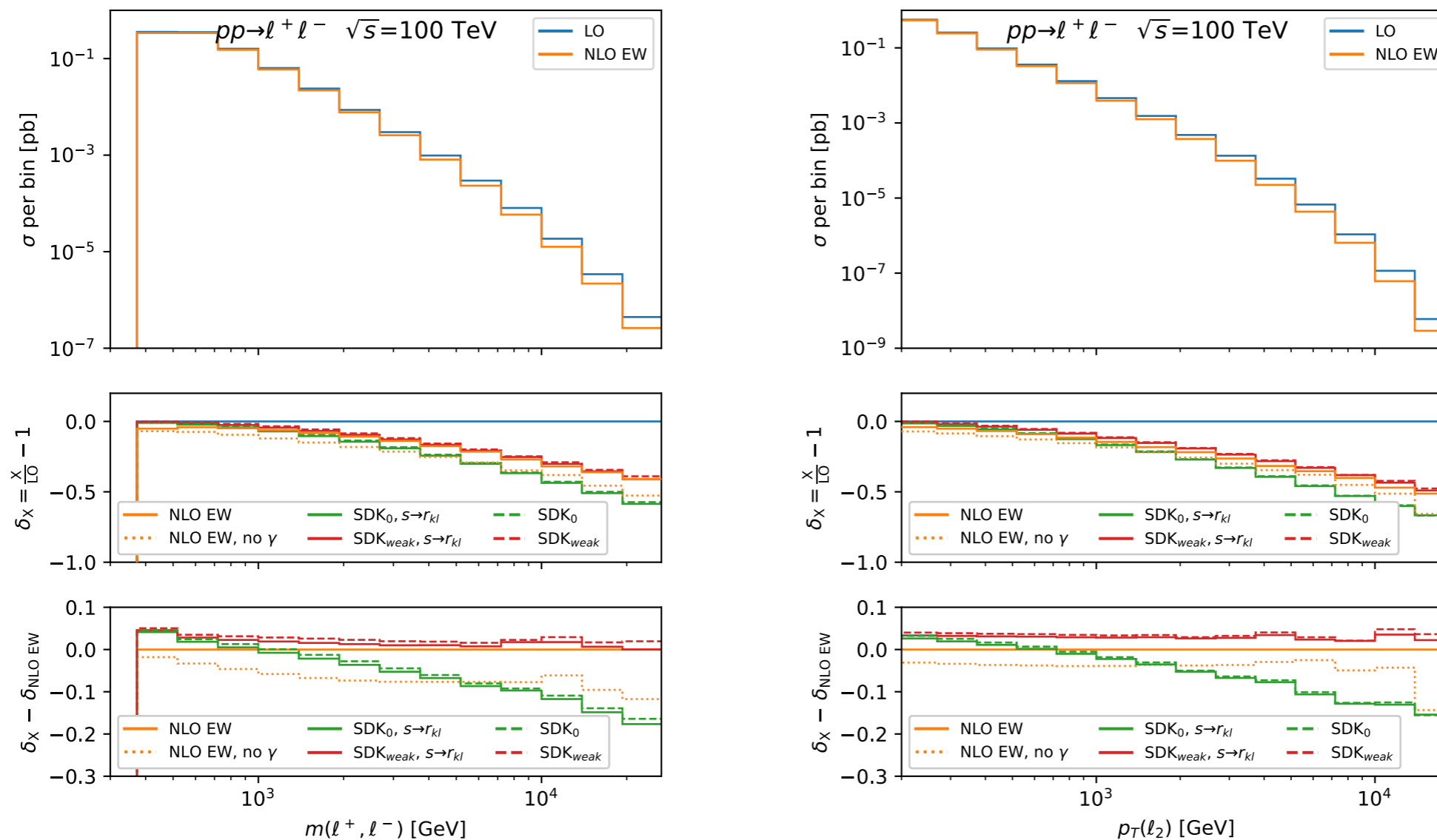


# Predictions for cross sections

- Setup:
  - 100 TeV pp collider
  - Charged particles are always clustered with photons within  $\Delta R=0.4$
  - Final-state particles required to be hard, central and separated (cuts are specific to each process considered)
- We compare exact NLO EW corrections (including and excluding initial-state photons) with
  - The Sudakov approximation as from D&P, excluding only the  $^{\text{em}}$  terms (**SDK<sup>0</sup>**)
  - Our approximation for the purely-weak Sudakov corrections (**SDK<sub>weak</sub>**)
  - Both cases are studied with or without the extra angular terms from  $\Delta^{s \rightarrow rkl}$

# Drell-Yan ( $pp \rightarrow e^+e^-$ )

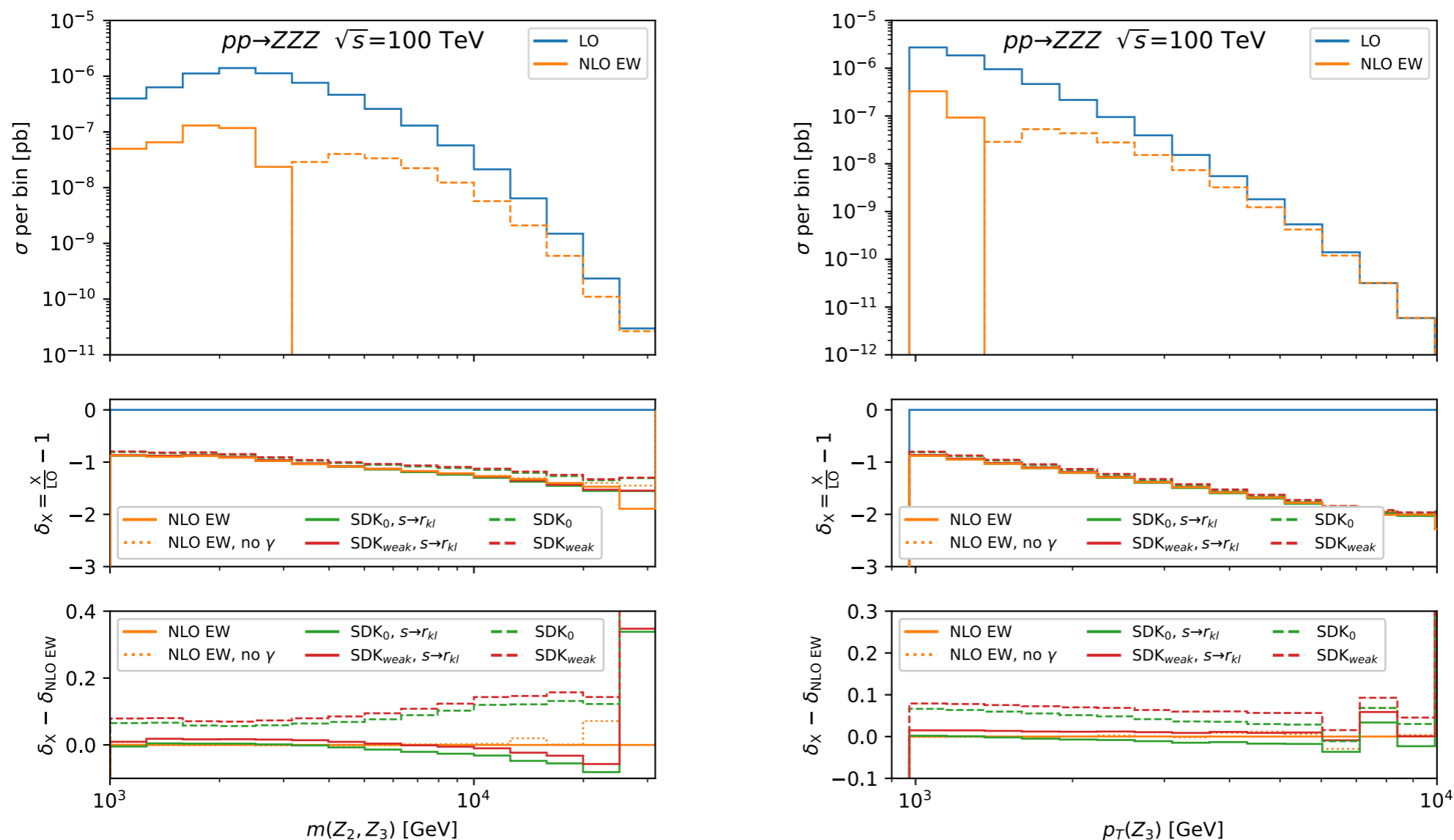
$$p_T(\ell^\pm) > 200 \text{ GeV}, \quad |\eta(\ell^\pm)| < 2.5, \quad m(\ell^+, \ell^-) > 400 \text{ GeV}, \quad \Delta R(\ell^+, \ell^-) > 0.5$$



- Charged FS: SDK<sub>weak</sub> much closer to EW corrections wrt SDK<sub>0</sub>
- $2 \rightarrow 2$  process with hard cuts: small effects due to  $\Delta^{s \rightarrow r_{kl}}$

# ZZZ production

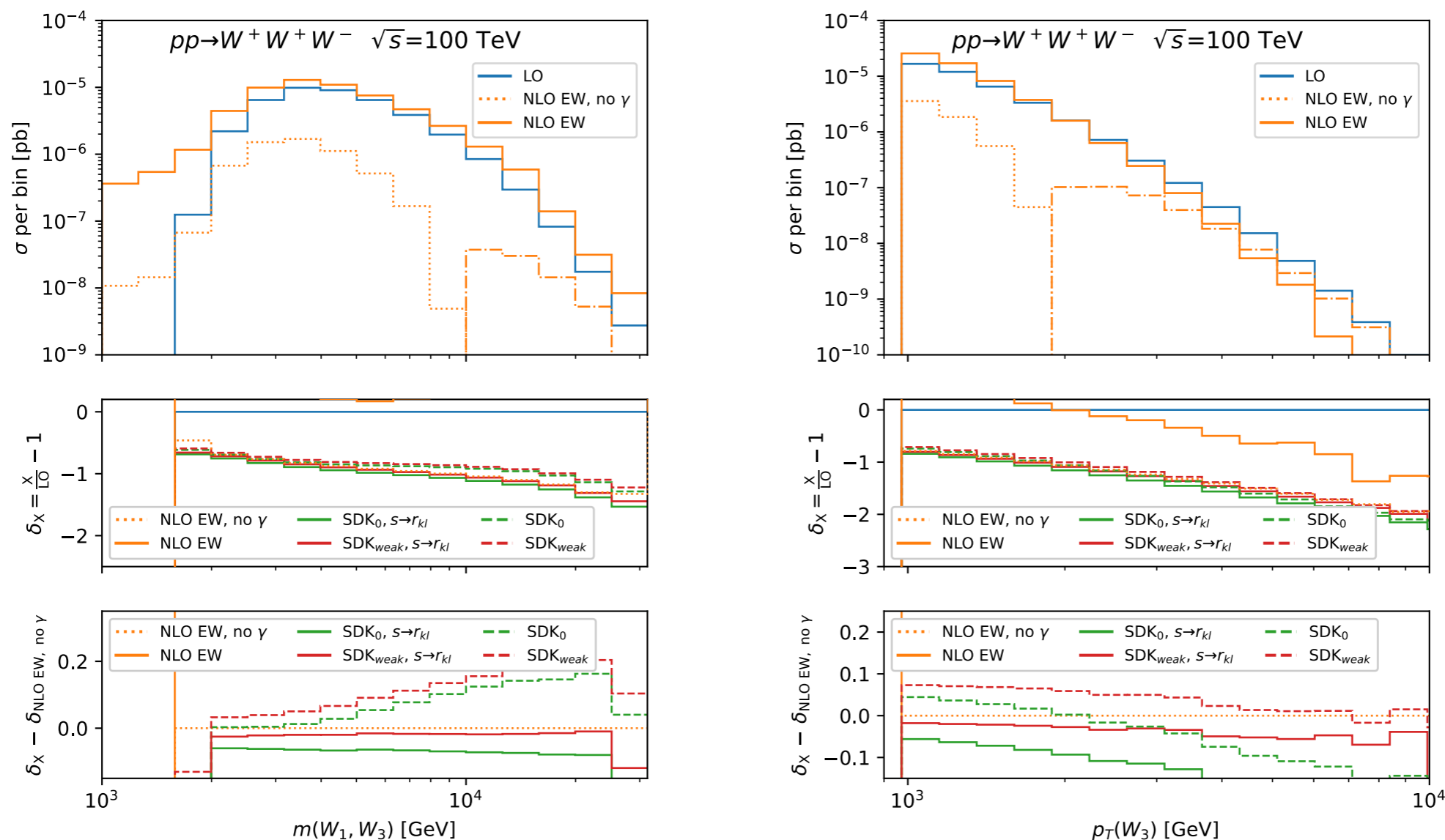
$$p_T(Z_i) > 1 \text{ TeV}, \quad |\eta(Z_i)| < 2.5, \quad m(Z_i, Z_j) > 1 \text{ TeV}, \quad \Delta R(Z_i, Z_j) > 0.5$$



- Neutral FS: small difference between  $SDK_{weak}$  and  $SDK_0$
- $2 \rightarrow 3$  process: inclusion of  $\Delta^{s \rightarrow r_{kl}}$  improves approximation of EW corrections
- EW corrections exceed 100%: need for their resummation

# WW production

$$p_T(W_i) > 1 \text{ TeV}, \quad |\eta(W_i)| < 2.5, \quad m(W_i, W_j) > 1 \text{ TeV}, \quad \Delta R(W_i, W_j) > 0.5$$



- $2 \rightarrow 3$  process with charged FS:  $\text{SDK}_{\text{weak}}$  with  $\Delta^{s \rightarrow r_{kl}}$  closest to exact EW corrections (without initial photons)
- Initial photons (from real radiation) have huge effects: not accounted for by Sudakov approx.



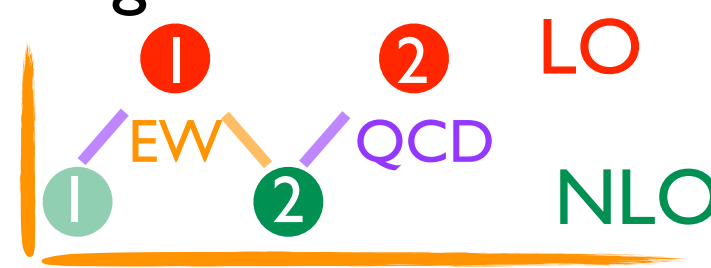


# Including EWSL in NLO+PS simulations

Pagani, Vitos, MZ, 2309.00452

# The problem

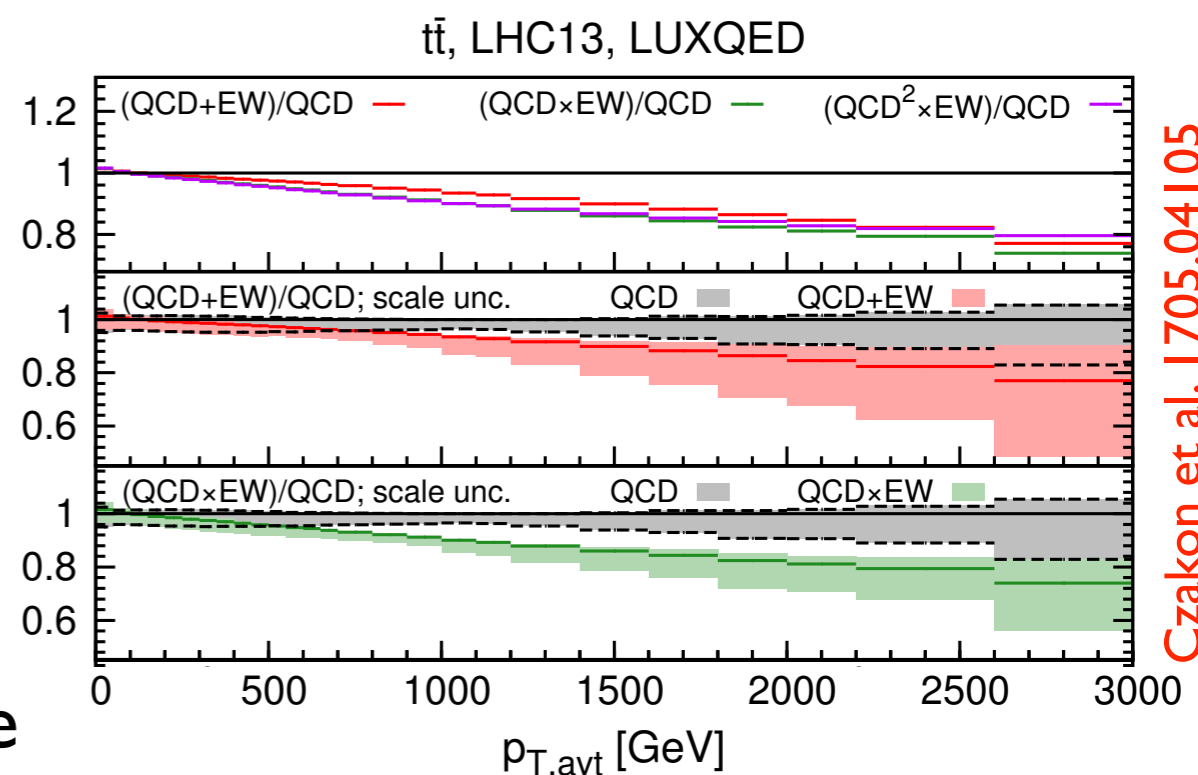
- Matching NLO EW to QED PS is not yet solved in general
  - Exact matching available only for processes with a single LO contribution  
 DY: Barzè et al, 1302.4606; HV(J): Granata et al, 1706.03522;  
 VBS: Chiesa et al, 1906.01863, VV: Chiesa et al, 2005.12146;  
 WZ@NNLO+PS: Lindert et al, 2208.12660
  - Approximate solutions exist, not formally NLO-accurate, but with a decent phenomenological description (when target accuracy is  $\sim 10\%$ )  
 VV(J): Brauer et al, 2005.12128; top: Gutschov et al, 1803.00950;  
 V+jets: Kallweit et al, 1511.08692, ...
- Main issue: how to assign colour-flows to interferences ( $LO_2$  is mostly an interference contribution)
  - However, quite often,  $LO_2/LO_1 \ll \alpha/\alpha_s$  so that these configurations can be somehow neglected
- EWSL are an excellent compromise for this problem:
 



  - They provide the bulk of the cross section, in a fast and stable manner
  - In the  $SDK_{\text{weak}}$  scheme, they can be supplemented by QED PS without double counting

# Including approximate EW corrections beyond NLO

- When combining NLO QCD EW corrections, one can approximate the mixed NNLO<sub>2</sub> term by the so-called multiplicative approach, if both are due to universal effects (soft emissions for QCD, EWSL for EW)
- This stabilises the scale-dependence of EW corrections, which is now NLO-like
- In the context of event-generation, EWSL can improve the multiplicative approach:
  - Each kind of events, Born-like (S) or Real-like (H), can be corrected by the EWSL corresponding to the event's multiplicity
  - Approach can be extended to multijet-merging [Bothmann et al, 2111.13453](#)
  - A smooth transition in the soft/collinear limit of H events must be ensured





# Including EWSL in NLO+PS samples

- in MG5\_aMC, S and H events are defined as follows:

$$d\sigma^{(S)} = d\phi_{n+1} \left[ (\mathcal{B} + \mathcal{V} + \mathcal{C}^{\text{int}}) \frac{d\phi_n}{d\phi_{n+1}} + (\mathcal{C}_{\text{MC}} - \mathcal{C}) \right]$$

$$d\sigma^{(H)} = d\phi_{n+1} (\mathcal{R} - \mathcal{C}_{\text{MC}}) ,$$

- Events from each class can be corrected by the corresponding EWSL

$$w_S \implies (1 + \delta_{(S)}^{\text{EWSL}}) w_S \quad w_H \implies (1 + \delta_{(H)}^{\text{EWSL}}) w_H$$

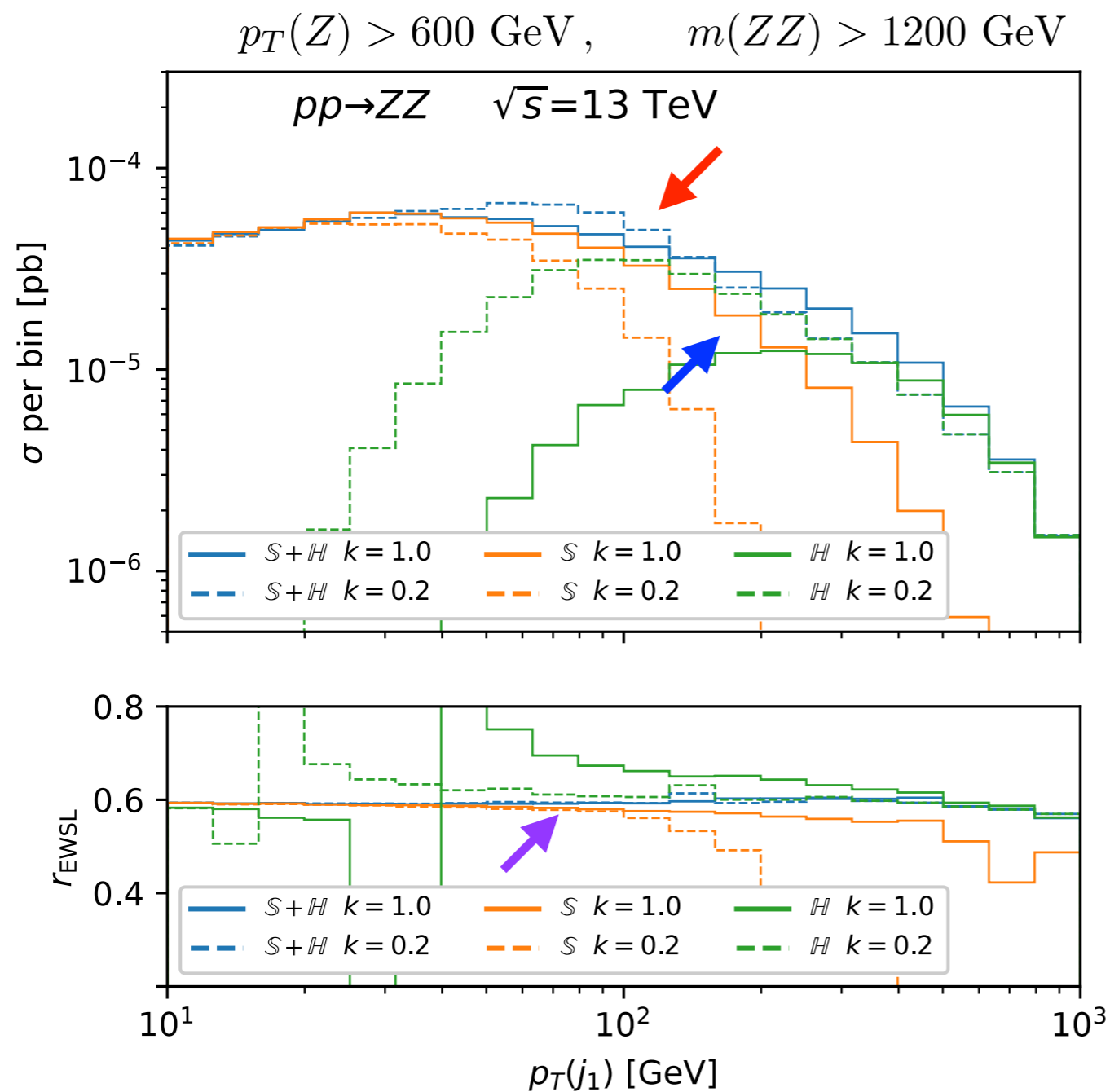
with

$$\delta_{(S)}^{\text{EWSL}} = \delta_{\text{LA}}^{\text{EW}} \Big|_{\text{SDK}_{\text{weak}}} (e_S) \quad \delta_{(H)}^{\text{EWSL}} \equiv \delta_{\text{LA}}^{\text{EW}} \Big|_{\text{SDK}_{\text{weak}}} (e_H) \text{ if } \forall |r_{kl}|/m_W > C$$

$$\delta_{(H)}^{\text{EWSL}} \equiv \delta_{\text{LA}}^{\text{EW}} \Big|_{\text{SDK}_{\text{weak}}} (e_S^{(\hat{k}, \hat{l})}) \quad \text{else}$$

- This enforces the proper IR behaviour of H events
- In principle, there is also an interplay between C and the shower scale  $\mu_s$ . In practice, relative impact of EWSL is independent on  $\mu_s$  variations even by large factors

# Shower-scale independence of relative corrections



$k=1$  (default  $\mu_s$ ) ———

vs

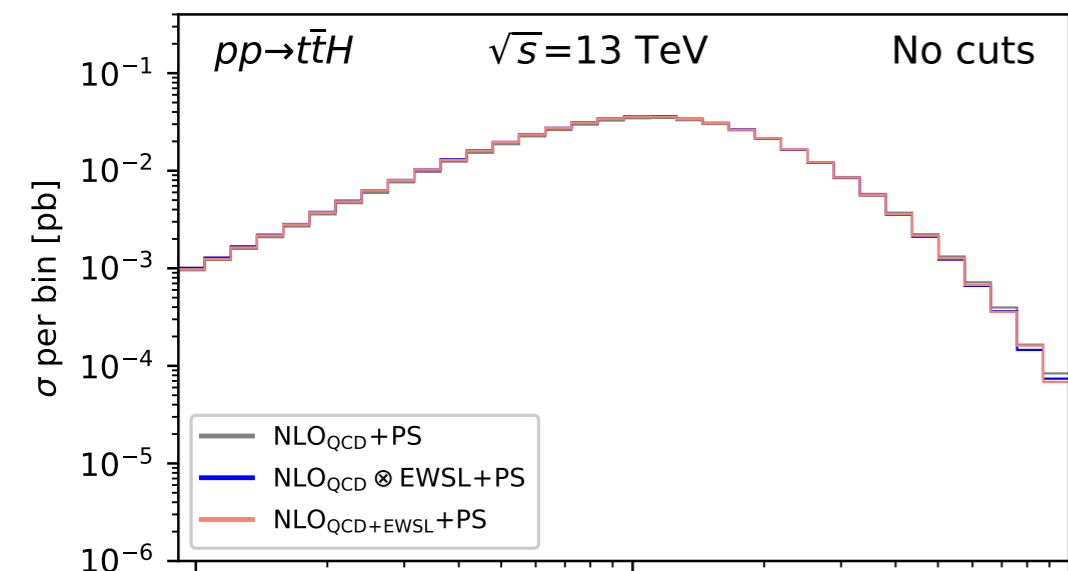
$k=0.2$  (1/5 reduction) - - - - -

Shower-scale variations have effect on rates

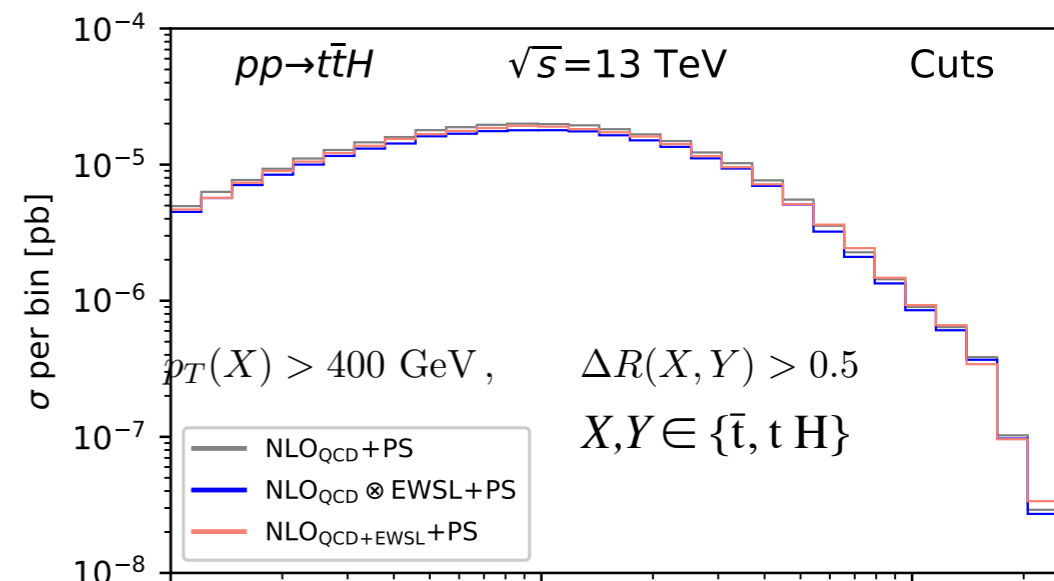
Huge effects on interplay between  $S$  and  $H$  events

Relative effect of EWSL stays the same for  $S+H$

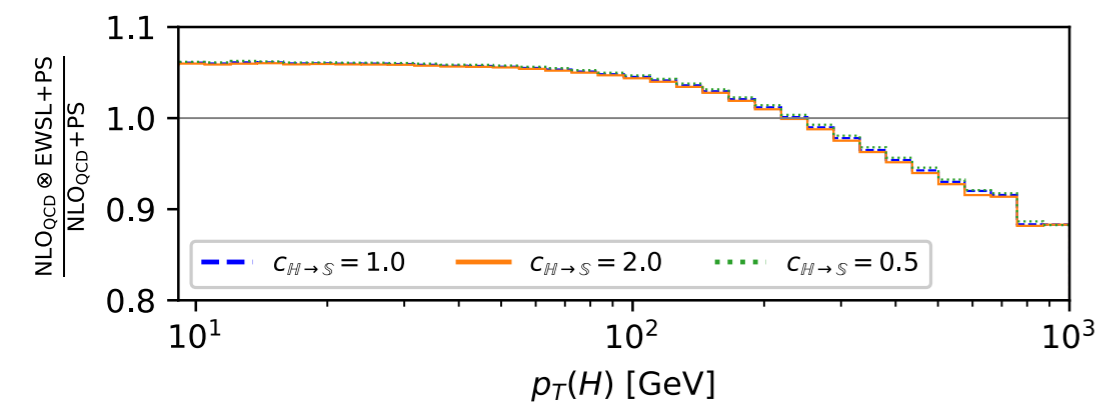
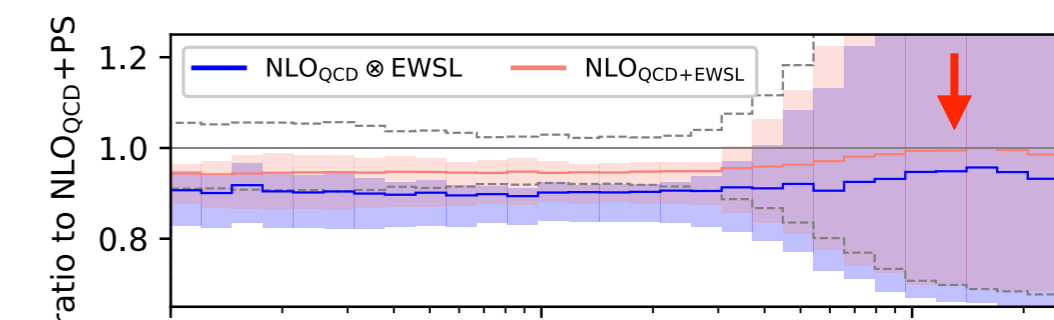
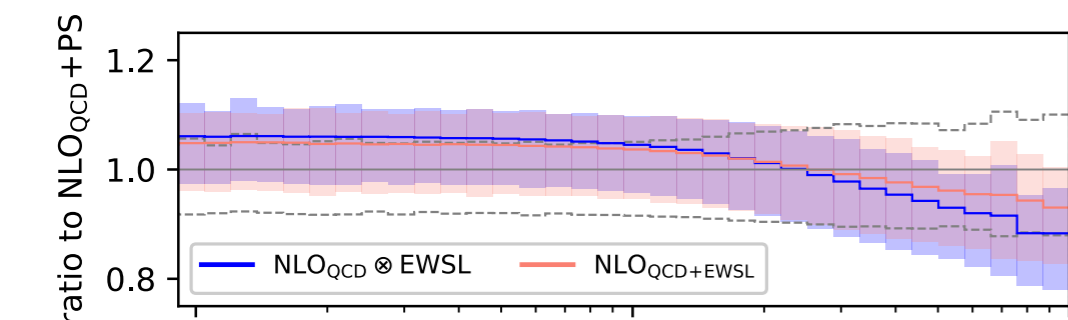
# Results: ttH



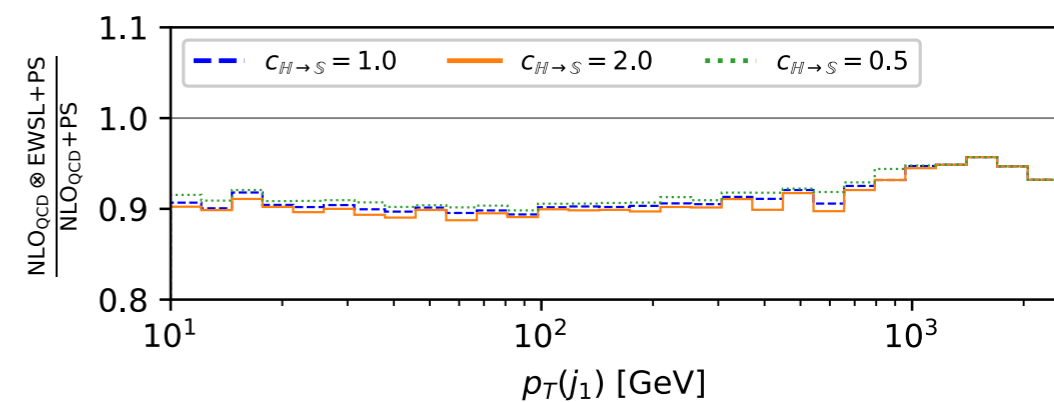
Effect of EW corr.  
rather mild for ttH



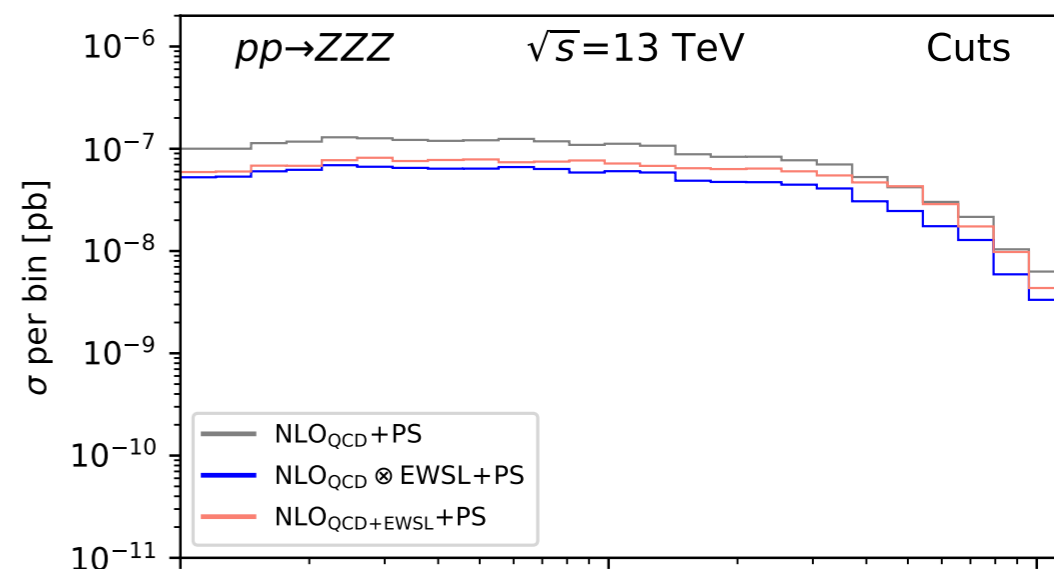
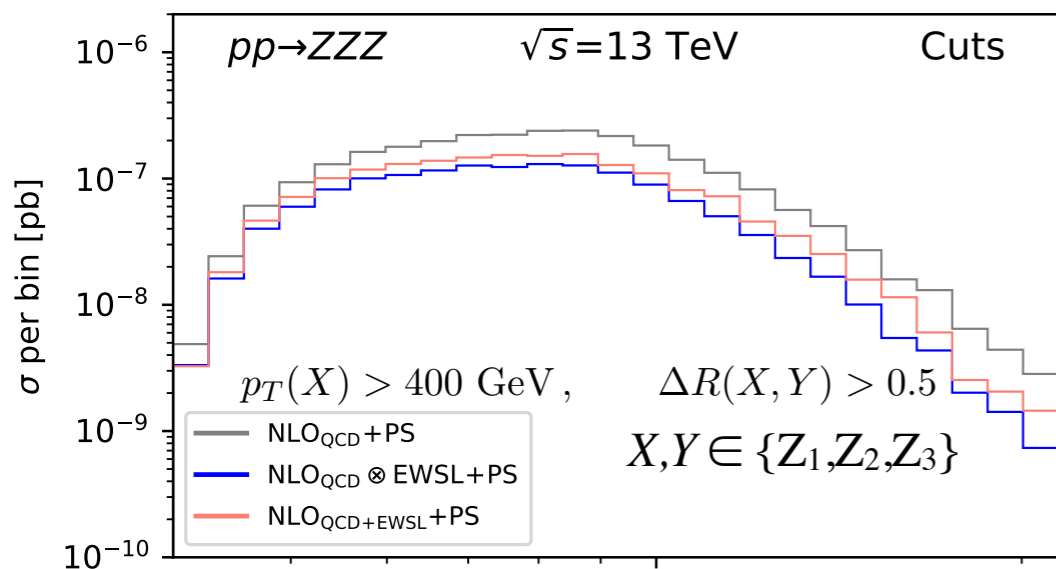
Difference between  
mult. and add.  
approach visible in  
regions dominated  
by hard radiation



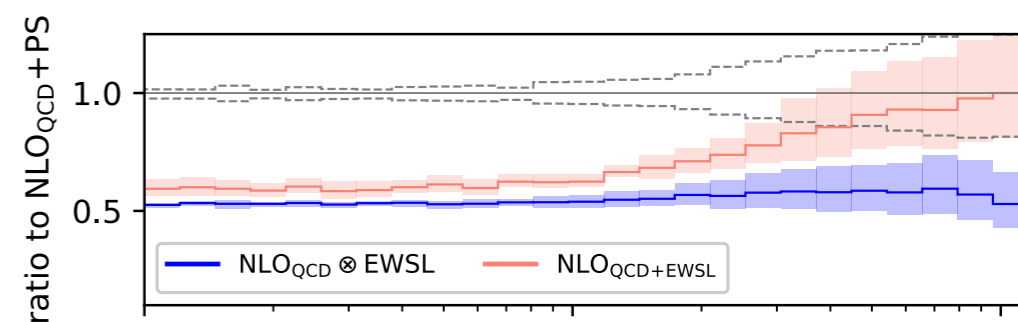
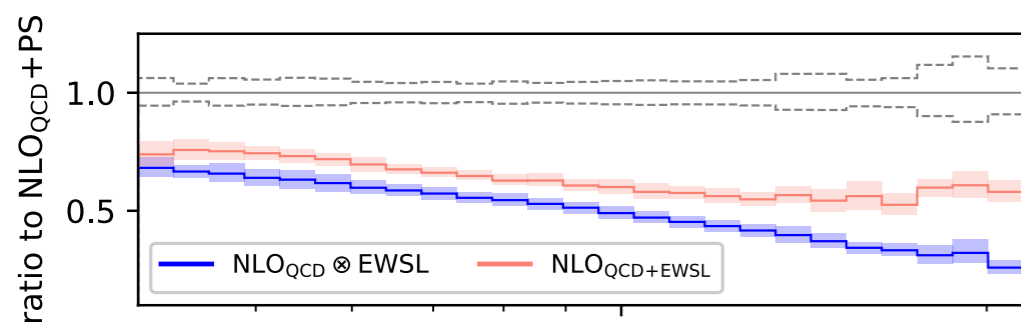
Effect of EWSL  
independent on C



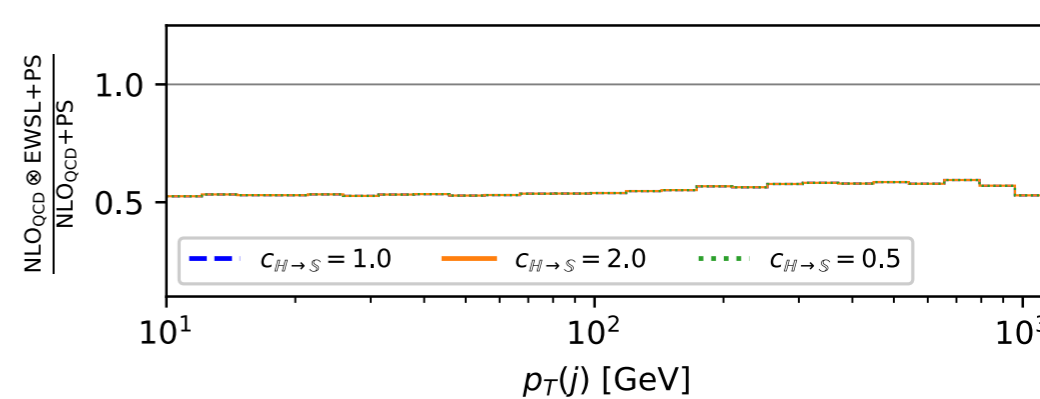
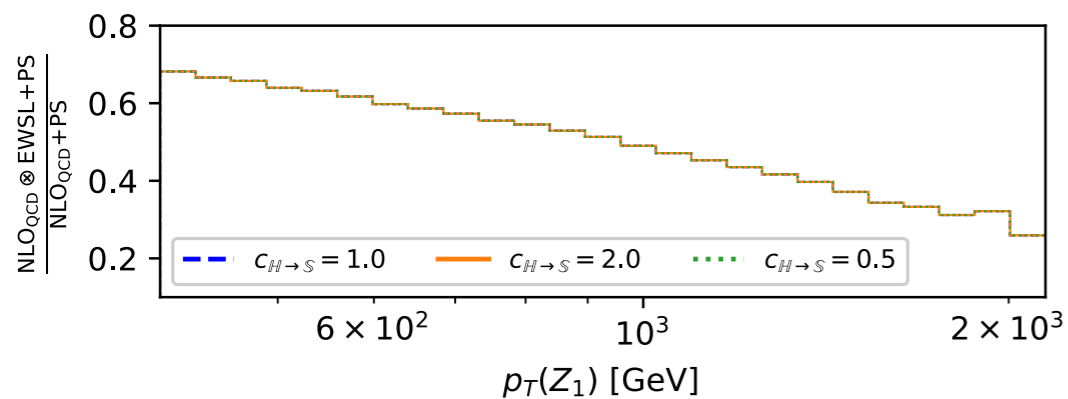
# Results: ZZZ



Effect of EW corr.  
(very) large for 3Z



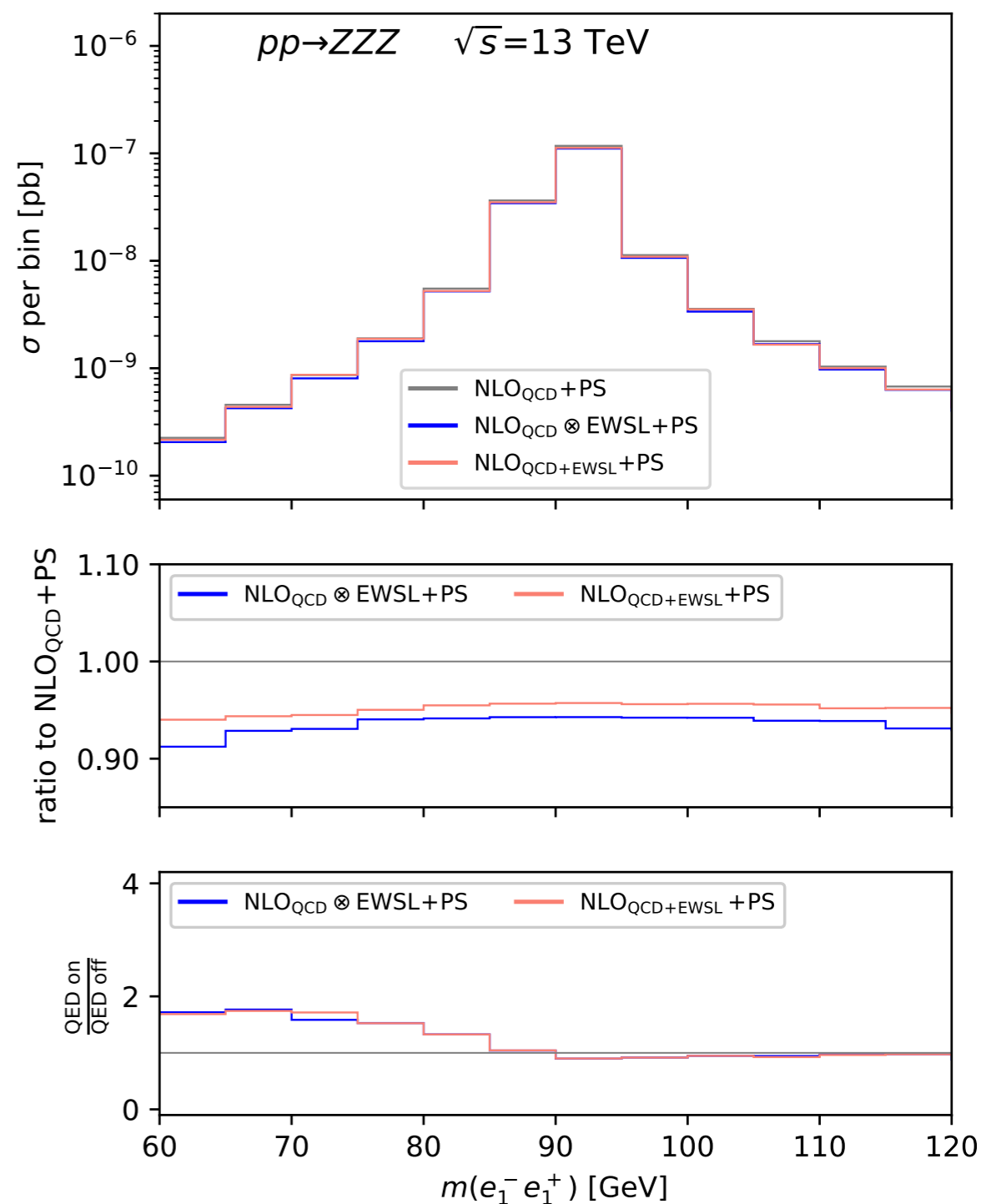
Difference between  
mult. and add.  
approach visible and  
larger than QCD  
scale unc.



Effect of EWSL  
independent on C

# Results: ZZZ (decayed)

- Events reweighted with the EWSL can be further processed with other tools, e.g. MadSpin [Artoisenet et al, 1212.3460](#)
- Decays are included keeping tree-level spin correlations (neglects non-resonant and virtual/EWSL-induced effects)
- The weak-only version of EWSL can be combined with QED PS without double counting







# EWSL for BSM simulations: top-pair production in the SMEFT

El-Faham, Mimasu, Pagani, Severi, Vryonidou, MZ, 24XX.YYYYYY



# EW corrections in the SMEFT

- Typical searches for BSM effects look at tails of distributions, where the high-energy behaviour may be different from the SM
- A comprehensive approach for BSM searches is the usage of Effective Theories, such as the SMEFT
- Currently, SMEFT is simulated without EW corrections
- Computing EW corrections in the SMEFT is a very challenging: so far, only available for very simple processes

**$\mu$  decay:** Pruna et al, 1408.3565

**H decay:** Hartmann et al, 1505.02646 & 1507.03568; Ghezzi et al, 1505.03706; Gauld et al, 1512.02508; Dawson et al, 1801.01136 & 1807.11504; Dedes et al, 1805.00302 & 1903.12046; Cullen et al, 1904.06358 & 2007.15238;

**Z/W pole obs.:** Hartmann et al, 1611.09879; Dawson et al, 1808.05948 & 1909.02000;

**Drell-Yan:** Dawson et al, 2105.05852

- Can we use EWSL in the SMEFT?



# Mass suppressed amplitudes in the SMEFT

- While in the SM processes with a mass-suppressed amplitude are very rare, they are quite common in the SMEFT

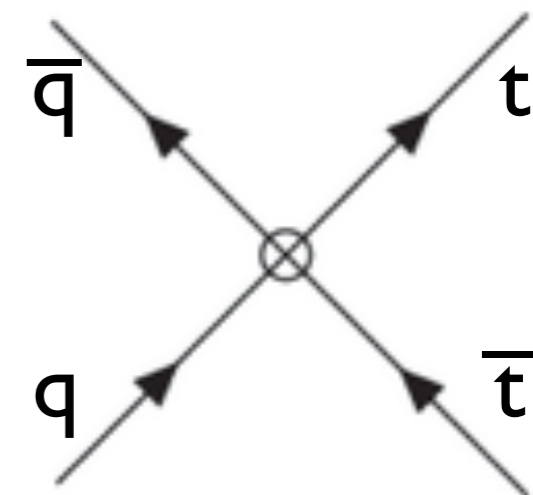
$$O_{tG} = \frac{g_S C_{tG}}{\Lambda^2} \bar{Q}_L \tilde{\phi} \sigma^{\mu\nu} G_{\mu\nu} t_R = \frac{g_S C_{tG} v}{\Lambda^2} \bar{t}_L \sigma^{\mu\nu} G_{\mu\nu} t_R + \frac{g_S C_{tG}}{\Lambda^2} h \bar{t}_L \sigma^{\mu\nu} G_{\mu\nu} t_R$$

↙ mass-supp.                      ↙ non mass-supp.

- The D&P algorithm works only for non-mass suppressed amplitudes
  - One cannot use EWSL in general for SMEFT processes
- However, for those operators which are not mass suppressed, EWSL can give us the bulk of EW corrections at high energy

# A class of non-mass-suppressed contributions: four fermion operators

- 4f operators are a class of non mass-suppressed operators
- They are relevant for Drell-Yan, top pair production, ...
- We can use them to validate D&P in a non-trivial BSM case, and to estimate, for the first time, the impact of EW corrections on these processes in the SMEFT.



Are EW corrections the same as in the SM?

- If we restrict ourselves to 4f operators, the 3-coupling expansion of the amplitudes (QCD, EW,  $1/\Lambda$ ) greatly simplifies
- In the case of  $q\bar{q} \rightarrow t\bar{t}$

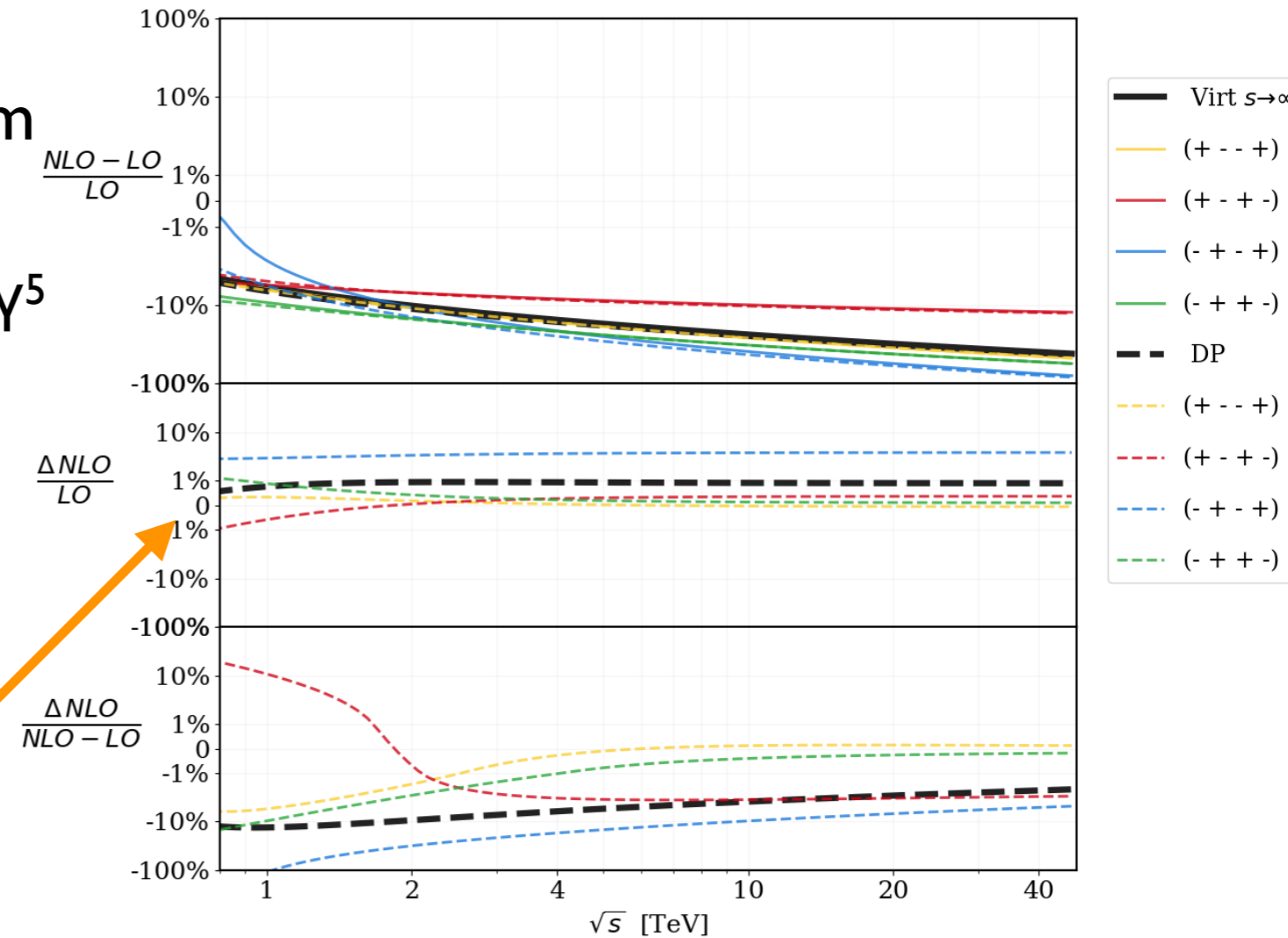
$$\begin{aligned} \mathcal{O}_{tu}^8 &= \sum_{f=1}^2 (\bar{t}\gamma_\mu T^A t)(\bar{u}_f\gamma^\mu T_A u_f), \\ \mathcal{O}_{td}^8 &= \sum_{f=1}^3 (\bar{t}\gamma_\mu T^A t)(\bar{d}_f\gamma^\mu T^A d_f), \\ \mathcal{O}_{tq}^8 &= \sum_{f=1}^2 (\bar{t}\gamma^\mu T^A t)(\bar{q}_f\gamma_\mu T_A q_f), \\ \mathcal{O}_{Qu}^8 &= \sum_{f=1}^2 (\bar{Q}\gamma_\mu T^A Q)(\bar{u}_f\gamma^\mu T^A u_f), \\ \mathcal{O}_{Qd}^8 &= \sum_{f=1}^3 (\bar{Q}\gamma_\mu T^A Q)(\bar{d}_f\gamma^\mu T^A d_f), \\ \mathcal{O}_{Qq}^{1,8} &= \sum_{f=1}^2 (\bar{Q}\gamma_\mu T^A Q)(\bar{q}_f\gamma^\mu T_A q_f), \\ \mathcal{O}_{Qq}^{3,8} &= \sum_{f=1}^2 (\bar{Q}\gamma_\mu T^A \sigma_I Q)(\bar{q}_f\gamma^\mu T_A \sigma^I q_f) \end{aligned}$$

$$\lim_{M_W^2/s \rightarrow 0} \mathcal{M}_1^{\text{NP}} = \delta\mathcal{M}^{\text{NP}} = \mathcal{M}_0^{\text{NP}} \delta_{\text{SM}}^{\text{EW}} + \cancel{\mathcal{M}_0^{\text{NP}'}} \delta_{\text{SM}}^{\text{QCD}} + \mathcal{M}_0^{\text{SM}} \delta_{\text{NP}}^{\text{EW}} + \cancel{\mathcal{M}_0^{\text{SM}'}} \delta_{\text{NP}}^{\text{QCD}}$$

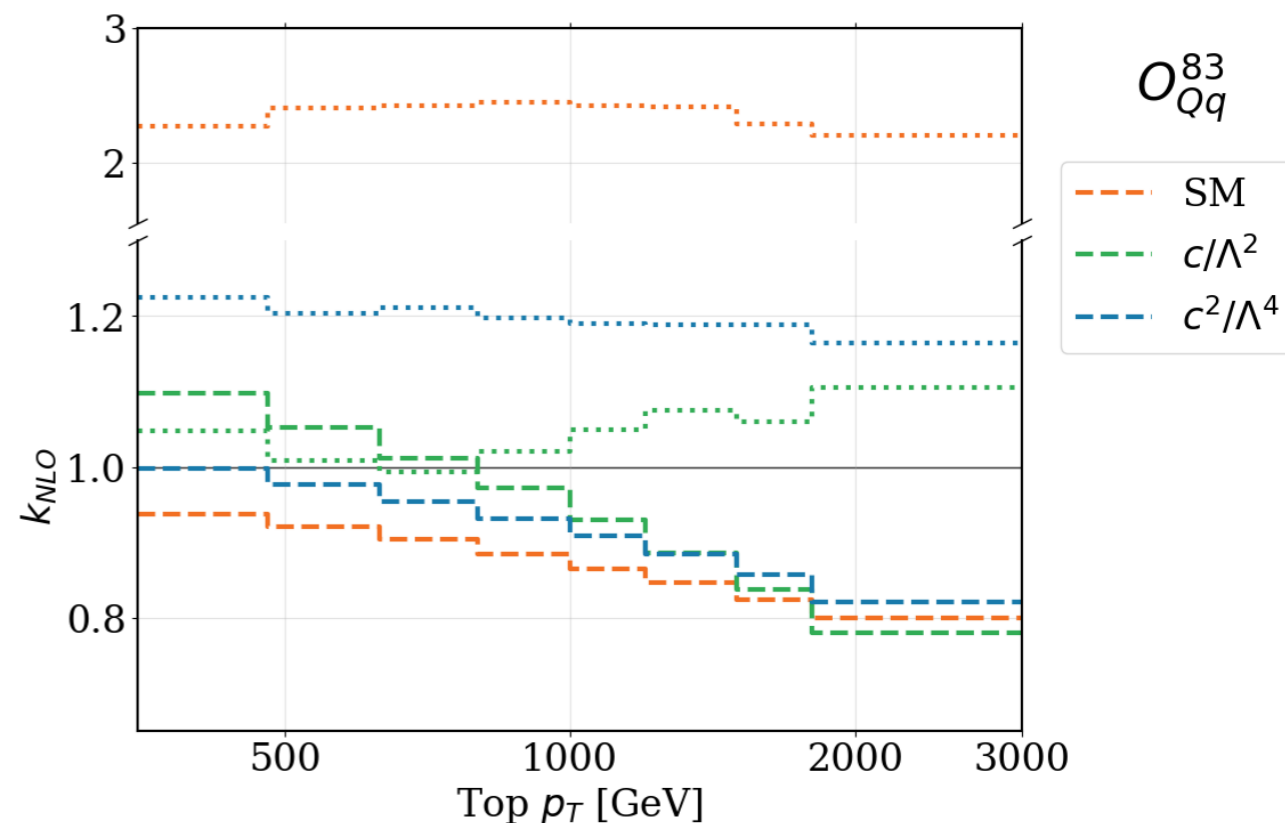
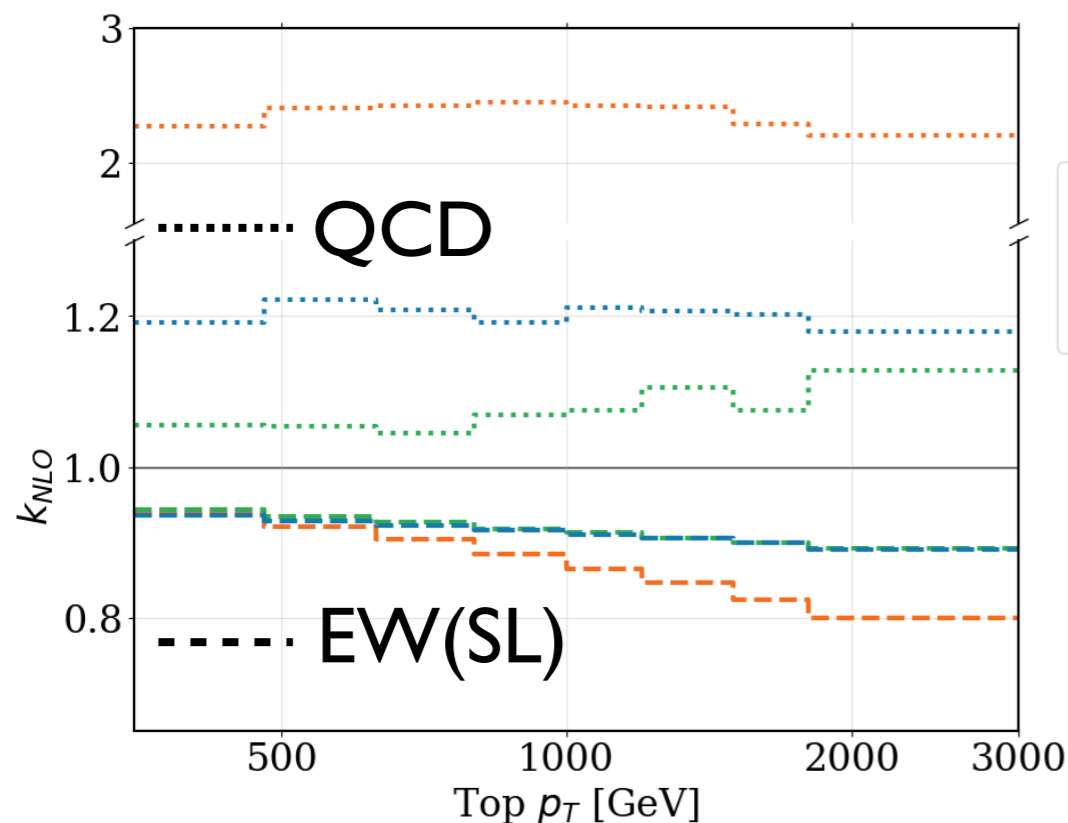
=0 for color octets
=0 for 4F op.
=0 for  $q \neq b$

# Validation of one-loop results

- We compute 1-loop EW corrections to  $u\bar{u} \rightarrow t\bar{t}$  and we compare with the D&P algorithm
- Loops are computed with FeynCalc+Feynarts+PackageX.  $\gamma^5$  is treated in the BHMV scheme
- EWSL are computed with MG5\_aMC, on top of the  $1/\Lambda^4$  Born
- Difference between EWSL and exact virtual approaches a constant



# EW Corrections to top-pair production

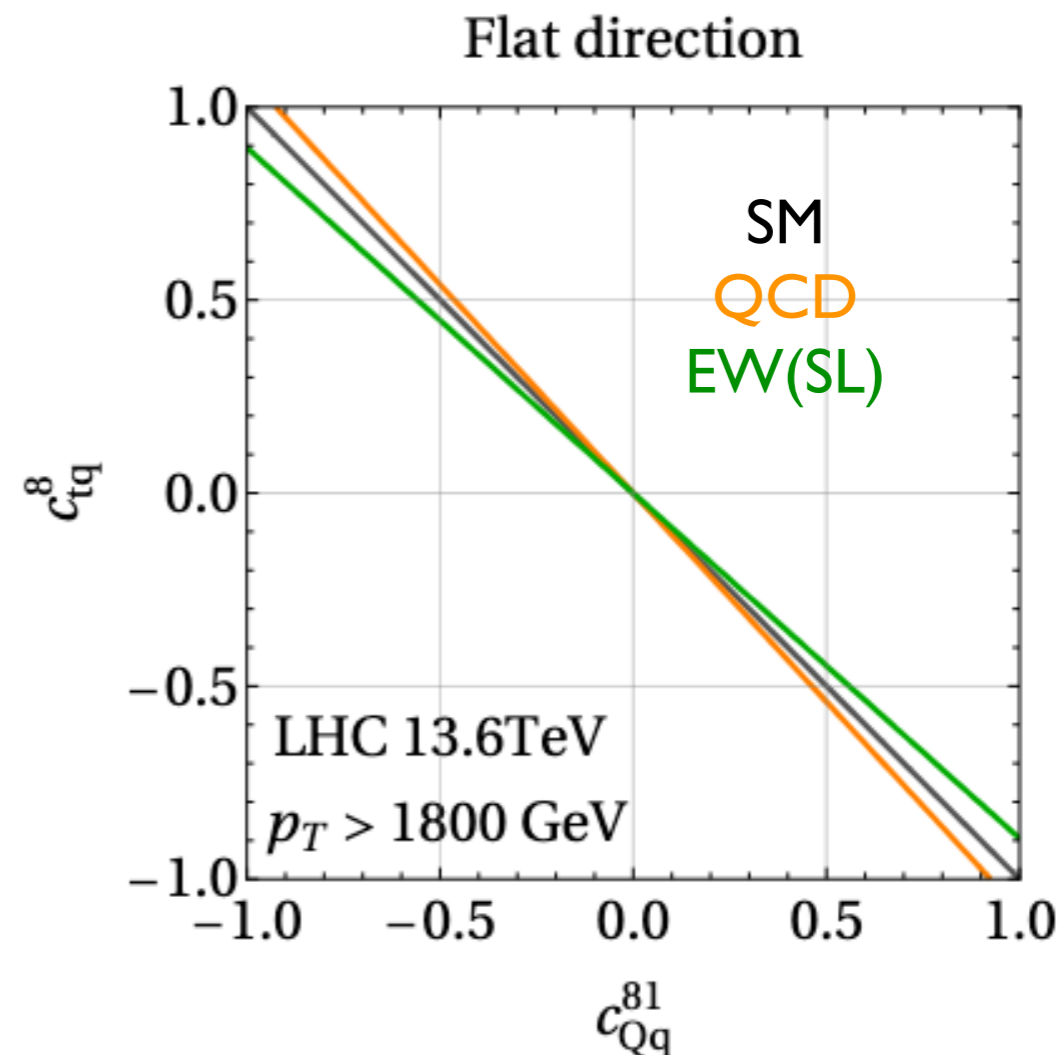


- Relative impact of EW(SL) is different between SM,  $1/\Lambda^2$ ,  $1/\Lambda^4$  terms. Pattern of corrections depend on operator
- Difference related to isospin-linked contributions (single-logarithmic)
- EFT contributions show cancelations between QCD and EW
- It is inaccurate to propagate SM K-factors to SMEFT contributions
- Impact of EW corrections about 10% at 1 TeV

# Lifting degeneracy between operators

Cross-section [pb]  
Top  $p_T > 1.8$  TeV

SM	Cross-section [pb]			
	LO	EWSL		
	$2.01 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$		
	Linear		Quadratic	
	LO	SUD	LO	SUD
$c_{Qq}^{1,8}$	$3.78 \cdot 10^{-4}$	$2.56 \cdot 10^{-4}$	$9.04 \cdot 10^{-4}$	$6.06 \cdot 10^{-4}$
$c_{Qq}^{3,8}$	$2.20 \cdot 10^{-4}$	$1.72 \cdot 10^{-4}$	$9.04 \cdot 10^{-4}$	$7.41 \cdot 10^{-4}$
$c_{tq}^8$	$3.78 \cdot 10^{-4}$	$2.86 \cdot 10^{-4}$	$9.04 \cdot 10^{-4}$	$6.80 \cdot 10^{-4}$
$c_{tu}^8$	$3.00 \cdot 10^{-4}$	$2.68 \cdot 10^{-4}$	$7.30 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$
$c_{Qu}^8$	$2.99 \cdot 10^{-4}$	$2.43 \cdot 10^{-4}$	$7.30 \cdot 10^{-4}$	$5.88 \cdot 10^{-4}$
$c_{td}^8$	$7.95 \cdot 10^{-5}$	$7.12 \cdot 10^{-5}$	$1.74 \cdot 10^{-4}$	$1.56 \cdot 10^{-4}$
$c_{Qd}^8$	$7.92 \cdot 10^{-5}$	$6.45 \cdot 10^{-5}$	$1.74 \cdot 10^{-4}$	$1.41 \cdot 10^{-4}$



- EW corrections lift degeneracy of different operators, removing flat directions in global fits



# Wrapping up...

- LHC is restarting: a challenging physics programme is awaiting us!
- Search for new physics relies on accurate knowledge of SM processes  
→ Inclusion of QCD and EW corrections crucial
- EW corrections dominated by Sudakov logarithms at high energies
- EWSL provide a fast and stable approximation for EW corrections, with some practical advantages
  - Possibility to deliver predictions at NLO+NLL EW
  - Easy matching/merging
  - Straightforward extension to BSM scenarios
- However, large EW effects can also come via other mechanisms (photon PDF, quasi-collinear configurations, etc)  
→ the validity of the EWSL approximation should be assessed process by process and observable by observable





# Conclusion & Outlook

- We have automated EWSL in MG5\_aMC, based on the work of Denner&Pozzorini, with a couple of extensions
- EWSL thoroughly validated vs exact virtual amplitude
- For physical cross-sections, we have devised a weak-only version of EWSL
- EWSL contributions can be included in NLO+PS samples via reweighting
  - For the moment, our method neglects terms originating from  $LO_2$ , therefore it can be applied only for processes where  $LO_2/LO_1 \ll \alpha/\alpha_s$
  - EWSL in the  $SDK_{\text{weak}}$  approach can be combined with QED PS
- WIP for the application of EWSL in the SMEFT
  - Care should be used to avoid mass-suppressed terms
  - Results for 4 fermion operators: simplest case
    - Relative impact is different on  $1/\Lambda^4$ ,  $1/\Lambda^2$ , and SM. Important for EFT fits
    - EW corrections lift degeneracy between operators



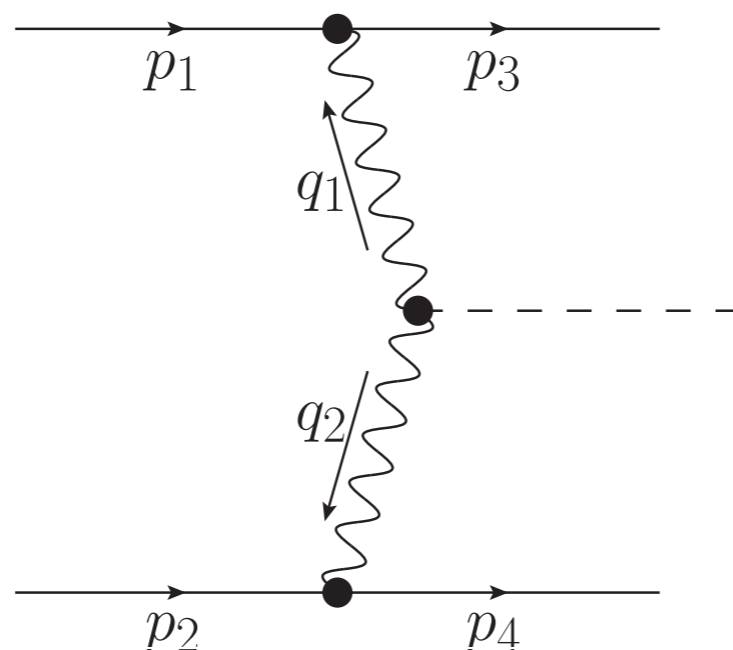
Thank You!



# Backup

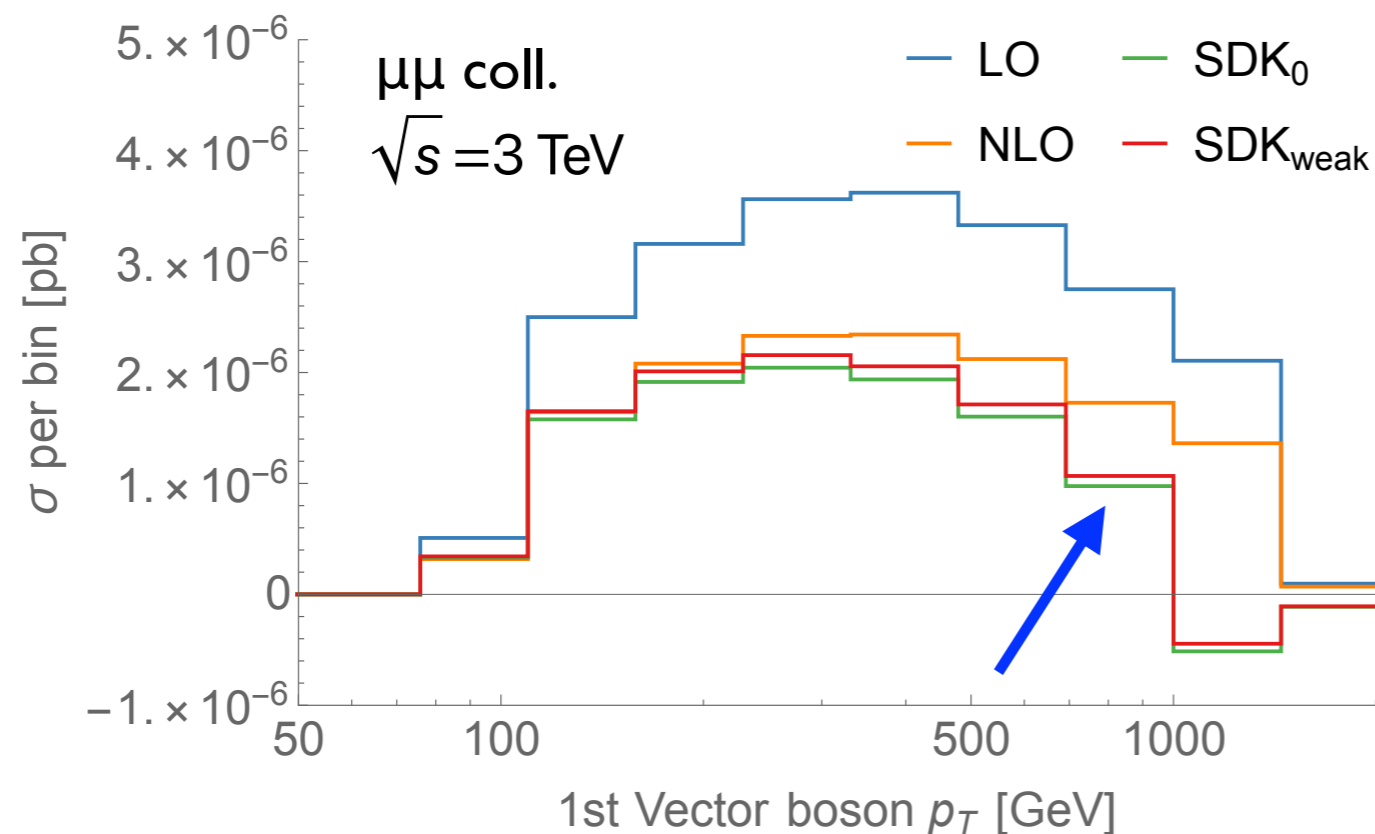
# Dominant helicities

- The derivation of D&P crucially relies on the amplitude not being mass-suppressed
- If  $d$  is the dimensionality of a squared matrix element (for  $2 \rightarrow n$ ,  $d=2-n$ ), the D&P algorithm applies only if  $|M|^2$  scales with  $s$  as  $s^d$
- A notable exception: Higgs VBF, for which  $|M|^2 \sim M_W^2/s^2$



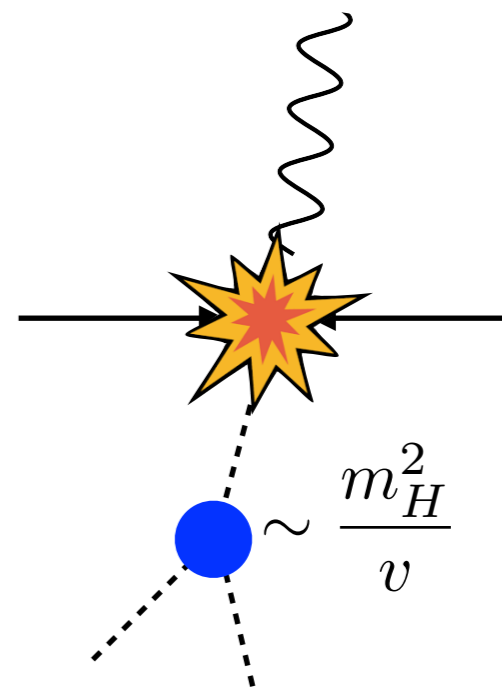
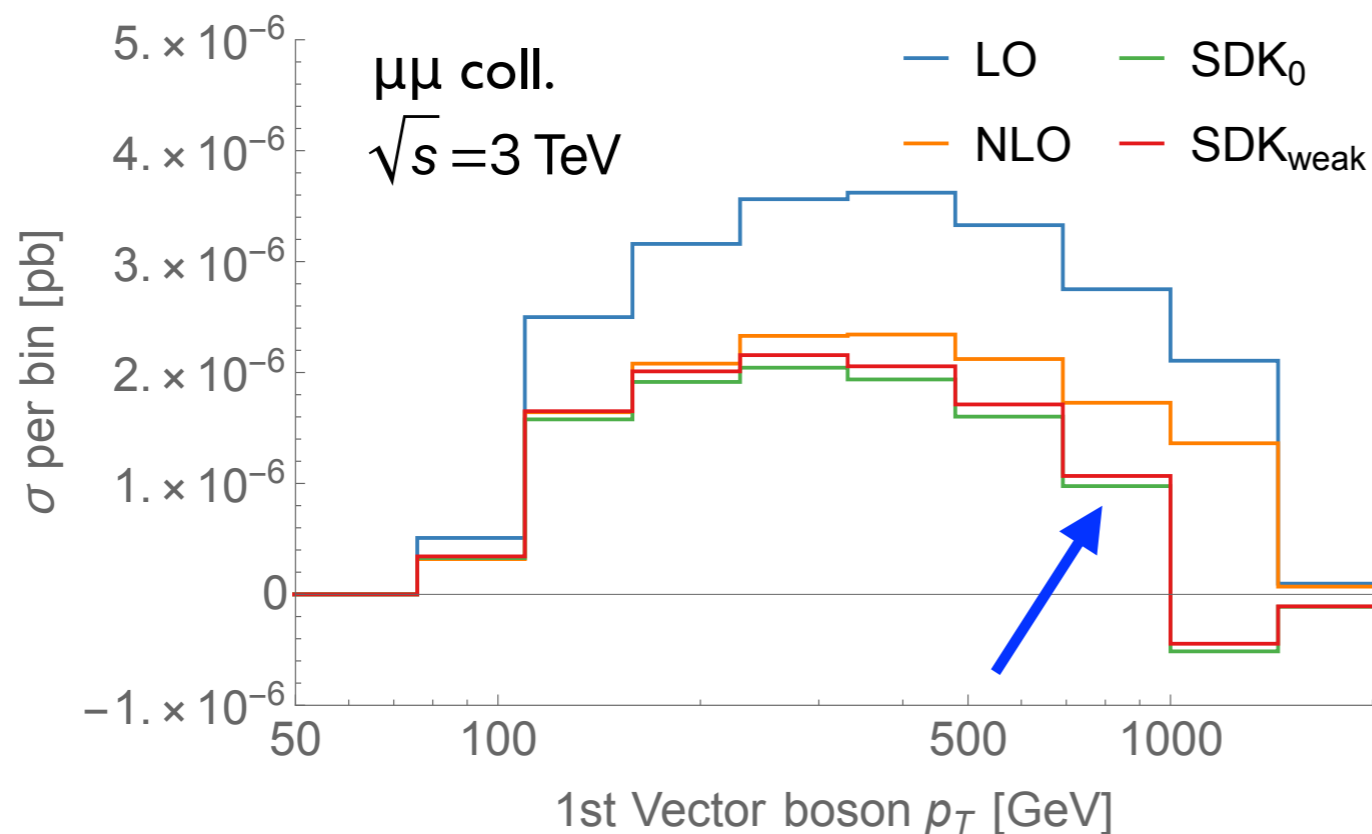
# Don't buy everything they sell

- In ZHH production, at *large*  $p_T(Z)$ , EWSLs fail to reproduce EW corrections



# Don't buy everything they sell

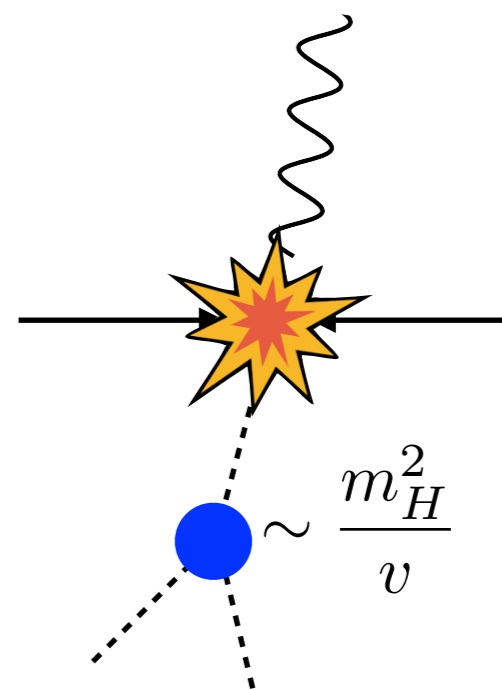
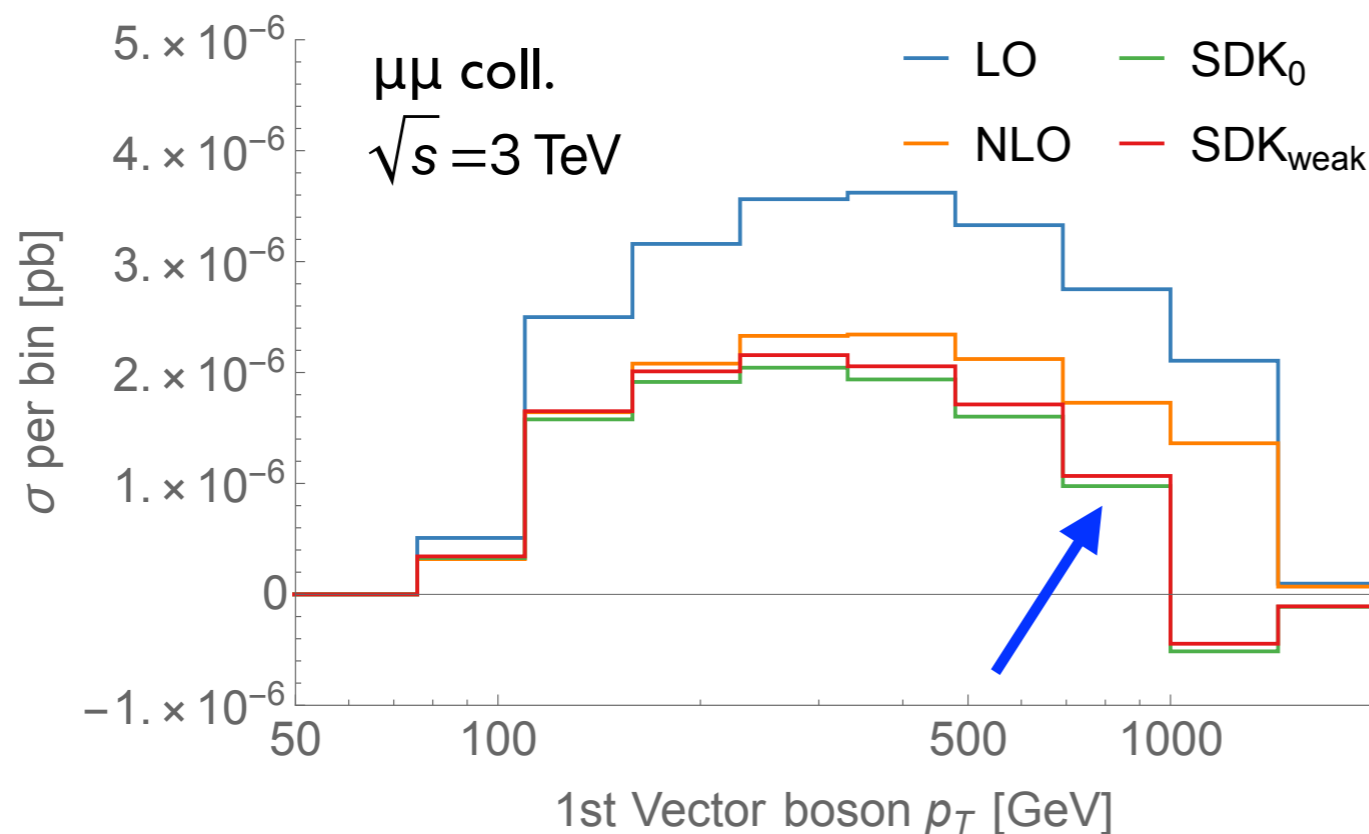
- In ZHH production, at *large*  $p_T(Z)$ , EWSLs fail to reproduce EW corrections



- These configurations are dominated by low  $M(HH)$  and are mass-suppressed (dominated by the trilinear diagram)

# Don't buy everything they sell

- In ZHH production, at *large*  $p_T(Z)$ , EWSLs fail to reproduce EW corrections

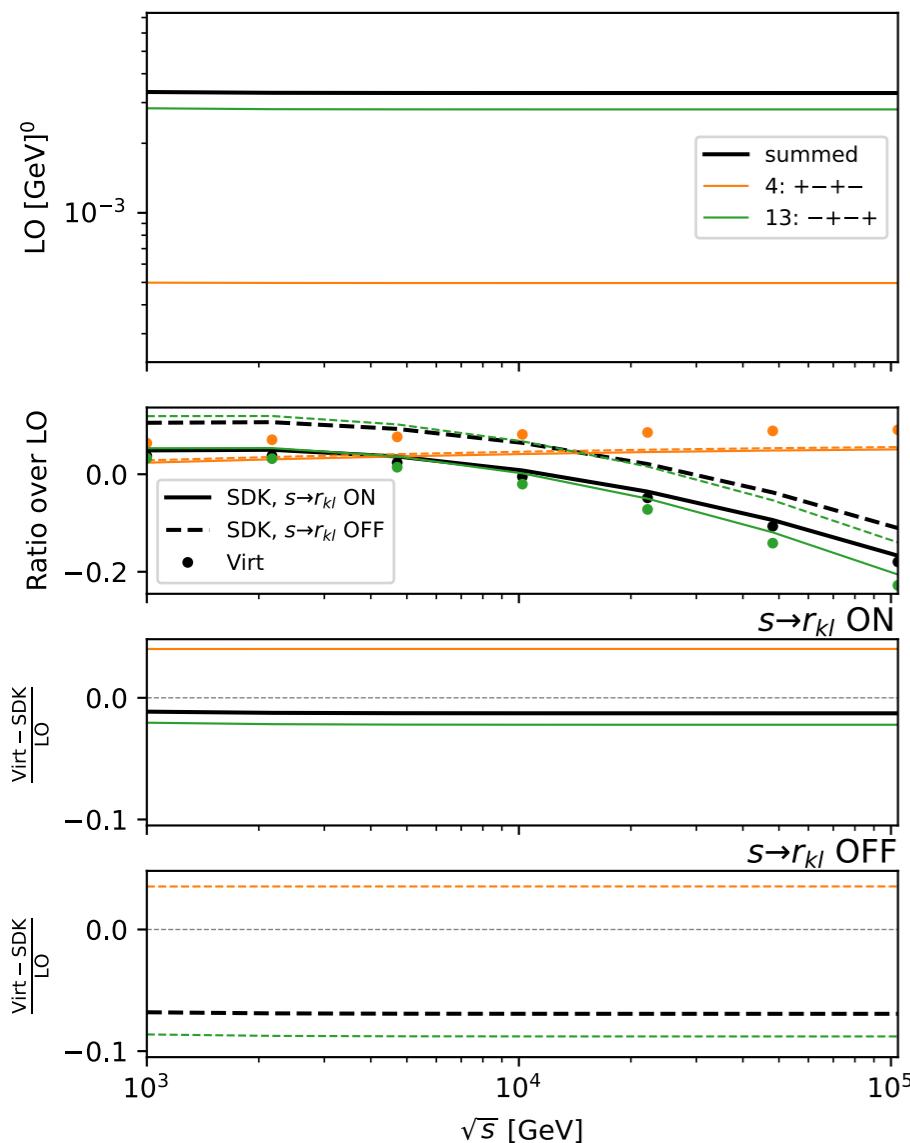


- These configurations are dominated by low  $M(HH)$  and are mass-suppressed (dominated by the trilinear diagram)

WIP with Y. Ma, D. Pagani

# Some first results, $2 \rightarrow 2$

$d\bar{d} \rightarrow e^- e^+$  LO  $O(\alpha^2)$



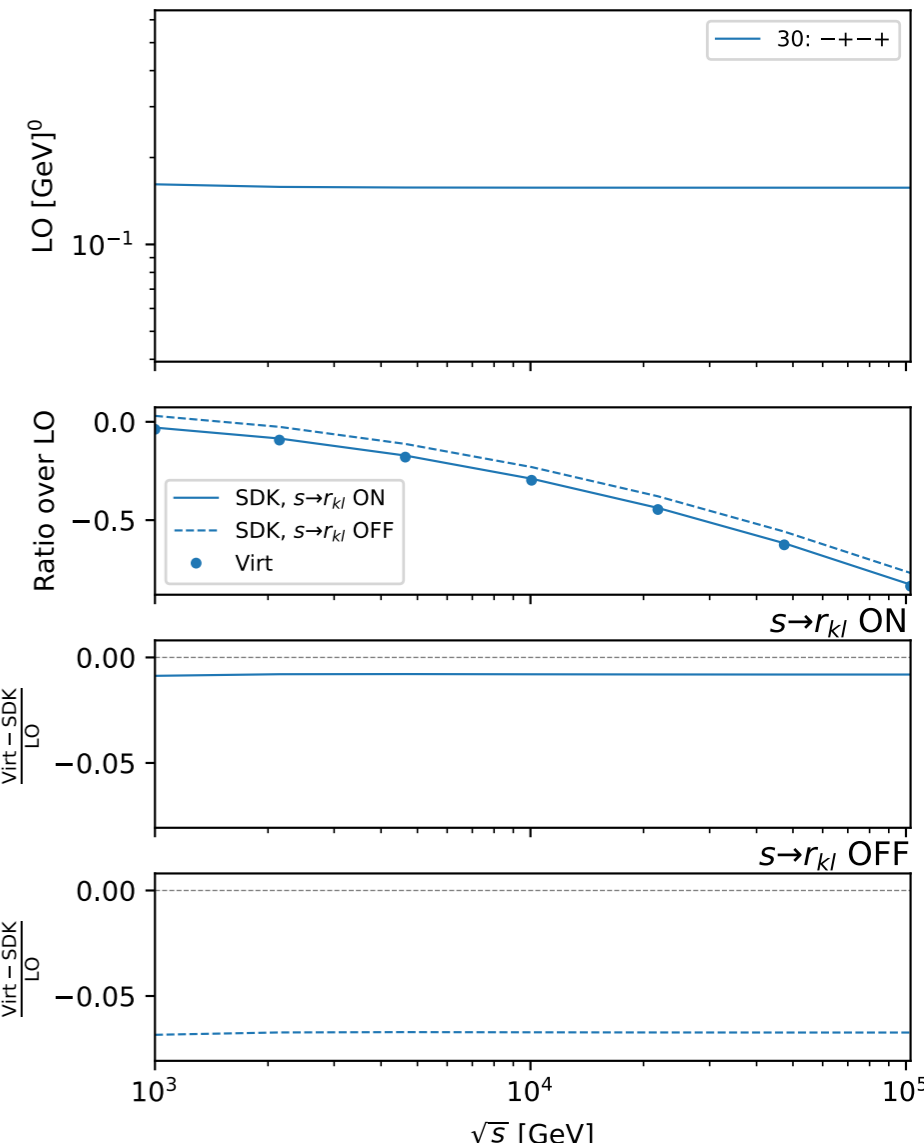
LO amplitude, for each leading helicity

Loop or Sudakov (in different approx.) over LO

(Loop-Sudakov)/LO must be a constant if logs are correct (this is verified also with a fit)

We see that the inclusion of the angular dependent  $\Delta^{s \rightarrow r_{kl}}$  term in general reduces the constant

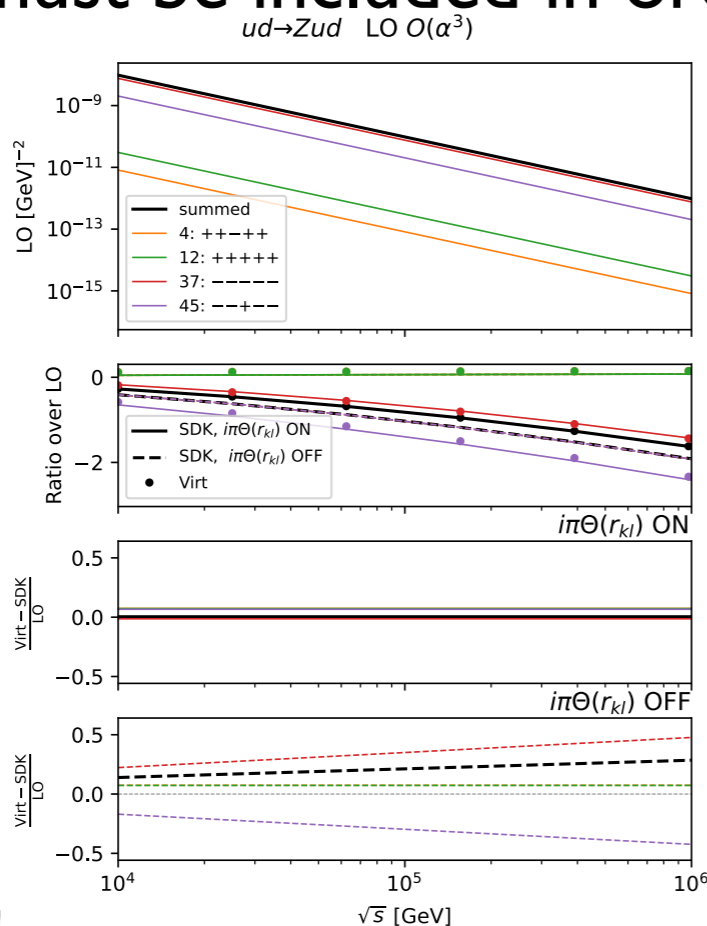
$u\bar{d} \rightarrow W^+ Z$  LO  $O(\alpha^2)$





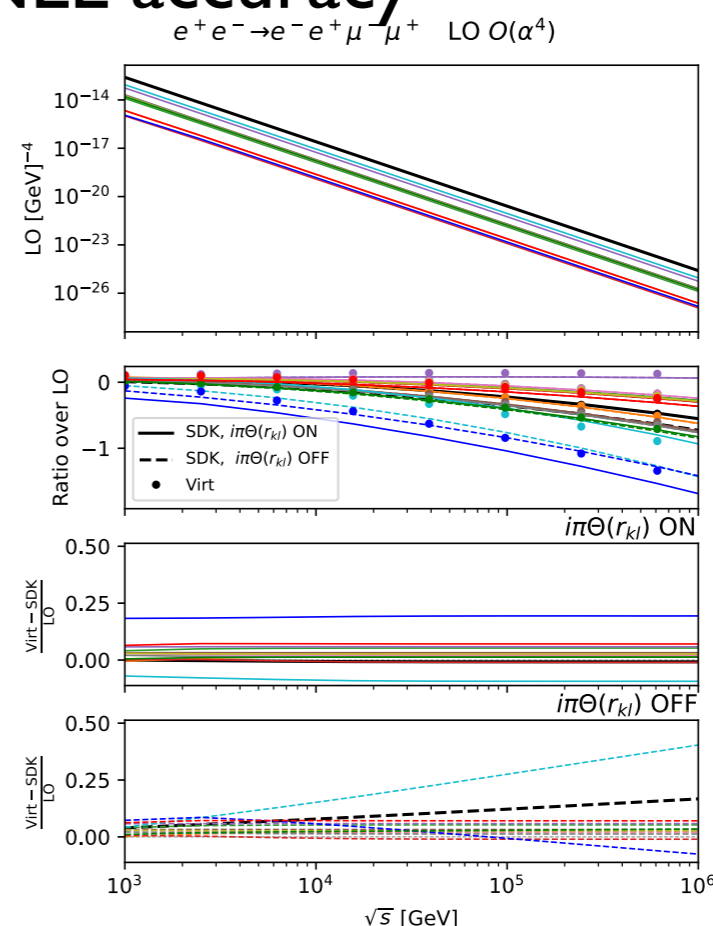
# The missing $i\pi\theta(r_{kl})$ factor

- $2 \rightarrow 2$  amplitudes (as those considered by D&P) are always real (optical theorem). Any missing imaginary part in the logs drops out when considering  $2\Re(BV^*) \simeq 2\Re(BB'^*\delta)$
- For  $2 \rightarrow n$ ,  $n \geq 3$ , imaginary parts from the logs can combine with those of  $BB'^*$ , giving rise to single-logarithmic terms
- They must be included in order to claim NLL accuracy



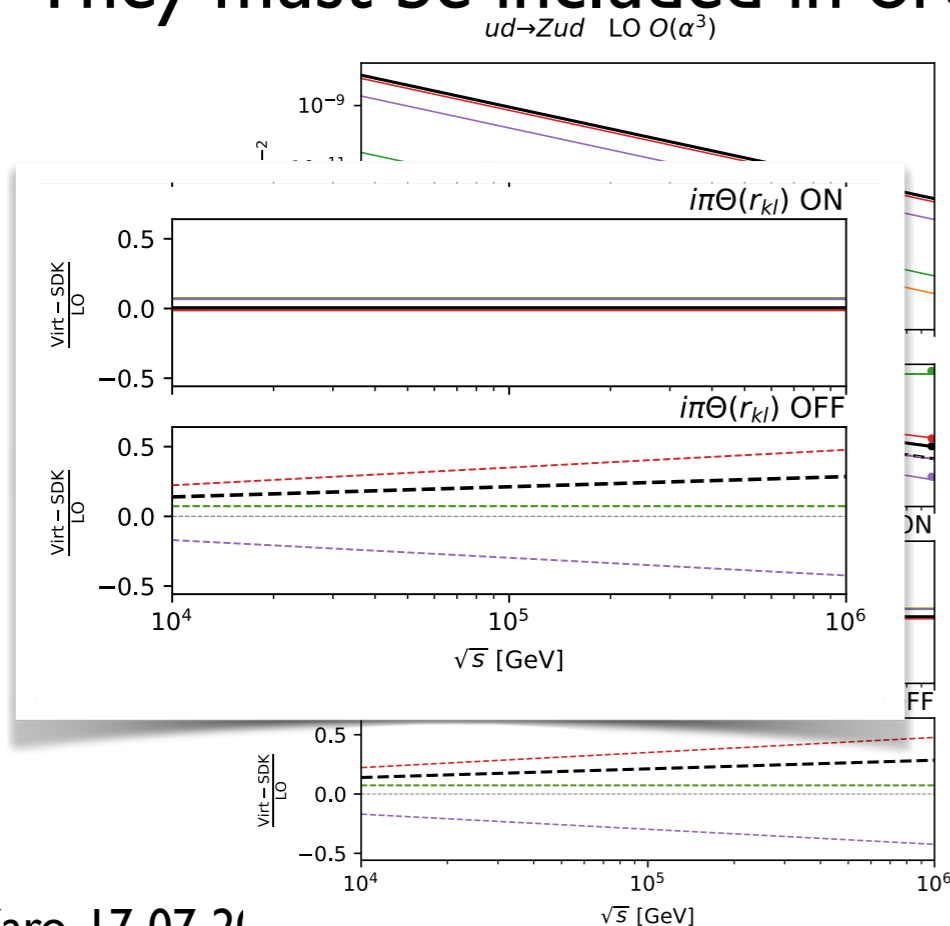
constant V-SDK  
helicity by helicity

V-SDK  $\sim \log(s)$   
if  $i\pi\theta(r_{kl})$  neglected



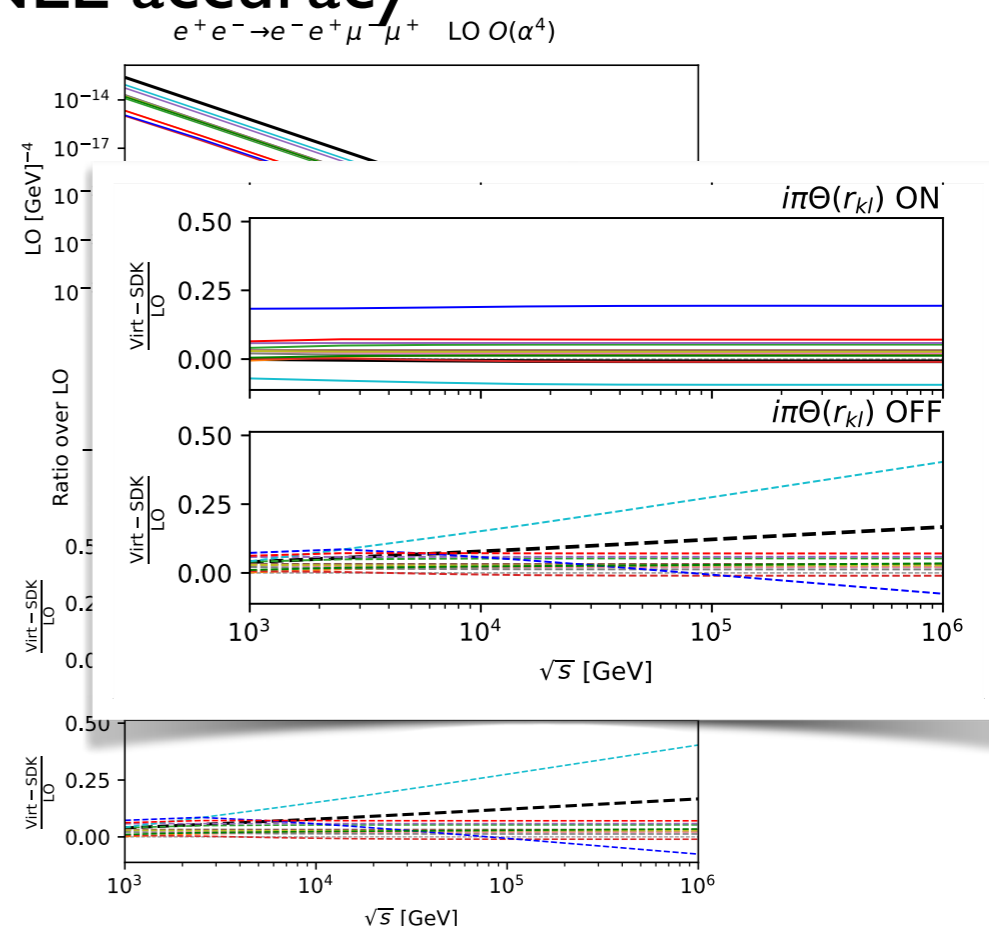
# The missing $i\pi\theta(r_{kl})$ factor

- $2 \rightarrow 2$  amplitudes (as those considered by D&P) are always real (optical theorem). Any missing imaginary part in the logs drops out when considering  $2\Re(BV^*) \simeq 2\Re(BB'^*\delta)$
- For  $2 \rightarrow n$ ,  $n \geq 3$ , imaginary parts from the logs can combine with those of  $BB'^*$ , giving rise to single-logarithmic terms
- They must be included in order to claim NLL accuracy



constant V-SDK  
helicity by helicity

V-SDK  $\sim \log(s)$   
if  $i\pi\theta(r_{kl})$  neglected



# From IR masses to Dim.Reg. (and the treatment of QED effects)

- Consider e.g. the LSC term, in the D&P formalism. A photon mass appears

$$\delta_{i'_k i_k}^{\text{LSC}}(k) = -\frac{1}{2} \left[ \underbrace{Q_k^2 + (I_Z^2)_k + (I_W^2)_k}_{\text{photon mass}} C_{i'_k i_k}^{\text{ew}}(k) L(s) - 2(I^Z(k))_{i'_k i_k}^2 \log \frac{M_Z^2}{M_W^2} l(s) + \delta_{i'_k i_k} Q_k^2 L^{\text{em}}(s, \lambda^2, m_k^2) \right]$$

- The QED contribution is split in two parts:
  - from  $M_W$  to  $M_W$  in  $L^{\text{em}}$   $L^{\text{em}}(s, \lambda^2, m_k^2) := 2l(s) \log \left( \frac{M_W^2}{\lambda^2} \right) + L(M_W^2, \lambda^2) - L(m_k^2, \lambda^2)$
  - from  $M_W$  to  $s$  in  $L(s)$
- Consider the divergent part:  $\lambda$  (and  $m_k$ ) acts as a regularisation scale for the IR divergences
- We can promote  $\lambda$  to  $Q$ , the IR regularisation scale of Dim.Reg., without losing any logarithmic term
- We can then set  $Q^2 = \mu_R^2$ , and compare the Sudakov approximation with the exact virtuals



# From IR masses to Dim.Reg. (and the treatment of QED effects)

- In the SSC terms, this leads to a vanishing contribution if the  $r_{ij}/s$  part is dropped

$$\delta_{i'_k i_k i'_l i_l}^{A,SSC}(k, l) = \left[ 2 (l(s) + l(M_W^2, Q^2)) \left( \log \frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl}) \right) + \Delta^{s \rightarrow r_{kl}}(r_{kl}, M_W^2) \right] I_{i'_k i_k}^A(k) I_{i'_l i_l}^A(l)$$

# More on the QED contribution (towards the extension to QCD)

$$\delta_{i'_k i_k}^{\text{LSC}}(k) = -\frac{1}{2} \left[ \underbrace{Q_k^2 + (I_Z^2)_k + (I_W^2)_k}_{\text{}} C_{i'_k i_k}^{\text{ew}}(k) L(s) - 2(I^Z(k))_{i'_k i_k}^2 \log \frac{M_Z^2}{M_W^2} l(s) + \delta_{i'_k i_k} Q_k^2 L^{\text{em}}(s, Q^2, m_k^2) \right]$$

$$L^{\text{em}}(s, \lambda^2, m_k^2) := 2l(s) \log \left( \frac{M_W^2}{\lambda^2} \right) + L(M_W^2, \lambda^2) - L(m_k^2, \lambda^2) \text{ remember: } \lambda^2 \rightarrow Q^2 = s$$

- In LSC, QED enters in  $L^{\text{em}}$  and in the term  $\sim L(s)$
- In the D&P formulation,  $M_W$  acts as a separator from the low-energy ( $\lambda^2 \rightarrow M_W^2$ ) to the high-energy regime ( $M_W^2 \rightarrow s$ )
- If  $Q^2=s$ , the QED contribution vanishes in LSC (also in SSC) for massless particles
  - Warning! This is not equivalent to just saying that  $L^{\text{em}}$  can be dropped
  - For massive particles (e.g. top), a term  $\sim L(s, m_t)$  remains
- QED contributions appear also in the C and PR terms
- QCD terms are analogous to those from QED (only top is massive)

# The inclusion of QCD effects



- Remember:  $NLO_i = LO_{i-1} \otimes EW + LO_i \otimes QCD$
- So far, we have focused on approximating EW corrections
- The corrections of QCD origin stemming on top of  $LO_2$  are analogous to the QED-type corrections
- Since in QCD we always cluster massless patrons into jets, a remarkably-simple structure appears

$$\delta\tilde{\mathcal{M}} \equiv \tilde{\mathcal{M}}_0 \left[ \underbrace{\left( n_t L^t(s) \right)}_{LSC} + \underbrace{n_{\alpha_S} l^{\alpha_S}(\mu_R^2)}_{PR, \alpha_S} - \underbrace{n_g l^{\alpha_S}(s)}_{C, g \rightarrow t\bar{t}} + \underbrace{\frac{\delta\tilde{\mathcal{M}}_0}{\delta m_t} (\delta m_t)^{QCD}}_{PR, m_t} \right]$$

with

$$L^t(s) \equiv \frac{C_F}{2} \frac{\alpha_S}{4\pi} \left( \log^2 \frac{s}{m_t^2} + \log \frac{s}{m_t^2} \right) \quad l^{\alpha_S}(\mu^2) \equiv \frac{1}{3} \frac{\alpha_S}{4\pi} \log \frac{\mu^2}{m_t^2} \quad (\delta m_t)^{QCD} \equiv -3C_F \frac{\alpha_S}{4\pi} \log \frac{s}{m_t^2}$$



# The inclusion of QCD effects: some results

