

Top-Quark Loops for Precision Higgs Physics

Marco Vitti (Karlsruhe Institute of Technology, TTP and IAP)

TUM/MPP Collider Phenomenology Seminar, Munich, 24 Jul 2024



Outline

1. Precision Higgs physics at the LHC
 2. Challenge: $2 \rightarrow 2$ with massive loops
 3. Example: top-mediated $gg \rightarrow ZZ$ @NLO QCD
 4. Towards NNLO QCD...
 5. Conclusions
-

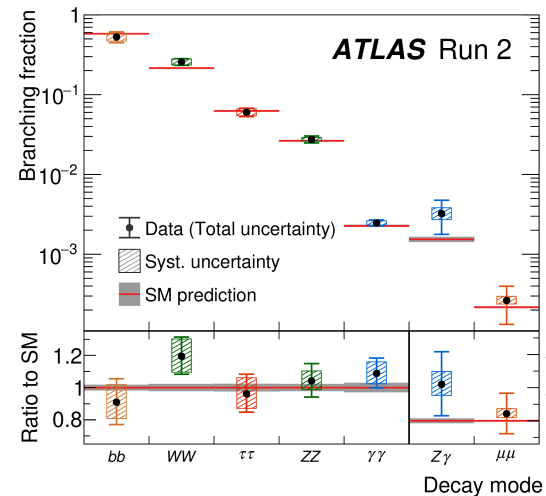
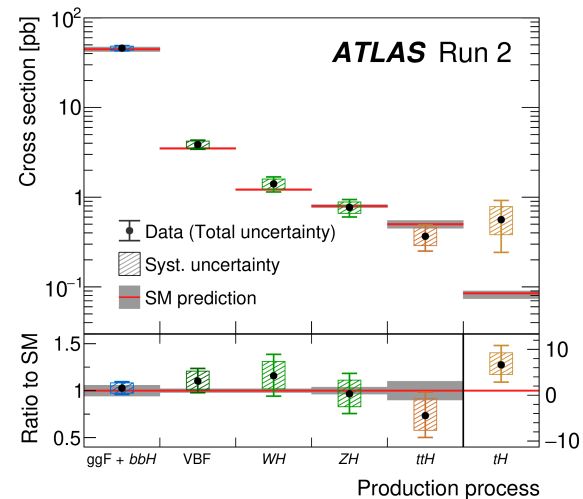
Higgs Physics at the LHC

Does the discovered Higgs boson behave as the SM predicts?

What we know after Run2 (139 fb^{-1})

- CP-even scalar
- Mass measured with **permille** precision
- Production and decay channels all compatible with SM predictions
- Experimental uncertainties in the 10-20% range

[ATLAS-2207.00092]



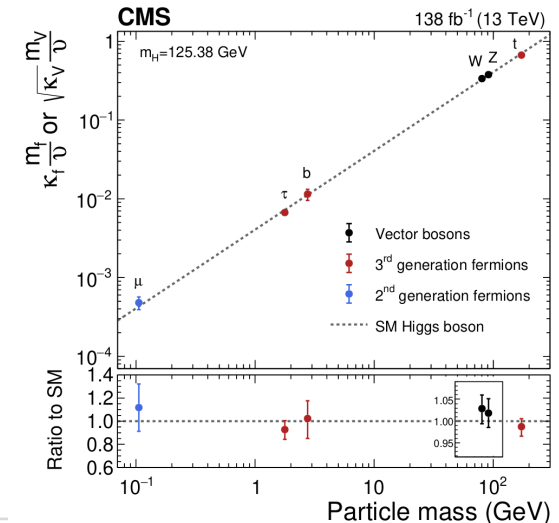
Higgs Physics at the LHC

Does the discovered Higgs boson behave
as the SM predicts?

$$V(h) = \frac{m_H^2}{2}h^2 + \lambda_3 v h^3 + \frac{\lambda_4}{4}h^4$$

What we still don't know

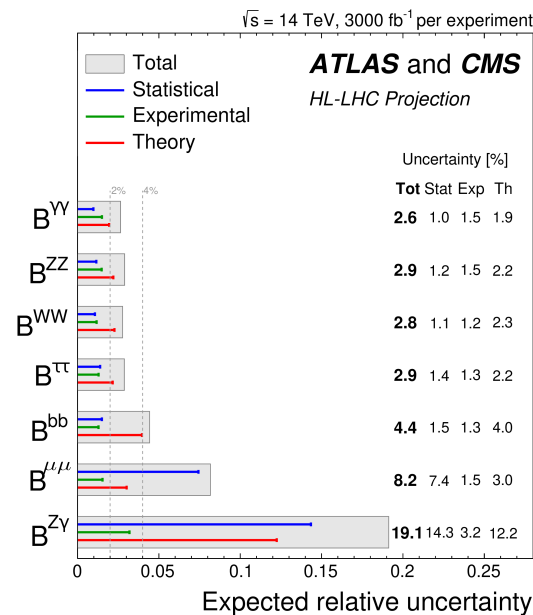
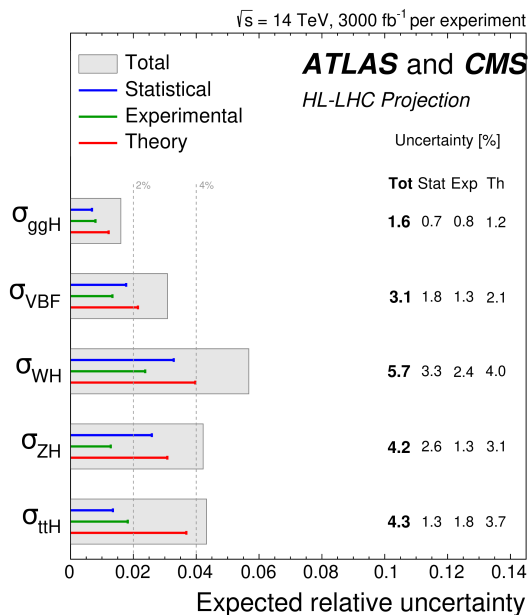
- Shape of the Higgs potential
- Yukawa couplings of first and second generation
- Higgs total decay width
- ...



[CMS - 2207.00043]

Projections for High-Luminosity LHC

■ **Systematic** uncertainties will play a very important role



[Cepeda et al. - 1902.00134]

■ Theory uncertainties need to be reduced \Rightarrow Improve predictions within the SM

Theory goal : **percent precision**

(Some) Theory Uncertainties

- Parametric uncertainties
- PDF determination
- Matching with parton showers

[THIS TALK]

- Missing higher orders in perturbative calculations
Conventionally estimated by varying **renormalization** and **factorization scales**

$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \hat{\sigma}_{ij}(x_1, x_2, Q, \mu_F, \mu_R) + \mathcal{O}(\Lambda_{QCD}/Q)$$

$$\hat{\sigma}_{ij}(\mu_F, \mu_R) = \alpha_S^k(\mu_R) \sum_{m=0}^n \hat{\sigma}_{ij}^{(m)}(\mu_F, \mu_R) \alpha_S^m(\mu_R)$$

Where to Look for Improvements?

Les Houches precision wishlist [Huss et al. - 2207.02122]

Table 1. Precision wishlist: Higgs boson final states. $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ means a calculation using the structure function approximation. $V = W, Z$.

Process	Known	Desired
$pp \rightarrow H$	$N^3\text{LO}_{\text{HTL}}$ $\text{NNLO}_{\text{QCD}}^{(f)}$ $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}^{(\text{HTL})}$ NLO_{QCD}	$N^4\text{LO}_{\text{HTL}}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(b,c)}$
$pp \rightarrow H + j$	NNLO_{HTL} NLO_{QCD} $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$	$\text{NNLO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow H + 2j$	$\text{NLO}_{\text{HTL}} \otimes \text{LO}_{\text{QCD}}$ $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NLO}_{\text{EW}}^{(\text{VBF})}$	$\text{NNLO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NNLO}_{\text{QCD}}^{(\text{VBF})}$
$pp \rightarrow H + 3j$	NLO_{HTL} $\text{NLO}_{\text{QCD}}^{(\text{VBF})}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow VH$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $\text{NLO}_{gg \rightarrow HZ}^{(t,b)}$	
$pp \rightarrow VH + j$	NNLO_{QCD} $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow HH$	$N^3\text{LO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}}$	NLO_{EW}
$pp \rightarrow HH + 2j$	$N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NLO}_{\text{EW}}^{(\text{VBF})}$	
$pp \rightarrow HHH$	NNLO_{HTL}	
$pp \rightarrow H + t\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ NNLO_{QCD} (off-diag.)	NNLO_{QCD}
$pp \rightarrow H + t/\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}

Table 3. Precision wishlist: vector boson final states. $V = W, Z$ and $V', V'' = W, Z, \gamma$. Full leptonic decays are understood if not stated otherwise.

Process	Known	Desired
$pp \rightarrow V$	$N^3\text{LO}_{\text{QCD}}$ $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$ NLO_{EW}	$N^3\text{LO}_{\text{QCD}} + N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$ $N^2\text{LO}_{\text{EW}}$
$pp \rightarrow VV'$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $+ \text{NLO}_{\text{QCD}}$ (gg channel)	NLO_{QCD} (gg channel, w/ massive loops) $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$
$pp \rightarrow V + j$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	hadronic decays
$pp \rightarrow V + 2j$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (QCD component) $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (EW component)	NNLO_{QCD}
$pp \rightarrow V + b\bar{b}$	NLO_{QCD}	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow VV' + 1j$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}
$pp \rightarrow VV' + 2j$	NLO_{QCD} (QCD component) $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (EW component)	Full $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$

Where to Look for Improvements?

Les Houches precision wishlist [Huss et al. - 2207.02122]

Table 1. Precision wishlist: Higgs boson final states. $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ means a calculation using the structure function approximation. $V = W, Z$.

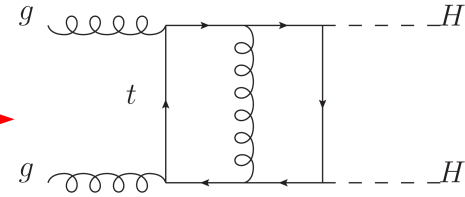
Process	Known	Desired
$pp \rightarrow H$	$N^3\text{LO}_{\text{HTL}}$ $\text{NNLO}_{\text{QCD}}^{(f)}$ $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}^{(\text{HTL})}$ NLO_{QCD}	$N^4\text{LO}_{\text{HTL}}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(b,c)}$
$pp \rightarrow H + j$	NNLO_{HTL} NLO_{QCD} $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$	$\text{NNLO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow H + 2j$	$\text{NLO}_{\text{HTL}} \otimes \text{LO}_{\text{QCD}}$ $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NLO}_{\text{EW}}^{(\text{VBF})}$	$\text{NNLO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NNLO}_{\text{QCD}}^{(\text{VBF})}$
$pp \rightarrow H + 3j$	NLO_{HTL} $\text{NLO}_{\text{QCD}}^{(\text{VBF})}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow VH$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $\text{NLO}_{gg \rightarrow HZ}^{(t,b)}$	
$pp \rightarrow VH + j$	NNLO_{QCD} $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow HH$	$N^3\text{LO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}}$	NLO_{EW}
$pp \rightarrow HH + 2j$	$N^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NLO}_{\text{EW}}^{(\text{VBF})}$	
$pp \rightarrow HHH$	NNLO_{HTL}	
$pp \rightarrow H + t\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ NNLO_{QCD} (off-diag.)	NNLO_{QCD}
$pp \rightarrow H + t/\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}

Table 3. Precision wishlist: vector boson final states. $V = W, Z$ and $V', V'' = W, Z, \gamma$. Full leptonic decays are understood if not stated otherwise.

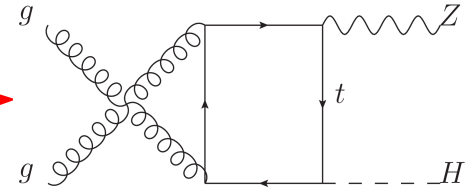
Process	Known	Desired
$pp \rightarrow V$	$N^3\text{LO}_{\text{QCD}}$ $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$ NLO_{EW}	$N^3\text{LO}_{\text{QCD}} + N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$ $\text{N}^2\text{LO}_{\text{EW}}$
$pp \rightarrow VV'$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $+ \text{NLO}_{\text{QCD}}$ (gg channel)	NLO_{QCD} (gg channel, w/ massive loops) $N^{(1,1)}\text{LO}_{\text{QCD} \otimes \text{EW}}$
$pp \rightarrow V + j$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	hadronic decays
$pp \rightarrow V + 2j$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (QCD component) $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (EW component)	NNLO_{QCD}
$pp \rightarrow V + b\bar{b}$	NLO_{QCD}	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \rightarrow VV' + 1j$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}
$pp \rightarrow VV' + 2j$	NLO_{QCD} (QCD component) $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ (EW component)	Full $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$

Diboson Production (gg -initiated)

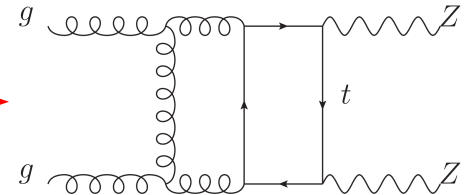
Process	Known	Desired
$pp \rightarrow HH$	N^3LO_{HTLQ} NLO_{QCD}	NLO_{EW}



Process	Known	Desired
$pp \rightarrow VH$	$NNLO_{\text{QCD}} + NLO_{\text{EW}}$ $NLO_{gg \rightarrow HZ}^{(t,b)}$	



Process	Known	Desired
$pp \rightarrow VV'$	$NNLO_{\text{QCD}} + NLO_{\text{EW}}$ $+ NLO_{\text{QCD}} (gg \text{ channel})$	$NLO_{\text{QCD}} (gg \text{ channel, w/ massive loops})$ $N^{(1,1)}LO_{\text{QCD} \otimes \text{EW}}$



Typically receive large QCD corrections!

Multi-scale (m_t, s, t, m_{ext}) two-loop integrals
No full analytic results available

Two-loop Boxes with Massive Lines

Numerical Evaluation (Sector Decomposition)

- Exact results
- Demanding in terms of computing resources and time
- Issues with flexibility of input

[Borowka et al. - 1604.06447]

[Chen et al. - 2011.12325]

[Agarwal et al. - 2011.15113]

Analytic Approximations: exploit hierarchies of masses/kinematic invariants

- Reduce the number of scales in Feynman integrals
- Proliferation of integrals
- Restricted to specific phase-space regions

■ Limit $m_t \rightarrow \infty$

[Dawson et al. - 9805244]

[Altenkamp et al. - 1211.50]

[Dowling, Melnikov – 1503.01274; Caola, et al. – 1605.04610]

■ Large mass expansion (LME)

[Grigo, Hoff, Steinhauser - 1508.00909]

[Hasselhuhn, Luthe, Steinhauser - 1611.05881]

■ High-energy expansion: $m_{\text{ext}}^2 \ll m_t^2 \ll \hat{s}, \hat{t}$

[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

[Davies, Mishima, Steinhauser – 2011.12314]

[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

■ Small-mass expansion: $m_{\text{ext}} \rightarrow 0$

[Wang et al. - 2010.15649]

[Wang, Xu, Xu, Yang - 2107.08206]

■ pT expansion: $m_{\text{ext}}^2, p_T^2 \ll m_t^2, \hat{s}$

[Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

[Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]

[Degrassi, Gröber, MV - 2404.15113]

$pp \rightarrow ZZ$ at the LHC

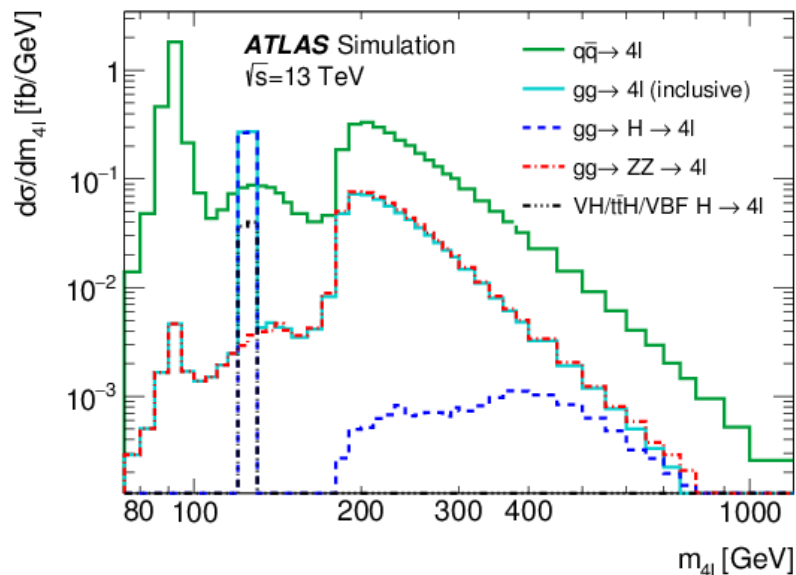
■ Probe of EW theory: polarisation measurements, “golden channel” for Higgs production

■ Indirect access to Higgs width

[Kauer, Passarino – 1206.4803]

[Caola, Melnikov – 1307.4935]

[Campbell, Ellis, Williams - 1311.3589]



[ATLAS – 1902.05892]

Compare on-shell and off-shell signal strengths

$$\mu_{\text{on}} = \frac{\kappa_{ggh}^2(m_h) \kappa_{hZZ}^2(m_h)}{\Gamma_h / \Gamma_h^{\text{SM}}}$$

$$\mu_{\text{off}} = \kappa_{ggh}^2(m_{ZZ}) \kappa_{hZZ}^2(m_{ZZ})$$

$$\Gamma_H = 3.2_{-1.7}^{+2.4} \text{MeV}$$

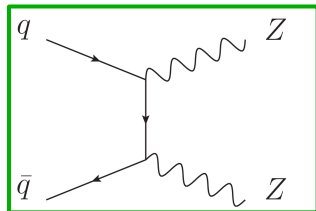
[CMS - 2202.06923]

$$\Gamma_H = 4.5_{-2.5}^{+3.3} \text{MeV}$$

[ATLAS - 2304.01532]

Accurate theoretical predictions
needed in both regions!

Theoretical Predictions



Dominant contribution

NNLO QCD

[Brown, Mikaelian – ('79); Ohnemus, Owens - ('91);
 Mele, Nason, Ridolfi - ('91); Cascioli et al. - 1405.2219;
 Heinrich et al. - 1710.06294; Gehrmann et al. - 1404.4853;
 Caola et al. - 1408.6409; Gehrmann et al. - 1503.04812;
 Grazzini et al. - 1507.06257;
 Kallweit, Wiesemann - 1806.05941]

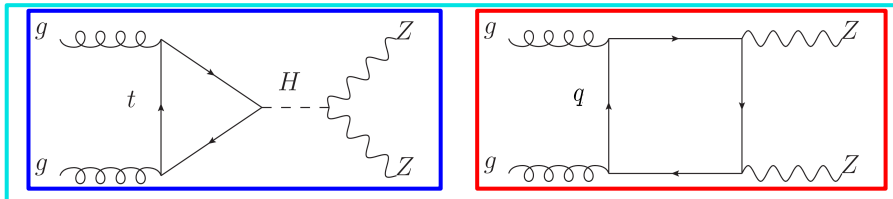
NLO EW

[Bierweiler et al. – 1305.5402; Baglio, Ninh, Weber – 1307.4331;
 Chiesa et al. - 2005.12146]

\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) $^{+2.4\%}_{-3.2\%}$	13.933(1) $^{+5.5\%}_{-6.4\%}$	-27.5%	-29.8%
NLO	11.2958(4) $^{+2.5\%}_{-2.0\%}$	19.8454(7) $^{+2.5\%}_{-2.1\%}$	0%	0%
$q\bar{q}$ NNLO	12.09(2) $^{+1.1\%}_{-1.1\%}$	21.54(2) $^{+1.1\%}_{-1.2\%}$	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) $^{+28.2\%}_{-20.9\%}$	2.0052(1) $^{+23.5\%}_{-17.9\%}$	0%	0%
gg NLO $_{gg}$	1.4787(4) $^{+15.9\%}_{-13.1\%}$	3.626(1) $^{+15.2\%}_{-12.7\%}$	+86.3%	+80.8%
gg NLO	1.3892(4) $^{+15.4\%}_{-13.6\%}$	3.425(1) $^{+13.9\%}_{-12.0\%}$	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) $^{+2.8\%}_{-2.2\%}$	23.55(2) $^{+3.0\%}_{-2.6\%}$	+14.0%	+18.7%
nNNLO	13.48(2) $^{+2.6\%}_{-2.3\%}$	24.97(2) $^{+2.9\%}_{-2.7\%}$	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions



LO loop-induced (α_s^2 correction)

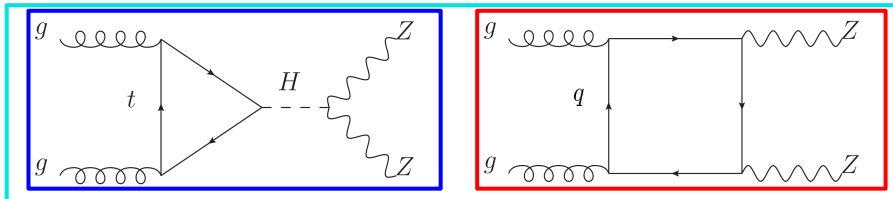
[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to $\sim 10\%$ of hadronic xsec

\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) ^{+2.4%} _{-3.2%}	13.933(1) ^{+5.5%} _{-6.4%}	-27.5%	-29.8%
NLO	11.2958(4) ^{+2.5%} _{-2.0%}	19.8454(7) ^{+2.5%} _{-2.1%}	0%	0%
$q\bar{q}$ NNLO	12.09(2) ^{+1.1%} _{-1.1%}	21.54(2) ^{+1.1%} _{-1.2%}	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) ^{+28.2%} _{-20.9%}	2.0052(1) ^{+23.5%} _{-17.9%}	0%	0%
gg NLO _{gg}	1.4787(4) ^{+15.9%} _{-13.1%}	3.626(1) ^{+15.2%} _{-12.7%}	+86.3%	+80.8%
gg NLO	1.3892(4) ^{+15.4%} _{-13.6%}	3.425(1) ^{+13.9%} _{-12.0%}	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) ^{+2.8%} _{-2.2%}	23.55(2) ^{+3.0%} _{-2.6%}	+14.0%	+18.7%
nNNLO	13.48(2) ^{+2.6%} _{-2.3%}	24.97(2) ^{+2.9%} _{-2.7%}	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions



LO loop-induced (α_s^2 correction)

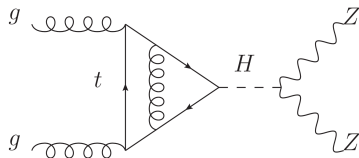
[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Higgs-mediated

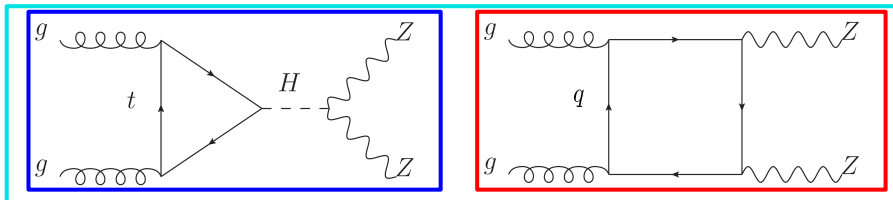
[Spira et al. - 9504378 ; Aglietti et al. - 0611266 ;
Harlander, Kant - 0509189; Anastasiou et al. - 0611236]



\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) ^{+2.4%} _{-3.2%}	13.933(1) ^{+5.5%} _{-6.4%}	-27.5%	-29.8%
NLO	11.2958(4) ^{+2.5%} _{-2.0%}	19.8454(7) ^{+2.5%} _{-2.1%}	0%	0%
$q\bar{q}$ NNLO	12.09(2) ^{+1.1%} _{-1.1%}	21.54(2) ^{+1.1%} _{-1.2%}	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) ^{+28.2%} _{-20.9%}	2.0052(1) ^{+23.5%} _{-17.9%}	0%	0%
gg NLO _{gg}	1.4787(4) ^{+15.9%} _{-13.1%}	3.626(1) ^{+15.2%} _{-12.7%}	+86.3%	+80.8%
gg NLO	1.3892(4) ^{+15.4%} _{-13.6%}	3.425(1) ^{+13.9%} _{-12.0%}	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) ^{+2.8%} _{-2.2%}	23.55(2) ^{+3.0%} _{-2.6%}	+14.0%	+18.7%
nNNLO	13.48(2) ^{+2.6%} _{-2.3%}	24.97(2) ^{+2.9%} _{-2.7%}	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions



LO loop-induced (α_s^2 correction)

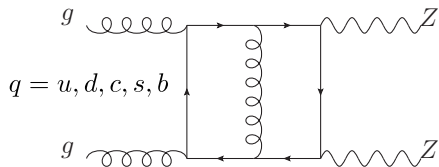
[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Non-resonant (light quarks)

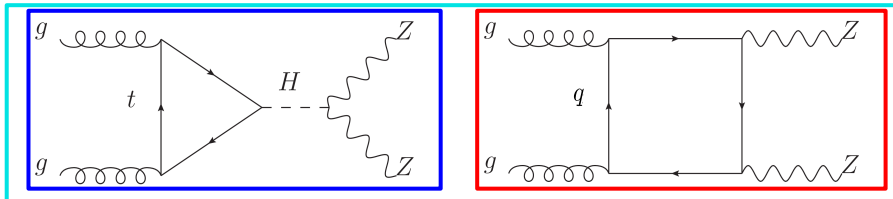
[von Manteuffel, Tancredi – 1503.08835;
Caola et al. - 1509.06734]



\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) $^{+2.4\%}_{-3.2\%}$	13.933(1) $^{+5.5\%}_{-6.4\%}$	-27.5%	-29.8%
NLO	11.2958(4) $^{+2.5\%}_{-2.0\%}$	19.8454(7) $^{+2.5\%}_{-2.1\%}$	0%	0%
$q\bar{q}$ NNLO	12.09(2) $^{+1.1\%}_{-1.1\%}$	21.54(2) $^{+1.1\%}_{-1.2\%}$	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) $^{+28.2\%}_{-20.9\%}$	2.0052(1) $^{+23.5\%}_{-17.9\%}$	0%	0%
gg NLO $_{gg}$	1.4787(4) $^{+15.9\%}_{-13.1\%}$	3.626(1) $^{+15.2\%}_{-12.7\%}$	+86.3%	+80.8%
gg NLO	1.3892(4) $^{+15.4\%}_{-13.6\%}$	3.425(1) $^{+13.9\%}_{-12.0\%}$	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) $^{+2.8\%}_{-2.2\%}$	23.55(2) $^{+3.0\%}_{-2.6\%}$	+14.0%	+18.7%
nNNLO	13.48(2) $^{+2.6\%}_{-2.3\%}$	24.97(2) $^{+2.9\%}_{-2.7\%}$	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions



LO loop-induced (α_s^2 correction)

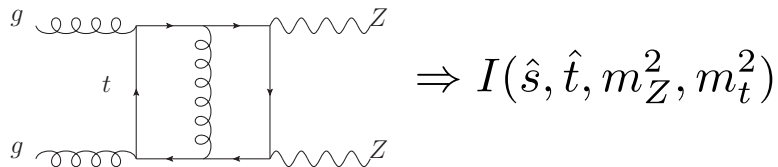
[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Non-resonant (top quark)

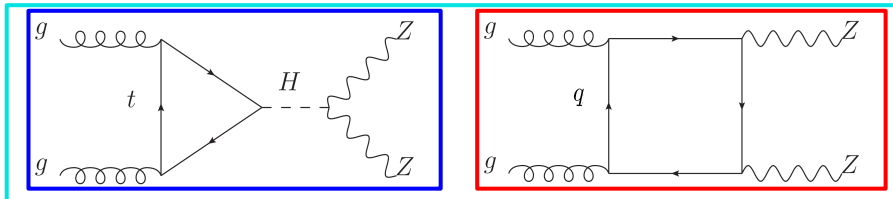
No exact results in full analytic form



\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) ^{+2.4%} _{-3.2%}	13.933(1) ^{+5.5%} _{-6.4%}	-27.5%	-29.8%
NLO	11.2958(4) ^{+2.5%} _{-2.0%}	19.8454(7) ^{+2.5%} _{-2.1%}	0%	0%
$q\bar{q}$ NNLO	12.09(2) ^{+1.1%} _{-1.1%}	21.54(2) ^{+1.1%} _{-1.2%}	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) ^{+28.2%} _{-20.9%}	2.0052(1) ^{+23.5%} _{-17.9%}	0%	0%
gg NLO _{gg}	1.4787(4) ^{+15.9%} _{-13.1%}	3.626(1) ^{+15.2%} _{-12.7%}	+86.3%	+80.8%
gg NLO	1.3892(4) ^{+15.4%} _{-13.6%}	3.425(1) ^{+13.9%} _{-12.0%}	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) ^{+2.8%} _{-2.2%}	23.55(2) ^{+3.0%} _{-2.6%}	+14.0%	+18.7%
nNNLO	13.48(2) ^{+2.6%} _{-2.3%}	24.97(2) ^{+2.9%} _{-2.7%}	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions



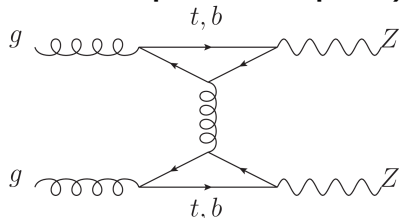
LO loop-induced (α_s^2 correction)

[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Double-Triangle diagrams
(Standard one-loop techniques)



\sqrt{s}	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
LO	8.1881(8) ^{+2.4%} _{-3.2%}	13.933(1) ^{+5.5%} _{-6.4%}	-27.5%	-29.8%
NLO	11.2958(4) ^{+2.5%} _{-2.0%}	19.8454(7) ^{+2.5%} _{-2.1%}	0%	0%
$q\bar{q}$ NNLO	12.09(2) ^{+1.1%} _{-1.1%}	21.54(2) ^{+1.1%} _{-1.2%}	+7.0%	+8.6%
	σ [fb]		$\sigma/\sigma_{\text{ggLO}} - 1$	
gg LO	0.79355(6) ^{+28.2%} _{-20.9%}	2.0052(1) ^{+23.5%} _{-17.9%}	0%	0%
gg NLO _{gg}	1.4787(4) ^{+15.9%} _{-13.1%}	3.626(1) ^{+15.2%} _{-12.7%}	+86.3%	+80.8%
gg NLO	1.3892(4) ^{+15.4%} _{-13.6%}	3.425(1) ^{+13.9%} _{-12.0%}	+75.1%	+70.8%
	σ [fb]		$\sigma/\sigma_{\text{NLO}} - 1$	
NNLO	12.88(2) ^{+2.8%} _{-2.2%}	23.55(2) ^{+3.0%} _{-2.6%}	+14.0%	+18.7%
nNNLO	13.48(2) ^{+2.6%} _{-2.3%}	24.97(2) ^{+2.9%} _{-2.7%}	+19.3%	+25.8%

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Importance of Top-Quark Effects

- Dominant contribution to Higgs-mediated/nonresonant interference for large invariant masses

$$2 \operatorname{Re} \left(\text{triangle diagram} * \text{square diagram} \right)$$

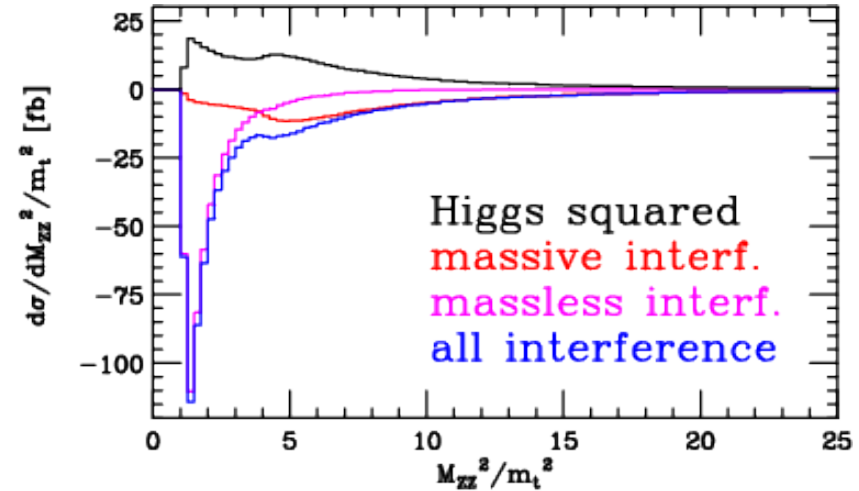
The diagram shows two Feynman diagrams for the process $gg \rightarrow H \rightarrow ZZ$. The first diagram is a triangle loop with a top quark (t) and a Higgs boson (H). The second diagram is a square loop with a top quark (t) and a Z boson (Z). The two diagrams are multiplied together and the real part is taken.

- Exact numerical results available

[Agarwal, Jones, von Manteuffel - 2011.15113 ;
Brønnum-Hansen, Wang – 2101.12095]

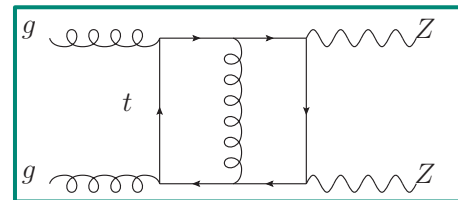
- Large effects found also at NLO QCD

[Agarwal, Jones, Kerner, von Manteuffel - 2404.05684]



[Campbell et al. - 1605.01380]

Analytic Approximations



■ Limit $m_t \rightarrow \infty$

[Dowling, Melnikov – 1503.01274; Caola, et al. – 1605.04610]

■ Large mass expansion (LME)

[Campbell et al. - 1605.01380; Gröber, Maier, Rauh – 1908.04061]

■ High-energy expansion: $m_Z^2 \ll m_t^2 \ll \hat{s}, \hat{t}$

[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

This talk: pT expansion $m_Z^2, p_T^2 \ll m_t^2, \hat{s}$ [Degrassi, Gröber, MV - 2404.15113]

Previously applied to

$gg \rightarrow HH$ [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

$gg \rightarrow ZH$ [Alasfar, Degrassi, Giardino, Gröber, MV - 2103.06225]

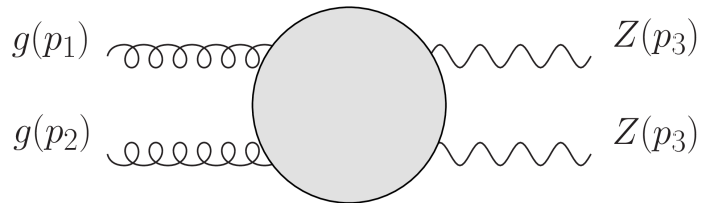
pT Expansion - Calculation Overview

1. Generation of Feynman diagrams (FeynArts [[Hahn - 0012260](#)])
2. Lorentz decomposition of the amplitude: contractions, Dirac traces...
(FeynCalc [[Shtabovenko et al. - 2001.04407](#)])

$$\mathcal{A}_{\mu\nu\rho\sigma} = \sum_{i=1}^{16} \mathcal{P}_{\mu\nu\rho\sigma}^{(i)} A^{(i)} \quad A^{(i)} = \sum_{i=1}^n C^{(i)} I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_t^2)$$

3. Expansion of form factors in the limit of small p_T, m_Z (Mathematica)
 4. Decomposition of scalar integrals using IBP identities (LiteRed [[Lee - 1310.1145](#)])
 5. Evaluation of master integrals
-

pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1 + p_3)^2 \rightarrow 0 \Leftrightarrow \hat{t} \rightarrow 0 \Rightarrow p_T \rightarrow 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1$$

$$\frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at
integrand level

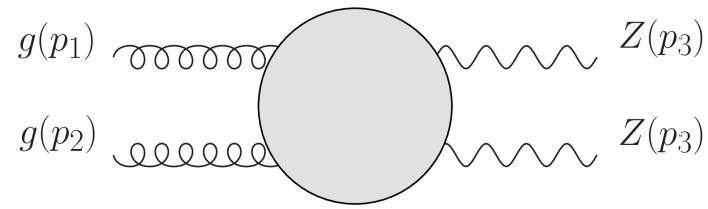
- After tensor + IBP reduction

$$\mathcal{A}_i = \mathcal{N}(p_T^2, m_Z^2) \sum_{N=0}^{\infty} \sum_{i+j=N} c_{ij} (\hat{s}/m_t^2) (p_T^2)^i (m_Z^2)^j$$

- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **single-scale integrals!**

$$I(\hat{s}, p_T^2, m_Z^2, m_t^2) \rightarrow \text{MI}(\hat{s}/m_t^2)$$

pT Expansion - Example



1) Consider a **one-loop** box integral

$$\int d^D q \frac{(q^2)^{n_1} (q \cdot p_1)^{n_2} (q \cdot p_2)^{n_3} (q \cdot p_3)^{n_4}}{(q^2 - m_t^2)[(q + p_2)^2 - m_t^2][(q - p_1 - p_3)^2 - m_t^2][(q - p_1)^2 - m_t^2]}$$

2) Focus on the p3-dependent part; explicit transverse momentum (Sudakov)

$$\frac{(q \cdot p_3)^{n_4}}{[(q - p_1 - p_3)^2 - m_t^2]}$$

$$p_3^\mu = -p_1^\mu - \frac{t'}{s'}(p_1 - p_2)^\mu + r_\perp^\mu$$

$$r_\perp^\mu = -p_T^\mu \quad \frac{t'}{s'} = -\frac{1}{2} \left\{ 1 - \sqrt{1 - 2 \frac{p_T^2 + m_Z^2}{s'}} \right\}$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

$$\int d^D q \frac{(q^2)^{n_1} (q \cdot p_1)^{n'_2} (q \cdot p_2)^{n'_3} (q \cdot r_\perp)^{n'_4}}{(q^2 - m_t^2)^{l_1} [(q + p_2)^2 - m_t^2][(q - p_1)^2 - m_t^2]}$$

4) Tensor + IBP reduction \rightarrow Dependence on r_\perp removed

pT Expansion - Two-Loop Master Integrals

■ 52 MIs: same basis for $gg \rightarrow HH$, $gg \rightarrow ZH$

■ 50 MIs expressed in terms of Generalized Harmonic Polylogarithms

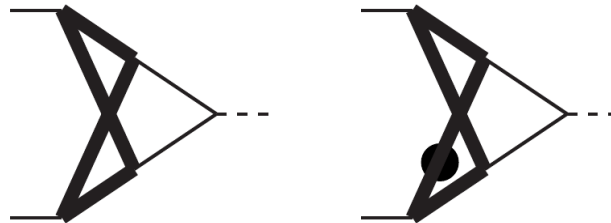
[Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

Evaluated using handyG [Naterop, Signer, Ulrich - 1909.01656]

■ Two elliptic integrals [von Manteuffel, Tancredi ('17)]

Re-evaluated using expansion of differential equations (semi-analytical)

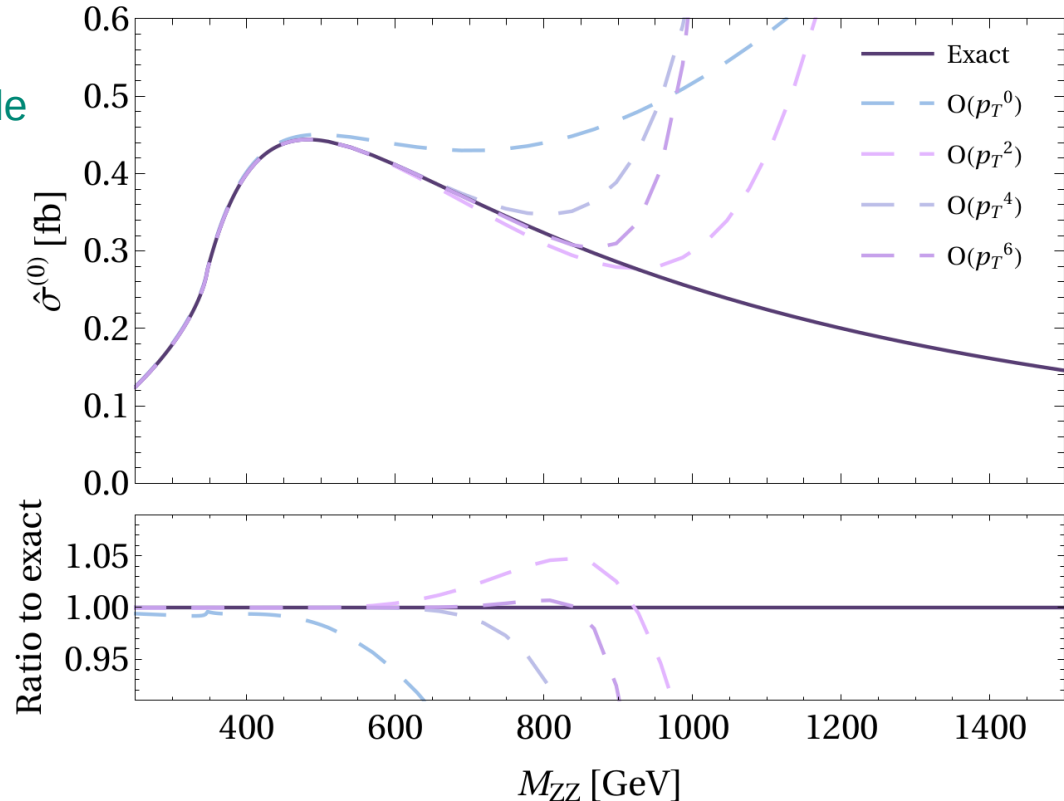
Implemented in FORTRAN routine [Bonciani, Degrassi, Giardino, Gröber - 1812.02698]



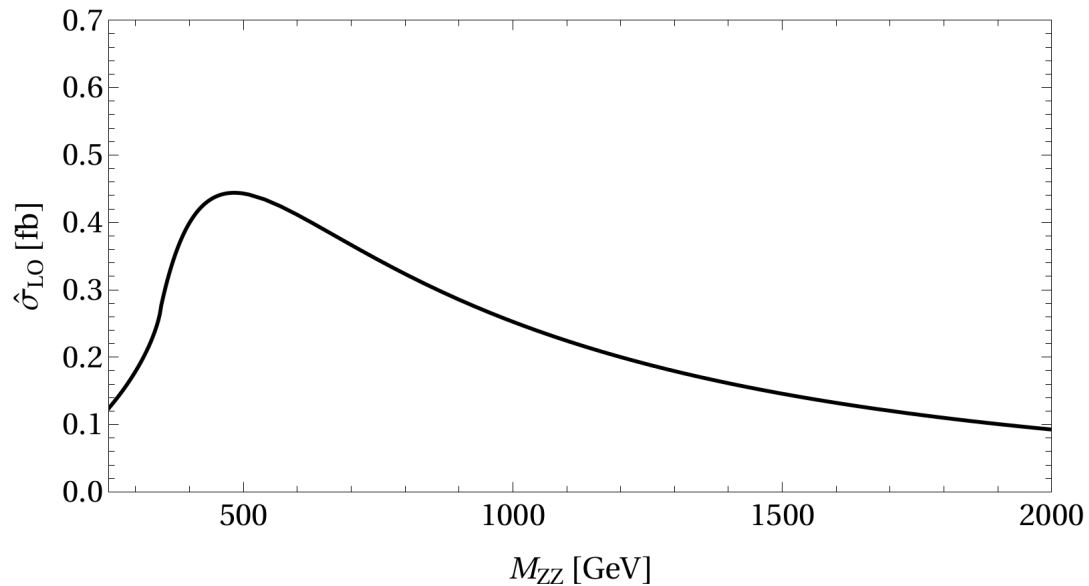
Validation at LO

■ Three orders sufficient for **permille** accuracy

■ For $M_{ZZ} \gtrsim 700$ GeV
the assumption $p_T^2 \ll 4m_t^2$
can be violated in a significant
part of the phase space



Complementing Phase-Space Coverage



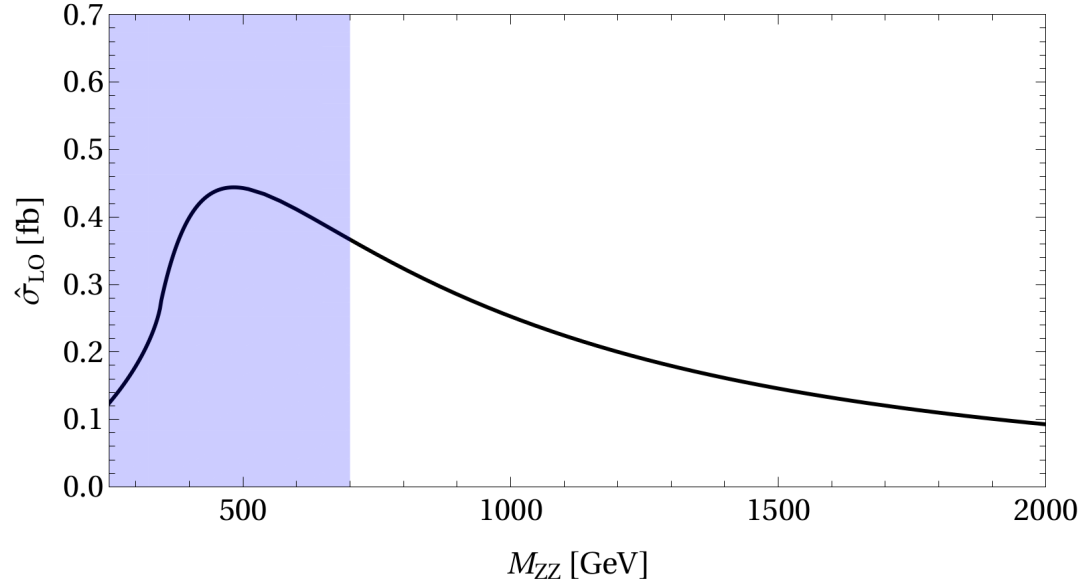
Complementing Phase-Space Coverage

■ p_T exp

$$p_T^2 \lesssim 4m_t^2$$

or

$$|\hat{t}| \lesssim 4m_t^2$$



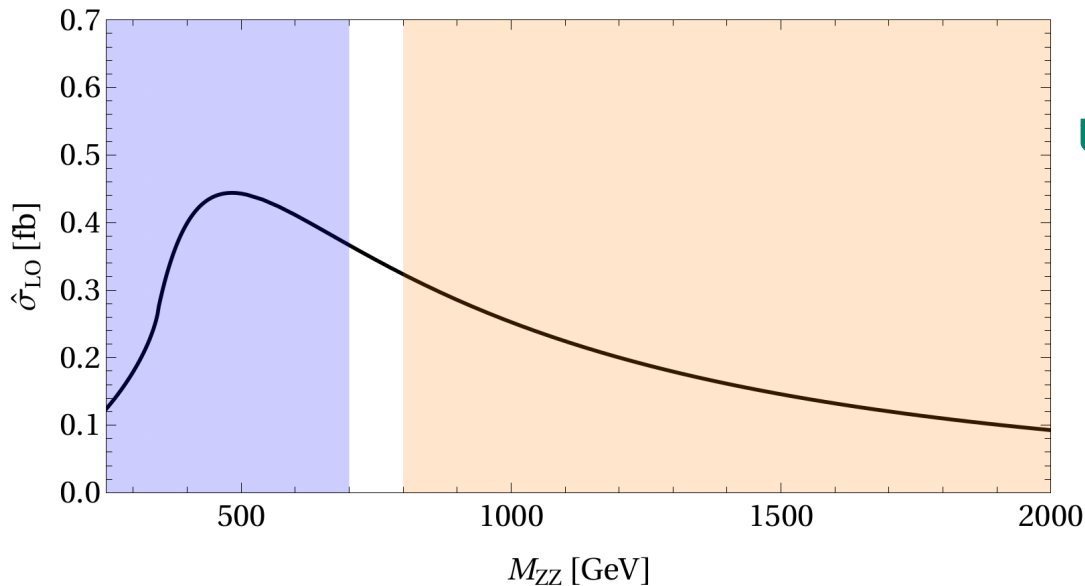
Complementing Phase-Space Coverage

■ p_T exp

$$p_T^2 \lesssim 4m_t^2$$

or

$$|\hat{t}| \lesssim 4m_t^2$$



■ High-Energy exp:

$$|\hat{t}| \gtrsim 4m_t^2$$

[Davies, Mishima,
Steinhauser, Wellmann
- 2002.05558]

The two expansions can be combined

Needed refinement using Padé approximants

[Bellafronte, Degrossi, Giardino, Gröber, MV -2103.06225]

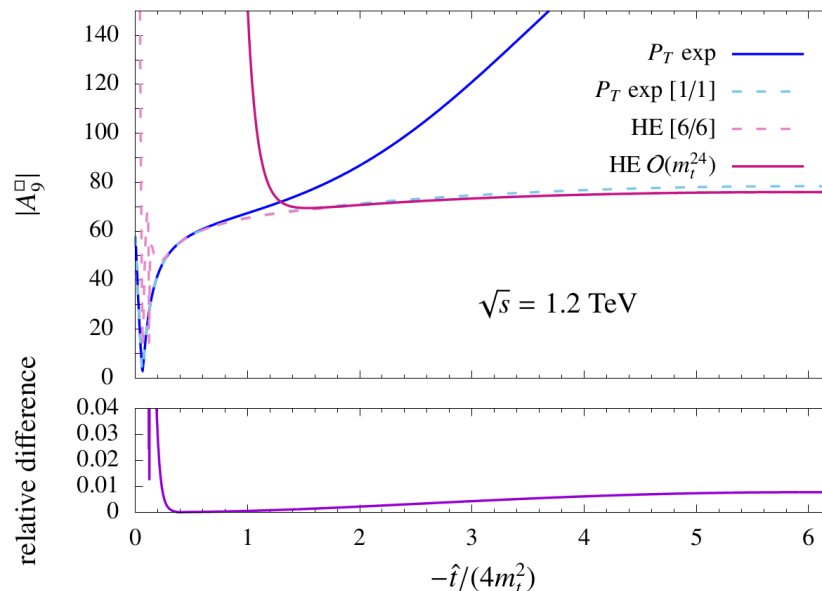
See also [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

Merging pT and HE Expansions at NLO

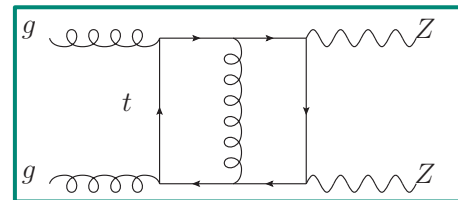
Improve the convergence of a series expansion by matching the coefficients of the **Padé approximant [m/n]** [e.g. Fleisher, Tarasov ('94) - Campbell et al. - 1605.01380]

$$f(x) \stackrel{x \rightarrow 0}{\simeq} c_0 + c_1 x + \dots + c_q x^q \quad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \dots + a_m x^m}{1 + b_1 x + \dots + b_n x^n} \quad (q = m + n)$$

- For each FF we merged the following results
 - pT exp improved by [1/1] Padé
 - HE exp improved by [6/6] Padé
[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]
- Padé results are stable and comparable in the region $|\hat{t}| \sim 4m_t^2 \rightarrow$ can switch without loss of accuracy
- Evaluation time for a phase-space point below 0.1 s \Rightarrow suitable for Monte Carlo



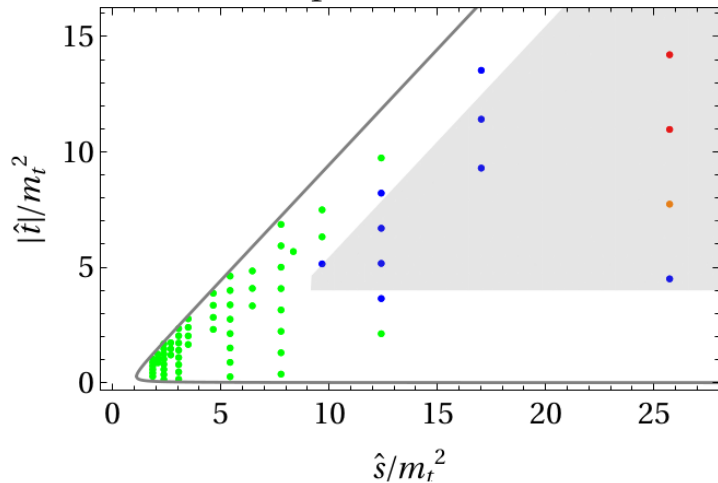
Comparing with Numerical Results



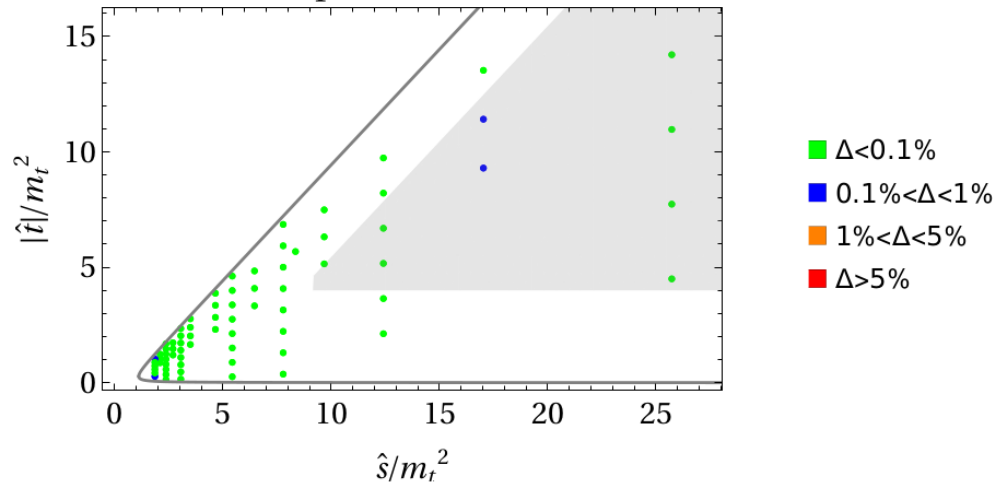
Comparison with helicity amplitudes of [\[Agarwal, Jones, von Manteuffel - 2011.15113\]](#)

$$\mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{\text{fin}} = \left(\frac{\alpha_s}{2\pi}\right) \mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{(2)} + \mathcal{O}(\alpha_s^3)$$

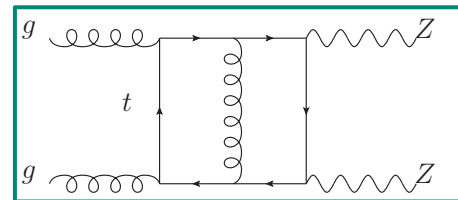
Re[++00]: pT-Pade vs numerical



Im[++00]: pT-Pade vs numerical



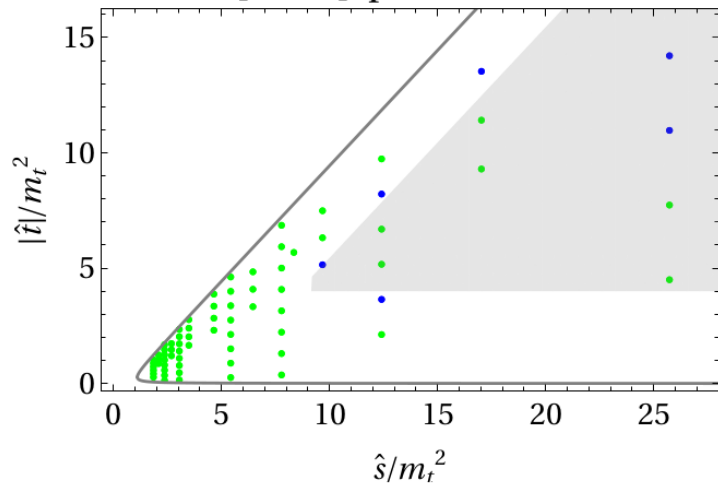
Comparing with Numerical Results



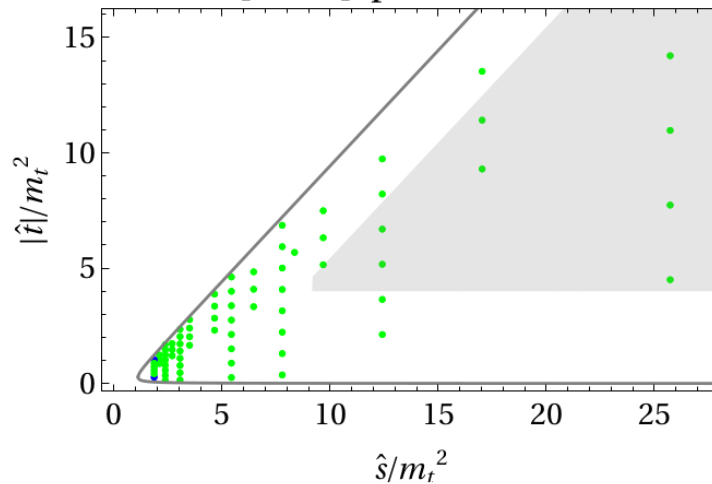
Comparison with helicity amplitudes of [\[Agarwal, Jones, von Manteuffel - 2011.15113\]](#)

$$\mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{\text{fin}} = \left(\frac{\alpha_s}{2\pi}\right) \mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{M}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Re[++00]: pT+HE vs exact



Im[++00]: pT+HE vs exact



- $\Delta < 0.1\%$
- $0.1\% < \Delta < 1\%$
- $1\% < \Delta < 5\%$
- $\Delta > 5\%$

Results

$$\mathcal{V}_{\text{fin}} = \frac{G_F^2 m_Z^4}{16} \left(\frac{\alpha_s}{\pi} \right)^2 \left\{ \sum_i \left| \mathcal{A}_i^{(0)} \right|^2 \frac{C_A}{2} \left(\pi^2 - \log^2 \left(\frac{\mu_R^2}{\hat{s}} \right) \right) + 2 \sum_i \text{Re} \left[\mathcal{A}_i^{(0)} \left(\mathcal{A}_i^{(1)} \right)^* \right] \right\}$$

$$\Delta\sigma_{\text{virt}} = \int_{\hat{t}^-}^{\hat{t}^+} d\hat{t} \frac{1}{2} \frac{1}{16\pi\hat{s}^2} \left(\frac{\alpha_s}{\pi} \right) \mathcal{V}_{\text{fin}}(\hat{t})$$

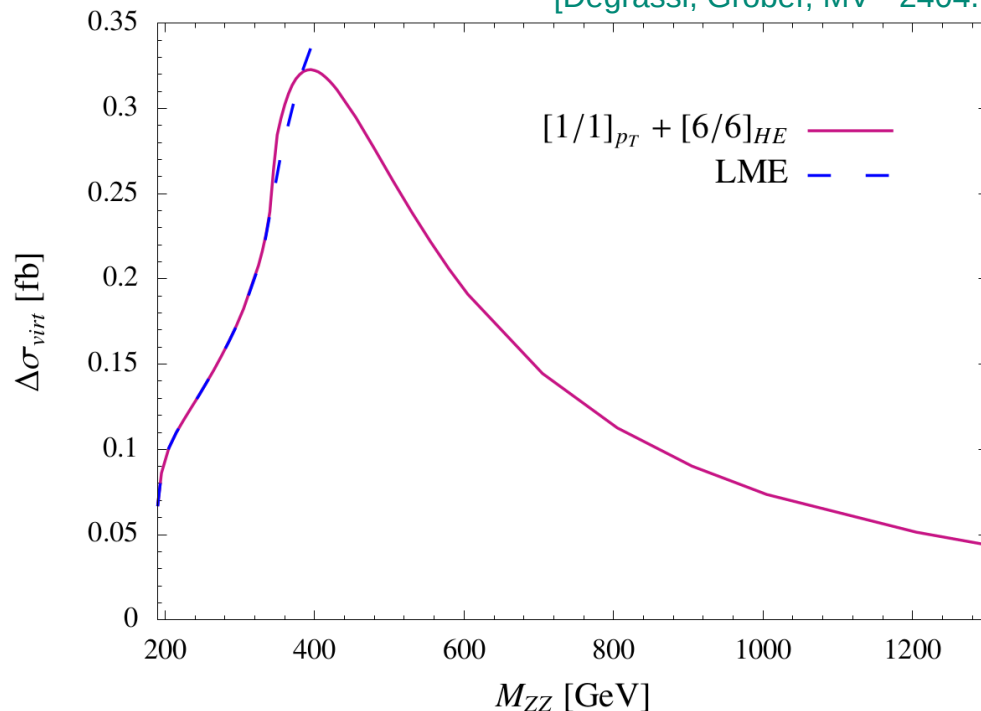
$$\mathcal{A}_i^{(0)} = \mathcal{A}_i^{(0,\Delta)} + \mathcal{A}_i^{(0,\square)}$$

$$\mathcal{A}_i^{(1)} = \mathcal{A}_i^{(1,\Delta)} + \mathcal{A}_i^{(1,\square)} + \mathcal{A}_i^{(1,\infty)}$$

LME from

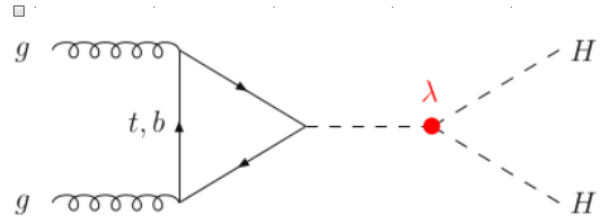
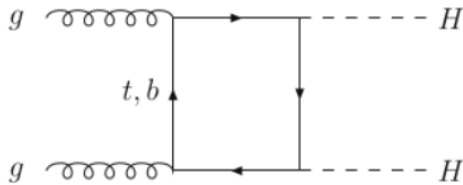
[Davies, Mishima, Steinhauser, Wellmann -
2002.05558]

[Degrassi, Gröber, MV - 2404.15113]

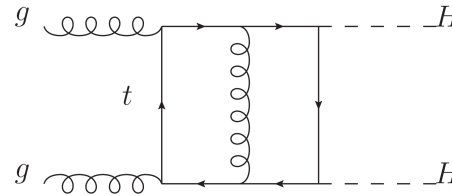


Going to NNLO QCD...

- Consider a “simpler” case: $gg \rightarrow HH$
(2 FFs, scalar identical final particles)
- Best chance to measure λ_3 at LHC



- Two loop boxes are a problem again

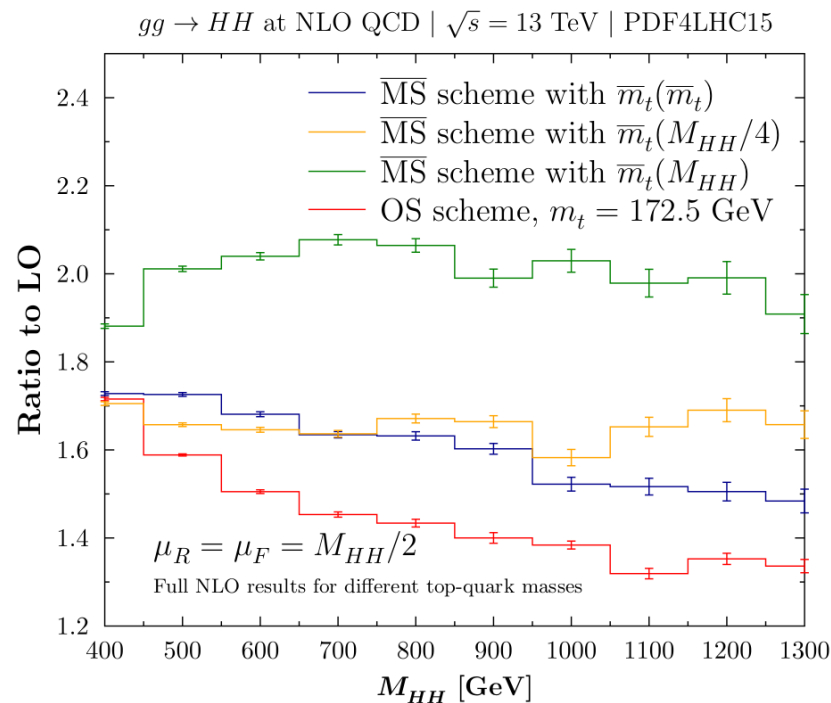


Going to NNLO QCD...

- Consider a “simpler” case: $gg \rightarrow HH$
(2 FFs, scalar identical final particles)
- Best chance to measure λ_3 at LHC

- Large uncertainty at NLO, due to choice of renormalization scheme and scale for the top mass
- NNLO would still be desirable

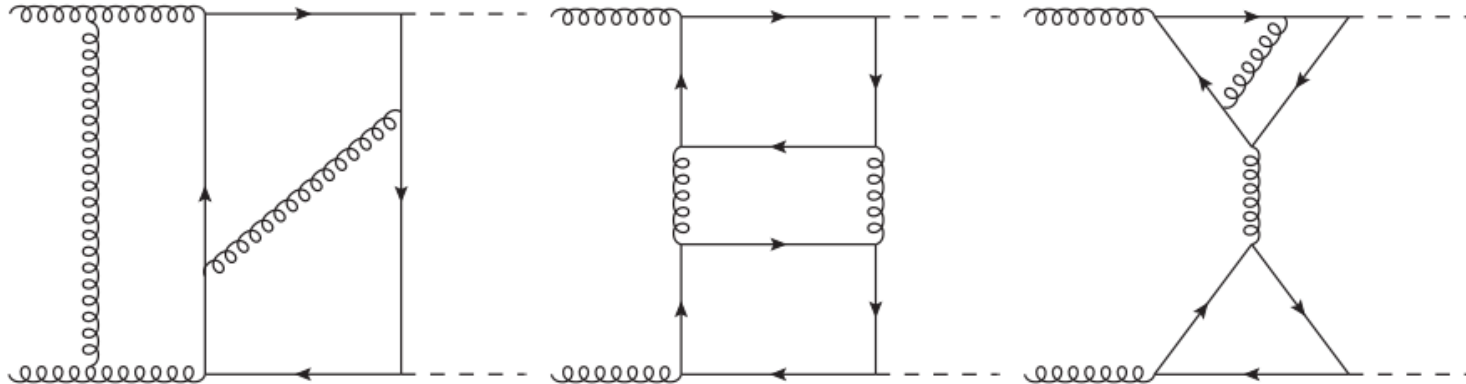
[Baglio et al. - 2008.11626]



Going to NNLO QCD...

Can we use the forward expansion for higher orders?

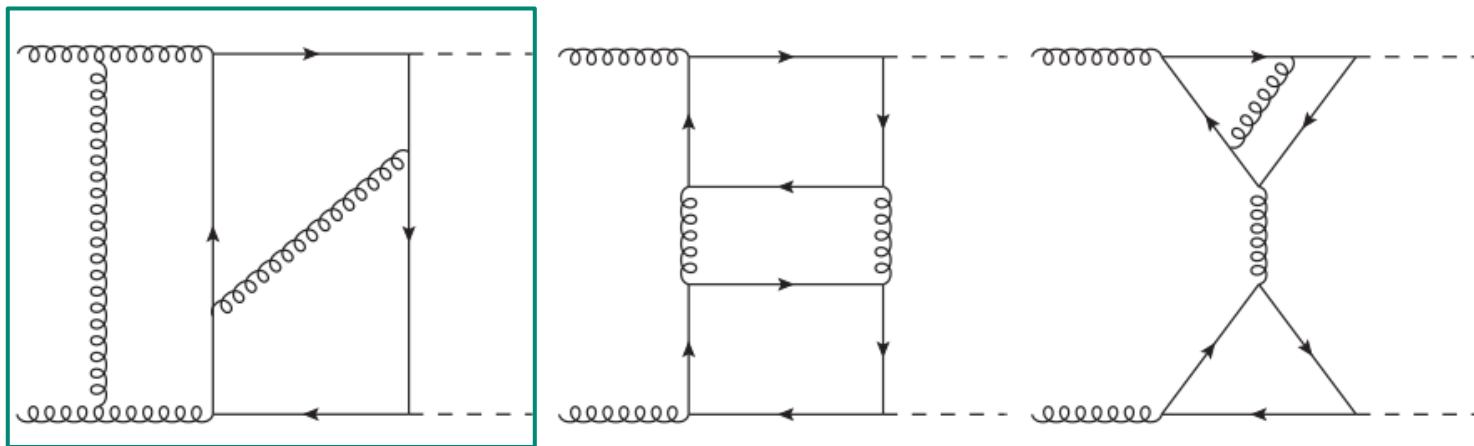
■ Classes of three loop diagrams



Going to NNLO QCD...

Can we use the forward expansion for higher orders?

■ Classes of three loop diagrams



Conceptually yes

Practical implementation promising

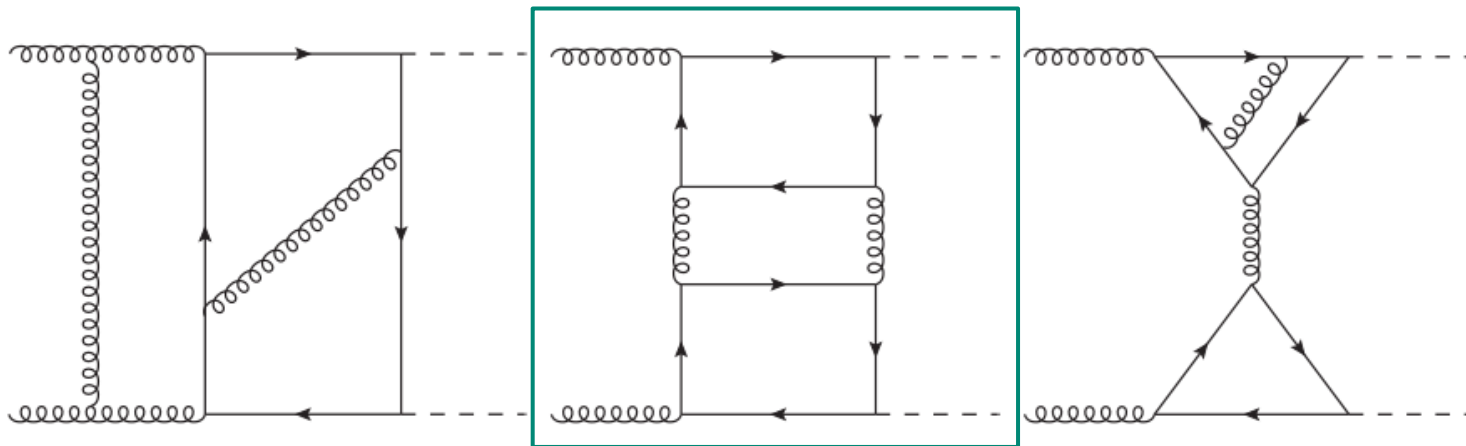
for the $t \rightarrow 0$ expansion $\{t^0, m_H^0\}$

[Davies, Schönwald, Steinhauser 2307.04796]

Going to NNLO QCD...

Can we use the forward expansion for higher orders?

■ Classes of three loop diagrams



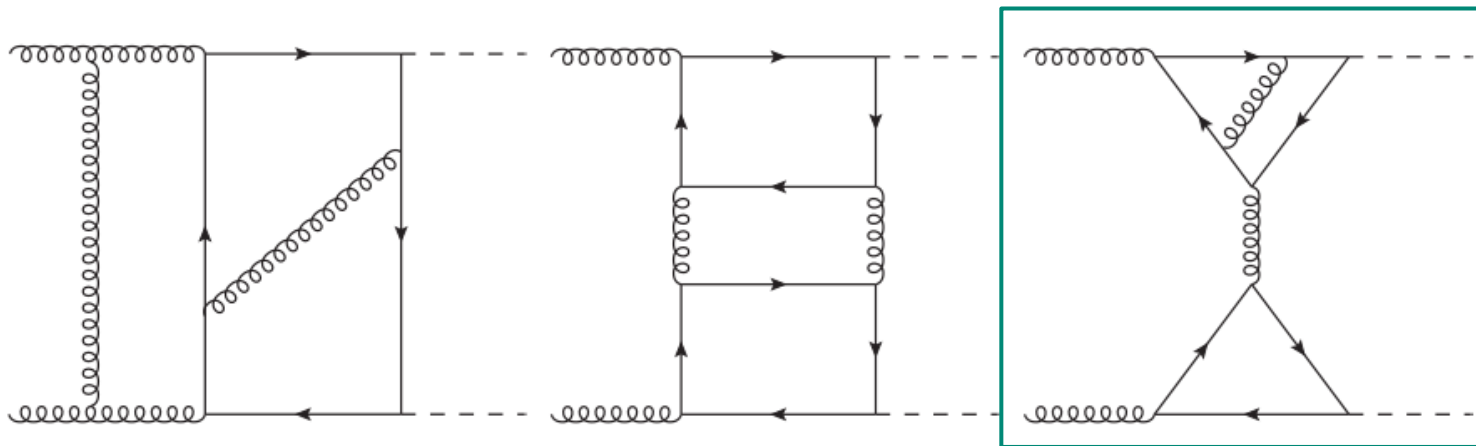
More involved due to t-channel cuts
through massless lines

⇒ A Taylor expansion is not sufficient

Going to NNLO QCD...

Can we use the forward expansion for higher orders?

■ Classes of three loop diagrams



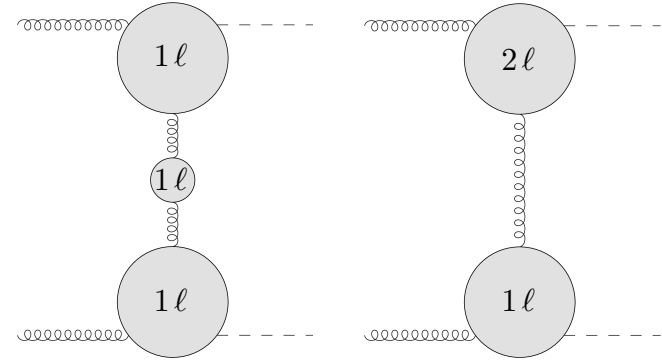
Start by studying the 1PR piece

1PR Contribution to $gg \rightarrow HH$ @ 3 Loops

[Davies, Schönwald, Steinhauser, MV - 2405.20372]

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0s(F_1A_1^{\mu\nu} + F_2A_2^{\mu\nu})$$

Goal: compute $F_1^{(3\ell, 1PR)}$ $F_2^{(3\ell, 1PR)}$

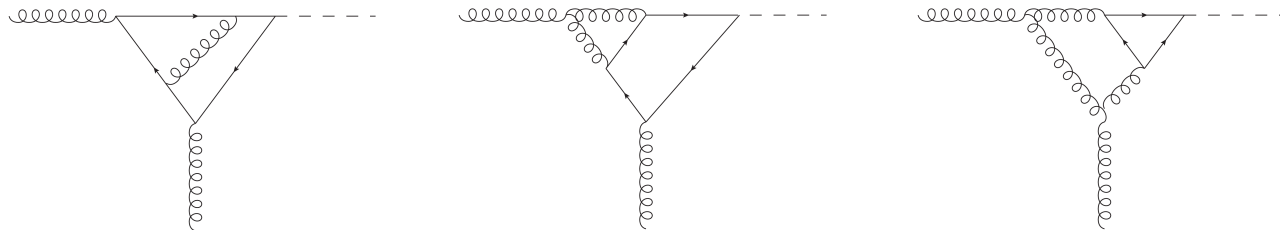


Approach: construct the $gg \rightarrow HH$ form factors from the 1PI gg^*H subamplitudes

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = F_a g^{\alpha\beta}(q_s \cdot q_2) + F_b q_s^\alpha q_2^\beta + F_c q_2^\alpha q_s^\beta + F_d q_s^\alpha q_s^\beta + F_e q_2^\alpha q_2^\beta$$

$$q_2^2 = 0, q_s^2 \neq 0$$

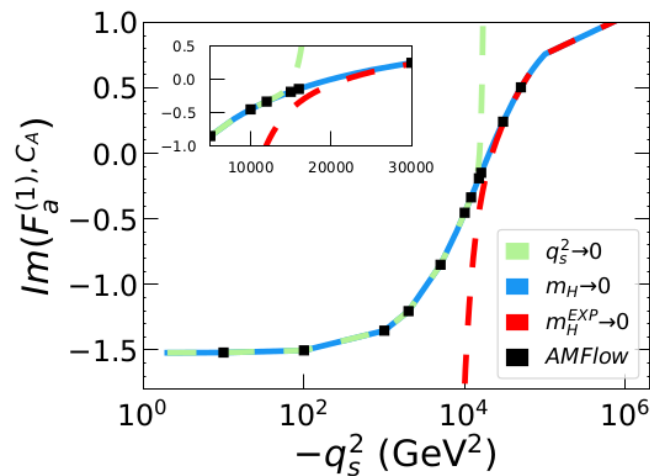
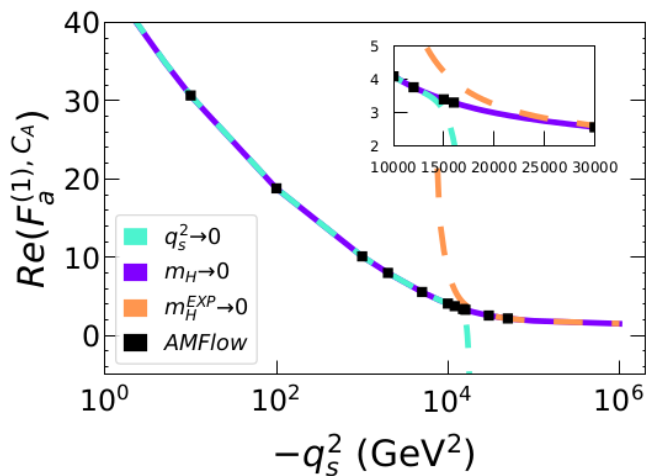
$$m_H \neq 0$$



1PR Contribution to $gg \rightarrow HH$ @ 3 Loops

Expansion by regions [Beneke, Smirnov ('98)]

- $m_H^2 \ll q_s^2, m_t^2$ ■ Use expanded MIs but keep coefficients exact ($m_H \rightarrow 0$)
- $q_s^2 \ll m_H^2, m_t^2$ ■ Results checked with AMFlow [Liu, Ma - 2201.11669]



Conclusions

- Top loops are crucial for precision Higgs physics
- At two-loops, an important class of gg -initiated diboson processes can be approximated (semi-)analytically using a combination of a **forward** expansion and a **high-energy** expansion
- Results are fast and precise over the complete phase space

Outlook

- Going to 3 loops is not straightforward \Rightarrow use asymptotic expansions?
 - Looking for new combinations of expansions to cover a significant part of the phase space
-

- “You can’t **always for now** get what you want”



- But if you try sometime / you might find / you get what you need (for phenomenology)
-

Thank you for your attention

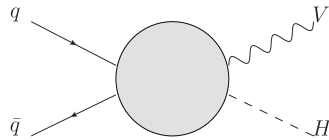
Backup

VH Production at the LHC

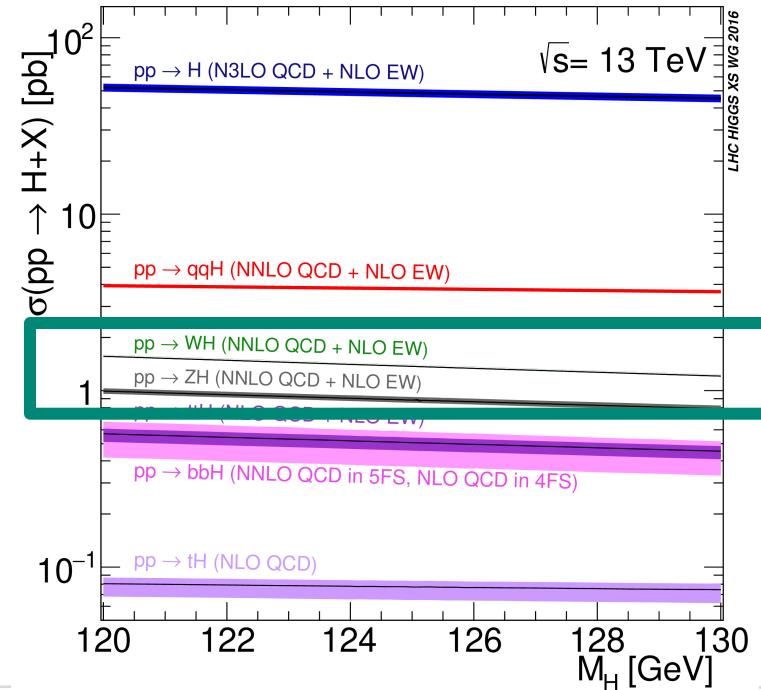
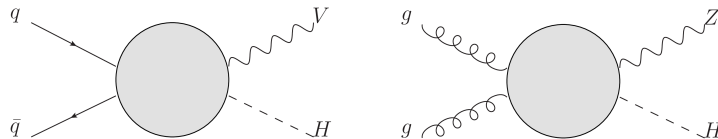
$pp \rightarrow VH$ is the most sensitive process to $H \rightarrow b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Work in progress on $H \rightarrow c\bar{c}$
[ATLAS-2201.11428, CMS-2205.0555]
- Probe of VVH coupling
- Partonic channels

$pp \rightarrow WH$



$pp \rightarrow ZH$



VH Production at the LHC

$pp \rightarrow VH$ is the most sensitive process to $H \rightarrow b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Work in progress on $H \rightarrow c\bar{c}$
[ATLAS-2201.11428, CMS-2205.0555]
- Probe of VVH coupling
- Larger scale uncertainties in ZH

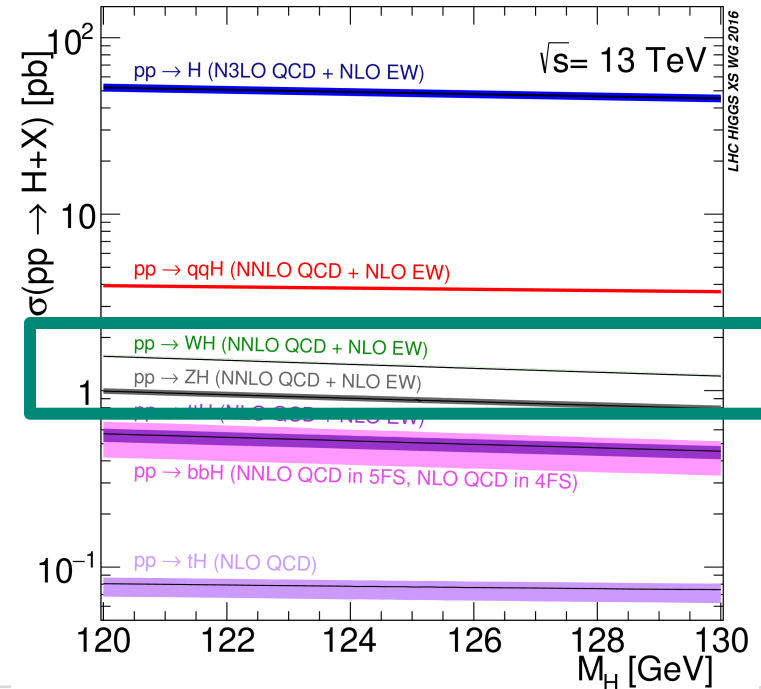
$pp \rightarrow WH$

\sqrt{s} [TeV]	$\sigma_{\text{NNLO QCD} \otimes \text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF} \oplus \alpha_s}$ [%]
13	1.358	+0.51 -0.51	1.35
14	1.498	+0.51 -0.51	1.35
27	3.397	+0.29 -0.72	1.37

$pp \rightarrow ZH$

\sqrt{s} [TeV]	$\sigma_{\text{NNLO QCD} \otimes \text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF} \oplus \alpha_s}$ [%]
13	0.880	+3.50 -2.68	1.65
14	0.981	+3.61 -2.94	1.90
27	2.463	+5.42 -4.00	2.24

[Cepeda et al. - 1902.00134]



$gg \rightarrow ZH$ @ NLO QCD

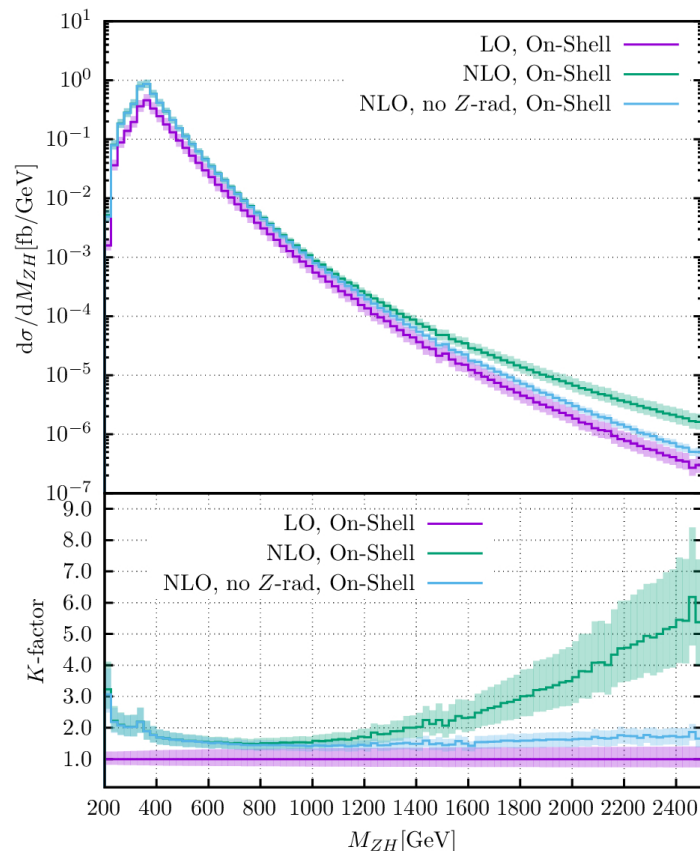
Inclusive cross section $\sqrt{s} = 13\text{TeV}$
 $\mu_r = \mu_f = M_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	64.01 ^{+27.2%} _{-20.3%}	—	118.6 ^{+16.7%} _{-14.1%}	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	59.40 ^{+27.1%} _{-20.2%}	0.928	113.3 ^{+17.4%} _{-14.5%}	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	57.95 ^{+26.9%} _{-20.1%}	0.905	111.7 ^{+17.7%} _{-14.6%}	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	54.22 ^{+26.8%} _{-20.0%}	0.847	107.9 ^{+18.4%} _{-15.0%}	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	49.23 ^{+26.6%} _{-19.9%}	0.769	103.3 ^{+19.6%} _{-15.6%}	0.871	2.10

- NLO corrections are the same size as LO ($K \sim 2$)
- Scale uncertainties reduced by 30% wrt LO

Invariant-mass distribution

- K -factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail ($M_{ZH} > 1\text{TeV}$)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \rightarrow ZH$ @ NLO QCD

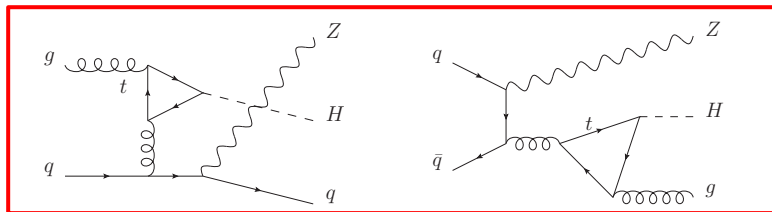
Inclusive cross section $\sqrt{s} = 13\text{TeV}$
 $\mu_r = \mu_f = M_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	64.01 ^{+27.2%} _{-20.3%}	—	118.6 ^{+16.7%} _{-14.1%}	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	59.40 ^{+27.1%} _{-20.2%}	0.928	113.3 ^{+17.4%} _{-14.5%}	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	57.95 ^{+26.9%} _{-20.1%}	0.905	111.7 ^{+17.7%} _{-14.6%}	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	54.22 ^{+26.8%} _{-20.0%}	0.847	107.9 ^{+18.4%} _{-15.0%}	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	49.23 ^{+26.6%} _{-19.9%}	0.769	103.3 ^{+19.6%} _{-15.6%}	0.871	2.10

■ NLO corrections are the same size as LO ($K \sim 2$)

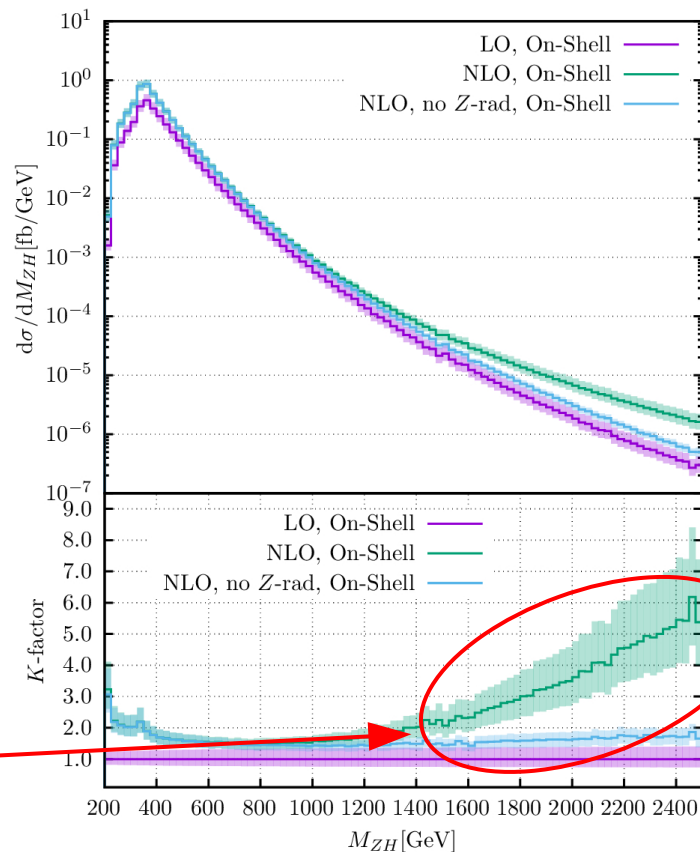
■ Scale uncertainties reduced by 30% wrt LO

Invariant-mass distribution

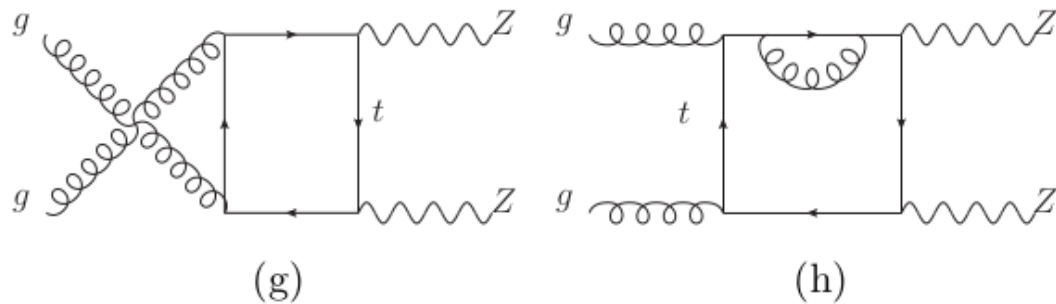
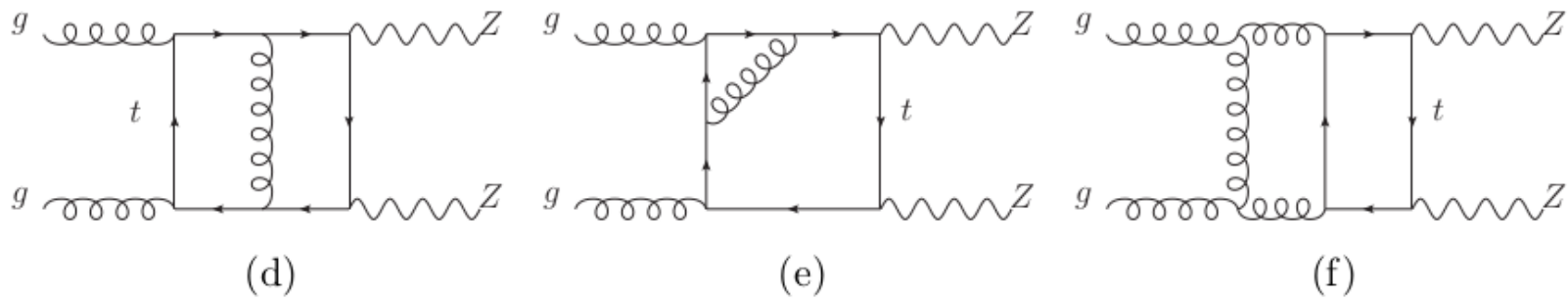
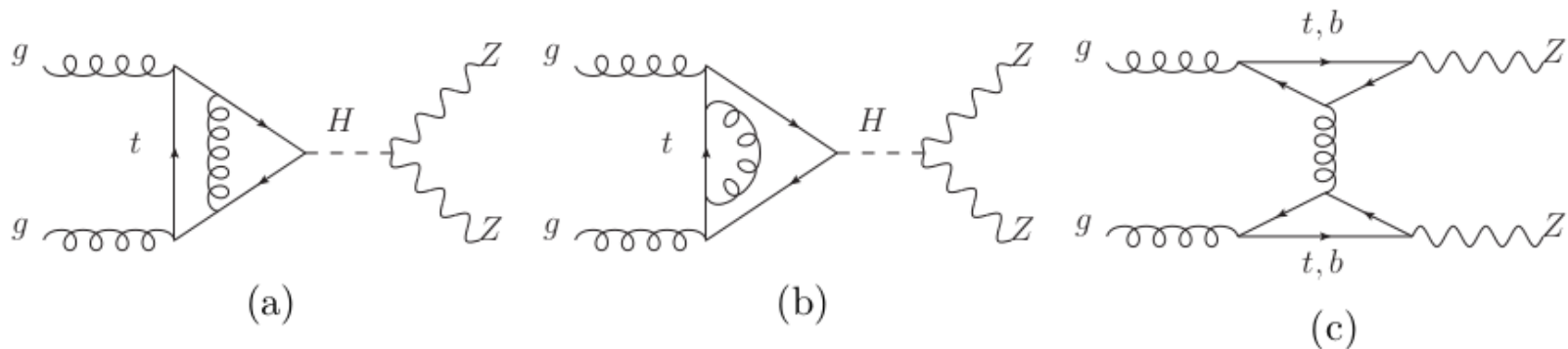


Dominant

PDF suppressed



[Degrassi, Gröber, MV, Zhao - 2205.02769]



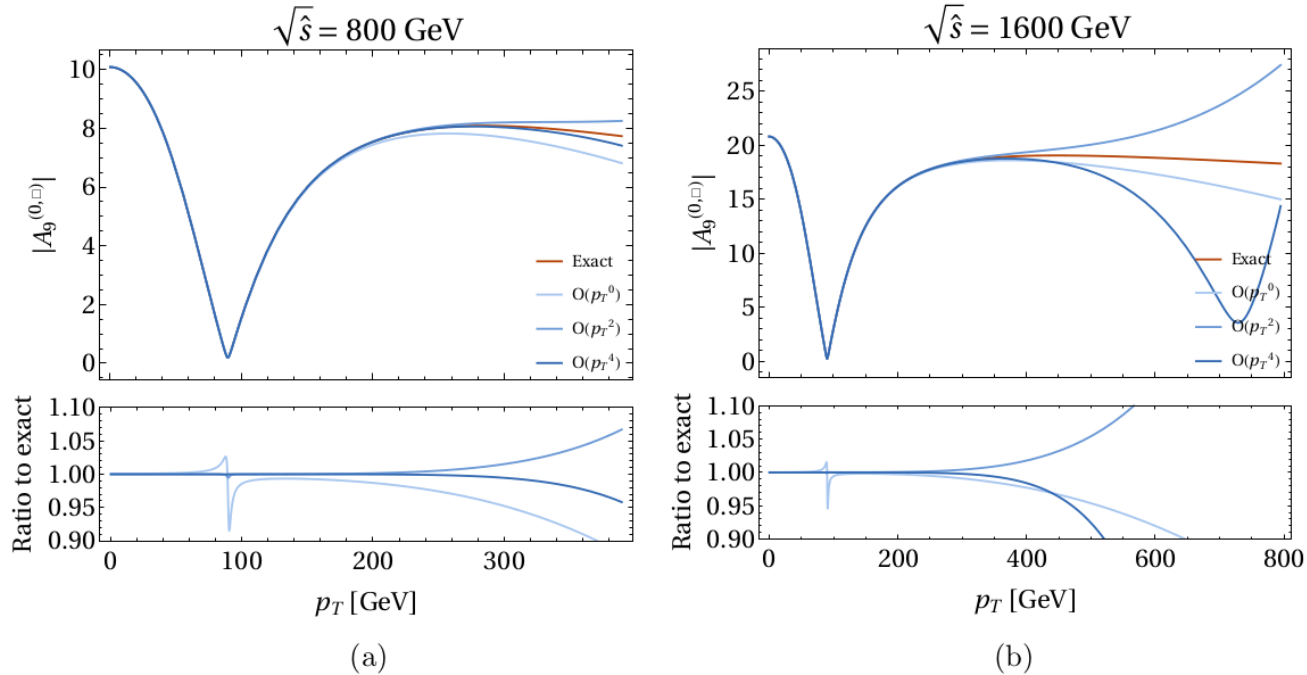


Figure 3: Absolute value of the form factor $\mathcal{A}_9^{(0, \square)}$ for moderate (a) and high (b) partonic centre-of-mass energies as a function of the transverse momentum. The exact result and the results obtained at various orders in the p_T expansion are shown.

1PR Contribution to $gg \rightarrow HH$ @ 3 Loops: Strategy

1. Generation of diagrams with qgraf [Nogueira, '93]
 2. Manipulation with Tapir [Gerlach, Herren, Lang - 2201.05618],
q2e/exp [Harlander, Seidensticker Steinhauser – '97], FORM [Ruijl, Ueda, Vermaseren - 1707.06453]
 3. IBP reduction (KIRA [Klappert, Lange, Maierhöfer, Usovitsch - 2008.06494])
 4. The MIs can be expanded for $m_H \rightarrow 0$ (LiteRed [Lee - 1310.1145])
 5. Results mapped onto single-scale “forward” topologies [Davies, Mishima, Schönwald,
Steinhauser - 2302.01356]
 6. Evaluated semi-analytically using “expand-and-match” approach
[Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]
- Two-loop: results in agreement with [Degrassi, Giardino, Gröber – 1603.00385]
 - Three-loop: agreement with LME result of [Davies, Steinhauser - 1909.01361]
-