

Classical radiation at one loop

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arXiv : 2303.06211



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NORDITA
The Nordic Institute for Theoretical Physics

Holonomic Techniques for Feynman Integrals,
Max Planck Institute,
17 October 2024

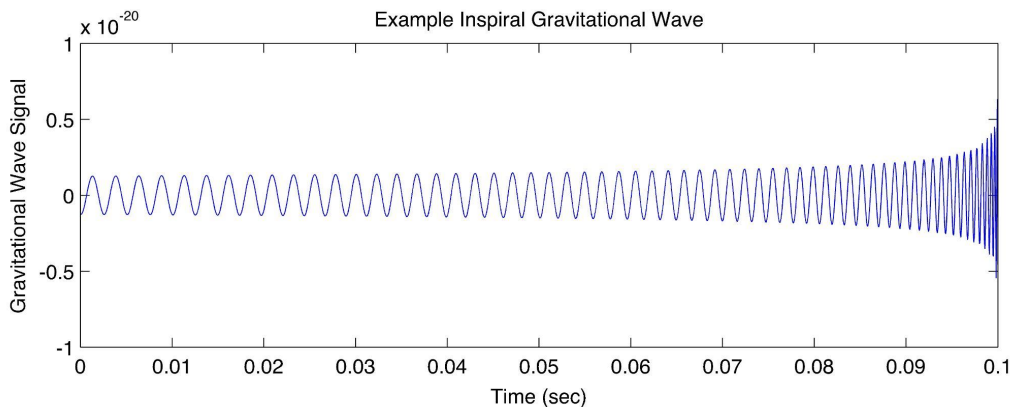


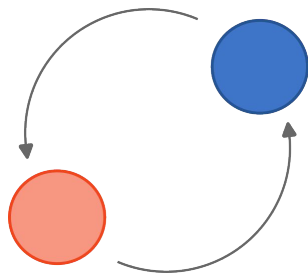
Elkhidir, O'Connell, Sergola,
IVH

Herderschee, Roiban, Teng

Brandhuber, Brown, Chen
De Angelis, Gowdy,
Travaglini

Georgoudis, Heissenberg,
IVH

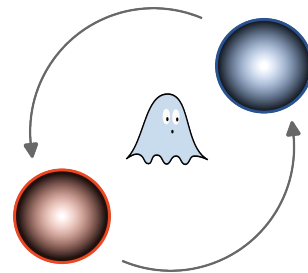




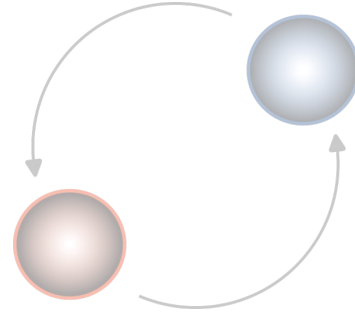
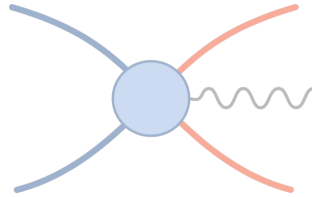
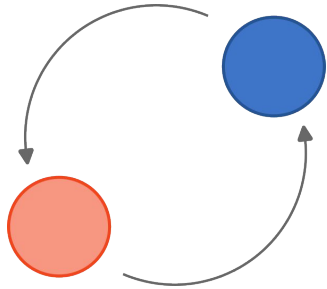
Express **radiation waveforms** in terms of scattering amplitudes

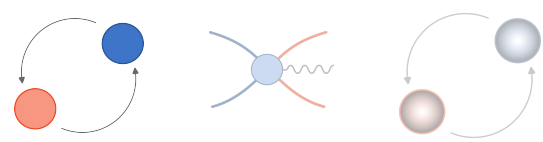
QCD amplitudes are bootstrapped using the color-kinematics duality

Gravity amplitudes are double copies of QCD amplitudes



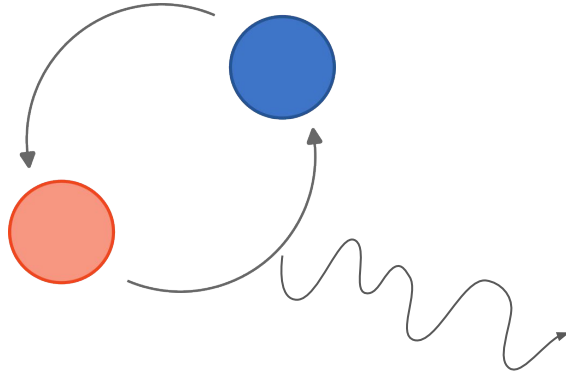
I. Radiation waveform





I. Radiation waveform

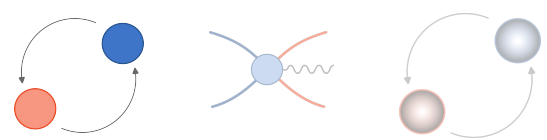
$$\langle F_{\mu\nu} \rangle \sim$$



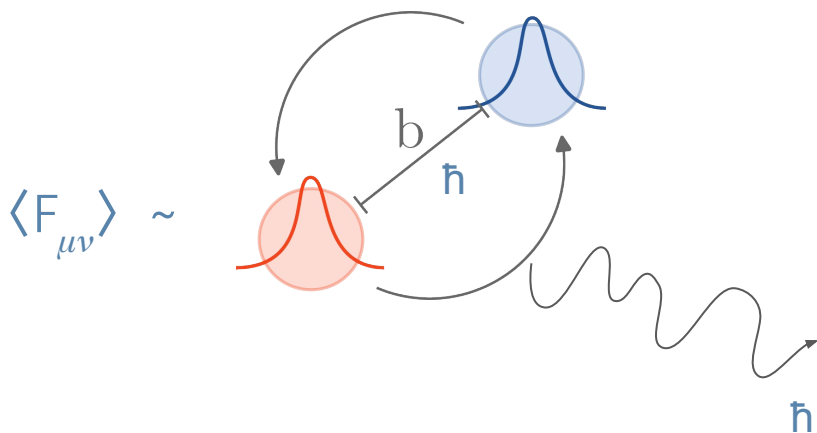
Radiation waveforms are associated with the expectation value of the **field strength/Riemann tensor**

Define an initial state and the expectation in the far future

Treat classically, restore \hbar



I. Radiation waveform



Radiation waveforms are associated with the expectation value of the **field strength/Riemann tensor**

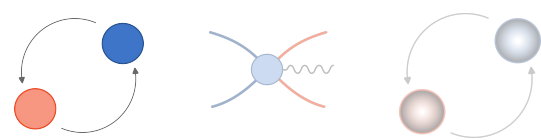
Define an initial state and the expectation in the far future

Treat classically, restore \hbar

$$|\psi\rangle_{in} = \int d\tilde{k} \text{ (wave packet) } e^{ib \cdot p_1/\hbar} |p_1 p_2\rangle_{in}$$

$$\langle \psi | S^\dagger F_{\mu\nu}(x) S | \psi \rangle$$

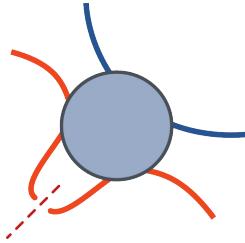
$$\langle p'_1 p'_2 | a_\eta(k) \text{Re } T + \frac{i}{2} ([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T]) | p_1, p_2 \rangle$$



I. Radiation waveform

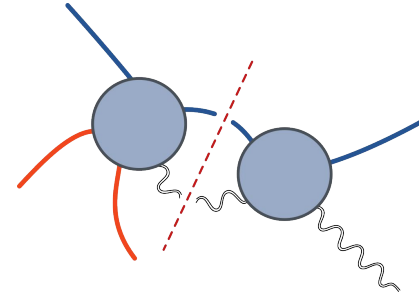
$$\langle p'_1 p'_2 | a_\eta(k) \text{Re} T + \frac{i}{2} ([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T]) | p_1, p_2 \rangle$$

Real

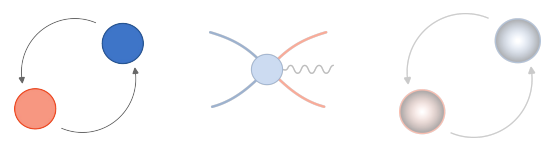


The real part is **radiation**, we use **1-particle cuts**

Imaginary



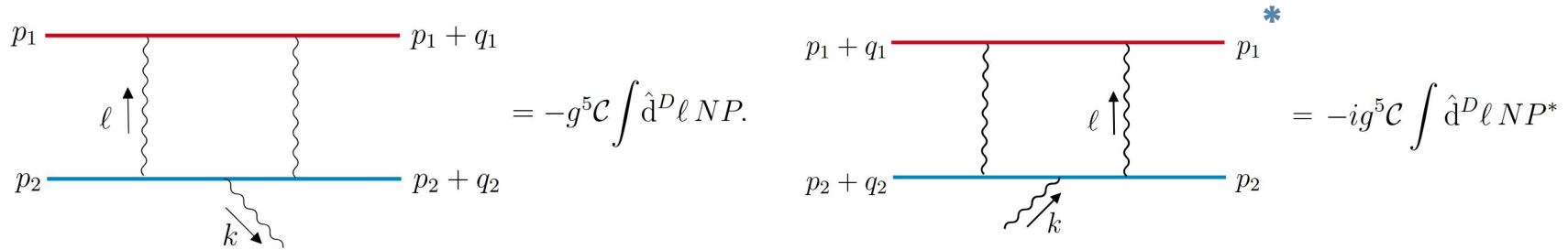
The imaginary part is **radiation reaction**, we use **2-particle cuts**



I. Radiation waveform

Real part is associated with radiation effects

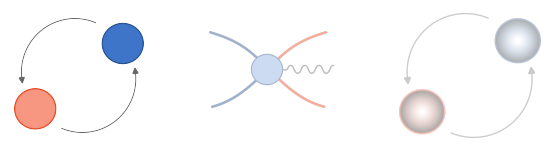
$$\text{Real} = \mathcal{A}_{(5,1)}(p_1, p_2 \rightarrow p'_1, p'_2, k_\eta) + \mathcal{A}_{(5,1)}^*(p'_1, p'_2, k_{-\eta} \rightarrow p_1, p_2)$$



$$P^{-1} = (\ell^2 + i\epsilon)[(q_1 - \ell)^2 + i\epsilon][(p_1 + \ell)^2 - m_1^2 + i\epsilon][(p_2 - \ell)^2 - m_2^2 + i\epsilon][(p_2 - \ell - k)^2 - m_2^2 + i\epsilon]$$

$$-g^5 \mathcal{C} \int \hat{d}^D \ell N [-i(P - P^*)] = -2g^5 \mathcal{C} \int \hat{d}^D \ell N \text{Im} P$$

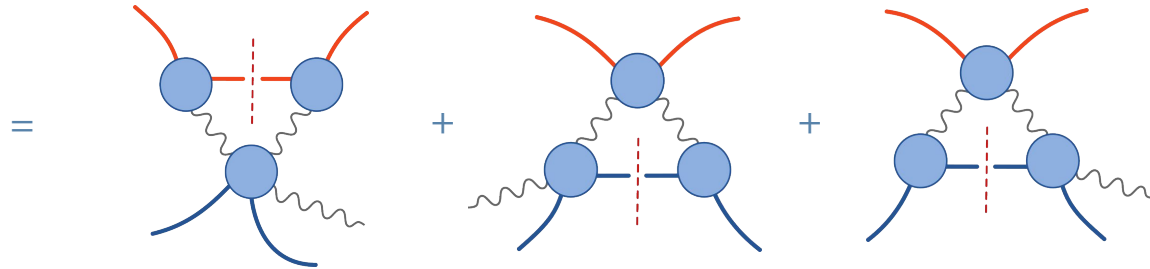
$$\frac{1}{p^2 - m^2 + i\epsilon} = \text{PV} \left(\frac{1}{p^2 - m^2} \right) - \frac{i}{2} \hat{\delta}(p^2 - m^2)$$



I. Radiation waveform

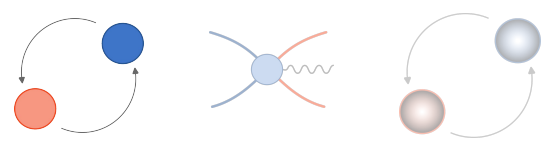
Real part is associated with radiation effects

$$\text{Real} = \mathcal{A}_{(5,1)}(p_1, p_2 \rightarrow p'_1, p'_2, k_\eta) + \mathcal{A}_{(5,1)}^*(p'_1, p'_2, k_{-\eta} \rightarrow p_1, p_2)$$



$$-g^5 \mathcal{C} \int \hat{d}^D \ell N[-i(P - P^*)] = -2g^5 \mathcal{C} \int \hat{d}^D \ell N \text{Im} P$$

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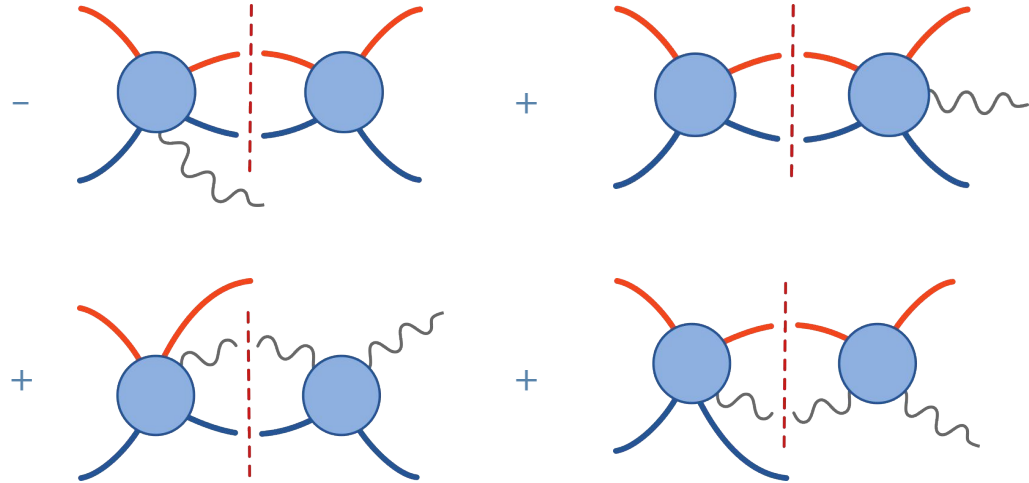


I. Radiation waveform

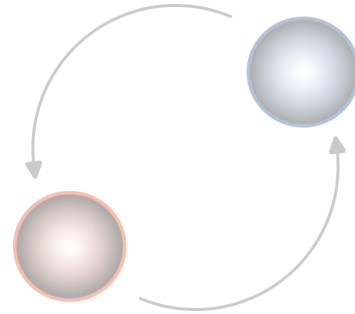
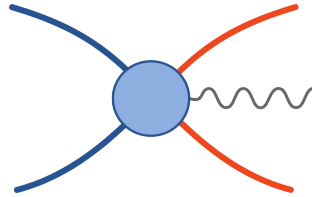
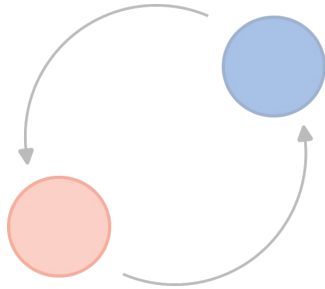
$$\langle p'_1 p'_2 | \frac{i}{2} ([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T]) | p_1, p_2 \rangle$$

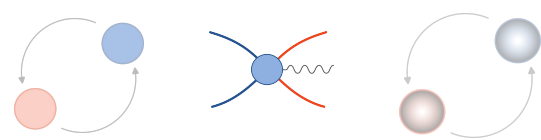
Imaginary part is associated with radiation reaction effects

Imaginary =



II. Amplitudes





II. Amplitudes

We bootstrap the amplitude (integrand) using **color-kinematics duality** and **unitarity cuts**

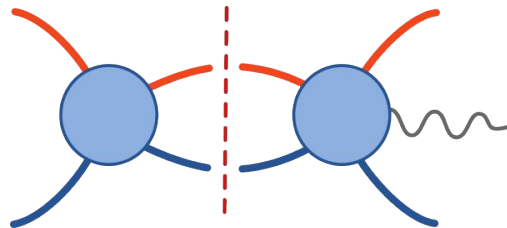
Bern, Carrasco, Johansson

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$

$$\begin{array}{c} b \\ \text{wavy} \\ a \end{array}
 \begin{array}{c} c \\ \text{wavy} \\ d \end{array}
 =
 \begin{array}{c} c \\ \text{wavy} \\ b \end{array}
 \begin{array}{c} d \\ \text{wavy} \\ a \end{array}
 +
 \begin{array}{c} a \\ \text{wavy} \\ c \end{array}
 \begin{array}{c} b \\ \text{wavy} \\ d \end{array}$$

$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$

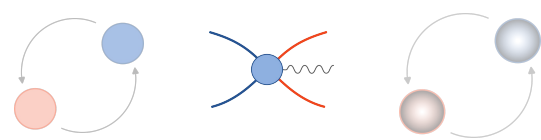
Gives us amplitudes we can
double copy



$$\frac{C_i n_i}{d_i} \rightarrow \frac{n_i n_j}{d_i}$$

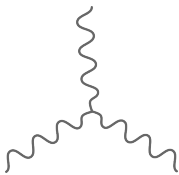
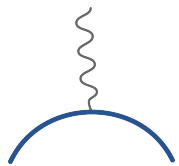
Gauge theory

Gravity

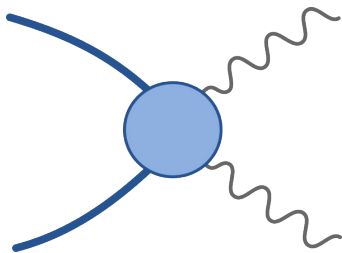


II. Amplitudes

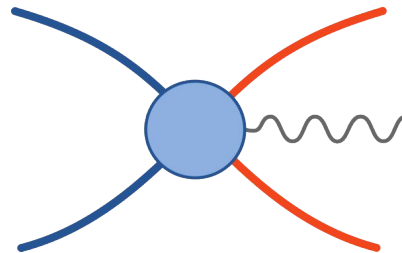
We bootstrap tree-level amplitudes using color-kinematics duality and factorisation



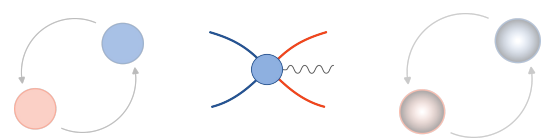
3-point



4-point

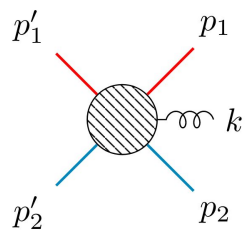


5-point

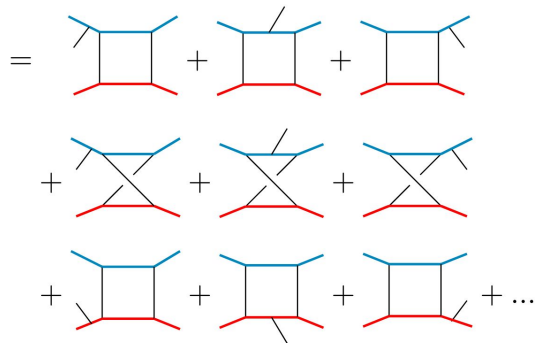


II. Amplitudes

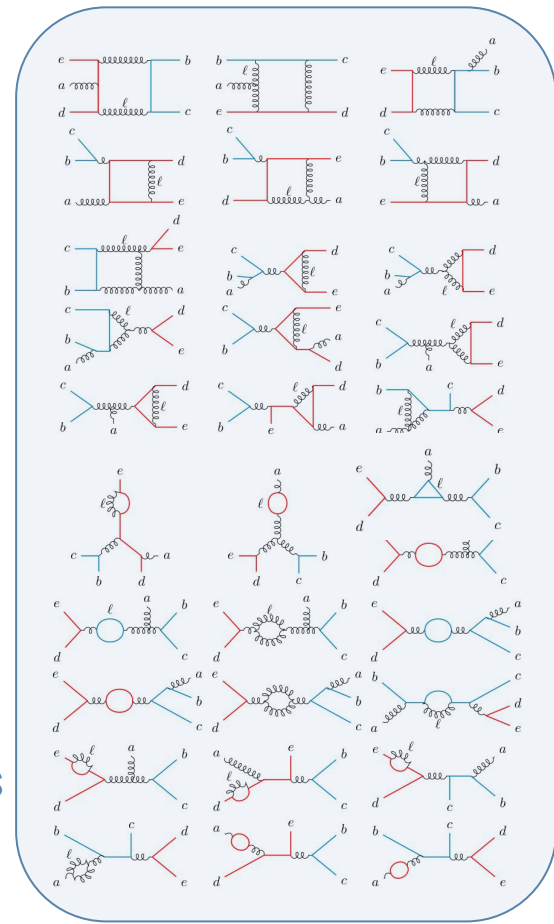
The 5-point one-loop amplitude has 116 graphs

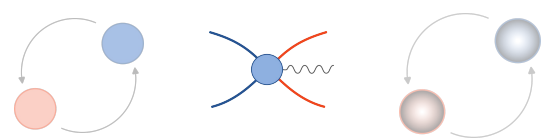


$$= \int d^4\ell \sum_{g=1}^{116} \frac{n_g c_g}{d_g}$$



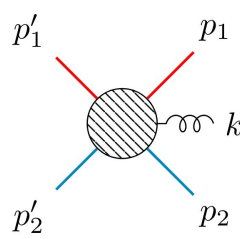
33 topologies





II. Amplitudes

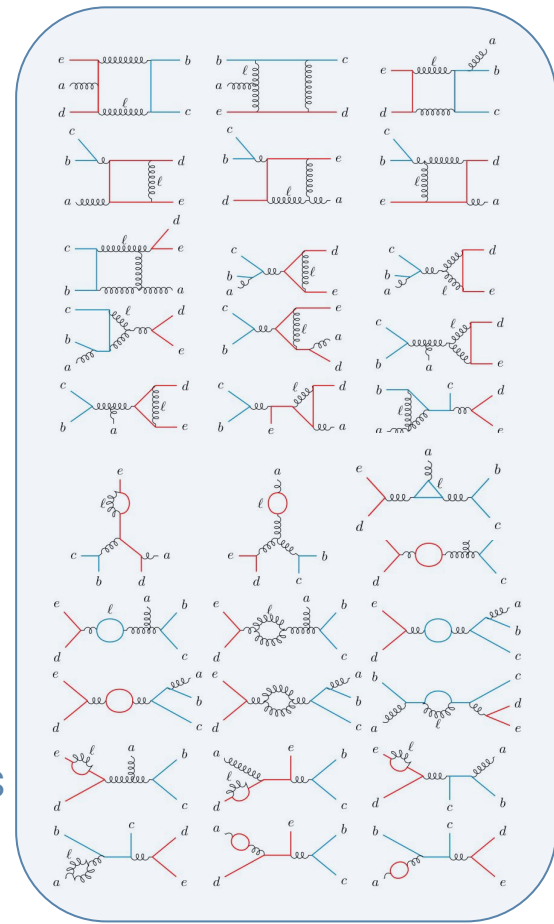
The 5-point one-loop amplitude has 116 graphs

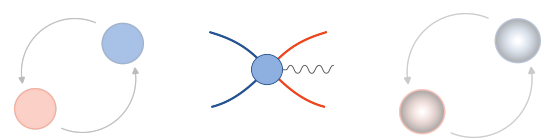


$$\begin{aligned}
 & \mathcal{C} \left(\text{Diagram 1} \right) A_1 + \mathcal{C} \left(\text{Diagram 2} \right) A_2 + \mathcal{C} \left(\text{Diagram 3} \right) A_3 \\
 & + \mathcal{C} \left(\text{Diagram 4} \right) A_4 + \mathcal{C} \left(\text{Diagram 5} \right) A_5 + \mathcal{C} \left(\text{Diagram 6} \right) A_6 \\
 & + \mathcal{C} \left(\text{Diagram 7} \right) A_7 + \mathcal{C} \left(\text{Diagram 8} \right) A_8 + \mathcal{C} \left(\text{Diagram 9} \right) A_9 \\
 & + \mathcal{C} \left(\text{Diagram 10} \right) A_{10} + \mathcal{C} \left(\text{Diagram 11} \right) A_{11} + \mathcal{C} \left(\text{Diagram 12} \right) A_{12} \\
 & + \mathcal{C} \left(\text{Diagram 13} \right) A_{13} + \mathcal{C} \left(\text{Diagram 14} \right) A_{14} + \mathcal{C} \left(\text{Diagram 15} \right) A_{15} \\
 & + \mathcal{C} \left(\text{Diagram 16} \right) A_{16}
 \end{aligned}$$

Look at A_3

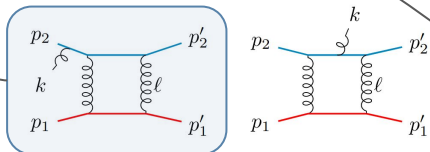
33 topologies



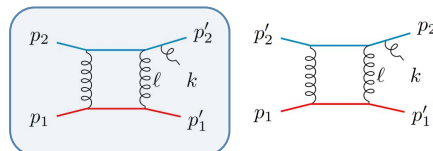
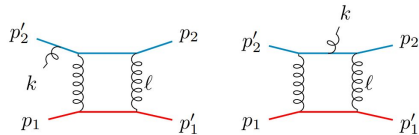


II. Amplitudes

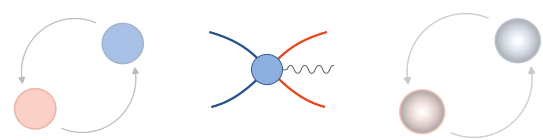
Partial amplitude A_3 $\mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a diagonal line from top-left to bottom-right]} \end{array} \right) A_3$



$$\mathcal{C} \left(\begin{array}{c} p_2 \quad p'_2 \\ k \\ p_1 \quad p'_1 \end{array} \begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a wavy line labeled 'k' on the top edge]} \end{array} \right) = \mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a diagonal line from top-left to bottom-right]} \end{array} \right)$$



$$\begin{aligned} \mathcal{C} \left(\begin{array}{c} p_2 \quad p'_2 \\ k \\ p_1 \quad p'_1 \end{array} \begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a wavy line labeled 'k' on the right edge]} \end{array} \right) &= -\mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a diagonal line from top-right to bottom-left]} \end{array} \right) + 2\hbar \mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a horizontal line across the middle]} \end{array} \right) - \frac{\hbar^3}{2} \mathcal{C} \left(\begin{array}{c} \text{[Diagram: A circle with three lines extending from it: one red, one blue, one white]} \end{array} \right) \\ &+ \hbar^2 \mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a red line on the top edge]} \end{array} \right) + \hbar^2 \mathcal{C} \left(\begin{array}{c} \text{[Diagram: Square with top and bottom edges in blue, left and right edges in red, and a blue line on the top edge]} \end{array} \right) - \frac{\hbar^2}{2} \mathcal{C} \left(\begin{array}{c} \text{[Diagram: A circle with two lines extending from it: one red, one blue]} \end{array} \right) \end{aligned}$$



II. Amplitudes

Partial amplitude $A_3 \mathcal{C} \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right) A_3$

(QED $Q_2^3 Q_1^2$)

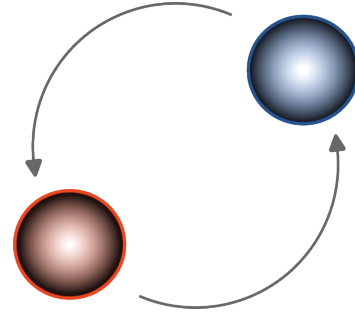
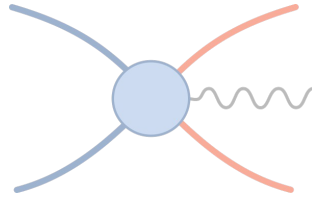
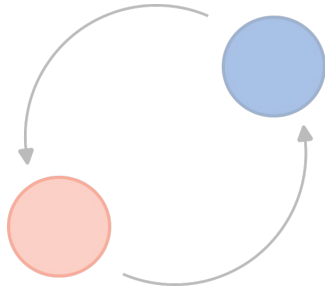
$$A_3(p_1 p'_1 p_2 p'_2 k) =$$

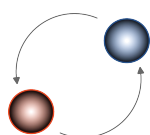
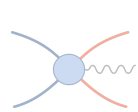
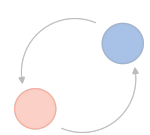
Now we easily find the radiation and reaction

Real =

Im =

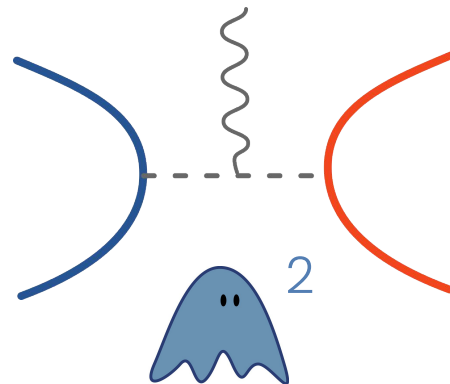
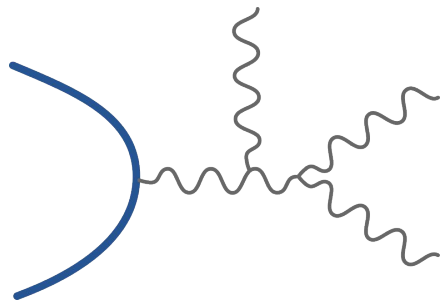
III. Gravity amplitude





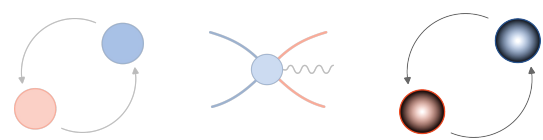
II. Gravity amplitude

The double copy gives extra massless states



Is there a constructive approach?

Is there a double copy approach?



II. Gravity amplitude

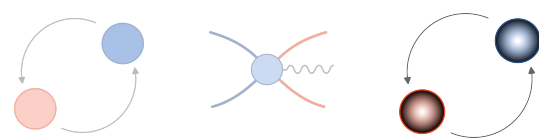
Is there a constructive approach?

$$\left[\text{Diagram} \right]^2 - \text{Diagram} = \text{Diagram}$$

The diagram on the left is a blue semi-circle connected to a red semi-circle by a wavy line. Two vertical dashed red lines are placed on the wavy line. The diagram on the right is identical but with a single vertical dashed red line on the wavy line.

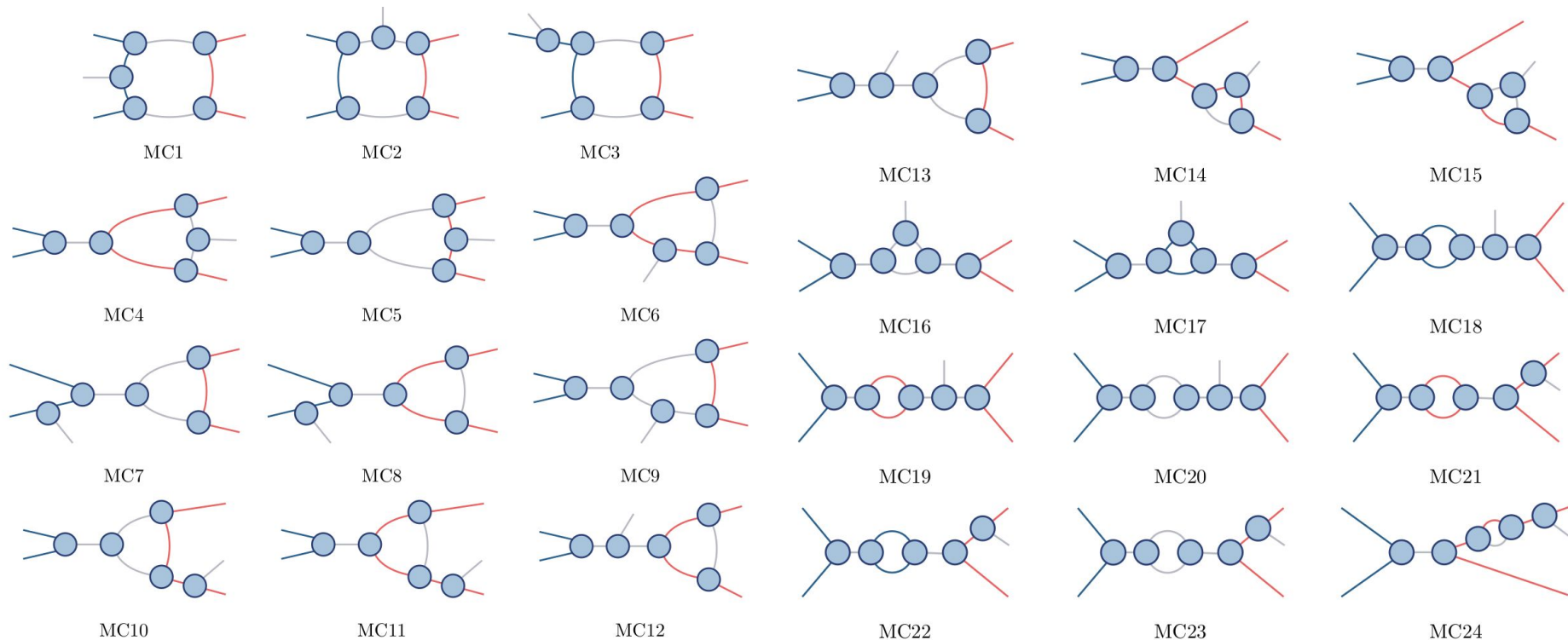
$$n \left[\text{Diagram} \right] = n \left[\text{Diagram} \right]^2 - n \left[\text{Diagram} \right]$$

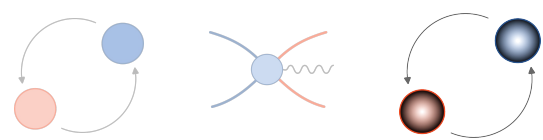
The diagram on the left is a blue semi-circle connected to a red semi-circle by a wavy line. The diagram in the middle is the same as in the first equation. The diagram on the right is the same as in the first equation.



II. Gravity amplitude

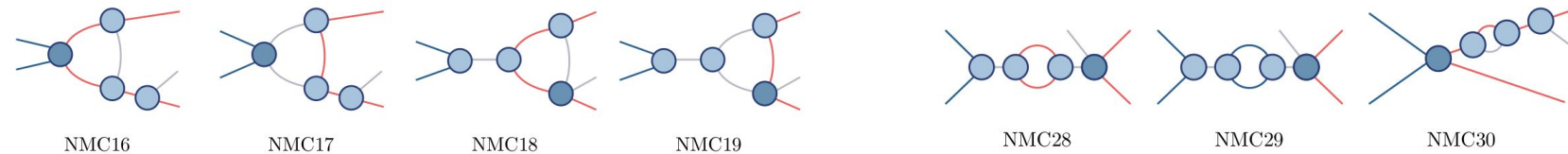
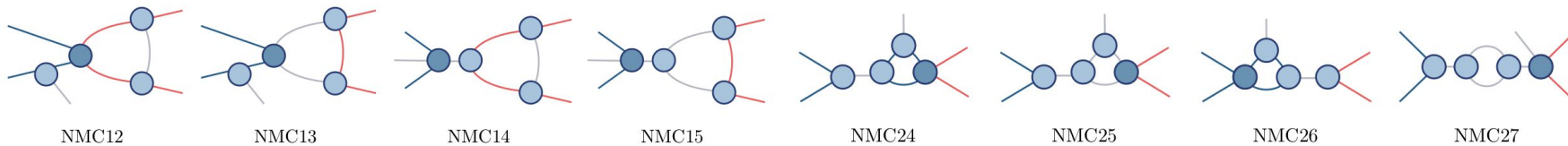
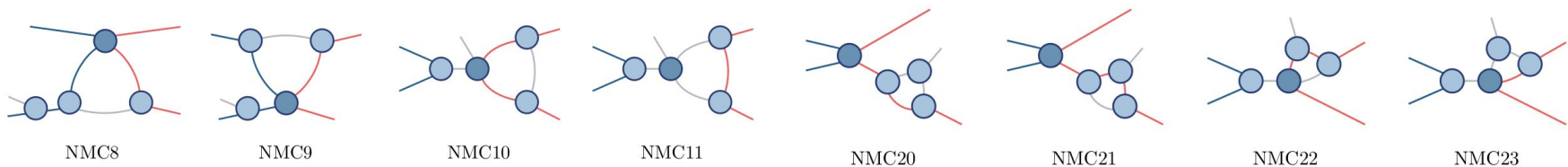
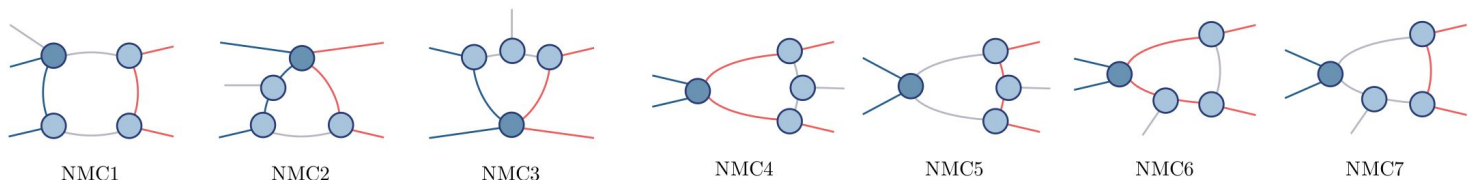
Projective double copy at loop-level

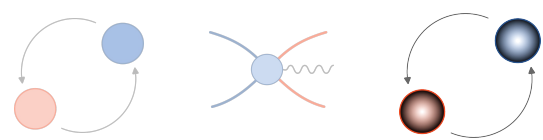




II. Gravity amplitude

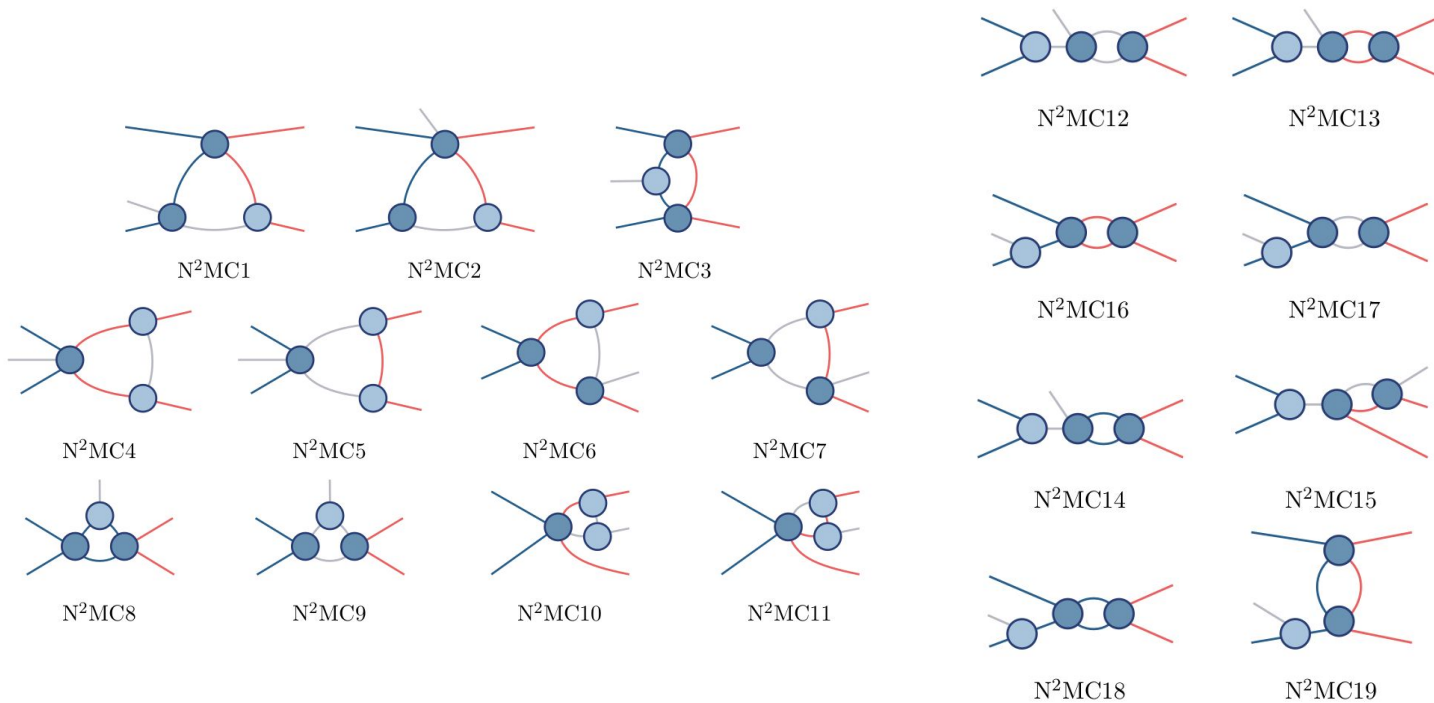
Projective double copy at loop-level

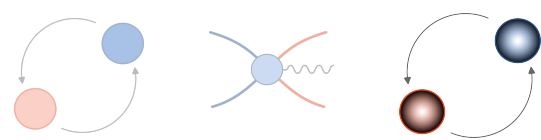




II. Gravity amplitude

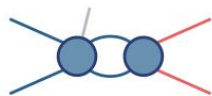
Projective double copy at loop-level



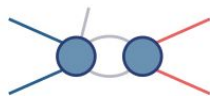


II. Gravity amplitude

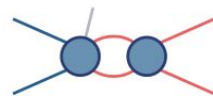
Projective double copy at loop-level



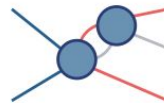
N^3MC1



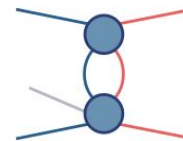
N^3MC2



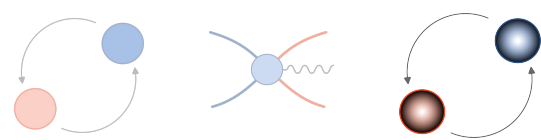
N^3MC3



N^3MC4



N^3MC5

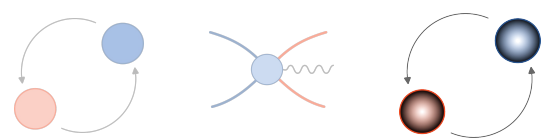


II. Gravity amplitude

Can we double copy **the dilaton**?

$$\left[\text{Diagram} \right] = (k_b \cdot k_d)^2 - \left((k_b \cdot k_d)^2 - \frac{m_1^2 m_2^2}{(D_s - 2)} \right)$$

$\sum_s \mathcal{M}_3^{\text{trees}}(a, b, q^s) \mathcal{M}_3^{\text{trees}}(-q^{\bar{s}}, c, d)$

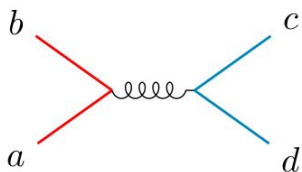


II. Gravity amplitude

Can we double copy **the dilaton**?

Yes!

Johansson, IVH (coming soon)

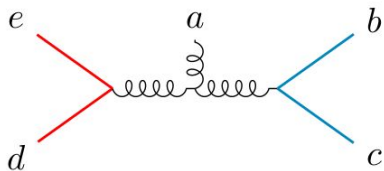


$$\mathcal{N} = 0$$

$$\frac{1}{4} (k_a \cdot k_b + 2 k_b \cdot k_c + m_1^2)^2$$

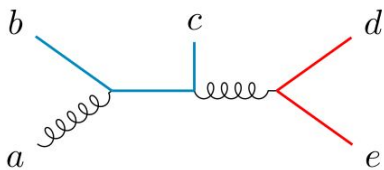


$$\frac{m_1^2 m_2^2}{D_s - 2}$$



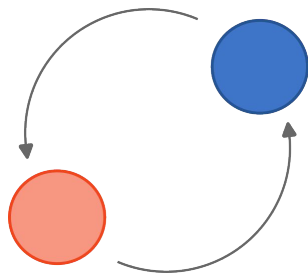
$$\frac{1}{16} \left[(k_{bc}^- \cdot k_1) (k_{de}^- \cdot \varepsilon_1) + 2 (k_c \cdot k_{de}^-) (k_b \cdot \varepsilon_a) - 2 (k_b \cdot k_{de}^-) (k_c \cdot \varepsilon_a) \right]^2$$

$$\frac{m_1^2 m_2^2}{D_s - 2} (k_{de} \cdot \varepsilon_a)^2$$



$$\frac{1}{16} \left[(k_{ce} \cdot k_{de}) (k_{de}^- \cdot \varepsilon_a) + 2 (k_c \cdot k_{de}^-) (k_b \cdot \varepsilon_a) \right]^2$$

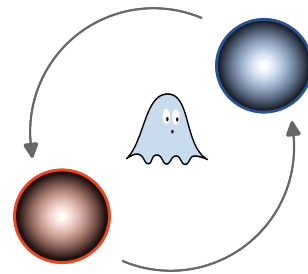
$$\frac{m_1^2 m_2^2}{D_s - 2} (k_b \cdot \varepsilon_a)^2$$



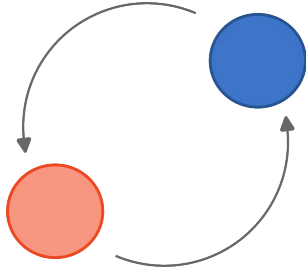
Express **radiation waveforms** in terms of scattering amplitudes

QCD amplitudes are bootstrapped using the color-kinematics duality

Gravity amplitudes are (modified) double copies of QCD amplitudes



What now?



Consider higher loops – waveform? Interesting physics?
Color-kinematics?

Spin?

Double copy dilatons?

