Classical radiation at one loop

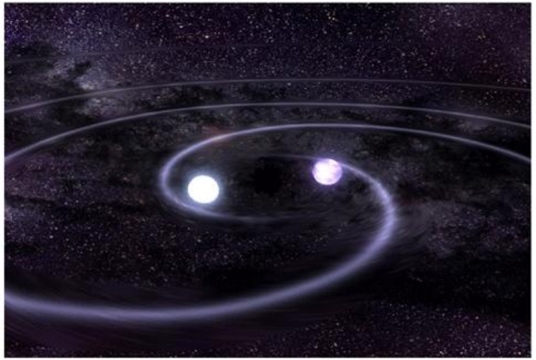
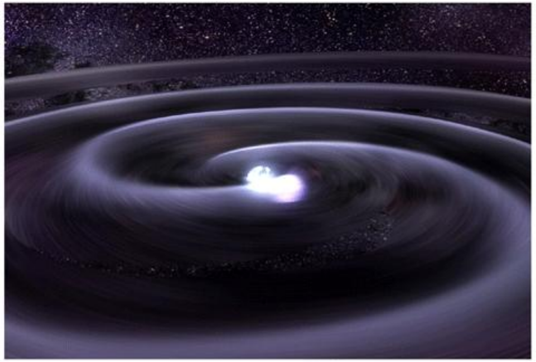
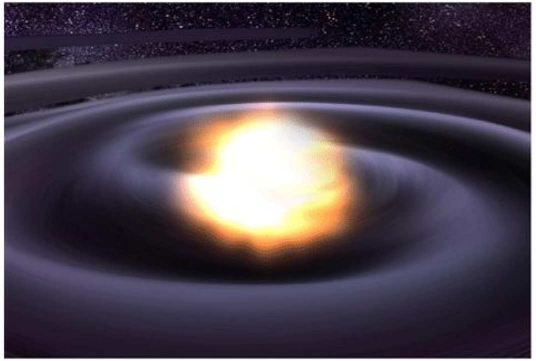
Ingrid A. Vazquez-Holm

Asaad Elkhidir, Donal O'Connell, Matteo Sergola
arXiv : 2303.06211

UPPSALA UNIVERSITET

NORDITA
The Nordic Institute for Theoretical Physics

Holonomic Techniques for Feynman Integrals,
Max Planck Institute,
17 October 2024

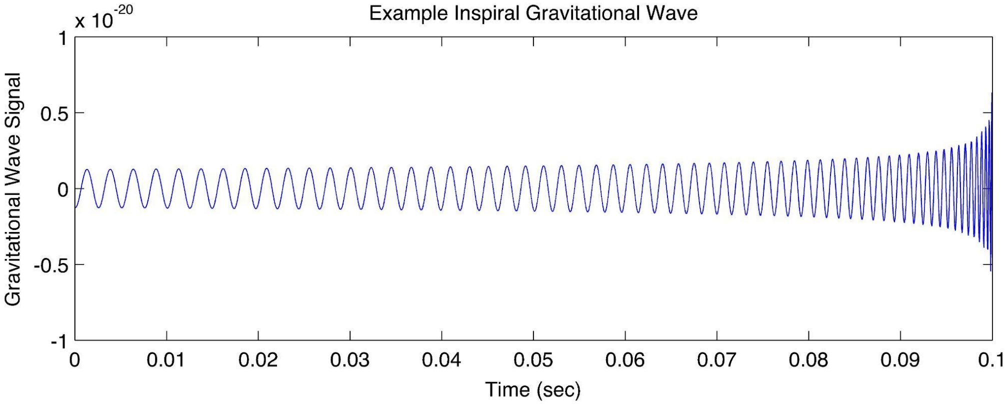




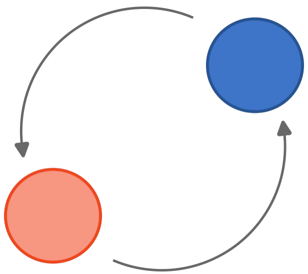
Elkhidir, O'Connell, Sergola, IVH

Herderschee, Roiban, Teng

Brandhuber, Brown, Chen
De Angelis, Gowdy, Travaglini

Georgoudis, Heissenberg, IVH

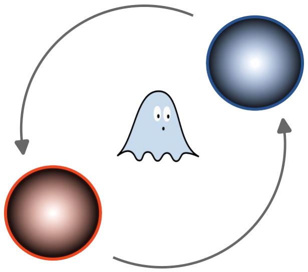




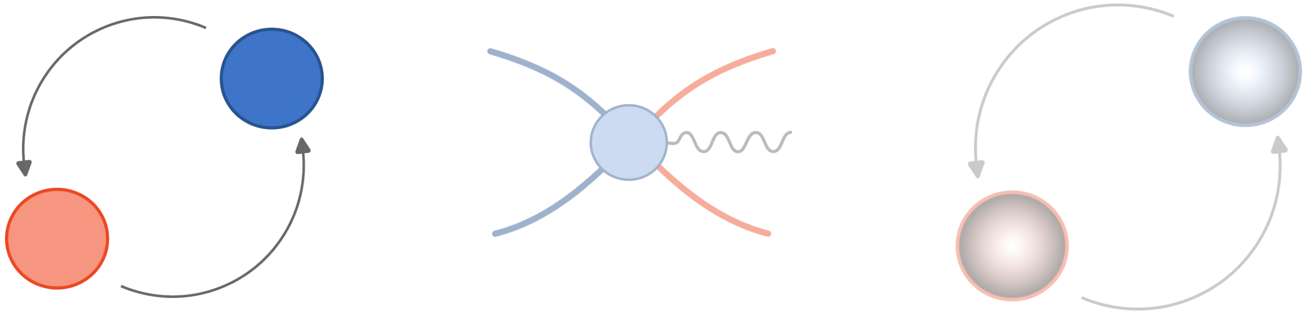
Express **radiation waveforms** in terms of scattering amplitudes

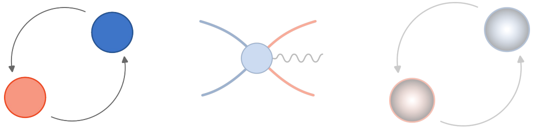
QCD amplitudes are bootstrapped using the color-kinematics duality

Gravity amplitudes are double copies of QCD amplitudes



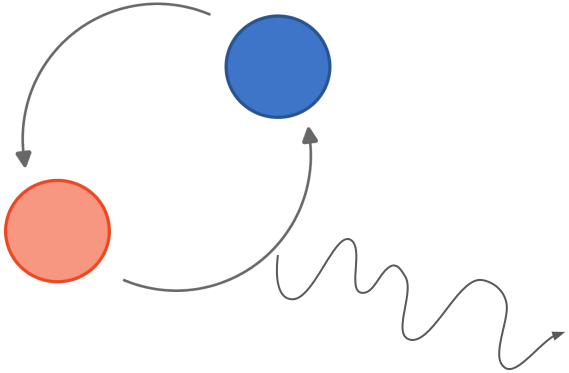
I. Radiation waveform





I. Radiation waveform

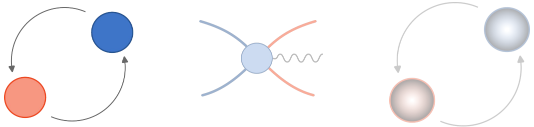
$\langle F_{\mu\nu} \rangle \sim$



Radiation waveforms are associated with the expectation value of the **field strength/Riemann tensor**

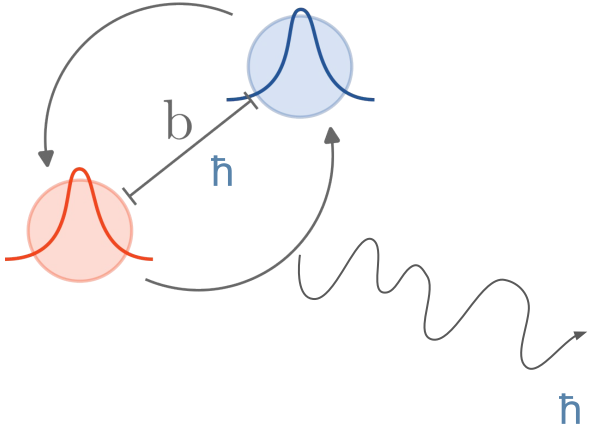
Define an initial state and the expectation in the far future

Treat classically, restore ħ



I. Radiation waveform

$\langle F_{\mu\nu} \rangle \sim$



Radiation waveforms are associated with the expectation value of the **field strength/Riemann tensor**

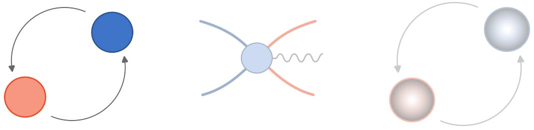
Define an initial state and the expectation in the far future

Treat classically, restore ħ

$$|\psi\rangle_{in} = \int d\tilde{k} \; e^{ib\cdot p_1/\hbar} |p_1 p_2\rangle_{in}$$

$$\langle \psi | \, S^\dagger F_{\mu\nu}(x) S \, |\psi\rangle$$

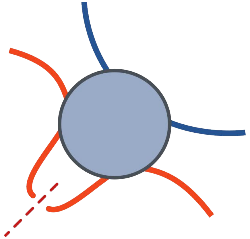
$$\langle p_1' p_2' | a_\eta(k) \, \mathrm{Re}\, T + \frac{i}{2} \left([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T] \right) |p_1, p_2\rangle$$



I. Radiation waveform

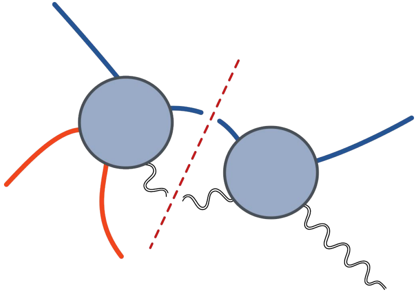
$$\langle p_1' p_2' | a_\eta(k)\, \mathrm{Re}\, T + \frac{i}{2}\left([a_\eta(k), T^\dagger]T - T^\dagger[a_\eta(k), T]\right) |p_1, p_2\rangle$$

Real

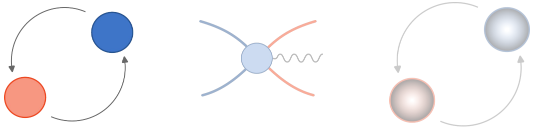


The real part is **radiation**, we use **1-particle cuts**

Imaginary



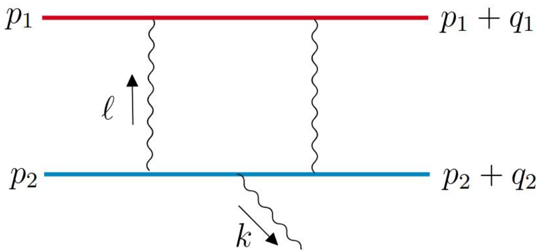
The imaginary part is **radiation reaction**, we use **2-particle cuts**



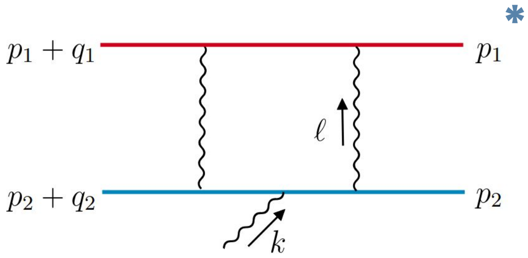
I. Radiation waveform

Real part is associated with radiation effects

Real = $\mathcal{A}_{(5,1)}(p_1, p_2 \to p_1', p_2', k_\eta) + \mathcal{A}^*_{(5,1)}(p_1', p_2', k_{-\eta} \to p_1, p_2)$



$= -g^5 \mathcal{C} \int \hat{\mathrm{d}}^D \ell \, N P.$

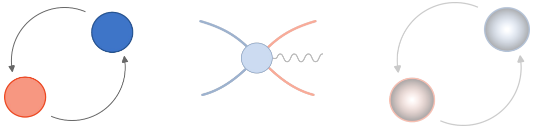


$= -ig^5 \mathcal{C} \int \hat{\mathrm{d}}^D \ell \, N P^*$

$$P^{-1} = (\ell^2 + i\epsilon)[(q_1 - \ell)^2 + i\epsilon][(p_1 + \ell)^2 - m_1^2 + i\epsilon][(p_2 - \ell)^2 - m_2^2 + i\epsilon][(p_2 - \ell - k)^2 - m_2^2 + i\epsilon]$$

$$-g^5 \mathcal{C} \int \hat{\mathrm{d}}^D \ell \, N \, [-i(P - P^*)] = -2g^5 \mathcal{C} \int \hat{\mathrm{d}}^D \ell \, N \, \mathrm{Im}\, P$$

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathrm{PV}\left(\frac{1}{p^2 - m^2}\right) - \frac{i}{2} \hat{\delta}(p^2 - m^2)$$



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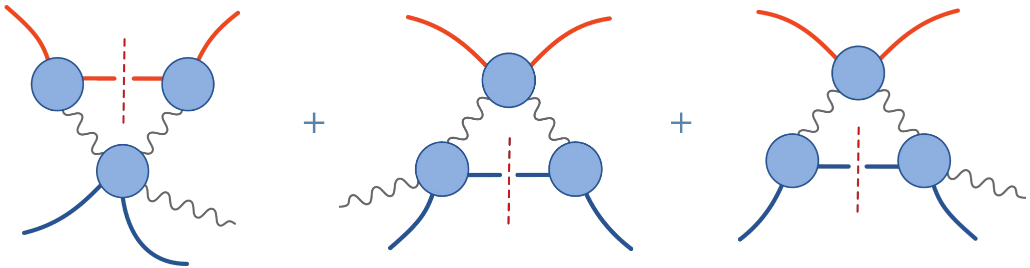
= 

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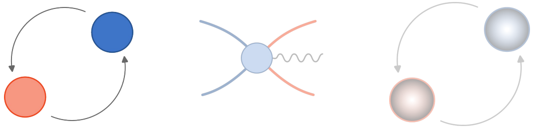
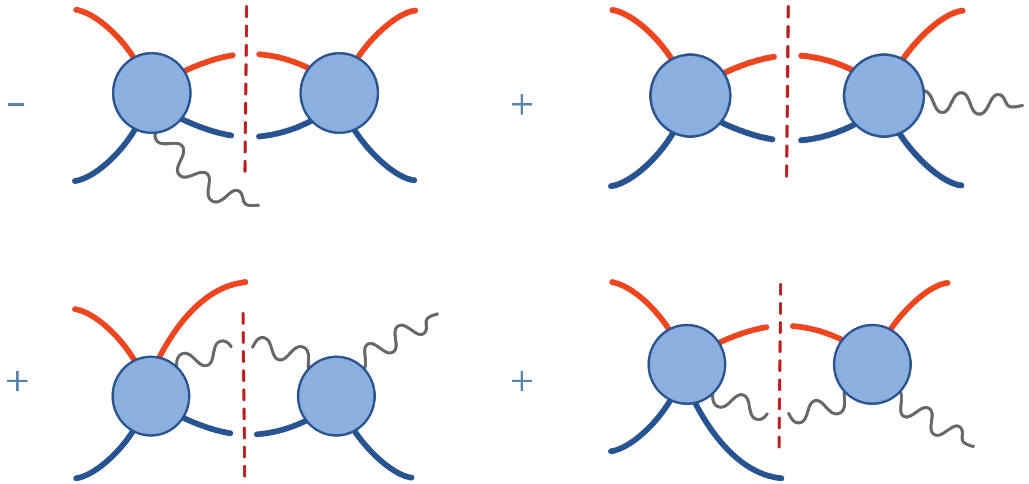


I. Radiation waveform

$$\langle p_1' p_2'| \frac{i}{2}\left([a_\eta(k), T^\dagger]T - T^\dagger[a_\eta(k),T]\right)|p_1,p_2\rangle$$

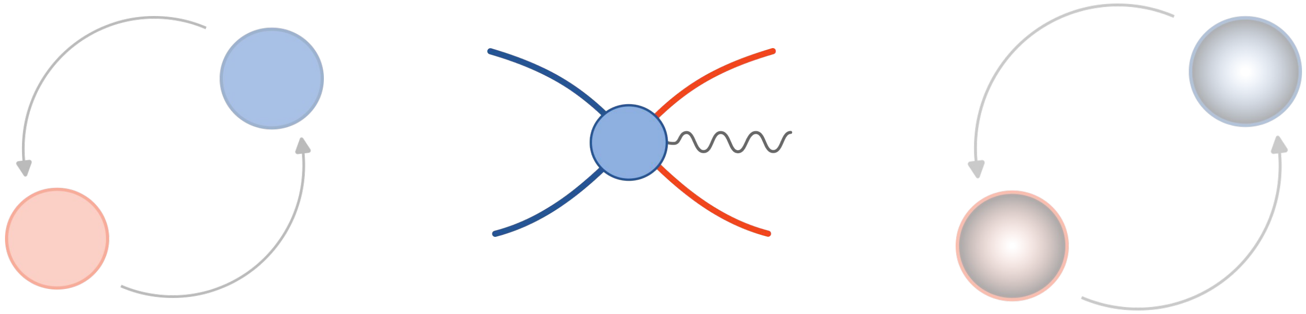
Imaginary part is associated with radiation reaction effects

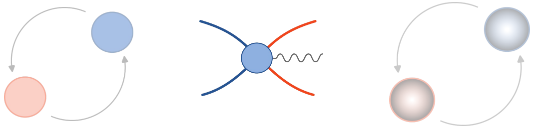
Imaginary =



Caron-Huot, Giroux, Hannesdottir, Mizera

II. Amplitudes



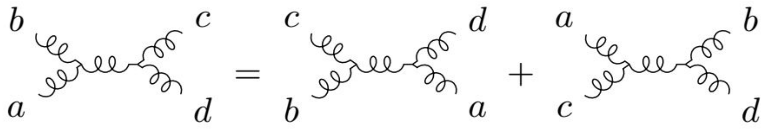


II. Amplitudes

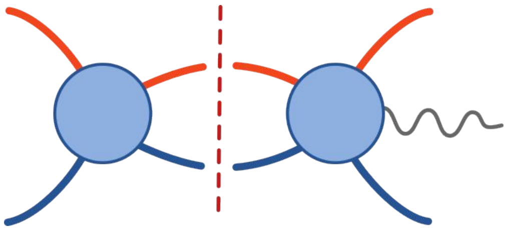
We bootstrap the amplitude (integrand) using **color-kinematics duality** and **unitarity cuts**

Bern, Carrasco, Johansson

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$



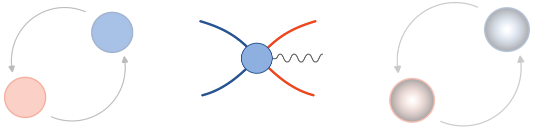
$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$



$$\frac{C_i n_i}{d_i} \rightarrow \frac{n_i n_j}{d_i}$$

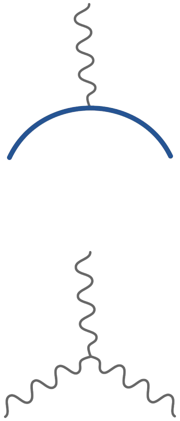
Gauge theory   Gravity

Gives us amplitudes we can **double copy**

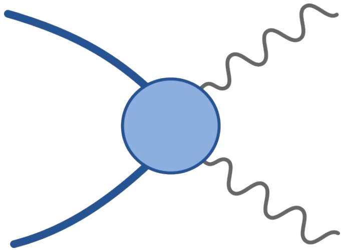


II. Amplitudes

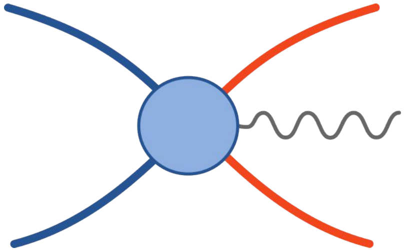
We bootstrap tree-level amplitudes using color-kinematics duality and factorisation



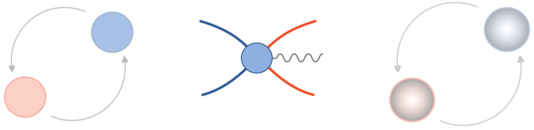
3-point



4-point

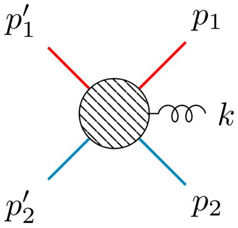


5-point

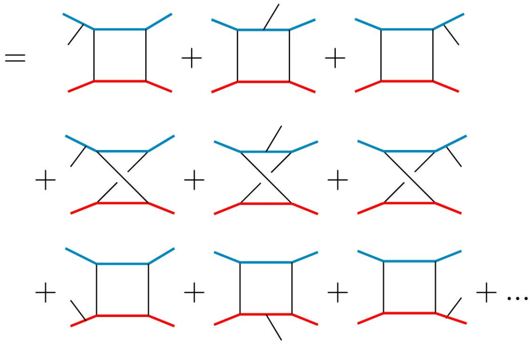


II. Amplitudes

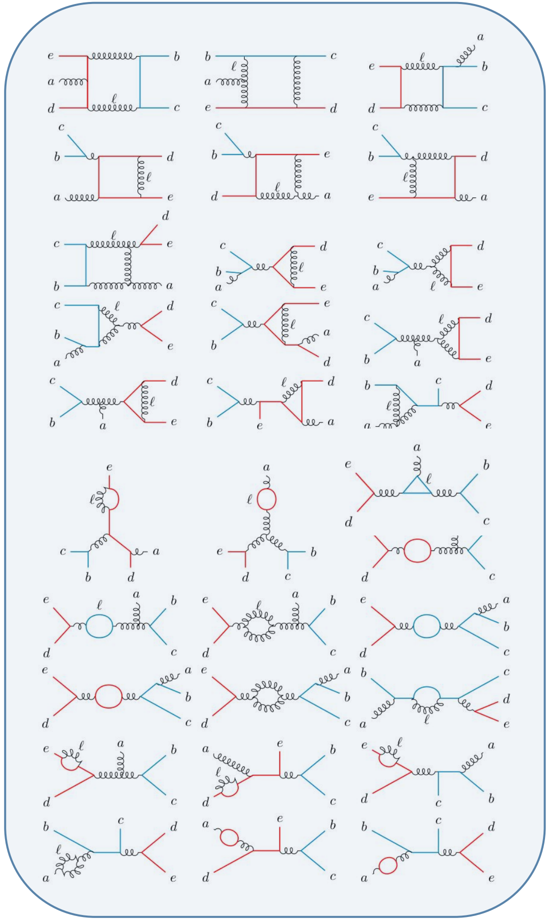
The 5-point one-loop amplitude has 116 graphs



$$= \int d^4\ell \sum_{g=1}^{116} \frac{n_g c_g}{d_g}$$



33 topologies



II. Amplitudes

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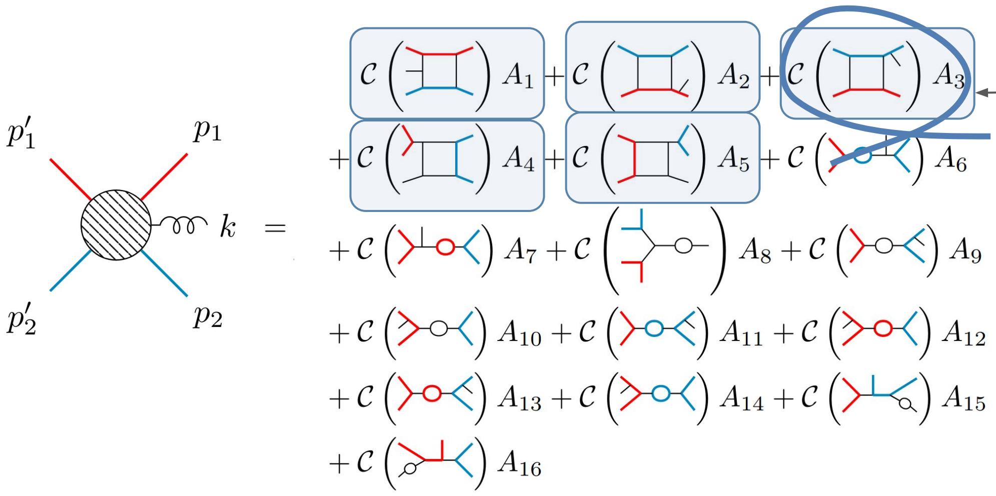
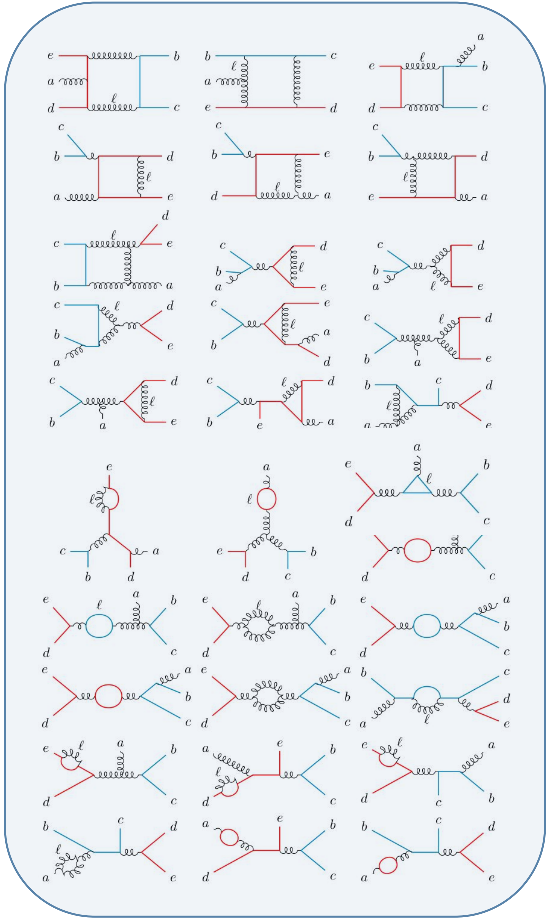


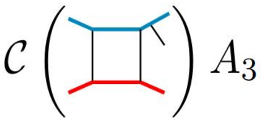
Look at A_3

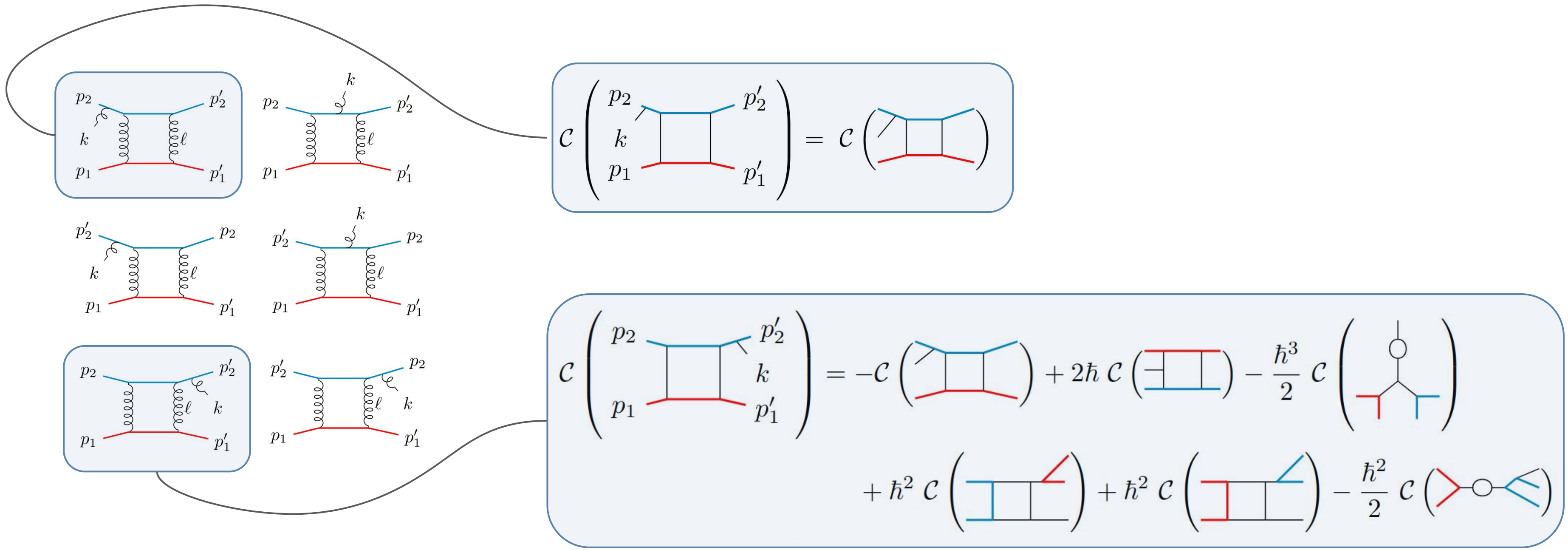
33 topologies

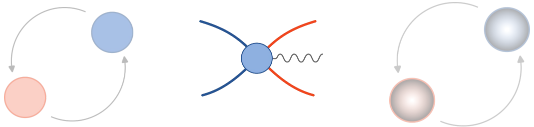


II. Amplitudes

Partial amplitude A_3



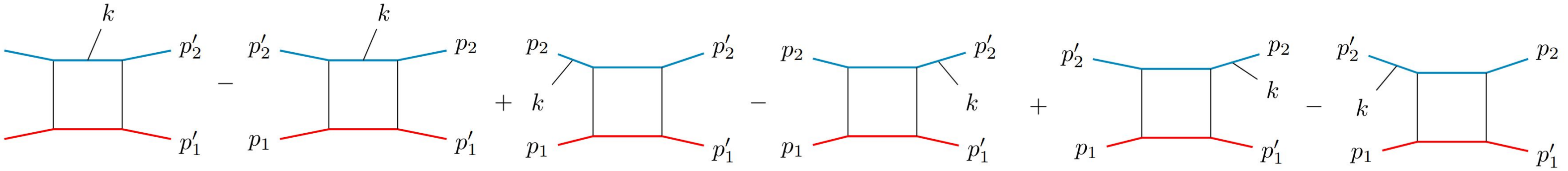




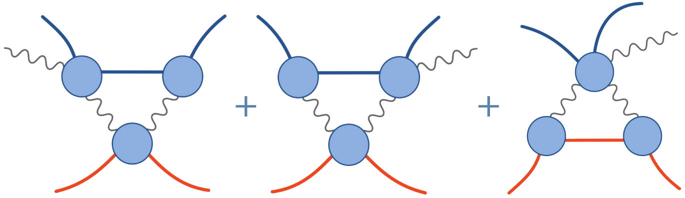
II. Amplitudes

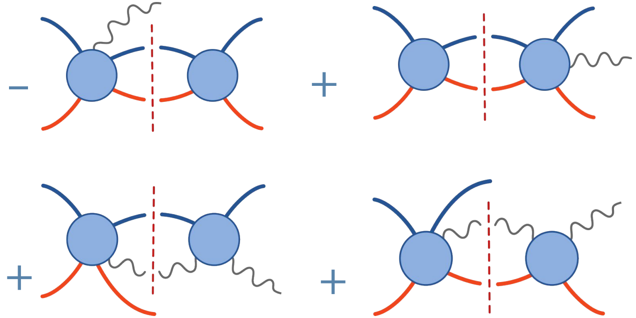
Partial amplitude A_3 $\mathcal{C}\Big($  $\Big) A_3$

(QED $Q_2^{\,3} Q_1^{\,2}$)

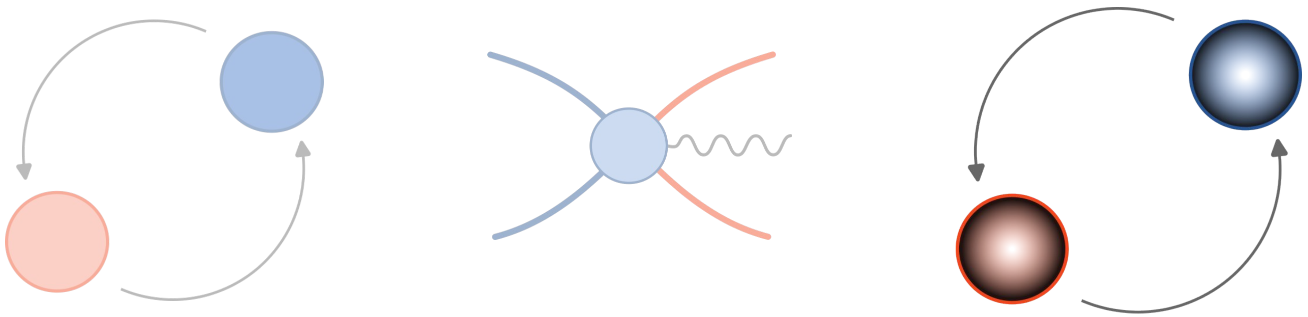


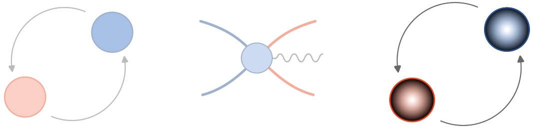
Now we easily find the radiation and reaction

Real = 

Im = 

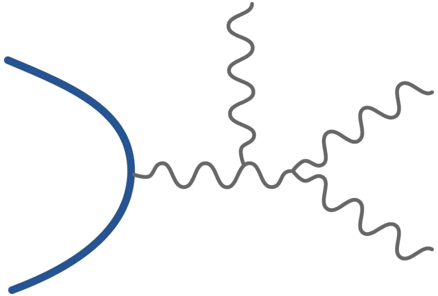
III. Gravity amplitude

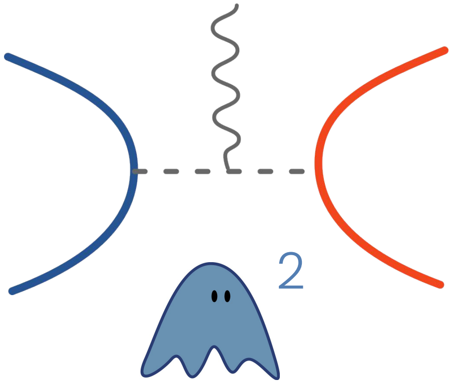




II. Gravity amplitude

The double copy gives extra massless states

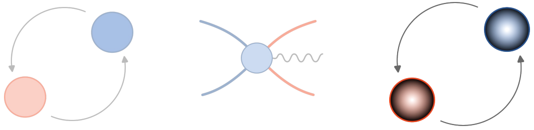




Is there a constructive approach?

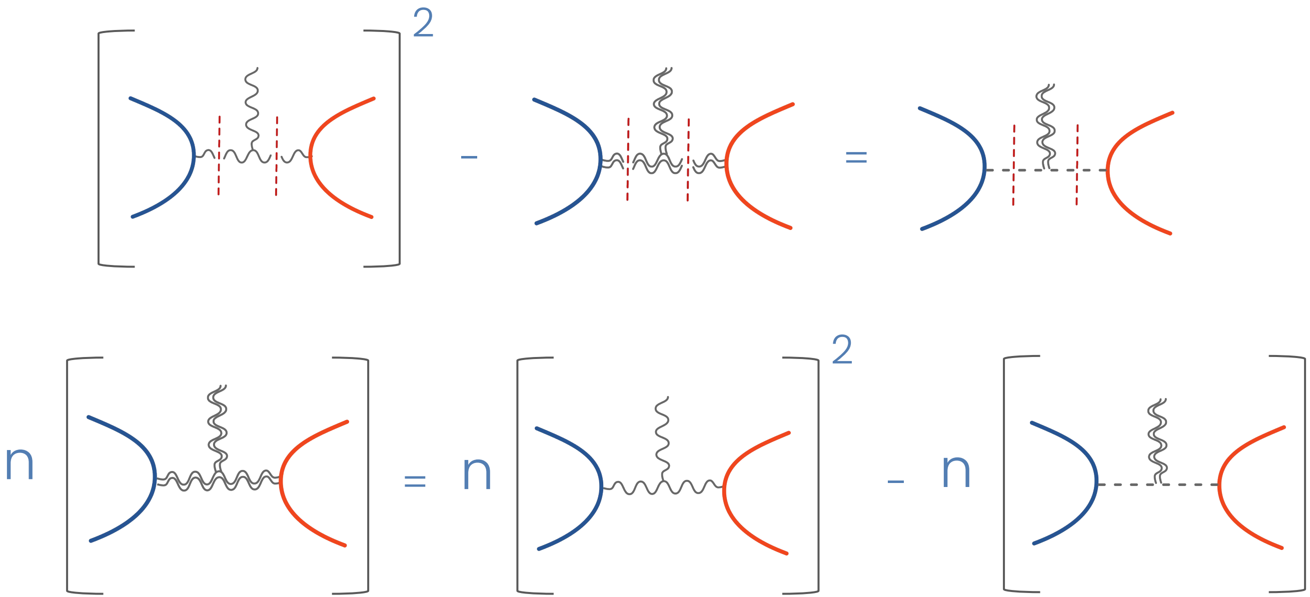
Is there a double copy approach?

Johansson, Ochirov ‘14
Luna, Nicholson, O’Connell, White ‘17



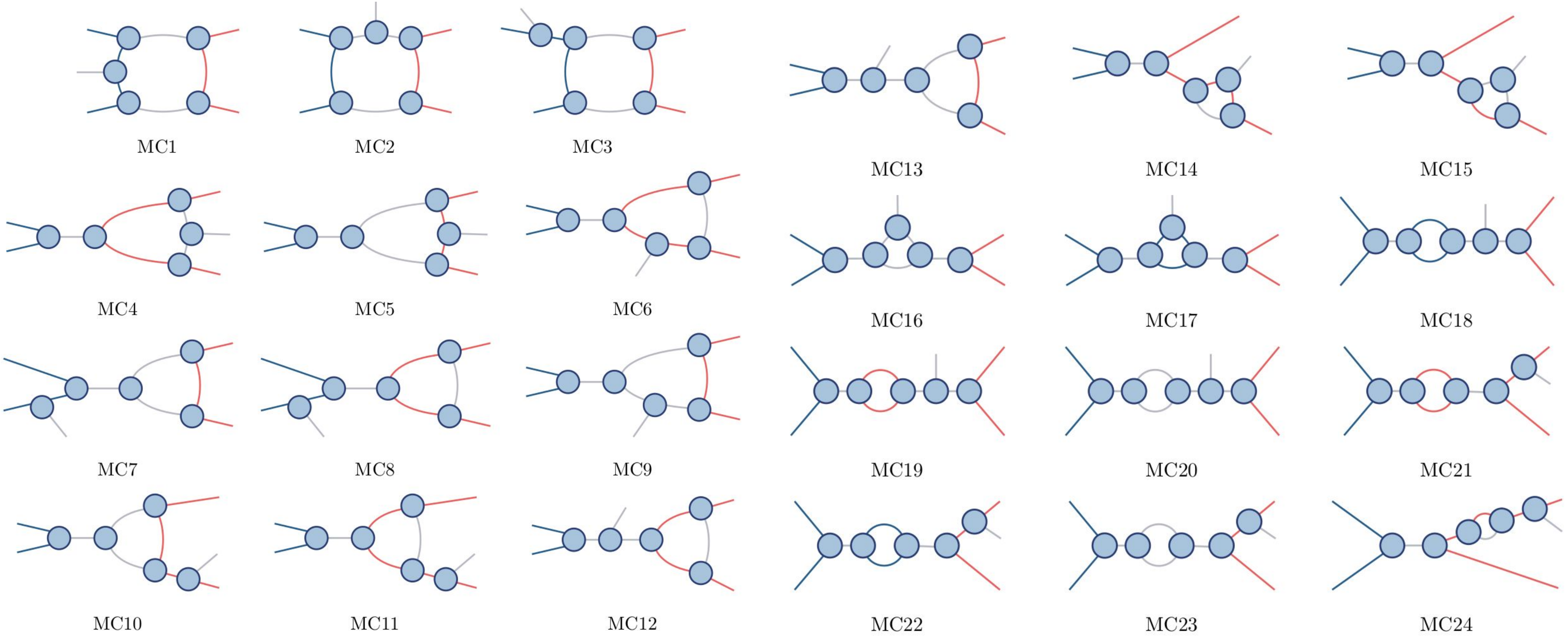
II. Gravity amplitude

Is there a constructive approach?



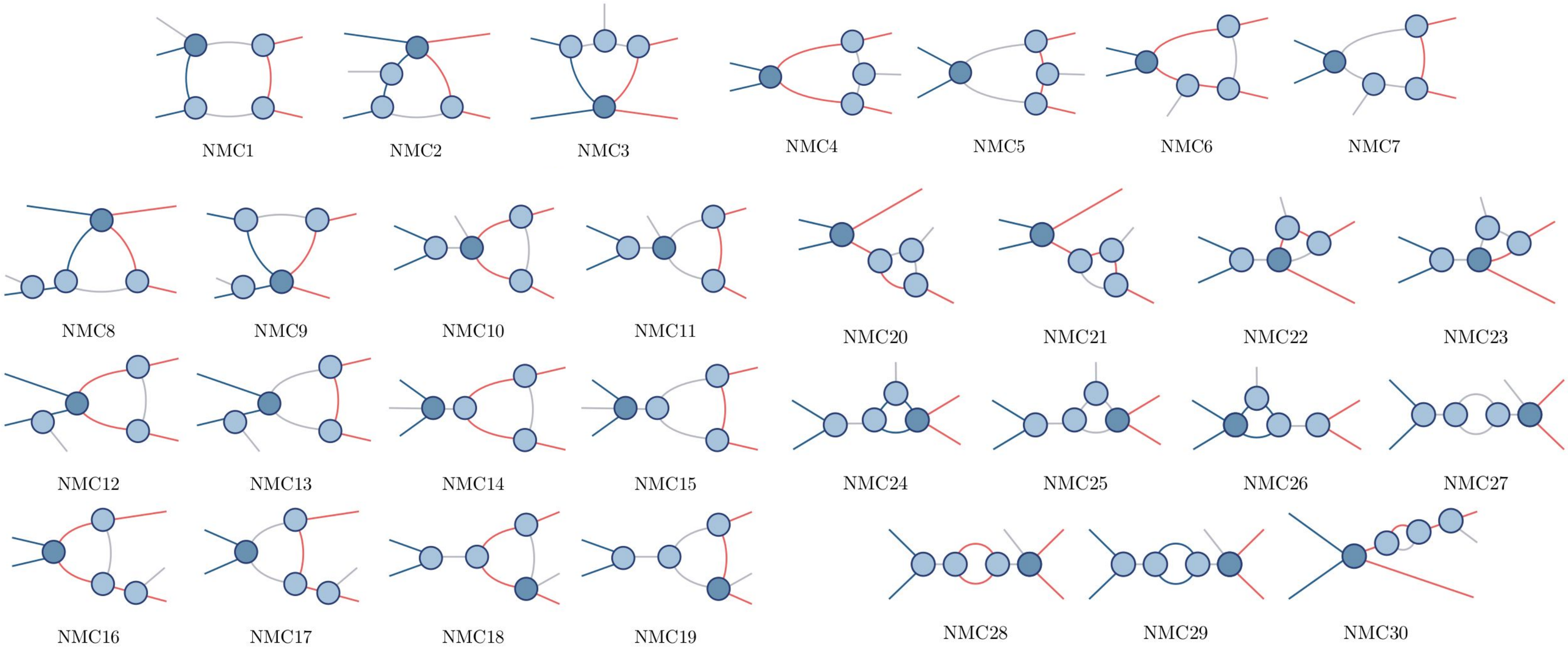
II. Gravity amplitude

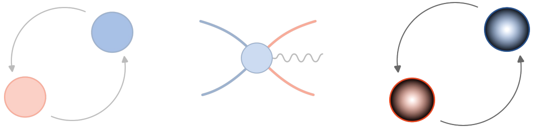
Projective double copy at loop-level



II. Gravity amplitude

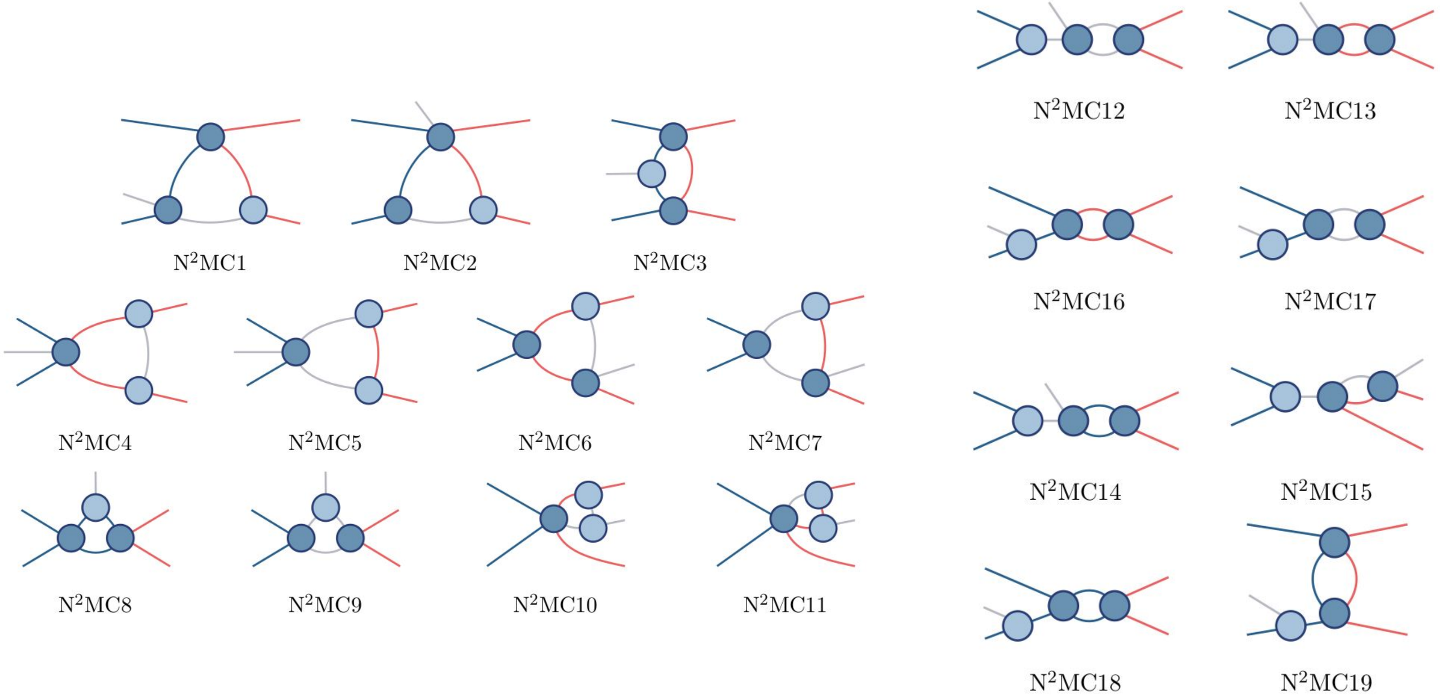
Projective double copy at loop-level

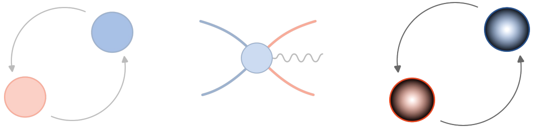




II. Gravity amplitude

Projective double copy at loop-level



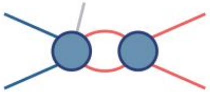


II. Gravity amplitude

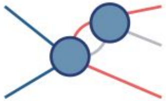
Projective double copy at loop-level

N^3MC1

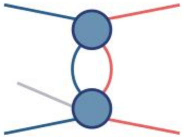
N^3MC2



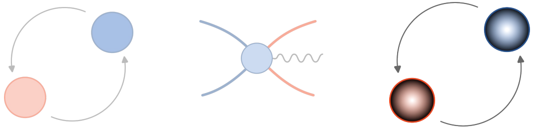
N^3MC3



N^3MC4

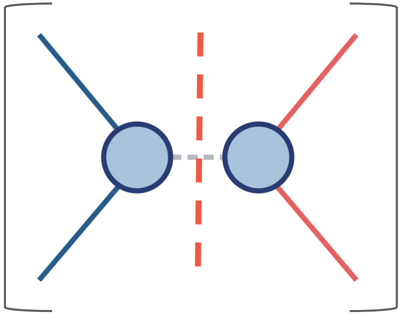


N^3MC5



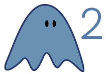
II. Gravity amplitude

Can we double copy **the dilaton**?

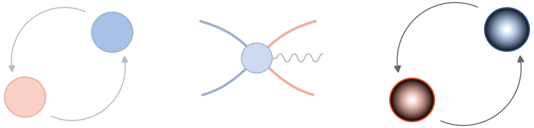
n 

$\mathcal{N} = 0$ | Einstein-Hilbert

$$= (k_b \cdot k_d)^2 - \left((k_b \cdot k_d)^2 - \frac{m_1^2 m_2^2}{(D_s - 2)} \right)$$



$$\sum_s \mathcal{M}_3^{\text{trees}}(a, b, q^s)\mathcal{M}_3^{\text{trees}}(-q^{\bar{s}}, c, d)$$

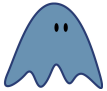


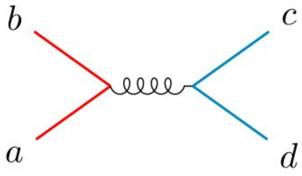
II. Gravity amplitude

Can we double copy **the dilaton**? Yes!

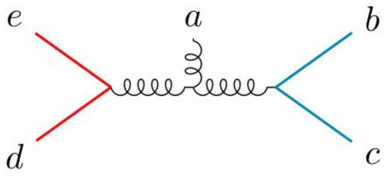
Johansson, IVH (coming soon)

$$\boxed{\mathcal{N}=0}$$

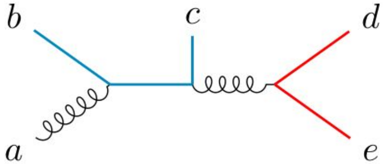




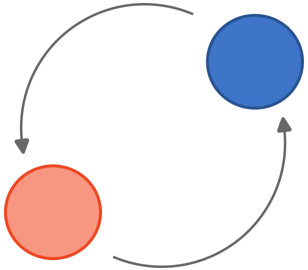
$$\frac{1}{4}(k_a\cdot k_b + 2\ k_b\cdot k_c + m_1^2)^2 \qquad \frac{m_1^2 m_2^2}{D_s-2}$$



$$\frac{1}{16}\Big[(k_{bc}^-\cdot k_1)(k_{de}^-\cdot\varepsilon_1) + 2(k_c\cdot k_{de}^-)(k_b\cdot\varepsilon_a) - 2(k_b\cdot k_{de}^-)(k_c\cdot\varepsilon_a)\Big]^2 \qquad \frac{m_1^2 m_2^2}{D_s-2}(k_{de}\cdot\varepsilon_a)^2$$



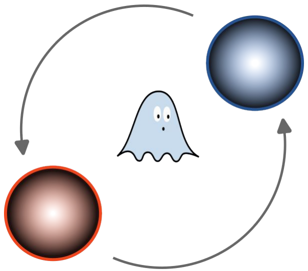
$$\frac{1}{16}\Big[(k_{ce}\cdot k_{de})\,(k_{de}^-\cdot\varepsilon_a) + 2\ (k_c\cdot k_{de}^-)(k_b\cdot\varepsilon_a)\Big]^2 \qquad \frac{m_1^2 m_2^2}{D_s-2}(k_b\cdot\varepsilon_a)^2$$



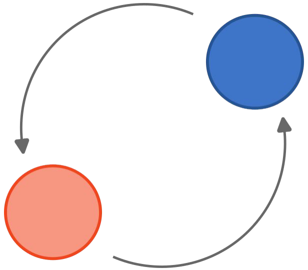
Express **radiation waveforms** in terms of scattering amplitudes

QCD amplitudes are bootstrapped using the color-kinematics duality

Gravity amplitudes are (modified) double copies of QCD amplitudes



What now?



Consider higher loops - waveform? Interesting physics?
Color-kinematics?

Spin?

Double copy dilatons?