Holonomic Techniques for Feynman Integrals

Report of Contributions

https://indico.mpp.mpg.de/e/10191

Contribution ID: **3** Type: **not specified**

Registration

Monday, 14 October 2024 08:30 (45 minutes)

Presenter: SCHOLTES, Sorana (Max-Planck-Institut für Physik)

Contribution ID: **4** Type: **not specified**

Welcome

Monday, 14 October 2024 09:15 (15 minutes)

Presenter: HENN, Johannes

Contribution ID: **5** Type: **not specified**

Parametric cycles

Monday, 14 October 2024 09:30 (45 minutes)

I will discuss integration contours for integrals over Feynman parameters that reveal discontinuities. I explore their variation in phase space, with a view towards deriving cut relations and understanding sequential discontinuities.

Presenter: BRITTO, Ruth (Trinity College Dublin)

Contribution ID: **6** Type: **not specified**

Coffee Break

Contribution ID: **7** Type: **not specified**

Effective homology and periods of algebraic varieties

Monday, 14 October 2024 11:00 (45 minutes)

The period matrix of a smooth complex projective variety encodes the isomorphism between its singular homology and its algebraic De Rham cohomology. Numerical approximations with sufficient precision of the entries of the period matrix allow to recover some algebraic invariants of the variety, such as the Néron-Severi group in the case of surfaces. In this talk, we will present a method relying on the computation of an effective description of the homology for obtaining such numerical approximations of periods of algebraic varieties, and showcase implementations for the case of hypersurfaces and elliptic surfaces.

Presenter: PICHON-PHARABOD, Eric

Contribution ID: **8** Type: **not specified**

Lunch Break

Contribution ID: **10** Type: **not specified**

Free Discussions

Monday, 14 October 2024 13:00 (45 minutes)

Contribution ID: **11** Type: **not specified**

Positive geometries and canonical forms via mixed Hodge theory

Monday, 14 October 2024 14:00 (45 minutes)

Prompted by Arkani-Hamed and Trnka's discovery of the amplituhedra, the concept of positive geometry recently emerged as an important tool in the study of scattering amplitudes and related quantities in physics. Roughly speaking, a positive geometry is a semi-algebraic domain whose boundary structure matches the residue structure of a unique logarithmic form, called its canonical form. The goal of this talk is to recast these notions as natural byproducts of Deligne's mixed Hodge theory, a central organizing principle in complex algebraic geometry which is intimately linked to the study of logarithmic forms and their residues.
This is joint work with Francis Brown.

Presenter: DUPONT, Clément

Contribution ID: **12** Type: **not specified**

Taming IBPs with Transverse Integration

Monday, 14 October 2024 15:00 (45 minutes)

Reduction of Feynman integrals to a basis of linearly independent master integrals is a crucial step in any perturbative calculation, but also one of its main bottlenecks. In this talk I will present an improvement over the traditional approach to IBP reduction, that exploits transverse integration identities. Given an integral family to be reduced, the key idea is to find sectors whose corner integrals correspond to either diagrams with fewer external legs or diagrams that can be factorized as products of lower-loop integrals. Then, using transverse integration identities, i.e. a tensor decomposition in the subspace that is transverse to the external momenta of the diagrams, we map integrals belonging to these sectors and their subsectors to (products of) integrals belonging to new and simpler integral families, characterized by either fewer generalized denominators, fewer external invariants, lower loops or combinations thereof. Integral reduction is thus drastically simpler for the newer families. I will include some applications to cutting-edge two-loop families which show significant improvements with respect to traditional methods.

Presenter: FONTANA, Gaia (University of Zürich)

Contribution ID: **13** Type: **not specified**

Moduli spaces of graphs

Monday, 14 October 2024 16:45 (45 minutes)

A natural setting for studying the n-loop contribution to a Feynman amplitude is the appropriate moduli space of graphs. I will describe these moduli spaces, then explain how methods from both geometric group theory and quantum field theory can be used to explore their structure.

Presenter: VOGTMANN, Karen (University of Warwick)

Contribution ID: **14** Type: **not specified**

Linear PDE with constant coefficients

Tuesday, 15 October 2024 09:30 (40 minutes)

We discuss practical methods for computing the space of solutions to an arbitrary homogeneous linear system of partial differential equations with constant coefficients. These rest on the Fundamental Principle of Ehrenpreis-Palamodov from the 1960's. Our audience will have a chance to gain hands-on experience with primary ideals and the schemes they represent.

Presenter: STURMFELS, Bernd

Contribution ID: **15** Type: **not specified**

Canonical differential equations for maximal cuts of hyperelliptic Feynman integrals

Tuesday, 15 October 2024 11:00 (45 minutes)

Feynman integrals are the building blocks of multi-loop scattering amplitudes and beyond one loop, many Feynman integrals are related to interesting geometries. In this talk, I will focus on Feynman integrals related to hyperelliptic curves and discuss ongoing work on finding canonical differential equations for maximal cuts of such Feynman integrals. This includes new ideas about the connection between the intersection matrix of a twisted cohomology group related to the maximal cut and the form of its differential equation.

Co-author: REDL, Julia

Presenter: PORKERT, Franziska (Universität Bonn)

Contribution ID: **16** Type: **not specified**

Relations in the differential equation for Feynman integrals

Tuesday, 15 October 2024 13:00 (45 minutes)

The differential equation for a system of Feynman integrals is encoded in a connection matrix. In this talk I will discuss that a specific choice of master integrals can lead to relations among the entries of the connection matrix. I will discuss self-duality and Galois symmetries.

Presenter: WEINZIERL, Stefan (Universität Mainz)

Contribution ID: **17** Type: **not specified**

Advancing diversity in math and physics

Tuesday, 15 October 2024 14:00 (45 minutes)

Presenters: STURMFELS, Bernd; LÓPEZ-FALCÓN, Diana; BRITTO, Ruth (Trinity College Dublin)

Contribution ID: **19** Type: **not specified**

A hypergeometric view on Landau singularity

Wednesday, 16 October 2024 09:30 (45 minutes)

Recent years have seen a resurgence of old topics in quantum field theory. One of them is Landau singularity, which should be defined as the singular locus of a Feynman integral. In this talk, we develop a way to understand and compute Landau singularity from the perspective of hypergeometric system, which is a special class of D-modules. This explains why Landau singularity is a natural generalization of principal A-determinant.

Presenter: MATSUBARA-HEO, Saiei

Contribution ID: **20** Type: **not specified**

Euler Discriminant of Complements of Hyperplanes

Wednesday, 16 October 2024 11:00 (45 minutes)

The Euler discriminant describes the locus of coefficients that cause a drop in the Euler characteristic of a very affine variety. In this talk, we focus on the case where the variety is the complement of hyperplanes. I will present formulas for two specific scenarios: when the coefficients are sparse and when they are restricted to a subspace of the parameter space. These formulas enable the computation of singularities in Euler integrals of linear forms, with applications in cosmology. This is joint work with Saiei Matsubara-Heo.

Presenter: FEVOLA, Claudia (Inria Saclay)

Contribution ID: **21** Type: **not specified**

Free Discussions

Wednesday, 16 October 2024 13:00 (45 minutes)

Contribution ID: **22** Type: **not specified**

IBP Reduction using Gröbner bases

Wednesday, 16 October 2024 14:00 (45 minutes)

Integral reduction based on integration-by-parts (IBP) identities are an indispensable tool for accomplishing higher-order calculations in perturbative quantum field theory. We approach this task from the point-of-view of algebraic geometry by solving the system of IBP equations symbolically. We formulate the problem as a non-commutative rational double-shift algebra Y and a left ideal that the IBP equations generate therein. For simple examples we compute the Gröbner basis of the left ideal in Y. For more complicated problems we compute so-called normal-form IBPs from a linear algebra ansatz, whose solution makes use of techniques from finite fields and rational reconstruction. Both, the Gröbner basis and the normal-form IBPs, allow for a fast and targeted reduction of integrals. We also report on open issues and ongoing developments, among which the extension of the algebra by including the dimensional shift operator. The presentation is based on arXiv2207.09275, arXiv2210.05347, and work in progress.

Presenter: HUBER, Tobias (Siegen U)

Contribution ID: **23** Type: **not specified**

A Survey on (un)Twisted Logarithmic Comparison Theorems

Wednesday, 16 October 2024 15:00 (45 minutes)

The comparison theorems of Grothendieck and Deligne tell us that to compute cohomology of the complement of a hypersurface with constant coefficients one must compute the cohomology of the meromorphic (or rational) de Rham complex. As this complex's objects lack finiteness, one wonders if a subcomplex of forms of order at most one along the hypersurface suffices. In verbiage: does the Logarithmic Comparison Theorem hold? We will survey the recent results on this problem, which, time permitting, may include: making the statement of the Logarithmic Comparison Theorem, and its twisted versions, explicit; verifying the (un)Twisted Logarithmic Comparison theorems for hyperplane arrangements, resolving a conjecture Terao from the 1970s; an explicit D-module theoretic formulation of the problem in the spirit of the Riemann Hilbert correspondence; applications to Bernstein—Sato polynomials; how to use the (un)Twisted Logarithmic Comparison theorem to turn computations into finite dimensional linear algebra.

Based on solo work by the speaker (arXiv: 2202.01462) and joint work with Morihiko Saito (arXiv: 2203.11716).

Presenter: BATH, Daniel (KU Leuven)

Contribution ID: **24** Type: **not specified**

Geometrizing Landau Analysis

Wednesday, 16 October 2024 16:45 (45 minutes)

In this talk, we will revisit Landau analysis for Feynman Integrals and their singularity study from a new geometrical viewpoint. The first part of the talk is about the foundation of our method. Through interpreting Landau loci by pinching of Schubert solutions, we will be able to uplift Landau singularities of an integral to its symbol letters automatically, and see our previous conjectural method for symbology study called Schubert analysis naturally arises. In the second part, we apply this method for studying alphabets of scattering amplitudes in N=4 Super Yang-Mills theory. Following amplituhedron picture and previous Landau diagrams study, we produce 2-loop MHV and NMHV amplitudes alphabet for all multiplicities nicely. As a byproduct conclusion, we also talk about cluster algebraic structures underlying the amplitude alphabets.

Presenter: YANG, Qinglin (Max Planck Institute for Physics)

Contribution ID: **25** Type: **not specified**

Triangulations of cosmological polytopes

Thursday, 17 October 2024 09:30 (45 minutes)

A cosmological polytope is defined for a given Feynman diagram, and its canonical form may be used to compute the contribution of the Feynman diagram to the wavefunction of certain cosmological models. Given a subdivision of a polytope, its canonical form is obtained as a sum of the canonical forms of the facets of the subdivision. The goal of this talk to report on specific types of triangulations of these polytopes, obtained via algebraic techniques. More precisely, we show that the toric ideal of every cosmological polytope a Gröbner basis with a squarefree initial ideal, yielding a regular unimodular triangulation of the polytope. In specific instances, including chains and cycles, we recover graphical characterizations of the facets of such triangulations that may be used to compute the desired canonical form. Choosing the term order wisely, for the chain, we find a triangulation that respects the graph symmetry and whose facets seem to be in bijection with certain graph tubings which used have been used by Arkani-Hamed, Baumann, Hillman, Joyce, Lee and Pimentel to use certain wavefunction coefficients.

Presenter: JUHNKE, Martina

Contribution ID: **26** Type: **not specified**

Completely monotone functions and applications

Thursday, 17 October 2024 11:00 (45 minutes)

Complete monotonicity of a smooth function on a convex cone is a strong property given by infinitely many sign conditions on the directional derivatives of the function. I will discuss results and questions around this concept that are motivated by research in convex optimization (interior-point methods), algebraic statistics (exponential families) and real algebraic geometry (hyperbolic and nonnegative polynomials).
The talk is based on joint works with M. Michalek, B. Sturmfels and J.-B. Lasserre.

Presenter: KOZHASOV, Khazhgali (Université Côté d'Azur)

Contribution ID: **27** Type: **not specified**

Free Discussions

Thursday, 17 October 2024 13:00 (45 minutes)

Contribution ID: **28** Type: **not specified**

Classical radiation at one loop

Thursday, 17 October 2024 14:00 (45 minutes)

We study classical radiation fields at next-to-leading order using the methods of scattering amplitudes. The fields of interest to us are sourced when two massive, point-like objects scatter inelastically, and can be computed from one-loop amplitudes. The real and imaginary parts of the amplitudes play important but physically distinct roles in the radiation field. We argue that the imaginary part captures the effects of radiation reaction. This aspect of radiation reaction is directly linked to cuts of one-loop amplitudes which expose Compton trees.

Presenter: VAZQUEZ-HOLM, Ingrid

Contribution ID: **29** Type: **not specified**

Symbol Alphabets from the Landau Singular Locus

Thursday, 17 October 2024 15:00 (45 minutes)

I present work which provides evidence through two loops that rational letters of polylogarithmic Feynman integrals are captured by the Landau equations, when the latter are recast as a polynomial of the kinematic variables of the integral, known as the principal A-determinant. Focusing on one loop, I further discuss how all square-root letters may also be obtained, by re-factorizing the principal A-determinant with the help of Jacobi identities. The letters are verified by explicitly constructing canonical differential equations for the one-loop integrals in both odd and even dimensions of loop momenta.

Presenter: DLAPA, Christoph (DESY)

Contribution ID: **30** Type: **not specified**

Symbolic summation and integration techniques to simplify Feynman integrals

Thursday, 17 October 2024 16:45 (45 minutes)

We present tools from symbolic summation and integration that are tailored for Feynman integrals. In particular, we will present algorithms that enable one to produce linear differential or difference equations that contain the input expression (e.g., in form of definite hypergeometric multisums, hyperexpontial multiintegrals or coupled systems of linear differential equations) as solution. In a nutshell, we show constructively that the given input expressions of Feynman integrals are holonomic. Given such equations one can then look for closed form solutions, e.g., in terms of iterative integrals and sums (which are again holonomic). This leads usually to rather compact expressions that can be used for asymptotic expansions or analytic continuation.
We will report on these different tools that we have applied within our long-term cooperation between RISC and DESY (Deutsches Elektronen-Synchrotron) and demonstrate their flexibility.

Presenter: SCHNEIDER, Carsten

Contribution ID: **31** Type: **not specified**

The arithmetic of resurgent topological strings

Friday, 18 October 2024 09:30 (45 minutes)

Factorially divergent power series naturally arise as perturbative expansions in quantum theories but do not uniquely determine the original functions due to hidden non-analytic terms. In favourable circumstances, these terms can be systematically understood within the framework of resurgence. Growing evidence indicates that this is the case for topological string theory. In this talk, I will discuss the resurgence of the strong and weak coupling limits of the spectral traces of a toric Calabi-Yau threefold, which are captured by the free energy of the refined topological string on the same background. In the case of the spectral trace of local P^2, a remarkable arithmetic structure unfolds, revealing an exact strong-weak resurgent symmetry exchanging the perturbative/non-perturbative sectors in the dual regimes. Guided by this example, I will propose a new perspective on the resurgence of particular formal power series, which are conjectured to possess specific summability and quantum modularity properties, leading us to introduce the general paradigm of modular resurgence. This talk is based on arXiv:2212.10606, 2404.10695, and 2404.11550.

Presenter: RELLA, Claudia (Institut des Hautes Études Scientifiques, Université Paris-Saclay)

Contribution ID: **32** Type: **not specified**

Single-valued Integration on the Elliptic Curve: Monodromy Relations and Twisted (Co)homology

Friday, 18 October 2024 11:00 (45 minutes)

Primary author: STIEBERGER, Stephan

Presenter: STIEBERGER, Stephan

Contribution ID: **33** Type: **not specified**

Free Discussions

Friday, 18 October 2024 13:00 (45 minutes)

Contribution ID: **34** Type: **not specified**

Non-perturbative Methods for Planar N=4 SYM at Conformal Point

Friday, 18 October 2024 14:00 (45 minutes)

In this pedagogical talk, we discuss non-perturbative methods to study planar N=4 SYM theory. Focusing on the four-point correlation function of the stress-energy tensor at the conformal point, we show how sum rules based on dispersion relations can be used to numerically bootstrap various objects in the theory, such as OPE coefficients, the four-point correlation function, and the energy-energy correlator. We show, for the first time, rigorous non-perturbative results for the planar OPE coefficients of single-trace operators, as well as the correlation function at various points in cross-ratio space. Additionally, focusing on the energy-energy correlator (EEC), we present rigorous bounds for its spin-2 and spin-3 Legendre coefficients, as well as the full EEC function at various angles. These results were obtained for a wide range of 't Hooft couplings, highlighting the power of the bootstrap in probing the non-perturbative aspects of planar N=4 SYM theory.

Presenter: ZAHRAEE, Zahra

Contribution ID: **35** Type: **not specified**

Closing Remarks

Friday, 18 October 2024 15:00 (15 minutes)

Presenter: SATTELBERGER, Anna-Laura (Max Planck Institute for Mathematics in the Sciences, Leipzig)

Contribution ID: **38** Type: **not specified**

Poster Session

Presenters: HASHEMI, Baran; RODRIGUEZ, Carlos; TELLANDER, Felix; BROCHET, Hadrien; SINGH, Kajal; BAUNE, Konstantin; BALDUF, Paul-Hermann; DITSCH, Sara (MPP)

Contribution ID: **39** Type: **not specified**

Statistics and correlations of primitive Feynman integrals

Tuesday, 15 October 2024 15:00 (15 minutes)

A Feynman integral is called "primitive" if it is superficially divergent and does not contain subdivergences. The "period" of a primitive graph is the coefficient of logarithmic energy dependence, or equivalently the simple pole in minimal subtraction. In recent work [2305.13506, 2403.16217] together with Kimia Shaban, we numerically computed the periods of 2 million Feynman integrals in phi^4 theory up to 18 loops. This allows us to examine their distribution, various statistical features, and the correlation between the value of the period and properties of the underlying Feynman graph. We show proof-of-concept results how those correlations can be used in a weighted Monte Carlo sampling algorithm to compute the sum of periods (which constitutes the primitive contribution to the beta function) very efficiently.

Primary author: BALDUF, Paul-Hermann (University of Oxford)

Presenter: BALDUF, Paul-Hermann (University of Oxford)

Session Classification: Poster Session

Contribution ID: **40** Type: **not specified**

Polylogarithms for all genera: numerics and identities

Tuesday, 15 October 2024 15:15 (15 minutes)

Polylogarithms on higher-genus Riemann surfaces are necessary for systematic calculations of certain Feynman integrals and loop amplitudes in string theory. Employing the Schottky uniformization of a Riemann surface we construct higher-genus generating functions of polylogarithmic integration kernels, coinciding with the set of meromorphic differentials defined by Enriquez. This allows for numerical evaluation of hyperelliptic polylogarithms, which we can relate to sums of iterated integrals over elliptic integration kernels.
In a second part, we investigate functional relations between higher-genus polylogarithms, which rely on identities for the integration kernels. We derive identities for generating series of Enriquez' meromorphic integration kernels, generalizing the genus-one Fay identities, and show that our set of three-point identities is exhaustive.
(Based on arXiv:2406.10051 and arXiv:2409.08208)

Primary author: BAUNE, Konstantin (ETH Zürich)

Presenter: BAUNE, Konstantin (ETH Zürich)

Session Classification: Poster Session

Contribution ID: **41** Type: **not specified**

Faster Integration of D-module

Tuesday, 15 October 2024 15:30 (15 minutes)

We consider the problem of computing all the linear relations between the integrals of the functions lying in a given holonomic D-module. We present in this poster a new integration algorithm designed for handling multiple integrals of holonomic functions. This novel algorithm can be regarded as both an extension of Lairez's reduction-based algorithm, which is limited to integrals of rational functions, and an enhancement of Takayama's algorithm based on D-module theory.
This is a joint work with my two PhD advisors Frédéric Chyzak and Pierre Lairez.

Primary author: BROCHET, Hadrien (Inria Saclay)

Presenter: BROCHET, Hadrien (Inria Saclay)

Session Classification: Poster Session

Contribution ID: **42** Type: **not specified**

Sara Ditsch

Presenter: DITSCH, Sara (MPP)

Session Classification: Poster Session

Contribution ID: **43** Type: **not specified**

D-modules and Griffiths theorem for Feynman Integrals

Tuesday, 15 October 2024 15:45 (15 minutes)

We investigate the D-module structure of Feynman integrals and Euler-Mellin integrals by means of Griffiths theorem. We present first applications to special mathematical functions and one-and two-loop integrals, and discuss the generation of corresponding Pfaffian equation they obey, via Macaulay matrix method.

Primary author: FLIEGER, Wojciech (University of Padova)

Presenter: FLIEGER, Wojciech (University of Padova)

Session Classification: Poster Session

Contribution ID: **44** Type: **not specified**

Can AI Perform Enumerative Geometry?

Tuesday, 15 October 2024 16:00 (15 minutes)

What happens when we let artificial intelligence tackle mathematical problems? This work explores how Transformer neural networks—initially designed for language processing—can learn and perform tasks in computational Algebraic Geometry. As a result, we introduce a neural network model that approximates psi-class intersection numbers on the moduli space of curves. Through our analysis, we discover that the network is independently learning Virasoro constraints among the intersections, meaning that the network is not only computing but revealing the hidden structures that generate them. We also explore the network's internal reasoning and find that it can assist in forming new mathematical conjectures. By interpreting its Causal "thought process" and abductive knowledge discovery, we investigate how it encodes information about the asymptotic behaviour of the intersection of psi-classes, offering insights that could guide human intuition. Our findings advocate an exciting possibility: AI could become collaborators in research-level Mathematics, providing data-driven hints and evidence for relationships that mathematicians suspect but haven't yet proven.

Primary author: HASHEMI, Baran (Origins Cluster)

Presenter: HASHEMI, Baran (Origins Cluster)

Session Classification: Poster Session

Contribution ID: **45** Type: **not specified**

Beyond Riemann-Wirtinger integrals

Tuesday, 15 October 2024 16:15 (15 minutes)

We study families of hypergeometric integrals given by one-dimensional integrals over a genus-g Riemann surface. These integrals are closely related to string amplitudes at loop-level in the chiral splitting formalism. Watanabe has studied the twisted cohomology of these integrals, and here we explore their twisted homology, with a goal of understanding the double copy at genus g > 1.

Presenter: RODRIGUEZ, Carlos

Session Classification: Poster Session

Contribution ID: **46** Type: **not specified**

Loops, Recursions, and Soft Limits for Fermionic Correlators in (A)dS

Tuesday, 15 October 2024 16:30 (15 minutes)

Study of correlation functions in AdS/CFT and in-in correlators in de Sitter space often requires the computation of Witten diagrams. Due to the complexity of evaluating radial integrals for these correlators, several indirect approaches have been developed to simplify computations. However, in momentum space, these methods have been limited to fields with integer spin. Here, we formulate tools for evaluating Witten diagrams with spin–1/2 fields in momentum space and discuss where they differ from the corresponding integer-spin analysis. We formulate our tools explicitly for massless fermions and present how appropriate Weight shifting operators with respect to the external kinematics can be used to obtain the generalization to fermions with integer mass. Further, we classify the nature of IR divergences encountered for interacting massive scalars and fermions. We also prove a novel Weinberg-like soft theorem for gauge fields coupled to matter in AdS and show that the universal terms in the leading soft factor are sensitive to the spin of the matter field. These generalize the recently discovered soft theorems for pure Yang-Mills to Yang-Mills with matter.

Primary author: SINGH, Kajal (Department of Mathematical Sciences, University of Liverpool)

Presenter: SINGH, Kajal (Department of Mathematical Sciences, University of Liverpool)

Session Classification: Poster Session

Contribution ID: **47** Type: **not specified**

Spectral Analysis of the Feynman Integral: Stratifying Wave Fronts and Landau Singularities

Tuesday, 15 October 2024 16:45 (15 minutes)

I will present a method to calculate the Landau singularities of a Feynman integrals using Whitney stratifications. Whitney stratifications themselves are microlocal in nature, meaning that they naturally do not only live in the space we are stratifying but in the cotangent bundle of this space. With this in mind I introduce a microlocal (or rather distributional) framework for Feynman integrals where singularities are now described by wave front sets and characteristic varieties; these sets not only describe where a distribution is singular but also the Fourier components causing them. The role of Whitney stratifications will be stressed throughout.

Primary author: TELLANDER, Felix (University of Oxford)

Presenter: TELLANDER, Felix (University of Oxford)

Session Classification: Poster Session