Scattering Amplitudes from Positive Geometry

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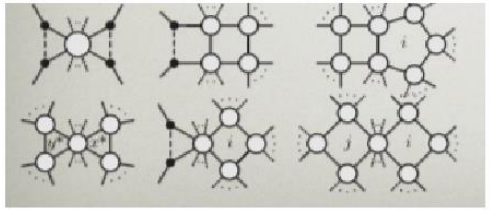
from Quantum Field Theory Group

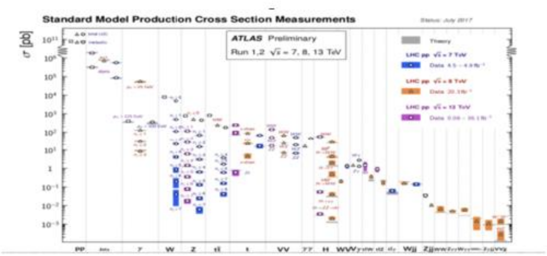
09.12

Scattering amplitudes study nowadays

New underlying properties for field theory



Formal QFT

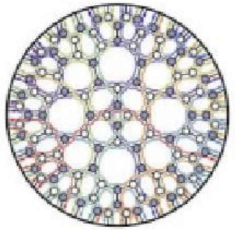


Mathematics

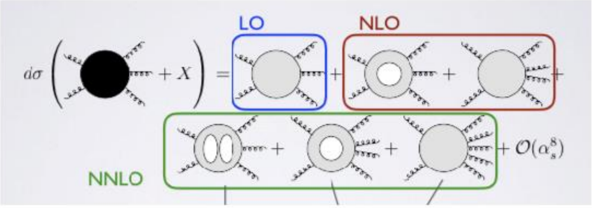
Amplitudes

Collider Phenomenology

precision frontier

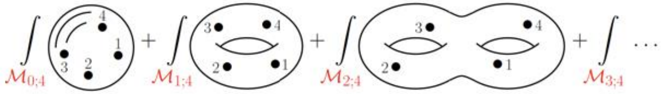


geometries, combinatorics, number theory ...



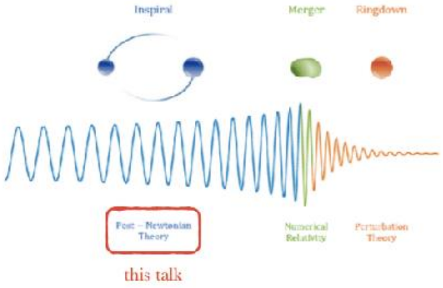
String Theory

Gravity



$$\mathcal{A}^{g\text{-loop}}_{\text{string}}(1,2,\ldots,n) \sim \int_{\mathcal{M}_{g;n}} \Big\langle \big(\prod_j \mathrm{PCO}(w_j)\big)\, V_1(z_1)\, V_2(z_2)\, \ldots\, V_n(z_n) \Big\rangle_g$$

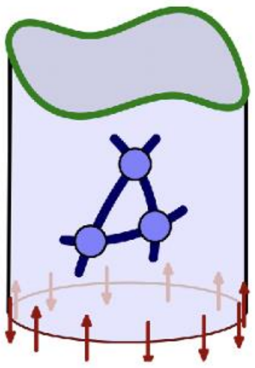
String-inspired structures in field theory

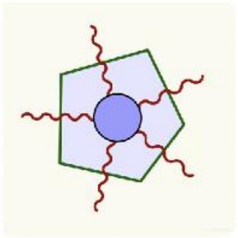


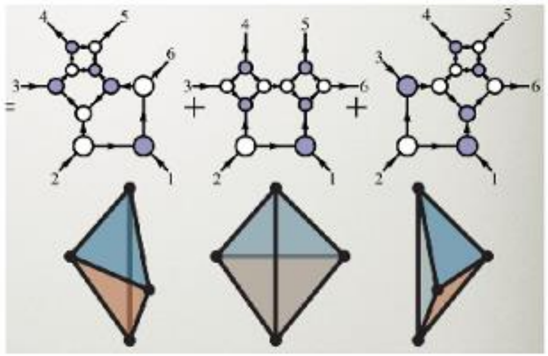
Black hole scattering and gravitational wave

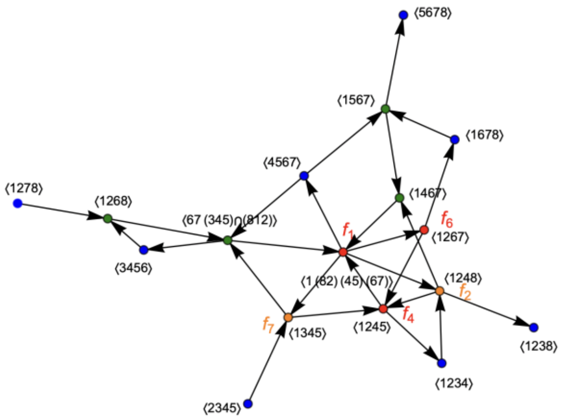
The simplest 4D QFT [Arkani-Hamed, Cachazo, Kaplan 08’]

- **4D (planar) N=4 Super Yang-Mills Theory**
- Fruitful playground for scattering amplitudes and Feynman Integrals
- Integrable theory and closed related to string theory (AdS/CFT, Wilson-loop duality, correlators, OPE..)
- New mathematical structures (Grassmannian geometry, all-loop recursion amplituhedron, cluster algebras..)





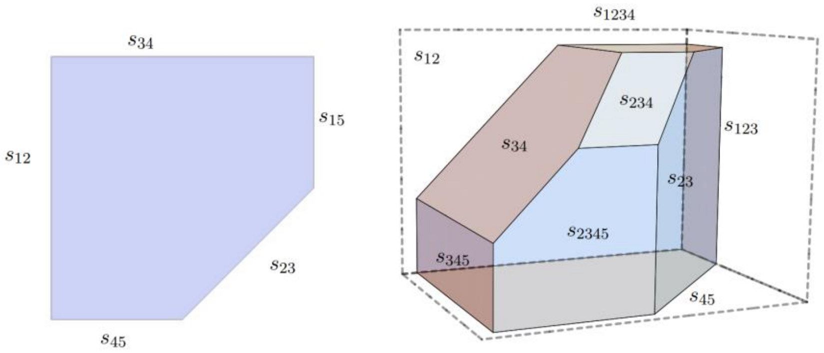




Amplituhedron & Positive Geometry (and UNIVERSE+)

Scattering amplitudes in certain special theories (planar N=4 sYM and more) can be thought of as the 'volume' of a new geometrical object called the 'amplituhedron'.
[Arkani-Hamed, Trnka, 13'] [Arkani-Hamed, Bai, Lam,17']

<u>Positive geometry:</u> A new framework for particle physics in which spacetime and quantum mechanics emerge from more basic mathematical concepts.



<u>Polytope examples: amplituhedra for 5- and 6-particle bi-adjoint phi^3 tree-level amplitudes</u>
[Arkani-Hamed, Bai, He, Yan,17]

Deeper geometrical origin is also found for correlation functions in cosmology
[Arkani-Hamed, Benincasa, Postnikov,17]

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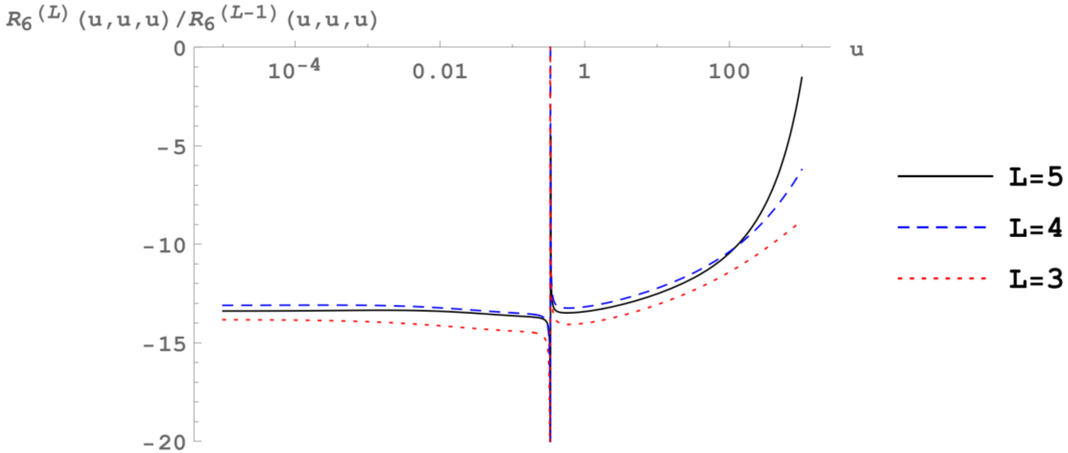
MAX PLANCK INSTITUTE FOR MATHEMATICS IN THE SCIENCES
UNIVERSITY OF AMSTERDAM

Steinmann/Cluster Bootstrap for amplitudes [2005.06735 & ref therein]

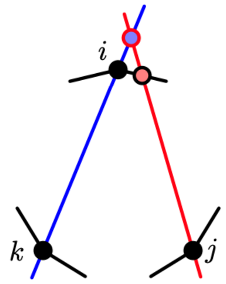
Compute n=6,7 amplitudes without computing integrals at all! (amplitudes with n less than 6 are fully determined by symmetry)

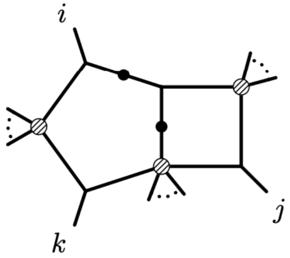
Predicted singularities + discontinuity + (Extended) Steinmann relation = huge reduction of function space

6-particle MHV/NMHV amplitudes determined up to 8 loops, 7-pt up to 4 loops



(a)





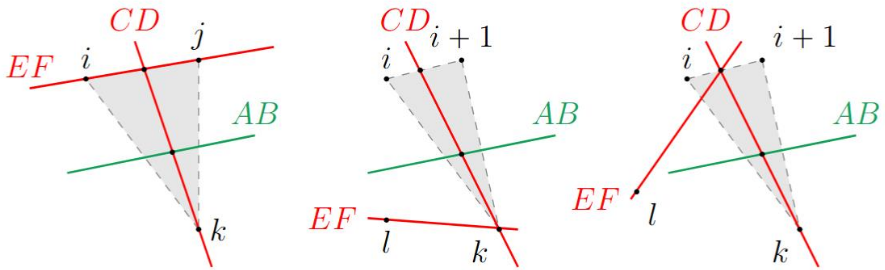
Q: Can geometrical pictures help us to detect the singularities ?

A: Landau singularities from boundary structures of the geometry!

Possible bootstrap for amplitudes of higher points/loops in the future.

Eg. Wilson-loop with Lagrangian insertion & Negative Geometries

$$\frac{1}{\pi^2} F_n(x_0; x_1, \ldots, x_n) = \frac{\langle W_{\mathrm{F}}[x_1, \ldots, x_n] \mathcal{L}(x_0) \rangle}{\langle W_{\mathrm{F}}[x_1, \ldots, x_n] \rangle},$$



Log of scattering amplitude (negative geometry from amplituhedra) **[Arkani-Hamed, Henn, Trnka, 21']**

After integration (L-loop with L-1 loop integrated):

4-pt: 1 scale, special topologies can be solved non-perturbatively (related to cusp anomalous dimension)

From 5-point, we try to detect their singularities from amplituhedra

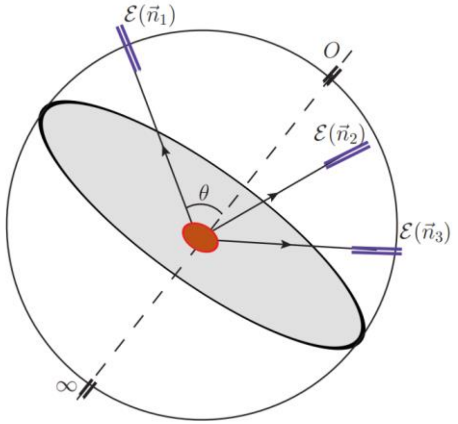
5-pt: 4 scales, recently solved for L=2 & 3, and some special topologies in L=4 (related to L=2 & 3 pentagon functions & all-plus YM amplitudes)

[Chicherin, Henn, 22'] [Chicherin, Henn, Trnka, Zhang 24']

6-pt: 7 scales, now in progress by bootstrap strategy (related to exploration for L=2 hexagon functions in the future)

Energy correlators from amplitudes

$$\mathrm{EEEC}(\chi_1,\chi_2,\chi_3)=\int\prod_{i=1}^{3}[d\Omega_{\vec{n}_i}\delta(\vec{n}_i\cdot\vec{n}_{i+1}-\cos\chi_i)]$$
$$\times\frac{\int d^4x\,\mathrm{e}^{iqx}\langle 0|O^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)O(0)|0\rangle}{(q^0)^3\int d^4x\,\mathrm{e}^{iqx}\langle 0|O^\dagger(x)O(0)|0\rangle}\,. \quad (1)$$



Energy correlators in N=4 SYM theory are closed related to square of scattering amplitudes (when detectors under colinear limit)

[Yan, Zhang 22'][Chicherin, Moult, Sokatchev, Yan, Zhu, 24']

+

Square of amplitudes can be computed from f-diagrams/amplituhedra up to n+L=11

[Bourjaily,Heslop,Tran 16']

Integrand of energy correlators are known for rather high point/loops

Integration? Non-MPL functions? Numerical computation?

Thanks for listening!