Small-x Resummation from Glauber SCET

Duff Neill, AP, Iain Stewart: *JHEP* **09 (2023) 089**
Anjie Gao, AP, Gherardo Vita: **In progress**

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MPP/TUM Collider Phenomenology Seminar,
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Outline

Motivation

Problems in the Small-*x* Resummation in DIS

The Glauber SCET Framework

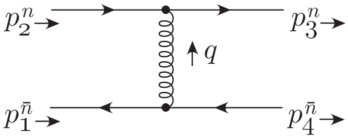
Small-*x* Factorization from Glauber SCET

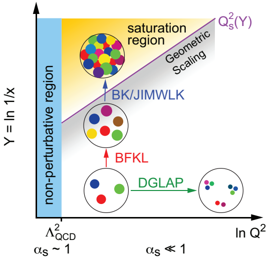
Towards NLL Small-*x* Resummation

Outline

The high energy limit

A storehouse of rich phenomenology.





- Three fundamental limits:

  soft, collinear and Regge.
- Forward scattering regime involves large logarithms of $x \sim \frac{t}{s}$
- The BFKL equation describes this rapidity evolution in x.

Two lines of attack to describe Regge behavior in cross-sections:

- Unitarization of cross-section leading to nonlinear generalizations of BFKL.
- Achieve consistent collinear and BFKL resummation: $\leftarrow$ This talk

Importance of higher order small-x_b resummation

Small-x data at N^{3}LO plays a crucial role in improving PDFs

- N^{3}LO pieces available from BFKL evolution of the splitting functions [McG+23]:

  P_{gg}: $\alpha_s^3 \ln^3 x$ ✓, $\alpha_s^3 \ln^2 x$ ✓, $\alpha_s^3 \ln x$ (?)

  P_{gq}: $\alpha_s^3 \ln^3 x$ ✓, $\alpha_s^3 \ln^2 x$ (?)

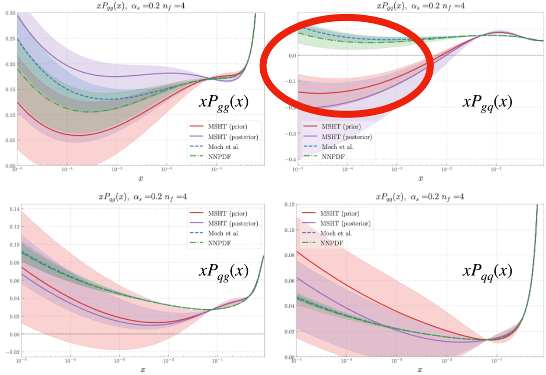
  P_{qg}: $\alpha_s^3 \ln^2 x$ ✓, $\alpha_s^3 \ln x$ (?)

  $P_{qq}^{\rm PS}$: $\alpha_s^3 \ln^2 x$ ✓, $\alpha_s^3 \ln x$ (?)

- Compare P_{gg} and P_{gq}:
  Difference in moments (MSHT, 4 vs. NNPDF, 5) only impacts large x region.
- P_{gq} has the highest power of missing logarithm $\to$ large uncertainty in the small-x region.

Comparison with MSHT and NNPDF versions

Slide from Robert Thorne, DIS 2024



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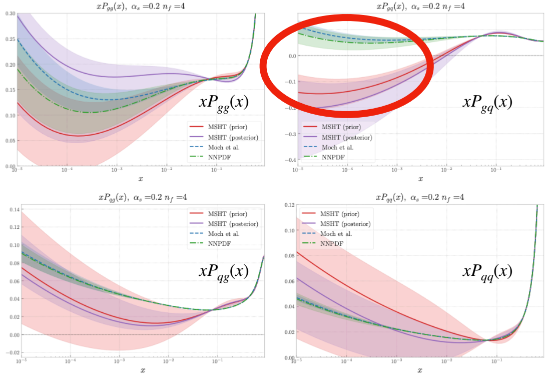
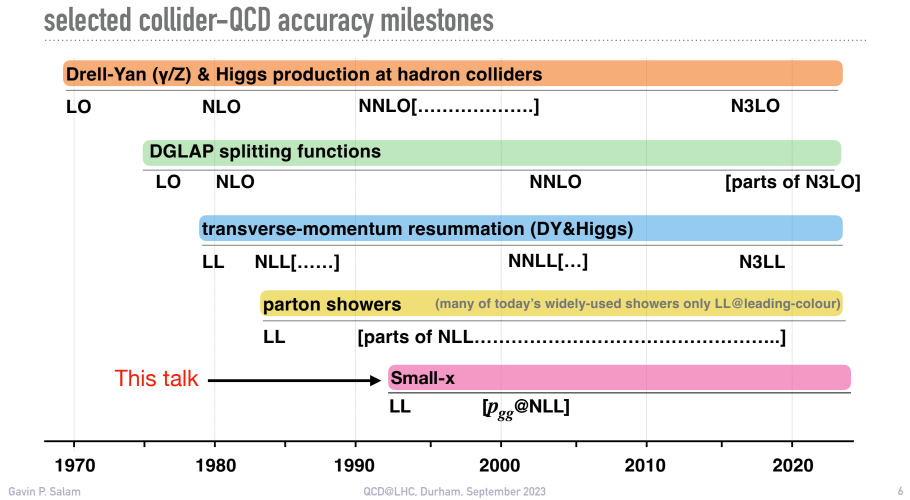


Why has been achieving NLL$_x$ P_{ij} so challenging?

State-of-the-art in small-x_b resummation



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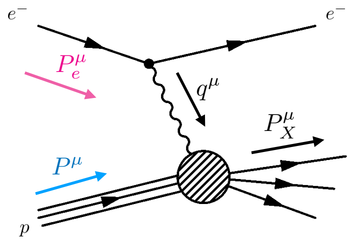
DIS review: Twist expansion

Consider unpolarized, inclusive DIS:

$$Q^2 = -q^2 > 0 \,, \qquad x_b = \frac{Q^2}{2P\cdot q} \,,$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_b\,\mathrm{d}Q^2}(e^- p \to e^- X) = \frac{2\pi y \alpha^2}{Q^4} L_{\mu\nu}(P_e, q)\, W^{\mu\nu}(P,q) \,.$$

$$W^{\mu\nu} = e_L^{\mu\nu} \frac{1}{x_b} F_L(x_b, Q^2) + e_2^{\mu\nu} \frac{1}{x_b} F_2(x_b, Q^2) \,.$$



Well-known twist-2 factorization:

$$\frac{1}{x_b} F_a(x_b, Q^2) = \sum_\kappa \int_{x_b}^1 \frac{\mathrm{d}\xi}{\xi}\, H_a^{(\kappa)}\!\left(\frac{x_b}{\xi}, Q, \mu\right) f_{\kappa/p}(\xi, \mu) + \mathcal{O}\!\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right) .$$

PDF absorbs all the IR divergences.

DIS review: Twist expansion

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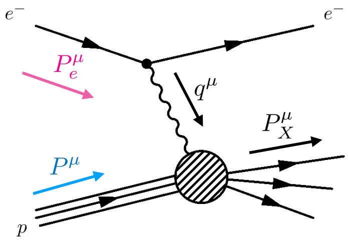

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Take Mellin Transform:

$$\bar{F}_p(N, Q^2) = \int_0^1 \frac{dx}{x} x^N \left(\frac{1}{x} F_p(x)\right) , \qquad \Rightarrow \qquad \bar{F}_p^{(\kappa)}(N) = \sum_{\kappa'} \bar{H}_p^{(\kappa')}(N) \times \underbrace{\bar{\Gamma}_{\kappa'\kappa}(N,\epsilon)}_{\text{PDF}} \, .$$

IR divergences are exponentiated into PDFs (transition functions),

$$\bar{\Gamma}_{\kappa'\kappa}(\alpha_s(\mu^2), N, \epsilon) \equiv \mathrm{P}\ \exp\left(\int_0^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\epsilon,\alpha)} \boldsymbol{\gamma}^s(\alpha, N)\right)_{\kappa'\kappa} \, .$$

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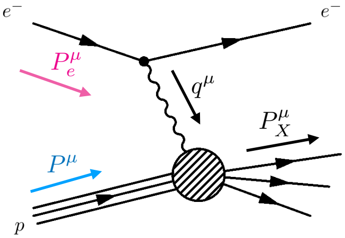

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$$\bar{\Gamma}_{\kappa'\kappa}(\alpha_s(\mu^2), n) = \mathrm{P}\exp\left(-\frac{1}{\epsilon}\int_0^{\alpha_s(\mu^2)} \frac{\mathrm{d}\alpha}{\alpha}\gamma^s(\alpha, n)\right)_{\kappa'\kappa} , \qquad \text{(fixed coupling: } \beta(\alpha_s,\epsilon) = -\epsilon\alpha_s)$$

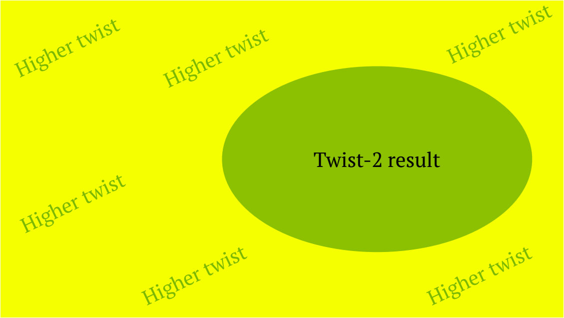
The leading and higher twist pieces

The box represents the full expression of the structure function (perturbative as well as nonperturbative):

The complete result for F_a

The leading and higher twist pieces

Terms that are retained at leading power in twist expansion $\left(\frac{\Lambda_{\mathrm{QCD}}}{Q} \ll 1\right)$



$$\frac{1}{x_b} F_a(x_b, Q^2) = \sum_\kappa \int_{x_b}^1 \frac{\mathrm{d}\xi}{\xi} H_a^{(\kappa)}\left(\frac{x_b}{\xi}, Q, \mu\right) f_{\kappa/p}(\xi, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right) .$$

Mellin Transform

Mellin transform is very useful: (set $N = n + 1$)

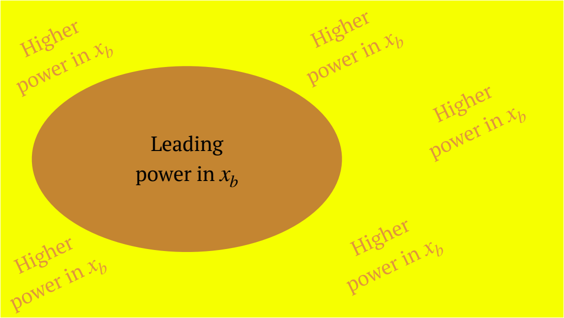
$$\bar{f}(n) \equiv \int_0^1 \frac{dx}{x} x^{n+1} f(x)$$

Let us note that

$f(x)$	$\bar{f}(n)$	singularity in $x \to 0$
$\frac{1}{x} \ln^{\ell-1}(x)$	$\sim \frac{1}{n^\ell}$	pole at $n = 0$, (leading power in x_b)
$x^{p-1} \ln^{\ell-1}(x)$	$\sim \frac{1}{(n+p)^\ell}$	pole at $n = -p$, (higher power in x_b)

Two subtleties with small-x_b resummation

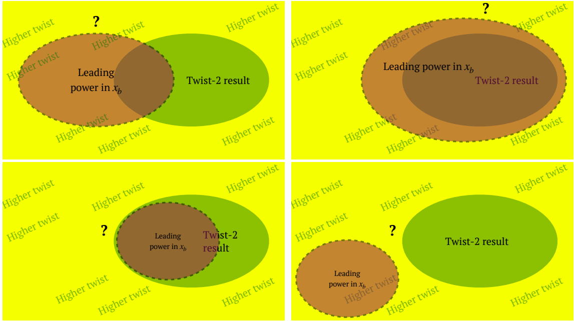
Terms that are leading power in $x_b \ll 1$ expansion ($\sim \frac{1}{x}$ or $n = 0$ pole)



Where is this relative to leading twist terms?

Two subtleties with small-x_b resummation

Which scenario is correct?

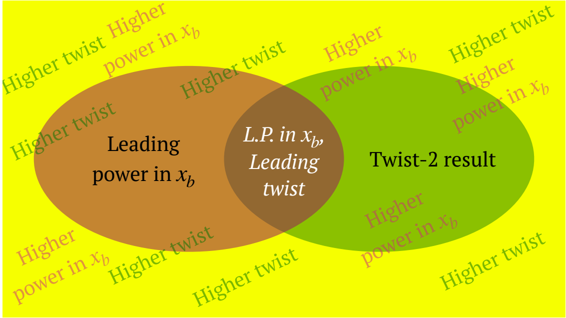


Can partly answer this by inspecting the perturbative series of the coefficient function.

$$H_L^{(g)}(x) \sim \alpha_s\, x(1-x) + \mathcal{O}\left(\frac{\alpha_s^2}{x}\right) \quad \Leftrightarrow \quad \bar{H}_L^{(g)}(x) \sim \alpha_s \left(\frac{1}{n+2} - \frac{1}{n+3}\right) + \mathcal{O}\left(\frac{\alpha_s^2}{n}\right)$$

Subtlety # 1: Non-trivial overlap between twist and small-x_b expansions

Small-x_b expansion is based on rapidity factorization and makes no reference to Λ_{QCD} (more on this later), and **hence includes leading as well as higher twist pieces**.



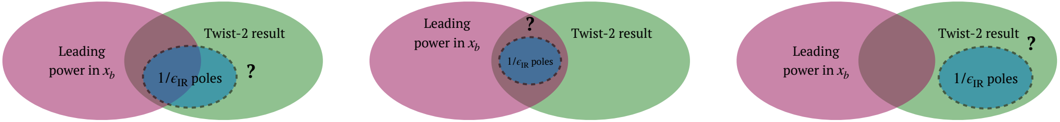
Our first goal is to compute the overlap between the two expansions.

Factorization of nonperturbative pieces

Using dim-reg $d = 4 - 2\epsilon$ and simple partonic states we can kill higher twist pieces.
IR divergences appear as $1/\epsilon_{\text{IR}}$ poles in the PDF.

$$\bar{\Gamma}_{\kappa'\kappa}(\alpha_s(\mu^2), n) = \mathsf{P}\,\mathsf{exp}\left(-\frac{1}{\epsilon}\int_0^{\alpha_s(\mu^2)} \frac{\mathsf{d}\alpha}{\alpha}\gamma^s(\alpha, n)\right)_{\kappa'\kappa}, \qquad \text{(fixed coupling: } \beta(\alpha_s, \epsilon) = -\epsilon\alpha_s)$$

Where are exactly all the leading twist-2 $1/\epsilon_{\text{IR}}$ poles?



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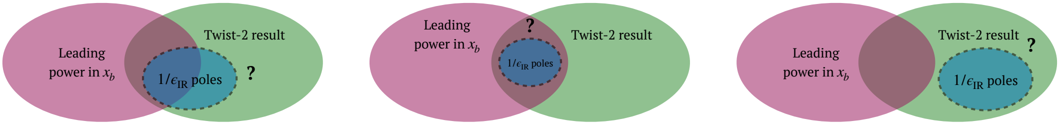


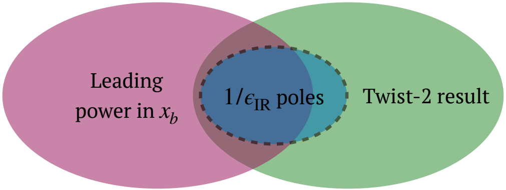
We can answer this by inspecting the perturbative series of the anomalous dimension:

$$\gamma_{gg}(n) = \frac{\alpha_s C_A}{\pi n} + \dots ,$$

$$\gamma_{qg}(n) = \frac{\alpha_s T_F}{3\pi} + \mathcal{O}\left(\frac{\alpha_s}{n}\right)$$

Subtlety # 2: Non-trivial overlap *also of IR poles* between twist and small-x_b expansions

There are IR divergences associated with small-x_b logs as well as those appearing at higher powers in $x_b \ll 1$ expansion:



The overlap $1/\epsilon_{\rm IR}$ terms are generated by *bad BFKL boundary conditions*.

Our second goal is to consistently factorize IR poles of these two origins into the twist-2 PDF.

The BFKL equation

Resummation of small-x_b logs invovles solving the BFKL equation. For a function

$$f(x, \boldsymbol{q}_\perp) \sim x^{p-1} (\text{logs of } x) \,,$$

that satisfies BFKL equation in 4 dimensions:

$$x \frac{d}{dx} f(x, \boldsymbol{q}_\perp) = (p-1) f(x, \boldsymbol{q}_\perp) + c \big[K \otimes_\perp f \big](\boldsymbol{q}_\perp)$$

where

$$K = \bar{\alpha}_s K_{\text{LO}} + \bar{\alpha}_s^2 K_{\text{NLO}} + \dots \,, \qquad \bar{\alpha}_s \equiv \frac{\alpha_s C_A}{\pi} \,,$$

$$\big[K_{\text{LO}} \otimes_\perp f \big](\boldsymbol{q}_\perp) \equiv (2\pi) \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \left\{ \frac{2 f(\boldsymbol{k}_\perp)}{(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)^2} - \frac{\boldsymbol{q}_\perp^2}{\boldsymbol{k}_\perp^2 (\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)^2} f(\boldsymbol{q}_\perp) \right\} ,$$

In the n-space we have an iterative equation

$$\bar{f}(n, \boldsymbol{q}_\perp) = \frac{1}{n+p} \times \underbrace{f(x=1, \boldsymbol{q}_\perp)}_{\text{Boundary condition}} - \frac{c}{n+p} \big[K \otimes_\perp \bar{f}(n) \big](\boldsymbol{q}_\perp)$$

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Eigenfunctions of BFKL Kernel:

$$\left[K_{\text{LO}} \otimes_\perp \left(\frac{1}{\boldsymbol{k}_\perp^{2(1-\gamma)}} e^{\mathrm{i} n \phi} \right) \right] (\boldsymbol{q}_\perp) = \chi(n, \gamma) \frac{1}{\boldsymbol{q}_\perp^{2(1-\gamma)}} e^{\mathrm{i} n \phi} \, , \qquad 0 < \text{Re}\, \gamma < 1 \, .$$

Bad boundary condition = IR divergence!

What happens for $\gamma\!=\!0$?

$$\gamma = 0 : \qquad \left[K_{\mathrm{LO}} \otimes_\perp \frac{1}{\boldsymbol{k}_\perp^2}\right](\boldsymbol{q}_\perp) = \frac{1}{\boldsymbol{q}_\perp^2}(2\pi)\int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \frac{\boldsymbol{q}_\perp^2}{\boldsymbol{k}_\perp^2 \left(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp\right)^2}$$

This Integral is divergent! But we can make sense of it in dimensional regularization:

$$\begin{aligned}(2\pi) I_\epsilon\left[\boldsymbol{q}_\perp^2\right] &\equiv (2\pi)\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{\mathrm{d}^{2-2\epsilon} k_\perp}{(2\pi)^{2-2\epsilon}} \frac{\boldsymbol{q}_\perp^2}{\boldsymbol{k}_\perp^2 \left(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp\right)^2} \\ &= \left(\frac{\boldsymbol{q}_\perp^2}{\mu^2}\right)^{-\epsilon} \Gamma(-\epsilon) e^{\epsilon\gamma_E} \frac{\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} \\ &= -\frac{1}{\epsilon} + \log\left(\frac{\boldsymbol{q}_\perp^2}{\mu^2}\right) + \mathcal{O}(\epsilon)\,.\end{aligned}$$

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This is relevant: Nature produces bad boundary conditions for the BFKL equation and these IR divergences go into the PDF, (but *not every* IR divergence is generated this way.)

Another bad boundary condition

Notice how BFKL kernel acts on $\delta^{(2-2\epsilon)}(\boldsymbol{q}_\perp)$:

$$K \otimes_\perp \delta^{(2-2\epsilon)}(\boldsymbol{q}_\perp) \sim \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-\epsilon}$$

$$K \otimes_\perp \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-\epsilon} \sim \frac{1}{\epsilon} \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-2\epsilon}$$

$$\vdots$$

$$K \otimes_\perp \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-\ell\epsilon} \sim \frac{1}{\ell\epsilon} \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-(\ell+1)\epsilon}$$

This generates an IR divergent series solution for $\bar{\mathcal{F}}_g^{(0)}$:

$$\delta^{(2-2\epsilon)}(\boldsymbol{q}_\perp) \to \delta^{(2-2\epsilon)}(\boldsymbol{q}_\perp) + \frac{1}{\boldsymbol{k}_\perp^{2-2\epsilon}} \sum_{\ell=1}^{\infty} c_\ell(\epsilon) \left(\frac{\bar{\alpha}_s}{n}\left(\frac{\boldsymbol{k}_\perp^2}{\mu^2}\right)^{-\epsilon}\right)^{\ell}, \qquad c_\ell(\epsilon) = \frac{1}{\ell!}\left(-\frac{1}{\epsilon}\right)^{\ell}\left(1+\mathcal{O}(\epsilon^2)\right)$$

> The action of BFKL generates the entire tower of $1/\epsilon$ IR poles associated with the high energy limit.

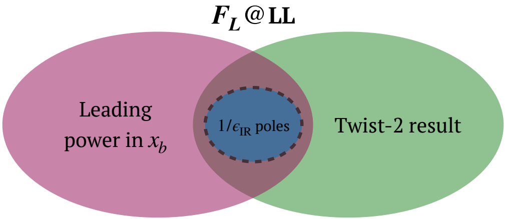
LL small-x resummation by Catani and Hautmann

- [CH94] resummed $\left(\frac{\alpha_s}{n}\right)^\ell$ terms in $\gamma_{gg}(\alpha_s, n)$ by inventing a *gluon Green's function* $\bar{\mathcal{F}}_g^{(0)}$:

$$\bar{\mathcal{F}}_g^{(0)}(n, \boldsymbol{q}_\perp) = \delta^{(2-2\epsilon)}(\boldsymbol{q}_\perp) + \frac{\bar{\alpha}_s}{n}\left[K \otimes_\perp \bar{\mathcal{F}}_g^{(0)}(n)\right](\boldsymbol{q}_\perp)\,, \qquad \bar{\alpha}_s \equiv \frac{\alpha_s C_A}{\pi}$$

  $\bar{\mathcal{F}}_g^{(0)}$ is determined completely by the $\delta^{(2-2\epsilon)}(q_\perp)$ boundary condition and iterations of the BFKL kernel.
- LL small-x_b resummation of F_L:

$$\bar{F}_L^{(g)}(n) = h_L(\gamma_{gg}) \times R(n) \times \left(\frac{Q^2}{\mu^2}\right)^{\gamma_{gg}} \times \bar{\Gamma}_{gg}\,,$$



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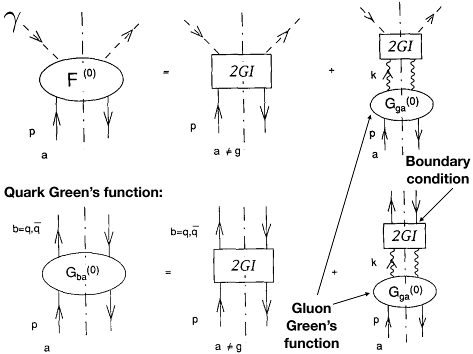
$$\bar{F}_L^{(g)}(n) = h_L(\gamma_{gg}) \times R(n) \times \left(\frac{Q^2}{\mu^2}\right)^{\gamma_{gg}} \times \bar{\Gamma}_{gg}\,,$$

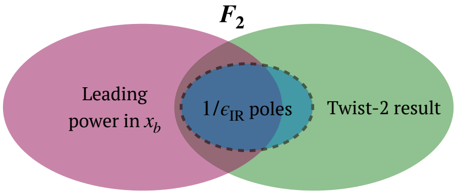
- At LL, the IR divergences in F_L^g appear in $\bar{\Gamma}_{gg}$.
- h_L: describes coupling with photon, IR finite, defined via an *off-shell* cross section.
- R: scheme chosen to factorize the IR divergences: $\bar{\Gamma}_{gg}$ absorbs the IR divergences in $\bar{\mathcal{F}}_g^{(0)}$.

$$\bar{\mathcal{F}}_g^{(0)}(n, \boldsymbol{q}_\perp) = \frac{1}{\pi \boldsymbol{k}_\perp^2} \times \gamma_{gg} \times \tilde{R}(n, \boldsymbol{k}_\perp, \epsilon) \times \bar{\Gamma}_{gg}\,.$$

LL small-x resummation by Catani and Hautmann

- Resummation of F_2 and γ_{qg} is not straightforward in this framework, because F_2 involves IR divergences NOT generated by BFKL evolution alone!
- They introduced a new *quark's Green's function* to capture this non-BFKL divergence.
- The approach of Catani and Hautmann [CH94] has not been extended beyond LL.





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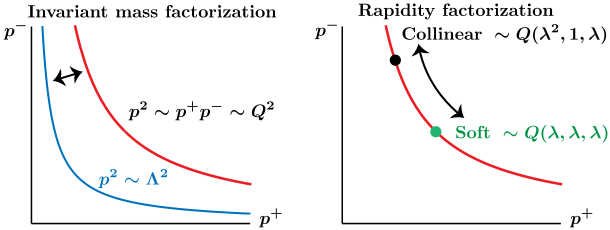
High Factorization = Rapidity Factorization

Light cone coordinates:

$$p^\mu = p^+ \frac{\bar{n}^\mu}{2} + p^- \frac{n^\mu}{2} + p_\perp^\mu\,, \qquad p^2 = p^+ p^- - \vec{p}_\perp^{\,2}\,, \qquad n^2 = \bar{n}^2 = 0\,, \qquad n\cdot\bar{n} = 2\,.$$



- Invariant mass factorization can be described via an OPE in QCD.
- Need to distinguish particles at different rapidities for high energy factorization.
- Soft collinear effective theory (SCET) is a powerful tool for dissecting collider QCD problems with multiple scales:

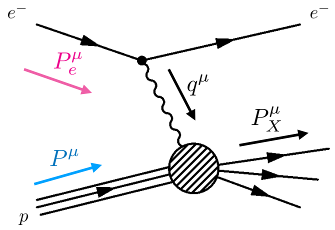
$$\mathcal{L}_{\text{SCET}} = \sum_{n_i} \mathcal{L}_{n_i}[\xi_{n_i}, A_{n_i}] + \mathcal{L}_s[\psi_s, A_s] + \mathcal{L}_{\text{int}}[\xi_{n_i}, A_n, \psi_s, A_s, \ldots]\,.$$

EFT modes and power counting

Center of mass light cone coordinates:

$$P^\mu = \frac{\sqrt{s}}{2} n^\mu \,, \qquad P_e^\mu = \frac{\sqrt{s}}{2} \bar{n}_\mu$$

Power counting parameters: $\lambda' \sim \frac{\Lambda_{\text{QCD}}}{Q}$ and $\lambda \sim x_b$.



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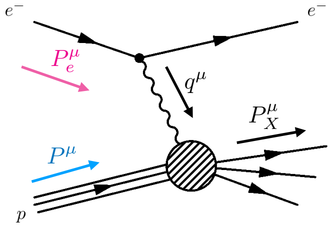


Two possible scenarios based on the scaling of the invariant mass of hadronic state:

	Hard scattering		***Forward scattering***
$\frac{\boldsymbol{P_X^2}}{\boldsymbol{s}} = \frac{(q+P)^2}{s} = \frac{Q^2}{s}\frac{(1-x_b)}{x_b}$	$\sim \lambda^0$	or	$\sim \lambda$
$\boldsymbol{q^\mu} = -\frac{Q^2}{\sqrt{s}}\frac{n^\mu}{2} + \frac{Q^2}{x_b\sqrt{s}}\frac{\bar{n}^\mu}{2} + q_\perp^\mu$	$\sim \sqrt{s}(1, \lambda, \sqrt{\lambda})$ (collinear to e^-)	or	$\sim \sqrt{s}(\lambda, \lambda^2, \lambda)$

Forward scattering: $P_X^2 \sim (p_n + p_s)^2 \sim p_n^- p_s^+ \sim s\lambda \,.$

EFT modes and power counting

Forward scattering

$$P_X^2/s \sim \lambda$$

$$q^\mu \sim \sqrt{s}(\lambda, \lambda^2, \lambda)$$

$$p_n^\mu \sim \sqrt{s}(\lambda^2, 1, \lambda)$$

$$p_{\bar{n}} \sim \sqrt{s}(\underbrace{1}_{p^+}, \underbrace{\lambda^2}_{p^-}, \underbrace{\lambda}_{p_\perp})$$

$$q_\gamma \downarrow \qquad \downarrow$$

$$k_s \sim \sqrt{s}(\underbrace{\lambda}_{p^+}, \underbrace{\lambda}_{p^-}, \underbrace{\lambda}_{p_\perp})$$

$$q_G \qquad \downarrow \qquad \downarrow$$

$$p_n \sim \sqrt{s}(\underbrace{\lambda^2}_{p^+}, \underbrace{1}_{p^-}, \underbrace{\lambda}_{p_\perp})$$

The photon cannot interact directly with collinear mode without knocking it offshell. The leading terms start at $\mathcal{O}(\alpha_s^2)$ due to intermediate soft sector:

$$p_s = (p_s^+, p_s^-, p_{s\perp}) \sim \sqrt{s}(\lambda, \lambda, \lambda)\,.$$

Need additional Glauber modes for soft-collinear interaction:

$$q_G^\mu = q'^\mu \sim \sqrt{s}(\lambda^2, \lambda, \lambda)\,.$$

SCET with Glauber operators

SCET Lagrangian:

$$\mathcal{L}_{\text{SCET}} = \sum_{n_i} \mathcal{L}_{n_i} + \mathcal{L}_s + \mathcal{L}_G \,.$$

Glauber operators derived in Rothstein and Stewart [RS16] account for forward scattering phenomena.

$$\mathcal{L}_G = \sum_{n_i,n_j} \mathcal{O}_{n_i} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{n_i} + \sum_{n_i} \mathcal{O}_{n_i} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{n_i}$$

$$S_G^{(n_i s)} = 8\pi\alpha_s \sum_{ij} \int \mathrm{d}^4x \int \mathrm{d}^4z \int \frac{\mathrm{d}^4q}{(2\pi)^4} \frac{e^{\mathrm{i}q\cdot(x-z)}}{\boldsymbol{q}_\perp^2} \mathcal{O}_{n_i}^{iA}(x)\, \mathcal{O}_s^{j n_i A}(z)$$

$\mathcal{O}_s^{i_n A}$

$\mathcal{O}_n^{iA}$

$$\mathcal{O}_s^{n_i,qA} = \overline{\psi}_S^{n_i} \mathbf{T}_i^A \frac{\not{n}_i}{2} \psi_S^{n_i} \,, \qquad \mathcal{O}_s^{n_i,gA} = \frac{1}{2} \mathcal{B}_{S\perp\mu}^{n_i B} (\mathrm{i} f^{ABC}) \frac{n_i}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{n_i C\mu} \,,$$

$$\mathcal{O}_{n_i}^{qA} = \overline{\chi}_{n_i} \mathbf{T}_i^A \frac{\not{\bar{n}}_i}{2} \chi_{n_i} \,, \qquad \mathcal{O}_{n_i}^{gA} = \frac{1}{2} \mathcal{B}_{n\perp\mu}^{B} (\mathrm{i} f^{ABC}) \frac{\bar{n}_i}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{C\mu} \,,$$

Dotted propagator represents insertion of operators from the Glauber Lagrangian.

Recent progress in Glauber SCET

Slide from Anjie Gao, SCET 2024

Factorization of the Regge amplitudes

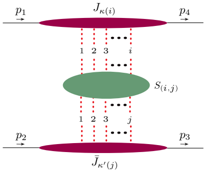
into jet & soft functions

A simpler organization

[our paper 2401.00931]

$$-i\mathcal{M} = \mathbf{J}_\kappa \cdot \mathbf{S} \cdot \bar{\mathbf{J}}_{\kappa'} = \sum_{ij} J^{\alpha}_{\kappa(i)}(\ell_{1\perp}, \dots, \ell_{i\perp}) \otimes_i S^{\alpha\beta}_{(i,j)}(\ell_{1\perp}, \dots, \ell_{i\perp}; \ell'_{1\perp}, \dots, \ell'_{j\perp}) \otimes_j \bar{J}^{\beta}_{\kappa'(j)}(\ell'_{1\perp}, \dots, \ell'_{j\perp})$$

$$= \sum_{ij}$$



- $\kappa, \kappa' = q, g, gg, \dots$: projectiles, $\alpha = a_1 \dots a_i$, $\beta = a'_1 \dots a'_j$: adjoint color indices
- $\otimes_i$: transverse 2d momentum convolution of i Glaubers
- 4d Glauber SCET $\Rightarrow$ 2d dynamics on the transverse plane

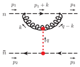
9

Recent progress in Glauber SCET

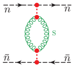
Slide from Anjie Gao, SCET 2024

Rapidity Renormalization for 1 Glauber

- Can be extracted either through **collinear** or **soft** loop calculation
  - At one loop ("=" in the sense of rapidity divergent piece)



$$= J^{[1]}_{(1)} S^{[0]}_{(1,1)} \bar{J}^{[0]}_{(1)} = \left(J^{[0]}_{(1)} Z^{[1]}_{J(1,1)} \right) S^{[0]}_{(1,1)} \bar{J}^{[0]}_{(1)}$$



$$= J^{[0]}_{(1)} S^{[1]}_{(1,1)} J^{[0]}_{(1)} = J^{[0]}_{(1)} \left(Z^{[1]}_{S(1,1)} S^{[0]}_{(1,1)} \right) J^{[0]}_{(1)} + J^{[0]}_{(1)} \left(S^{[0]}_{(1,1)} Z^{[1]}_{S(1,1)} \right) J^{[0]}_{(1)}$$

$$\Rightarrow \quad \nu \partial_\nu J_{(1)} = \omega_G(q_\perp) J_{(1)} \qquad \omega_G(q_\perp) = -\alpha_s N_c \int \frac{d\!\!{}^-{}^{d'} k_\perp \, \vec{q}_\perp^{\,2}}{\vec{k}_\perp^2 (\vec{q}_\perp - \vec{k}_\perp)^2}$$

Resummation gives Regge pole solution

$$\boxed{J^{8_A}_{(1)} \sim e^{\omega_G(q_\perp) \log \frac{s}{\nu^2}}}$$

11

Outline

Motivation

Problems in the Small-x Resummation in DIS

The Glauber SCET Framework

Small-x Factorization from Glauber SCET

Towards NLL Small-x Resummation

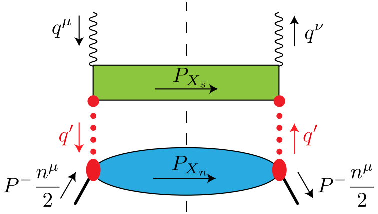
Small-x factorization formula

We include *two insertions* of the ns Glauber action:

$$S_G^{ns} = 8\pi\alpha_s \sum_{i,j,A} \int \mathrm{d}^d y \int \mathrm{d}^d x \int \frac{\mathrm{d}^d q'}{(2\pi)^d} \frac{e^{\mathrm{i}(x-y)\cdot q'}}{q'^2_\perp} \mathcal{O}_n^{iA}(x)\mathcal{O}_s^{j_n A}(y)\,.$$

Factorization formula at NLL:

$$W^{\alpha\beta}(q,P) = \int \mathrm{d}^{d-2}q'_\perp\, S^{\alpha\beta}\left(q, q'_\perp, \frac{\nu}{x_b P^-}, \epsilon\right) C\left(q'_\perp, P, \frac{\nu}{P^-}, \epsilon\right) + \dots\,.$$



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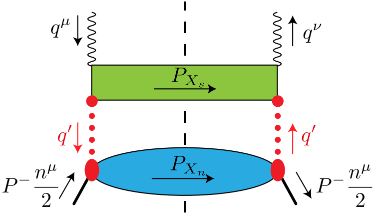


Factorization formula at NLL:

$$W^{\alpha\beta}(q,P) = \int \mathrm{d}^{d-2} q_\perp' \, S^{\alpha\beta}\Big(q, q_\perp', \frac{\nu}{x_b P^-}, \epsilon\Big) \, C\Big(q_\perp', P, \frac{\nu}{P^-}, \epsilon\Big) + \ldots\,.$$

The collinear and soft functions are defined as

$$C \equiv \frac{1}{\pi\nu} \frac{1}{\boldsymbol{q}_\perp'^2} \sum_{i,j,A} \int \frac{\mathrm{d}q'^+}{2\pi} \int \mathrm{d}^d x \, e^{\mathrm{i}\frac{x^- q'^+}{2} + \mathrm{i} x_\perp \cdot q_\perp'} \langle P|\mathcal{O}_n^{i\,A}(x)\mathcal{O}_n^{j\,A}(0)|P\rangle_\nu\,,$$

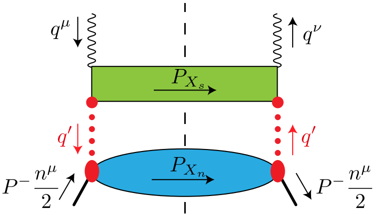
$$S^{\alpha\beta} \equiv \frac{\nu}{\boldsymbol{q}_\perp'^2} \frac{(2\pi\iota\mu^2)^{4-d}\big(8\pi\alpha_s(\mu^2)\big)^2}{16\pi^2\,(N_c^2-1)} \sum_{i,j,A} \int \frac{\mathrm{d}q'^-}{4\pi} \int \mathrm{d}^d z \, e^{\mathrm{i}z\cdot q} \int \mathrm{d}^d y_L \mathrm{d}^d y_R$$

$$\times e^{-\mathrm{i}\frac{q'^-(y_L^+ - y_R^+)}{2} - \mathrm{i}q_\perp' \cdot (y_{L\perp} - y_{R\perp})} \langle 0|\bar{T}\{J^\alpha(z)\mathcal{O}_s^{i_n\,A}(y_L)\}T\{J^\beta(0)\mathcal{O}_s^{j_n\,A}(y_R)\}|0\rangle_\nu\,.$$

Small-x factorization formula

We include *two insertions* of the ns Glauber action:

$$S_G^{ns} = 8\pi\alpha_s \sum_{i,j,A} \int \mathrm{d}^d y \int \mathrm{d}^d x \int \frac{\mathrm{d}^d q'}{(2\pi)^d} \frac{e^{\mathrm{i}(x-y)\cdot q'}}{q'^2_\perp} \mathcal{O}_n^{iA}(x)\mathcal{O}_s^{j_n A}(y)\,.$$

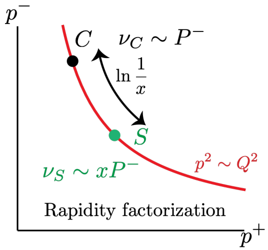


Factorization formula at NLL:

$$W^{\alpha\beta}(q,P) = \int \mathrm{d}^{d-2}q'_\perp \; S^{\alpha\beta}\Big(q, q'_\perp, \frac{\nu}{x_b P^-}, \epsilon\Big) \; C\Big(q'_\perp, P, \frac{\nu}{P^-}, \epsilon\Big) + \ldots$$

Here small-x_b logs are resummed via *rapidity evolution* for $\nu_S \sim x_b P^-$ and $\nu_C \sim P^-$

$$\frac{\nu_S}{\nu_C} = x_b$$



Process independence and the BFKL equation

Rothstein and Stewart [RS16] showed that for $\boldsymbol{pp} \to \boldsymbol{pp}$ forward scattering

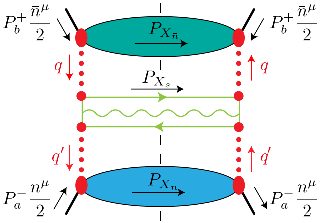
$$\sigma^{pp\to pp} \sim C_n \otimes S^{pp} \otimes C_{\bar{n}}$$

and S^{pp} satisfies the BFKL equation:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} S^{pp} \sim +2\bar{\alpha}_s \iota^{\epsilon} K \otimes_{\perp} S^{pp}$$

The collinear function is process independent and is expected to satisfy the BFKL equation from RG consistency:

$$\nu \frac{d}{d\nu} C = -C - \bar{\alpha}_s \iota^{\epsilon} K \otimes_{\perp} C\,.$$



Drell-Yan

Collinear function at NLO

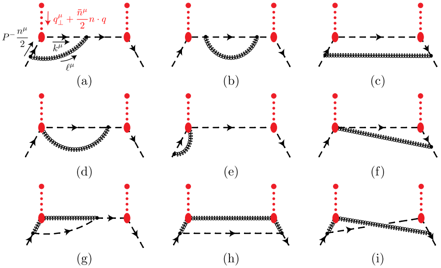
We computed the collinear function at NLO

$$C_\kappa^{\text{LO}}(q'_\perp) = \frac{P^-}{\nu}\frac{c_\kappa}{\pi \boldsymbol{q}'^2_\perp}\,, \qquad c_\kappa = C_F, C_A \qquad \text{(bad boundary condition!)}$$

$$C_q^{\text{NLO}} = \bar{\alpha}_s C_q^{\text{LO}} \times (-2\pi)\, I_\epsilon[\boldsymbol{q}'^2_\perp] \left(\frac{1}{\eta} + \ln\left(\frac{\nu}{P^-}\right) + \frac{3}{4}\right),$$

$$C_g^{\text{NLO}} = \bar{\alpha}_s C_g^{\text{LO}} \times (-2\pi)\, I_\epsilon[\boldsymbol{q}'^2_\perp]$$

$$\times \left(\frac{1}{\eta} + \ln\left(\frac{\nu}{P^-}\right) + \frac{11}{12} - \frac{n_f T_R}{4C_A}\left(1 - \frac{1}{3(1-\epsilon)}\right)\right),$$



$$(2\pi) I_\epsilon[\boldsymbol{r}^2_\perp] = -\frac{1}{\epsilon} + \ln\left(\frac{\boldsymbol{r}^2_\perp}{\mu^2}\right) + \mathcal{O}(\epsilon)\,, \qquad \bar{\alpha}_s \equiv \frac{\alpha_s C_A}{\pi}$$

The one-loop contribution is IR divergent and exhibits a rapidity divergence consistent with BFKL.

More IR divergences

$$\frac{1}{x_b} F_a(q, P) = \int_0^\infty \mathrm{d}^{d-2} q'_\perp \, S_a\left(q, q'_\perp, \frac{\nu}{x_b P^-}, \epsilon\right) C\left(q'_\perp, \frac{\nu}{P^-}, \epsilon\right), \qquad [S^{\mu\nu}] = 4 - d\,, \qquad [C] = -2\,.$$

The convolution itself generates IR divergences as nothing prevents $q'_\perp$ from entering the IR region. To see this explicitly, let us note that the SCET$_{\text{II}}$ collinear function has the all-orders expansion:

$$C\left(q'_\perp, \frac{\nu}{P^-}, \alpha_s(\mu^2), \epsilon\right) = \frac{1}{\boldsymbol{q}'^2_\perp} \sum_{\ell=0}^{\infty} C^{(\ell)}\left(\alpha_s(\mu^2), \frac{\nu}{P^-}, \epsilon\right) \left(\frac{\boldsymbol{q}'^2_\perp}{\mu^2}\right)^{-\ell\epsilon}.$$

Alternative form of the factorization formula:

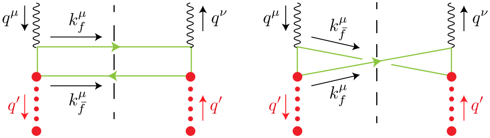
$$\frac{1}{x_b} F_a = \sum_{\ell=0}^{\infty} \left(\frac{\boldsymbol{q}^2_\perp}{\mu^2}\right)^{-(\ell+2)\epsilon} C^{(\ell)} \times \tilde{S}_a(\,\gamma = -\ell\epsilon\,)\,.$$

The γ-transform of the soft function:

$$\tilde{S}_a(\gamma) \sim \int \frac{\mathrm{d}^{2-2\epsilon} q'_\perp}{\boldsymbol{q}'^2_\perp} \left(\frac{\boldsymbol{q}'^2_\perp}{\mu^2}\right)^{\gamma} S_a(q_\perp, q'_\perp, \epsilon)\,.$$

More IR divergences

Leading order soft function calculated from



is IR finite for $\gamma \neq 0$:

$$\tilde{S}_2^{\text{LO}}(\gamma) = \alpha_s^2 n_f T_F \left(\frac{\nu}{x_b P^-}\right)\left(\frac{\pi^2\big(-3\gamma^2+3\gamma+2\big)\csc^2(\pi\gamma)}{8\Gamma(\frac{5}{2}-\gamma)\Gamma(\frac{3}{2}+\gamma)}\right) + \mathcal{O}(\epsilon)\,,$$

$$\tilde{S}_L^{\text{LO}}(\gamma) = \alpha_s^2 n_f T_F \left(\frac{\nu}{x_b P^-}\right)\left(\frac{\pi^2\big(-\gamma+1\big)\csc^2(\pi\gamma)}{4\Gamma(\frac{5}{2}-\gamma)\Gamma(\frac{3}{2}+\gamma)}\right) + \mathcal{O}(\epsilon)\,.$$

More IR divergences

In the convolution the collinear function forces us to set $\gamma = -\ell\epsilon$,

$$\frac{1}{x_b} F_a = \sum_{\ell=0}^{\infty} \left(\frac{\boldsymbol{q}_\perp^2}{\mu^2}\right)^{-(\ell+2)\epsilon} C^{(\ell)} \times \tilde{S}_a\big(\,\gamma = -\ell\epsilon\,\big)\,.$$

which implies

$$\lim_{\epsilon\to 0} \tilde{S}_2^{\text{LO}}(-\ell\epsilon) = \frac{2\alpha_s^2 n_f T_F}{3\pi} \frac{1}{(\ell+1)(\ell+2)} \left(\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right),$$

$$\lim_{\epsilon\to 0} \tilde{S}_L^{\text{LO}}(-\ell\epsilon) = \frac{2\alpha_s^2 n_f T_F}{3\pi} \frac{1}{(\ell+1)} \left(-\frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right).$$

The $\tilde{S}_a$ soft function will also contribute to the PDF despite being a vacuum matrix element.

More IR divergences

In the convolution the collinear function forces us to set $\gamma = -\ell\epsilon$,

$$\frac{1}{x_b} F_a = \sum_{\ell=0}^{\infty} \left(\frac{\boldsymbol{q}_\perp^2}{\mu^2}\right)^{-(\ell+2)\epsilon} C^{(\ell)} \times \tilde{S}_a\left(\gamma = -\ell\epsilon\right).$$

We find that for $\gamma \neq 0$, $\tilde{S}_L$ and $\tilde{S}_2$ are proportional to the off-shell cross section that appear in [CH94]:

$$\tilde{S}_2(\gamma, \epsilon = 0) = \left(\frac{\nu}{x_b P^-}\right) \alpha_s \frac{h_2(\gamma)}{\gamma^2}, \qquad (1)$$

$$\tilde{S}_L(\gamma, \epsilon = 0) = \left(\frac{\nu}{x_b P^-}\right) \alpha_s \frac{h_L(\gamma)}{\gamma}.$$

This is not the right limit for us and the full ϵ dependence is needed to perform small-x_b resummation.

> In [CH94] these IR divergences were separately captured in the gluon and quark Green's functions.

Leading log small-x_b resummation

Setting $\nu = \nu_S$ trivializes rapidity logs in the soft function:

$$\frac{1}{x_b} F_a^\kappa(x_b, Q^2) = \int \mathrm{d}^{d-2} q'_\perp S_a(1, q_\perp, q'_\perp, \epsilon) C_\kappa(x_b, q'_\perp, \epsilon)$$

Mellin space :

$$\bar{C}_\kappa(n, q'_\perp, \epsilon) = \frac{c_\kappa}{n\pi \boldsymbol{q}'^2_\perp} + \frac{\bar{\alpha}_s \iota^\epsilon}{n} K \otimes_\perp \bar{C}_\kappa(n, q'_\perp, \epsilon) \quad , \quad c_\kappa = C_F, C_A \, , \quad \bar{\alpha}_s = \frac{\alpha_s C_A}{\pi}$$

Solve for $\bar{C}_\kappa$ as a power series as before:

$$\bar{C}_{\kappa,\mathrm{LL}}(n, q'_\perp, \epsilon) = \frac{1}{n} \frac{c_\kappa}{\pi \boldsymbol{q}'^2_\perp} \sum_{\ell=0}^{\infty} c_{\ell+1}(\epsilon) \left(\frac{\bar{\alpha}_s}{n} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{\boldsymbol{q}'^2_\perp}{\mu^2} \right)^{-\epsilon} \right)^\ell , \qquad c_\ell(\epsilon) = \frac{1}{\ell!} \left(\frac{-1}{\epsilon} \right)^\ell \left(1 + \mathcal{O}(\epsilon^2) \right)$$

Now include the soft contribution to arrive at small-x_b resummed structure functions:

$$\bar{F}^\kappa_{a,\mathrm{LL}}(n, Q^2) = \frac{c_\kappa}{n\pi} \left(\frac{\boldsymbol{q}^2_\perp}{\mu^2} \right)^{-2\epsilon} \sum_{\ell=0}^{\infty} d_{a,\ell+1}(\epsilon) \left(\frac{\bar{\alpha}_s}{n} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{\boldsymbol{q}^2_\perp}{\mu^2} \right)^{-\epsilon} \right)^\ell \quad , \quad d_{a,\ell+1}(\epsilon) \equiv c_{\ell+1}(\epsilon) \tilde{S}_a(1, -\ell\epsilon, \alpha_s, \epsilon)$$

BFKL Resummation of F_L

We set $\mu^2 = Q^2$ and start with formula involving unknown pieces (HP = higher power)

$$\bar{F}^g_{L,\text{HP}} + \bar{F}^g_{L,\text{LL}}(n) = \bar{H}^{(g)}_L\Big(n, \frac{Q^2}{\mu^2} = 1, \alpha_s\Big)\bar{\Gamma}_{gg}(\alpha_s, n) \ .$$

Parameterize the the terms we want to determine for LL results as

$$\bar{H}^{(g)}_L = \frac{\alpha_s}{\pi}\sum_{k=0}^{\infty}\epsilon^k h^{(0,k)}_{L,g} + \frac{\alpha_s}{\pi}\sum_{\ell=1}^{\infty}\Big(\frac{\alpha_s}{\pi n}\Big)^{\ell}\sum_{k=0}^{\infty}\epsilon^k h^{(\ell,k)}_{L,g} \,,$$

$$\gamma_{gg} = \sum_{\ell=1}^{\infty}\gamma_{gg,\ell-1}\Big(\frac{\bar{\alpha}_s}{\pi}\Big)^{\ell} \,,$$

$$\bar{F}^g_{L,\text{HP}} = \frac{\alpha_s}{\pi}\sum_{k=-1}^{\infty}\epsilon^k f^{(k)}_{L,g} \,.$$

We have truncated the higher power pieces to $\mathcal{O}(\alpha_s)$ which is sufficient for LL resummation in small-x_b.

BFKL Resummation of F_L

We set $\mu^2 = Q^2$ and start with formula involving unknown pieces (HP = higher power)

$$\bar{F}^g_{L,\text{HP}} + \bar{F}^g_{L,\text{LL}}(n) = \bar{H}^{(g)}_L\Big(n, \frac{Q^2}{\mu^2} = 1, \alpha_s\Big)\bar{\Gamma}_{gg}(\alpha_s, n) \;.$$

By sequentially comparing the coefficients of $(\alpha_s/\epsilon)^\ell$, $\alpha_s(\alpha_s/\epsilon)^\ell, \ldots$ terms we find

$$\gamma_{gg} = \frac{\bar{\alpha}_s}{n} + 2\zeta_3\Big(\frac{\bar{\alpha}_s}{n}\Big)^4 + \ldots\,,$$

$$\bar{H}^{(g)}_L = \frac{2\alpha_s n_f T_F}{3\pi}\bigg(1 - \frac{1}{3}\frac{\bar{\alpha}_s}{n} + \Big(\frac{34}{9} - \zeta_2\Big)\Big(\frac{\bar{\alpha}_s}{n}\Big)^2 + \Big(-\frac{40}{27} + \frac{\pi^2}{18} + \frac{8}{3}\zeta_3\Big)\Big(\frac{\bar{\alpha}_s}{n}\Big)^3 + \ldots\bigg)\,,$$

$$\bar{F}^g_{L,\text{HP}} = \frac{2\alpha_s n_f T_F}{3\pi}\bigg(1 + 3\epsilon + \Big(6 - \frac{1}{2}\zeta_2\Big)\epsilon^2 + \Big(12 - \frac{\pi^2}{4} - \frac{7}{3}\zeta_3\Big)\epsilon^3 + \ldots\bigg)\,.$$

- ✓ Series agree with LL results in Catani and Hautmann [CH94]. Interestingly, we *simultaneously* determine the LL results for γ_{gg} and $\bar{H}^{(g)}_L$.
- ✓ We determined the unknown power suppressed pieces self-consistently!
- ✓ $F^g_{L,\text{HP}}$ has no IR poles $\rightarrow$ All the poles in F_L channel generated through BFKL evolution.

Resummation of F_2

For F_2, we write

$$\bar{F}^{g}_{2,\text{HP}} + \bar{F}^{g}_{2,\text{LL}}(n) = 2n_f\, \bar{\Gamma}_{qg} + \bar{H}^{(g)}_2\, \bar{\Gamma}_{gg}$$

Following the same steps as before, we find

$$\gamma_{qg} = \frac{\alpha_s T_F}{3\pi}\left(1 + \frac{5}{3}\frac{\bar{\alpha}_s}{n} + \frac{14}{9}\left(\frac{\bar{\alpha}_s}{n}\right)^2 + \left(\frac{82}{81} + 2\zeta_3\right)\left(\frac{\bar{\alpha}_s}{n}\right)^3 + \dots\right),$$

$$\bar{H}^{(g)}_2 = \frac{\alpha_s n_f T_F}{3\pi}\left(1 + \left(\frac{43}{9} - 2\zeta_2\right)\frac{\bar{\alpha}_s}{n} + \left(\frac{1234}{81} - \frac{13}{3}\zeta_2 + \frac{4}{3}\zeta_3\right)\left(\frac{\bar{\alpha}_s}{n}\right)^3 + \dots\right),$$

$$\bar{F}^{g}_{2,\text{HP}} = \frac{\alpha_s n_f T_F}{3\pi}\left(-\frac{2}{\epsilon} + 1 + (1+\zeta_2)\epsilon + \left(1 - \frac{1}{2}\zeta_2 + \frac{14}{3}\zeta_3\right)\epsilon^2 + \dots\right).$$

The IR pole in $\bar{F}^{g}_{2,\text{HP}}$ does not result from BFKL evolution. This required [CH94] to introduce a new auxiliary object, *the quark Green's function*. For us it results straightforwardly from our soft function $\tilde{S}_2$.

Comparison with previous work

The Glauber-SCET approach is special: it avoids many problems in the k_T-factorization framework.

Feature	k_T-Factorization	Glauber-SCET
Gauge Invariance	off-shell cross sections: gauge inv. only at LO	Constructed from gauge invariant Glauber ops.
Power Counting	Obscured due to attempting simultaneous twist and high-energy factorization	Manifest power counting from the start
Objects	Requires auxiliary Green's functions	Nothing beyond collinear and soft functions
Isolating $1/\epsilon_{\text{IR}}$	Additional non-trivial conversion for $1/\epsilon_{\text{IR}}$ in $\overline{\text{MS}}$	$S^{\alpha\beta}$ and C can be computed with $1/\epsilon_{\text{IR}}$ in $\overline{\text{MS}}$
Handling collinear (non-BFKL) $1/\epsilon_{\text{IR}}$	New non-BFKL $1/\epsilon_{\text{IR}} \rightarrow$ new Green's functions	No mixing of BFKL and collinear $1/\epsilon_{\text{IR}}$ poles.

Outline

Motivation

Problems in the Small-x Resummation in DIS

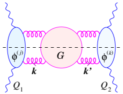
The Glauber SCET Framework

Small-x Factorization from Glauber SCET

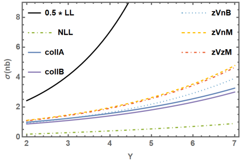
Towards NLL Small-x Resummation

The Collinear function

Avoids **two more problems** in the k_T-factorization framework



- For extension to NLL, k_T-Factorization requires computation of Impact Factors:
- Impact factors are process-dependent.
- Finite pieces are ambiguous: factorization scheme dependence.



BFKL exchange contribution to $\gamma^*\gamma^*$ scattering in Colferai, Li and Stasto '23 [CLS24]

In the EFT approach, the collinear function is universal and has a well-defined operator definition.

The Collinear function

Avoids **two more problems** in the k_T-factorization framework

> For extension to NLL, k_T-Factorization requires computation of Impact Factors:

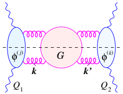
> Impact factors are process-dependent.

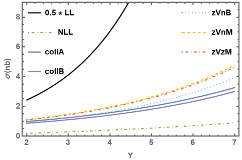
> Finite pieces are ambiguous: factorization scheme dependence.

In the EFT approach, the collinear function is universal and has a well-defined operator definition.

> For NLL small x, we would like to go to two loops.

> Find over 1000 diagrams!!!





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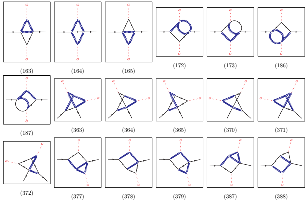


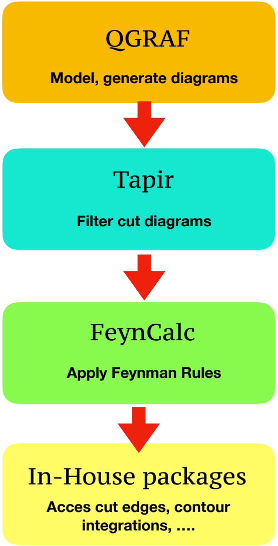
Need for automation

At two loops computing by hand becomes impractical. However, standard tools for Feynman Diagrams do not straightforwardly apply:

- SCET Lagrangian is not Lorentz Invariant
- Presence of Wilson lines leads to vertices with arbitrary valency
- Nontrivial problem to represent operators: $\chi_n, \mathcal{B}^{\mu}_{n\perp}, \mathcal{O}_n, \ldots$

The `SCETCalc` framework:

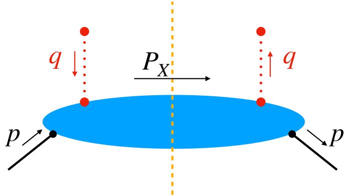
- Being built on `FeynCalc` [SMO20] development version that supports light-cone vectors.
- `QGRAF` [Nog93] to build the SCET “model”
- Automate all SCET Feynman rules
- Combine with `Tapir` [GHL23] for filtering cut diagrams
- In-house software in `Mathematica` for accessing and playing with cut edges.



Collinear function @ NNLO: First Step

Operator definition:

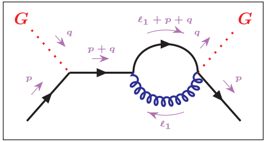
$$C\Big(\frac{\nu}{\bar n\cdot p}, q_\perp, \epsilon\Big) \equiv \frac{1}{\boldsymbol{q}_\perp^2}\frac{1}{\pi\nu}\sum_{i,j,A}\int\frac{\mathrm{d}n\cdot q}{2\pi}\int \mathrm{d}^d x\, e^{\mathrm{i}x\cdot q}\langle P|\mathcal{O}_n^{i\,A}(x)\mathcal{O}_n^{j\,A}(0)|P\rangle_\nu$$



A more useful starting point for computation:

$$C_\kappa\Big(\frac{\nu}{\bar n\cdot p}, q_\perp, \epsilon\Big) = 2\int\frac{\mathrm{d}n\cdot q}{2\pi}\,\mathrm{Im}\,\mathcal{C}_\kappa\Big(\frac{\nu}{\bar n\cdot p}, q^\mu, \epsilon\Big)$$

$$\mathcal{C}_\kappa\Big(\frac{\nu}{\bar n\cdot p}, q^\mu, \epsilon\Big) \equiv \frac{1}{\boldsymbol{q}_\perp^2}\frac{1}{\pi\nu}\frac{1}{2N_\kappa}\sum_{i,j=q,g}\sum_A \mathrm{i}\int \mathrm{d}^d x\, e^{\mathrm{i}x\cdot q}\langle\kappa(P)|\mathrm{T}\left\{\mathcal{O}_n^{iA}(x)\mathcal{O}_n^{jA}(0)\right\}|\kappa(P)\rangle\,,$$



Make use of the Glauber collapse rule by integrating over $q^+ = n\cdot q$:

$$\int \mathrm{d}q^+\mathrm{d}\ell^+\,\frac{1}{(p+q)^2+\mathrm{i}0}\,\frac{1}{(\ell+p+q)^2+\mathrm{i}0}\,\frac{1}{\ell^2+\mathrm{i}0}$$

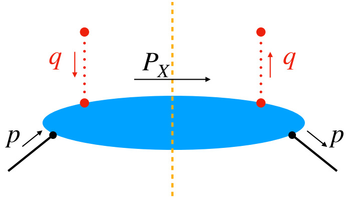
$$= \int \mathrm{d}q^+\mathrm{d}\ell^+\,\frac{1}{p^-q^+ + q_\perp^2+\mathrm{i}0}\,\frac{1}{(\ell^-+p^-)(\ell^++q^+)+(\ell_\perp+q_\perp)^2+\mathrm{i}0}\,\frac{1}{\ell^-\ell^++\ell_\perp^2+\mathrm{i}0}$$

$$= \Big(\frac{-\mathrm{i}}{p^-}\Big)\int \mathrm{d}\ell^+\,\frac{\Theta(-(\ell^-+p^-))}{(\ell^-+p^-)(\ell^++q^+)+(\ell_\perp+q_\perp)^2+\mathrm{i}0}\,\frac{1}{\ell^-\ell^++\ell_\perp^2+\mathrm{i}0}\Bigg|_{q^+\to\frac{-q_\perp^2}{p^-}} = 0\,.$$

Collinear function @ NNLO: First Step

Operator definition:

$$C\left(\frac{\nu}{\bar{n}\cdot p}, q_\perp, \epsilon\right) \equiv \frac{1}{\boldsymbol{q}_\perp^2}\frac{1}{\pi\nu}\sum_{i,j,A}\int\frac{\mathrm{d}n\cdot q}{2\pi}\int \mathrm{d}^d x\, e^{\mathrm{i}x\cdot q}\langle P|\mathcal{O}_n^{i\,A}(x)\mathcal{O}_n^{j\,A}(0)|P\rangle_\nu$$



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> Take the imaginary part after doing $n\cdot q$ integral:

$$C_\kappa\left(\frac{\nu}{\bar{n}\cdot p}, q_\perp, \epsilon\right) = 2\,\mathrm{Im}\int\frac{\mathrm{d}n\cdot q}{2\pi}\left|\frac{n\cdot q}{\nu'}\right|^{-\eta'}\mathcal{C}_\kappa\left(\frac{\nu}{\bar{n}\cdot p}, q^\mu, \epsilon\right)$$

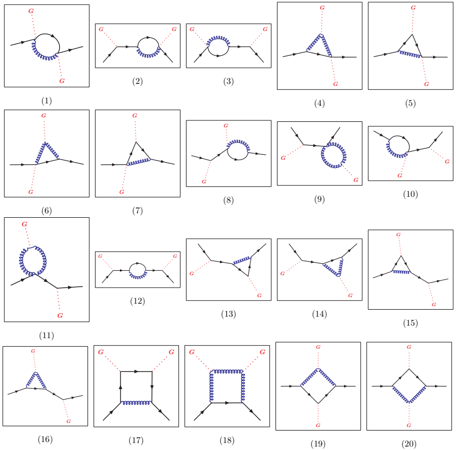
This requires a non-analytic regulator.

Filter graphs using collapse rule

- Exploiting the collapse rule allows us to filter a range of graphs.
- Previously these one-loop graphs were computed via taking cuts:

$$C_q^{(1),\text{real}}\left(\frac{\nu}{\bar{n}\cdot P},\vec{q}_\perp^{\,2}\right) = \frac{2\alpha_s}{\pi\vec{q}_\perp^{\,2}}\frac{\bar{n}\cdot P}{\nu}I_\epsilon[\vec{q}_\perp^{\,2}]\int_0^1 \mathrm{d}z\, P_{gq}^{\eta}(z)$$
$$\times\left[\left(C_F^2-\frac{C_FC_A}{2}\right)z^{-2\epsilon}+\frac{C_FC_A}{2}\left((1-z)^{-2\epsilon}+1\right)\right].$$

- All pieces except for the highlighted term cancel against virtual graphs.



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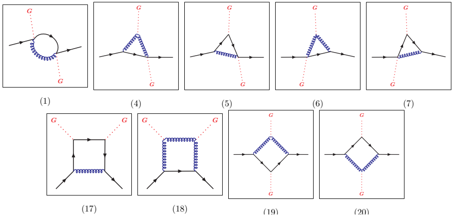

- This can be avoided by invoking the collapse rule (implemented as ℓ_1^+ contour integration).

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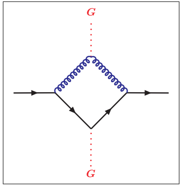

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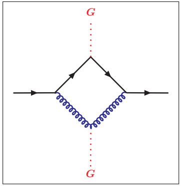


(20)

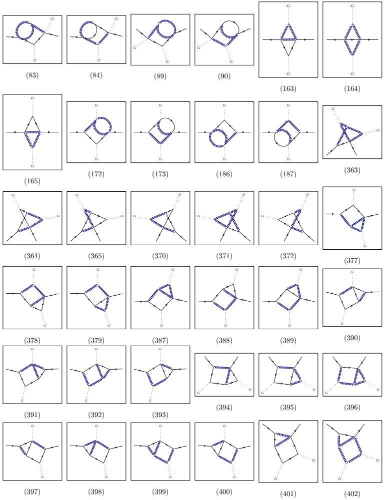
- All pieces except for the highlighted term cancel against virtual graphs.
- This can be avoided by invoking the collapse rule (implemented as ℓ_1^+ contour integration).
- Further drop scaleless integrals:
- Left with a single graph!

$$\mathcal{C}_q^{[1]} = C_F C_A g^2 \frac{P^-}{\pi\nu}\int_0^1 \mathrm{d}z \frac{1+(1-z)^2}{z}\int\frac{\mathrm{d}^{d-2}\ell_\perp}{\ell_\perp^2(\ell_\perp+q_\perp)^2},$$

NNLO graphs: What's left?

> At two loops this is a must. Start with 763 (of 1866) graphs that have valid cuts

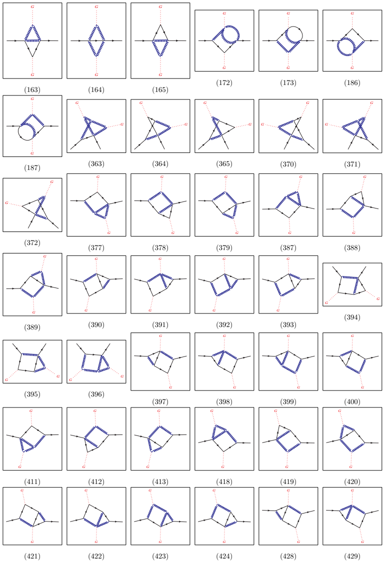
> Applying collapse rule results in 179 graphs.



NNLO graphs: What's left?

- At two loops this is a must. Start with 763 (of 1866) graphs that have valid cuts
- Applying collapse rule results in 179 graphs.
- Further drop scaleless graphs with dangling Glaubers:

  $\rightarrow$ Left with 155 graphs.



Conclusion

- Glauber SCET is a promising framework for reliably and efficiently computing NLL small-x corrections.
- Offers several advantages over previous approaches.
  - Factorization functions gauge invariant to all orders.
  - No separate Green's functions needed to be calculated.
  - Off-shell cross sections replaced by one soft function $S^{\alpha\beta}$ for all DIS channels.
  - Manifest power counting.
  - No factorization or scheme dependencies.
  - Universal, process independent, collinear-function.
- This work provides a new framework for extending resummed calculations for coefficient functions and anomalous dimensions to higher logarithmic orders.
- First step towards NLL small-x resummation: two loop computation in SCET requires automation. Aim is to create a much more general tool-set for the community.

Thank you!

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