Three-dimensional gravity and logarithmic conformal field theories

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INSTITUTE FOR THEORETICAL PHYSICS

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• LCFTs appear in condensed matter physics
  to describe systems with quenched disorder
  (spin glasses, quenched random magnets, percolation)

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LCFT – a logarithmic pair

- the Hamiltonian does not diagonalize
- two operators have degenerate conformal weights and form a logarithmic pair

$$H \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O} \end{pmatrix} = \begin{pmatrix} E & 1 \\ 0 & E \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O} \end{pmatrix} \qquad J \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O} \end{pmatrix} = \begin{pmatrix} j & 0 \\ 0 & j \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O} \end{pmatrix}$$

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- the rank of the Jordan cell denotes the level of degeneration

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- there are logs in the correlators

$$\langle \mathcal{O}(z)\, \mathcal{O}(0) \rangle = \frac{0}{2z^{2h}}$$

$$\langle \mathcal{O}(z)\, \mathcal{O}^{\log}(0,0) \rangle = \frac{b}{2z^{2h}}$$

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- non-vanishing correlators are given by the new anomaly b

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- R. . . Ricci scalar

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- R... Ricci scalar
- λ... cosmological constant, to have black hole solutions

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- $L_{CS} \ldots$ Chern–Simons term, adding a graviton

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- R... Ricci scalar
- λ... cosmological constant, to have black hole solutions
- L_{CS}... Chern–Simons term, adding a graviton
- K... quadratic in R, another graviton

GMG – equations of motion

- linearize around AdS: $g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$
- the action is of fourth order in derivatives
- we can write the equations of motion as

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- consider all possible degenerations
  that we get by tuning m_1 and m_2

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- $D^{m_1} = D^L$:
  T_{zz} has log-partner!
- $D^{m_1} = D^{m_2}$:
  $\mathcal{O}^M$ has log-partner!
- $D^{m_1} = D^{m_2} = D^L$:
  Rank three Jordan cell!
- $c_L = c_R = 0$: log-NMG
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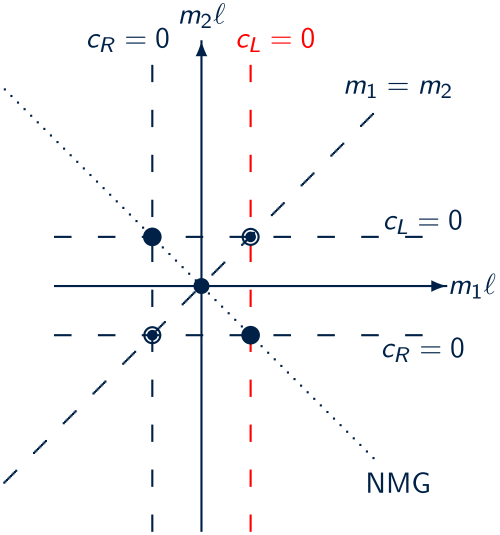

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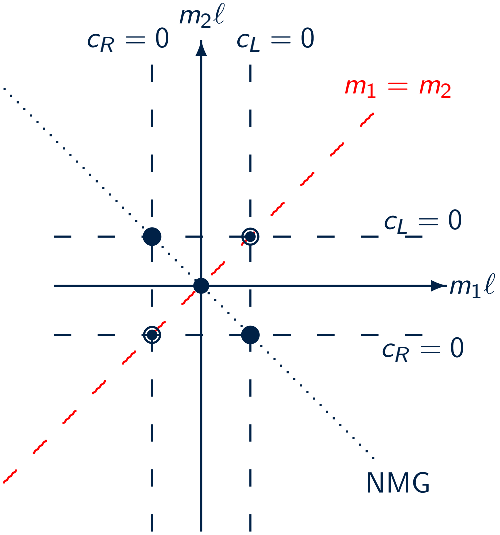

GMG – rank two LCFT dual

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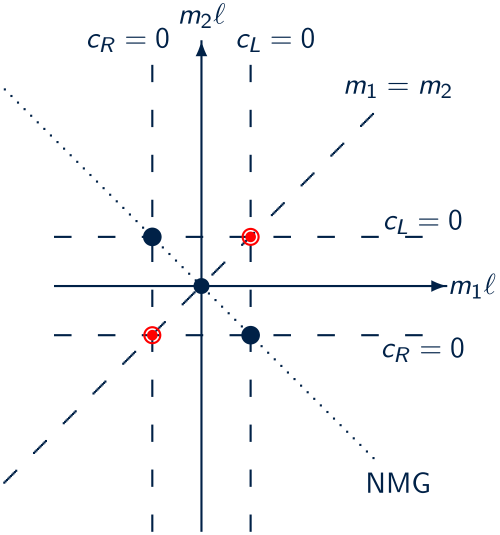

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and $diag(2,2,2)$ for J.

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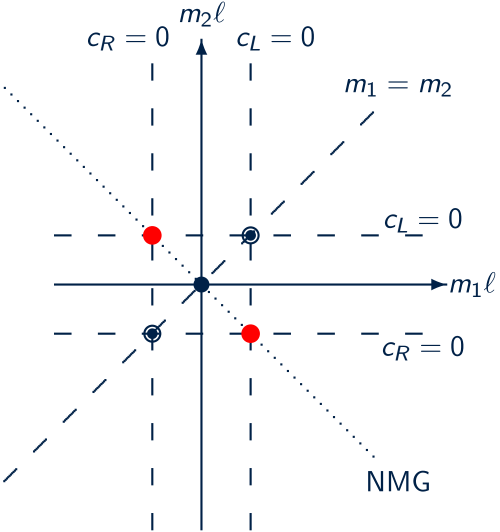

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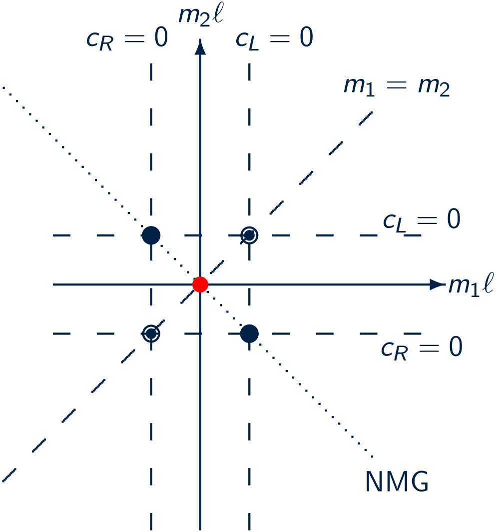

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Thank you for your attention!

Thank you!

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- E. A. Bergshoeff, O. Hohm, and P. K. Townsend, "Massive Gravity in Three Dimensions," *Phys. Rev. Lett.* **102** (2009) 201301, 0901.1766.
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where

$$L_{CS} = \frac{1}{2}\varepsilon^{\lambda\mu\nu}\Gamma^{\alpha}_{\lambda\sigma}\left[\partial_\mu\Gamma^{\sigma}_{\alpha\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\alpha}\right] \tag{3}$$

$$K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\,. \tag{4}$$

Expressions for $\mathcal{D}$ and $m_\pm$

$$(\mathcal{D}^L \mathcal{D}^R \mathcal{D}^{m_+} \mathcal{D}^{m_-} h)_{\mu\nu} = 0$$

with

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu{}^\nu \pm \ell \varepsilon_\mu{}^{\alpha\nu} \nabla_\alpha \quad (5)$$

$$(\mathcal{D}^{m_\pm})_\mu{}^\nu = \delta_\mu{}^\nu + \frac{1}{m_\pm} \varepsilon_\mu{}^{\alpha\nu} \nabla_\alpha \quad (6)$$

and

$$\ell \, m_\pm = \frac{m^2 \ell^2}{2\mu\ell} \pm \sqrt{\frac{m^4 \ell^4}{4\mu^2 \ell^2} - \sigma m^2 \ell^2 + \frac{1}{2}}\,. \quad (7)$$

CFT – obtaining a logarithmic partner operator

Again we obtain the logarithmic operator $\mathcal{O}^{\log}$ by considering the difference between $\mathcal{O}^m$ and $\mathcal{O}^L$:

$$\mathcal{O}^m(\epsilon = 0) = \mathcal{O}^L \tag{8}$$

$$\big(h(\epsilon), \bar{h}(\epsilon)\big) = \big(2 + \epsilon, \epsilon\big) \tag{9}$$

and define

$$\mathcal{O}^{\log} := \lim_{\epsilon \to 0} \frac{\mathcal{O}^m(\epsilon) - \mathcal{O}^L}{\epsilon} \tag{10}$$