Counterterm Contributions to the Anomalous Magnetic Moment of the Muon in the Minimal Supersymmetric Standard Model

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München,
06. Dezember 2010

TECHNISCHE UNIVERSITÄT DRESDEN

Content

What is a_μ?

experimental point of view:

electrically charged particle in a homogeneous magnetic field:
Hamiltonian: $H = -2\,(1 + a_\mu)\,\frac{e}{2m}\vec{S} \cdot \vec{B}$

$$\text{circular motion: } \omega_\mathrm{c} = \frac{Q}{m} B$$

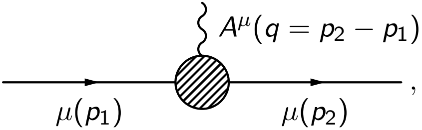
$$\text{spin precession: } \omega_\mathrm{s} = \frac{2\,(1 + a_\mu)\,Q}{2m} B$$

$$\text{measurement: } \omega_\mathrm{a} = \omega_\mathrm{s} - \omega_\mathrm{c} = a_\mu \frac{Q}{m} B$$

What is a_μ?

compare with Quantum Field Theory:

$M(x; p_1, p_2) =$



,

$$\tilde{M}(q; p_1, p_2) = \mathrm{i}e\bar{u}(p_2)\left[\gamma^\mu F_\mathrm{E}(q^2) + \mathrm{i}\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_\mathrm{M}(q^2)\right] u(p_1),$$

result: $a_\mu \equiv F_\mathrm{M}(0)$.

Standard Model Contributions to a_μ

QED	$(11\,658\,471.809 \pm 0.015) \cdot 10^{-10}$
[T. Kinoshita, M. Nio, Physical Review D 73 (2006)]	
electroweak	$(15.4 \pm 0.1 \pm 0.2) \cdot 10^{-10}$
[A. Czarnecki, W. J. Marciano, A. Vainshtein, Physical Review D 67 (2003)], [R. Jackiw, S. Weinberg, Physical Review D 5 (1972)]	
hadronic	$(693.0 \pm 4.2 \pm 2.6 \pm 0.09) \cdot 10^{-10}$
[M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan, Z. Zhang, arXiv:1010.4180 (2010)], [K. Hagiwara, A. D. Martin, D. Nomura, T. Teubner, Physics Letters B 649 (2007)], [J. Prades, E. de Rafael, A. Vainshtein (2009)]	

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experiment: $$a_\mu^{\text{BNL}} = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10}$$

[G. W. Bennett et al., Physical Review D 73 (2006)],

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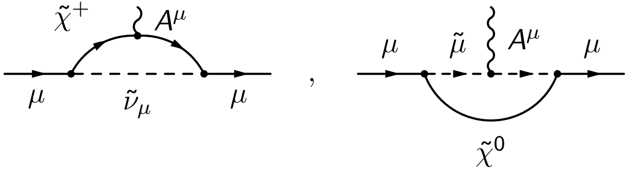
Is there physics beyond the Standard Model?

loops with Chargino $\tilde{\chi}^+$ or Neutralino $\tilde{\chi}^0$



$$a_\mu^{\tilde{\chi}_i} = \frac{e^2}{16\pi^2 \sin^2(\theta_W)} \frac{m_\mu^2}{m_{\tilde{\ell}}^2} \left[F_1^{(0)}(x_i) C_1^{\tilde{\chi}_i} + m_{\tilde{\chi}_i} F_2^{(0)}(x_i) C_2^{\tilde{\chi}_i} \right]$$

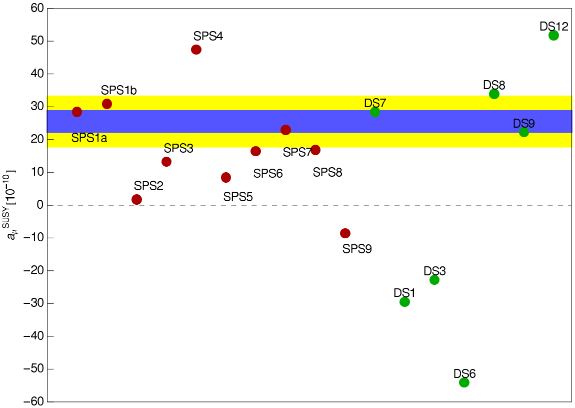
with $C_j^{\tilde{\chi}_i}$: containing all couplings,

$F_i^{(k)}$: formfactors, depending on $x_i = \frac{m_{\tilde{\chi}_i}^2}{m_{\tilde{\ell}}^2}$

discrepancy between experiment and Standard Model prediction can be explained completely by the MSSM

Predictions from a_μ in the MSSM

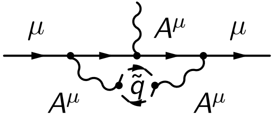
a_μ useful for measuring MSSM parameters, complementary to the LHC



plot: [D. Stöckinger (2010)],

green points: [Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas (2010)]

MSSM Contributions to a_μ: Two-Loop Corrections

1. MSSM corrections to Standard Model diagrams e. g. 

[S. Heinemeyer, D. Stöckinger, G. Weiglein, Nuclear Physics B 690 (2004)]

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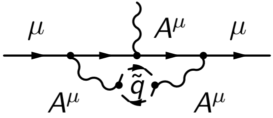
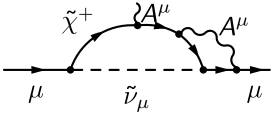


2. corrections to diagrams with supersymmetric particles:
   - photonic corrections
     resulting in
     leading logarithms e. g.



[G. Degrassi, G. F. Giudice, Physical Review D 58 (1998)]

[P. von Weitershausen, M. Schäfer, and H. Stöckinger-Kim, and D. Stöckinger, (2009)]

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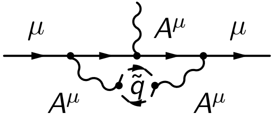

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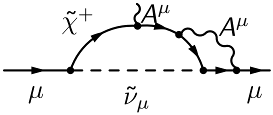
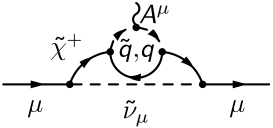


   - corrections with quarks, leptons and their superpartners e. g.



[M. Schäfer, diploma thesis (2009)],
huge algebraical results $\Rightarrow$ integration by parts method fails

MSSM Contributions to a_μ: Two-Loop Corrections

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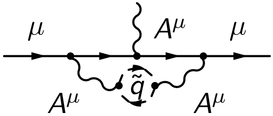

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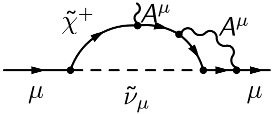

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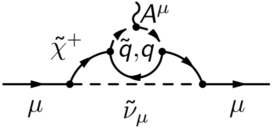
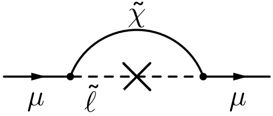


task in my diploma thesis:
renormalisation of this two-loop diagrams

Renormalisation and Counterterm Contributions to a_μ

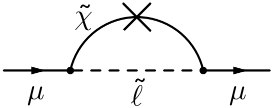
Three Different Classes of Counterterm Corrections

1. corrections to Smuon or Sneutrino propagator



divergent!

2. corrections to Chargino or Neutralino propagator



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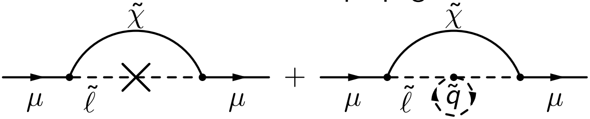

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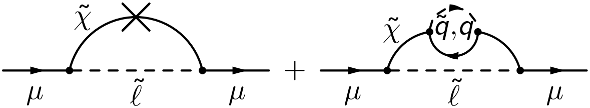

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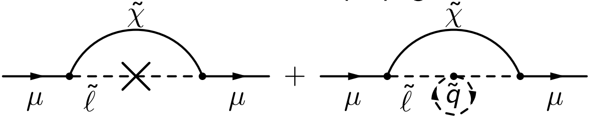

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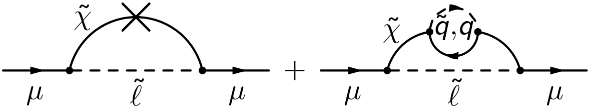
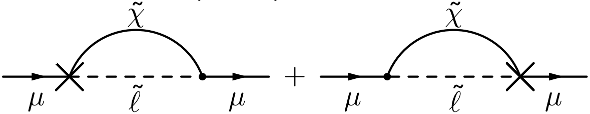


3. corrections to vertices (finite!)



Analytical Result

factorisation to

$$
\begin{aligned}
a_\mu^{\tilde{\chi}_i} = \frac{e^2}{16\pi^2 \sin^2(\theta_{\mathrm{W}})} \frac{m_\mu^2}{m_{\tilde{\ell}}^2} & \\
\lim_{\epsilon \to 0} \Big[\Big(F_1^{(0)}(x_i) + \epsilon F_1^{(1)}(x_i) \Big) \, \delta C_1^{\tilde{\chi}_i} & \\
+ \, m_{\tilde{\chi}_i^+} \Big(F_2^{(0)}(x_i) + \epsilon F_2^{(1)}(x_i) \Big) \, \delta C_2^{\tilde{\chi}_i} \Big] &
\end{aligned}
$$

with $\delta C_j^{\tilde{\chi}_i}$: containing all couplings and renormalisation constants

Analytical Result

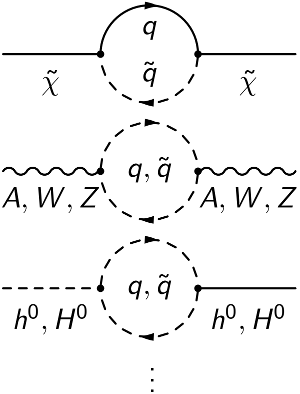
factorisation to

$$a_\mu^{\tilde{\chi}_i} = \frac{e^2}{16\pi^2 \sin^2(\theta_W)} \frac{m_\mu^2}{m_{\tilde{\ell}}^2} \lim_{\epsilon \to 0} \left[\left(F_1^{(0)}(x_i) + \epsilon \boldsymbol{F_1^{(1)}(x_i)} \right) \delta C_1^{\tilde{\chi}_i} + m_{\tilde{\chi}_i^+} \left(F_2^{(0)}(x_i) + \epsilon \boldsymbol{F_2^{(1)}(x_i)} \right) \delta C_2^{\tilde{\chi}_i} \right]$$

with $\boldsymbol{\delta C_j^{\tilde{\chi}_i}}$: containing all couplings **and renormalisation constants**

Renormalisation Constants

different renormalisation constants neccessary:

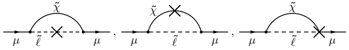
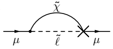


for $\delta m_\chi, \delta Z_\chi$

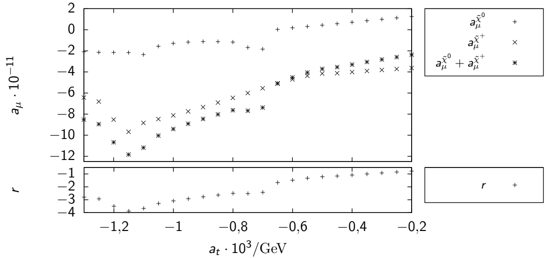
for $\delta M_W, \delta M_Z, \delta \sin\theta_W, \delta Z_e$

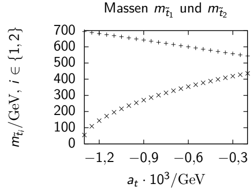
for $\delta \tan\beta$

Results

- all counterterm diagrams () and necessary renormalisation constants have been calculated
- vertex corrections () are finite indeed
- dependence on different MSSM parameters has been investigated

Results





dependence of the vertex corrections on a_t with the other MSSM parameters being fixed at the SPS 1a values, $r = \left(a_\mu^{\tilde{\chi}^0} + a_\mu^{\tilde{\chi}^+}\right) / a_\mu^{\text{1-loop}} \cdot 100$.

Outlook

- combination of the counterterm diagrams and two-loop diagrams (currently being evaluated at the IKTP, TUD)
- calculate more classes of two-loop diagrams and corresponding counterterm diagrams
- add all two-loop predictions to the one-loop result

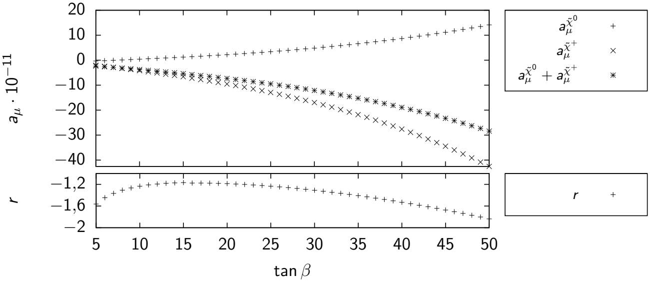
$\Rightarrow$ error on a_μ^{MSSM} lower than $1 \cdot 10^{-10}$

$\Rightarrow$ new experiment / more precise Standard Model prediction become more valuable (restrictions on MSSM parameter space)

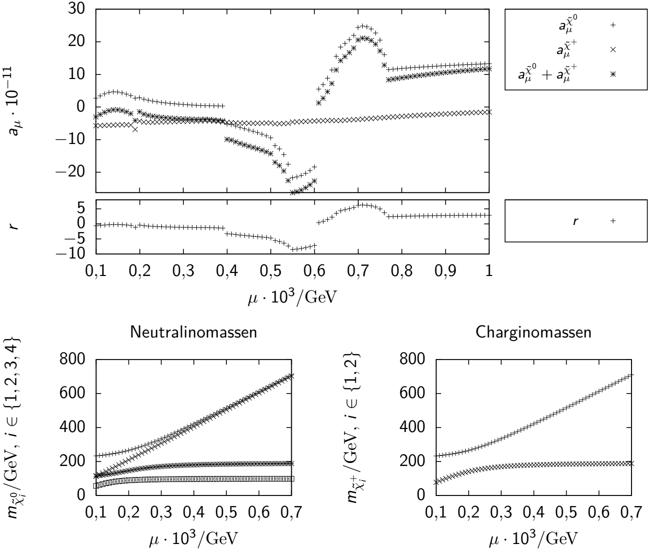
Thanks for your attention!

Renormalisation Scheme

electroweak sector: $\delta M_W, \delta M_Z, \delta \sin\theta_{\mathrm{W}}, \delta Z_e$	renormalised on-shell
Chargino-/Neutralino-sector: $\delta M_1, \delta M_2, \delta\mu$	both Charginos and one Neutralino renormalised on-shell
Sfermion-sector: $\delta m_{\tilde{f},1}, \delta m_{\tilde{f},2}, \delta Y_{\tilde{f}}$	renormalised on-shell, $\delta Y_{\tilde{f}}$ related to δA_f
Higgs-sector: $\delta \tan\beta$	renormalising vacuum expectation values, $\delta Z_{H^0} - \delta Z_{h^0}$



dependence of the vertex corrections on $\tan\beta$ with the other MSSM parameters being fixed at the SPS 1a values, $r = \left(a_\mu^{\tilde{\chi}^0} + a_\mu^{\tilde{\chi}^+}\right) / a_\mu^{\text{1-loop}} \cdot 100$.



dependence of the vertex corrections on μ with the other MSSM parameters being fixed at the SPS 1a values, $r = \left(a_\mu^{\tilde\chi^0} + a_\mu^{\tilde\chi^+}\right) / a_\mu^{\text{1-loop}} \cdot 100$.