# Flavour Structure of SUSY *SO*(10) Grand Unified Theories with Extended Matter Sector

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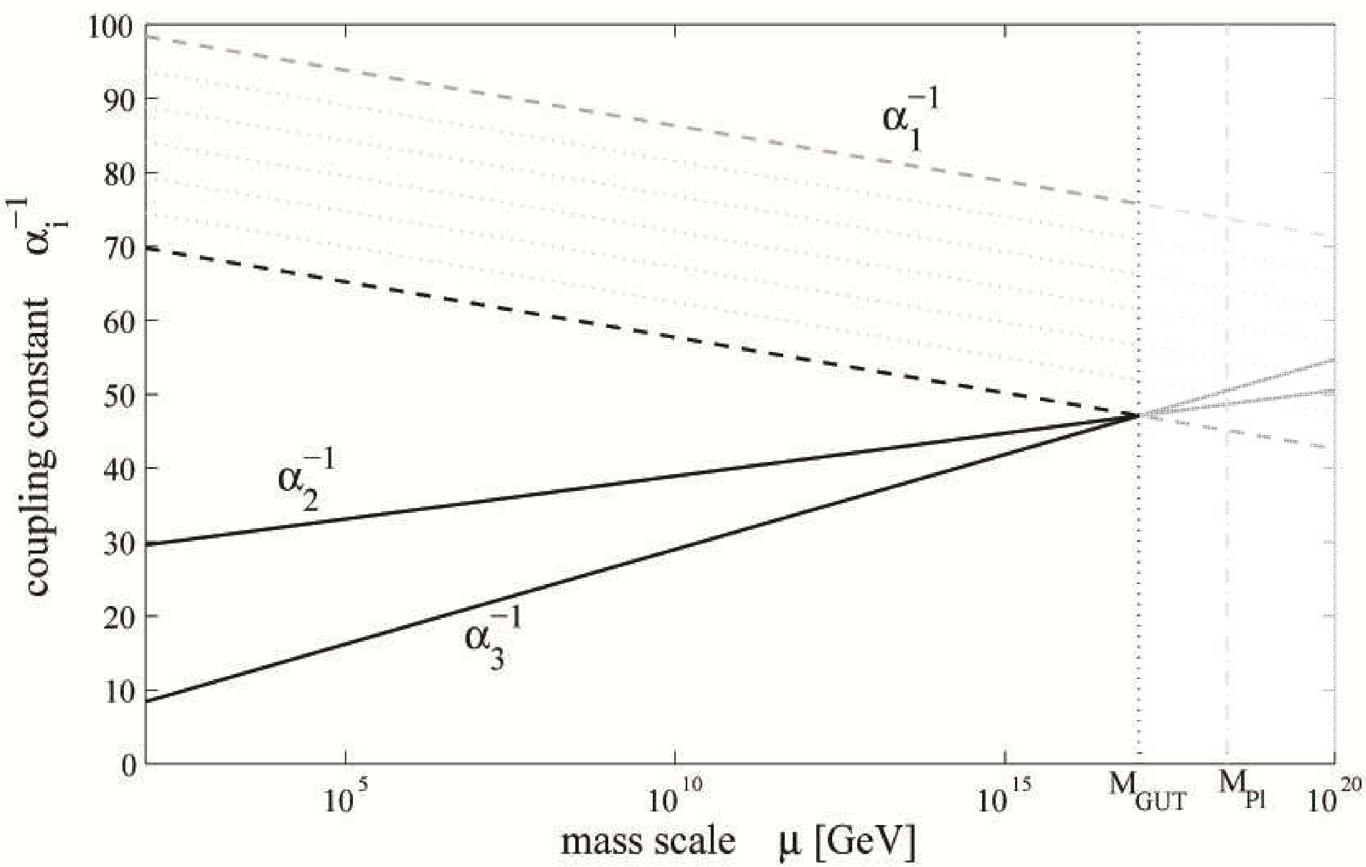
# The Standard Model (SM):

- Contra:
  - in SM: neutrinos have zero mass, $m_\nu^i = 0$
  - measured: $\left|\Delta m_{atm}^2\right| \equiv \left|(m_\nu^3)^2 - (m_\nu^1)^2\right| \approx 2.4 \cdot 10^{-3} eV^2$
- Strange:
  - ratios of hypercharges $Y$ fractional, $\sum Y = 0$
  - generally: anomaly cancellation in SM
  - $B$ and $L$ accidental symmetry
  - $B-L$ non-anomalous ⇨ $\ldots \otimes U(1)_{B-L}$?

# Gauge Unification




⇨ couplings match at $M_{GUT} \approx 10^{16-17} GeV$

# Grand Unified Theories (GUTs)

at $M_{GUT}$ all three SM forces could unify:

$G_{SM}$ subgroup of a simple(?) gauge group $G_{GUT}$

⇨ SM low-energy eff. theory after $G_{GUT}$ broken

⇨ fractional $Y$, (often) SM anomaly-free,

neutrino masses, monopoles, **proton decay**

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<u>Georgi-Glashow model (1974):</u>

H. Georgi and S. L. Glashow,
Phys. Rev. Lett. 32 (1974) 438

$G_{GUT} = SU(5)$, every generation of SM matter in two irreducible representations of $SU(5)$

$$\overline{5}_M = D_L^c \oplus L_L \qquad 10_M = U_L^c \oplus Q_L \oplus E_L^c$$

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minimal setting ruled out

# $SO(10)$ GUTs

gauge group: $G_{GUT} = SO(10)$ (anomaly-free)

subgroups: $SU(5) \otimes U(1)_X$

$G_{PS} \equiv SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$

matter sector: (usually) only one $16_M$ per generation

$$16_M = \underbrace{Q_L \oplus U_L^c \oplus E_L^c}_{} \oplus \underbrace{D_L^c \oplus L_L}_{} \oplus N_L^c \text{ (under } G_{SM}\text{)}$$

$$= \quad 10_M \quad \oplus \quad \bar{5}_M \quad \oplus 1_M \text{ (under } SU(5)\text{)}$$

# $SO(10)$ GUTs

Higgs sector: many possible choices

minimal SUSY GUT: $10_H \oplus 126_H \oplus \overline{126}_H \oplus 210_H$

C.S. Aulakh et. al., Phys. Lett. B588 (2004) 196, hep-ph/0306242

Problems with minimal SUSY GUT:

- not possible to accommodate realistic Yukawas!

  C.S. Aulakh and S.K. Garg, Nucl. Phys. B757 (2006) 47, hep-ph/0512224
  S. Bertolini et. al., Phys. Rev. D73 (2006) 115012, hep-ph/0605006

- 472-dimensional Higgs sector!

⇨ Landau pole: $g_{GUT}(\mu) \to \infty$ (for $\mu < M_{Pl}$)

# $SO(10)$ GUTs

why not $16_H \oplus \overline{16}_H$ instead of $126_H \oplus \overline{126}_H$ ?

⇨ $SU(5)$ breaking not communicated to matter sector at renormalizable level

⇨ need non-ren. operators to deviate from $SU(5)$-like flavor unification $Y_d = Y_e^T$

# SUSY *SO*(10) GUT with Extended Matter Sector

<u>matter sector:</u> three $16_M$'s and $n$ $10_M$'s of $SO(10)$

$$16_M = Q_L \oplus U_L^c \oplus E_L^c \oplus D_L^c \oplus L_L \oplus N_L^c$$

$$10_M = \Delta_L \oplus \Lambda_L^c \oplus \Delta_L^c \oplus \Lambda_L$$

- up-type quarks only in $16_M$'s
- remaining SM particles are mixed

# SUSY *SO*(10) GUT with Extended Matter Sector

Higgs sector: $10_H \oplus 16_H \oplus \overline{16}_H \oplus 45_H \oplus 54_H$

⇨ low dim. ⇨ Landau pole hidden behind $M_{Pl}$

Yukawa superpotential:

$$W_Y = Y\, 16_M 16_M 10_H + F\, 16_M 10_M 16_H + M_{10} 10_M 10_M$$
$$+ \lambda\, 10_M 10_M 54_H + \eta\, 10_M 10_M 45_H$$

- $10_M$'s feel $SU(5)$ breaking in $45_H$ and $54_H$
- $16_M$'s and $10_M$'s mixed by $F\, 16_M 10_M 16_H$

# Results and Outlook

- low-energy observables driven by GUT physics
- pseudo-Dirac neutrinos

⇨ need (≥) three matter singlets $1_M$ ( $E_6$ case)

- calculable triplet neutrino mass contribution!

⇨ Assumption: $M_\nu \approx M_\nu^\Delta$ ( $\Leftrightarrow$ $M_\nu^\Delta >> M_\nu^I$ )

- however, highly non-linear matrix equations

⇨ only analytical solution for minimal case: **one** $10_M$

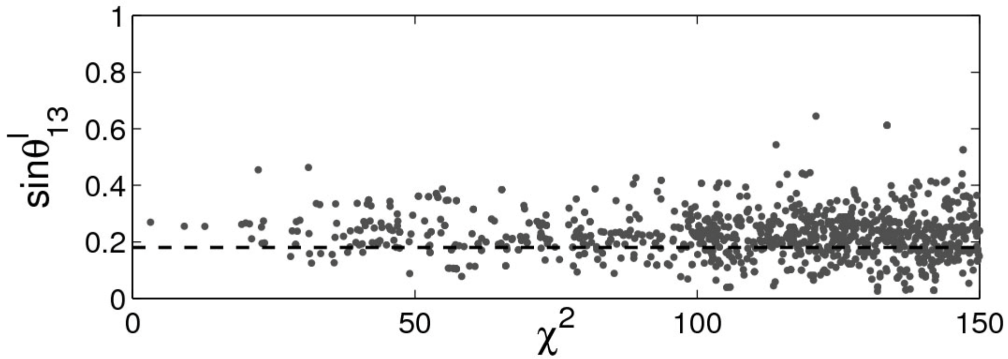
  - fails already for the charged sector only

    M Malinsky, Phys. Rev. D77 (2008) 055016, arXiv:0710.0581 [hep-ph]

# Results and Outlook

next-to-minimal case: **two** $10_M$'s

- $\chi^2$-fit of charged sector (for $\eta \to 0$):
  - $\chi^2 \leq 1$ easily possible for moderate decoupling
- $\chi^2$-fit of all observables:
  - difficult, best solution: $\chi^2 \approx 3.2$ (for $\eta \neq 0$)
  - small $\sin\theta_{13}^l$ preferred, though not fitted!



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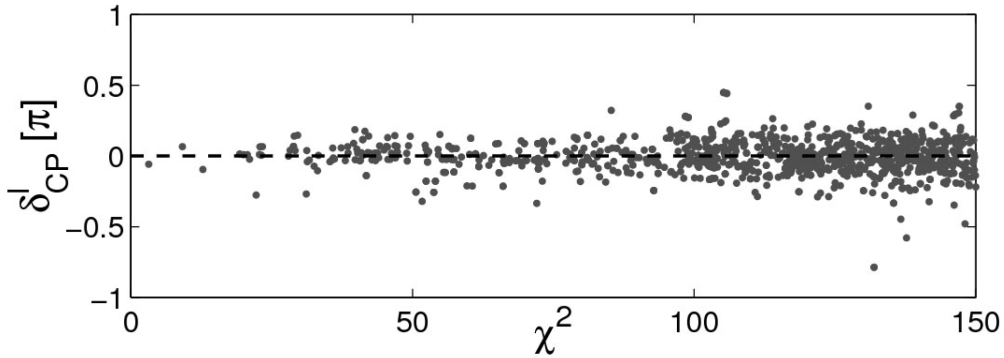

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  - leptonic Dirac CP phase $\delta_{CP}^l \approx 0$ preferred
  - no preference for lept. Majorana CP phase

*Thank you for your attention!*

# Gauge Unification

- couplings $g_i$ depend on energy scale $\mu$, they get renormalized, they “run“
- Renormalization Group Equations at one loop:

$$\alpha_i \equiv g_i^2 / 4\pi; \quad t \equiv \frac{1}{2\pi} \log(\mu / M_Z)$$

$$\Rightarrow \alpha_i^{-1}(t) = \alpha_i^{-1}(t_0) - b_i (t - t_0)$$ (linear equation)

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⇨ $\alpha_i^{-1}(t) = \alpha_i^{-1}(t_0) - b_i (t - t_0)$ (linear equation)

- normalization of $U(1)_Y$ coupling $g_i$ not fixed

⇨ line $\alpha_1^{-1}(t)$ can be shifted up and down, e.g. such that all $\alpha_i^{-1}$ meet at one scale

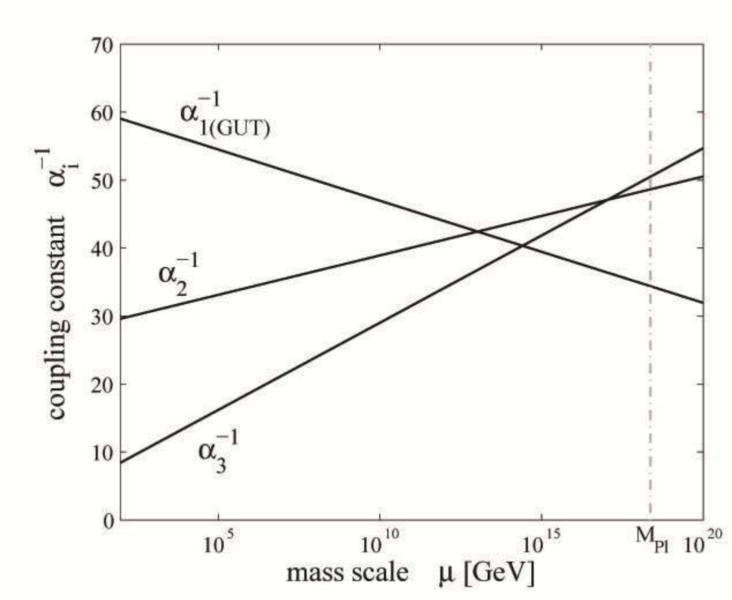
# Why SUSY?

- cancelation of two-loop corrections

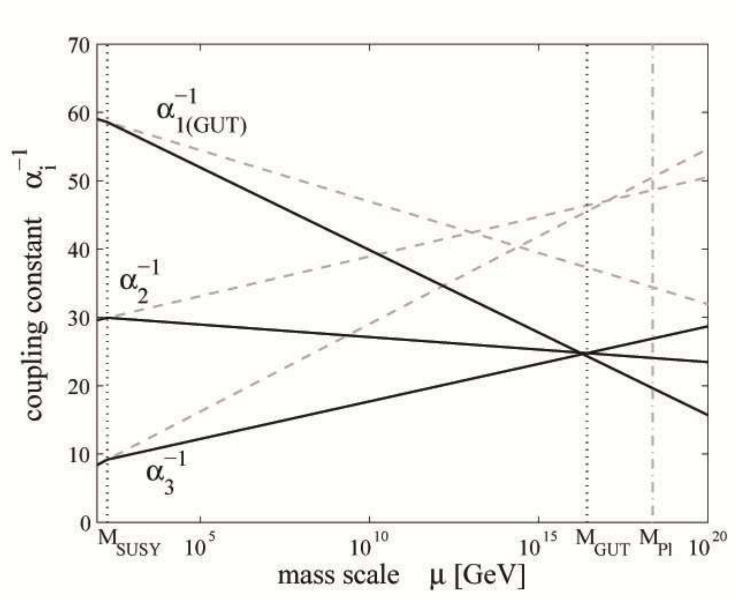
⇨ stabilization of the hierarchy problem

- additional constrains on the Higgs sector
- running of couplings, e.g. for $SU(5)$:

without SUSY:



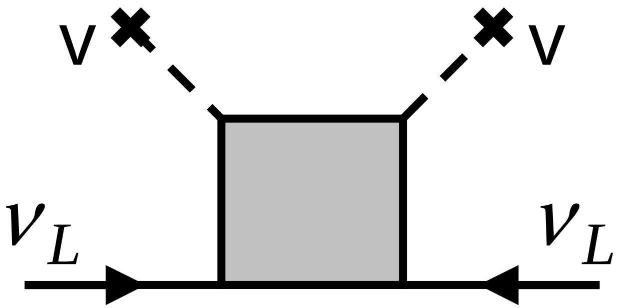

with SUSY:



# The Seesaw Mechanisms

How can SM neutrinos obtain mass?

Weinberg dim.-5 operator:




$$\frac{\kappa}{\Lambda} l_L l_L HH \xrightarrow{EWSB} \frac{v^2}{2\Lambda} \kappa \, \nu_L \nu_L$$

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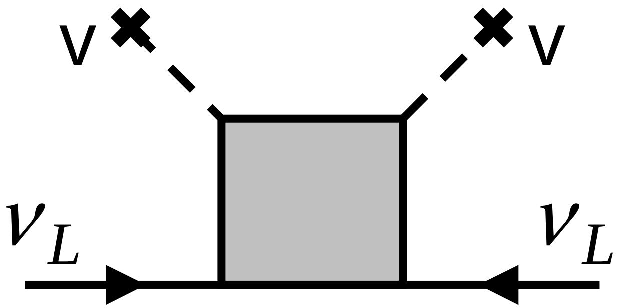
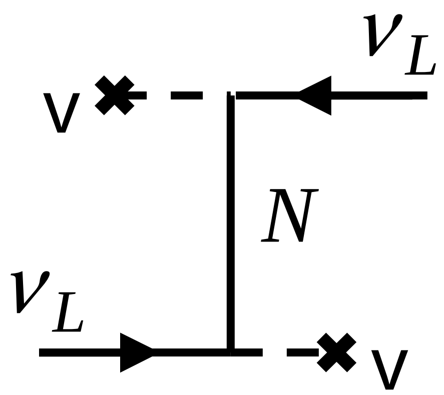



type-I:

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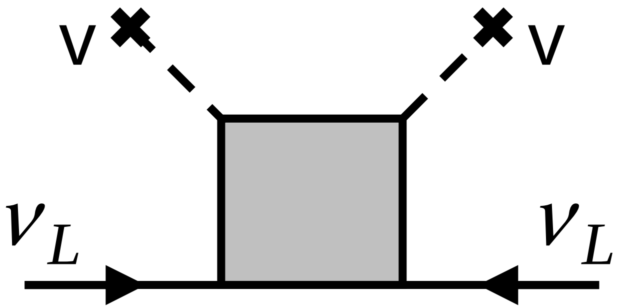


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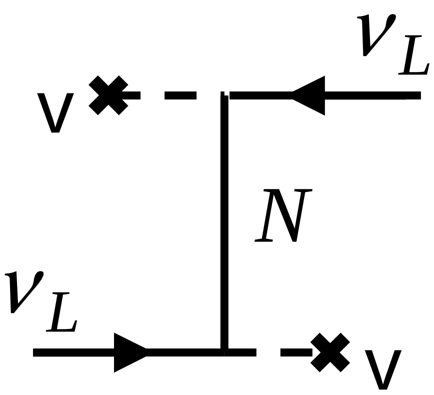


- type-I seesaw mechanism:
  SM neutrinos obtain tiny masses by exchange of a heavy fermionic singlet $\boxed{m_\nu^i \propto v^2 / M_N}$
- type-II: exchange of heavy scalar triplet
- type-III: exchange of heavy fermionic triplet

# Georgi-Glashow Model

- <u>gauge group:</u> $G_{GUT} = SU(5)$

⇨ <u>gauge sector:</u> gauge fields form adjoint of $SU(5)$

$$24 = (8,1,0) \oplus (1,3,0) \oplus (1,1,0) \oplus \underbrace{(3,2,\tfrac{-5}{3}) \oplus (\bar{3},2,\tfrac{+5}{3})}_{}$$

| 8 gluons, 3 vector bosons, 1 boson<br>SM gauge sector | 12 new gauge bosons<br>"Leptoquarks" X, Y |
|---|---|

<u>matter sector:</u> one $\bar{5}_M$ and one $10_M$ per generation

$$\bar{5}_M : \Psi_{La} = \begin{pmatrix} d^c_{L1} \\ d^c_{L2} \\ d^c_{L3} \\ e_L \\ -\nu_L \end{pmatrix} \qquad 10_M : \Psi_L^{\{ab\}} = \begin{pmatrix} 0 & u^c_{L3} & -u^c_{L2} & u_{L1} & d_{L1} \\ -u^c_{L3} & 0 & u^c_{L1} & u_{L2} & d_{L2} \\ u^c_{L2} & -u^c_{L1} & 0 & u_{L3} & d_{L3} \\ -u_{L1} & -u_{L2} & -u_{L3} & 0 & e_R \\ -d_{L1} & -d_{L2} & -d_{L3} & -e^c_L & 0 \end{pmatrix}$$

# Georgi-Glashow Model

- Higgs sector:
  - one $24_H$ to break $G_{GUT} = SU(5)$ to $G_{SM}$
  - one $5_H$ ,including SM Higgs doublet $H = (1,2,+1)$
- Yukawas: $Y_5^{ij} 10_M^i \overline{5}_M^j 5_H^* + Y_{10}^{ij} 10_M^i 10_M^j 5_H$

  down-type quarks & charged leptons | up-type quarks

- $Y_5 \propto \boxed{Y_d = Y_e^T}$ **flavor unification!** $Y_{10} \propto Y_u$

  but: $m_d^i \neq m_e^i$ even at $M_{GUT}$, neutrino masses zero,
  proton decay: $\tau_p^{(GG)} \leq 3 \cdot 10^{30}\, yr$ vs. $\tau_p^{(\exp)} \geq 10^{33}\, yr$
- higher Higgses (e.g. $45_H$) and/or non. ren. operators

# Mass Matrices

up-type quarks: $M_u = Y\, v_u^{10}$

down-type quarks: $M_d = \begin{pmatrix} Y\, v_d^{10} & \boxed{F\, v_d^{16}} \\ \boxed{F^T V^{16}} & M_\Delta \end{pmatrix}$

(with $M_\Delta \equiv M_{10} - \lambda\, V^{54} + \eta\, V^{45}$)

- off-diagonal GUT-scale entry ⇨ block diagonalize

$$M'_d \equiv M_d U_d^\dagger = \begin{pmatrix} Y\, v_d^{10} & F\, v_d^{16} \\ F^T V^{16} & M_\Delta \end{pmatrix} \begin{pmatrix} A_d^\dagger & C_d^\dagger \\ B_d^\dagger & D_d^\dagger \end{pmatrix} = \begin{pmatrix} \tilde{M}_d & \cdot \\ 0 & \bullet \end{pmatrix}$$

- only GUT-scale entry in 22-entry submatrix of $M'_d$

⇨ 11-entry of $M'_d$ is effective SM mass matrix $\tilde{M}_d$

# Mass Matrices

<u>charged leptons:</u> simular to down-type quarks

<u>neutrinos:</u> very simular to charged leptons

- correspondence between enties of $U_e$ and $U_\nu$
- but (even after block diagonalization):
  $\nu_L$ mixes only with $N_R$ by Dirac mass $\propto v_u^{10}$
  and $N_R$ Majorana mass is zero

⇨ electroweak-scale pseudo-Dirac neutrinos ↯

# Mass Matrices

electroweak-scale pseudo-Dirac neutrinos

solutions:

- include effective non. ren. operators
- include at least 3 new matter singlets ⇨ $E_6$ GUTs

$$E_6 \supset 27_M = 16_M \oplus 10_M \oplus 1_M \subset SO(10)$$

⇨ three $16_M$'s ⇨ three $10_M$'s and three $1_M$'s

both cases: $N_R$ obtain GUT-scale Majorana mass

⇨ type-I seesaw mechanism works;

$\nu_L$ obtain tiny type-I seesaw contribution $\tilde{M}_\nu^I$

# Effective Mass Matrices

eff. up-type quark mass matrix $\widetilde{M}_u$ taken as input

$$\widetilde{M}_d = \left( \frac{v_d^{10}}{v_d^{10}} \widetilde{M}_u - v_d^{16} V^{16} F \, (M_\Delta)^{-1} F^T \right) V_d^\dagger$$

$$\widetilde{M}_e^T = \left( \frac{v_d^{10}}{v_d^{10}} \widetilde{M}_u - v_d^{16} V^{16} F \, (M_\Lambda^T)^{-1} F^T \right) V_e^\dagger$$

$$\widetilde{M}_\nu^{II} \propto V_e^* F \, (M_\Lambda)^{-1} \lambda \, (M_\Lambda^T)^{-1} F^T V_e^\dagger$$

with

$$V_d = V_d^\dagger \equiv \left( \mathbf{1} + \left| V^{16} \right|^2 F^* (M_\Delta M_\Delta^\dagger)^{-1} F^T \right)^{-1/2}$$

$$V_e = V_e^\dagger \equiv \left( \mathbf{1} + \left| V^{16} \right|^2 F^* (M_\Lambda^T M_\Lambda^*)^{-1} F^T \right)^{-1/2}$$

# Effective Mass Matrices

<u>Assumption:</u> type-II seesaw contribution dominates

$$\widetilde{M}_\nu = \widetilde{M}_\nu^{I} + \widetilde{M}_\nu^{II} \approx \widetilde{M}_\nu^{II} \quad \text{… since calculable}$$

---

- highly non-linear matrix equarions
- only analytic solution for one $10_M$
  but not even possible to fit to charged sector only
- for $n > 1$ of $10_M$‘s no analytic solutions

⇨ numerical solution:
  $\chi^2$-fit of the model to the physical observables

# Parameter Counting

case with $n > 1$ $10_M$'s:

$\boxed{2n^2 + 7n + 3}$ parameters determine

- 3 down-type quark and 3 charged lepton masses
- 3 CKM and 3 PMNS angles
- 1 CKM and 1 PMNS Dirac CP phase
- 2 neutrino mass ratios

16 physical observables

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$\boxed{2n^2 + 7n + 3}$ parameters determine

- 3 down-type quark and 3 charged lepton masses
- 3 CKM and **2** PMNS angles
- 1 CKM and **0** PMNS Dirac CP phase
- **1** neutrino squared mass difference ratio

**13** measured physical observables

$\boxed{n = 2:\quad \textbf{25}\text{ model parameters}}$

~~$n = 3$: **42** model parameters ($E_6$ GUT case)~~

# Results and Outlook

<u>next-to-minimal case:</u> **two** $10_M$'s

⇨ keep tracked of decoupling of the heavier $10_M$

$$W_Y = Y\, 16_M 16_M 10_H + F\, 16_M 10_M 16_H + M_{10} 10_M 10_M$$

$$\cancel{+\lambda\, 10_M 10_M 54_H + \eta\, 10_M 10_M 45_H}$$

$$M_{10} \equiv t \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}$$

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$$M_{10} \equiv t\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} \quad \lambda V_{54} \equiv t\begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix}$$

decoupling is parameterized by

$$P \equiv p / \max\left\{ |c_{12}|^2, |c_{22}| \right\}$$

# Results and Outlook

- need for **three** $10_M$'s?

fundamental $27$ of $E_6$ branches under $SO(10)$ as:

$$27 = 16 \oplus 10 \oplus 1$$

⇨ **three** $16_M$'s demands **three** $10_M$'s and **three** $1_M$'s

$E_6$ and $E_8$ gauge groups motivated by string theory

**But:**
- highest eigenvalue of $M_{10}$ close to $M_{Pl}$ (?)
- for $n = 3$ model loses predictivity

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- assumption $M_\nu^{II} >> M_\nu^{I}$ wrong?

⇨ flavour model building, e.g. Froggatt-Nielsen