Concepts of Experiments at Future Colliders II

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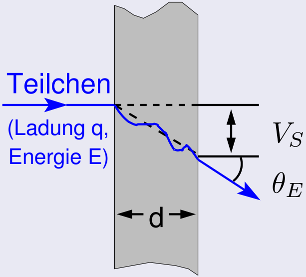
Recapitulation of the previous lecture

Monte Carlo method

- The probability density describing, for example, the outcome of a proton-proton collision measurement is composed of many probability densities and generally cannot be given analytically.
- The probability distribution can be obtained using the so-called Monte Carlo method.
  - In the Monte Carlo method, the overall process is broken down into subprocesses $T_1, \dots T_n$, for which the probability densities are known.
  - Using a random number generator, an outcome of T_1 is generated according to its probability density.
  - For this outcome of T_1, the outcome of T_2 is generated accordingly, and this procedure is continued up to T_n.
  - If this is repeated very often, the probability distribution for the overall process is gradually obtained.

Example: Scattering of muons in thick layers

Scattering of heavy charged particles in thin layers



Strahlungslänge der Schicht: X_0

- Energy loss in the layer is negligible.
- $V_S \approx \frac{1}{2} d\theta_E$.
- θ_E is approximately normally distributed around 0 with the standard deviation

$$\theta_0 := \frac{13.6\ \mathrm{MeV}}{E}\sqrt{\frac{d}{X_0}}.$$

Example: Scattering of muons in thick layers

Goal: Scattering in thick layers

Differences

- $V_S \neq \frac{1}{2} d\theta_E$.
- Energy loss is not negligible.

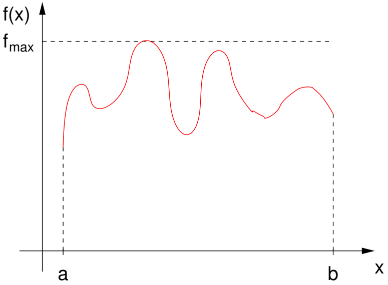
Solution

- Divide the thick layer into many thin layers.
- Describe the passage through thick layers as a sequence of random processes, namely as a sequence of scatterings in the thin layers.

Recapitulation of the previous lecture

Random number generators

- (Deterministic) random number generators refer to computer programs that can generate a sequence of pseudorandom numbers.
- They are called pseudorandom numbers because, although the generated numbers appear random, they are produced using a deterministic algorithm.
- If you have a random number generator that produces random numbers uniformly distributed in an interval $[a, b]$, you can generate random numbers according to any probability distribution.



- First, generate a uniformly distributed random number $x \in [a, b]$.
- Then, generate a uniformly distributed random number $y \in [0, f_{max}]$.
- If $y < f(x)$, keep the random number x; otherwise, discard it and generate a new number $x \in [a, b]$ until $y < f(x)$.

Interaction of particles with matter A recapitulation

Interaction of heavy charged particles with matter

Two effects when charged particles pass through matter

- Energy loss.
- Deflection from the original trajectory.

Responsible processes

- Inelastic collisions with atomic electrons in the material.
- Elastic scattering off the atomic nuclei in the material.
- Emission of Čerenkov radiation.
- Nuclear reactions.
- Bremsstrahlung.

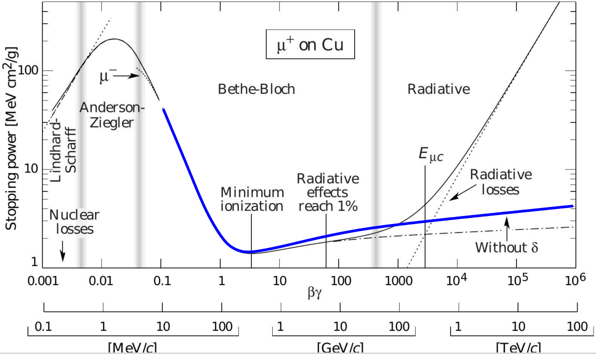
For heavy charged particles, the first two processes are dominant.
Heavy charged particles include $\mu^{\pm}$, $\pi^{\pm}$, p, $\bar{p}$, α particles, and light nuclei.

Energy loss of heavy charged particles

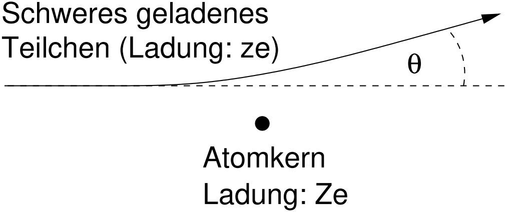
Heavy charged particles lose energy through excitation and ionization of atoms. The energy loss per unit path length is described by the Bethe-Bloch formula:

$$-\frac{dE}{dx} = \frac{4\pi n z^2}{m_e c^2 \beta^2} \cdot \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2 m_e c^2 \beta^2}{I(1-\beta^2)} - \beta^2\right)\right];$$

$\beta = v/c$: Velocity of the particle. E: Energy of the particle.
z: Charge of the particle. e: Elementary charge. n: Electron density of the material.
I: Mean excitation potential of the material.



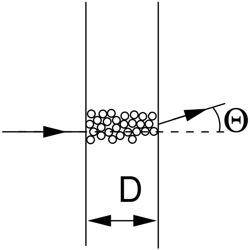
Multiple scattering



Scattering off a single nucleus:

$$\theta = \frac{\Delta p}{p} \propto \frac{z \cdot Z}{p}.$$

$$< \theta >= 0, 0 \neq \theta_0^2 := Var(\theta) \propto \frac{z^2 \cdot Z^2}{p^2}.$$



Scattering off many nuclei:

$$< \Theta > \; = \; 0$$

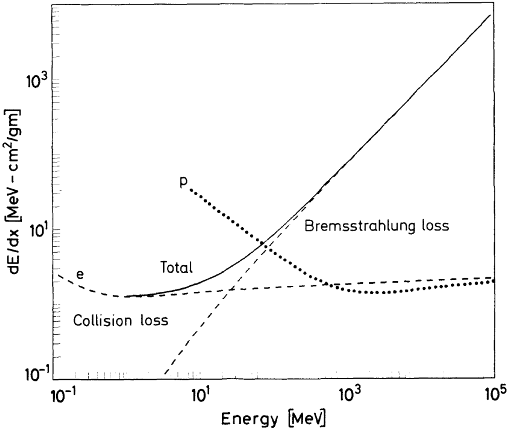
$$\Theta_0^2 := Var(\Theta) = \sum_{collisions} \theta_0^2 \propto D \cdot \frac{z^2 \cdot Z^2}{p^2}.$$

Thus, one obtains $\boxed{\Theta_0 \propto \frac{\sqrt{D}}{p}.}$

Energy loss of electrons and positrons

The mass m_e is so small that the acceleration experienced by electrons or positrons in collisions with atomic nuclei is significant enough to emit bremsstrahlung photons.

$$\left.\frac{dE}{dx}\right|_{e^\pm} = \left.\frac{dE}{dx}\right|_{collisions} + \left.\frac{dE}{dx}\right|_{bremsstrahlung}.$$



Critical Energy E_k

$$\left.\frac{dE}{dx}\right|_{collisions}(E_k) = \left.\frac{dE}{dx}\right|_{bremsstrahlung}(E_k).$$

$E_k \approx \frac{800\ \text{MeV}}{Z+1/2}$, hence above $E_{e^\pm} > 1$ GeV, bremsstrahlung is dominant.

Recapitulation of the previous lecture

Interaction of photons with matter

Main Processes

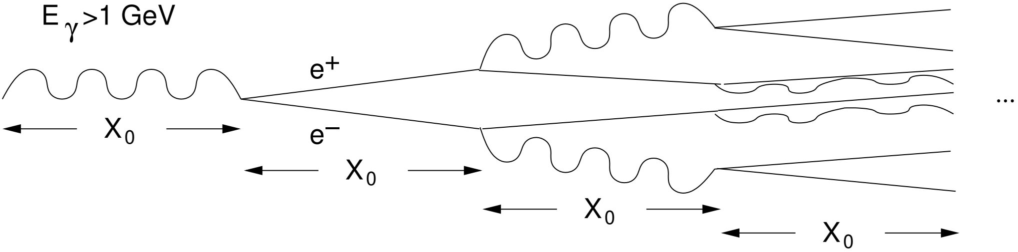
- Photoelectric effect in the energy range E_γ ∼keV.
- Compton scattering in the energy range E_γ ∼MeV.
- Electron-positron pair production for $E_\gamma \gtrsim 10$ MeV.

Consequence for high-energy photons

Formation of so-called electromagnetic showers in the traversed material.

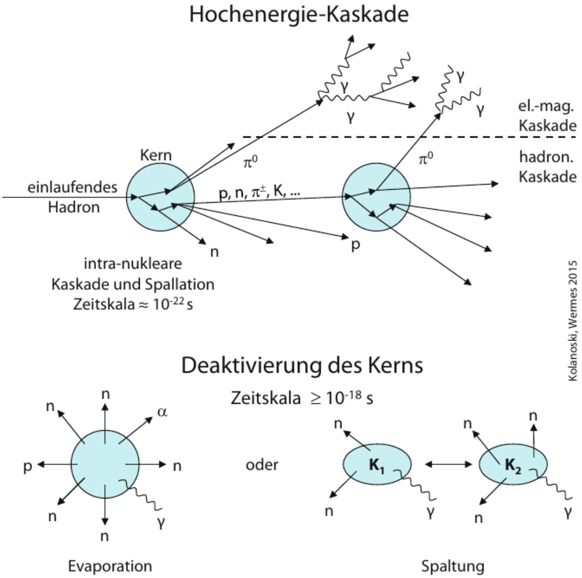
Detailed examination in the next lecture.

Electron-photon shower



- After traveling a distance of $n \cdot X_0$: 2^n particles with energy $E_n \approx \frac{E_\gamma}{2^n}$.
- Cascade (shower) ends when $E_n = E_k$: $n = \frac{\ln \frac{E_\gamma}{E_k}}{\ln 2}$.
- Length of the cascade: $n \cdot X_0 = X_0 \cdot \frac{\ln \frac{E_\gamma}{E_k}}{\ln 2}$.
- Transverse extent of the cascade independent of E_γ: $L_\perp \approx 4R_M = 4X_0 \frac{21.2\ \text{MeV}}{E_k}$.

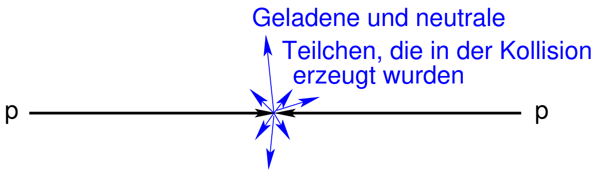
Hadron shower



Qualitatively similar behavior to electromagnetic showers:

- Shower length proportional to $\lambda_A \approx 35\ \mathrm{g\,cm^{-2}} \frac{A^{1/3}}{\rho} \gg X_0$.
- Transverse extent independent of the energy of the primary hadron: λ_A.

Reconstruction of pp collision events



Particles producible in the final state of a collision

Leptons

- Neutrinos: stable, only weakly charged. ⇒ No interaction leading to a measurable electrical signal in the detector components.
- Electrons: stable, electrically charged. ⇒ Electrical signals in the detector components.
- Muons: unstable, but due to being ultrarelativistic, they are long-lived in the laboratory frame and do not decay in the detector; electrically charged. ⇒ Electrical signals in the detector components.
- τ leptons: unstable. ⇒ Detectable only through their decay products.

Topology of a pp collision event

Additional particles producible in the final state of a collision

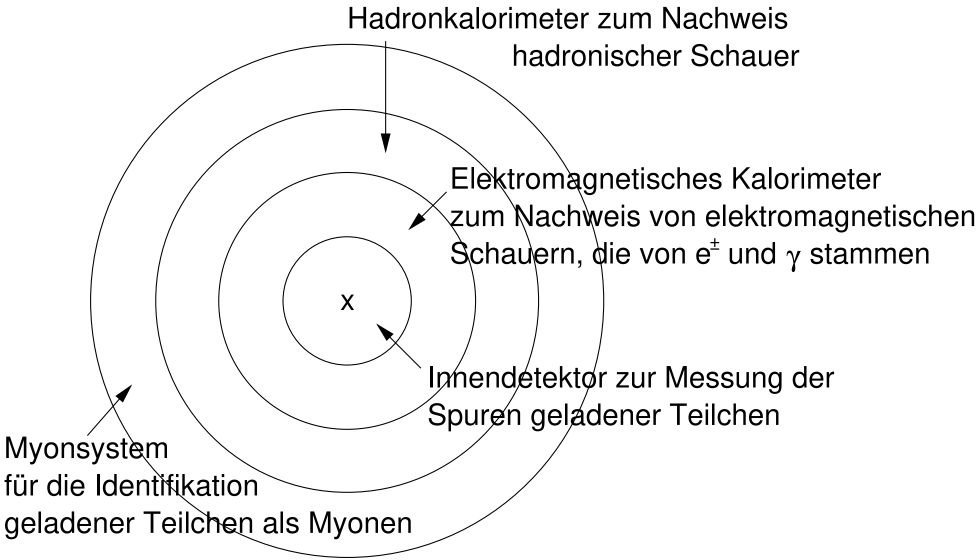
Hadrons

- In the elementary collision, quarks and gluons are initially produced. Due to confinement, these are not seen directly, but rather as so-called jets of hadrons, which originate from the quarks and gluons.
- Special role of two quarks:
  b-quarks form long-lived b-hadrons, which allows the identification of b-quark jets.
  t-quarks are so short-lived that they cannot form hadrons. They are detectable through their decay $t \to Wb$.

Photons

Photons are stable. Although they are electrically neutral, they can produce electromagnetic showers in matter, which can be detected by the detector.

Basic structure of a particle detector at a hadron collider



Reconstruction of muon tracks in the inner detector

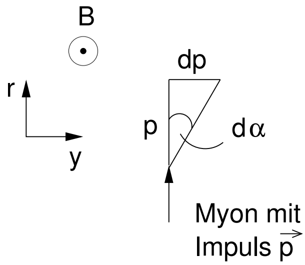
Reconstruction of muon trajectories is in a certain sense the simplest, because the energy loss of muons in the inner detector is negligible, and the trajectory therefore depends only on the following parameters:

- $\vec{x}_0$, $\vec{p}$ at the interaction point.
- Magnetic field in the inner detector.
- Multiple scattering in the inner detector.

The reconstruction of muon tracks, as well as particle tracks in general, occurs in two interconnected steps: the so-called pattern recognition, where the hit points in the inner detector corresponding to the particle track are found, and the so-called track fitting, where the trajectory is calculated from the hit points selected during pattern recognition.

Trajectory in a magnetic field

$$d\alpha = \frac{dp}{p} = \frac{qvBdt}{p} = \frac{q}{p} B \underbrace{vdt}_{=ds=dr} = \frac{q}{p} Bds.$$



Thus, we obtain

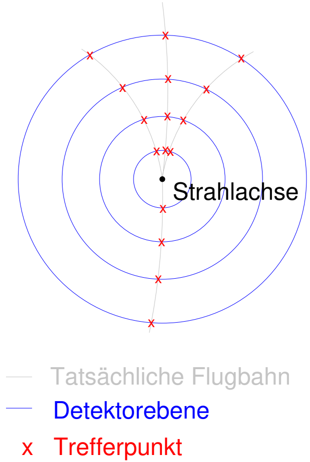
$$\alpha(r) \approx \frac{q}{p} \int_{r_0}^{r} B(s) ds$$

and

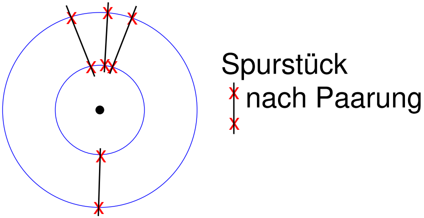
$$y(r) = \int_{r_0}^{r} \alpha(r') dr' = \frac{q}{p} \int_{r_0}^{r} \int_{r_0}^{r'} B(s)\, ds\, dr'.$$

Example. $p = 1$ GeV. $r_0 = 0$. $B = 2$ T.
$\alpha(10 \text{ cm}) = 60$ mrad. $y(10 \text{ cm}) = 3$ mm.
$\alpha(1 \text{ m}) = 0.6$ rad. $y(1 \text{ m}) = 30$ cm.

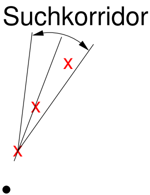
A possible method for pattern recognition



1. Consider all pairs of hits near the beamline sequentially.

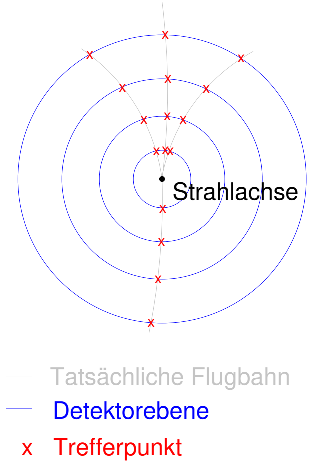


2. Search for hits in search corridors around the extrapolated track segments outward.

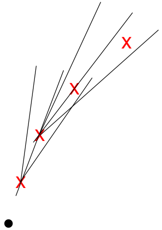


The size of the search corridor determines the smallest measurable momentum p.

A possible method for pattern recognition



3. Continue the extrapolation to the outermost measurement layer of the inner detector.



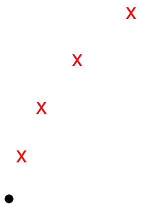
Two possibilities:

(a) Size of the search corridor constant.

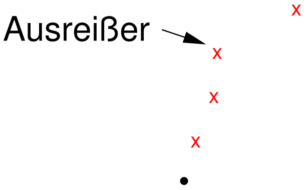
(b) Size of the search corridor depends on the trajectory of the hits found so far.

Now consider all clusters of hits found during pattern recognition one by one.

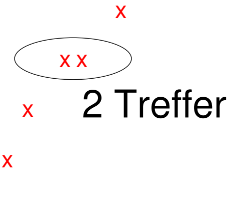
Ideal case: Only one hit in each detector layer, no outliers.



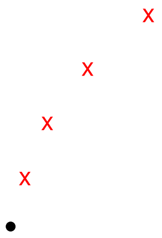
Situation with so-called outliers



Situation with more than one hit in a layer



Track fitting in the ideal case



- Hit coordinates: $\vec{x}_1, \ldots, \vec{x}_n$.
- Position uncertainties: $\sigma_1, \ldots, \sigma_n$.
- Uncertainties in detector positions, known as alignment uncertainties, lead to non-zero off-diagonal elements in the covariance matrix $Cov(x_k, x_\ell)$.

- Incorporate the influence of multiple scattering by introducing scattering centers, where the trajectory can bend.
- Track function: $\vec{y}_k = \vec{y}_k(\frac{q}{p}, \vec{x}_0, \hat{\vec{p}})$.
- Determination of $\frac{q}{p}$, $\vec{x}_0$, and $\hat{\vec{p}}$ using the method of least squares:

$$Q^2 = \sum_{k,\ell=1}^{n} (\vec{x}_k - \vec{y}_k)^t \, Cov(\vec{x}_k, \vec{x}_\ell)(\vec{x}_\ell - \vec{y}_\ell).$$

Handling outliers and ambiguities

Handling outliers

Option 1. Iterative approach: Track fitting including outliers. Then identify outliers from this track. Repeat track fitting excluding these identified outliers.

Option 2. $\sigma_k = \bar{\sigma}_k$ for $|\vec{x}_k - \vec{y}_k| < \delta$, $\sigma_k \to \infty$ for $|\vec{x}_k - \vec{y}_k| \geq \delta$. This makes the contribution of outliers to Q^2 negligibly small.

Handling ambiguities

$\sigma_k = \sigma(|\vec{x}_k - \vec{y}_k|)$ as above, including all hits.

Alternatively, perform track fitting with all possible hit combinations and select the track with the smallest Q^2.

Reconstruction of pion tracks in the inner detector

- $m_{\pi^\pm} \approx m_{\mu^\pm} \Rightarrow$ Pion tracks are very similar to muon tracks.
- $\pi \to \mu\nu_\mu$ decays are very rare in the lab frame due to time dilation. However, because so many $\pi^\pm$ are produced in pp collisions, it happens with non-negligible frequency that a charged pion decays within the inner detector. At the decay point, the track bends.
- The size of this bend must be taken into account, at least in the size of the hit search corridors.

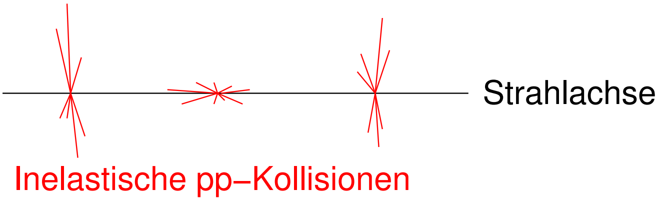
Reconstruction of electron tracks in the inner detector

$m_{e^\pm} \ll m_{\pi/\mu}$. $\Rightarrow$ Energy loss in the inner detector is not negligible!
Two effects must be considered:

- Continuous energy loss due to synchrotron radiation.
- Discrete, large energy loss due to bremsstrahlung after scattering on atomic nuclei of the detector material.

Common Procedure. If tracks reconstructed with the standard algorithm for pions can be associated with a cluster of energy depositions in the electromagnetic calorimeter, these tracks are reconstructed again under the assumption that they are electron tracks. This process takes into account the hit search and the track model for continuous and discrete energy loss.

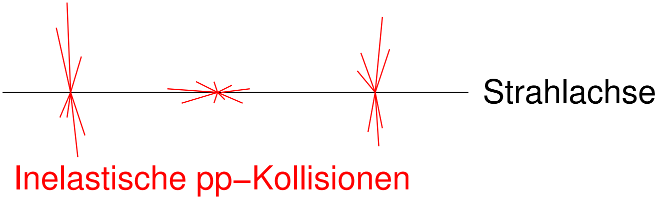
Determination of vertices



Position of a pp Collision: Primary vertex candidate.

Determination of a Primary Vertex Candidate

- Collection of reconstructed particle tracks that cluster along the beam axis at a specific point.
- Determination of the precise vertex position using the method of least squares for the tracks' distance from the vertex.



Selection of the primary vertex

In inelastic pp collisions, tracks are typically produced at small angles with respect to the beam axis. Therefore, in these collisions, the sum Σ of the transverse momentum magnitudes of the reconstructed particle tracks is small. This contrasts with collisions where a heavy particle is produced, where the transverse momenta of the decay products of this particle are large. Therefore, the primary vertex is usually selected as the primary vertex candidate with the maximum Σ.