Mixed NNLO QED-QCD corrections to the Higgs-boson decay $H \to b\bar{b}$

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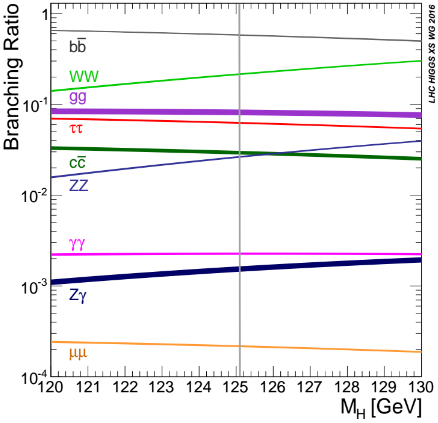
Contents

Motivation

- $H \to b\bar{b}$ dominant decay channel
  branching ratio $\mathrm{BR} = \frac{\Gamma_{H\to b\bar{b}}}{\Gamma_H} = 0.58$
- uncertainty of $\Gamma_{H\to b\bar{b}}$ significant for total decay width Γ_H & for all BRs
- data available from LHC (Aaboud et al. 2018; Sirunyan et al. 2018)

$\Rightarrow$ compute higher-order corrections



Branching ratios for intermediate Higgs-boson masses (Florian et al. 2016)

Parameters and definitions

- $m_\mathrm{H} = 125.09\,\mathrm{GeV}$, $m_\mathrm{b} = 4.92\,\mathrm{GeV}$ on-shell
- def. partial decay width

$$\Gamma_{X\to f} = \frac{1}{2m_X}\int |\mathcal{A}_{X\to f}|^2\,\mathrm{d}\Phi_{PS(f)}$$

- leading order decay width

$$\Gamma_{\mathrm{LO}} = \frac{1}{2m_\mathrm{H}}\int |\mathcal{A}^{(0)}_{b\bar{b}}|^2\,\mathrm{d}\Phi_2 = \frac{e^2 N_C m_\mathrm{H}^2 m_\mathrm{b}^2}{32\pi s_\mathrm{w}^2 m_\mathrm{W}^2}\left(1-\frac{4m_\mathrm{b}^2}{m_\mathrm{H}^2}\right)^{3/2} = 5.7\,\mathrm{MeV}$$

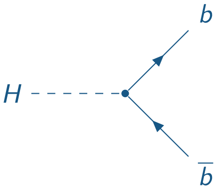
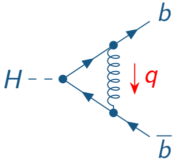


diagram for $\mathcal{A}^{(0)}_{b\bar{b}}$

NLO QCD correction - infrared divergences

virtual correction

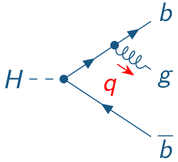


- massless propagator attached to massive on-shell leg

  $\Rightarrow$ loop integral divergent for $q \to 0$

  $\Rightarrow$ IR singularity

real correction



- massless particle radiated off massive on-shell leg

  $\Rightarrow$ amplitude diverges in **soft** region $q \to 0$

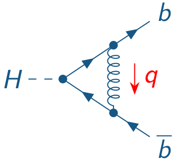
  $\Rightarrow$ phase-space integral diverges

  $\Rightarrow$ IR singularity

$\Rightarrow$ regularize with dimensional reg.: $4 \to D = 4 - 2\epsilon$

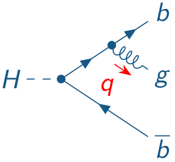
$\Rightarrow$ IR singularities show up as poles in ϵ

virtual correction



$$\Gamma_{\text{virt}} = \frac{1}{2m_H}\int \underbrace{\mathcal{M}^{(1)}_{b\bar{b}}}_{\text{IR-divergent one-loop amplitude}} \mathrm{d}\Phi_2$$

real correction



$$\Gamma_{\text{real}} = \frac{1}{2m_H}\int \underbrace{\mathcal{M}^{(0)}_{b\bar{b}g}\,\mathrm{d}\Phi_3}_{\text{IR-divergent phase-space integral}}$$

- KLN theorem (Kinoshita 1962; Lee and Nauenberg 1964):

$$\Gamma^{(\text{NLO})} = \Gamma_{\text{virt}} + \Gamma_{\text{real}} = \text{finite!}$$

- valid order by order: $\Gamma^{(\text{NNLO})} = \Gamma_{\text{double-virt}} + \Gamma_{\text{real-virt}} + \Gamma_{\text{double-real}} = \text{finite!}$
  $\Rightarrow$ extract IR singularities, use **subtraction scheme**

NLO QCD correction: FKS subtraction scheme - real correction

FKS subtraction scheme (Frixione, Kunszt, and Signer 1996; Engel, Signer, and Ulrich 2020)

- integration over real-emission amplitude in dimensional regularization $(D = 4 - 2\epsilon)$

$$\begin{aligned} \mathrm{d}\Phi_3 \mathcal{M}^{(0)}_{b\bar{b}g} &= \mathrm{d}\Phi_2\, \mathrm{d}\Phi_g \mathcal{M}^{(0)}_{b\bar{b}g} \\ &= \mathrm{d}\Phi_2\, \mathrm{d}\Upsilon\, \mathrm{d}\xi\; \xi^{1-2\epsilon} \mathcal{M}^{(0)}_{b\bar{b}g} \\ &= \mathrm{d}\Phi_2\, \mathrm{d}\Upsilon\, \mathrm{d}\xi\; \xi^{-1-2\epsilon} \xi^2 \mathcal{M}^{(0)}_{b\bar{b}g} \end{aligned}$$

ξ = scaled gluon energy

- use distributions to split gluon phase-space integration

$$\xi^{-1-2\epsilon} = \underbrace{-\frac{\xi_c{}^{-2\epsilon}}{2\epsilon}\delta(\xi)}_{\to \text{ soft region}} + \underbrace{\left(\frac{1}{\xi^{1+2\epsilon}}\right)_c}_{\to \text{ hard region}}$$

ξ_c = fixed cut parameter

$$\int_0^{\xi_{\max}} \left(\frac{1}{\xi^{1+2\epsilon}}\right)_c f(\xi)\, \mathrm{d}\xi = \int_0^{\xi_{\max}} \frac{f(\xi) - f(0)\theta(\xi_c - \xi)}{\xi^{1+2\epsilon}}\, \mathrm{d}\xi$$

NLO QCD correction: FKS subtraction scheme - real correction

⇒ split phase-space integration $\mathrm{d}\Phi_3 = \mathrm{d}\Phi_{3,\mathrm{h}} + \mathrm{d}\Phi_{3,\mathrm{s}}$

hard region: $$\mathrm{d}\Phi_{3,\mathrm{h}}\,\mathcal{M}^{(0)}_{b\bar{b}g} = \mathrm{d}\Phi_2\,\mathrm{d}\Upsilon\,\mathrm{d}\xi\left(\frac{1}{\xi^{1+2\epsilon}}\right)_c \xi^2 \mathcal{M}^{(0)}_{b\bar{b}g}$$

⇒ integrate numerically with $\epsilon = 0$

soft region: $$\mathrm{d}\Phi_{3,\mathrm{s}}\,\mathcal{M}^{(0)}_{b\bar{b}g} = \mathrm{d}\Phi_2\,\mathrm{d}\Upsilon\,\mathrm{d}\xi\,\frac{-\xi_c^{-2\epsilon}}{2\epsilon}\delta(\xi)\xi^2\mathcal{M}^{(0)}_{b\bar{b}g}$$

⇒ take soft limit analytically using **eikonal factorization**

NLO QCD correction: FKS subtraction scheme - real correction

- eikonal factorization in soft limit

$$\lim_{\xi \to 0} \xi^2 \mathcal{M}^{(0)}_{b\bar{b}g} = \underbrace{\mathcal{E}}_{\text{eikonal factor}} \times \underbrace{\mathcal{M}^{(0)}_{b\bar{b}}}_{\substack{\text{born-level} \\ \text{amplitude}}}$$

$$\mathcal{E} = 4\pi\alpha_s C_F \sum_{j,k=1}^{2} \frac{p_j \cdot p_k}{(p_j \cdot n_g)(p_k \cdot n_g)} (-1)^{j+k+1}$$

- analytical integration over angular gluon variables

$$\begin{aligned} \mathrm{d}\Phi_{3,s}\, \mathcal{M}^{(0)}_{b\bar{b}g} &= \mathrm{d}\Phi_2\, \mathrm{d}\Upsilon \frac{-\xi_c^{-2\epsilon}}{2\epsilon} \mathcal{E} \mathcal{M}^{(0)}_{b\bar{b}} \\ &= \mathrm{d}\Phi_2 \underbrace{\hat{\mathcal{E}}(\xi_c)}_{\text{contains } 1/\epsilon \text{ poles}} \mathcal{M}^{(0)}_{b\bar{b}} \end{aligned}$$

NLO QCD correction: FKS subtraction scheme - real correction

- integrated eikonal $\hat{\mathcal{E}}(\xi_c) = \sum_{j,k=1}^{2} \hat{\mathcal{E}}_{jk}(\xi_c)$

$$\hat{\mathcal{E}}_{jk}(\xi_c) = \frac{-\alpha_s C_F}{2\pi} \left[\left(\Delta_{\mathrm{IR}} + \ln \frac{\mu^2}{\xi_c^2 m_{\mathrm{H}}^2} \right) \frac{1}{2 v_{jk}} \ln \frac{1+v_{jk}}{1-v_{jk}} - \frac{a_{jk} p_j \cdot p_k}{2 m_{\mathrm{b}}^2 (a_{jk}^2 - 1)} \left(J(a_{jk} E_{\mathrm{b}}, \nu_{jk}) - J(E_{\mathrm{b}}, \nu_{jk}) \right) \right],$$

$$J(x,z) = \ln^2 \frac{1-\beta}{1+\beta} + 4\,\mathrm{Li}_2\left(1 - \frac{x(1+\beta)}{z}\right) + 4\,\mathrm{Li}_2\left(1 - \frac{x(1-\beta)}{z}\right),$$

$$\Delta_{\mathrm{IR}} = \frac{(4\pi)^{\epsilon}\Gamma(1+\epsilon)}{\epsilon}, \quad v_{jk} = \sqrt{1 - \left(\frac{m_{\mathrm{b}}^2}{p_j \cdot p_k}\right)^2}, \quad a_{jk} = (1+v_{jk}) \frac{p_j \cdot p_k}{m_{\mathrm{b}}^2}, \quad \nu_{jk} = \frac{m_{\mathrm{b}}^2}{2 E_{\mathrm{b}}}(a_{jk}+1)$$

(Engel, Signer, and Ulrich 2020)

NLO QCD correction: FKS subtraction scheme

⇒ add integrated soft part $\mathrm{d}\Phi_2\,\hat{\mathcal{E}}(\xi_c)\mathcal{M}^{(0)}_{b\bar{b}}$ to virtual correction!

$$\Gamma_{\text{virt}}(\xi_c) = \frac{1}{2m_{\text{H}}}\int \mathrm{d}\Phi_2 \underbrace{\left(\mathcal{M}^{(1)}_{b\bar{b}} + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}^{(0)}_{b\bar{b}}\right)}_{\text{free of } 1/\epsilon \text{ poles}}$$

$$\Gamma_{\text{real}}(\xi_c) = \frac{1}{2m_{\text{H}}}\int \mathrm{d}\Phi_3 \left(\frac{1}{\xi}\right)_c \xi\mathcal{M}^{(0)}_{b\bar{b}g}$$

$$\Rightarrow \quad \Gamma^{(\alpha_s)} = \underbrace{\Gamma_{\text{virt}}(\xi_c) + \Gamma_{\text{real}}(\xi_c)}_{\text{dependence on } \xi_c \text{ cancels}}$$

- numerical result

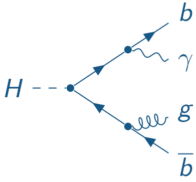
$$\Gamma^{(\alpha_s)} = -2.099\,\text{MeV}$$

$$\delta^{(\alpha_s)} = \frac{\Gamma^{(\alpha_s)}}{\Gamma_{\text{LO}}} = -36.8\,\%$$

Mixed NNLO QED-QCD corrections to $H \to b\bar{b}$

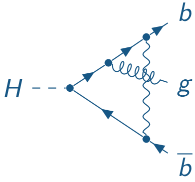
- four classes of diagrams

double-real $H \to b\bar{b}g\gamma$



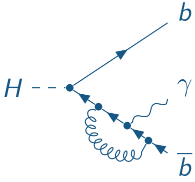
$\mathcal{M}^{(0)}_{b\bar{b}g\gamma}$

real-virtual $H \to b\bar{b}g$



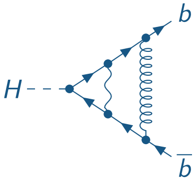
$\mathcal{M}^{(\gamma)}_{b\bar{b}g}$

real-virtual $H \to b\bar{b}\gamma$



$\mathcal{M}^{(g)}_{b\bar{b}\gamma}$

double-virtual $H \to b\bar{b}$



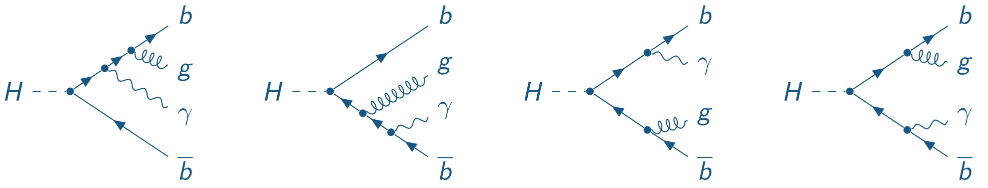
$\mathcal{M}^{(2)}_{b\bar{b}}$

- properties of $H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$
  - only abelian couplings (no gluon self-interaction)
  - simple factorization of double-real-emission amplitudes
  - overlapping soft singularities $\Rightarrow$ ϵ^{-2} & ϵ^{-1} poles present

$\Rightarrow$ generalize FKS subtraction to NNLO

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Double-real correction

double-real correction

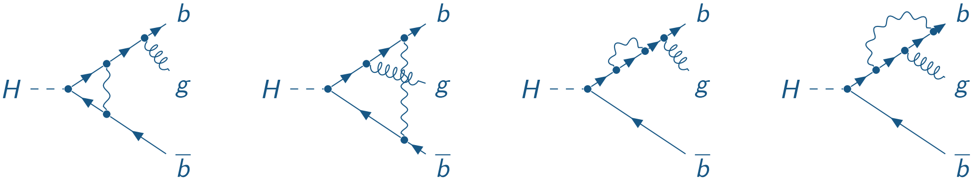


exemplary diagrams for $\mathcal{M}^{(0)}_{b\bar{b}\gamma g}$

- split photon & gluon phase space in soft and hard parts, respectively
  $\Rightarrow$ four contributions $\Gamma^{(\alpha\alpha_s)}_{b\bar{b}\gamma g} = \Gamma^{\mathrm{ss}}_{b\bar{b}\gamma g} + \Gamma^{\mathrm{hs}}_{b\bar{b}\gamma g} + \Gamma^{\mathrm{sh}}_{b\bar{b}\gamma g} + \Gamma^{\mathrm{hh}}_{b\bar{b}\gamma g}$
  $\Rightarrow$ dependence on two cut parameters ξ_a, ξ_c
- use eikonal factorization for integration of the soft parts

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Real-virtual correction

real-virtual contribution

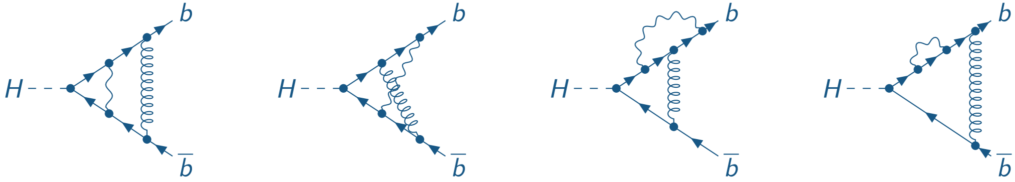


exemplary diagrams for $\mathcal{M}^{(\gamma)}_{b\bar{b}g}$

- eikonal factorizes in the soft limit despite one-loop structure
- FKS phase-space splitting via distributions & cut parameters ξ_a, ξ_c
- treatment of $\mathcal{M}^{(g)}_{b\bar{b}\gamma}$ analogous

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Double-virtual correction

two-loop amplitude contribution



exemplary diagrams for $\mathcal{M}^{(2)}_{b\bar{b}}$

- use analytical expression from literature (Bernreuther et al. 2005)

  $\Rightarrow \quad \mathcal{M}^{(2)}_{b\bar{b}} = [\ldots] \times \frac{1}{\epsilon^2} + [\ldots] \times \frac{1}{\epsilon} + \text{finite}$

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Combination

expressions after FKS subtraction

$$\Gamma^{(\alpha\alpha_s)} = \Gamma^{(\alpha\alpha_s)}_{b\bar{b}} + \Gamma^{(\alpha\alpha_s)}_{b\bar{b}\gamma} + \Gamma^{(\alpha\alpha_s)}_{b\bar{b}g} + \Gamma^{(\alpha\alpha_s)}_{b\bar{b}\gamma g}$$

$$\Gamma^{(\alpha\alpha_s)}_{b\bar{b}} = \frac{1}{2m_\mathrm{H}} \int \mathrm{d}\Phi_2 \left(\mathcal{M}^{(2)}_{b\bar{b}} + \mathcal{M}^{(\gamma g)}_{b\bar{b}} + \hat{\mathcal{E}}_\alpha(\xi_a)\mathcal{M}^{(g)}_{b\bar{b}} + \hat{\mathcal{E}}_{\alpha_s}(\xi_c)\mathcal{M}^{(\gamma)}_{b\bar{b}} + \hat{\mathcal{E}}_\alpha(\xi_a)\hat{\mathcal{E}}_{\alpha_s}(\xi_c)\mathcal{M}^{(0)}_{b\bar{b}} \right)$$

$$\Gamma^{(\alpha\alpha_s)}_{b\bar{b}\gamma} = \frac{1}{2m_\mathrm{H}} \int \mathrm{d}\Phi_3 \left(\frac{1}{\xi_\gamma}\right)_a \left(\xi_\gamma \left(\mathcal{M}^{(g)}_{b\bar{b}\gamma} + \hat{\mathcal{E}}_{\alpha_s}(\xi_c)\mathcal{M}^{(0)}_{b\bar{b}\gamma} \right) \right)$$

$$\Gamma^{(\alpha\alpha_s)}_{b\bar{b}g} = \frac{1}{2m_\mathrm{H}} \int \mathrm{d}\Phi_3 \left(\frac{1}{\xi_g}\right)_c \left(\xi_g \left(\mathcal{M}^{(\gamma)}_{b\bar{b}g} + \hat{\mathcal{E}}_{\alpha}(\xi_a)\mathcal{M}^{(0)}_{b\bar{b}g} \right) \right)$$

$$\Gamma^{(\alpha\alpha_s)}_{b\bar{b}\gamma g} = \frac{1}{2m_\mathrm{H}} \int \mathrm{d}\Phi_4 \left(\frac{1}{\xi_\gamma}\right)_a \left(\frac{1}{\xi_g}\right)_c \left(\xi_\gamma \xi_g \mathcal{M}^{(0)}_{b\bar{b}\gamma g} \right)$$

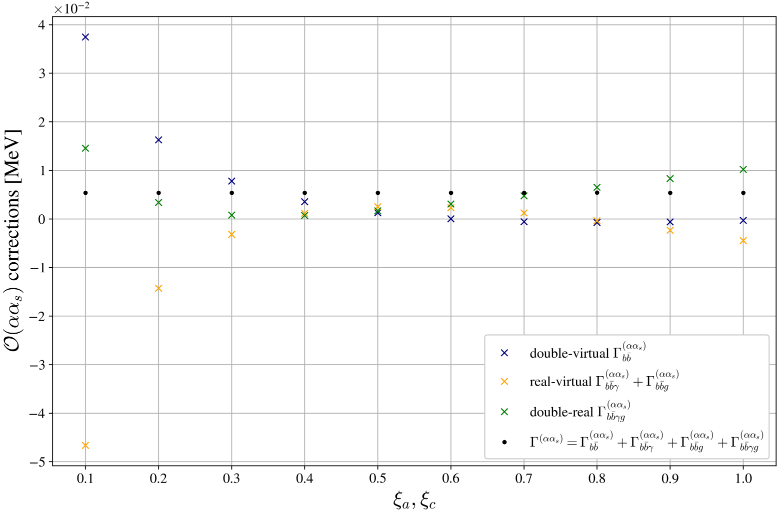
$\Rightarrow$ ϵ^{-2} & ϵ^{-1} poles cancel in $\Gamma^{(\alpha\alpha_s)}_{b\bar{b}}$

$\Rightarrow$ each integrand finite

$\Rightarrow$ dependence on ξ_a & ξ_c drops out in sum $\Gamma^{(\alpha\alpha_s)}$

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Numerical implementation & results

- numerical check of cancellation of ξ_a, ξ_c dependence in $\Gamma^{(\alpha\alpha_s)}$



- set $\xi_a = \xi_c$
- vary over domain $\xi_a = \xi_c = 0.1, 0.2, \ldots, 1.0 \Rightarrow$ no dependence on ξ_a, ξ_c

$H \to b\bar{b}$ at $\mathcal{O}(\alpha\alpha_s)$: Numerical implementation & results

- final result as average

$$\Gamma^{(\alpha\alpha_s)} = 5.400\,\mathrm{keV}$$

$$\delta^{(\alpha\alpha_s)} = \frac{\Gamma^{(\alpha\alpha_s)}}{\Gamma_{\mathrm{LO}}} = 0.0947\,\%$$

- massless result deviates by a factor of ~ 1.8 (too high) (Mihaila, Schmidt, and Steinhauser 2015)

Conclusion

summary

- dominant decay channel $H \to b\bar{b}$ with full quark mass dependence
- simple factorization of soft singularities at $\mathcal{O}(\alpha\alpha_s)$
- generalization of FKS to NNLO
- fully inclusive, IR finite prediction for $\Gamma_{H\to b\bar{b}}$ at $\mathcal{O}(\alpha\alpha_s)$
- results for relative correction
  - $\delta^{(\alpha_s)} = -36.8\,\%$
  - $\delta^{(\alpha\alpha_s)} = 0.0947\,\%$

future prospects

- clarify source of discrepancy compared to massless result
- generalize to $\overline{\mathrm{MS}}$ scheme
- calculate NNLO QCD-elctroweak correction with full mass dependence

Thank you for your attention!

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