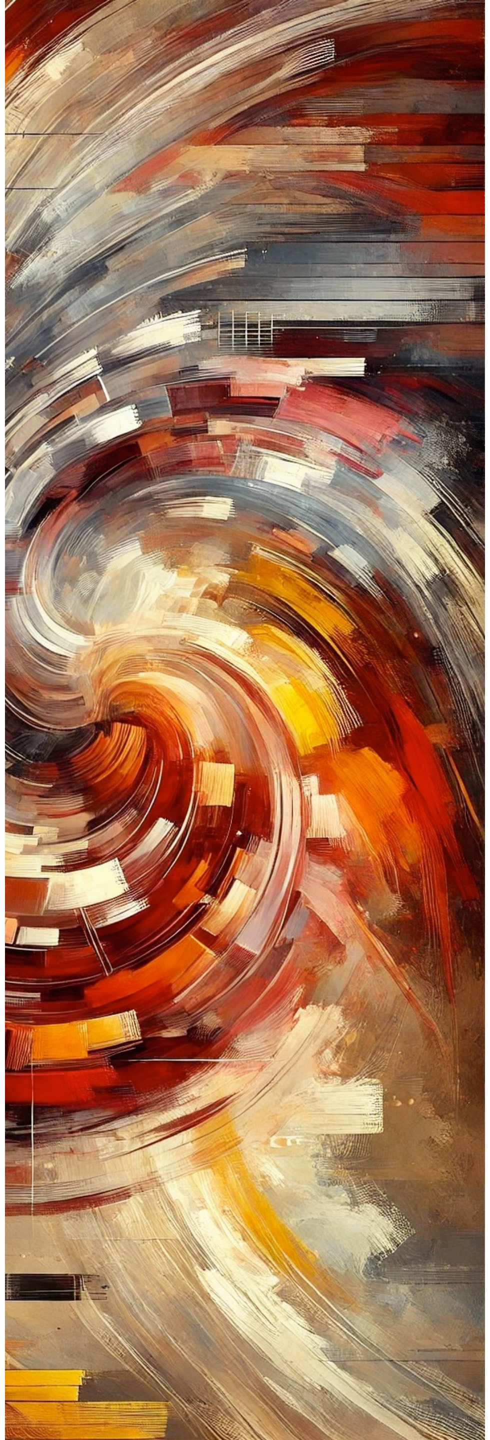
“Give Me the Numbers” Approach to Theoretical Physics

MPP Colloquium
5 November 2024

Sebastian Mizera
Princeton University

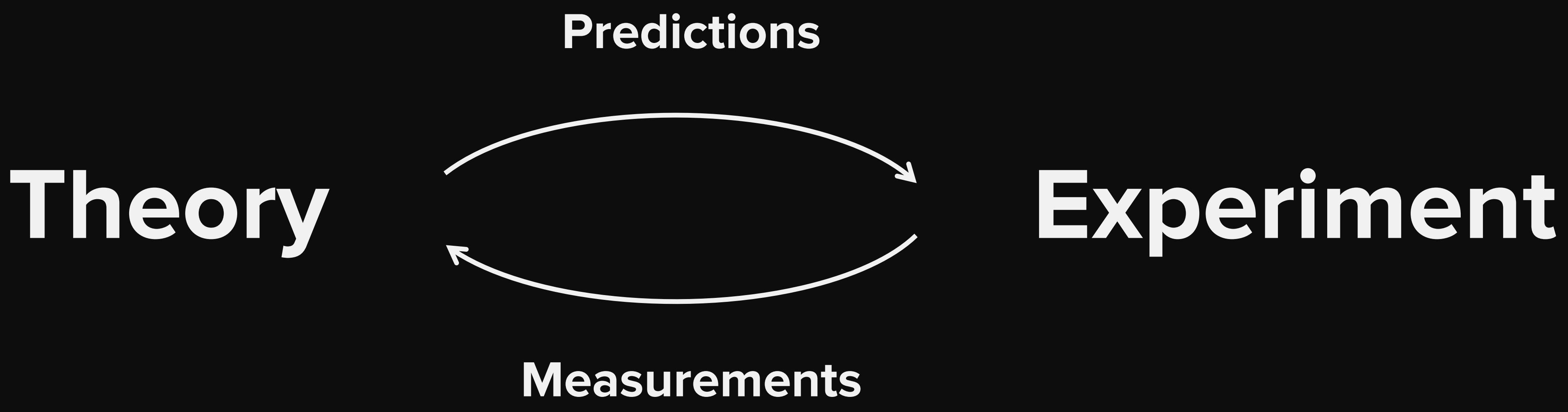


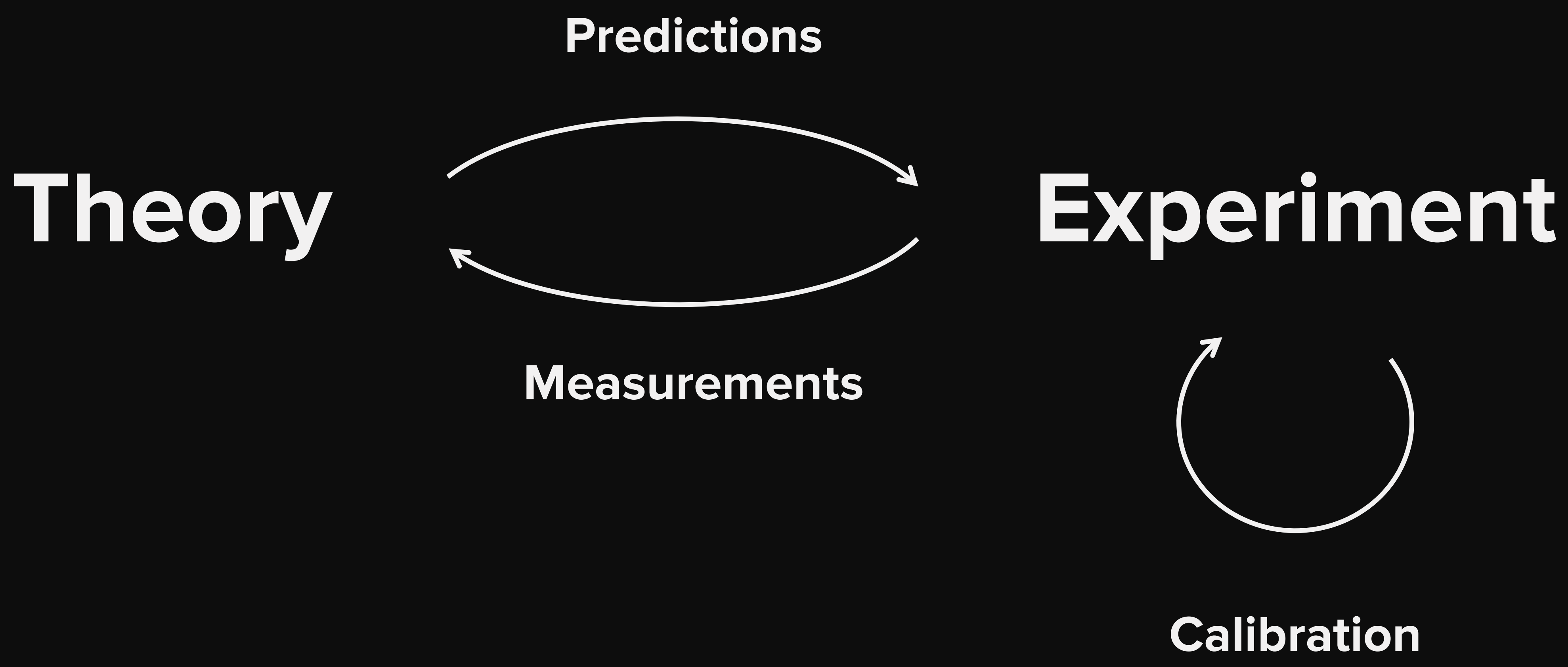
Theory

Experiment

Predictions

Theory → Experiment


Predictions
Theory
Experiment
Measurements


Predictions
Theory
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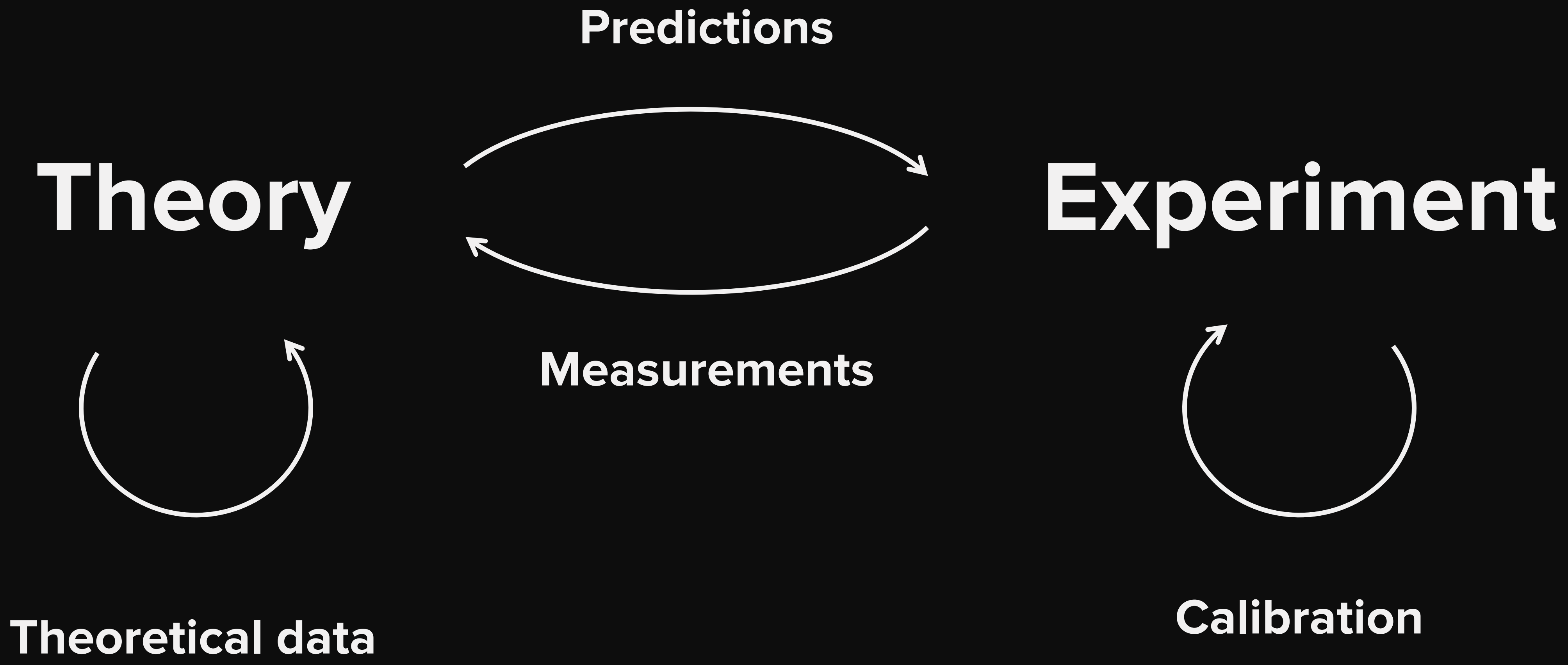
Calibration


Predictions
Theory
Experiment
Measurements
Theoretical data
Calibration

Theoretical data

- "Data" obtained from a <u>theoretical</u> construction, collected to enhance <u>theoretical</u> understanding

Theoretical data

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- Growing paradigm in many areas of theoretical physics

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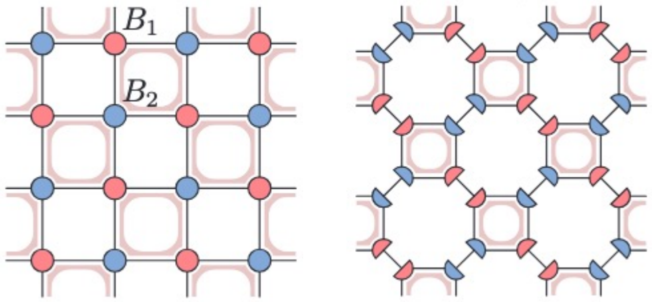
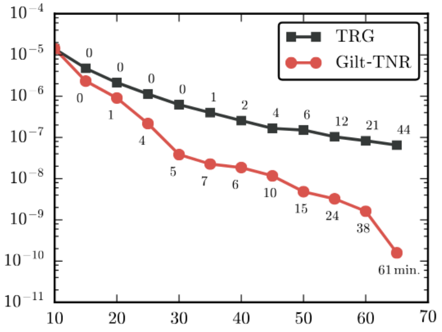


Tensor networks



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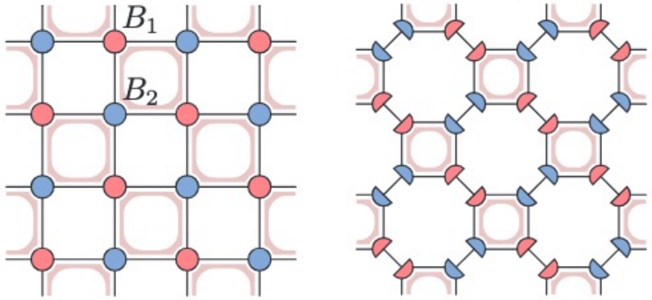
Theoretical data

- "Data" obtained from a <u>theoretical</u> construction, collected to enhance <u>theoretical</u> understanding

- Growing paradigm in many areas of theoretical physics



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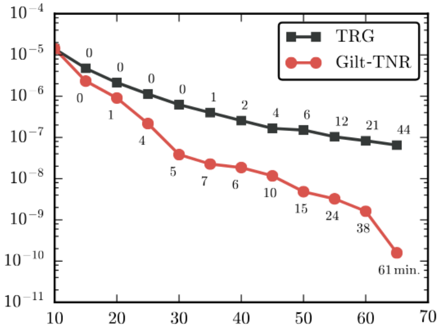

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- This talk: mathematical physics, formal quantum field theory, string theory

“Give me the numbers” approach

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Theory

Experiment

Generate
theoretical data

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Generate theoretical data

- Consistency checks
- Connections to other fields
- Paradigm shift

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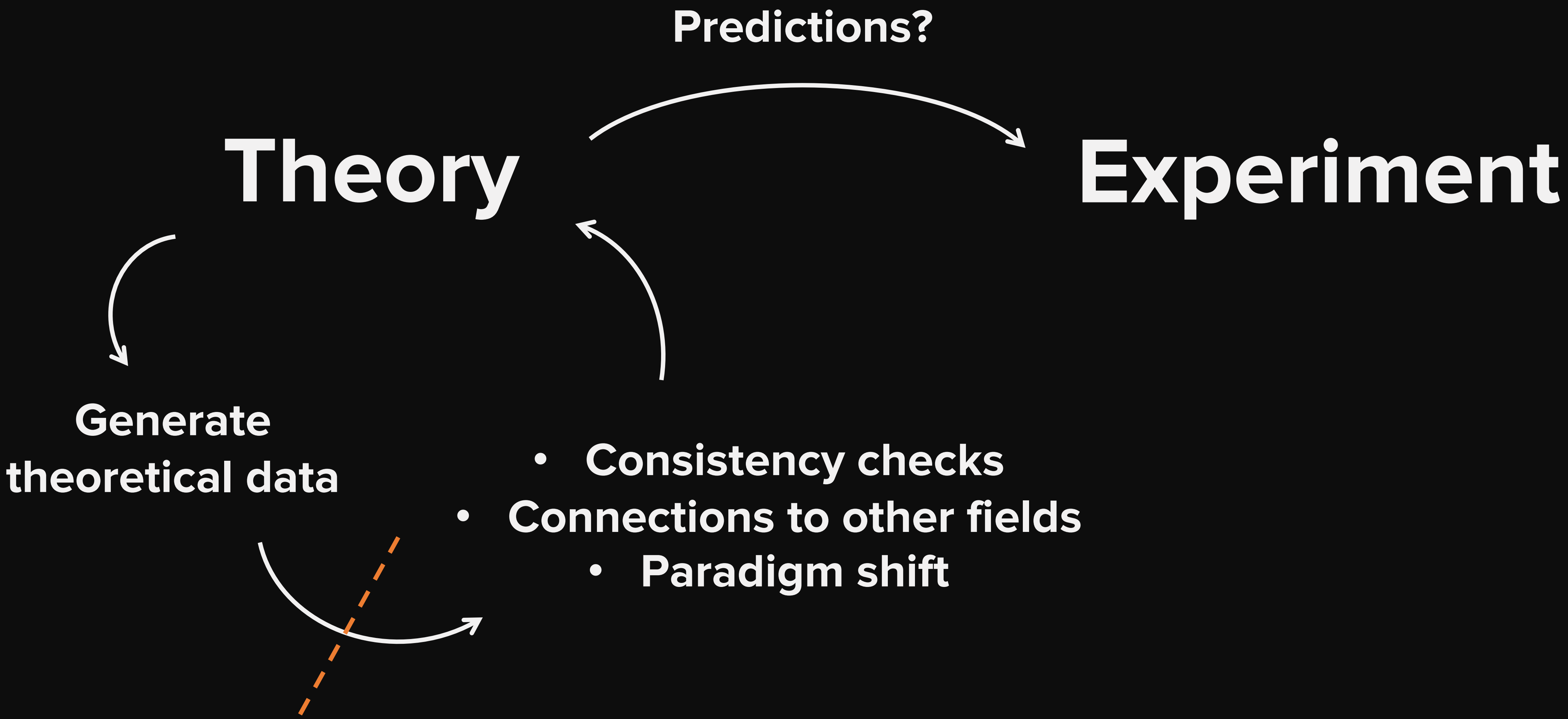
“Give me the numbers” approach



Historical example

Particle physics

String theory

Gravitational physics

Historical example: Logistic map

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$$x_{n+1} = r x_n (1 - x_n)$$

Historical example: Logistic map

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↑

Normalized population $x_n \in [0, 1]$ at time n

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Reproduction

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Competition

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Normalized population $x_n \in [0, 1]$ at time n

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Fertility rate

Normalized population $x_n \in [0, 1]$ at time n

Population modelling (1960's)

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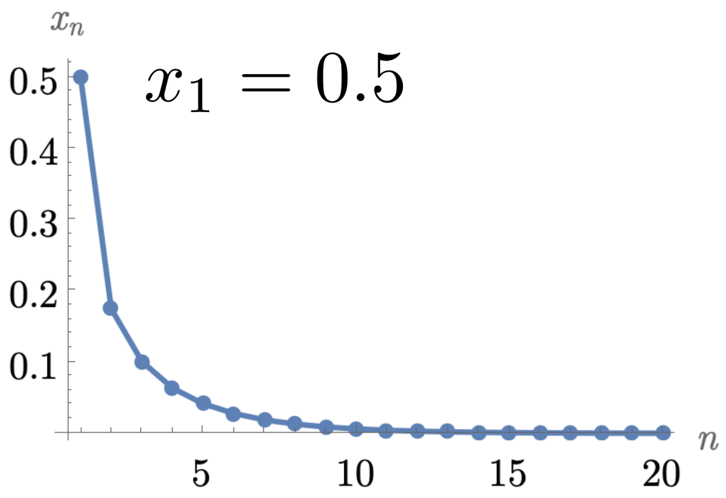
Low fertility

$$r = 0.7$$



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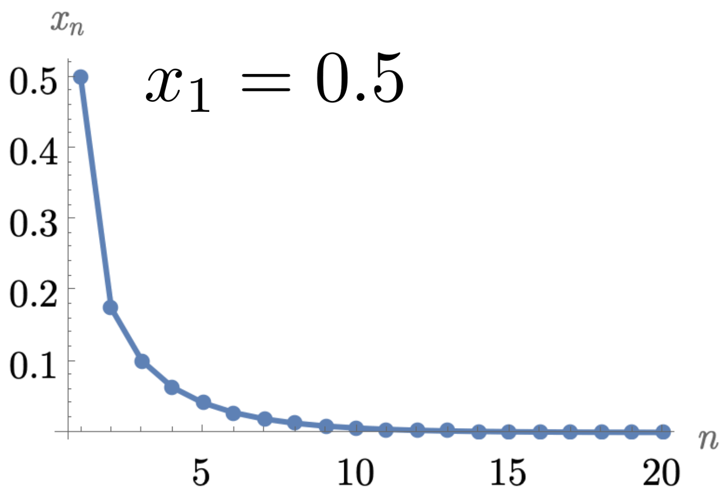

Population dies out

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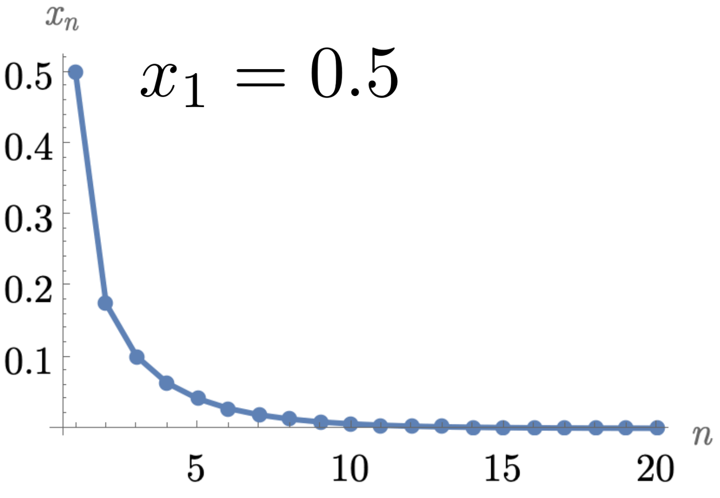
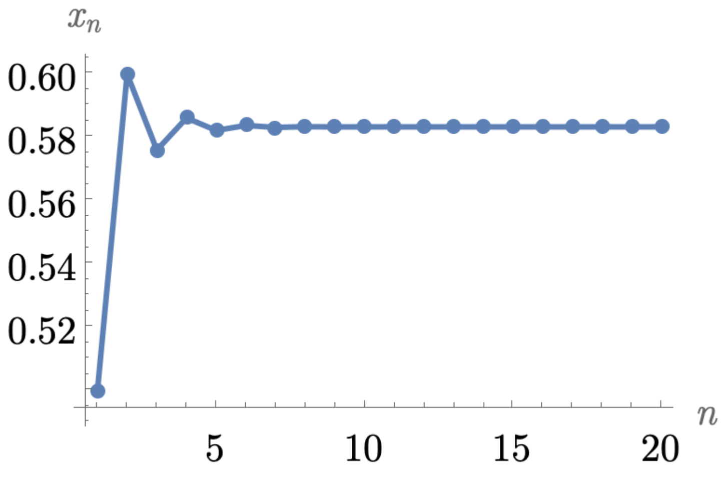


Population dies out

Medium fertility

$$r = 2.4$$



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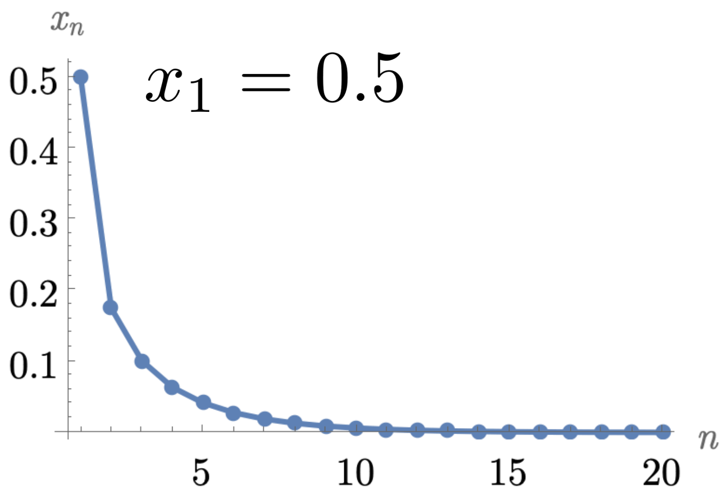

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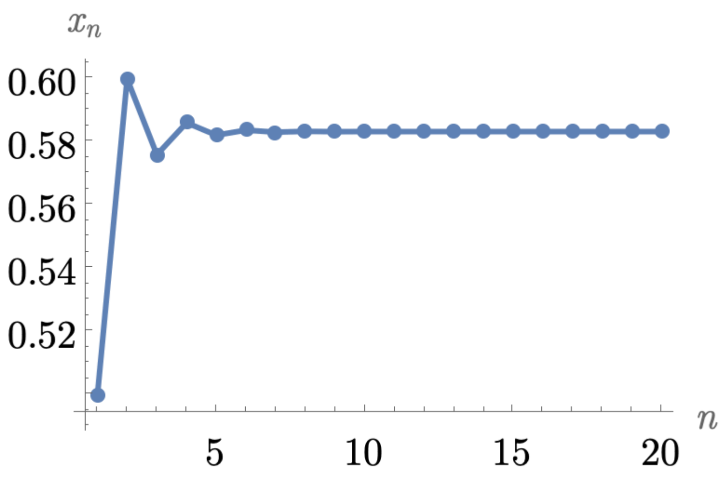

Population stabilizes

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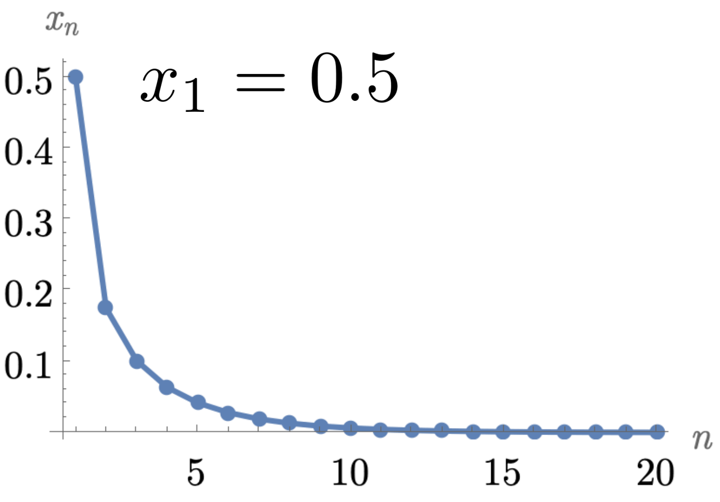

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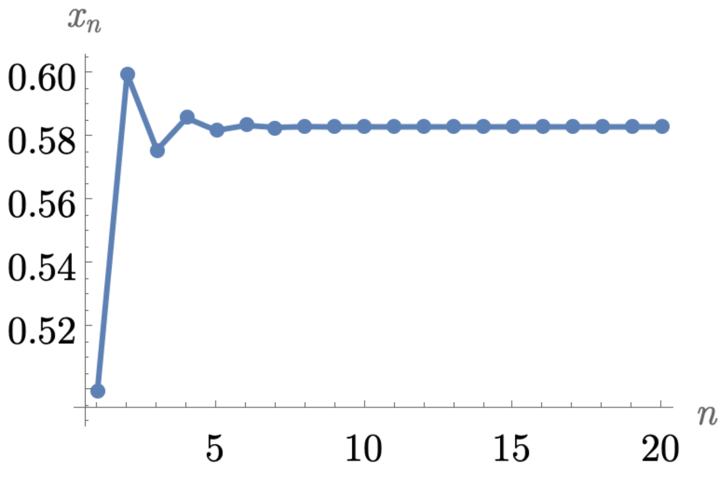

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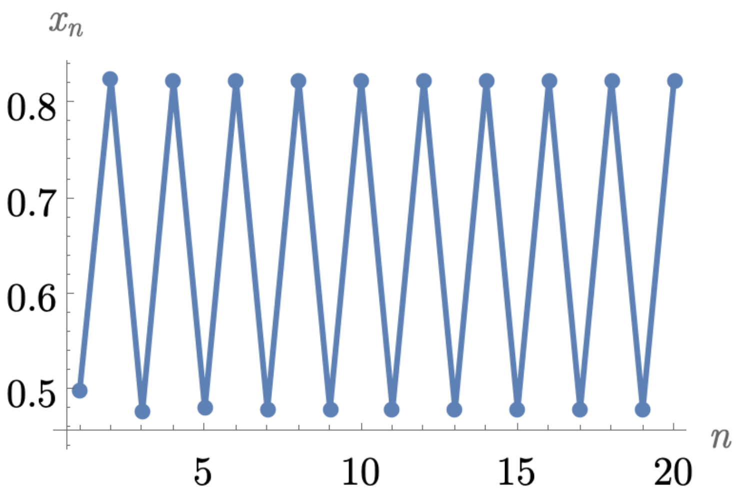

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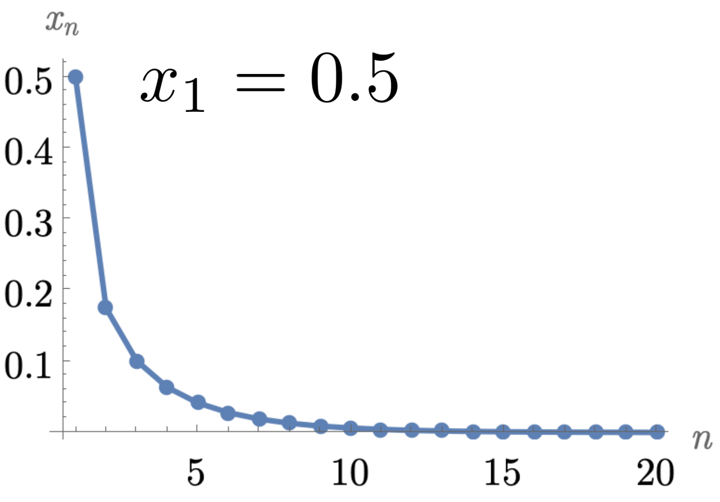

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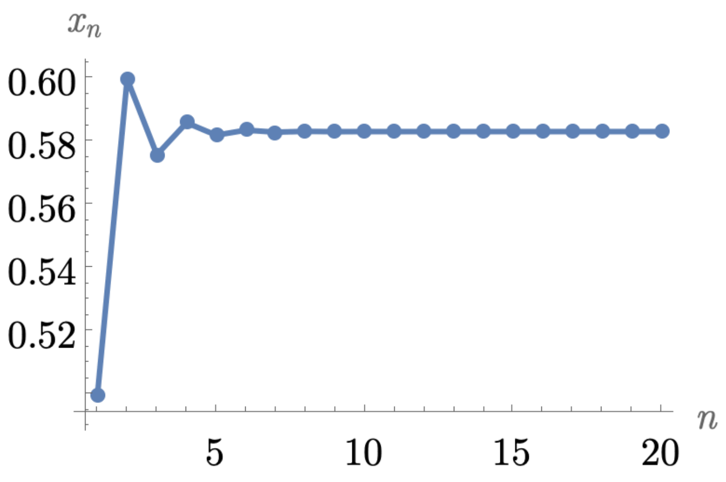

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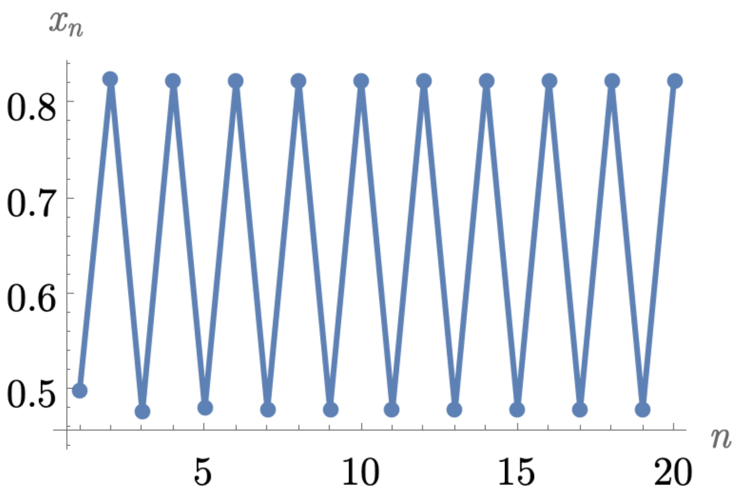

Two-year cycle equilibrium

Feigenbaum (1970’s): Collecting theoretical data

[Feigenbaum, J. Stat. Phys. **19** (1978)]

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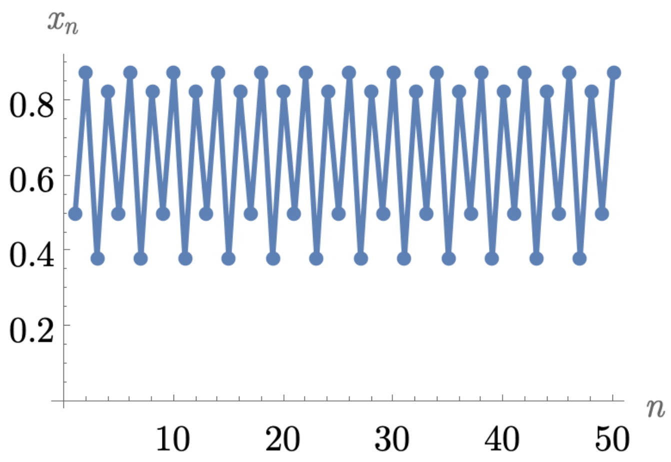

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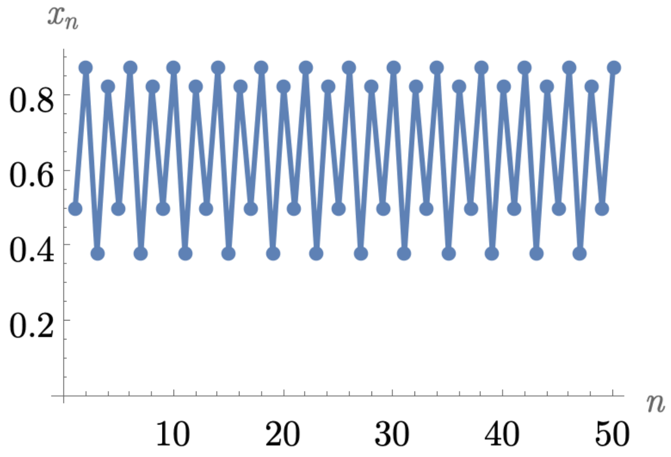

Four-year cycle

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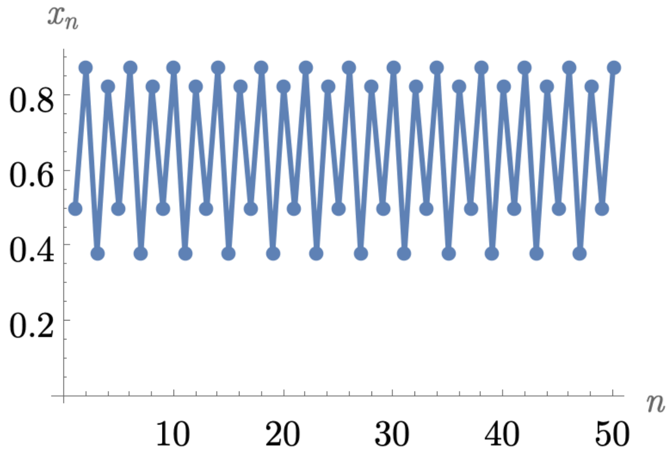
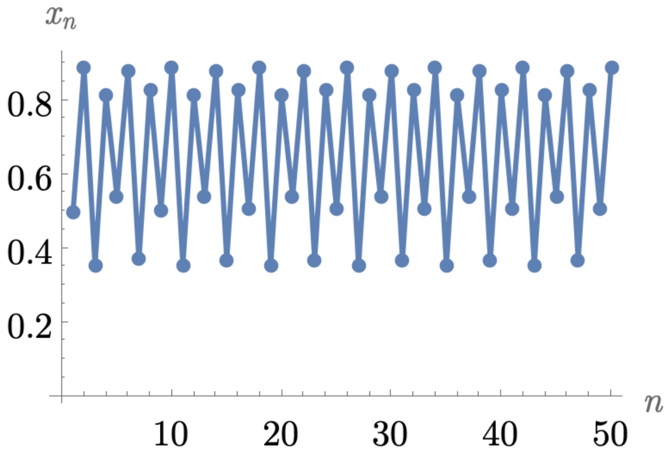


$r = 3.55$



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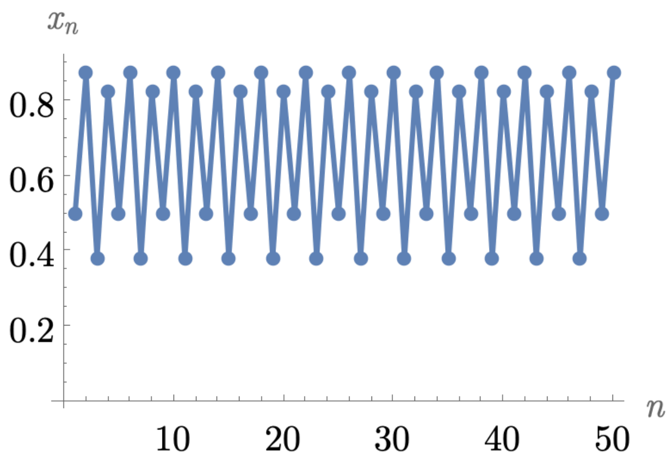

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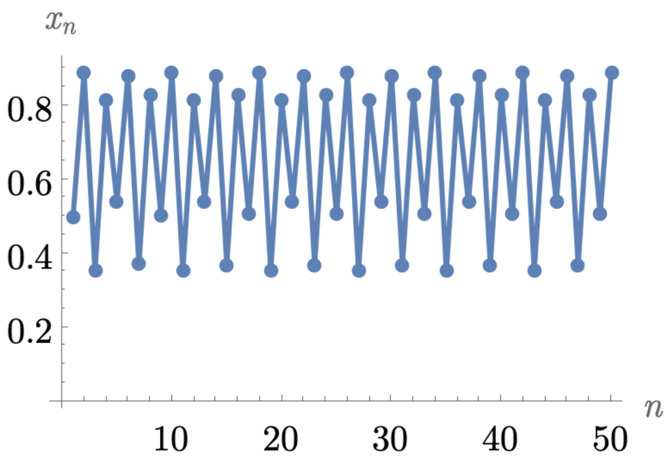

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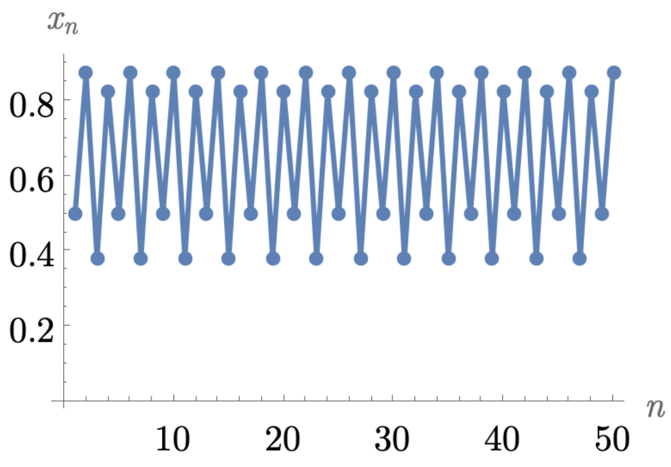

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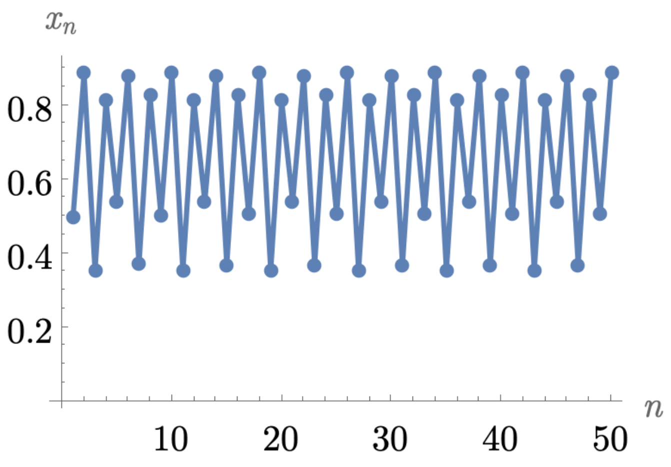
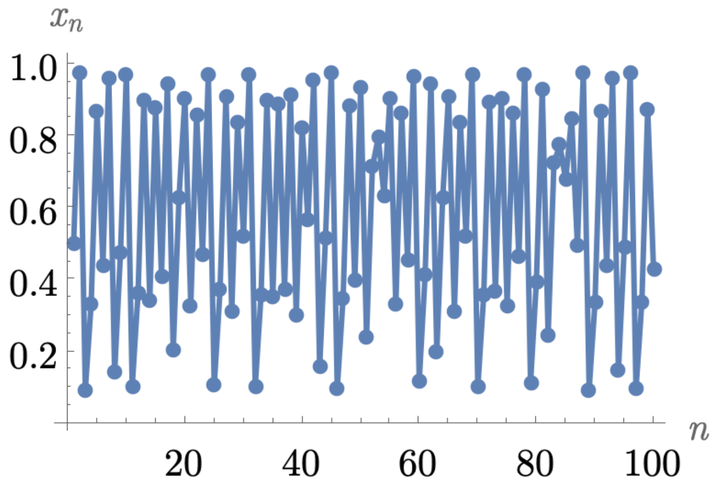


$r = 3.9$



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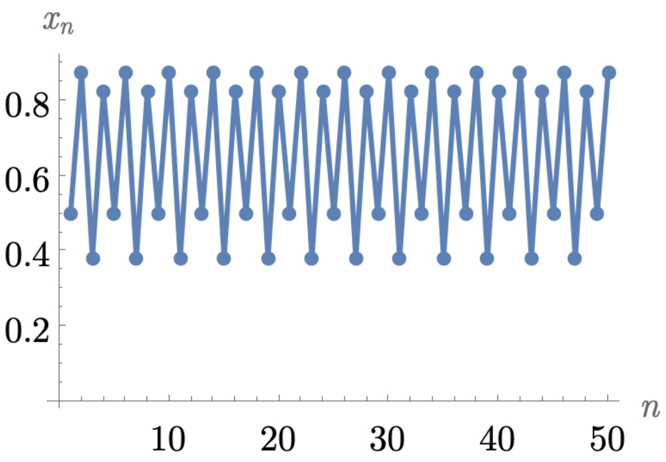

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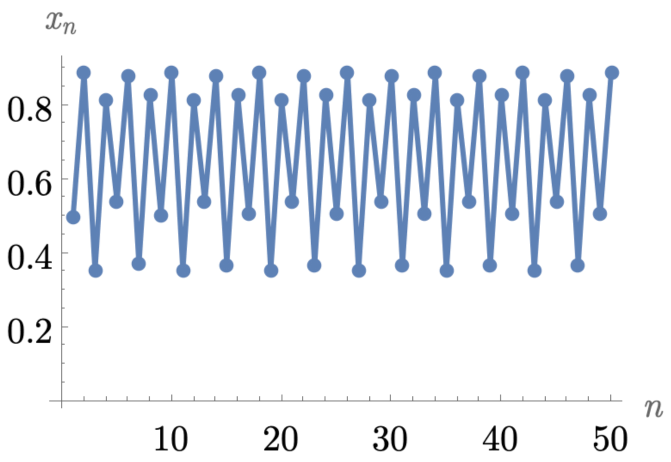

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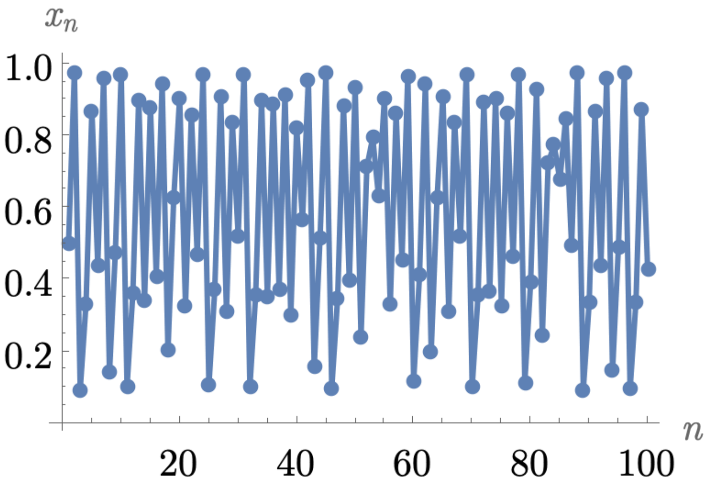
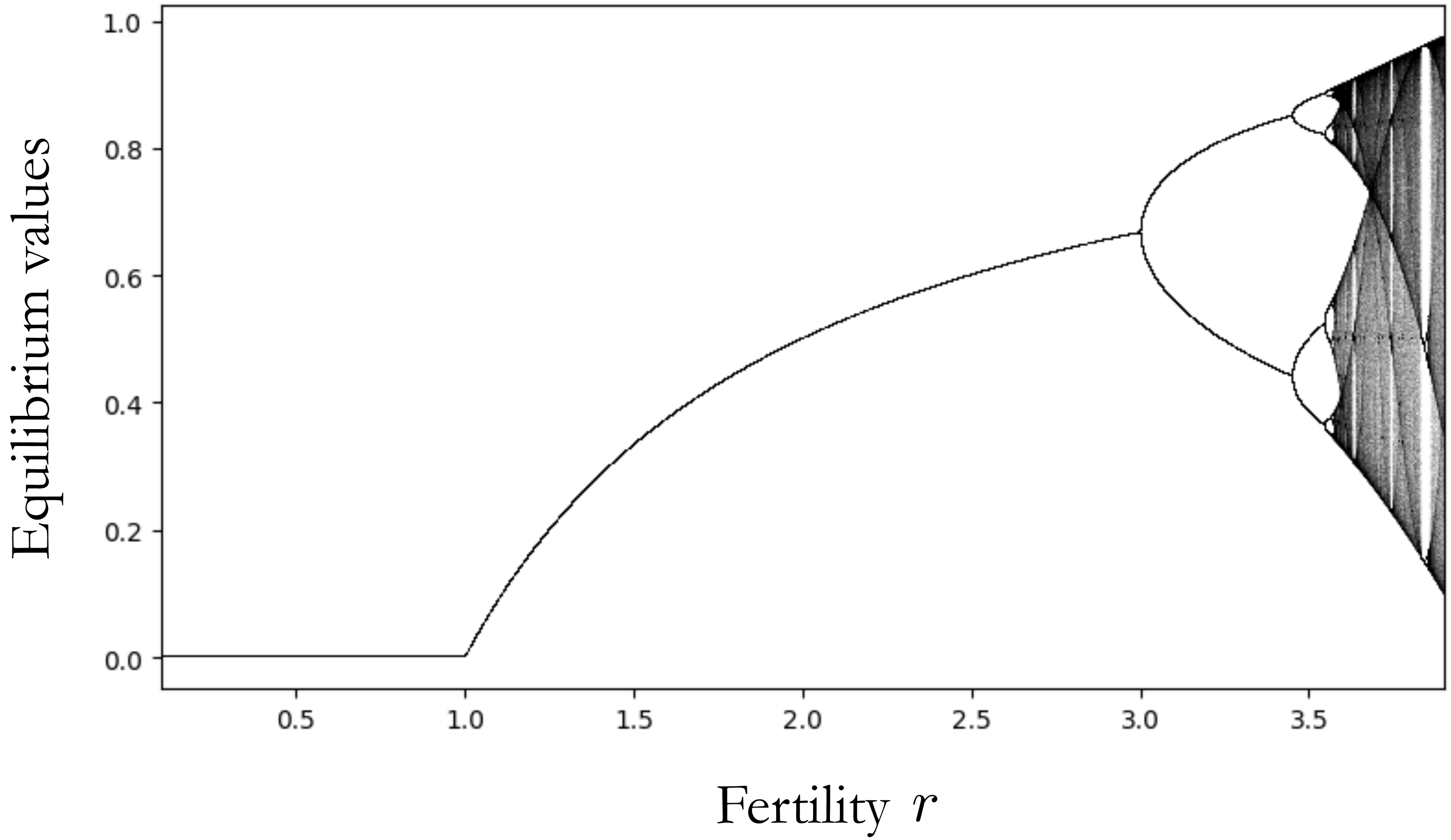


Chaos

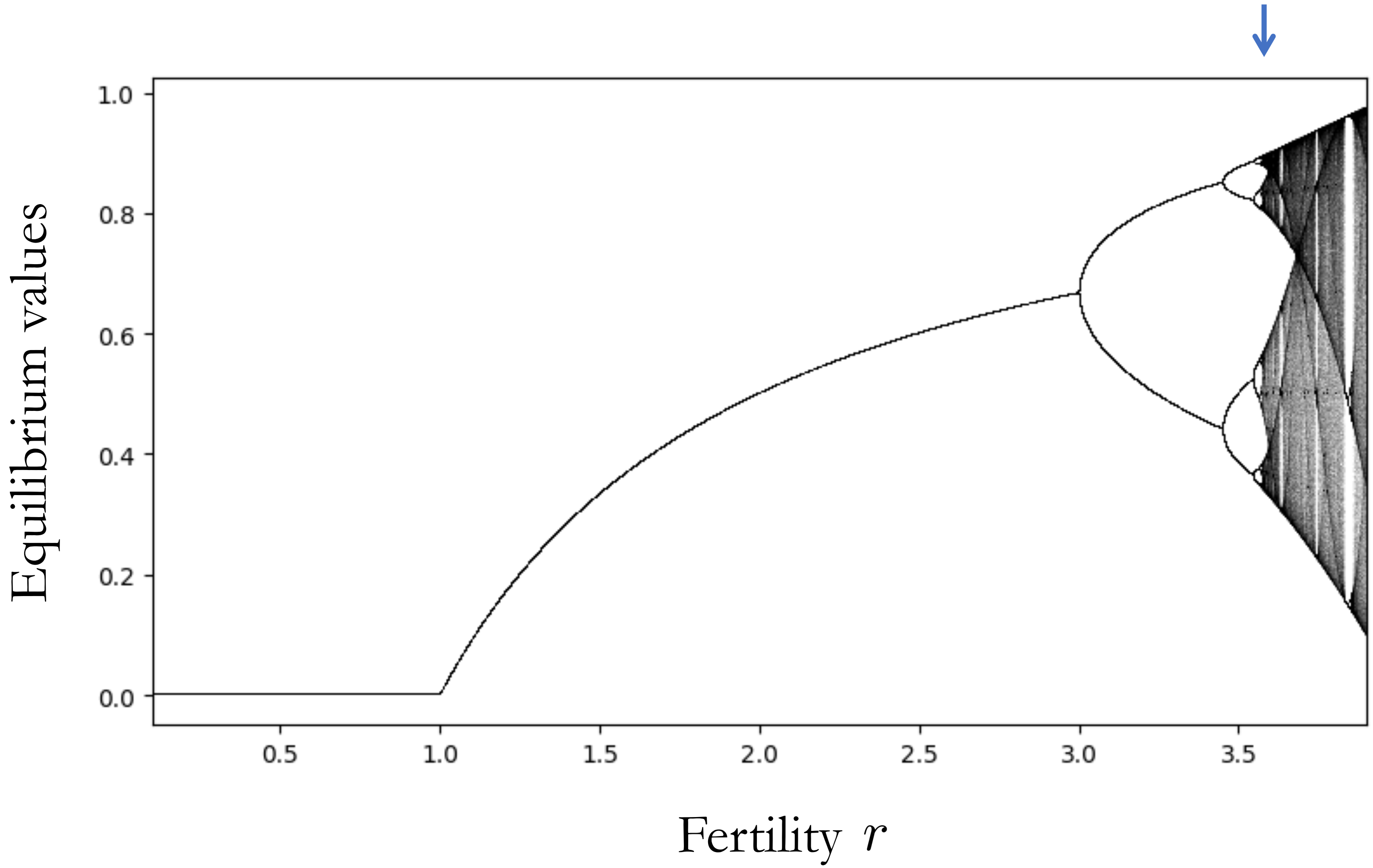
Bifurcation diagram

Bifurcation diagram



Bifurcation diagram

Transition into chaos



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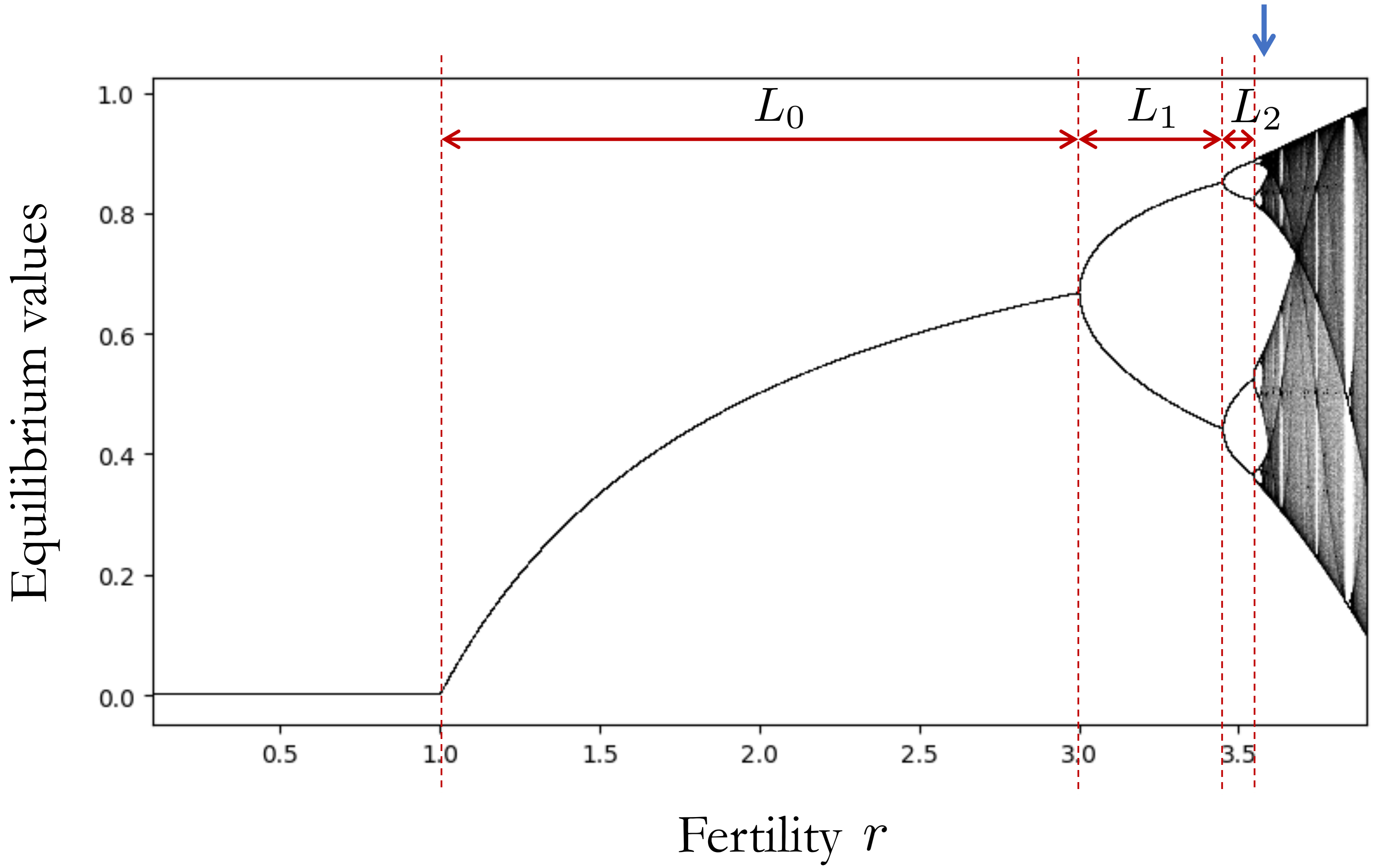

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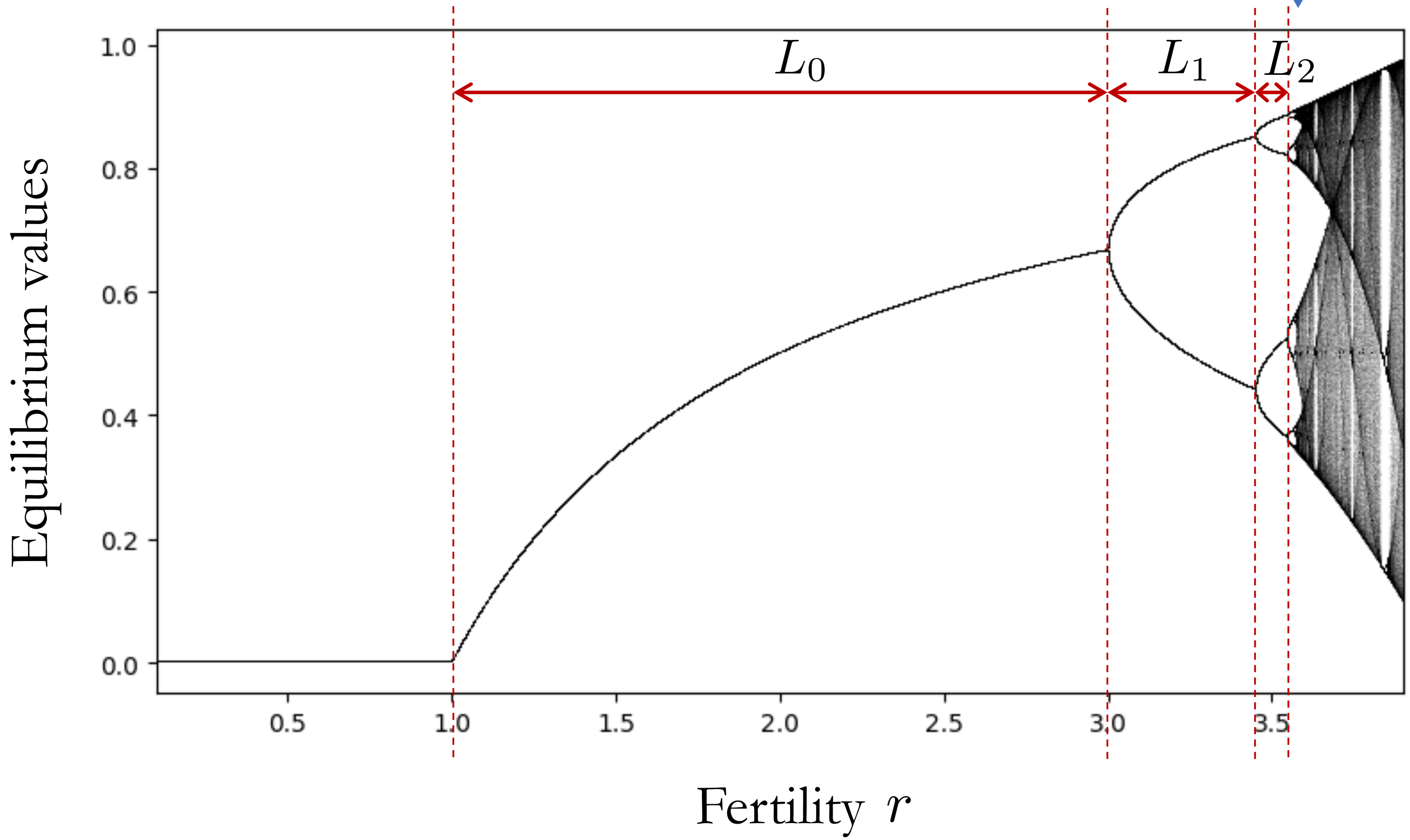
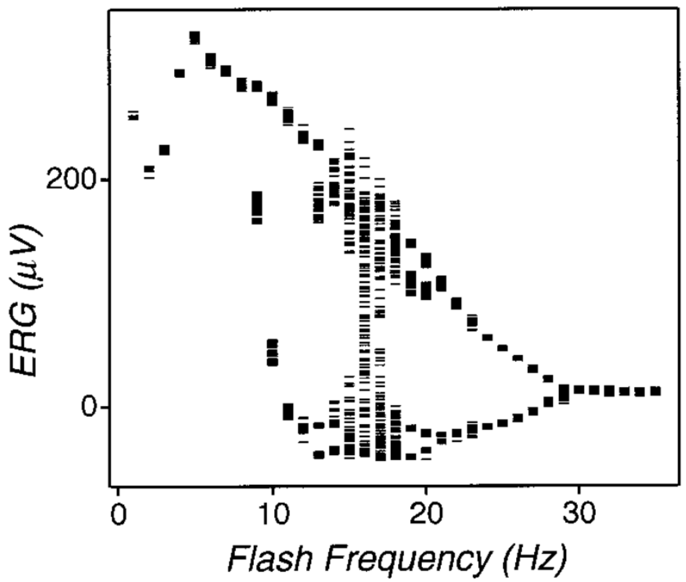


Feigenbaum constant

$$\frac{L_i}{L_{i+1}} = 4.669\ldots$$

Enormous impact on physics and other disciplines

- Non-linear dynamics
- Cloud evolution
- Electronic circuits
- Fractal geometry
- …
- Salamander vision



[Crevier, Meister, J. Neurophysiol. 79:4 (1998)]

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Theory

Experiment

Logistic map

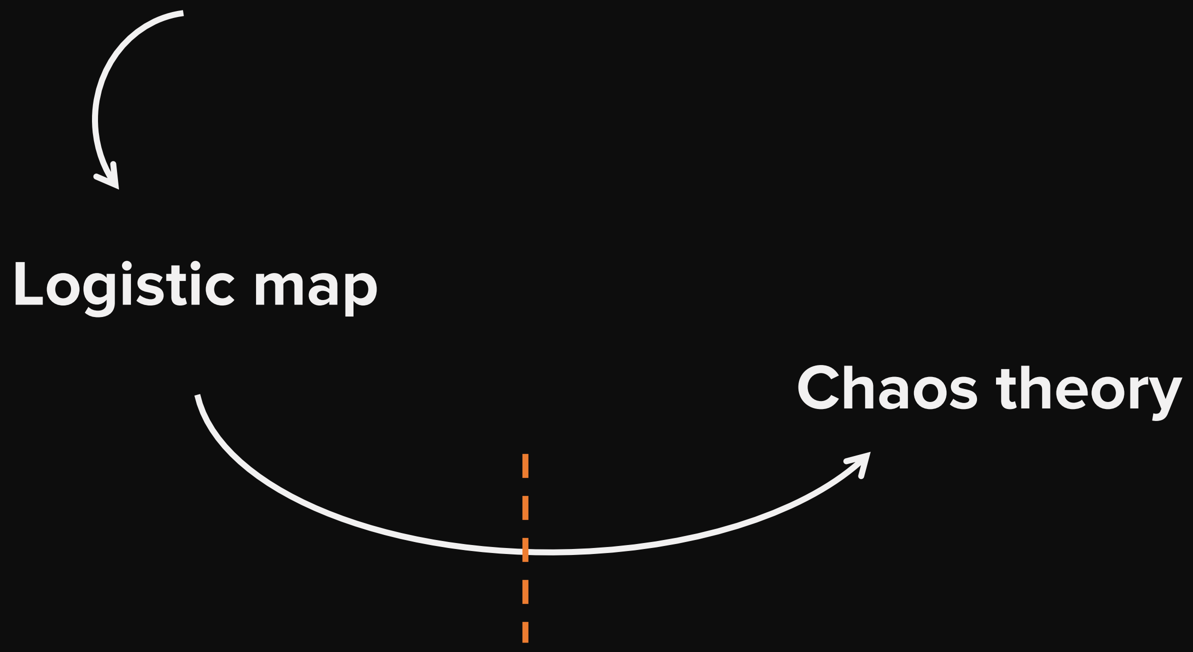
Theory

Experiment

Logistic map

Theory

Experiment



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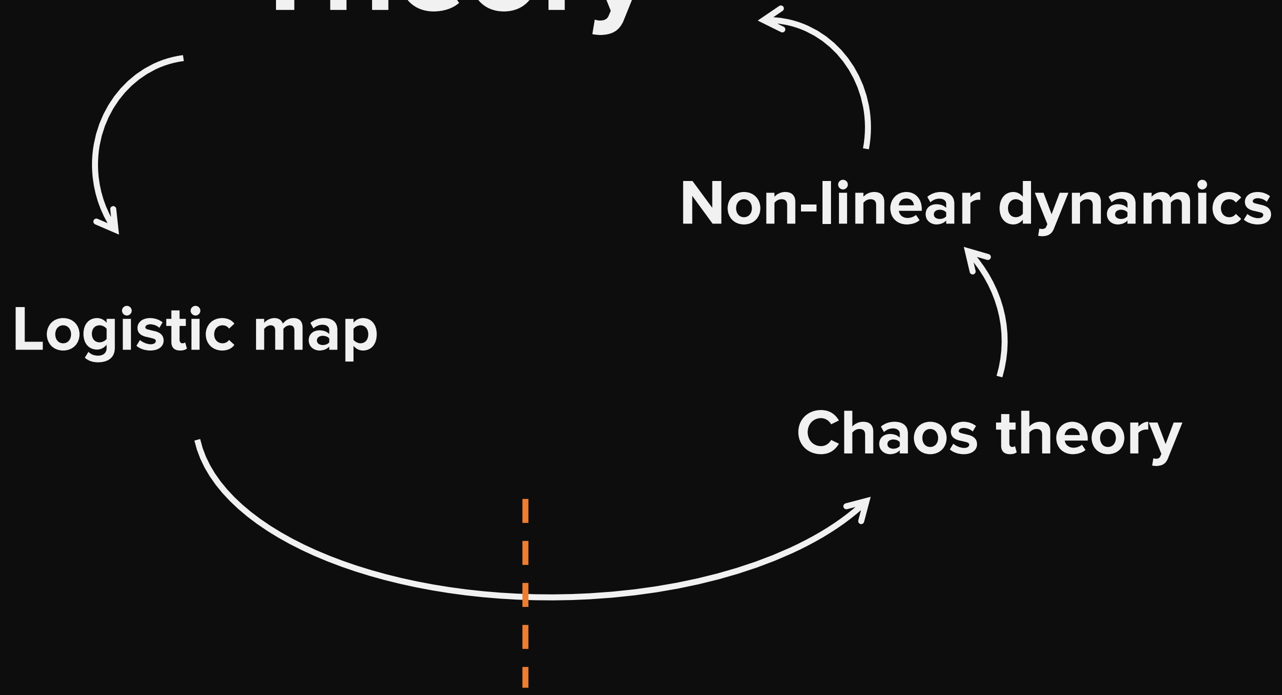

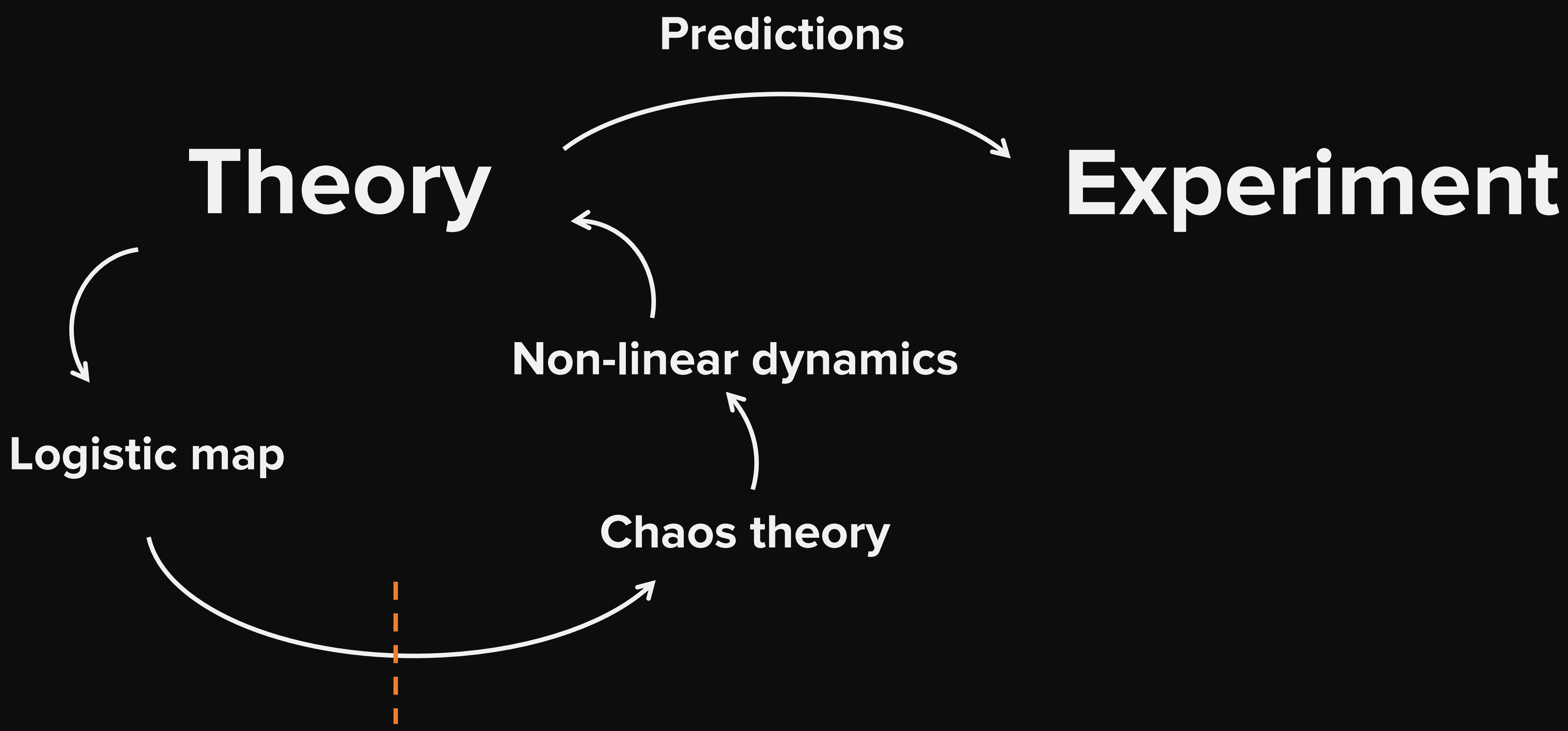
Predictions
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Chaos theory

Historical examples

Particle physics

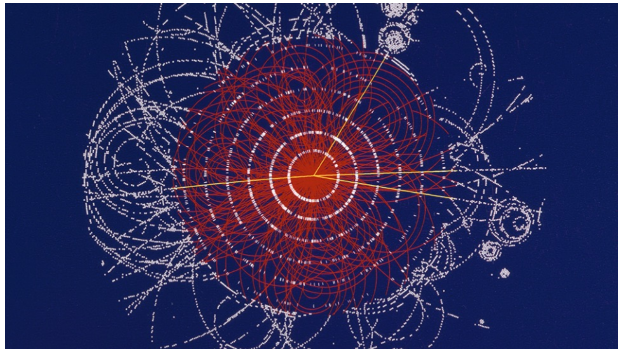
String theory

Gravitational physics

Particle physics

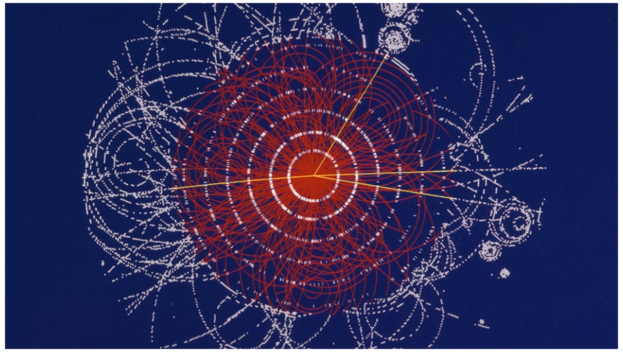
Particle physics

Perturbative expansion in quantum field theory



Particle physics

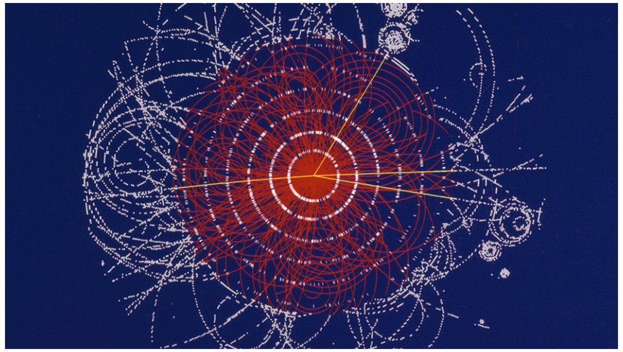
Perturbative expansion in quantum field theory



$$= \sum_{\substack{\text{Feynman} \\ \text{diagrams}}} \int \mathrm{d}^4\ell_1 \, \mathrm{d}^4\ell_2 \cdots \left(\begin{matrix} \text{Feynman} \\ \text{integral} \end{matrix} \right)$$

Particle physics

Perturbative expansion in quantum field theory



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Particle physics

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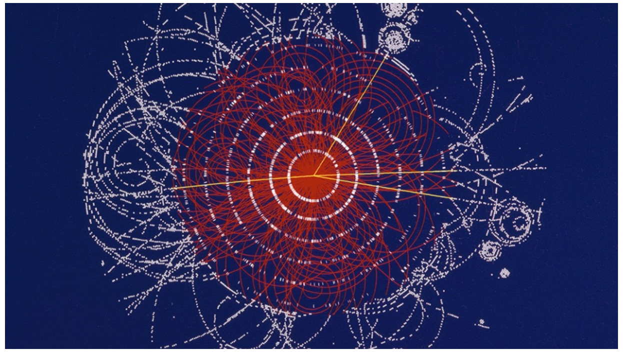

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Particle physics

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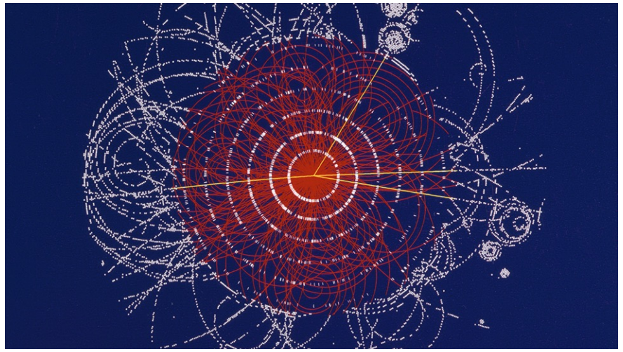

$\mathcal{O}(10^5)$

$$= \sum_{\substack{\text{Feynman} \\ \text{diagrams}}} \int \mathrm{d}^4\ell_1\, \mathrm{d}^4\ell_2 \cdots \left(\begin{array}{c}\text{Feynman} \\ \text{integral}\end{array} \right)$$

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$\mathcal{O}(10^3)$

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“Give me the numbers” approach: What is χ?

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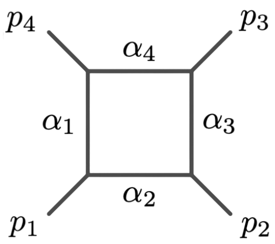
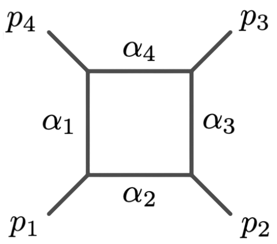


$$\chi = 3$$

“Give me the numbers” approach: What is χ ?



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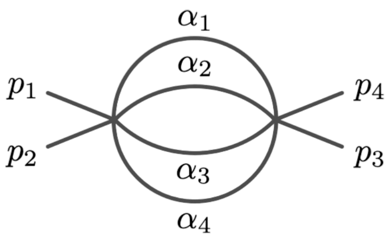
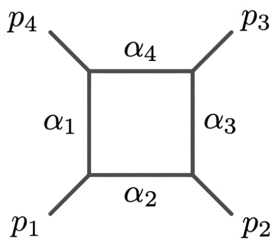
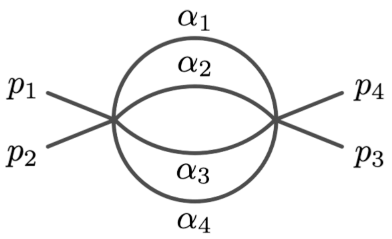


$\chi = 1$

“Give me the numbers” approach: What is χ?



$\chi = 3$



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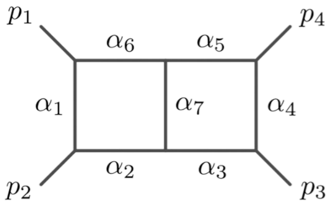
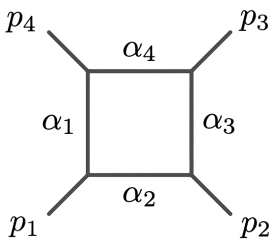


$\chi = 12$

“Give me the numbers” approach: What is χ ?



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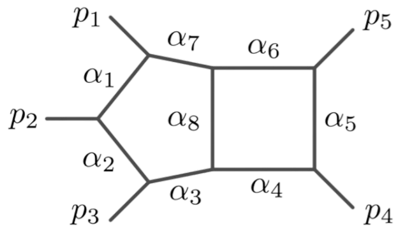
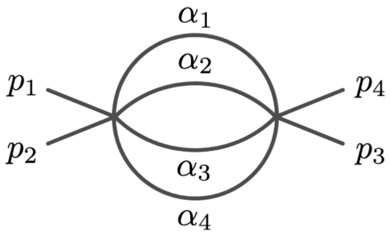
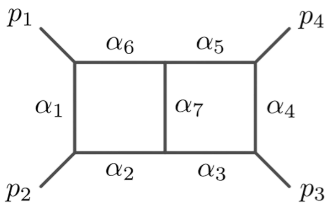


$$\chi = 62$$

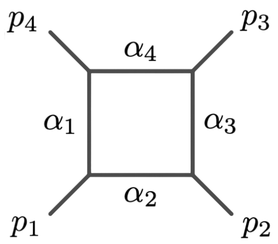


$$\chi = 1$$

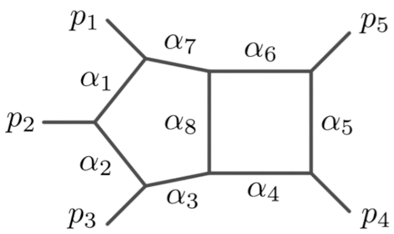


$$\chi = 12$$

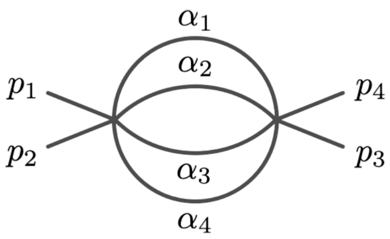
“Give me the numbers” approach: What is χ?



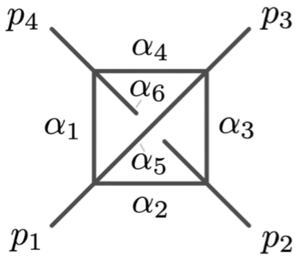
$\chi = 3$



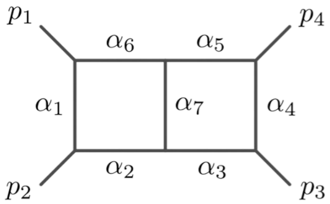
$\chi = 62$



$\chi = 1$

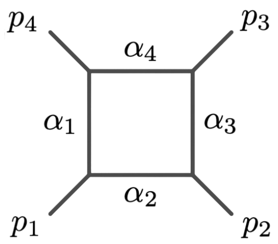


$\chi = 10$

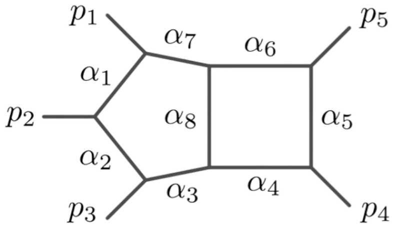


$\chi = 12$

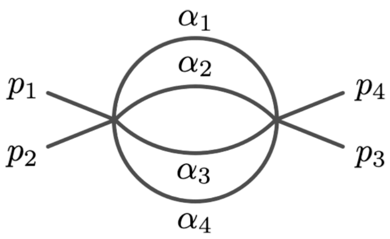
"Give me the numbers" approach: What is χ?



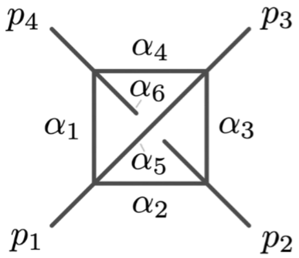
$$\chi = 3$$



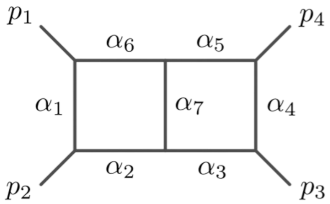
$$\chi = 62$$



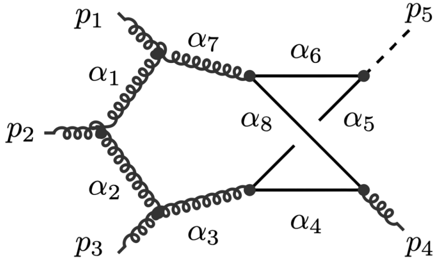
$$\chi = 1$$



$$\chi = 10$$

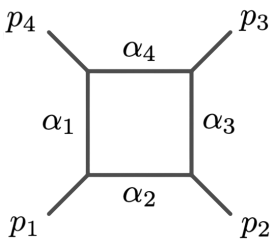


$$\chi = 12$$

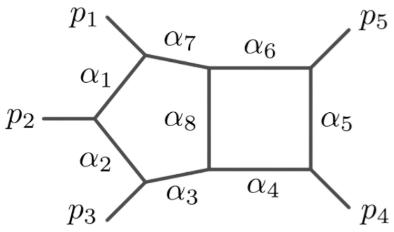


$$\chi = 330$$

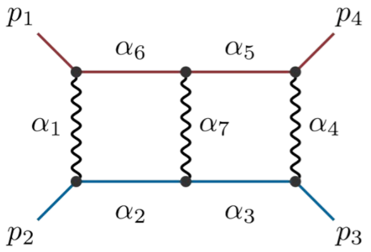
"Give me the numbers" approach: What is χ?



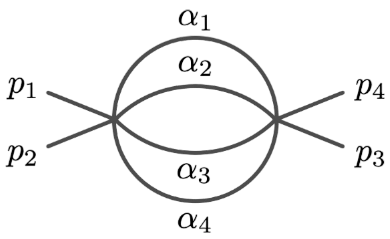
$\chi = 3$



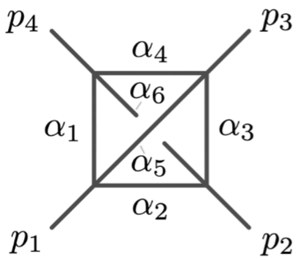
$\chi = 62$



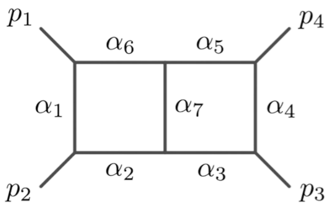
$\chi = 64$



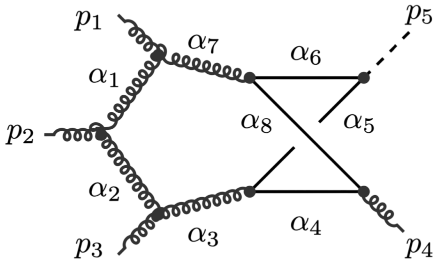
$\chi = 1$



$\chi = 10$

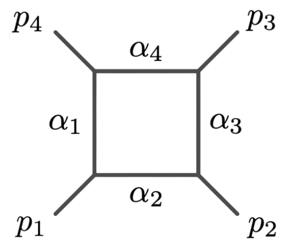


$\chi = 12$

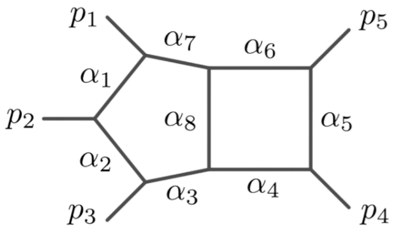


$\chi = 330$

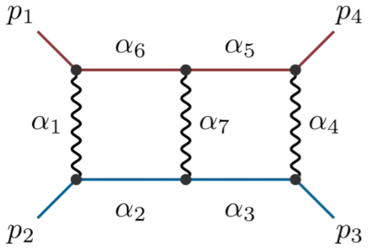
“Give me the numbers” approach: What is χ ?



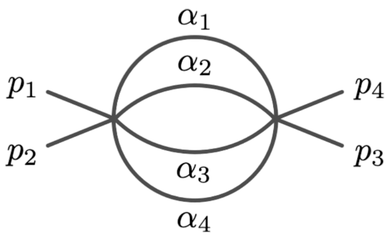
$\chi = 3$



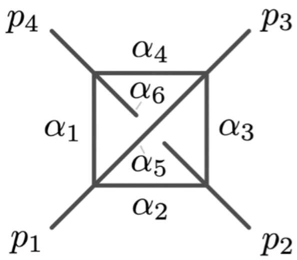
$\chi = 62$



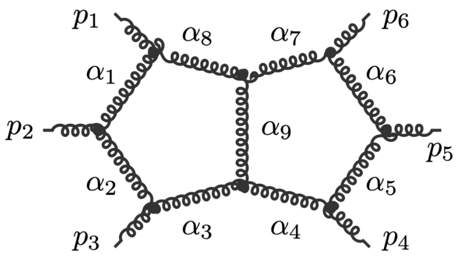
$\chi = 64$



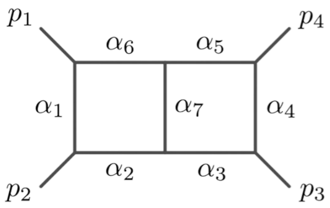
$\chi = 1$



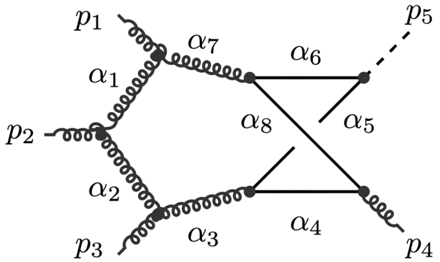
$\chi = 10$



$\chi = 281$

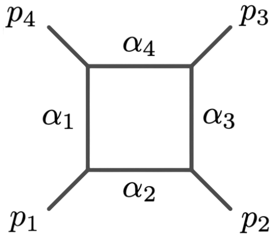


$\chi = 12$

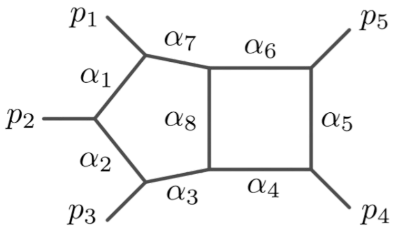


$\chi = 330$

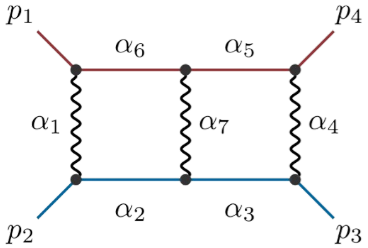
“Give me the numbers” approach: What is χ?



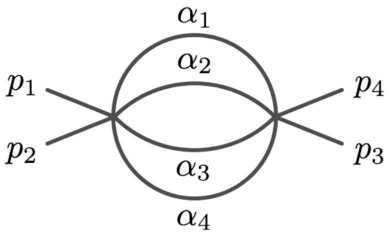
$\chi = 3$



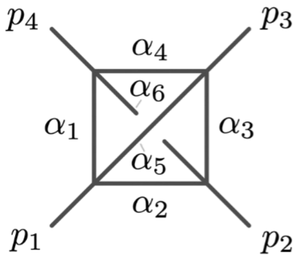
$\chi = 62$



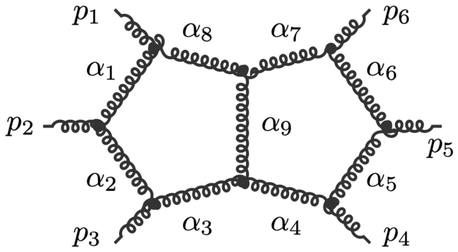
$\chi = 64$



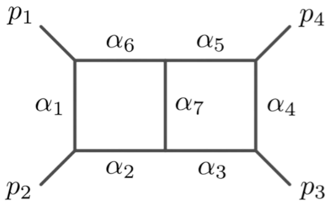
$\chi = 1$



$\chi = 10$



$\chi = 281$



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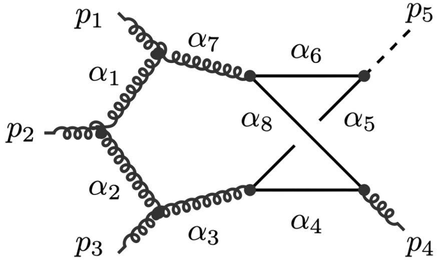
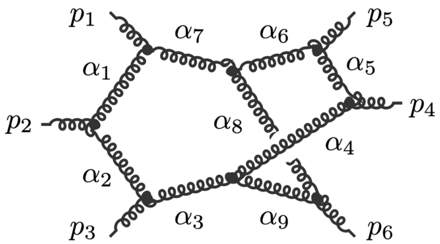


$\chi = 330$



$\chi = ?$

Vector space of a family of Feynman diagrams

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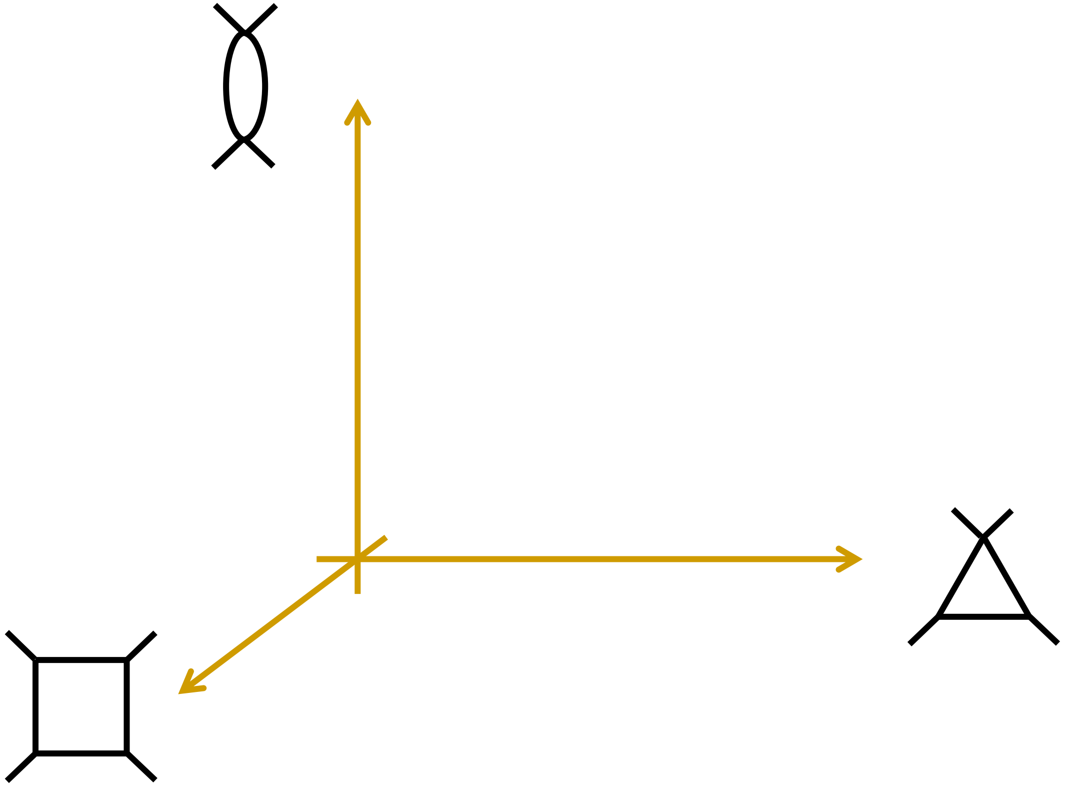
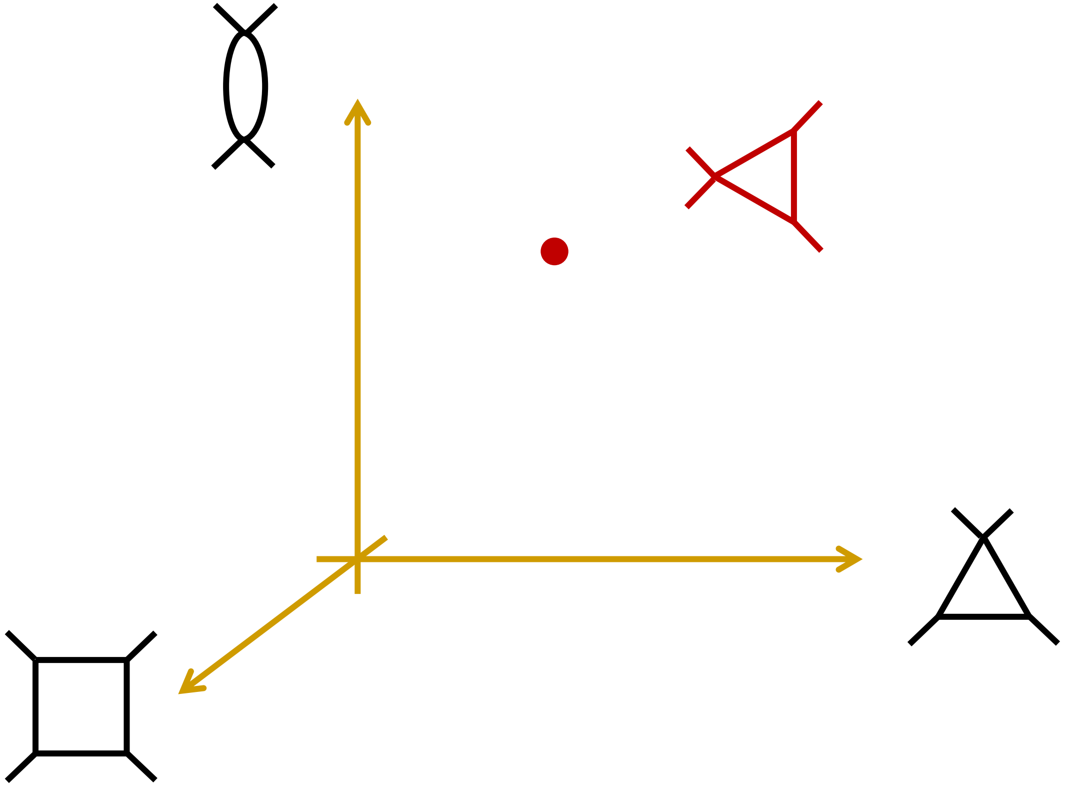
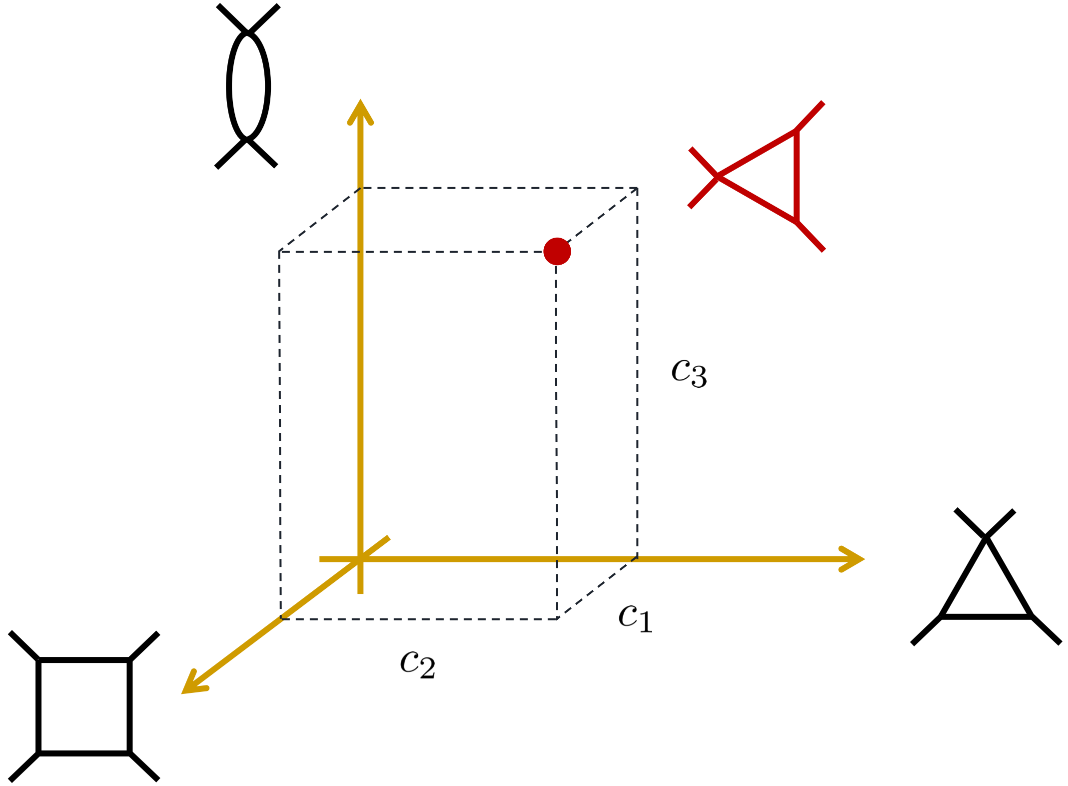


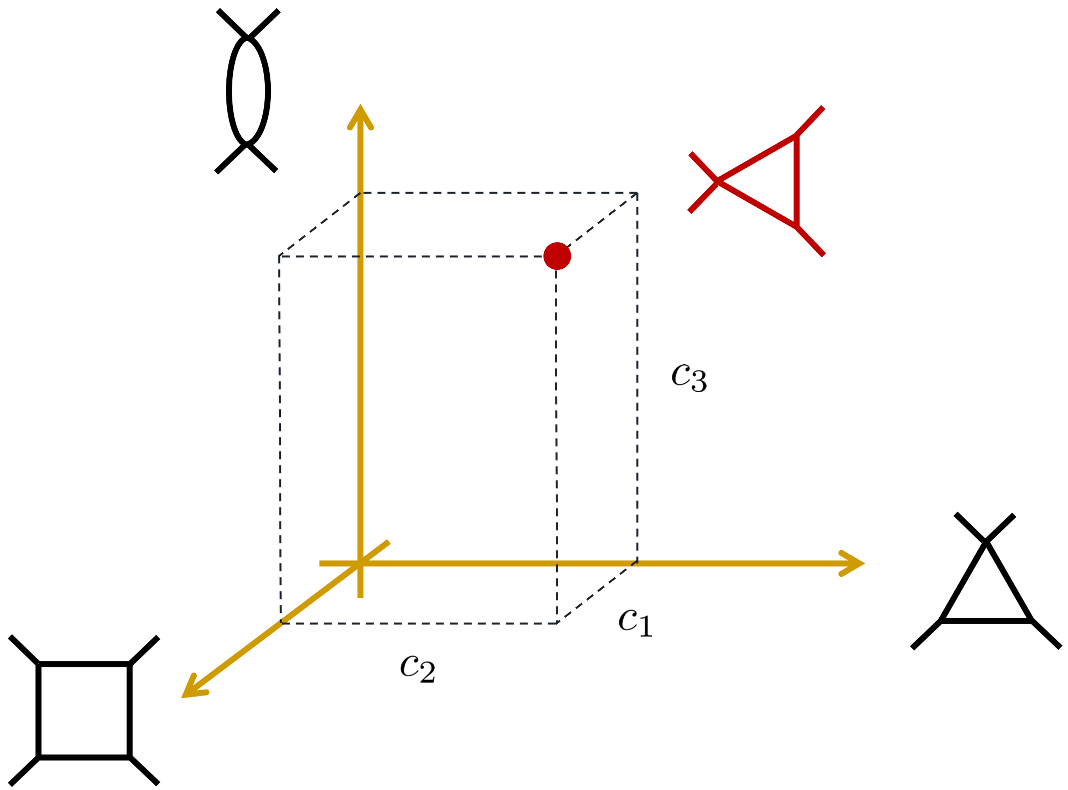
Vector space of a family of Feynman diagrams

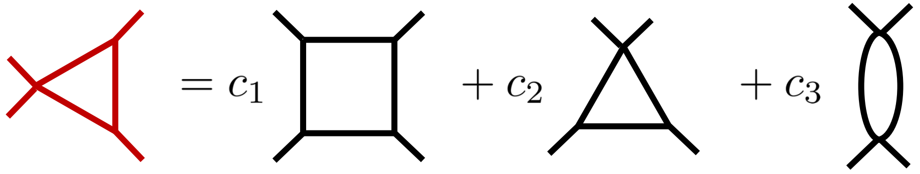


Vector space of a family of Feynman diagrams



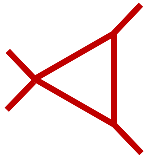
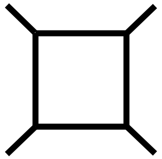
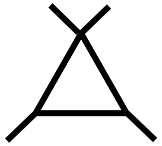
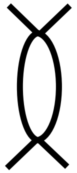
Vector space of a family of Feynman diagrams





What is the vector space?

What is the vector space?

$0 = \int \mathrm{d}\Big(\text{something}\Big) =$  $- c_1$  $- c_2$  $- c_3$ 

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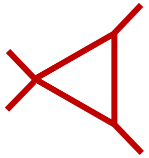
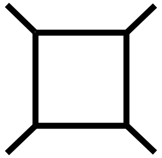
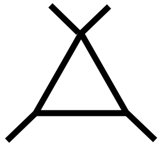
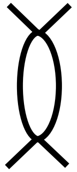
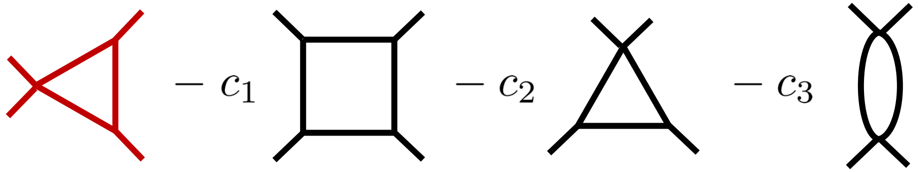
$$0 = \int \mathrm{d}\left(\text{something}\right) =$$  $$- c_1$$  $$- c_2$$  $$- c_3$$ 

$$\int \left(\begin{array}{c}\text{Feynman}\\ \text{integrand}\end{array}\right) = \int \left[\left(\begin{array}{c}\text{Feynman}\\ \text{integrand}\end{array}\right) + \mathrm{d}\left(\text{anything}\right)\right]$$

What is the vector space?

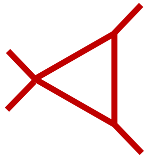
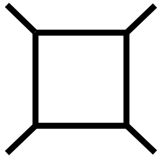
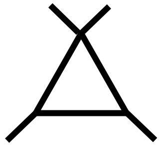
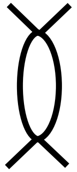
$$0 = \int \mathrm{d}\left(\text{something} \right) =$$



$$\int \left(\begin{matrix} \text{Feynman} \\ \text{integrand} \end{matrix} \right) = \int \left[\left(\begin{matrix} \text{Feynman} \\ \text{integrand} \end{matrix} \right) + \mathrm{d}\left(\text{anything} \right) \right]$$

Equivalence class of Feynman integrands:

What is the vector space?

$$0 = \int \mathrm{d}\left(\text{something} \right) =$$  $- c_1$  $- c_2$  $- c_3$ 

$$\int \left(\begin{array}{c} \text{Feynman} \\ \text{integrand} \end{array} \right) = \int \left[\left(\begin{array}{c} \text{Feynman} \\ \text{integrand} \end{array} \right) + \mathrm{d}\left(\text{anything} \right) \right]$$

Equivalence class of Feynman integrands:

$$H \equiv \frac{\{\text{space of possible loop integrads}\}}{\{\text{total derivatives}\}}$$

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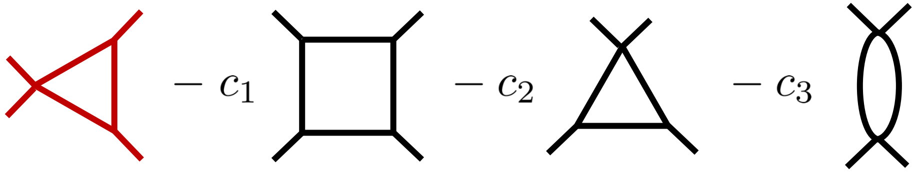

$$\int \left(\begin{array}{c} \text{Feynman} \\ \text{integrand} \end{array} \right) = \int \left[\left(\begin{array}{c} \text{Feynman} \\ \text{integrand} \end{array} \right) + \mathrm{d}\left(\text{anything} \right) \right]$$

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Known to mathematicians as the "**twisted cohomology group**"

[Deligne, Aomoto, Gelfand, Kita, Yoshida, Cho, Matsumoto, … 1960-70's]

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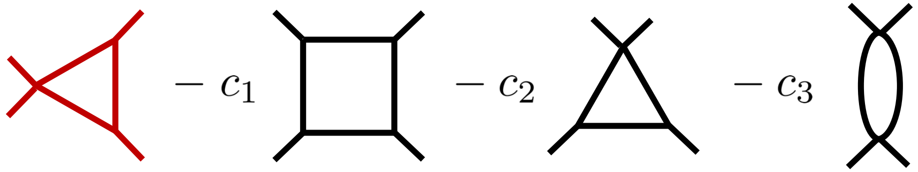

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Equivalence class of Feynman integrands:

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Dimensional regularization

Known to mathematicians as the "**twisted cohomology group**"

[Deligne, Aomoto, Gelfand, Kita, Yoshida, Cho, Matsumoto, … 1960-70's]

Connection with algebraic topology

Connection with algebraic topology

Dimension of the vector space is a topological invariant called the **signed Euler characteristic** $\chi = \dim H$

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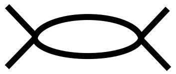
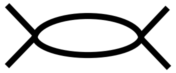


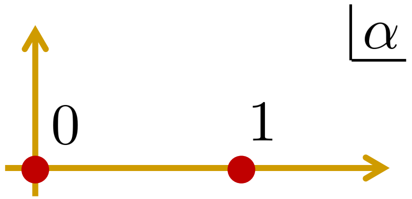
$$= \int_0^1 \frac{\mathrm{d}\alpha}{[s\alpha(1-\alpha)]^{3-\mathrm{D}/2}}$$

Connection with algebraic topology

Dimension of the vector space is a topological invariant called the **signed Euler characteristic** $\chi = \dim H$



$$= \int_0^1 \frac{\mathrm{d}\alpha}{[s\alpha(1-\alpha)]^{3-\mathrm{D}/2}}$$



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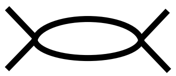
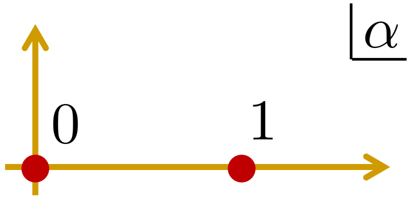


$$= \int_0^1 \frac{\mathrm{d}\alpha}{[s\alpha(1-\alpha)]^{3-\mathrm{D}/2}}$$

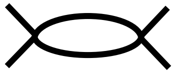


$$\chi = \left|\chi\left(\mathbb{C} - \{2 \text{ points}\}\right)\right| = 1$$

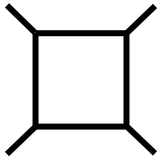
1 2

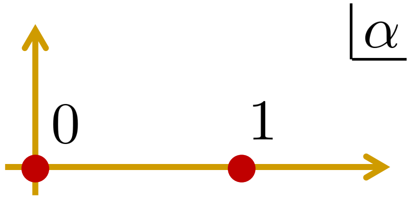
Connection with algebraic topology

Dimension of the vector space is a topological invariant called the
signed Euler characteristic $\chi = \dim H$

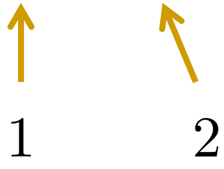


$$= \int_0^1 \frac{\mathrm{d}\alpha}{[s\alpha(1-\alpha)]^{3-\mathrm{D}/2}}$$





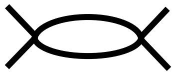
$$\chi = \left|\chi\left(\mathbb{C} - \{2 \text{ points}\}\right)\right| = 1$$



$$\chi = \left|\chi\left((\mathbb{C}^*)^3 - \{\text{quadric surface}\}\right)\right| = 3$$

Connection with algebraic topology

Dimension of the vector space is a topological invariant called the **signed Euler characteristic** $\chi = \dim H$



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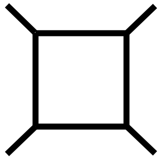

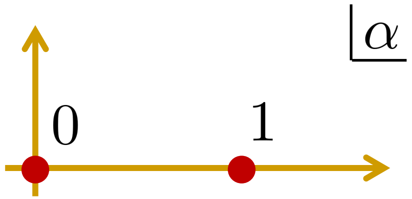

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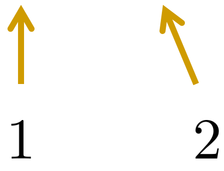
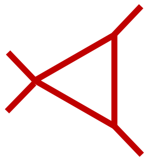
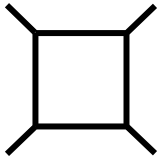
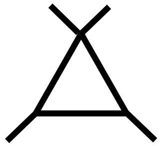
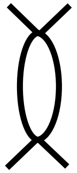


[Fevola, **SM**, Telen; PRL **132** (2024) 10]

[Bitoun, Bogner, Klausen, Panzer; Lett. Math. Phys. **109** (2019) 3]

Inner product between Feynman diagrams

Inner product between Feynman diagrams

 $= c_1$  $+ c_2$  $+ c_3$ 

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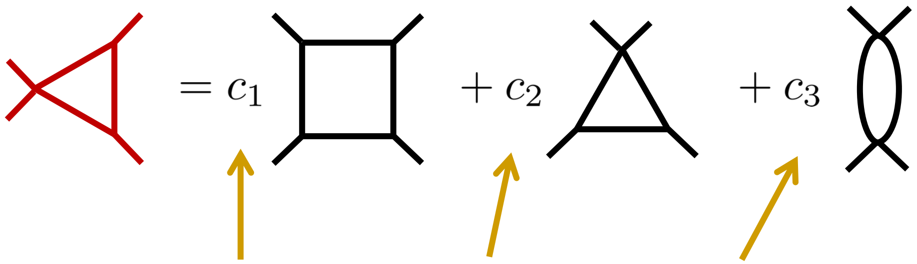
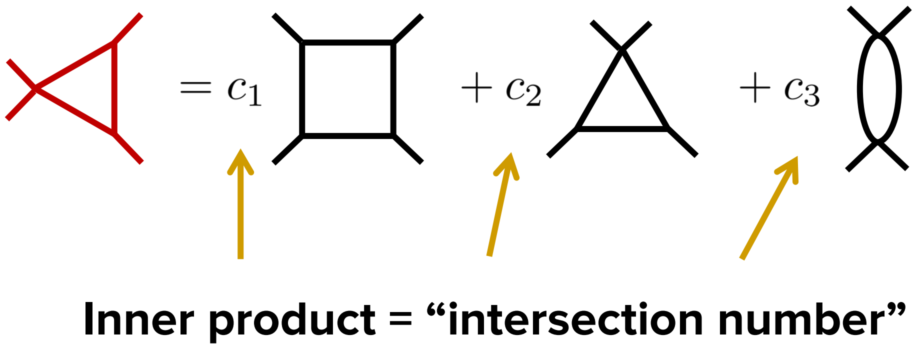
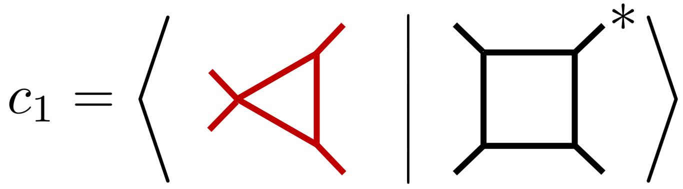


Inner product = “intersection number”

Inner product between Feynman diagrams





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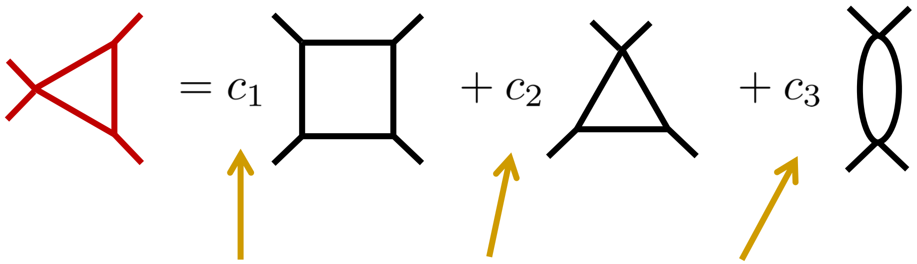

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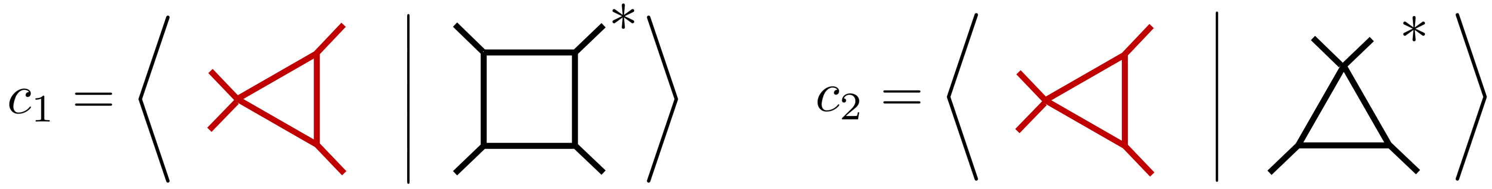
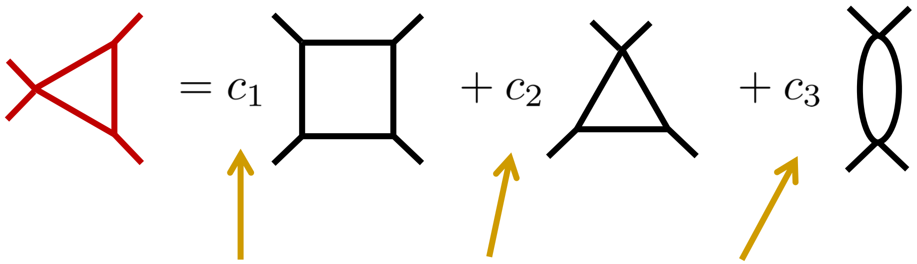


Inner product between Feynman diagrams



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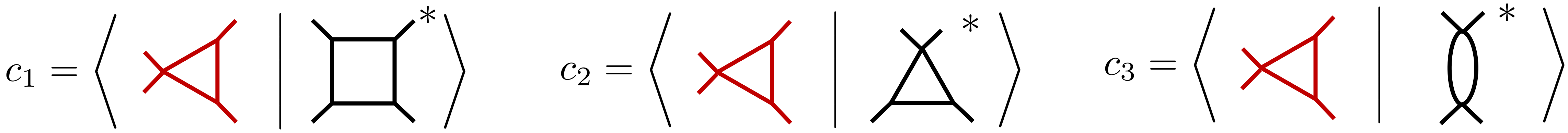

[**SM**; PRL **120** (2018) 14]

[Mastrolia, **SM**; JHEP **02** (2019) 139]

[Frellesvig, Gasparotto, Mandal, Mastrolia, Mattiazzi, **SM**; PRL **123** (2019) 20]

Opens a new avenue in perturbative computations

Connections & applications to

- QCD scattering amplitudes
- Post-Minkowskian expansions
- Generalized unitarity
- String theory
- Finite-field methods
- Hyperplane arrangements
- Matroid theory
- …

[Aomoto, Argeri, Arkani-Hamed, Baikov, Bai, Barucchi, Bern, Bitoun, Bosma, Britto, Brønnum-Hansen, Broedel, Caron-Huot, Chawdhry, Chetyrkin, Cho, Duhr, Febres Cordero, Frellesvig, Gasparotto, Gardi, Georgoudis, Giroux, Gluza, Goto, Grozin, Harley, Hartanto, Kajda, Kita, Klausen, Kotikov, Lam, Laporta, Larsen, Lee, Lim, Lo Presti, Maierhöfer, Mandal, Marcolli, Mastrolia, Matsumoto, Mattiazzi, Mazloumi, Mirabella, Mitov, **SM**, Moriello, Page, Panzer, Peraro, Pokraka, Pomeransky, Ponzano, Remiddi, Schabinger, Schönemann, Sogaard, Stieberger, Studerus, Tarasov, Tkachov, Usovitsch, Uwer, Weinzierl, Zeng, Zhang]

Theory

Experiment

Counting
master integrals

Theory

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Twisted cohomology

Theory

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Euler characteristic

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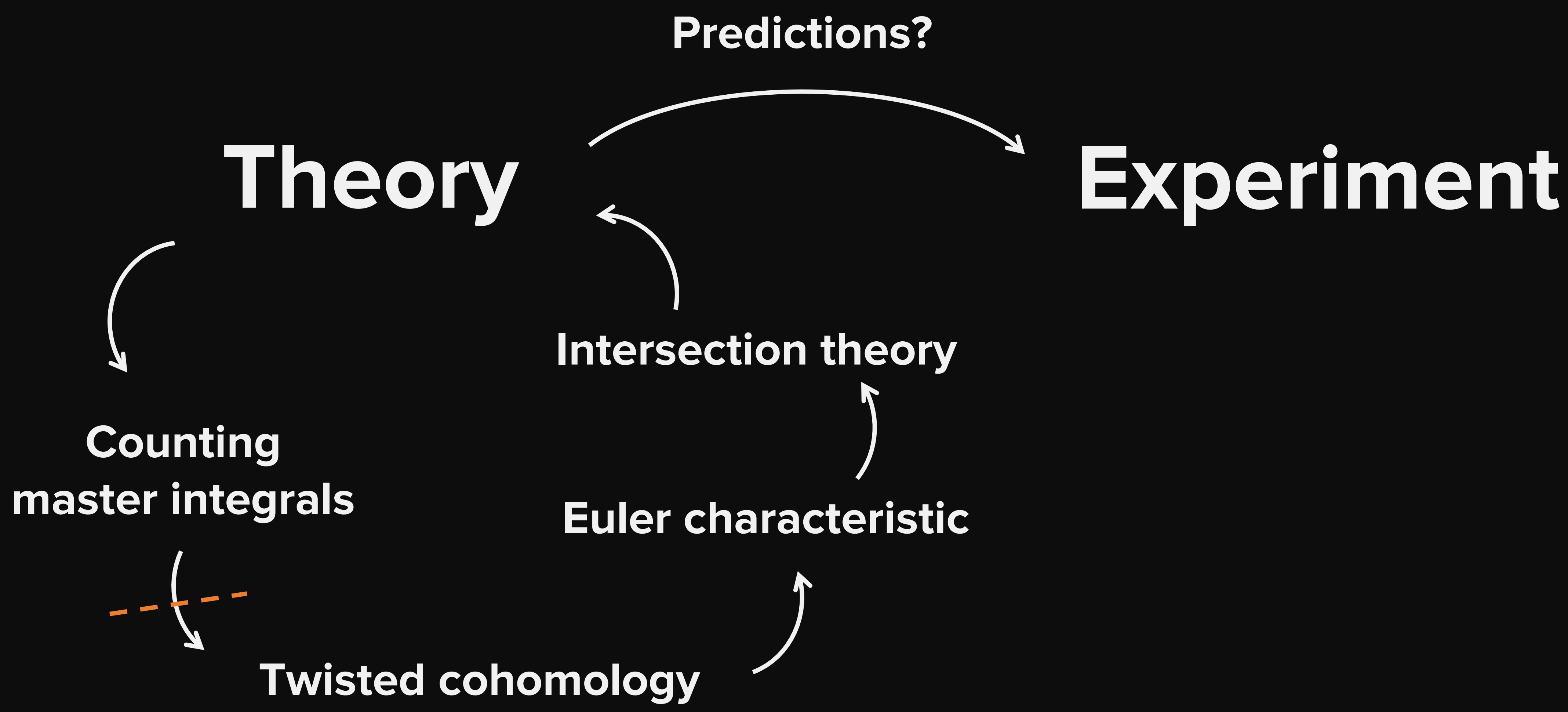
Predictions?
Theory
Experiment
Intersection theory
Counting
master integrals
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Twisted cohomology

Historical examples

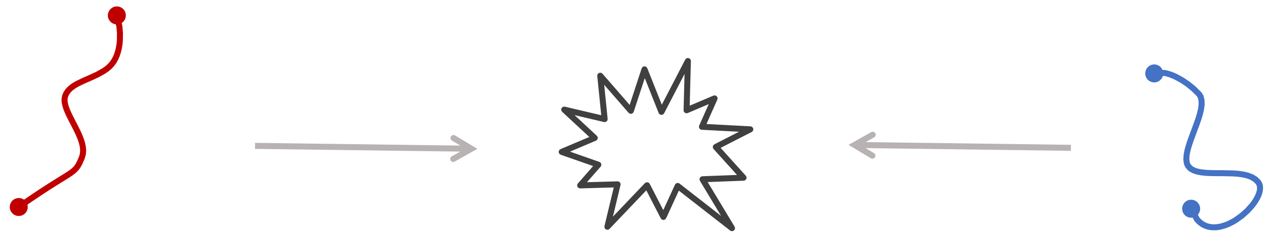
Particle physics

String theory

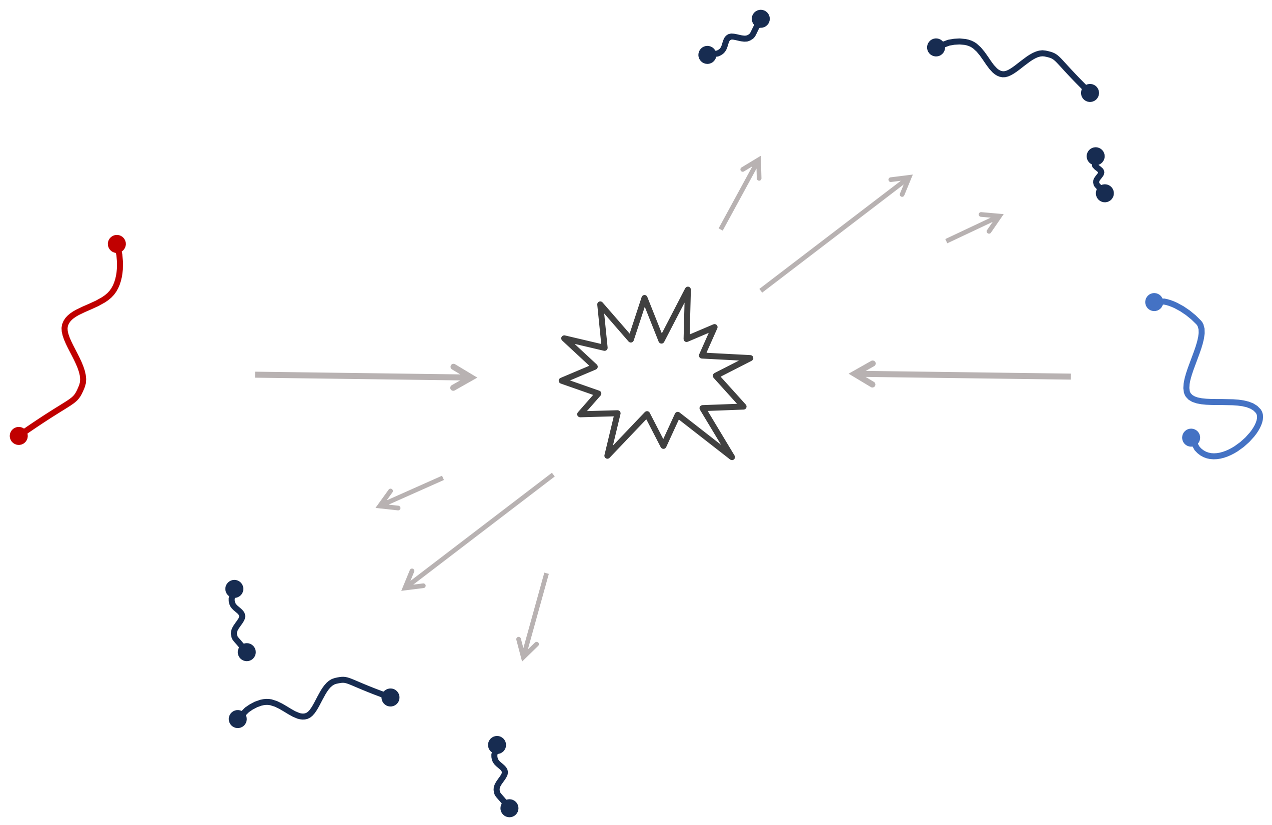
Gravitational physics

String theory: Rare window into quantum gravity

String theory: Rare window into quantum gravity



String theory: Rare window into quantum gravity



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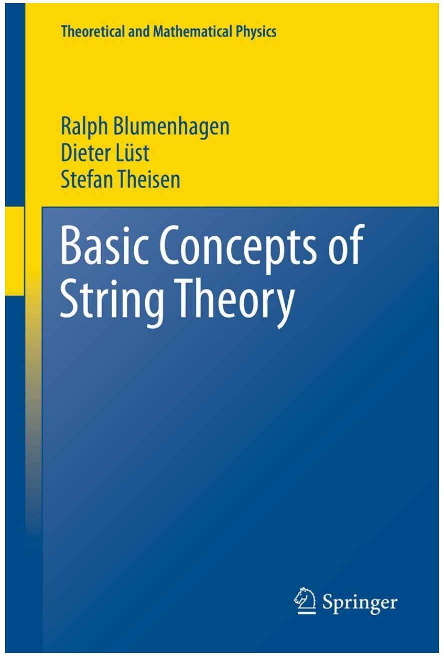

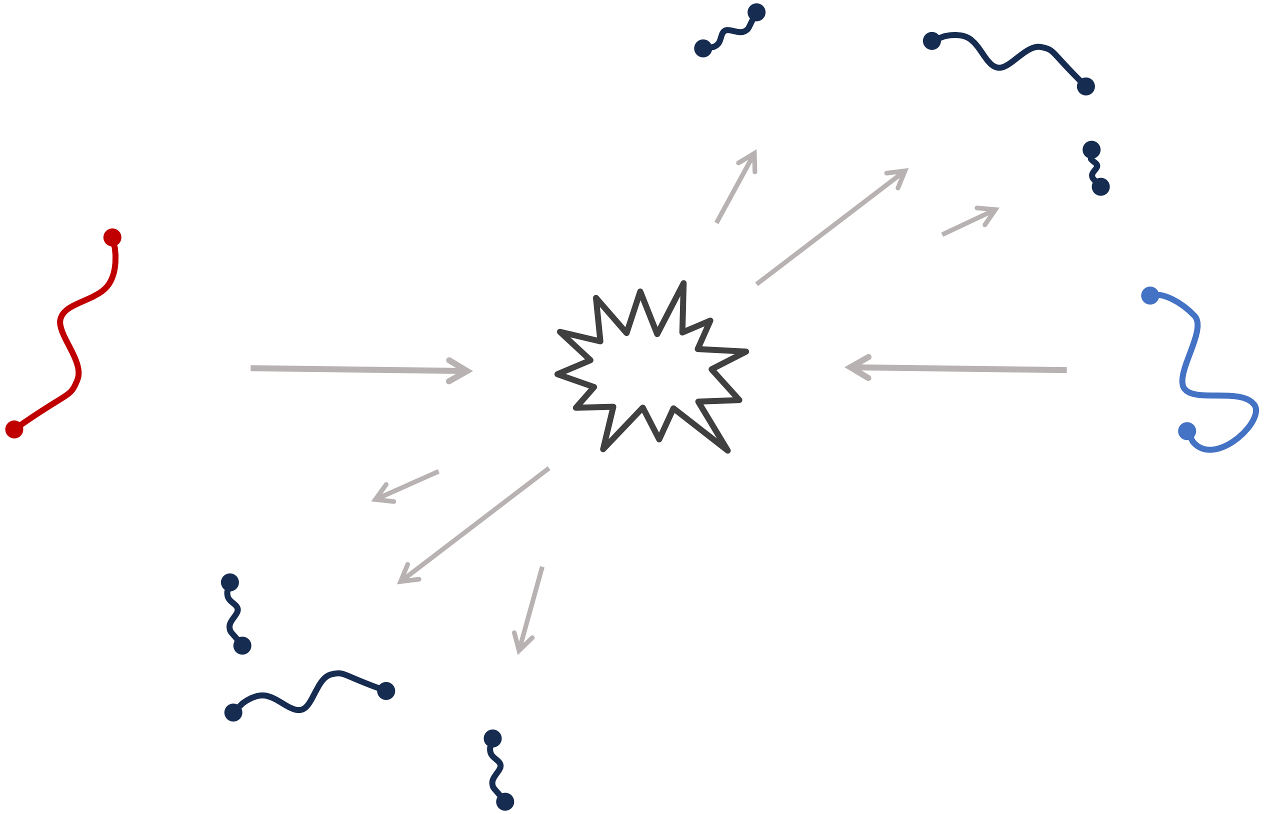

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"Give me the numbers" approach: Exclusive $2 \to 2$ scattering

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Center of mass energy

$s = (p_1 + p_2)^2$

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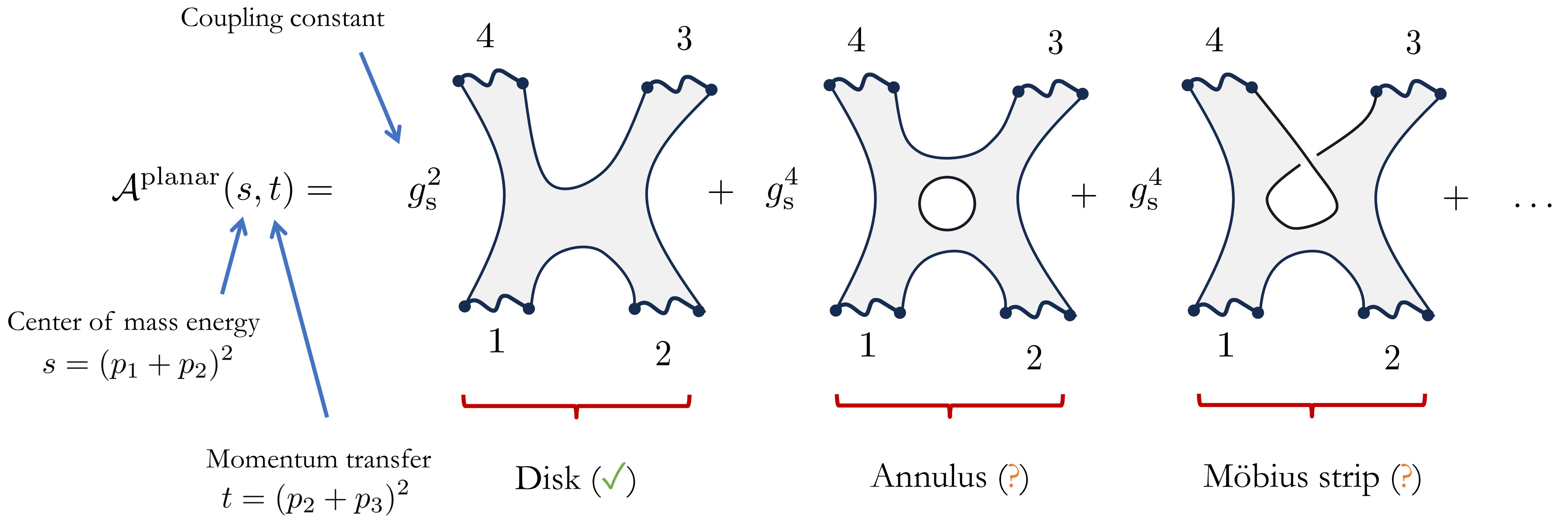
Center of mass energy
$s = (p_1 + p_2)^2$

Momentum transfer
$t = (p_2 + p_3)^2$

"Give me the numbers" approach: Exclusive $2 \to 2$ scattering



Tree level: Veneziano amplitude

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$$\mathcal{A}_{\text{tree}}^{\text{planar}}(s,t) = -t_8 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$$

Tree level: Veneziano amplitude

Polarization dependence $t_8 = s\, p_1{\cdot}\epsilon_2\, p_2{\cdot}\epsilon_1\, \epsilon_3{\cdot}\epsilon_4 + \ldots = 1$

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Inverse string tension

$\alpha' = 1$

One-loop level: Integral expression (for the experts)

By now, a textbook result [Green, Schwarz 1982]

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$$\mathcal{A}^{\text{planar}}_{\text{annulus}}(s,t) \overset{?}{=} -it_8 \int_0^{i\infty} \mathrm{d}\tau \int_{0<z_1<z_2<z_3<1} \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3 \left(\frac{\theta_1(z_2-z_1,\tau)\theta_1(z_4-z_3,\tau)}{\theta_1(z_3-z_1,\tau)\theta_1(z_4-z_2,\tau)}\right)^{-s} \left(\frac{\theta_1(z_3-z_2,\tau)\theta_1(z_4-z_1,\tau)}{\theta_1(z_3-z_1,\tau)\theta_1(z_4-z_2,\tau)}\right)^{-t}$$

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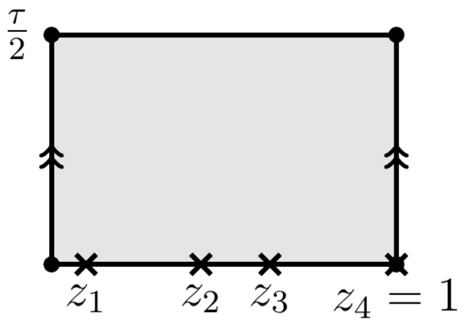

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Positions of punctures

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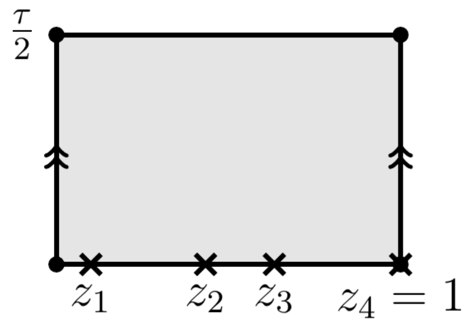

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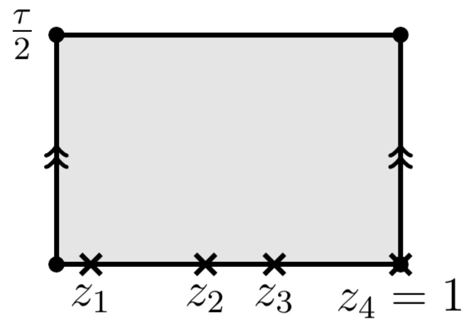

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Polarization dependence

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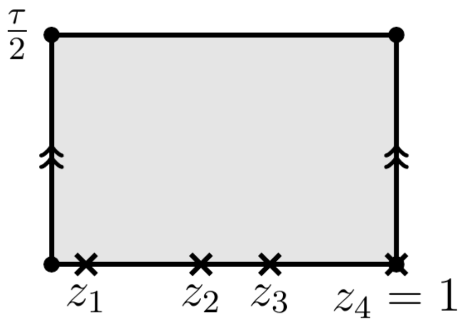

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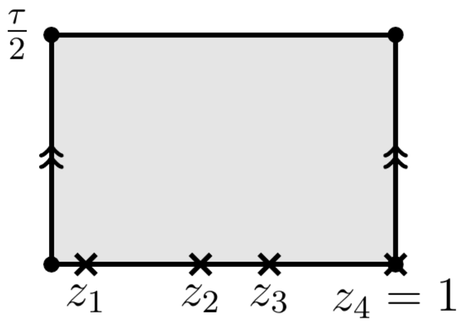
Polarization dependence

Result of computing the correlator

$$\mathcal{A}^{\text{planar}}_{\text{Mobius}}(s,t) \overset{?}{=} \frac{32}{N} it_8 \int_{\frac{1}{2}}^{\frac{1}{2}+i\infty} \cdots$$



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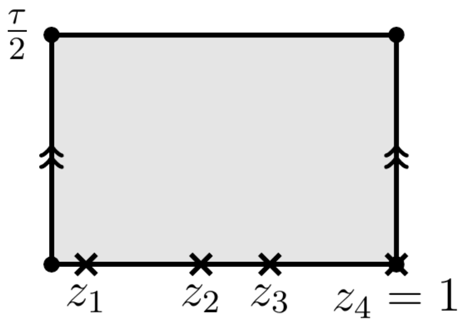
Shifted contour



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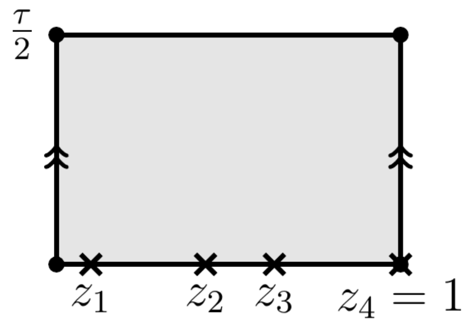
Shifted contour

Gauge group SO(N)



Let's get the numbers

In physical kinematics, $s > -t > 0$

Let’s get the numbers

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$$
\texttt{NIntegrate}\Bigg[\left(\frac{\theta_1[z_2 - z_1,\ \tau]\ \theta_1[z_4 - z_3,\ \tau]}{\theta_1[z_3 - z_1,\ \tau]\ \theta_1[z_4 - z_2,\ \tau]}\right)^{-s}\left(\frac{\theta_1[z_3 - z_2,\ \tau]\ \theta_1[z_4 - z_1,\ \tau]}{\theta_1[z_3 - z_1,\ \tau]\ \theta_1[z_4 - z_2,\ \tau]}\right)^{-t} -
$$

$$
\left(\frac{\theta_1[z_2 - z_1,\ \tau + 1/2]\ \theta_1[z_4 - z_3,\ \tau + 1/2]}{\theta_1[z_3 - z_1,\ \tau + 1/2]\ \theta_1[z_4 - z_2,\ \tau + 1/2]}\right)^{-s}\left(\frac{\theta_1[z_3 - z_2,\ \tau + 1/2]\ \theta_1[z_4 - z_1,\ \tau + 1/2]}{\theta_1[z_3 - z_1,\ \tau + 1/2]\ \theta_1[z_4 - z_2,\ \tau + 1/2]}\right)^{-t}\ /.
$$

$$
\{s \to 3/2,\ t \to -1/2,\ z_4 \to 1,\ \tau \to i\ \mathrm{Im}\tau\},\ \{\tau,\ 0,\ \infty\},\ \{z_1,\ 0,\ 1\},\ \{z_2,\ z_1,\ 1\},\ \{z_3,\ z_2,\ 1\}\Bigg]
$$

Let's get the numbers

In physical kinematics, $s > -t > 0$

```
NIntegrate[
 ((θ1[z2 - z1, τ] θ1[z4 - z3, τ]) / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-s ((θ1[z3 - z2, τ] θ1[z4 - z1, τ]) / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-t -
   ((θ1[z2 - z1, τ + 1 / 2] θ1[z4 - z3, τ + 1 / 2]) / (θ1[z3 - z1, τ + 1 / 2] θ1[z4 - z2, τ + 1 / 2]))^-s ((θ1[z3 - z2, τ + 1 / 2] θ1[z4 - z1, τ + 1 / 2]) / (θ1[z3 - z1, τ + 1 / 2] θ1[z4 - z2, τ + 1 / 2]))^-t /.
  {s → 3 / 2, t → -1 / 2, z4 → 1, τ → ⅈ Imτ}, {τ, 0, ∞}, {z1, 0, 1}, {z2, z1, 1}, {z3, z2, 1}]
```

NIntegrate : Catastrophic loss of precision in the global error estimate due to insufficient WorkingPrecision or divergent integral.

Let’s get the numbers

In physical kinematics, $s > -t > 0$

$$
\texttt{NIntegrate}\Big[
\left(\frac{\theta_1[z_2 - z_1, \tau]\,\theta_1[z_4 - z_3, \tau]}{\theta_1[z_3 - z_1, \tau]\,\theta_1[z_4 - z_2, \tau]}\right)^{-s}
\left(\frac{\theta_1[z_3 - z_2, \tau]\,\theta_1[z_4 - z_1, \tau]}{\theta_1[z_3 - z_1, \tau]\,\theta_1[z_4 - z_2, \tau]}\right)^{-t} -
$$

$$
\left(\frac{\theta_1[z_2 - z_1, \tau + 1/2]\,\theta_1[z_4 - z_3, \tau + 1/2]}{\theta_1[z_3 - z_1, \tau + 1/2]\,\theta_1[z_4 - z_2, \tau + 1/2]}\right)^{-s}
\left(\frac{\theta_1[z_3 - z_2, \tau + 1/2]\,\theta_1[z_4 - z_1, \tau + 1/2]}{\theta_1[z_3 - z_1, \tau + 1/2]\,\theta_1[z_4 - z_2, \tau + 1/2]}\right)^{-t} \;/.
$$

$$
\{s \to 3/2,\ t \to -1/2,\ z_4 \to 1,\ \tau \to i\ \mathrm{Im}\tau\},\ \{\tau, 0, \infty\},\ \{z_1, 0, 1\},\ \{z_2, z_1, 1\},\ \{z_3, z_2, 1\}\Big]
$$

··· NIntegrate: Catastrophic loss of precision in the global error estimate due to insufficient WorkingPrecision or divergent integral.

Getting the numbers forces us to rethink the problem

Euclidean vs. Lorentzian time evolution

Euclidean vs. Lorentzian time evolution

$$\mathcal{A}_{\text{1-loop}}^{\text{planar}} \sim i \int_0^{i\infty} \mathrm{d}\tau \, (\text{real integrand}) = \infty$$

Euclidean vs. Lorentzian time evolution

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↖ real

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Not compatible with space-time **unitarity**:

$$S^\dagger S = \mathbb{1}$$

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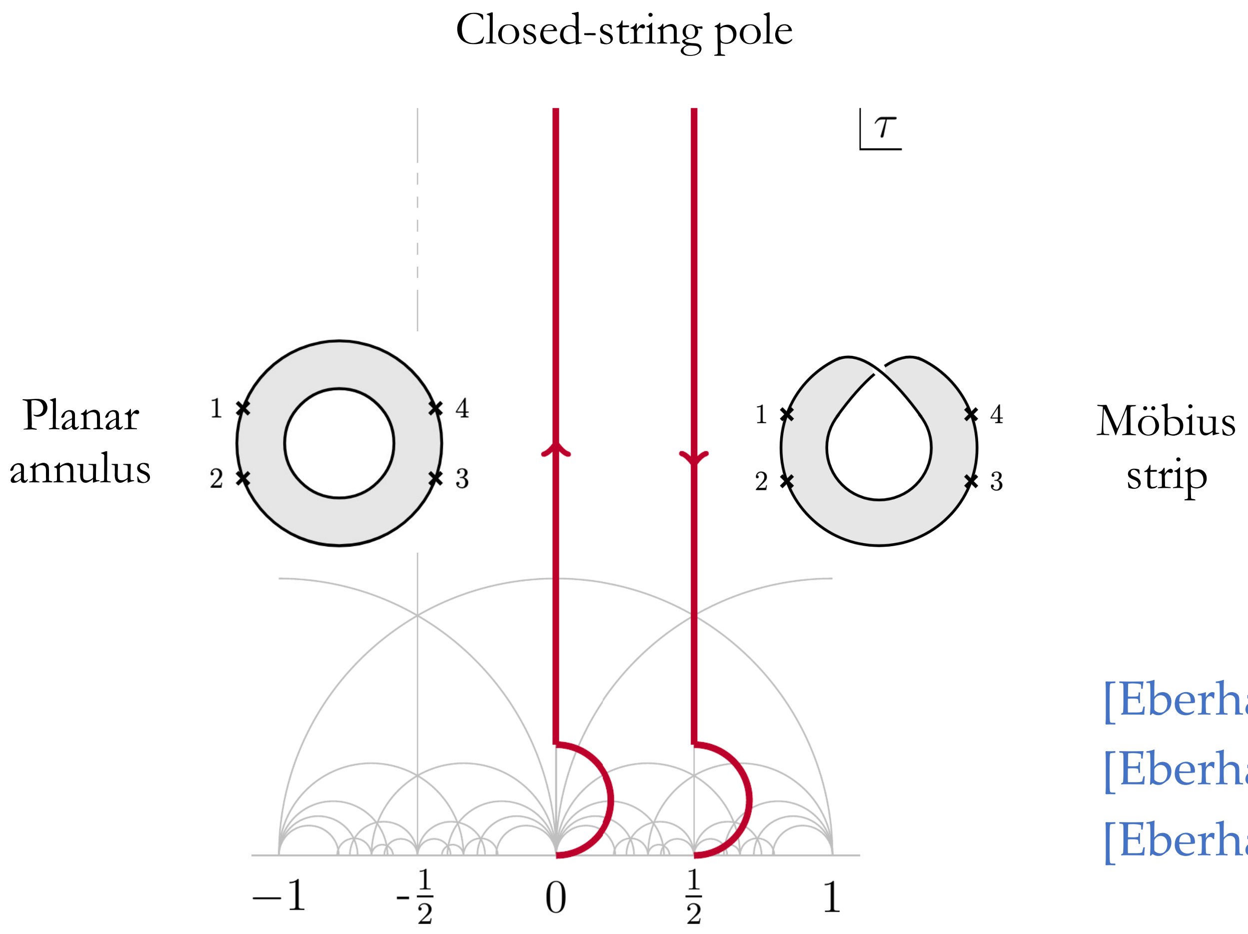
We need the **causal** $i\varepsilon$ prescription

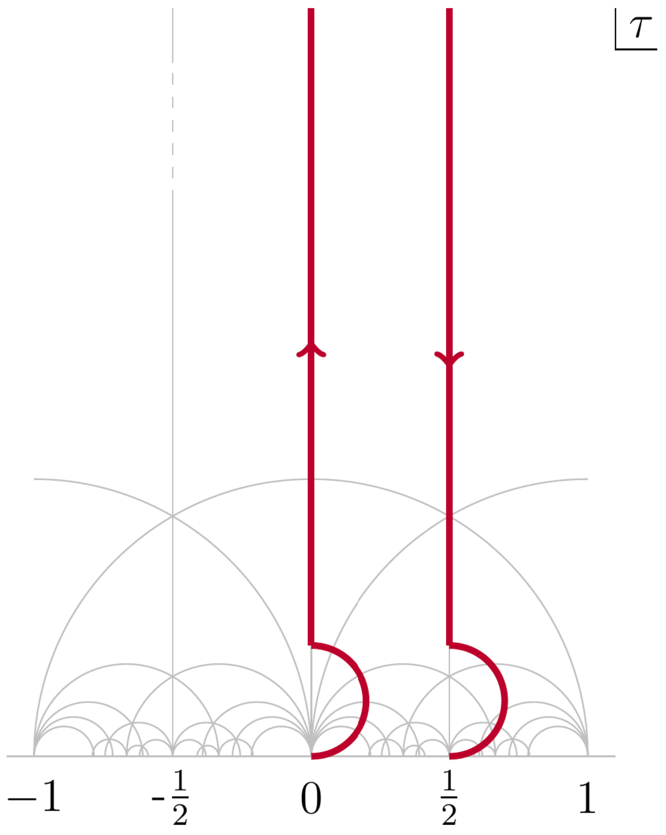
[Witten; JHEP **04** (2015) 055]

Correct integration contour: Lorentzian time evolution



[Eberhardt, **SM**; SciPost Phys. **14** (2023) 015]
[Eberhardt, **SM**; SciPost Phys. **15** (2023) 119]
[Eberhardt, **SM**; SciPost Phys. **17** (2024) 078]

Connection to analytic number theory



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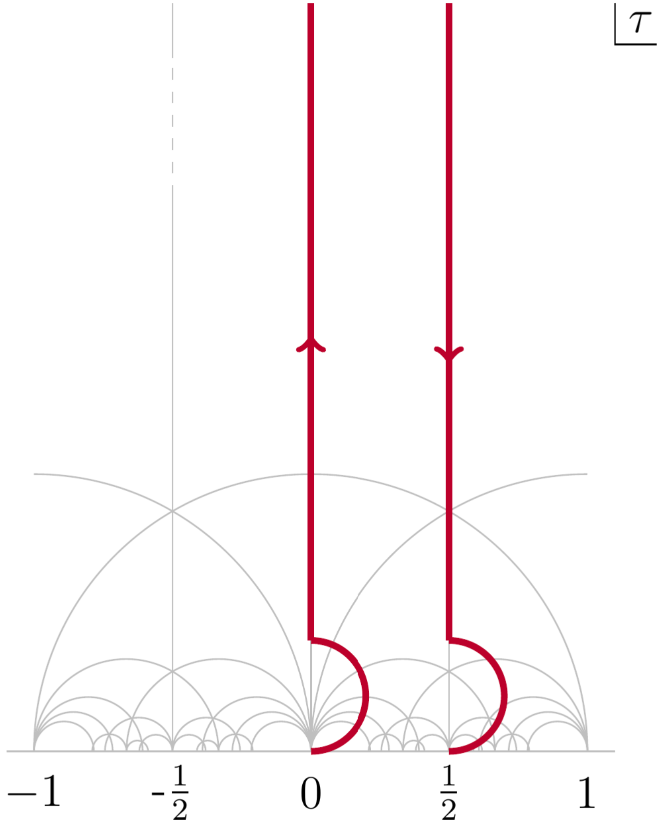
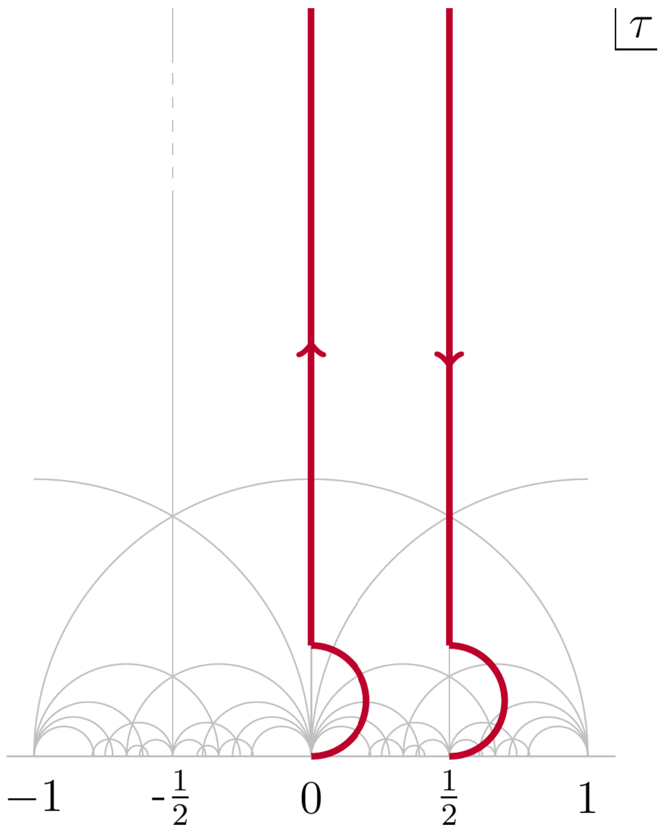


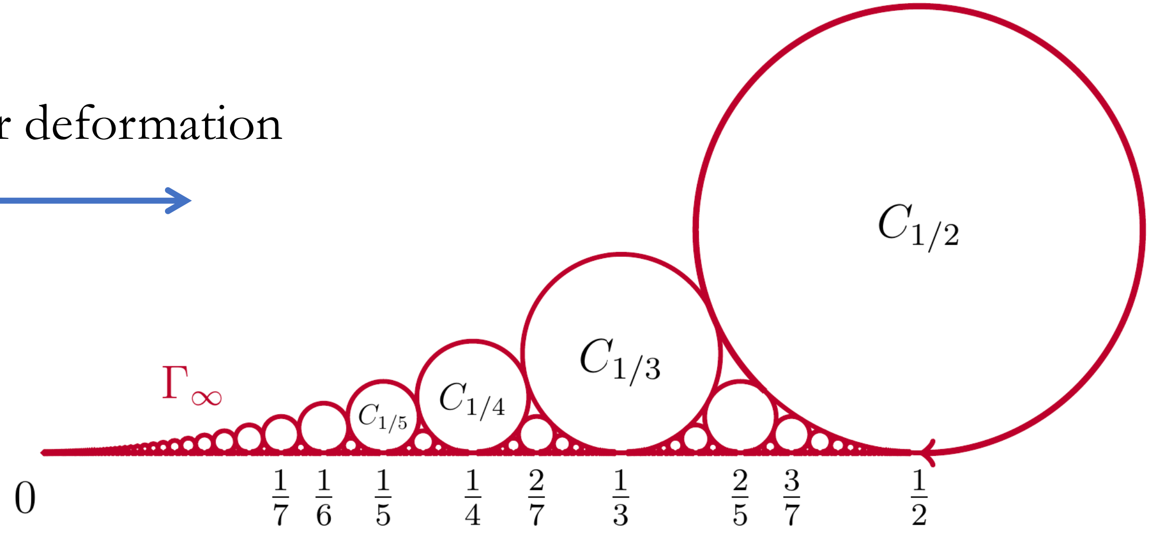
Crazy contour deformation

Connection to analytic number theory

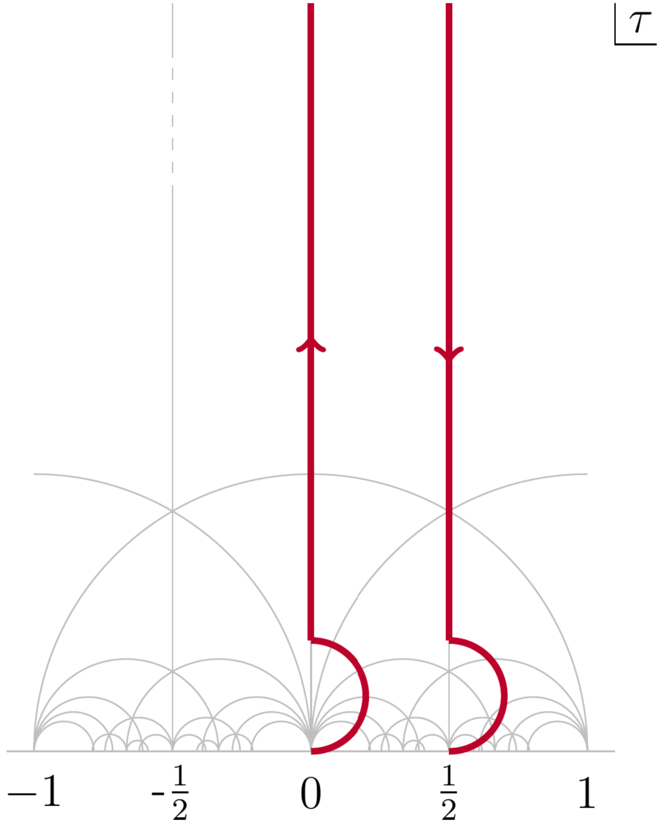


Crazy contour deformation

Hardy-Ramanujan-Rademacher contour



Connection to analytic number theory



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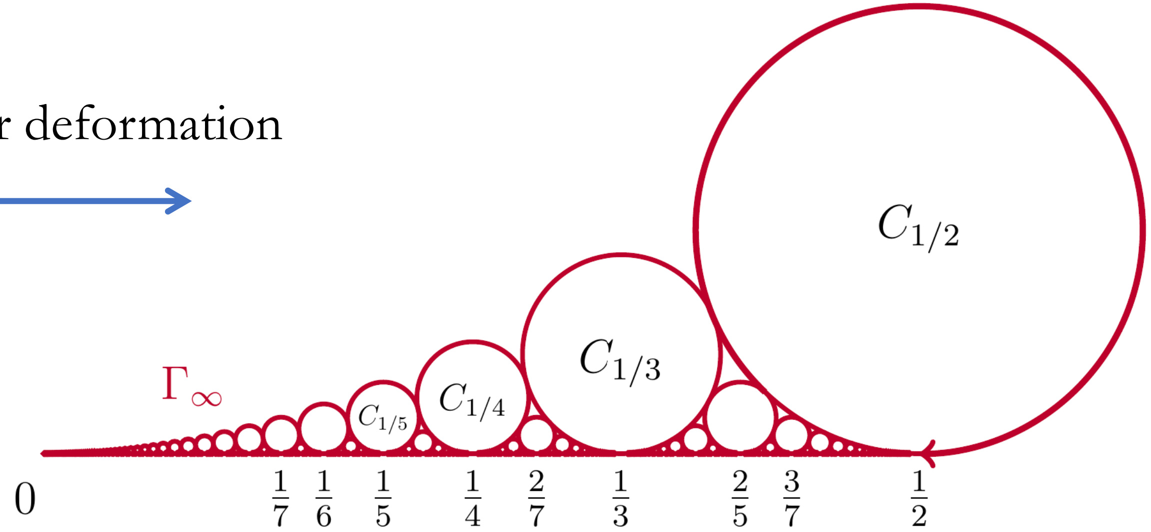
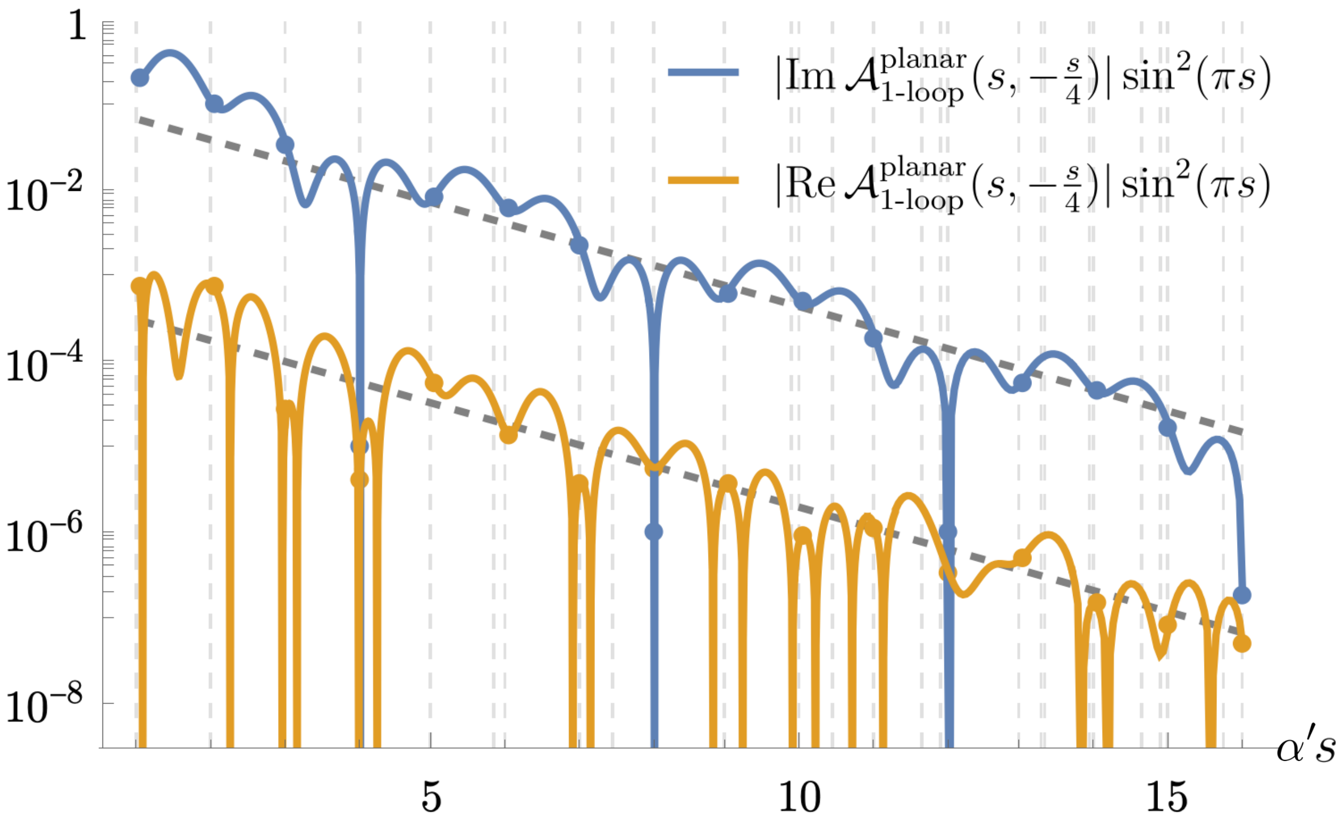


Complicated integral → **Infinite sum**

Result: First numerical evaluation of a quantum string amplitude



[Eberhardt, **SM**; SciPost Phys. **15** (2023) 119]

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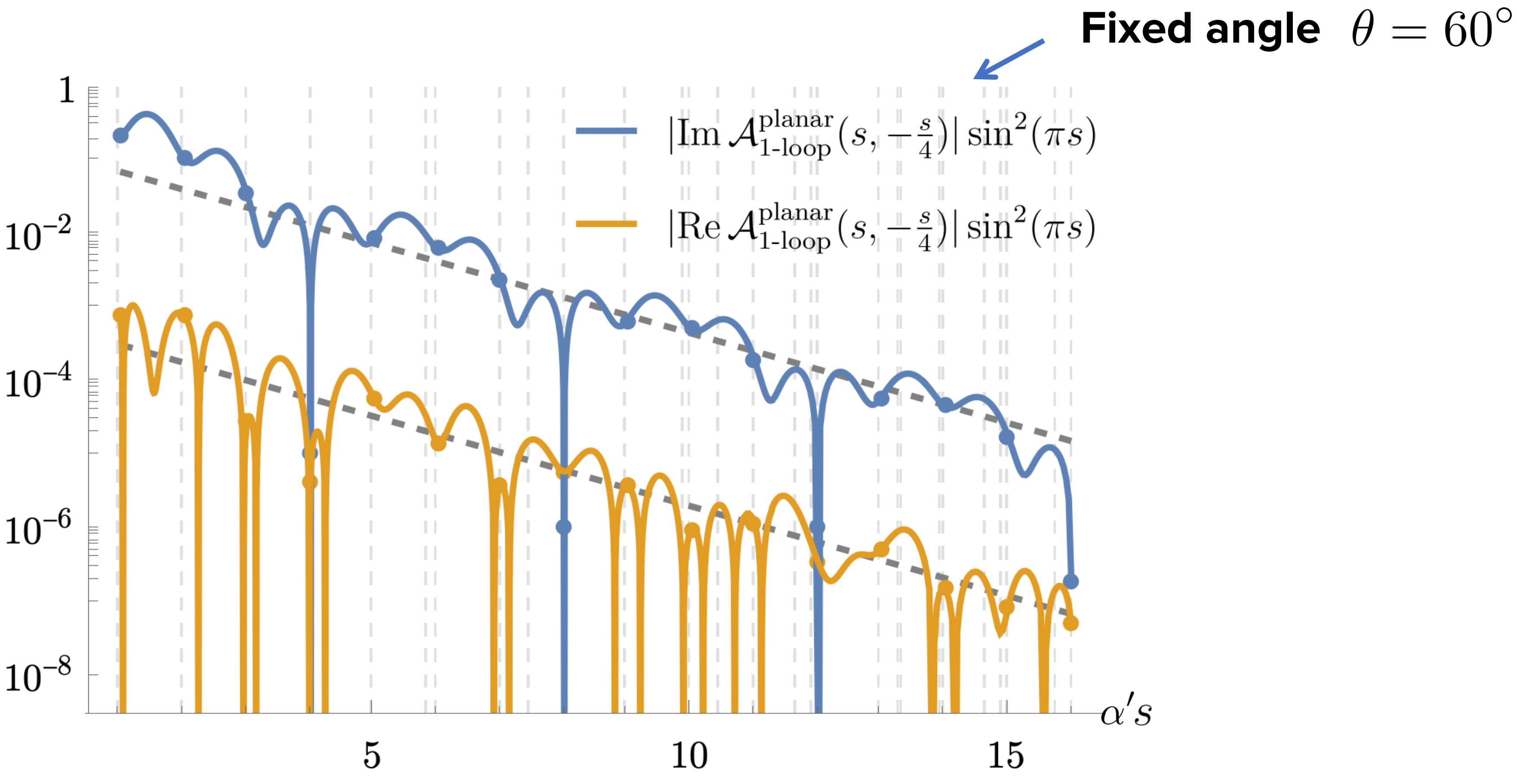
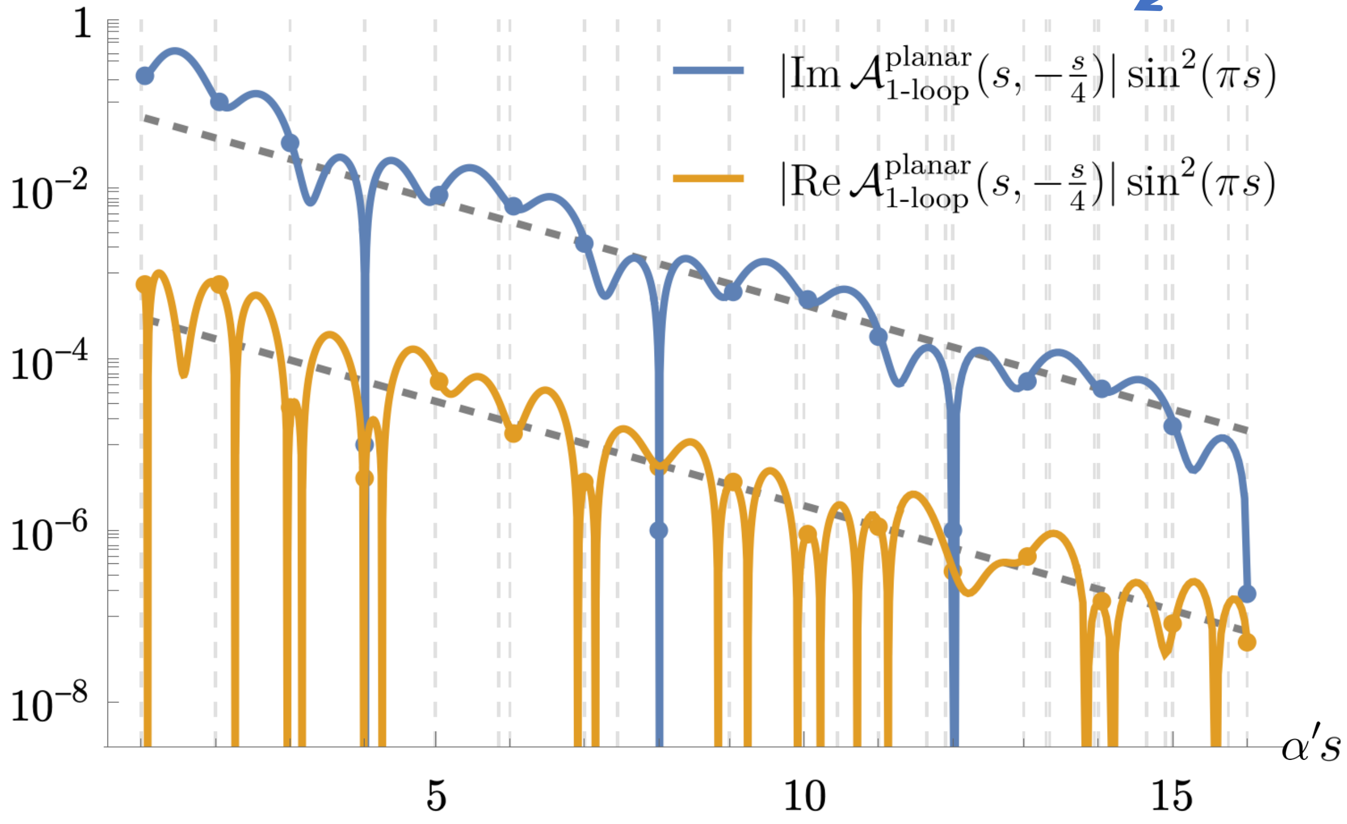


[Eberhardt, **SM**; SciPost Phys. **15** (2023) 119]

Result: First numerical evaluation of a quantum string amplitude

Fixed angle $\theta = 60^\circ$

Logarithmic scale



[Eberhardt, **SM**; SciPost Phys. **15** (2023) 119]

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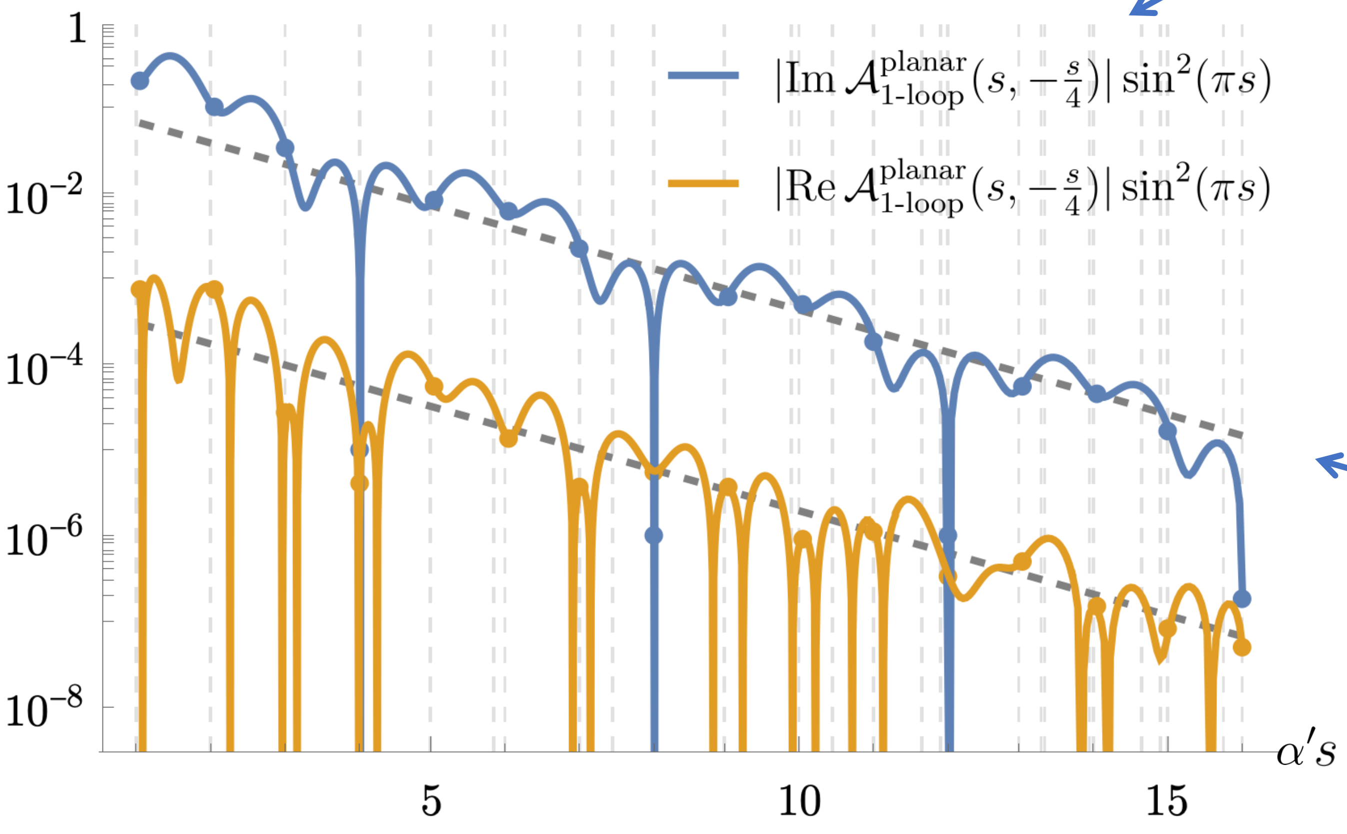


Exponential suppression

[Gross, Mende; Nucl. Phys. B **303** (1988)]

[Gross, Mañes; Nucl. Phys. B **326** (1989)]

[Eberhardt, **SM**; SciPost Phys. **15** (2023) 119]

Theory

Experiment

Evaluating string scattering

Theory

Experiment

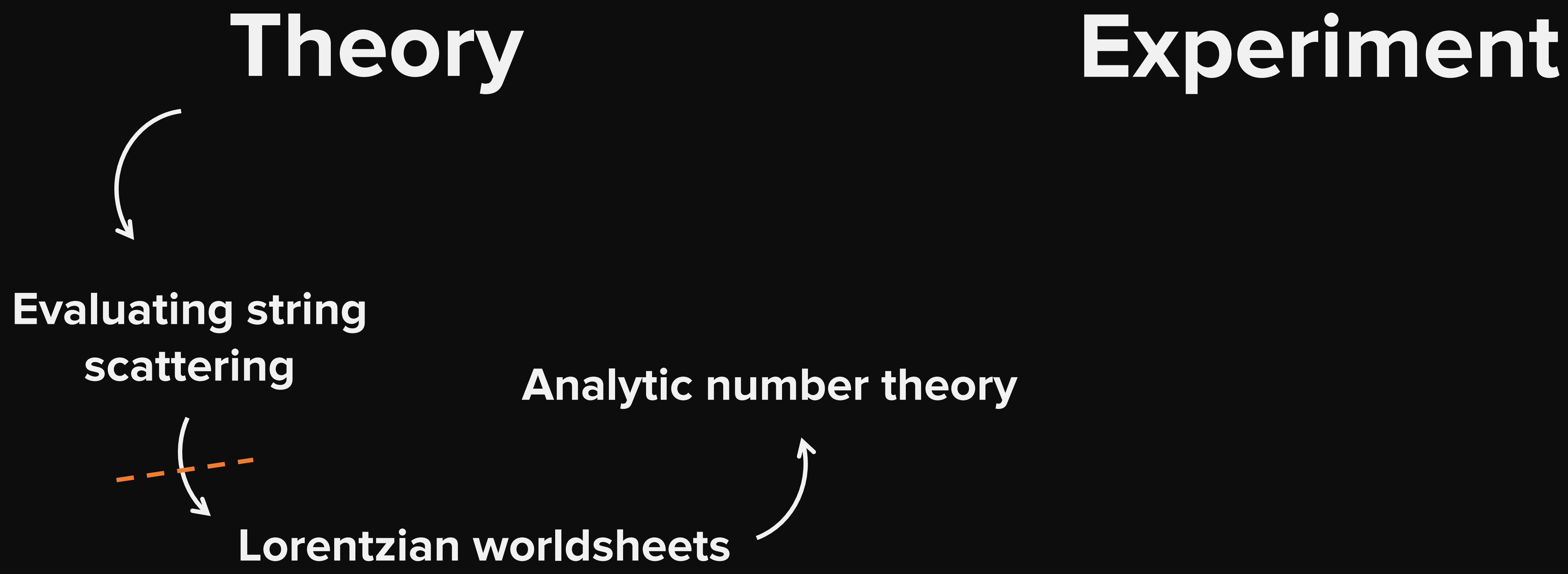
Evaluating string scattering

Theory

Experiment

Evaluating string scattering

Lorentzian worldsheets


Theory
Experiment
Evaluating string scattering
Analytic number theory
Lorentzian worldsheets

Theory

Experiment

High-energy scattering

Evaluating string scattering

Analytic number theory

Lorentzian worldsheets

Historical examples

Particle physics

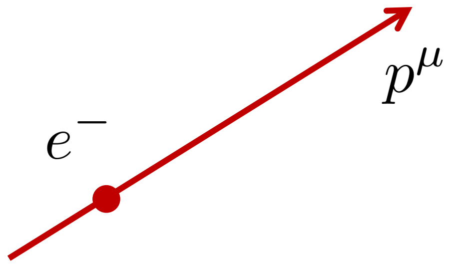
String theory

Gravitational physics

CPT theorem vs. crossing

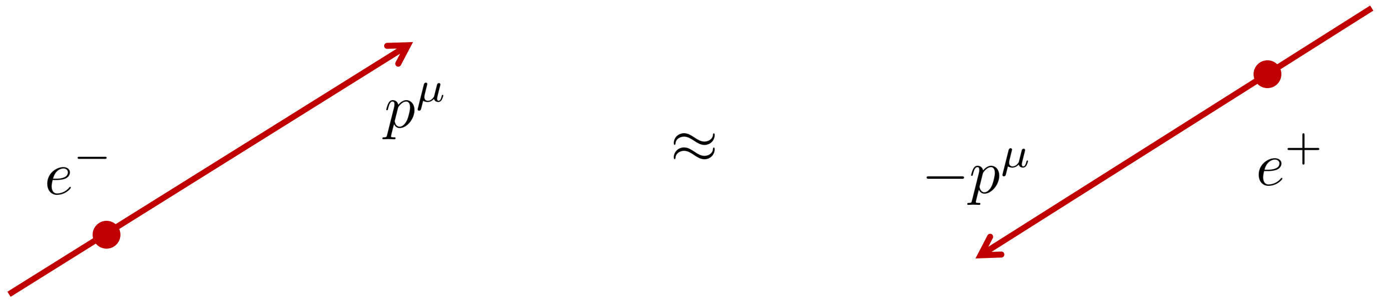
CPT theorem vs. crossing

Without interactions:



CPT theorem vs. crossing

Without interactions:



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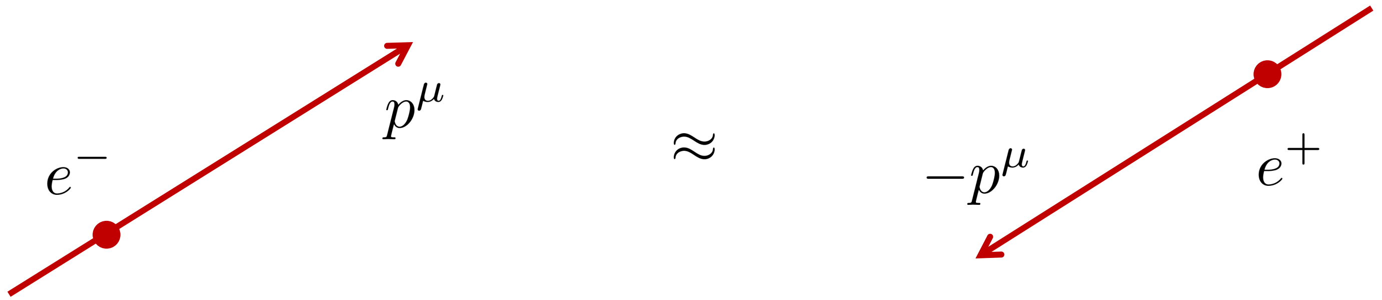
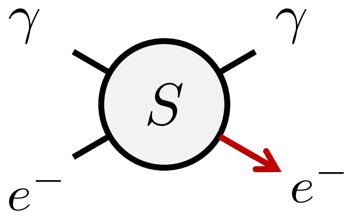


With interactions:



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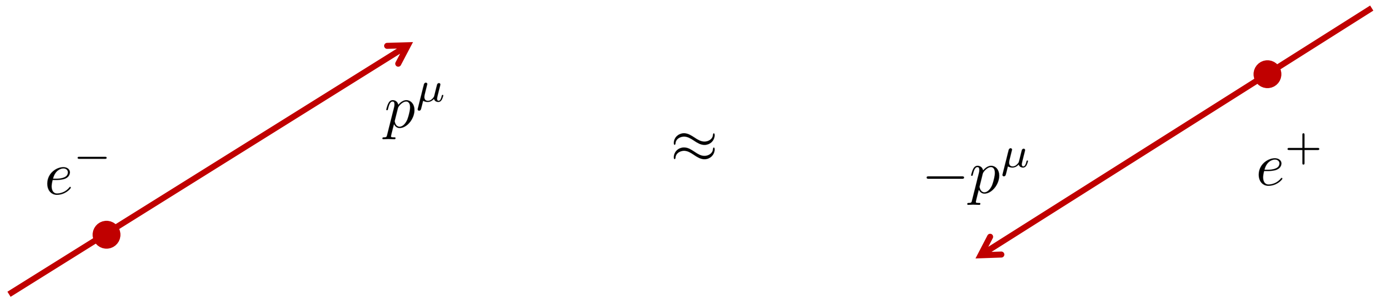

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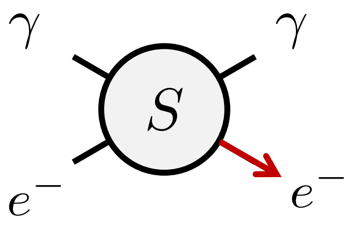


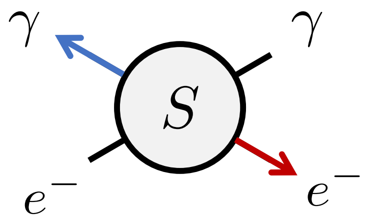
Can't deform without violating momentum conservation

$$\sum_{i=1}^{4} p_i = 0$$

Crossing two momenta at the same time

Crossing two momenta at the same time

Compton scattering



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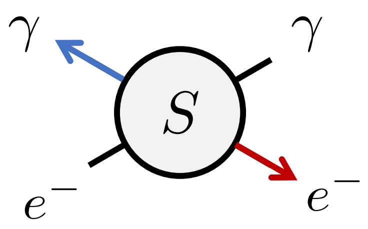
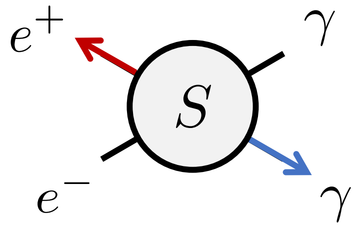


Electron-positron annihilation



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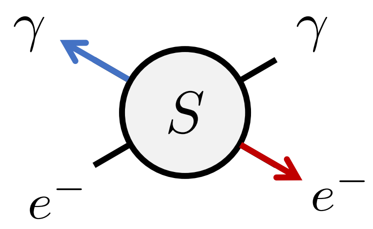

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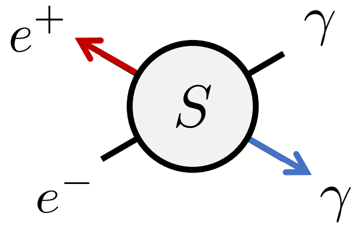

Explicit deformation

$$p_{e^-} = (\quad p^+, \quad p^-, \quad p^\perp_{e^-})$$

$$p_\gamma = (- \; p^+, - \; p^-, \quad p^\perp_\gamma)$$

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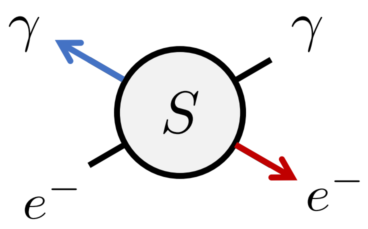
Compton scattering



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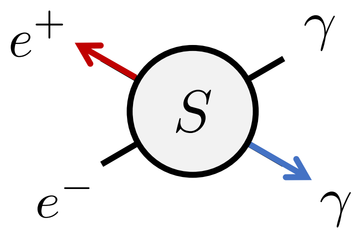

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Light-cone coordinates

$$p^2 = p^+ p^- - (p^\perp)^2$$

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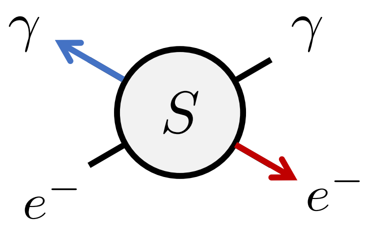

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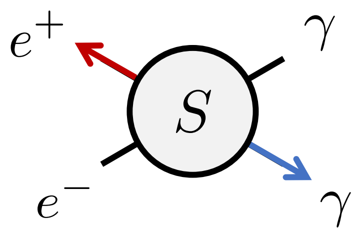

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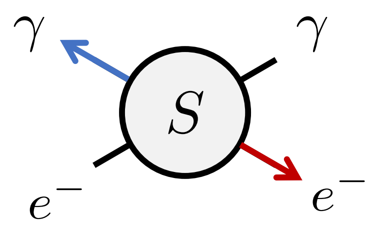

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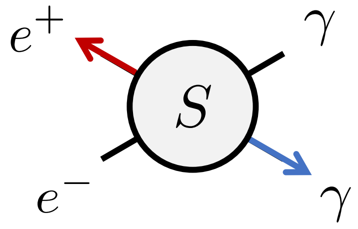

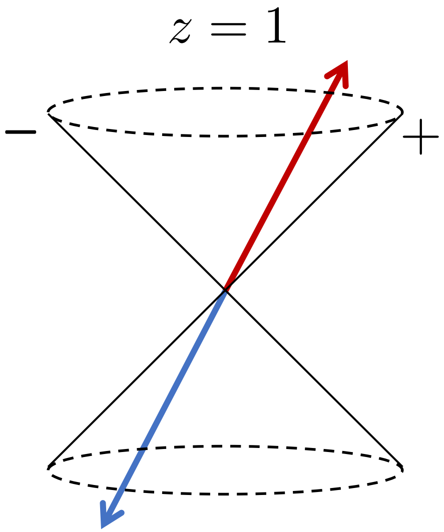

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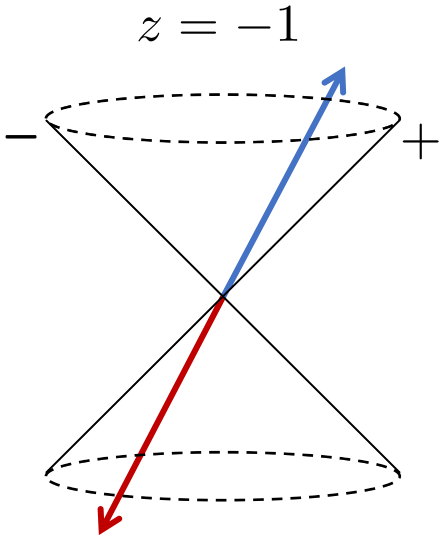
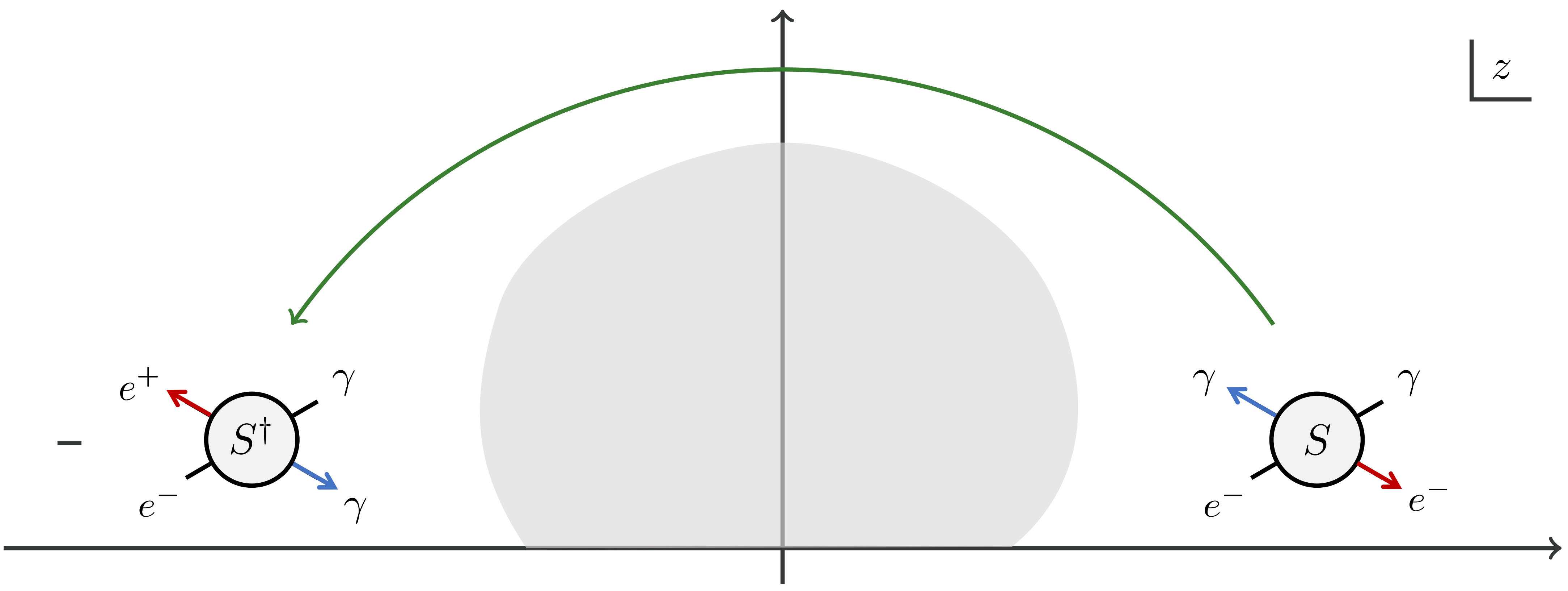
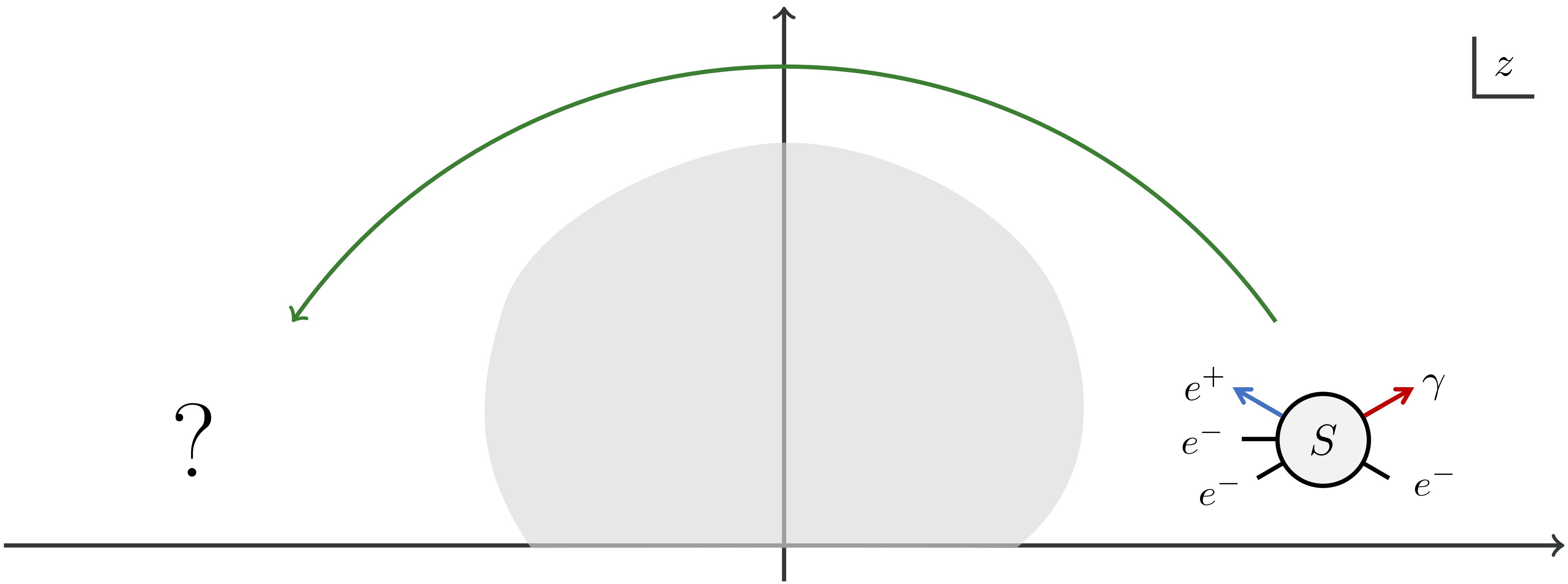


Analytic continuation in the energy



[Bros, Epstein, Glaser; Commun. Math. Phys. 1 (1965)]

How does it generalize?



Open problem since the 80's [Bros; Phys. Rept. **134** (1986) 325]

“Give me the numbers”: Tree-level ϕ^3 crossing

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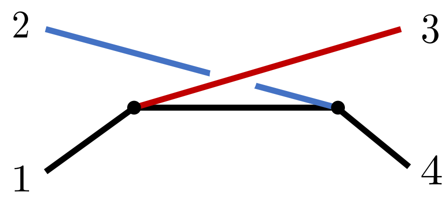
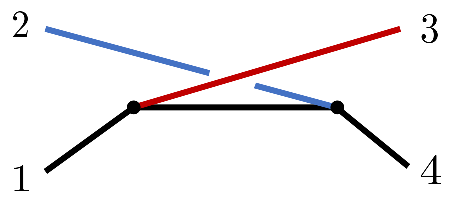
$$i\mathcal{M}_{12\to 34} = \frac{-ig^2}{(p_1 + p_3)^2 - m^2} + \dots$$



“Give me the numbers”: Tree-level ϕ^3 crossing

$$i\mathcal{M}_{12\to 34} = \frac{-ig^2}{\underbrace{(p_1 + p_3)^2}_{<0} - m^2} + \ldots$$



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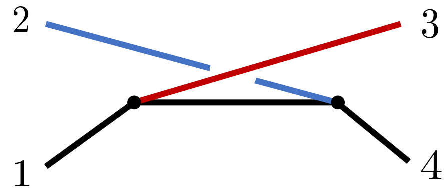
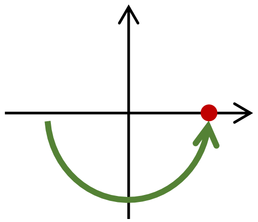
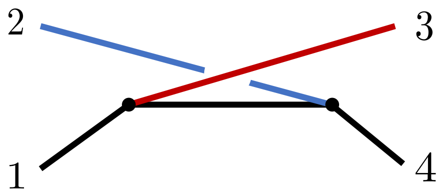


Analytic continuation $(p_1 + p_3(z))^2 \approx z p_3^+ p_1^-$



"Give me the numbers": Tree-level ϕ^3 crossing

$$i\mathcal{M}_{1{\color{blue}2}\to{\color{red}3}4} = \frac{-ig^2}{\underbrace{(p_1 + {\color{red}p_3})^2}_{<0} - m^2} + \ldots$$



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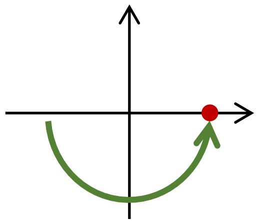
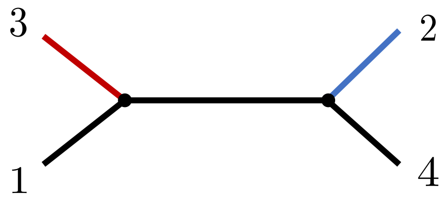


$$-(i\mathcal{M}_{1{\color{red}3}\to{\color{blue}2}4})^* = \frac{-ig^2}{(p_1 + {\color{red}p_3})^2 - m^2 - i\varepsilon} + \ldots$$



“Give me the numbers”: Tree-level ϕ^3 crossing

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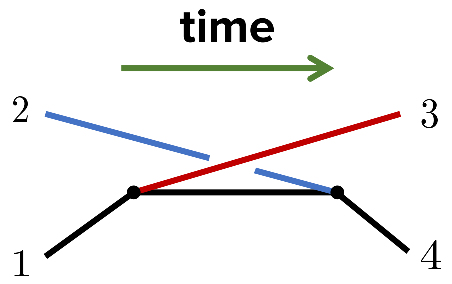

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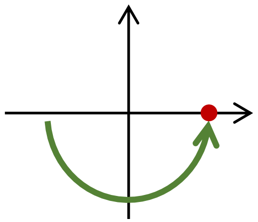

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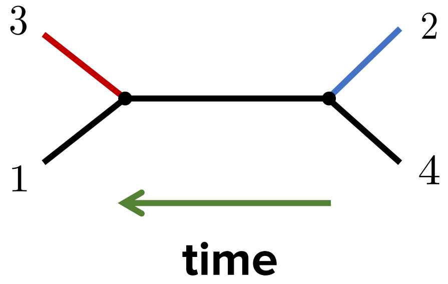
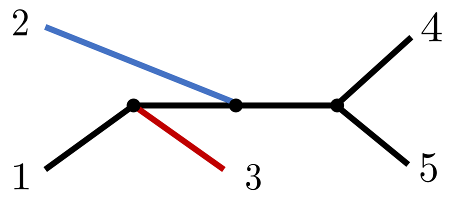


More theoretical data

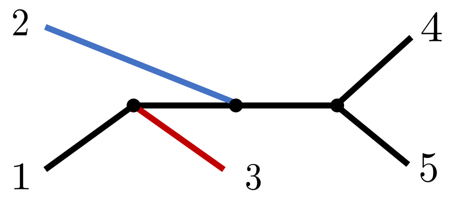
More theoretical data

$$i\mathcal{M}_{12\to 345} = \frac{-ig^3}{[(p_1 + p_3)^2 - m^2][(p_4 + p_5)^2 - m^2 + i\varepsilon]} + \dots$$



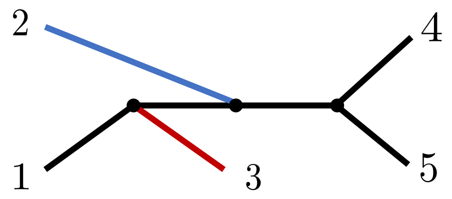
More theoretical data

$$i\mathcal{M}_{12\to345} = \frac{-ig^3}{\underbrace{[(p_1 + p_3)^2}_{<0} - m^2]\underbrace{[(p_4 + p_5)^2}_{>0 \text{ fixed}} - m^2 + i\varepsilon]} + \ldots$$



More theoretical data

$$i\mathcal{M}_{12\to 345} = \frac{-ig^3}{[\underbrace{(p_1 + p_3)^2}_{<0} - m^2][\underbrace{(p_4 + p_5)^2}_{>0 \text{ fixed}} - m^2 + i\varepsilon]} + \ldots$$



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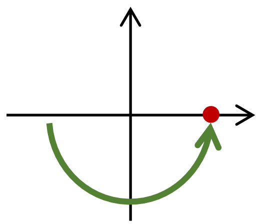

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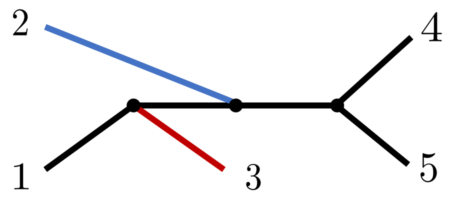
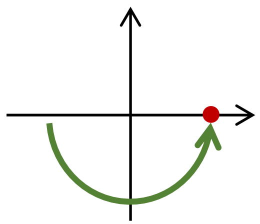
$$i\mathcal{M}_{12\to 345} = \frac{-ig^3}{[(p_1 + p_3)^2 - m^2][(p_4 + p_5)^2 - m^2 + i\varepsilon]} + \dots$$

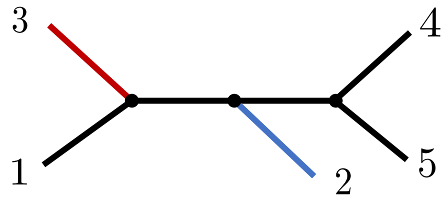


$(p_1 + p_3)^2 < 0$, $(p_4 + p_5)^2 > 0$ fixed

Analytic continuation $(p_1 + p_3(z))^2 \approx z p_3^+ p_1^-$

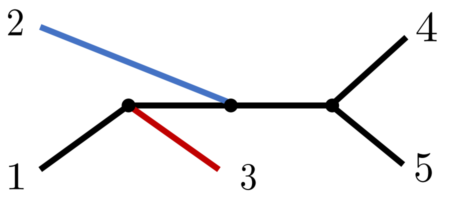


$$\frac{-ig^3}{[(p_1 + p_3)^2 - m^2 - i\varepsilon][(p_4 + p_5)^2 - m^2 + i\varepsilon]} + \dots$$



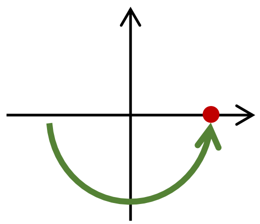
More theoretical data

$$i\mathcal{M}_{1{\color{blue}2}\to{\color{red}3}45} = \frac{-ig^3}{[(p_1 + {\color{red}p_3})^2 - m^2][(p_4 + p_5)^2 - m^2 + i\varepsilon]} + \ldots$$

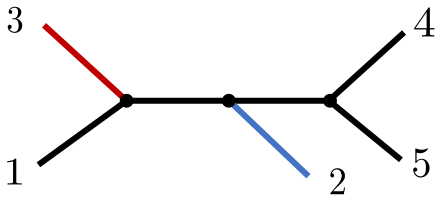


$\underbrace{(p_1 + p_3)^2}_{<0}$ $\quad$ $\underbrace{(p_4 + p_5)^2}_{>0 \text{ fixed}}$

Analytic continuation $\quad (p_1 + p_3(z))^2 \approx z p_3^+ p_1^-$

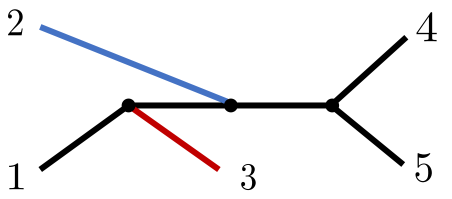


$$-(i\mathcal{M}^*_{13\to 245}) \neq \frac{-ig^3}{[(p_1 + {\color{red}p_3})^2 - m^2 - i\varepsilon][(p_4 + p_5)^2 - m^2 + i\varepsilon]} + \ldots$$

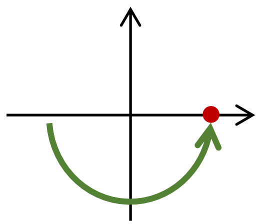


More theoretical data

$$i\mathcal{M}_{12\to345} = \frac{-ig^3}{[\underbrace{(p_1 + p_3)^2}_{<\,0} - m^2][\underbrace{(p_4 + p_5)^2}_{>\,0\text{ fixed}} - m^2 + i\varepsilon]} + \dots$$



Analytic continuation $(p_1 + p_3(z))^2 \approx z p_3^+ p_1^-$



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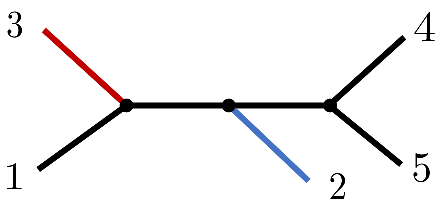
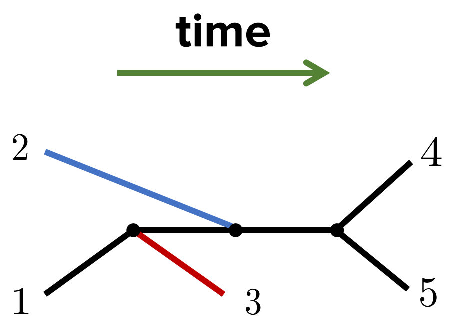


In-in expectation value

More theoretical data



$$i\mathcal{M}_{12\to345} = \frac{-ig^3}{[\underbrace{(p_1+p_3)^2}_{<0} - m^2][\underbrace{(p_4+p_5)^2}_{>0 \text{ fixed}} - m^2 + i\varepsilon]} + \dots$$

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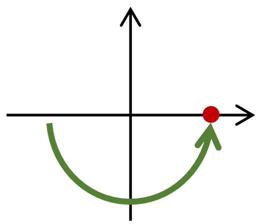

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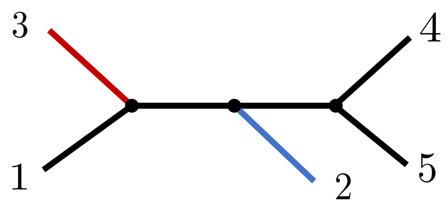

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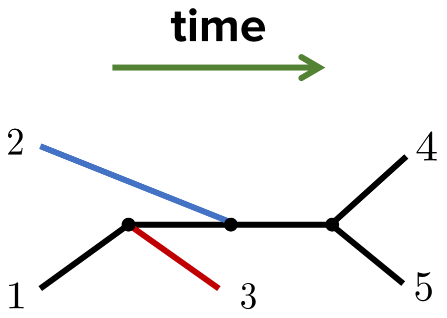

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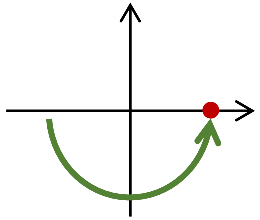

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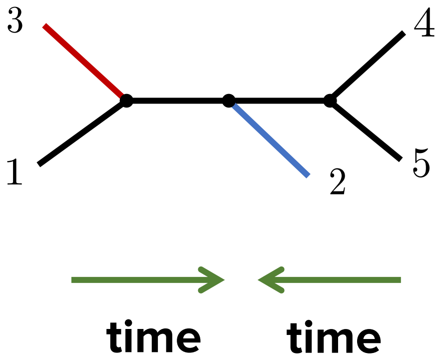
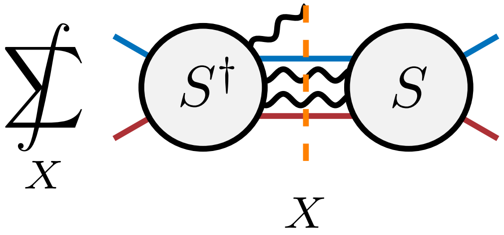
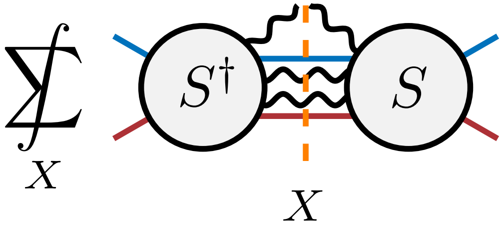


In-in expectation value

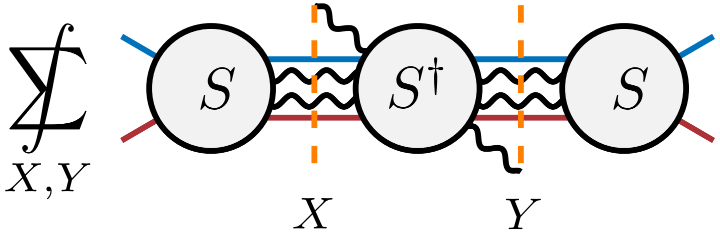
Zoo of "asymptotic observables"



Expectation value



Inclusive cross-section



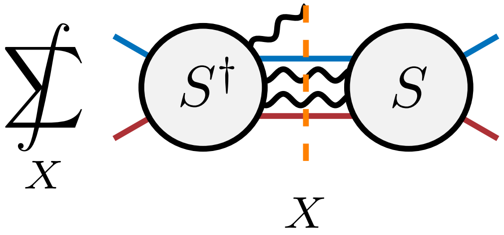
Out-of-time-order correlator

Connections to thermal physics & the Schwinger—Keldysh formalism:

New computational tools

[Caron-Huot, Giroux, Hannesdottir, **SM**; JHEP **01** (2024) 139]

Zoo of "asymptotic observables"



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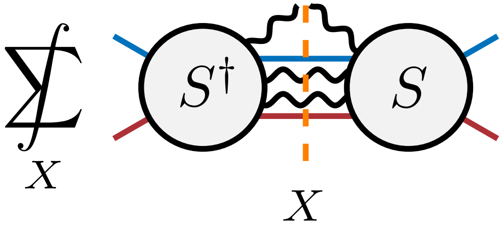

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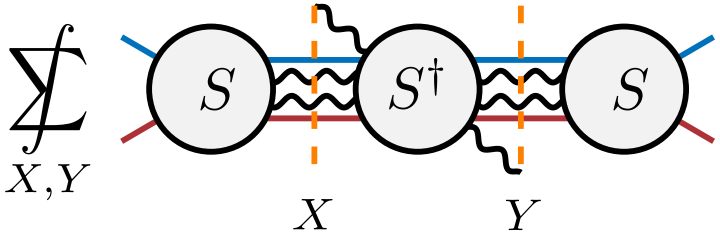

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Expectation value of gravitational radiation

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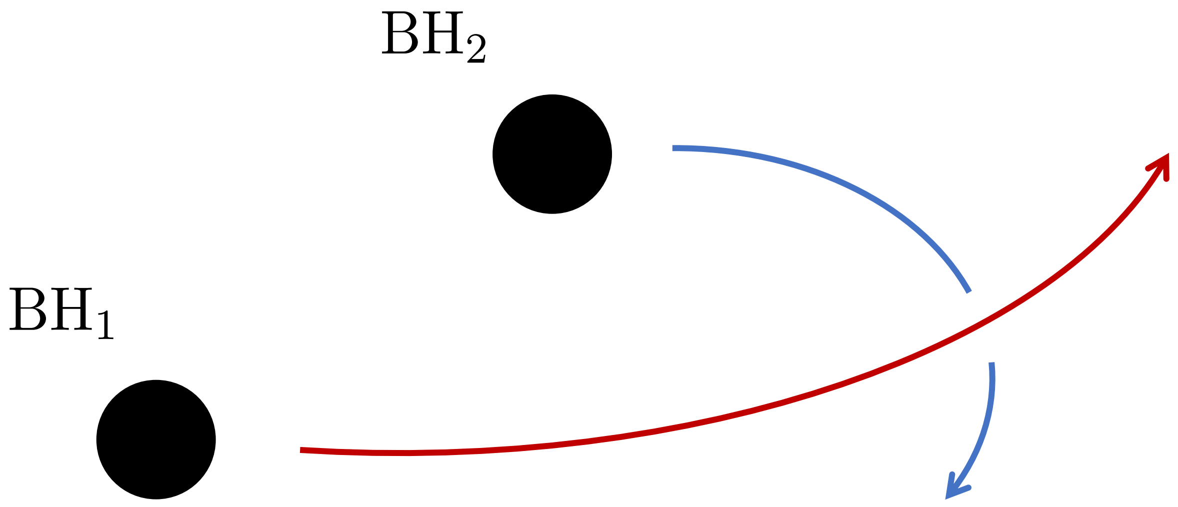
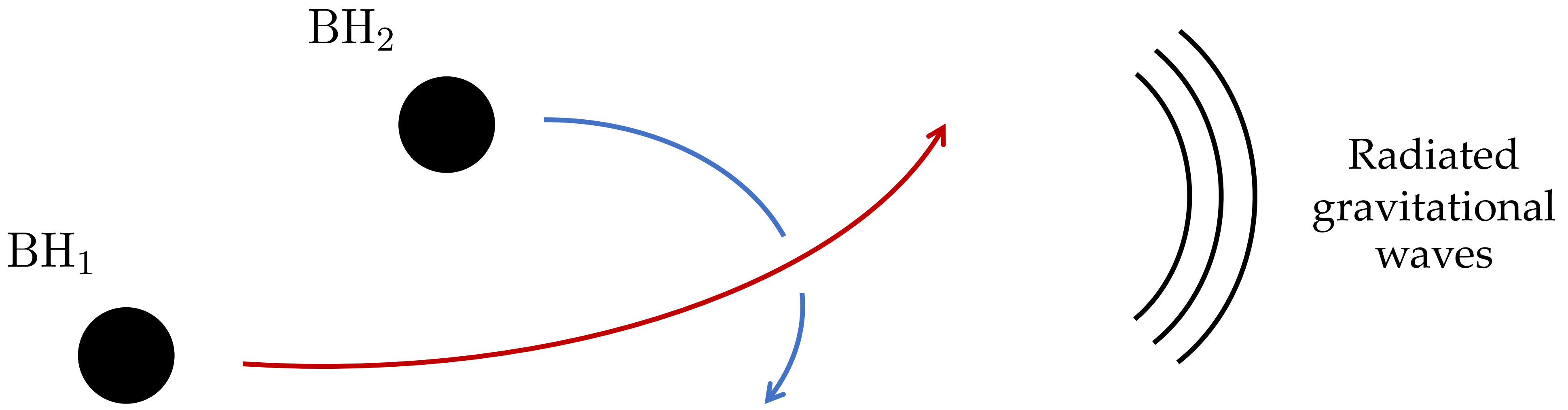


Expectation value of gravitational radiation



Expectation value of gravitational radiation

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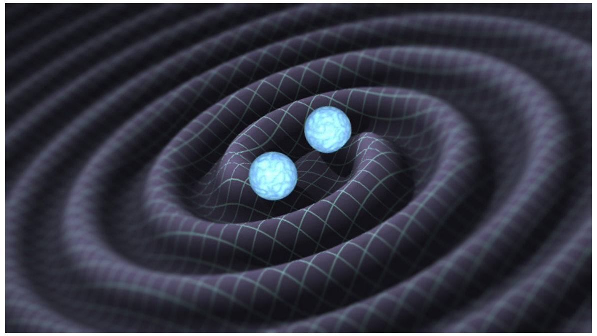

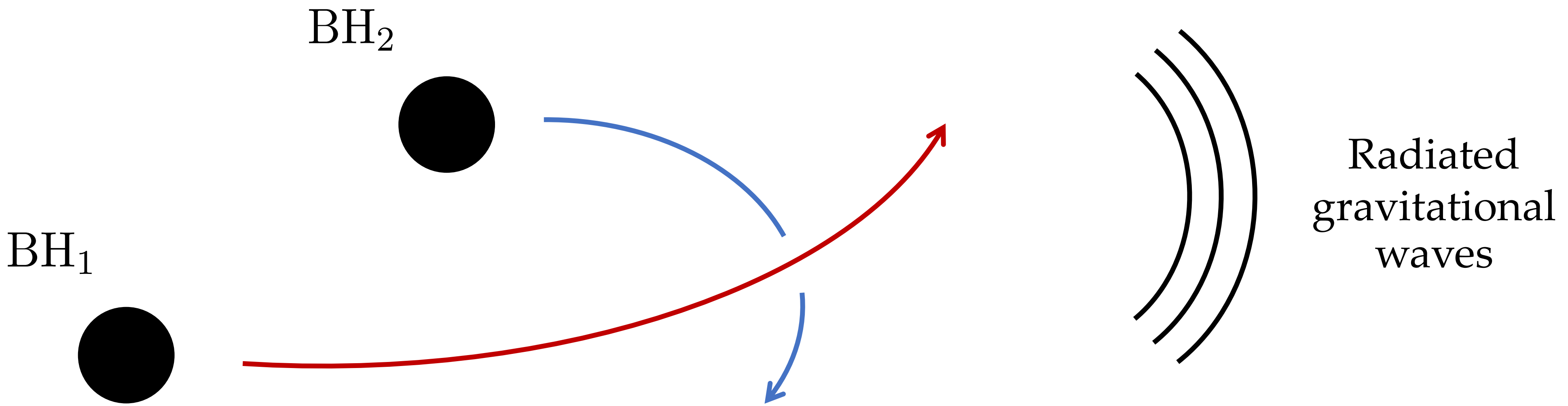

Expectation value of gravitational radiation

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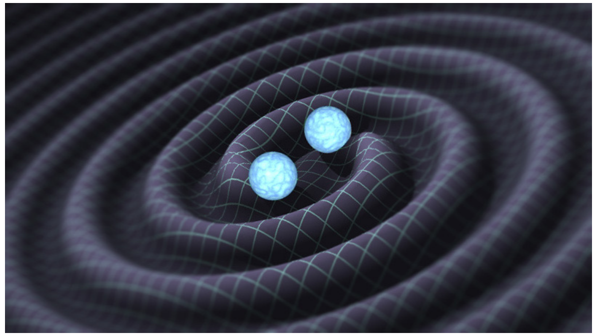

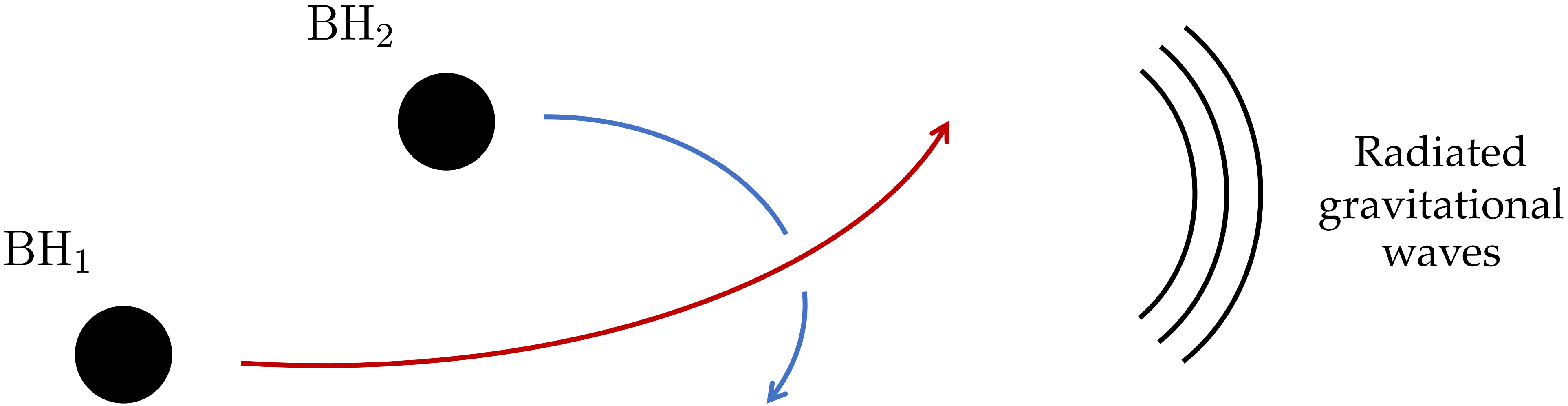

Leading order in G_N computed in [Kovacs, Thorne; Astrophys. J. 224 (1978)]

First correct computation of the gravitational waveform at NLO

[Herderschee, Roiban, Teng; JHEP **06** (2023) 004]
[Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini; JHEP **06** (2023) 048]
[Georgoudis, Heissenberg, Vazquez-Holm; JHEP **06** (2023) 126]

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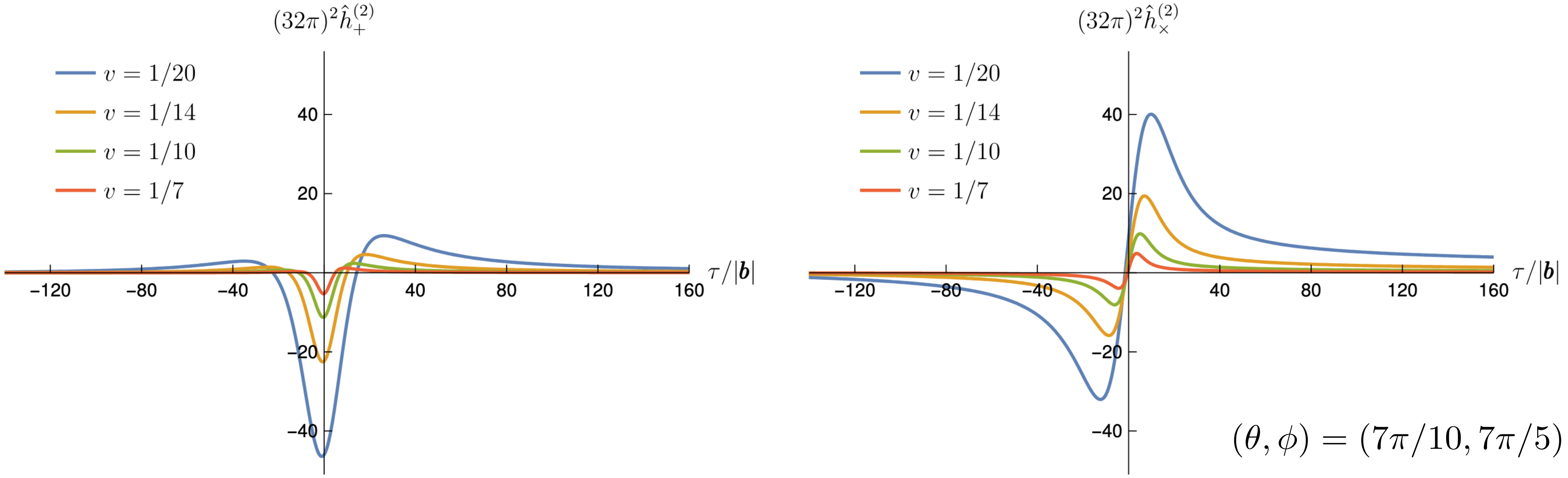

[Caron-Huot, Giroux, Hannesdottir, **SM**; JHEP **04** (2024) 060]

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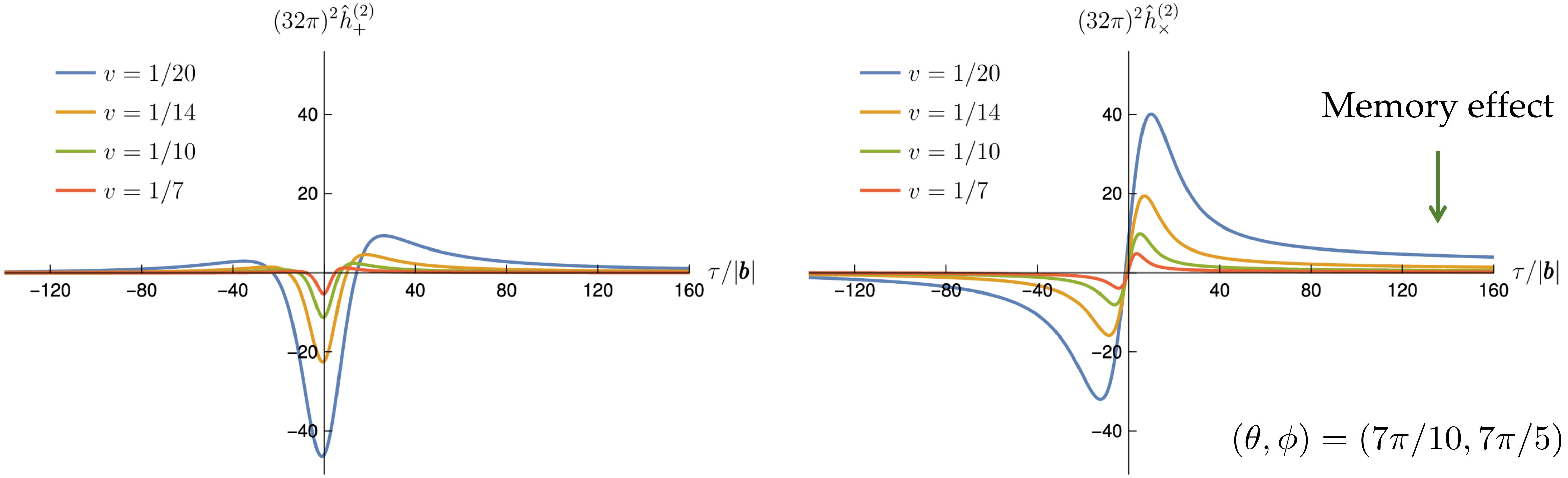

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Theory

Experiment

Crossing particles

Theory

Experiment

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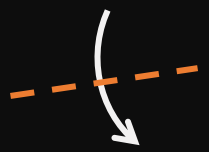

Theory

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Crossing particles

Asymptotic observables

Theory

Experiment

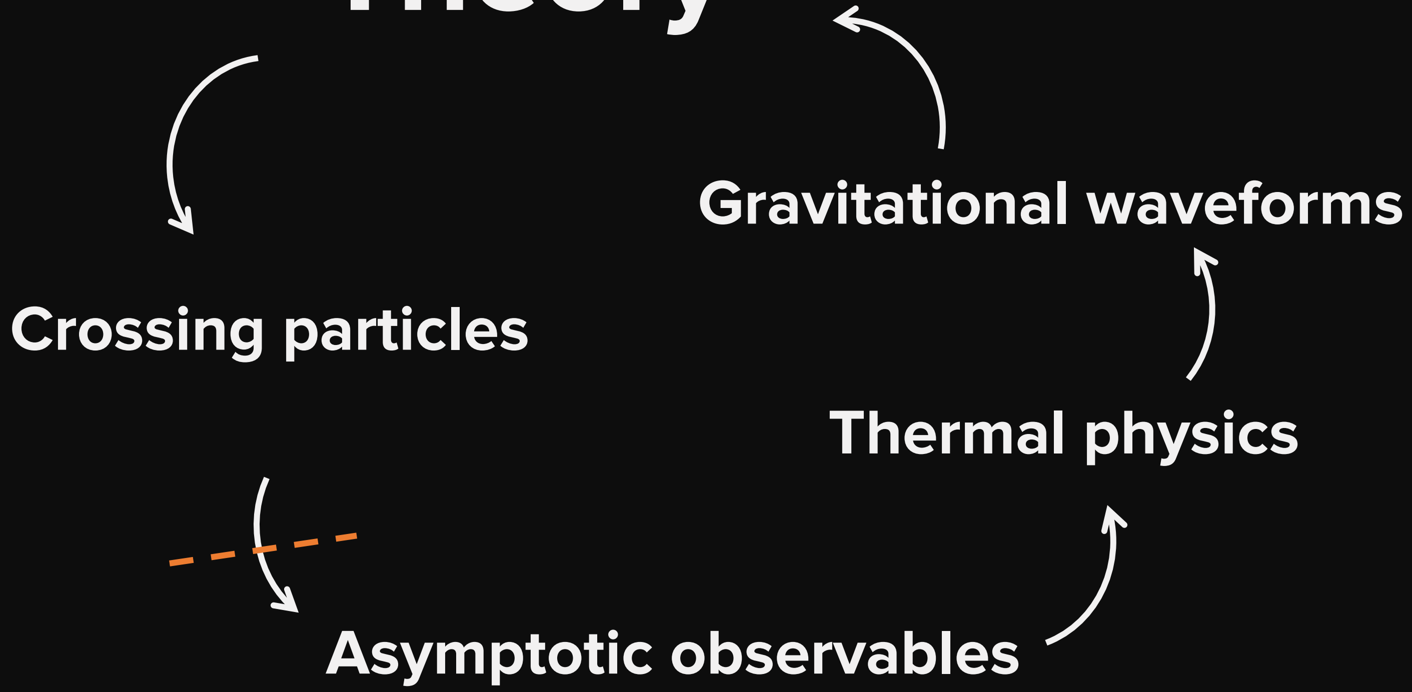
Crossing particles

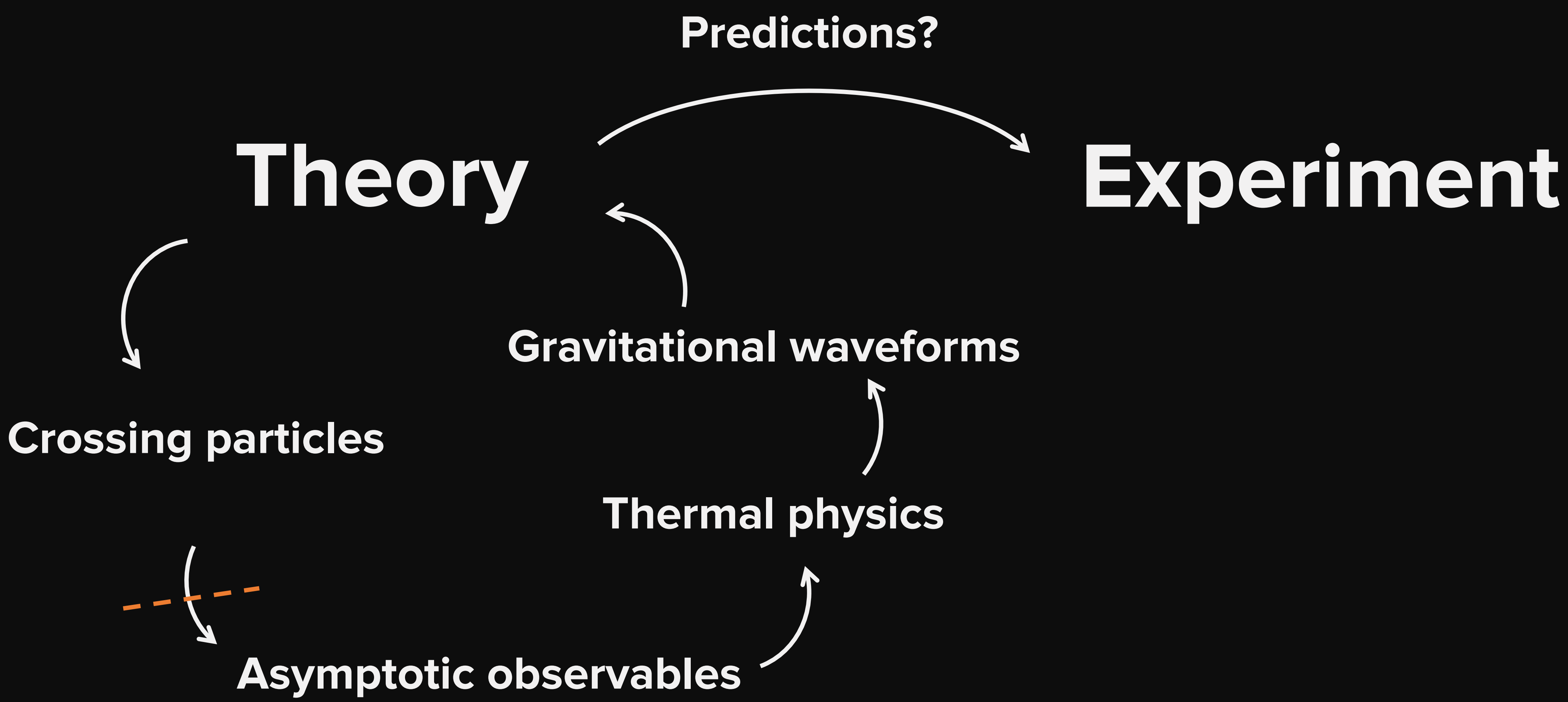
Thermal physics

Asymptotic observables

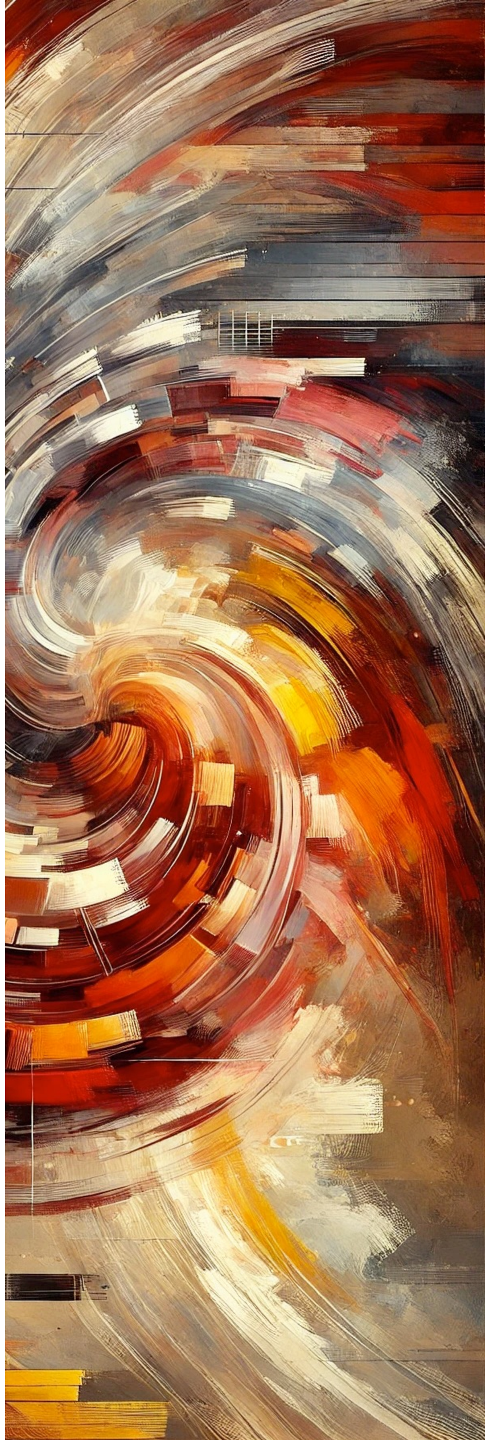
Theory

Experiment




Predictions?
Theory
Experiment
Gravitational waveforms
Crossing particles
Thermal physics
Asymptotic observables

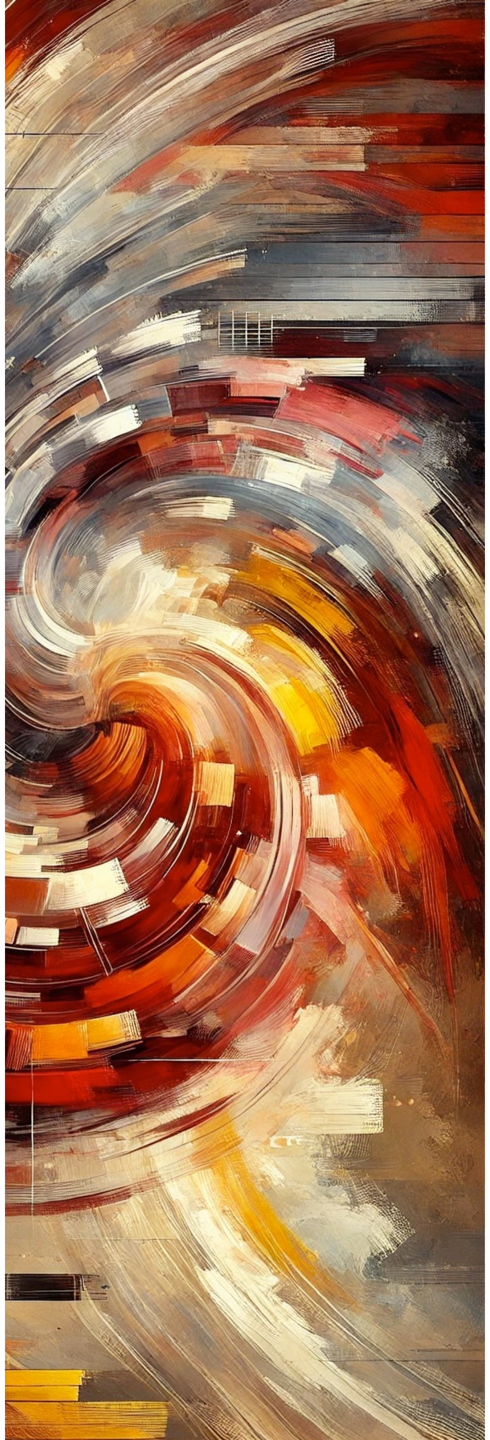
“Give me the numbers” approach



“Give me the numbers” approach

TL;DR

Creating artificial demand for “theoretical data” pushes us to develop the theory, which often leads to new results and unexpected cross-disciplinary connections



“Give me the numbers” approach

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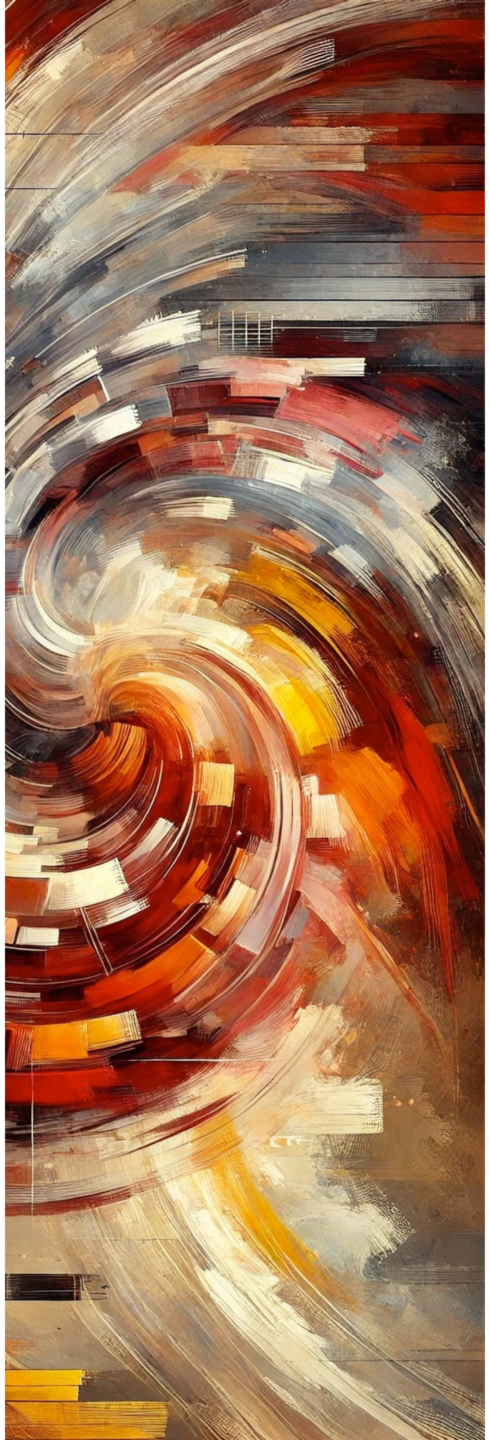
Historical example

Particle physics

String theory

Gravitational physics



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Thank you!