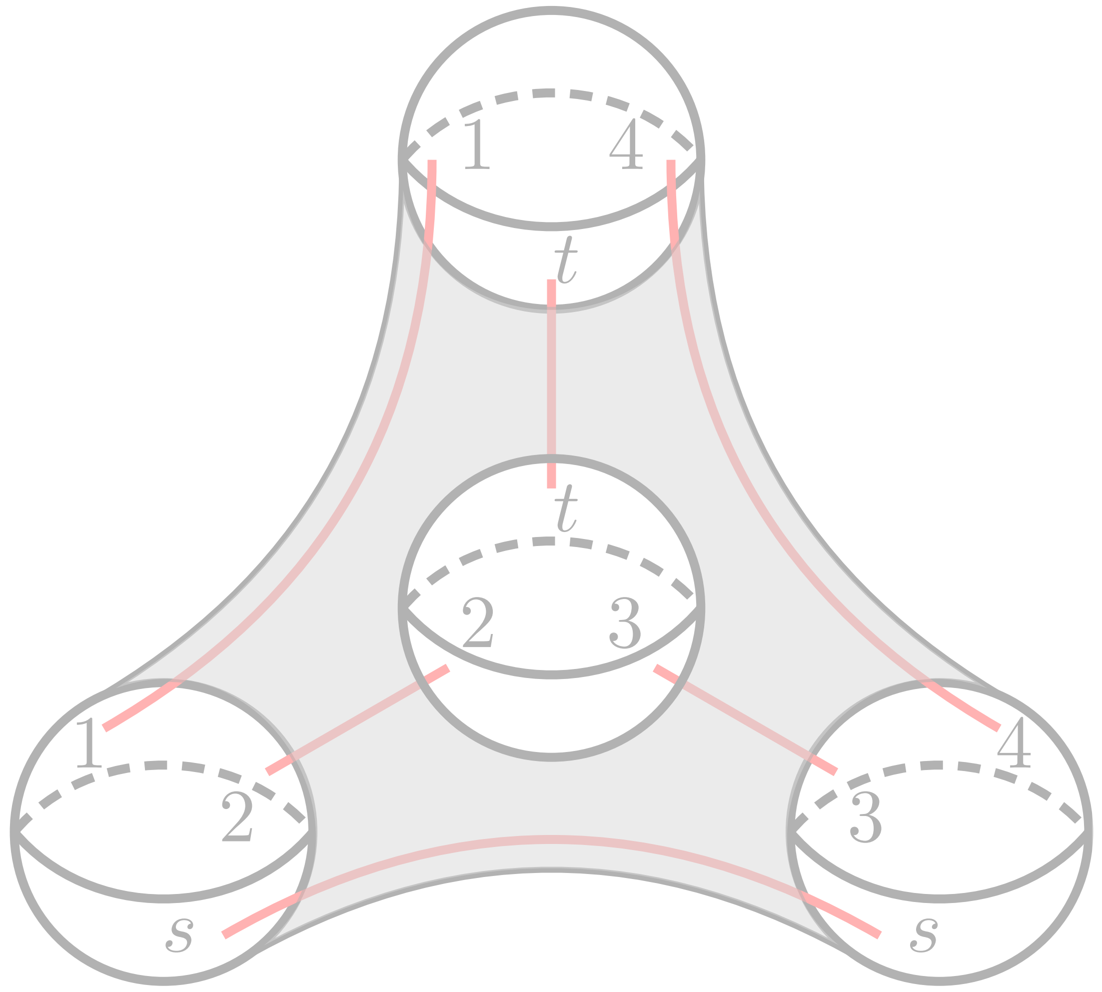
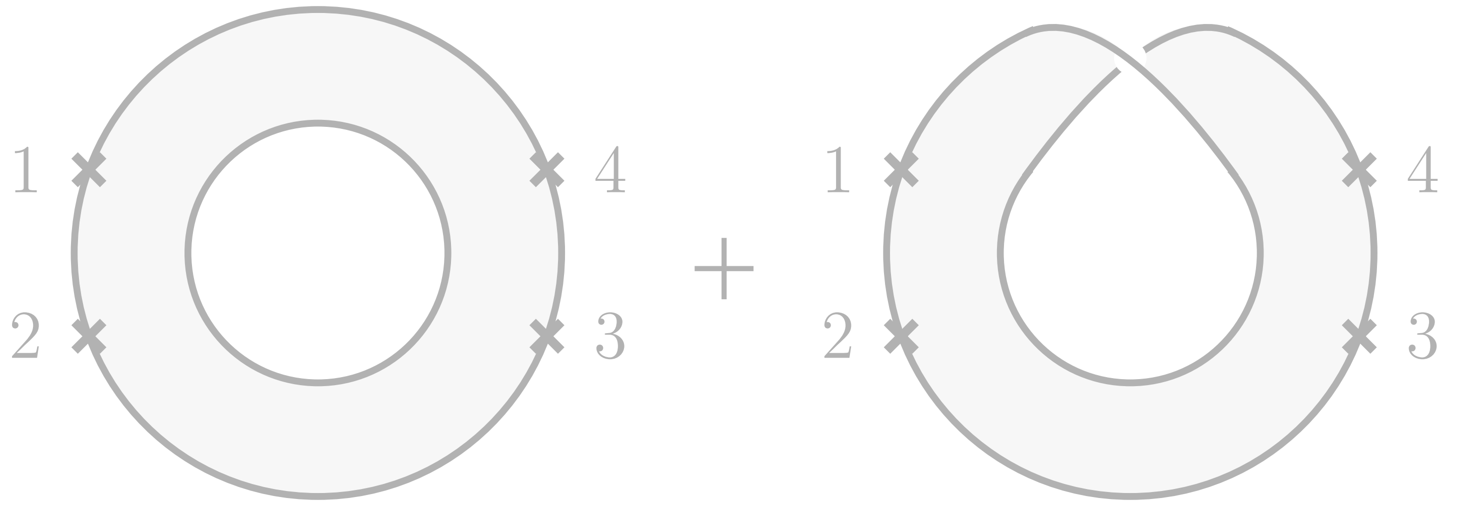
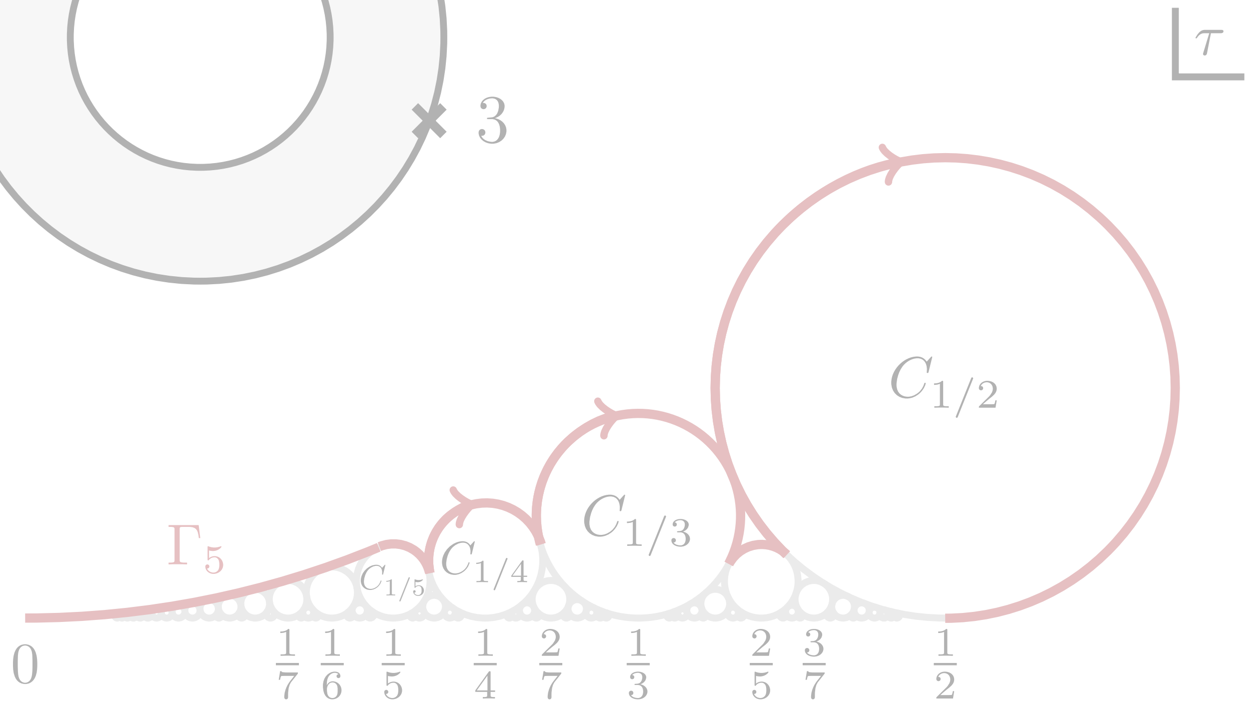
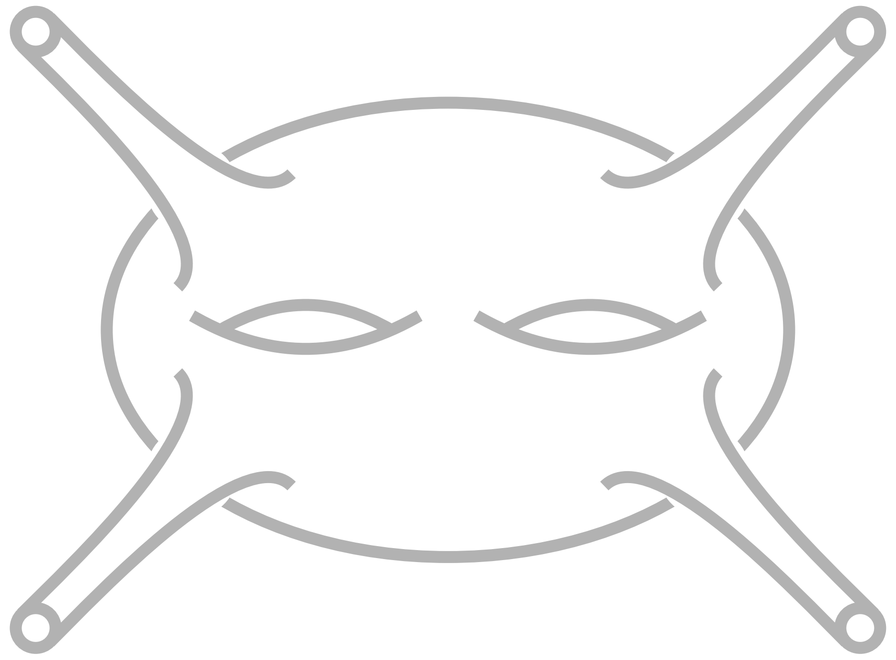
Solvable Quantum Gravity

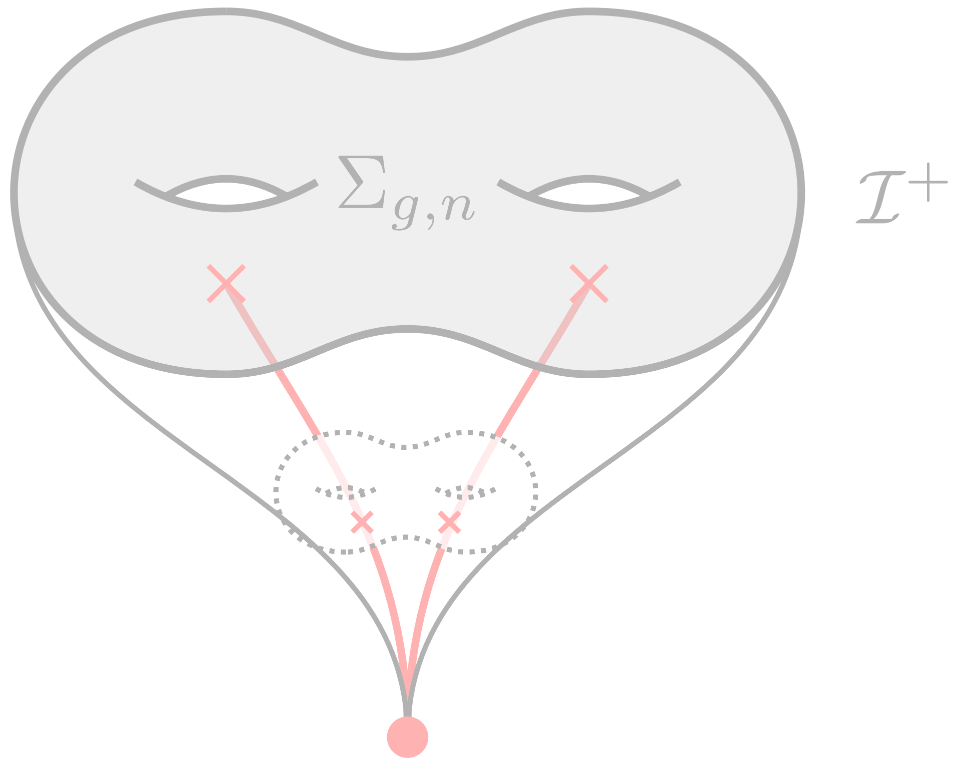






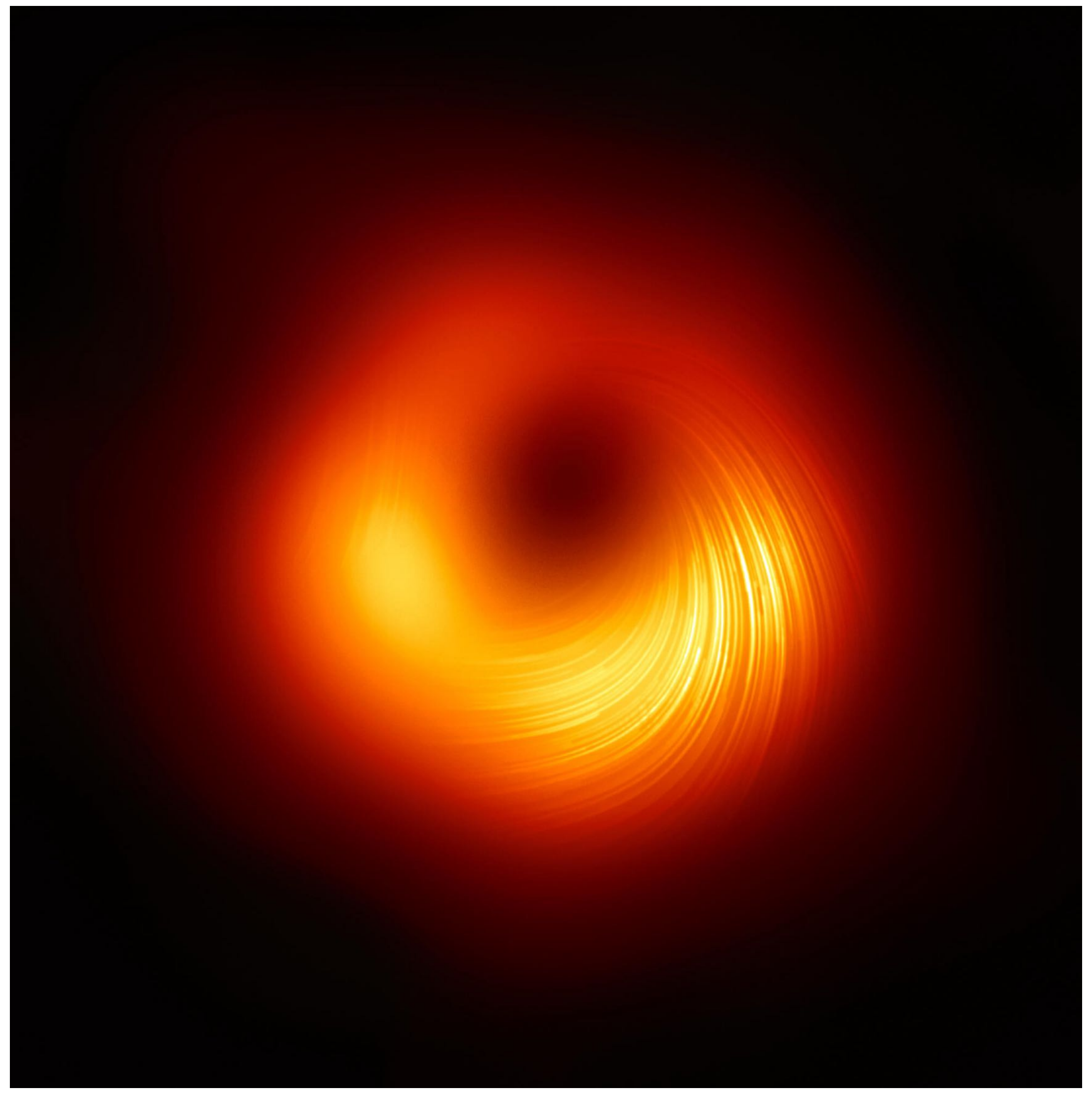
Lorenz Eberhardt
University of Amsterdam





Munich, 12.11.2024

What do we know about QG?



Difficult to measure quantum gravitational effects experimentally

Should we give up?

Our existence depends on it:

- Initial conditions of the universe
- the fate of our universe
- black hole physics
- internal consistency

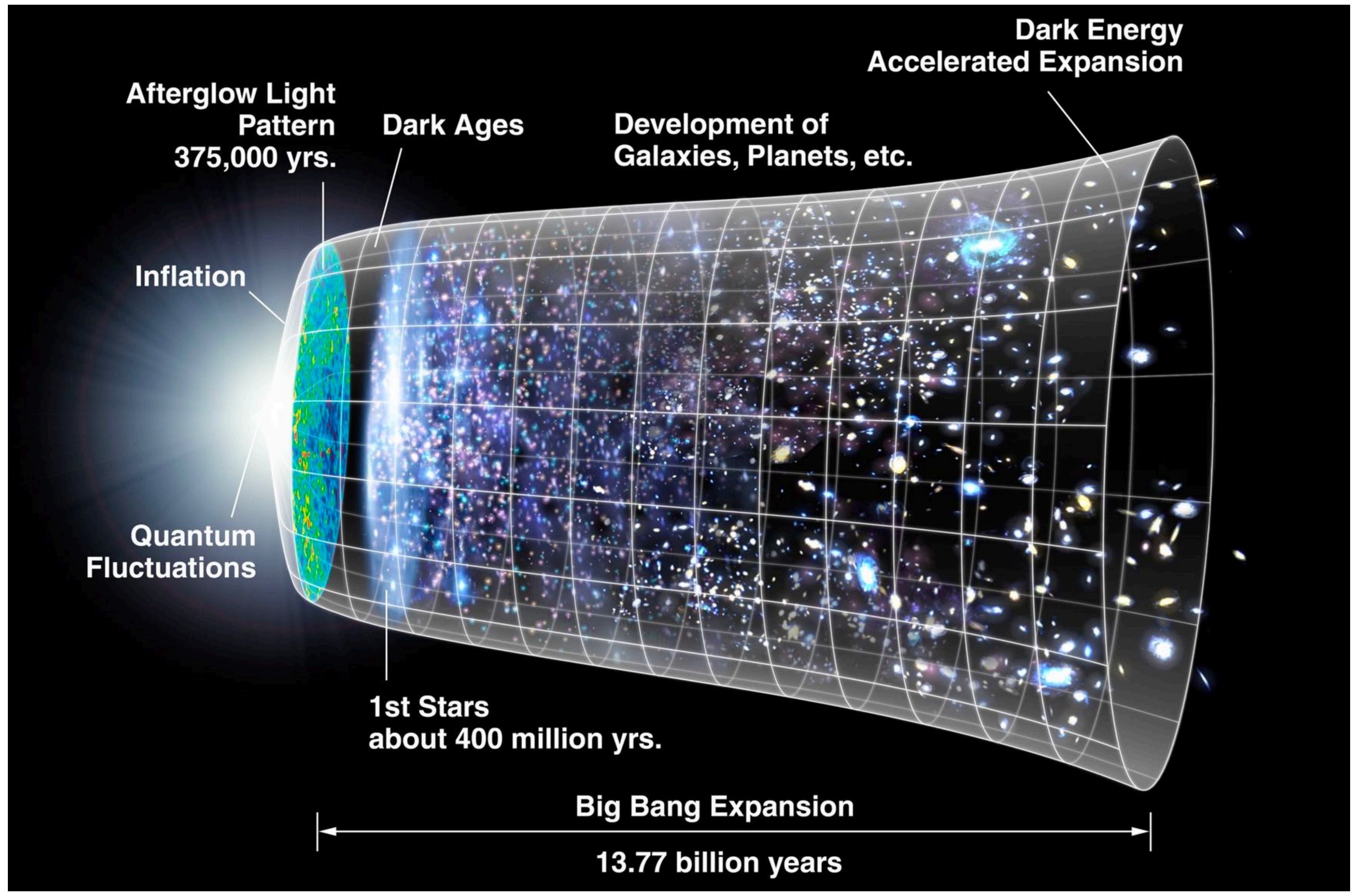


Image credit: Event Horizon telescope, NASA

Theory to the rescue

Any theory of quantum gravity has to be

- compatible with all known experimental data
- internally consistent

This is a very tall order

Not anything goes

- The only convincing possibility is string theory
- This is not for lack of trying!

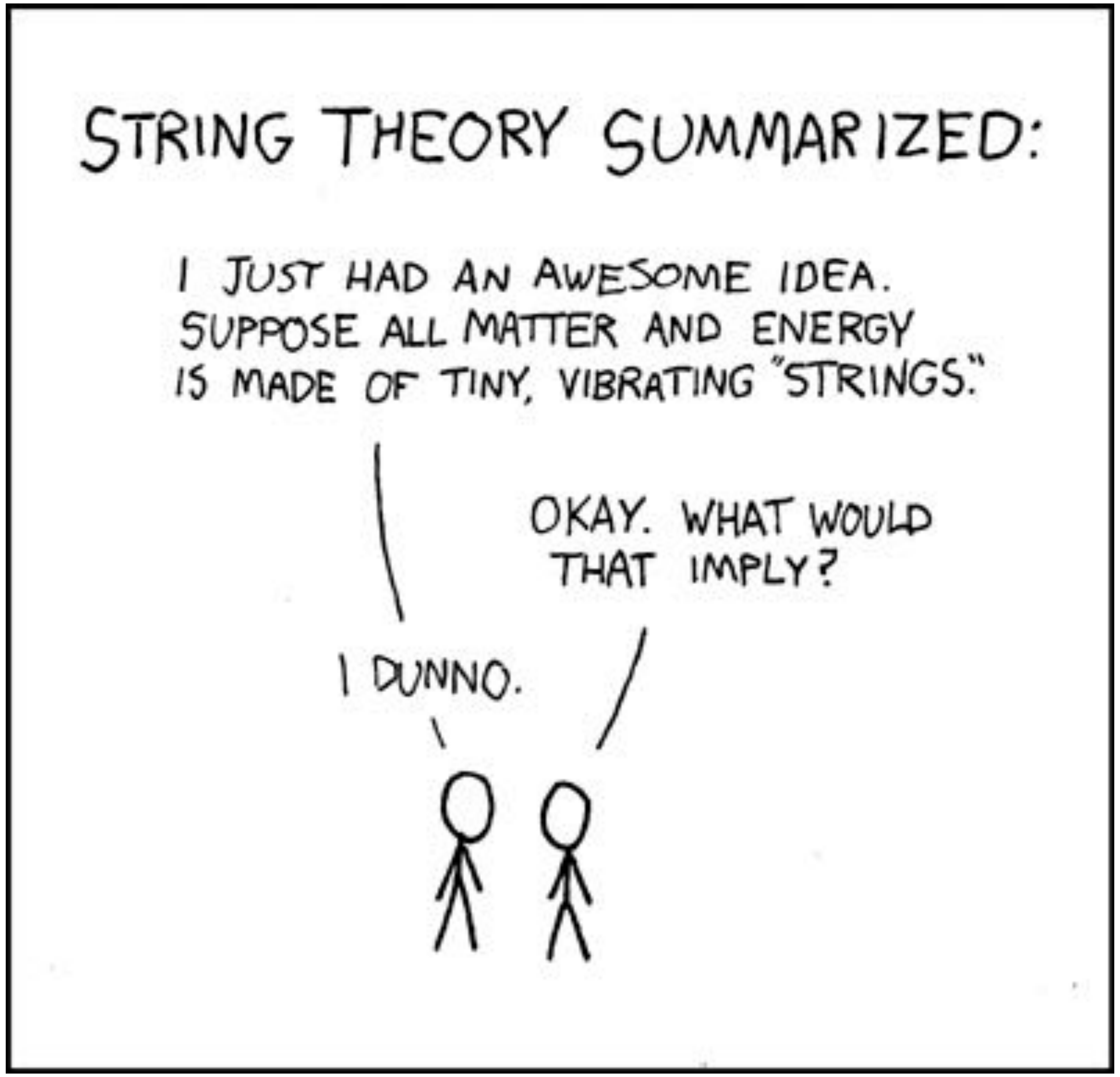
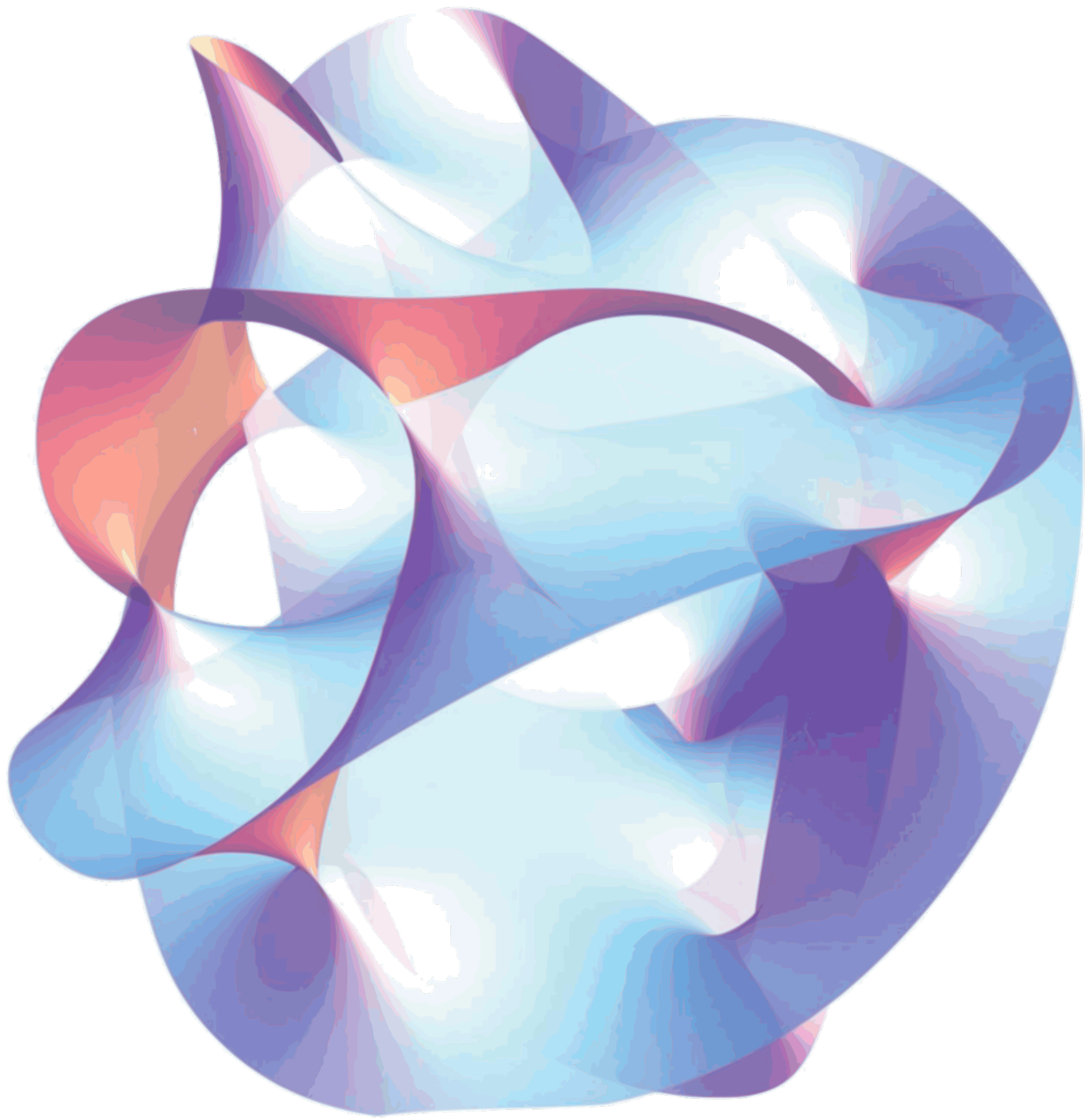


Image credit: Randall Munroe/xkcd

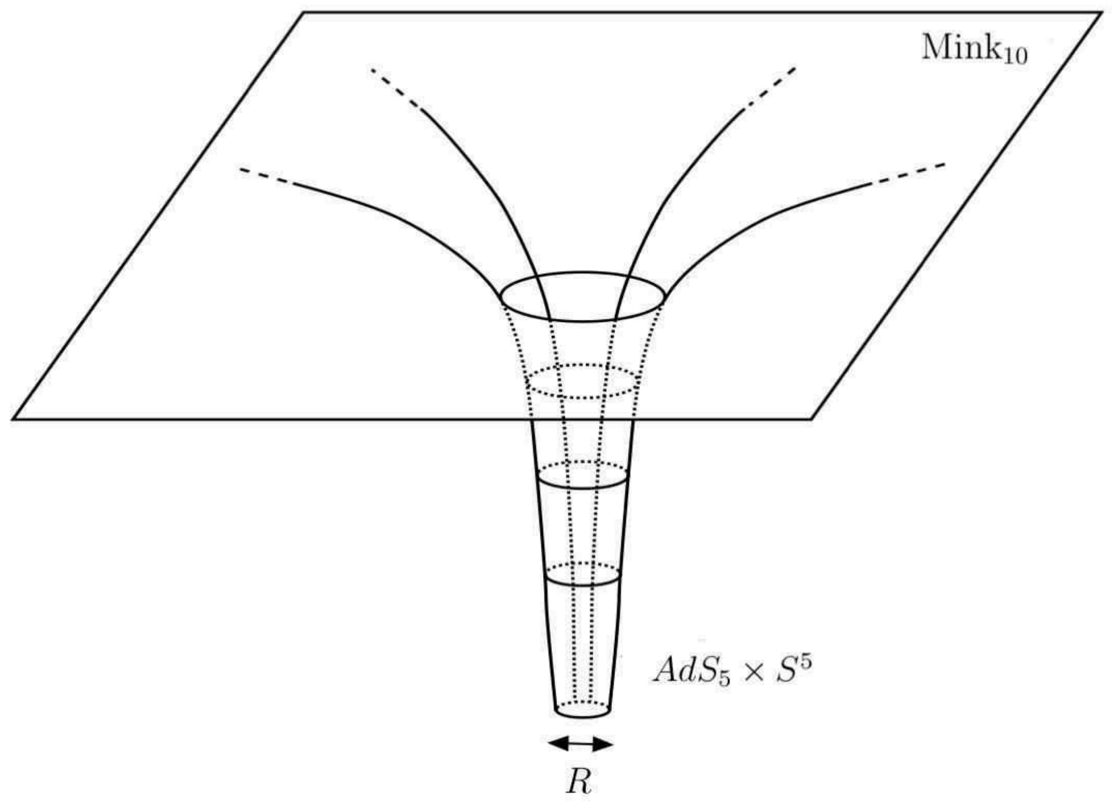
Are we done?

- Famously, string theory has MANY different solutions corresponding to different possible universes
- Some of them seem to look close to our real world
- Computations in realistic string compactifications are not under good enough control to know for sure



Solvable Theories

- Study simpler models of quantum gravity
  - Highly (super)symmetric string backgrounds
  - QG in low dimensions
- Sharpen our computational tools to tackle more realistic models
- Extract general lessons from the theory that hold regardless of the precise realization of our real world



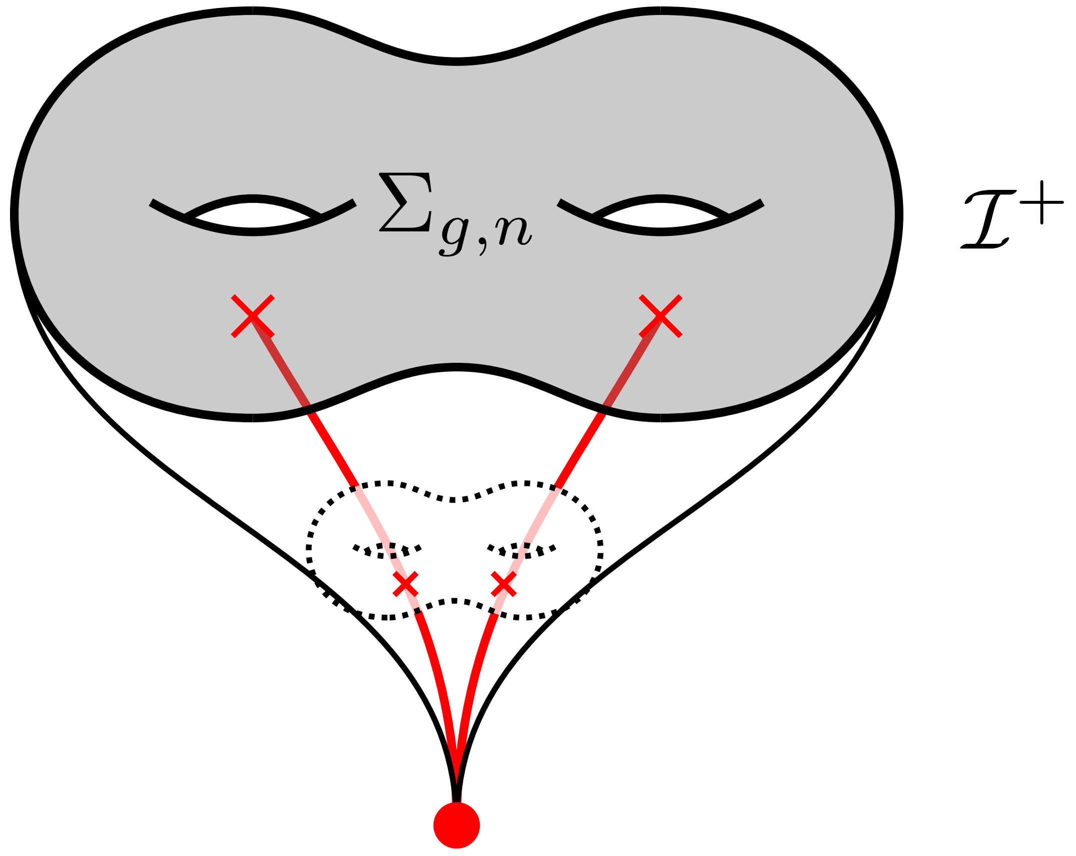
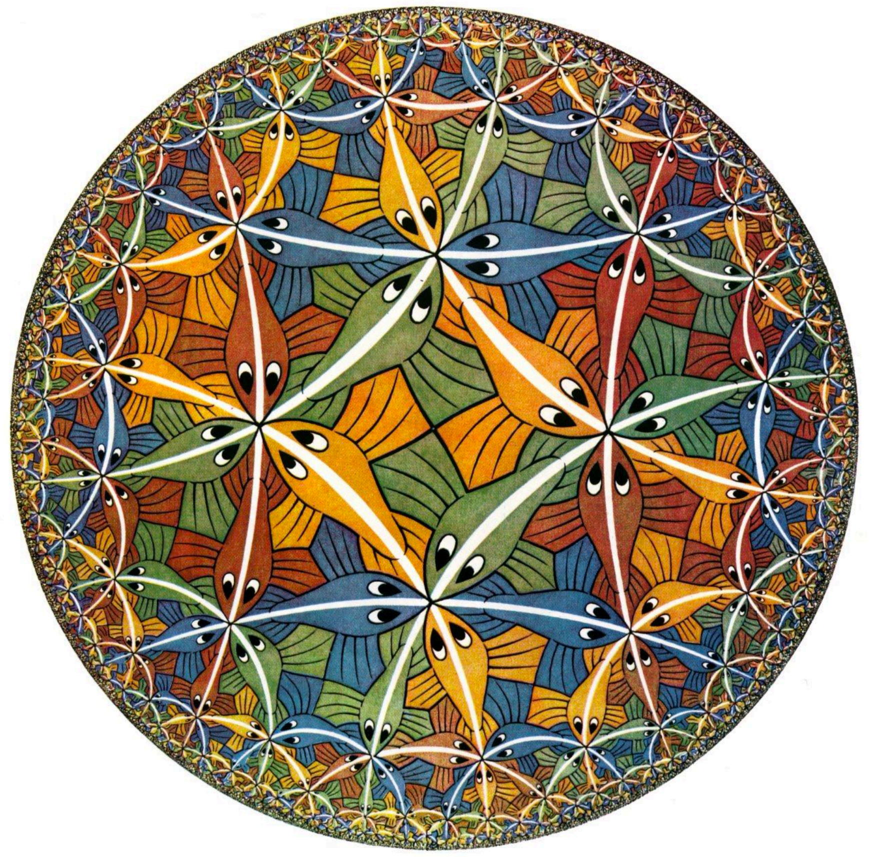


Image credit: D. Mateos

Λ

Quantum gravity behaves very differently depending on the sign of Λ:



	Observables
$\Lambda < 0$	AdS/CFT
$\Lambda = 0$	S-matrix
$\Lambda > 0$	Cosmological correlators

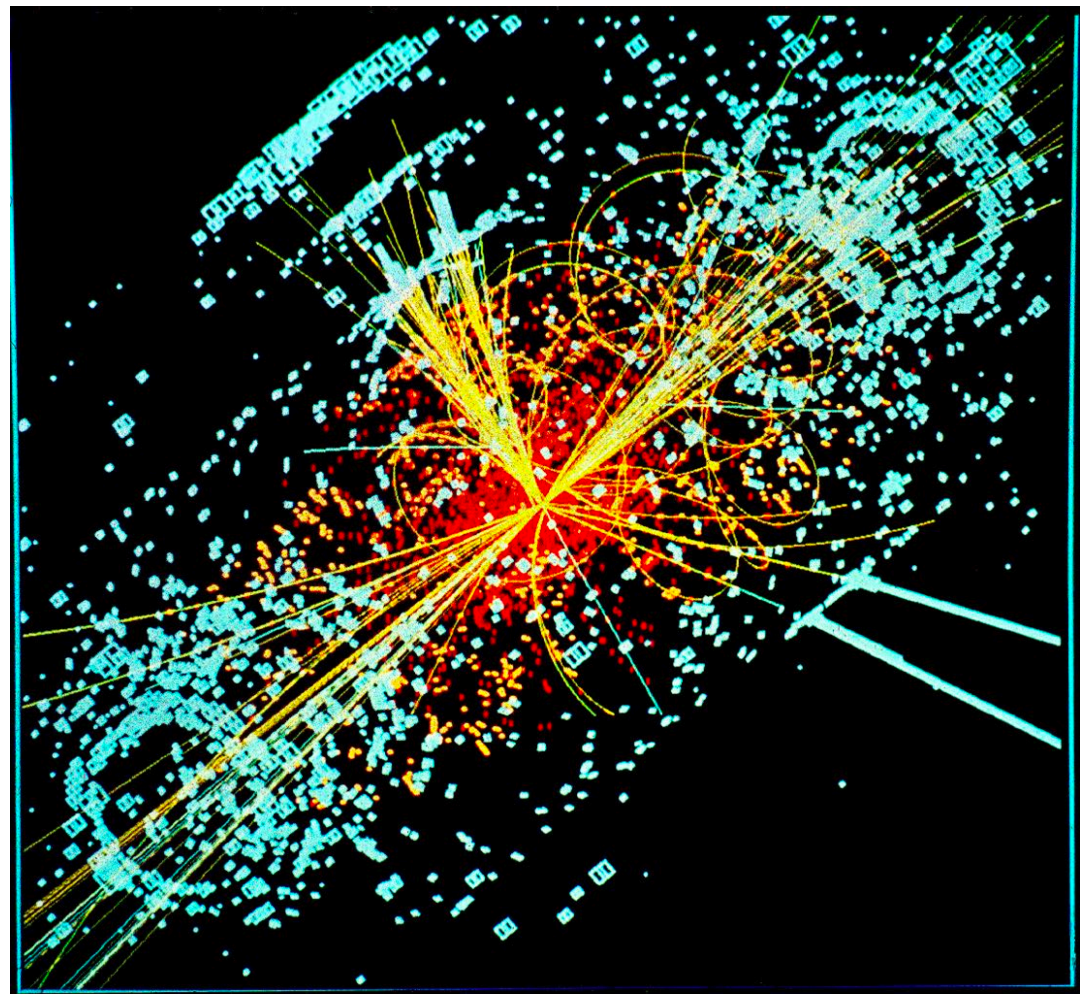
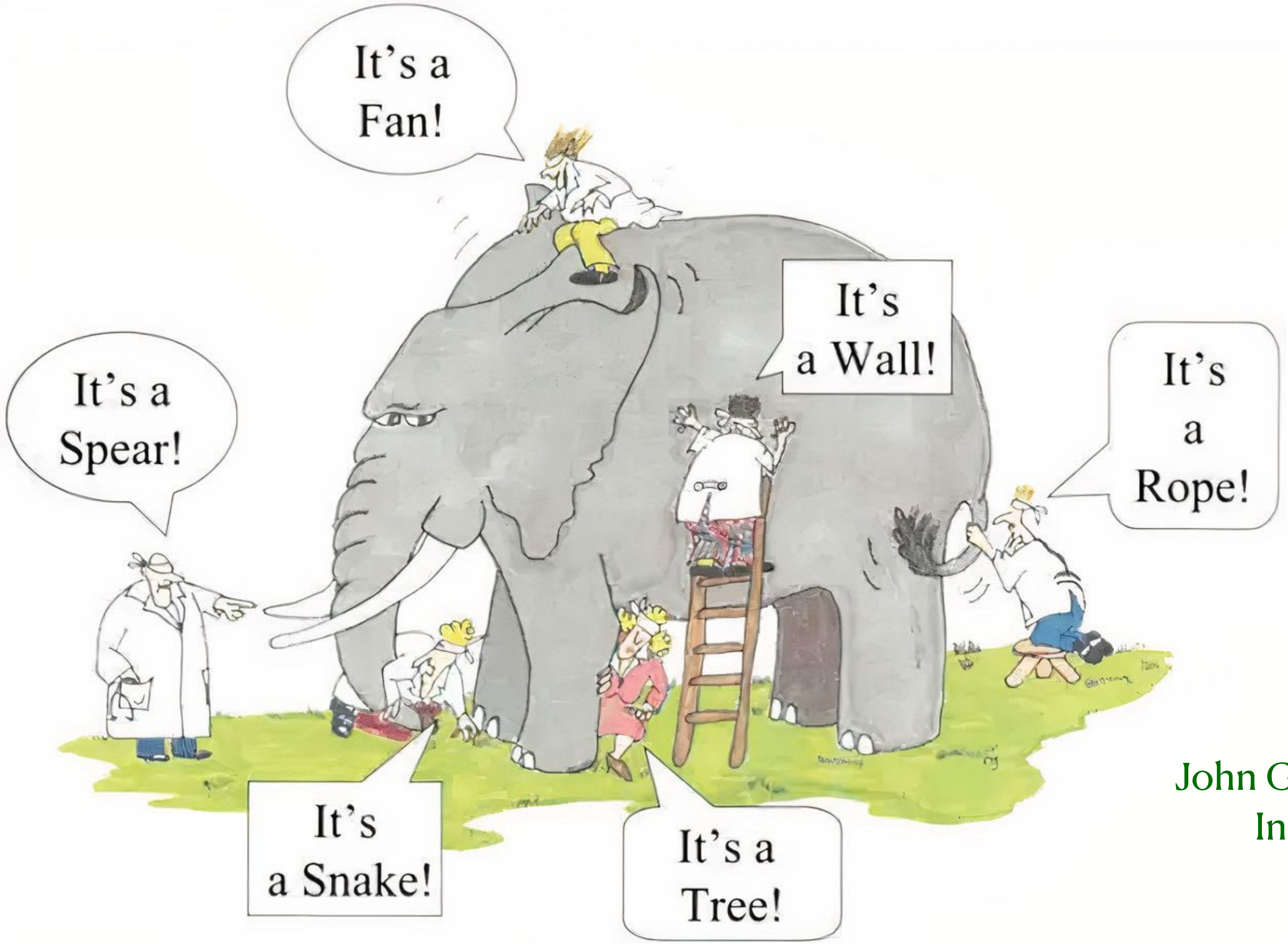


Image credit: M. C. Escher, Lucas Taylor/CERN, Quanta magazine

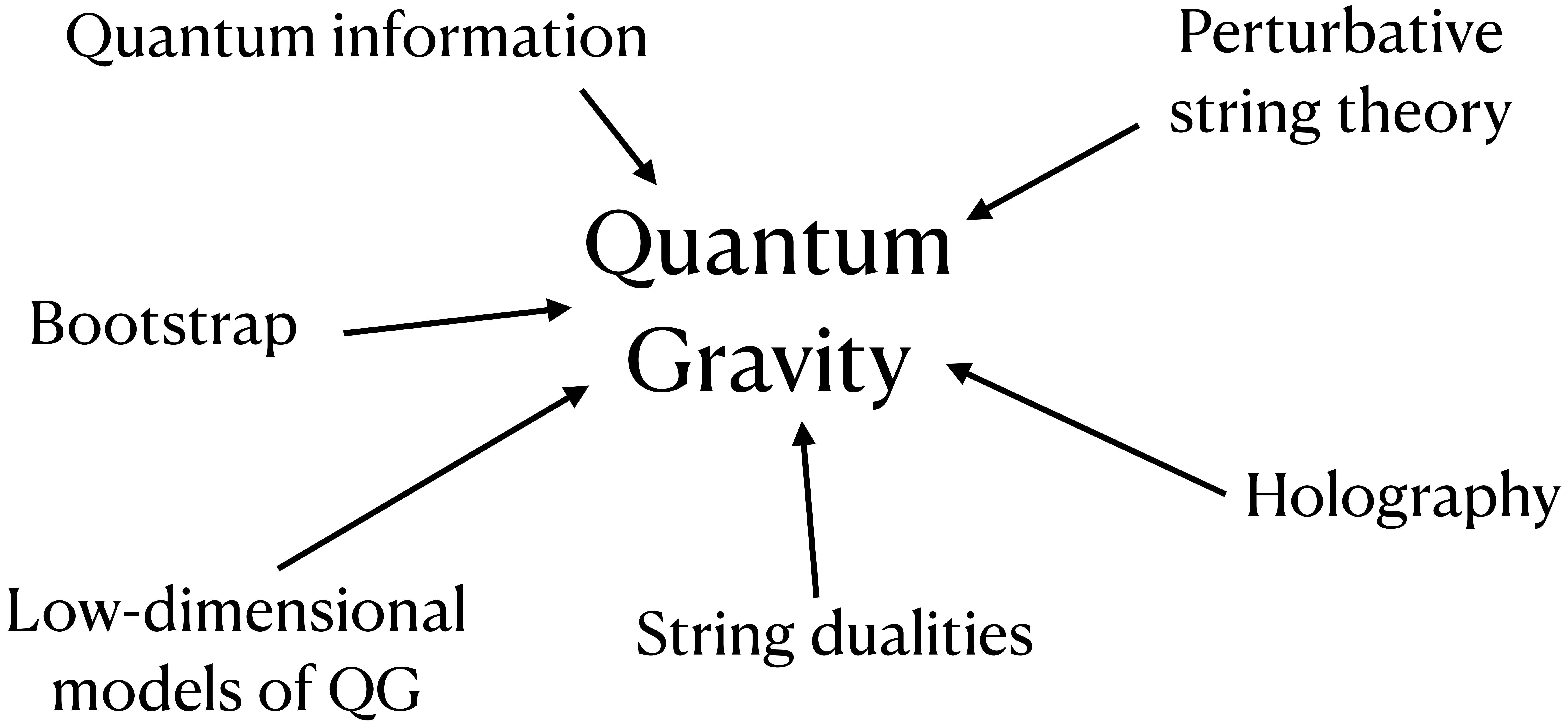
What is QG?



John Godfrey Saxe 1872,
Indian parable

Image credit: ItsALearningLife/Renee Guzlas

The theoretician's toolbox



Tools need to be developed, sharpened and combined

Collaborations with

I. 2+1d Einstein Gravity

Belin, Collier, Liska,
Post, Zhang '22 - ...

II. Stringy AdS_3/CFT_2

Datta, Dei, Gaberdiel,
Gopakumar, Ferreira, Li,
Zadeh '17-'22

III. String Scattering Amplitudes

Arkani-Hamed, Baccianti, Banerjee,
Hillman, Huang, Mizera '20 - ...

I. 2+1d Einstein Gravity

2+1d Einstein gravity

$$S = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^3x \sqrt{-g}\,(R - 2\Lambda)$$

- Much simpler than 3+1d: no gravitational waves
- Classically all solutions look locally the same, but still interesting global structure
- Holography?

2+1d gravity and topological quantum field theory

- 2+1d gravity is similar to a three-dimensional topological theory: Witten '88,...
  - $H = 0$
  - No gravitational waves
- For $\Lambda < 0$: precise correspondence with Virasoro TQFT Collier, **LE**, Zhang '23
- Evaluates the Euclidean gravitational path integral exactly in G_{N} for a fixed spacetime topology

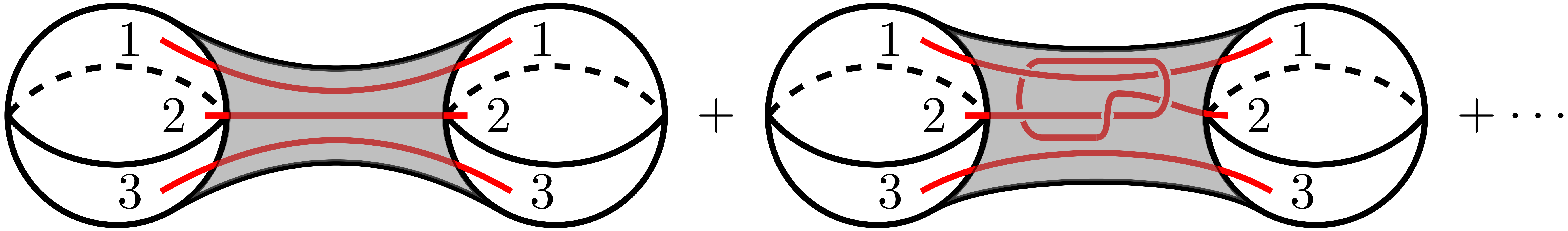
The gravitational path integral: $\Lambda < 0$

- Sum over all topologies

Maloney, Witten ’07,...

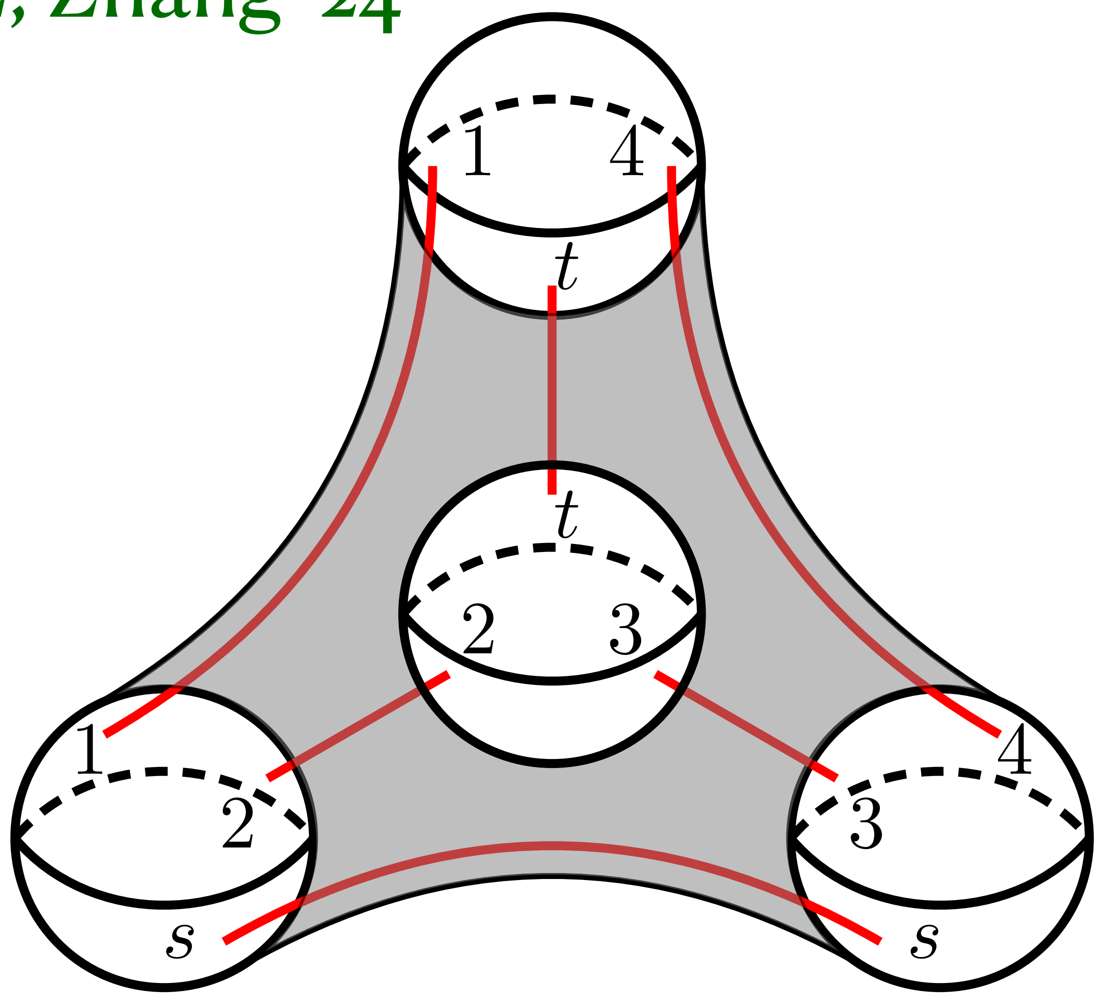
$$Z_{\text{grav}}(\partial M) = \sum_{\text{manifolds with fixed } \partial M} Z_{\text{grav}}(M)$$ ← Computable

- Systematic asymptotic expansion in $e^{-\ell_{\text{AdS}}/G_N}$, i.e. two three-punctured sphere boundaries



A holographic picture

- Is there a dual conformal field theory?
- Emerging picture: Gravity computes universal characteristics of CFTs, not of a single CFT Belin, de Boer '20; Cotler, Jensen '20; Chandra, Collier, Hartman, Maloney '22; Belin, de Boer, Jafferis, Nayak, Sonner '23; ...
- This makes very precise and quantitative predictions that can be checked Collier, **LE**, Zhang '24



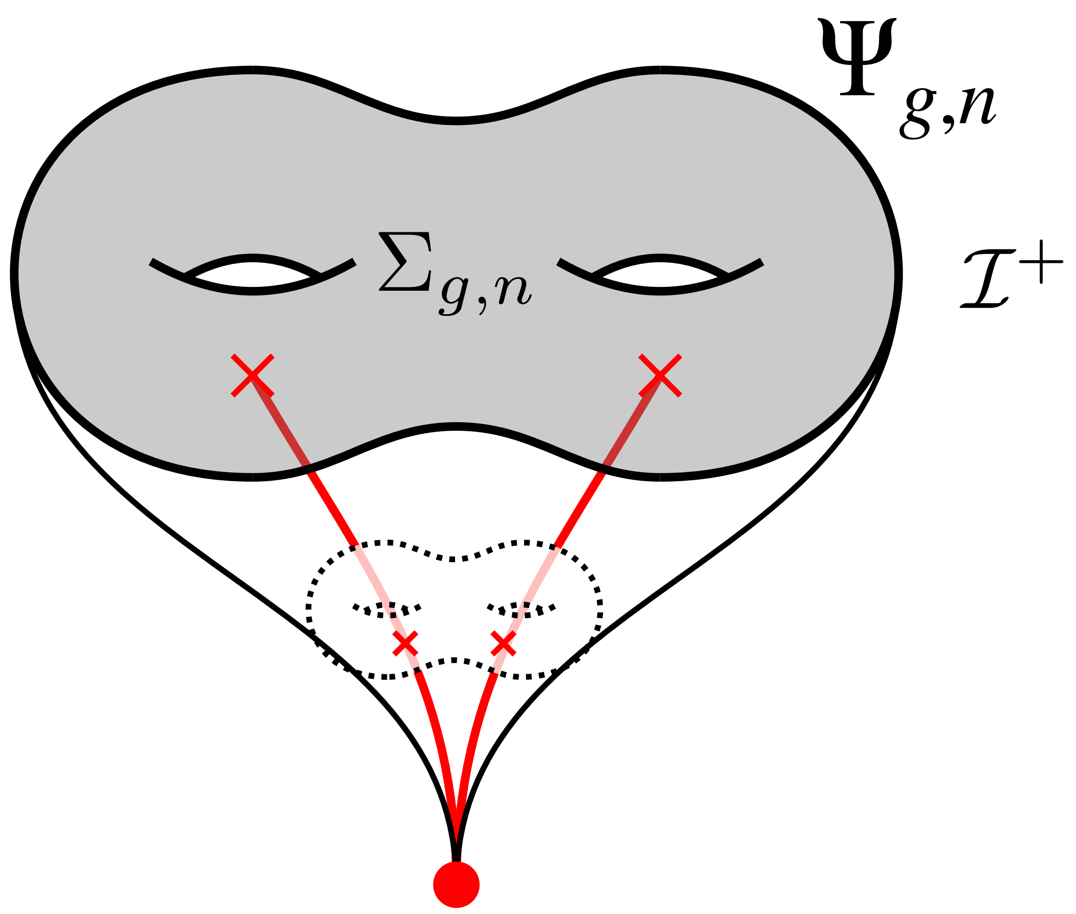
$$= \left| \begin{Bmatrix} h_1 & h_2 & h_s \\ h_3 & h_4 & h_t \end{Bmatrix} \right|^2$$

Virasoro 6j symbol Ponsot, Teschner '99

$\Lambda > 0$: The wave function of the universe

- A universe is specified by its wave function

Hartle, Hawking '83



- Consistency conditions or evaluation of the gravitational path integral on an expanding universe

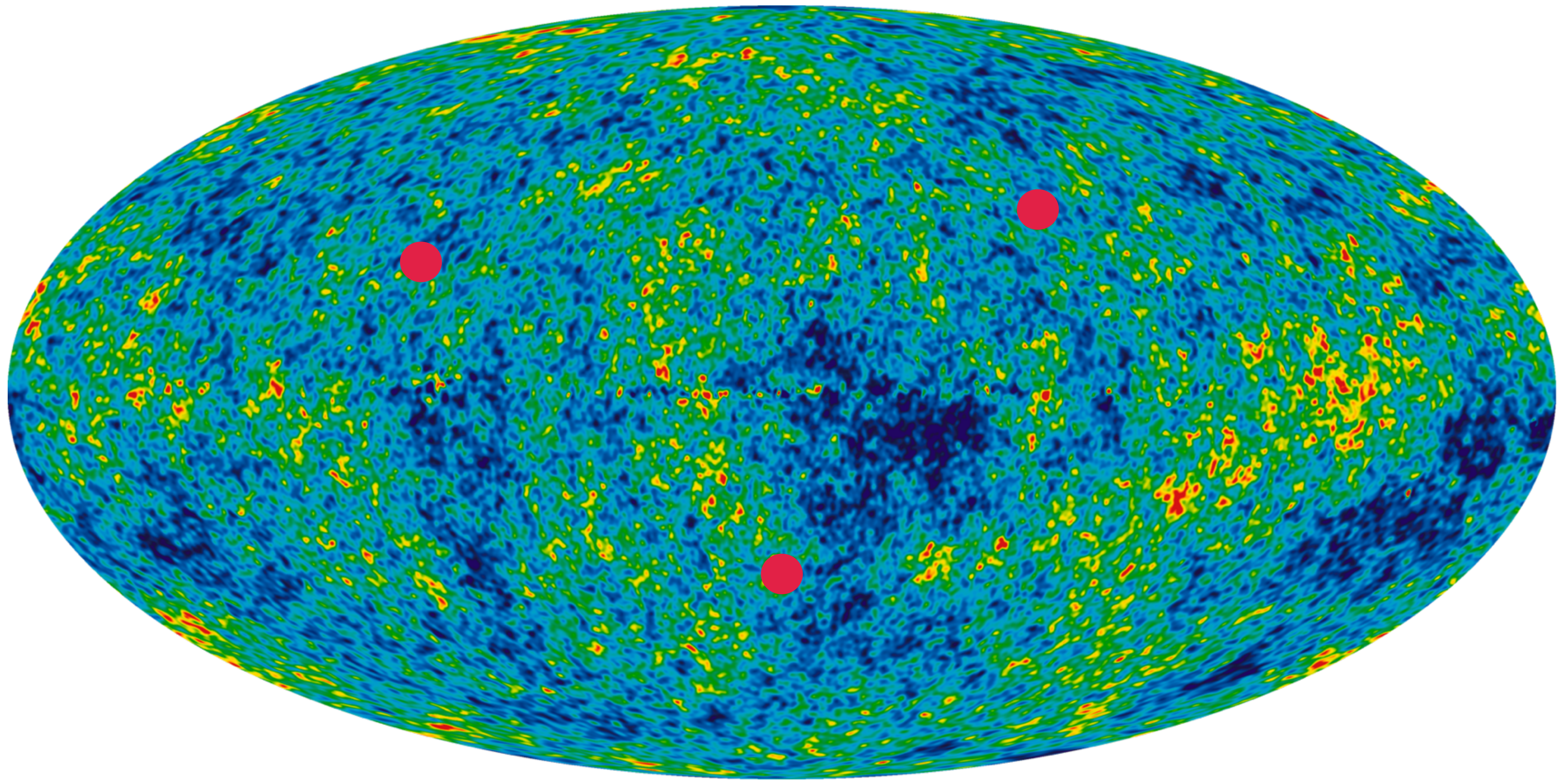
Collier, **LE**, Mühlmann WIP

$$\Psi_{g,n} = \langle V_{m_1}(z_1)\ldots V_{m_n}(z_n)\rangle_g$$

Correlator of Liouville CFT

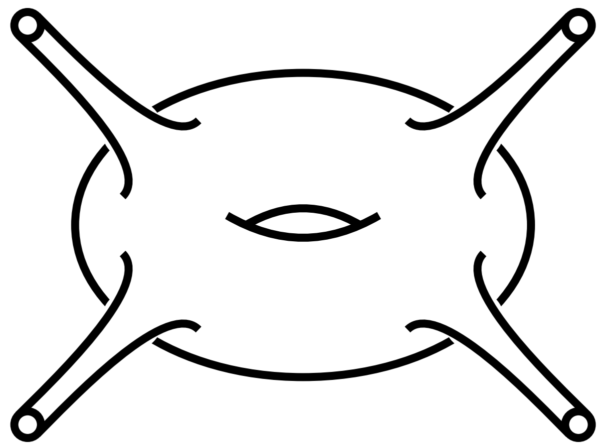
Cosmological correlators

- A person looking at the (gravitational) CMB in this model would measure cosmological correlators:



$$= \int_{\text{metrics at future infinity}} \mathscr{D}h \, |\Psi_{g,n}(h)|^2$$

Maldacena '02

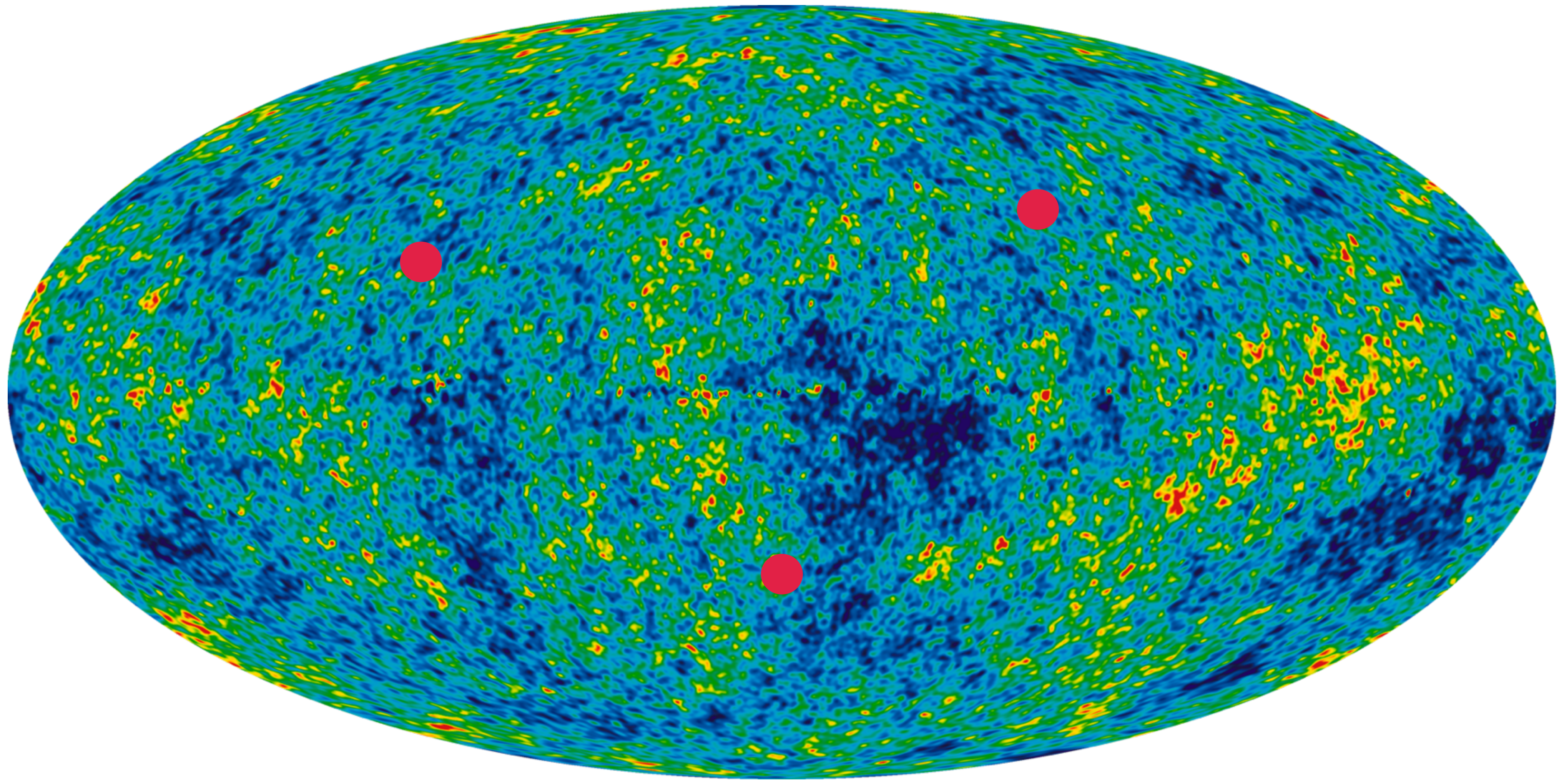


- String theorists are good at computing these integrals!

Image credit: NASA/WMAP

A microscopic dual of dS_3

- The answer is computed by a matrix model



Collier, **LE**, Mühlmann, Rodriguez '24

$$= \int_{N\times N} [\mathrm{d}M_1]\,[\mathrm{d}M_2]\,\mathrm{e}^{-V(M_1,M_2)}\,\mathcal{O}_{m_1}(M_1)\cdots\mathcal{O}_{m_n}(M_1)$$

- The matrix model has precisely the right number of microstates as predicted by the de Sitter entropy!

Image credit: NASA/WMAP

Lessons

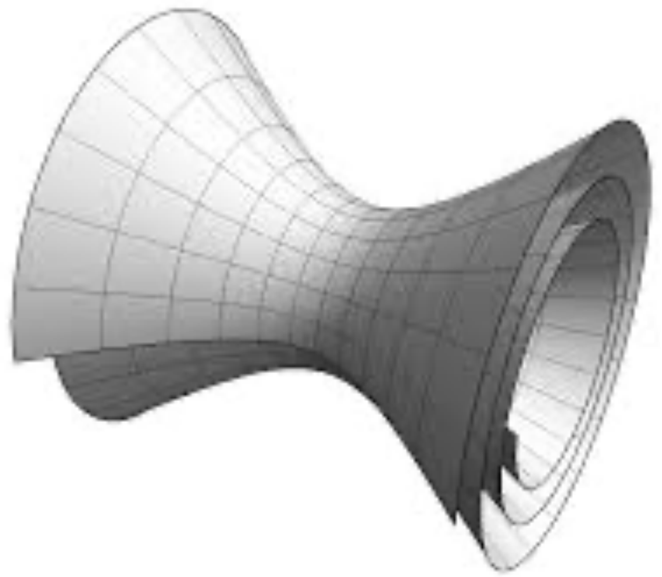
- Einstein quantum gravity in low dimensions makes sense, both for $\Lambda < 0$ and $\Lambda > 0$
- It computes universal or averaged quantities:
    - Universal CFT data for $\Lambda < 0$
    - Matrix average for $\Lambda > 0$
- Good observables drastically differ depending on the sign of the cosmological constant

II. Stringy AdS/CFT

The AdS/CFT correspondence

Maldacena '97

String theory on $AdS_5 \times S^5$



$\times$

$=$

$\mathcal{N} = 4 \; SU(N)$
Super-Yang Mills theory

Quantum gravity

Ordinary QFT

Image credit: Beisert et al.

N^{-1} ↑

Perturbative QFT

$\lambda \sim 0$

$\lambda \sim \ell_{\text{AdS}}/\ell_{\text{s}}$

Classical Gravity

$\lambda \to \infty$

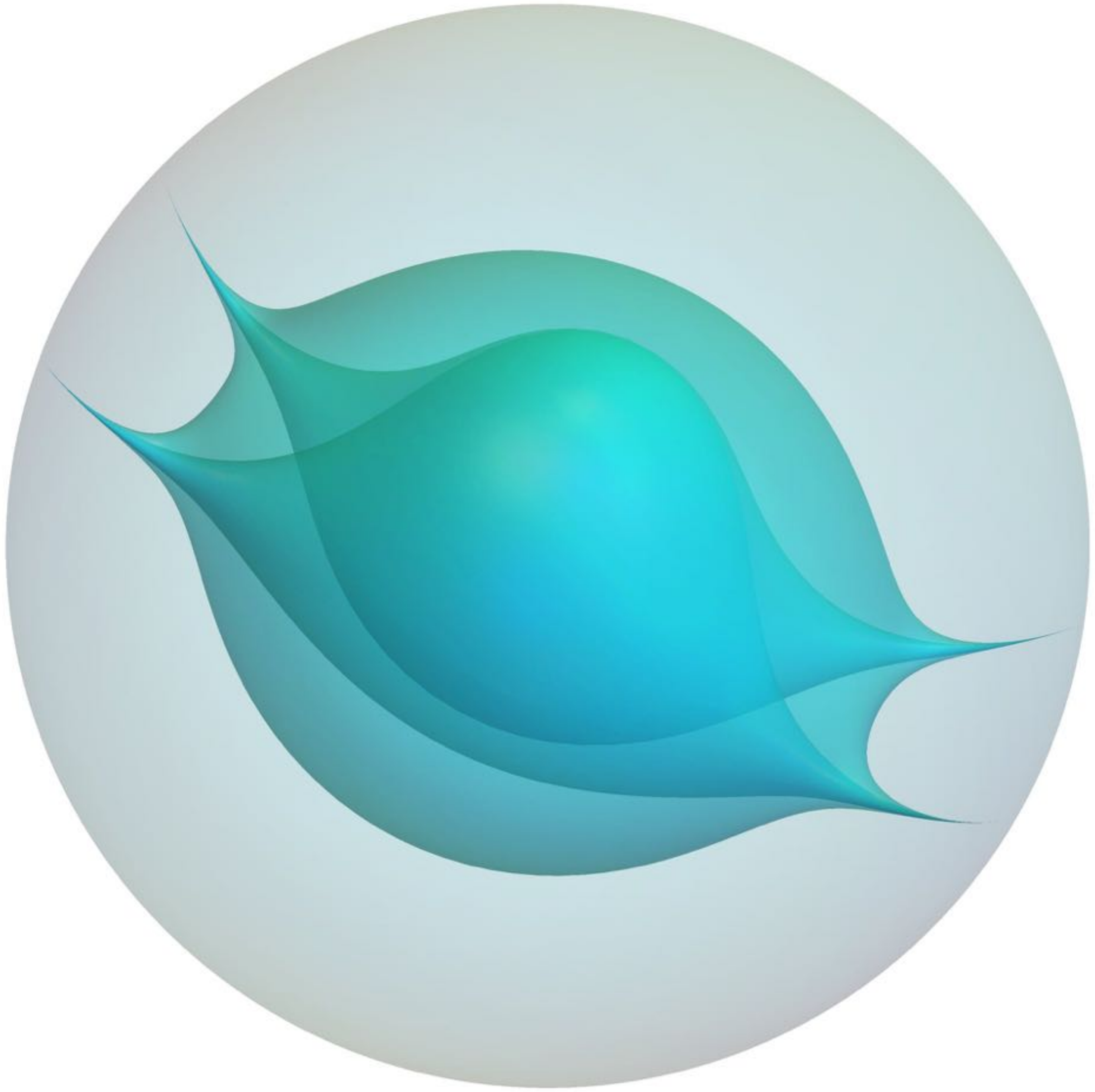
Image credit: iStock, DreamWorks Animation

AdS_3 backgrounds

- Testing (or deriving) this is very hard
- Relies usually on supersymmetry or integrability
- Look at simpler background first: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$
- Tractable by more standard perturbative string theory (NS-NS background)
- But less clear expectation about the holographic dual   Maldacena '97; Larsen, Martinec '99, ...
- Useful starting points to understand the inner workings of the AdS/CFT correspondence

Shut up and calculate…

- Compute all string correlators
- The theory is completely solved for genus 0 correlators Teschner ’99 - ’01; Maldacena, Ooguri ’00 - ’02; Dei, **LE** ’21 - ’22
- Main technical hurdle: winding correlators
- Result is intricate, but has hidden structure



Finding the needle in the haystack...

LE '21

Strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ $=$ Deformed symmetric orbifold

$$\frac{(\mathbb{R}^4_Q \times \mathbb{T}^4)^N}{S_N} + \text{marginal operator}$$

- Checked way beyond what is possible for $\mathrm{AdS}_5 \times \mathrm{S}^5$ (unprotected quantities)
- Shows **why** this works

The tensionless limit

- The correspondence becomes even simpler for very small AdS: $\ell^2_{\text{AdS}}/\ell^2_{\text{string}} \sim k = 1$
  LE, Gaberdiel, Gopakumar '18

Strings on $\text{AdS}_3 \times \text{S}^3 \times \mathbb{T}^4$ for $k = 1$ $=$ Symmetric orbifold $(\mathbb{T}^4)^N / S_N$

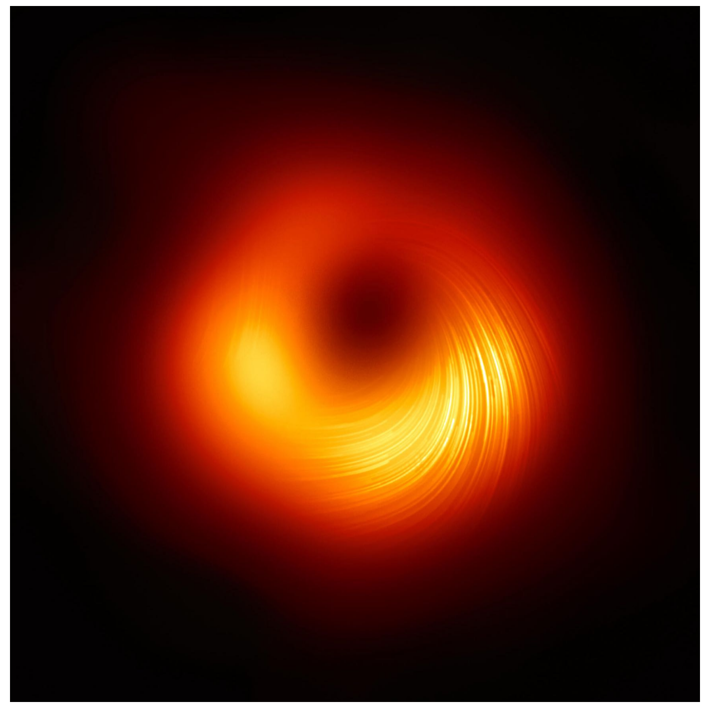
- Mechanism: localization of the string path integral
  LE, Gaberdiel, Gopakumar '19; **LE** '20 - '21; Dei, Gaberdiel, Gopakumar, Knighton '20; Knighton '20; ...

$$\int_{\text{worldsheets}} \mathscr{D}g \longrightarrow \sum$$

Lessons

- Exact holographic map **LE**, Gaberdiel, Gopakumar '18 - '19; Dei, Gaberdiel, Gopakumar, Knighton '20; Knighton '20 ; **LE** '20 - '21; Gaberdiel, Knighton, Vošmera '21; Dei, **LE** '21 - '22; Dei, Knighton, Naderi '23; Hikida, Schomerus '23; ...
- Exposes the inner workings of the duality
- Black holes are perturbative string states: black hole/string transition

  Susskind '94; Horowitz, Polchinski '96; Giveon, Kutasov, Rabinovici, Sever '05; **LE** '20



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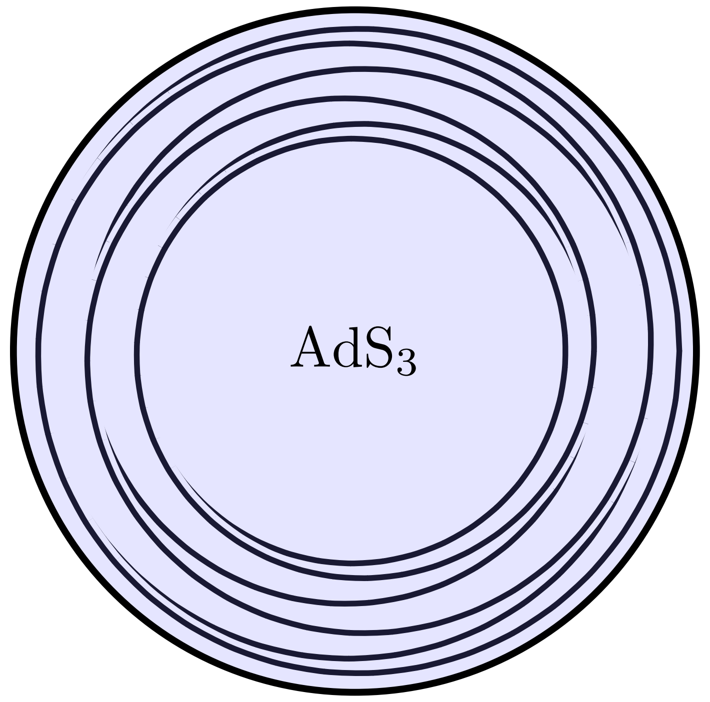
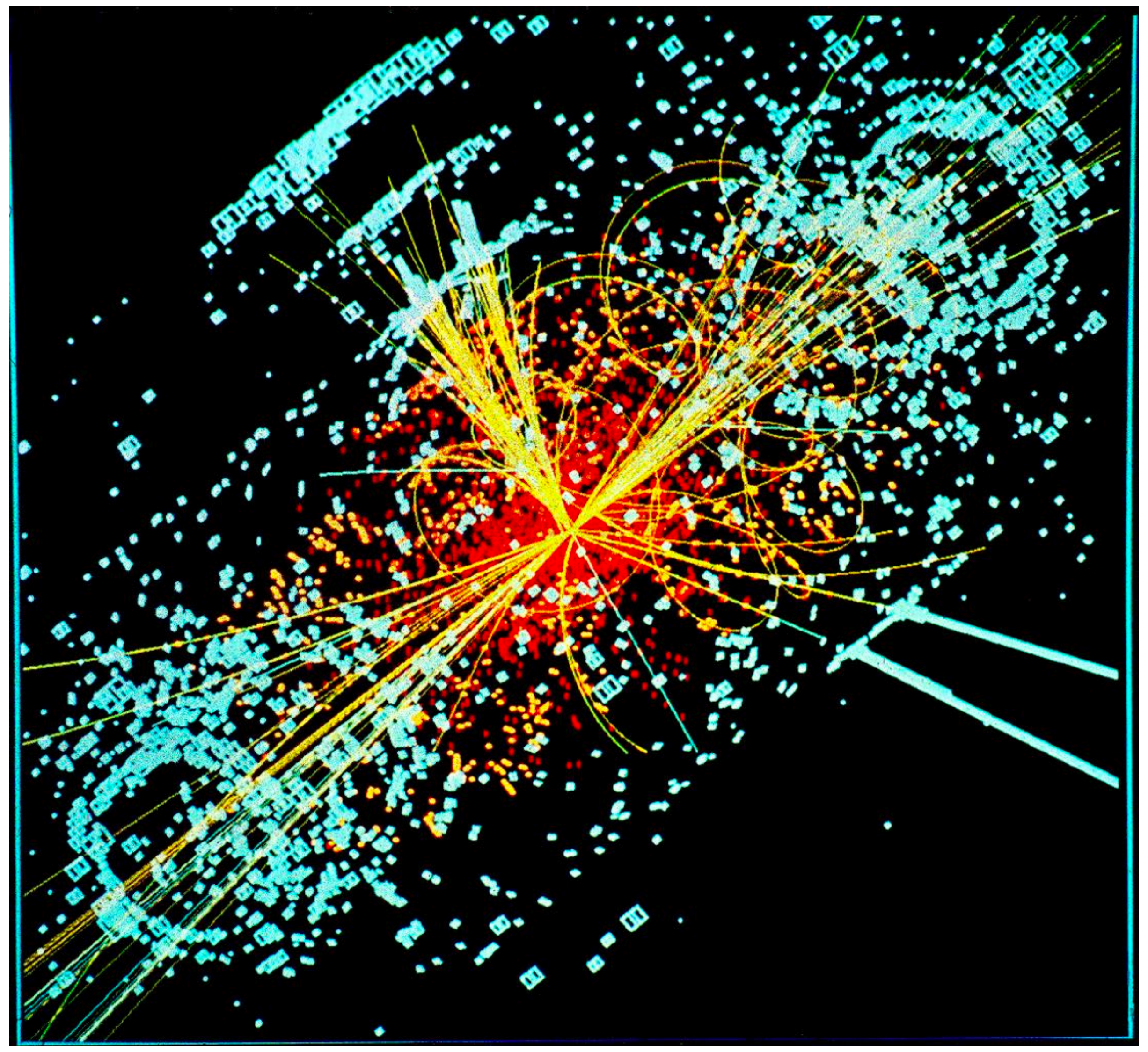


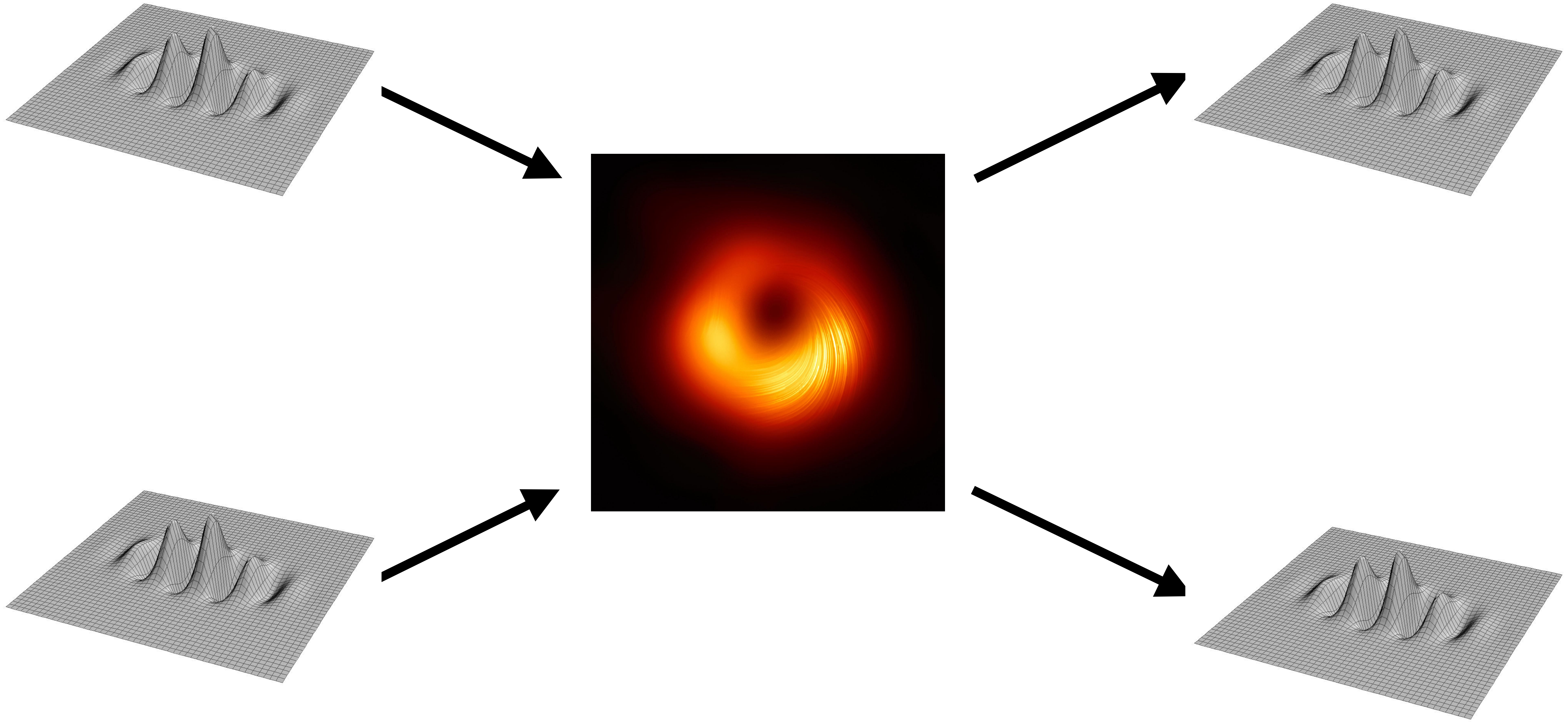
- Background independent **LE** '21, Knighton '24

III. String Scattering Amplitudes

Quantum gravity in the collider

Our universe is flat on short distances, so to good approximation





S-matrix elements

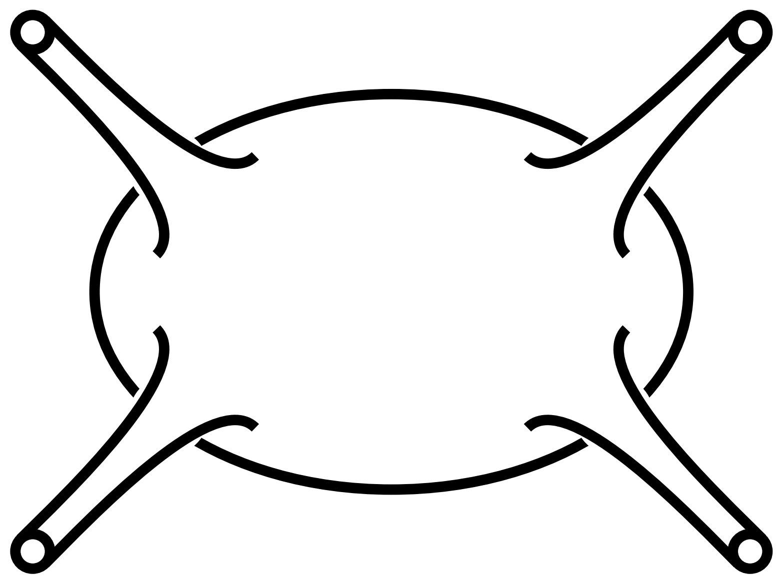
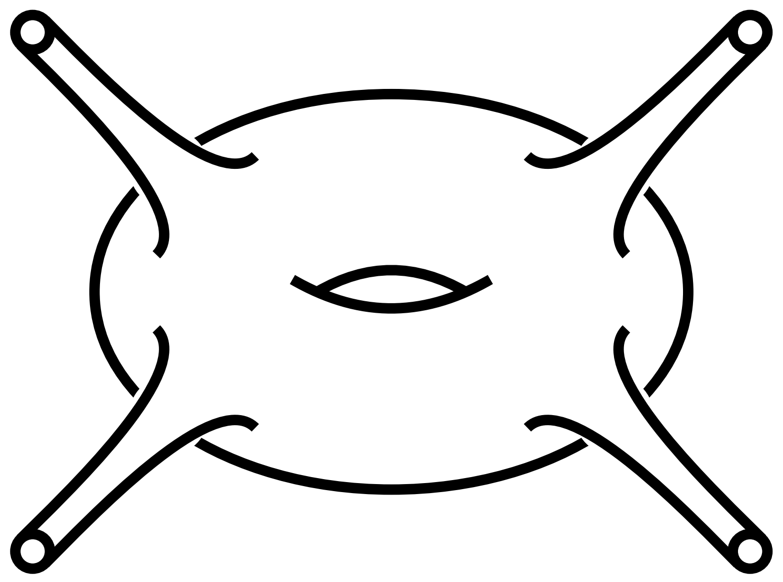
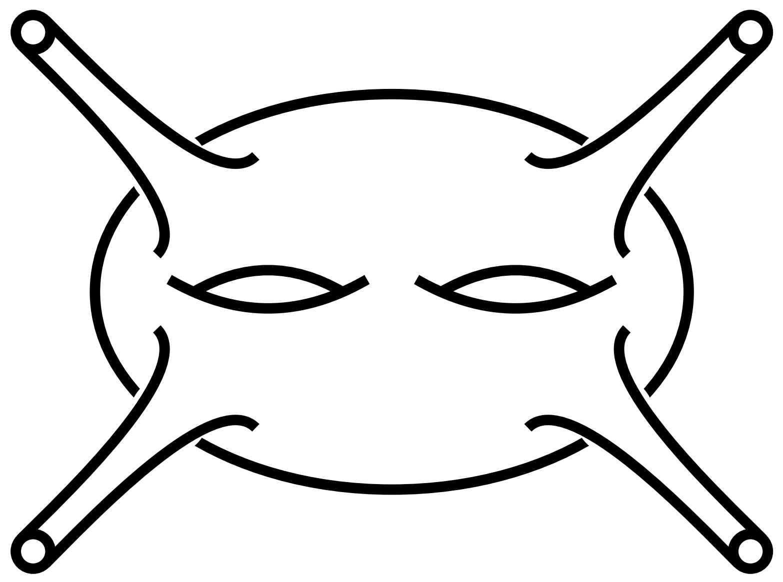
This collider will remain theoretical...

- String theory makes predictions about the scattering of gravitational waves
- But it is difficult to compute them

Virasoro '69, Shapiro '70

$$A_{\text{tree}}(s,t,u) = t_8 \tilde{t}_8 \frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$

- Perturbative string theory makes very precise predictions for higher string loop corrections to string scattering amplitudes (at least in 10d...)

$\mathcal{A} \sim g_s^2$  $+ \; g_s^4$  $+ \; g_s^6$  $+ \; \cdots$

String Amplitudes

Particle Physics

Feynman diagrams

Elliptic Integrals

QFT Scattering Amplitudes

S-matrix bootstrap

Gravity=Gauge theory^2

Mathematics

Analytic number theory

Algebraic geometry

Quantum Gravity and String Theory

Black hole production

AdS/CFT correspondence

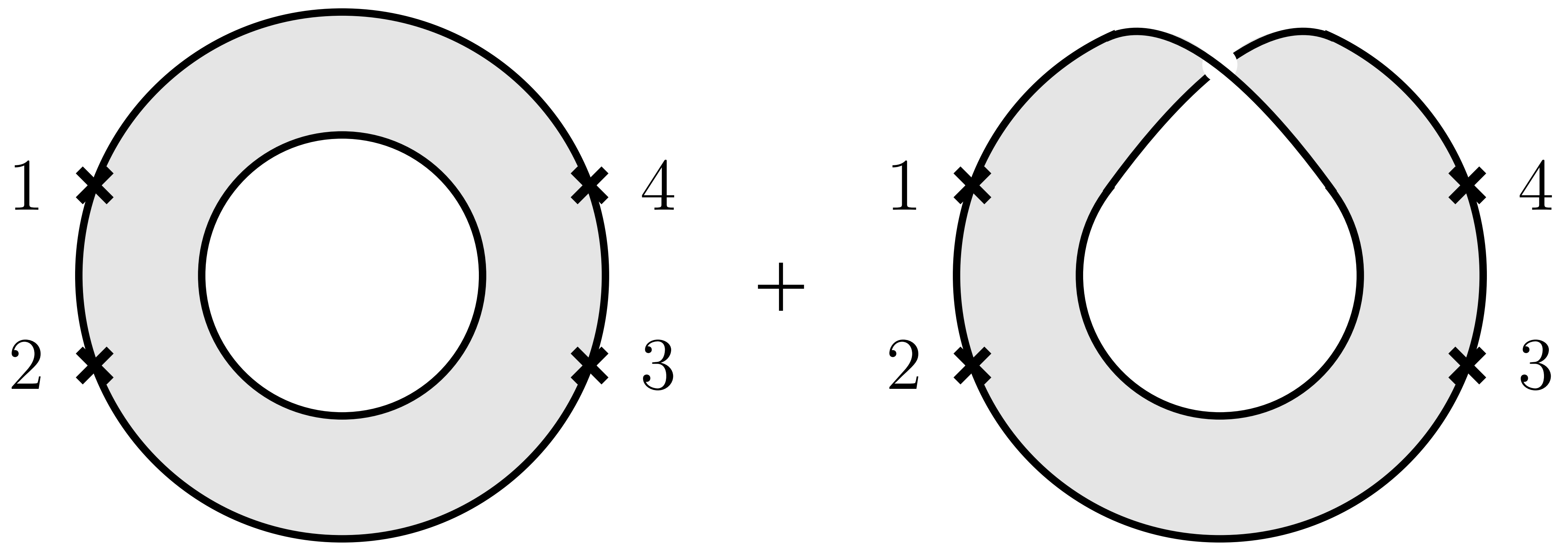
UV-behaviour of gravity

Evaluating the amplitudes

- Probe quantum gravity at weak coupling, but very high energy: universal behavior
- The expressions for higher loop amplitudes are difficult to evaluate
- Physical lessons are hard to extract
- Despite 40 years of effort, we know very little about the actual behavior of these amplitudes Green, Schwarz '80 - ; D'Hoker, Phong '85 - ; ...
- But there are many expectations...
Gross, Mende '87 - '88; Amati, Ciafaloni, Veneziano '87 - '88; Susskind '93; Horowitz, Polchinski '96 - '97; Veneziano '04; Giddings '06; Giddings, Gross, Maharana '08; ...

What's the problem?

- The simplest higher loop correction: scattering of four gauge bosons



- Deceptively simple integral representation:

Green, Schwarz '82

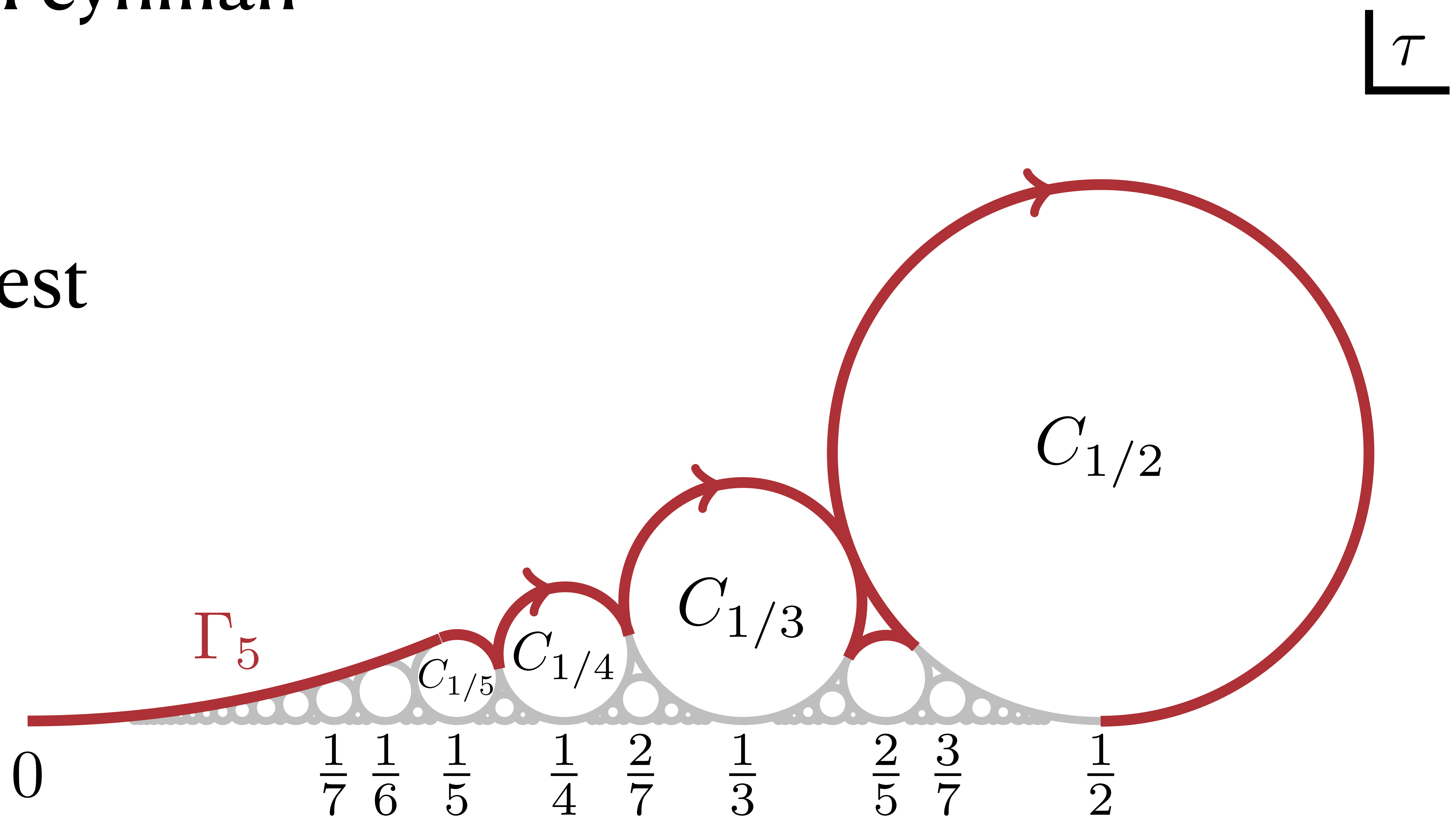
$$\int_\Gamma \mathrm{d}\tau\,\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,\left(\frac{\vartheta_1(z_{21},\tau)\vartheta_1(z_{43},\tau)}{\vartheta_1(z_{31},\tau)\vartheta_1(z_{42},\tau)}\right)^{-s}\left(\frac{\vartheta_1(z_{21},\tau)\vartheta_1(z_{41},\tau)}{\vartheta_1(z_{32},\tau)\vartheta_1(z_{42},\tau)}\right)^{-t}$$

IR divergent

- `NIntegrate[...]` doesn't work...

Analytic number theory is your friend

- Crazy contour deformation of Γ inspired from number theory   Hardy-Ramanujan-Littlewood 1916; Rademacher 1943; **LE**, Mizera '22 -
- Completely different than the strategy for Feynman diagrams
- Makes a lot of the analytic structure manifest (discontinuities, poles)

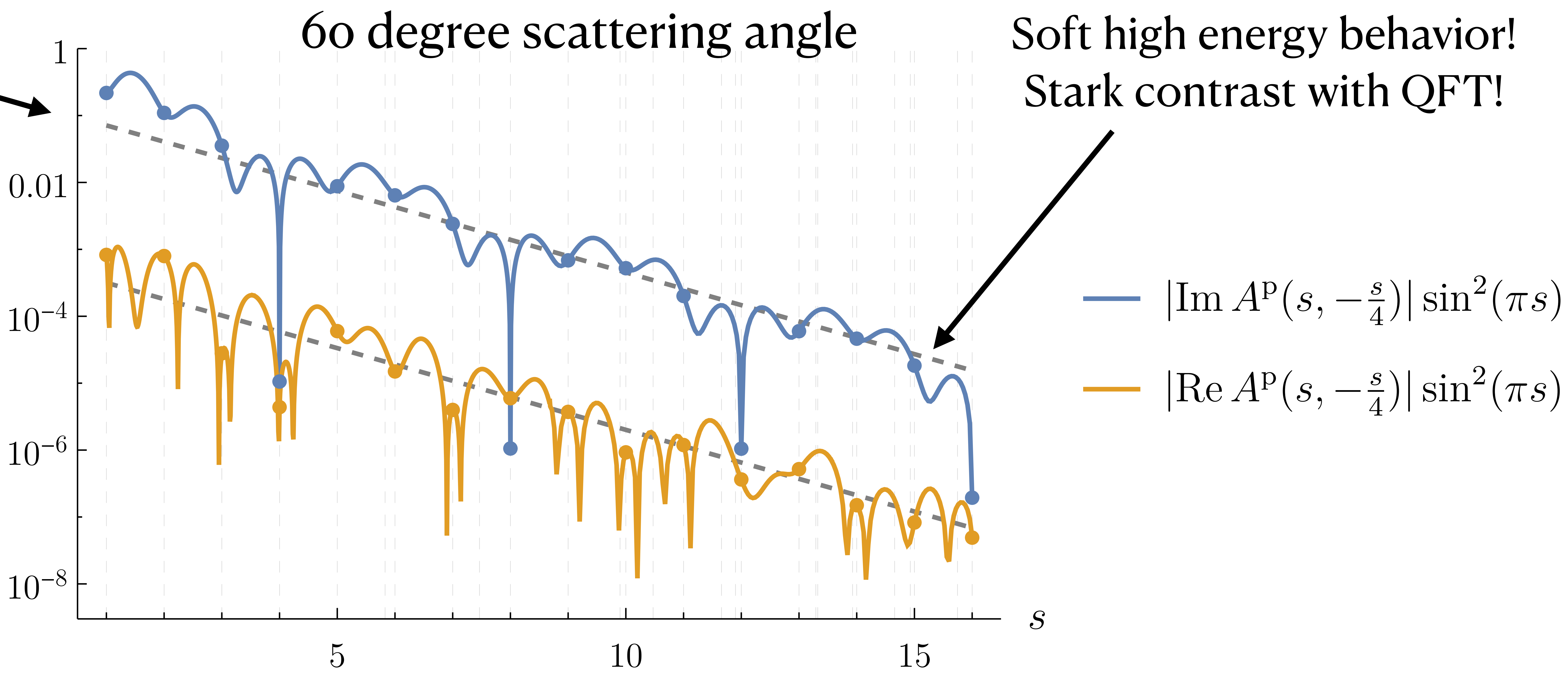


First string scattering amplitude @ 1-loop

LE, Mizera ’23

Field theory + α' corrections

enormous literature: Green, Schwarz, Gross, Veneziano, Amati, Ciafaloni, Di Vecchia, Koba, Nielsen, D’Hoker, Phong, Martinec, Bern, Dixon, Polyakov, Kosower, Vanhove, Schlotterer, Mafra, Stieberger, Brown, Broedel, Hohenegger, Kleinschmidt, Gerken, Roiban, Lipstein, Mason, Monteiro, ...



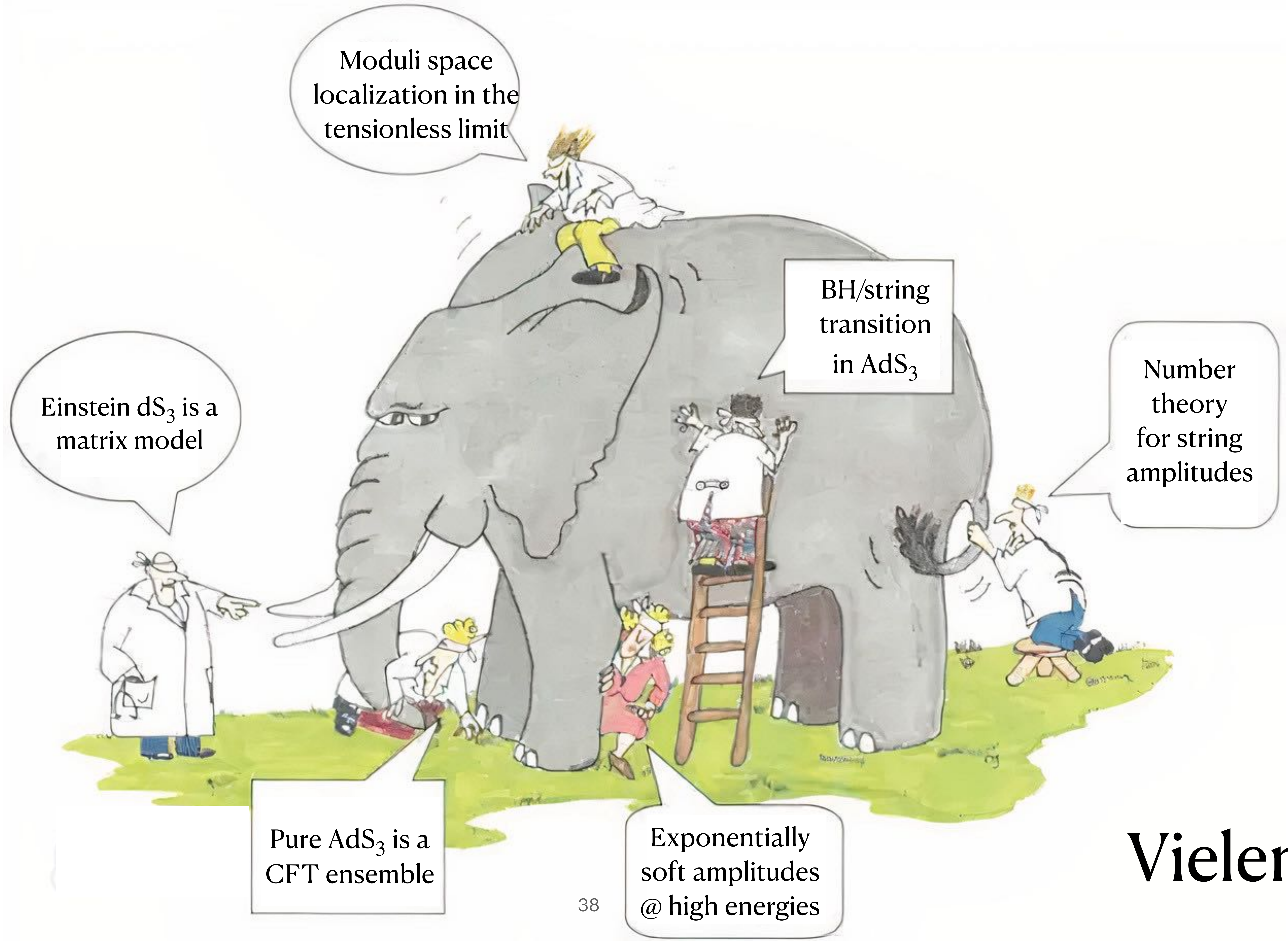
Soft high energy behavior!
Stark contrast with QFT!

Envelope conjectured by Gross, Mende, Mañes ‘89

Lessons

- String amplitudes are an old topic, but
  - cornerstone of quantum gravity
  - rigorous and precise
- Powerful mathematical machinery can be used to evaluate them: lots of hidden structure
- String amplitudes are exponentially soft in the UV, even at loop order

Conclusions


Moduli space localization in the tensionless limit
BH/string transition in AdS_3
Einstein dS_3 is a matrix model
Number theory for string amplitudes
Pure AdS_3 is a CFT ensemble
Exponentially soft amplitudes @ high energies

Vielen Dank!