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Phases of matter and spontaneous spacetime symmetry breaking

(based on many papers, with many collaborators, over the last ~15 years)

Usually

condensed matter theory = ~~relativity~~

1. $v \ll c$ $(= 1)$
2. preferred frame

Note: relativity = Lorentz or Galilei

For us instead

condensed matter theory = ~~relativity~~ spontaneously

Note: relativity = Lorentz or Galilei

Or equivalently

condensed matter theory = relativity
non-linearly
realized

Note: relativity = Lorentz or Galilei

Symmetries

Symmetry = Transformation of dynamical variables under which dynamics are invariant

Continuous symmetry ⇒ { Noether theorem, Goldstone theorem }

Noether theorem

Symmetries ⇄ Conservation laws

Time translations: Energy

Spatial translations: Momentum

Rotations: Angular momentum

etc.

Goldstone theorem

IF symmetry broken by state of the system

⇓

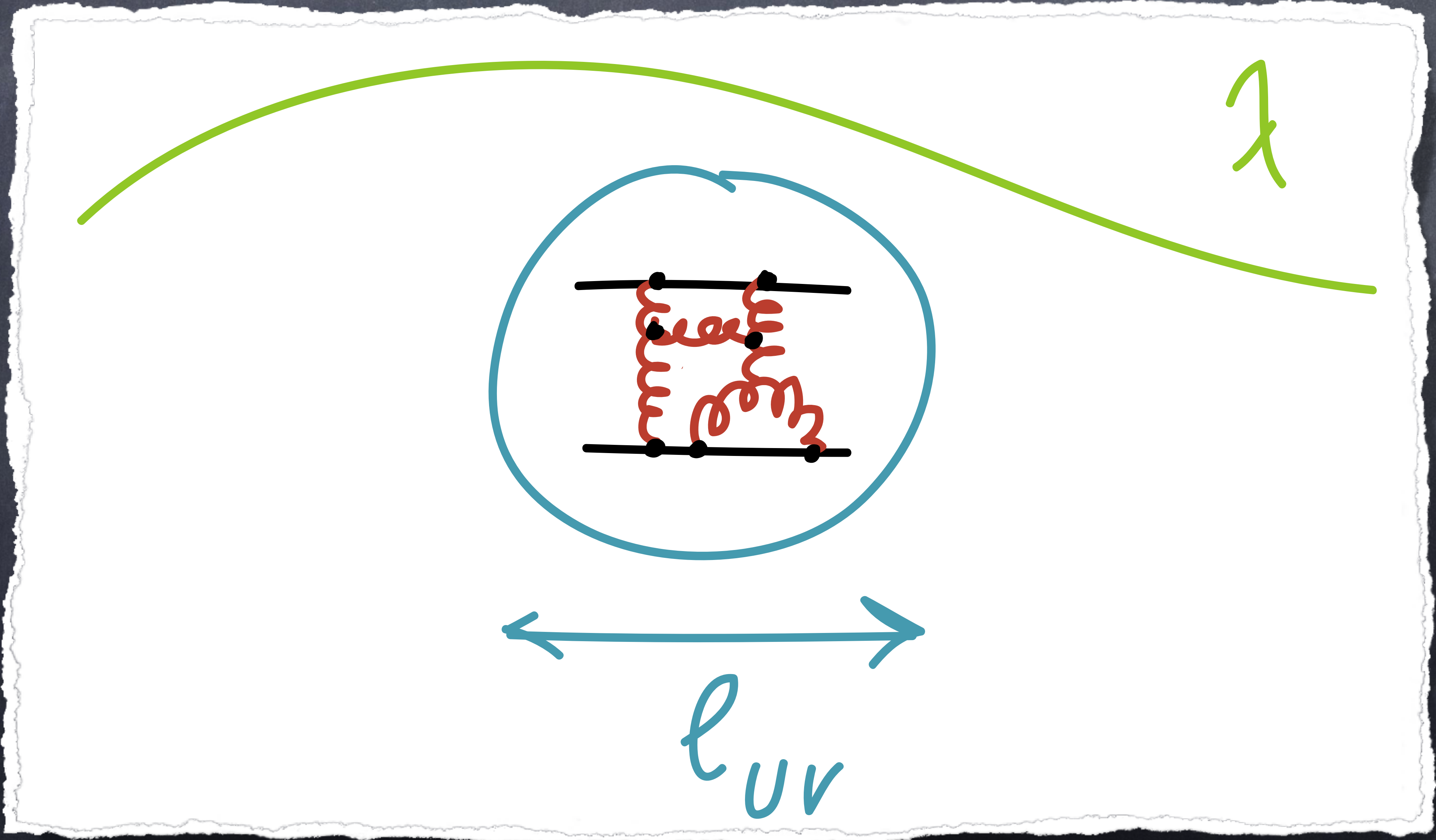
Gapless excitations

Gapless = { Zero energy (QM)
Zero frequency (classical) }

Dynamics of "Goldstones" constrained by symmetries

Effective field theory (EFT)

$$\lambda \gg \ell_{UV} \qquad \omega \ll \omega_{UV}$$



Low-energy degrees of freedom + symmetries:

$$\Phi \to G[\Phi]$$

Lagrangian:

$$\mathcal{L} = f(\Phi, \partial)$$

Derivative expansion:

$$(\ell_{UV} \cdot \vec{\nabla})^n \ , \qquad (1/\omega_{UV} \cdot \partial_t)^n$$

Low-energy degrees of freedom + symmetries:

$$\Phi \to G[\Phi]$$

Lagrangian:

$$\mathcal{L} = f(\Phi, \partial)$$

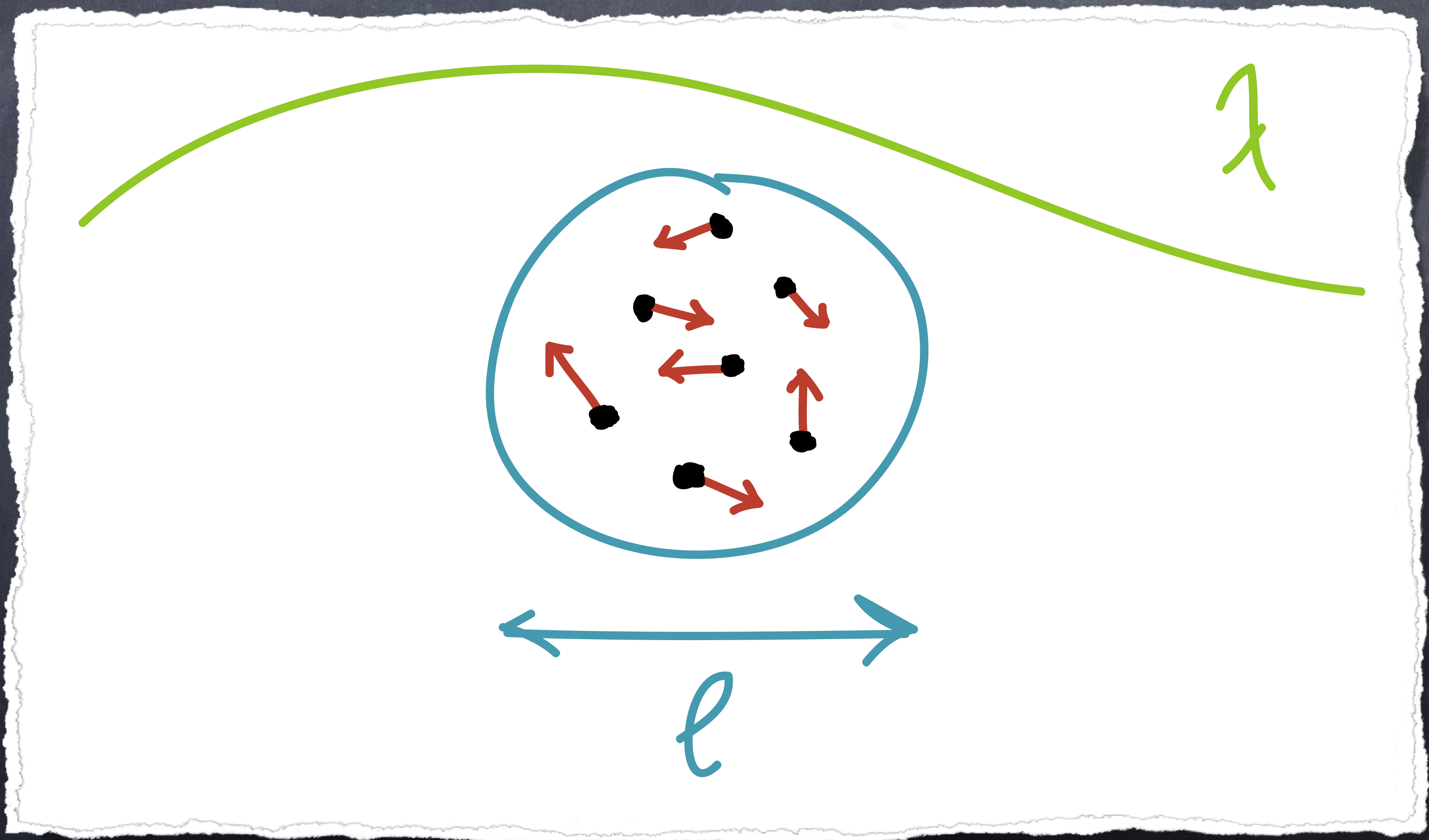
systematic

Derivative expansion:

$$(\ell_{UV} \cdot \vec{\nabla})^n \,, \qquad (1/\omega_{UV} \cdot \partial_t)^n$$

Example 1: Hydrodynamics

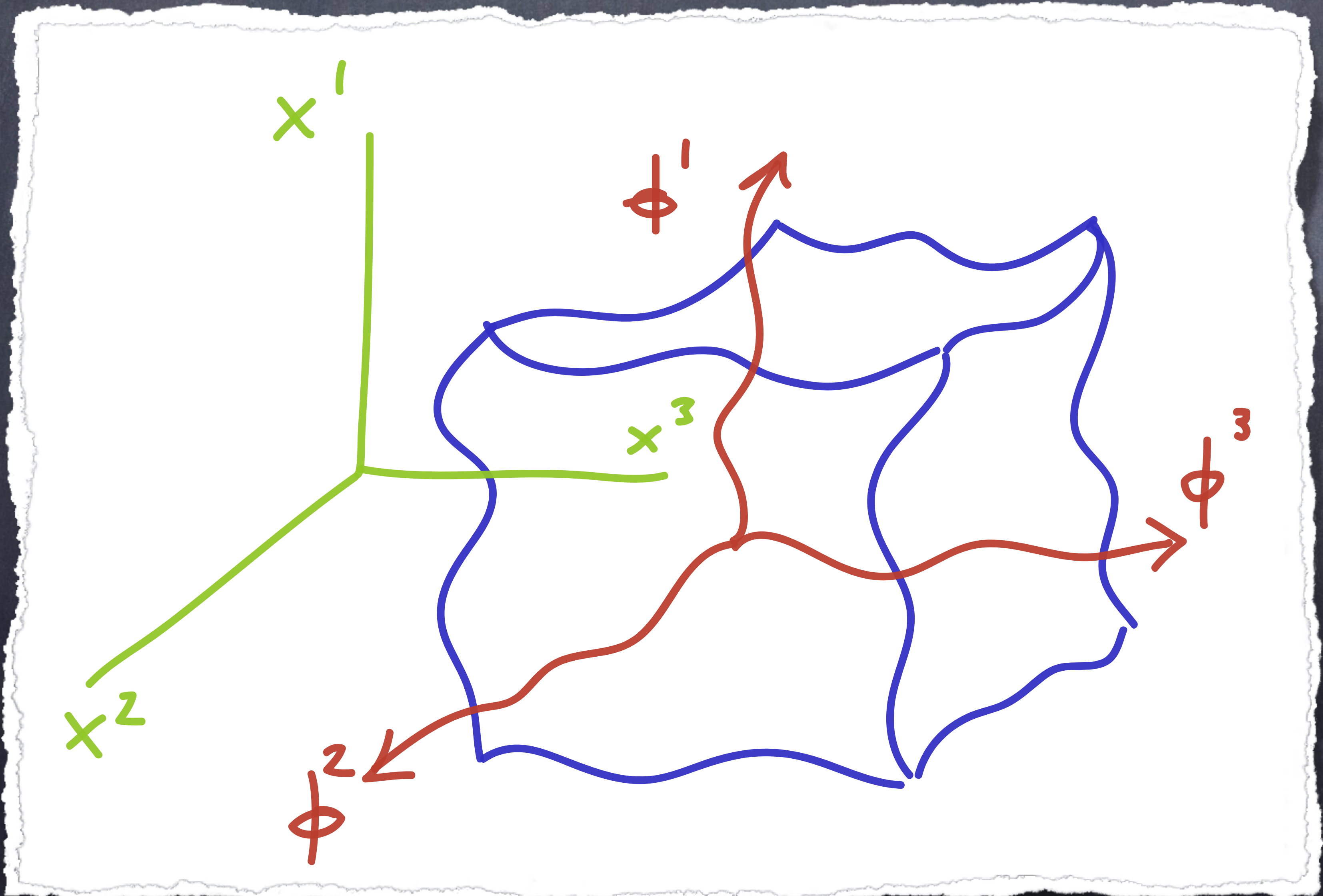
$\lambda \gg \ell$ $\omega \ll 1/\tau$



EFT for fluids

Dof: volume elements' positions

$$\phi^I(\vec{x},t) \qquad I = 1,2,3$$



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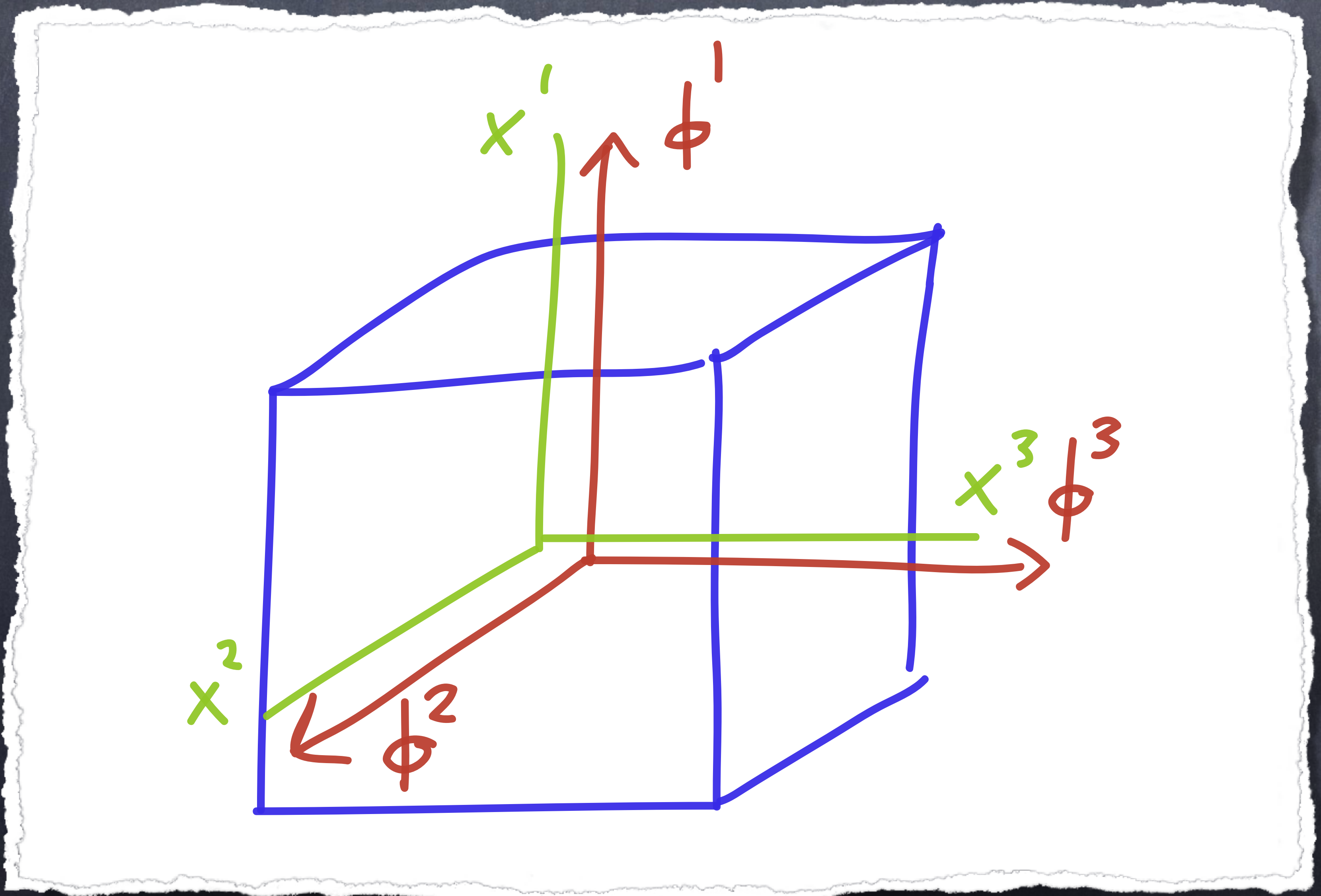


$$\langle \phi^I \rangle_{\rm eq} = x^I$$

Symmetries: Poincaré + internal

$$\left.\begin{array}{l}\phi^I \to \phi^I + a^I \\ \phi^I \to SO(3)\,\phi^I\end{array}\right\}$$ recover homogeneity/isotropy

$\langle \phi^I \rangle_{\rm eq} = x^I$ preserves diagonal combinations

$$\phi^I \to \xi^I(\phi) \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1$$ fluid vs solid

(Soper 1976)
(Dubovsky, Gregoire, Nicolis, Rattazzi 2005)

Action: $S = \int d^4x \, F(b)$ $b = \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J}$

Correct hydrodynamics ($T_{\mu\nu}$ + eom) with:

$$\begin{aligned} \rho &= -F \\ p &= F - F' \, b \\ u^\mu &= \frac{1}{6\,b} \, \epsilon\epsilon \, \partial\phi\partial\phi\partial\phi \end{aligned}$$

Relativistic, non-linear

ground state (at given p): $\phi^I = x^I$

Nambu-Goldstone modes: $\phi^I = x^I + \pi^I$

$$\mathcal{L} \to \left(\dot{\pi}^I\right)^2 - c_s^2\left(\partial_I \pi^I\right)^2 + \text{interactions}$$

longitudinal = sound $\omega = c_s k$

transverse = vortices $\omega = 0$

Application: Vortex-sound interactions

Subsonic regime ($v \ll c_s$)

Fluid nearly incompressible

sound waves difficult to excite

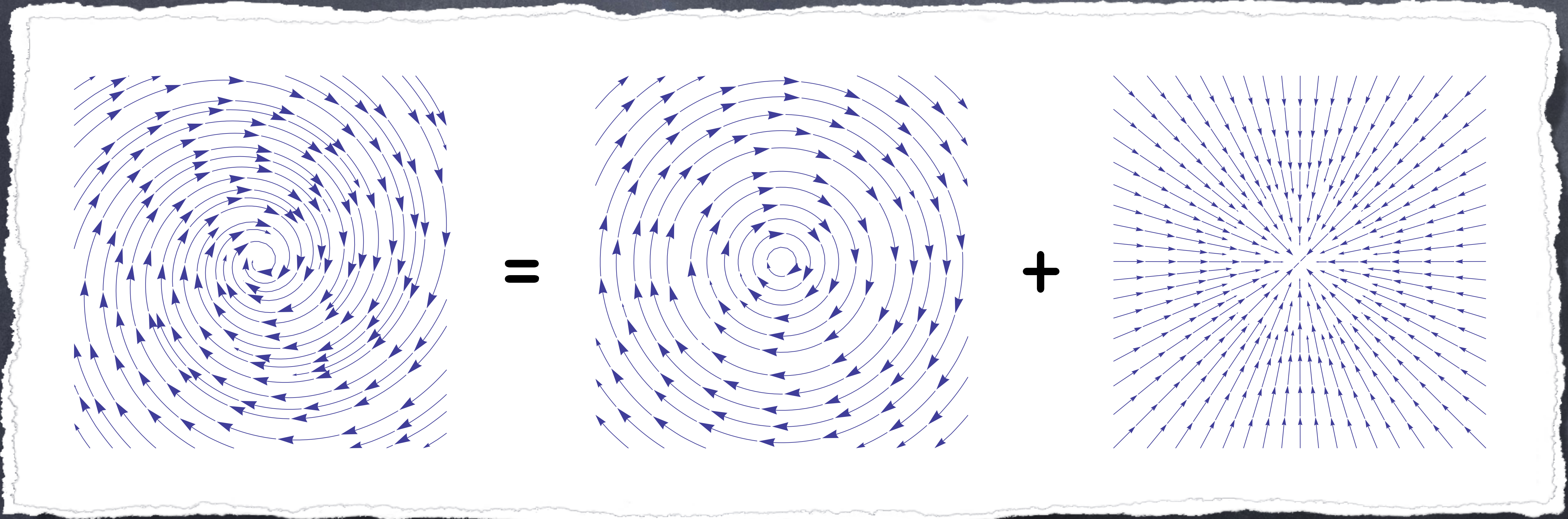
treat vortices non-linearly

treat sound perturbatively

integrate it out

(Endlich, Nicolis 2013)

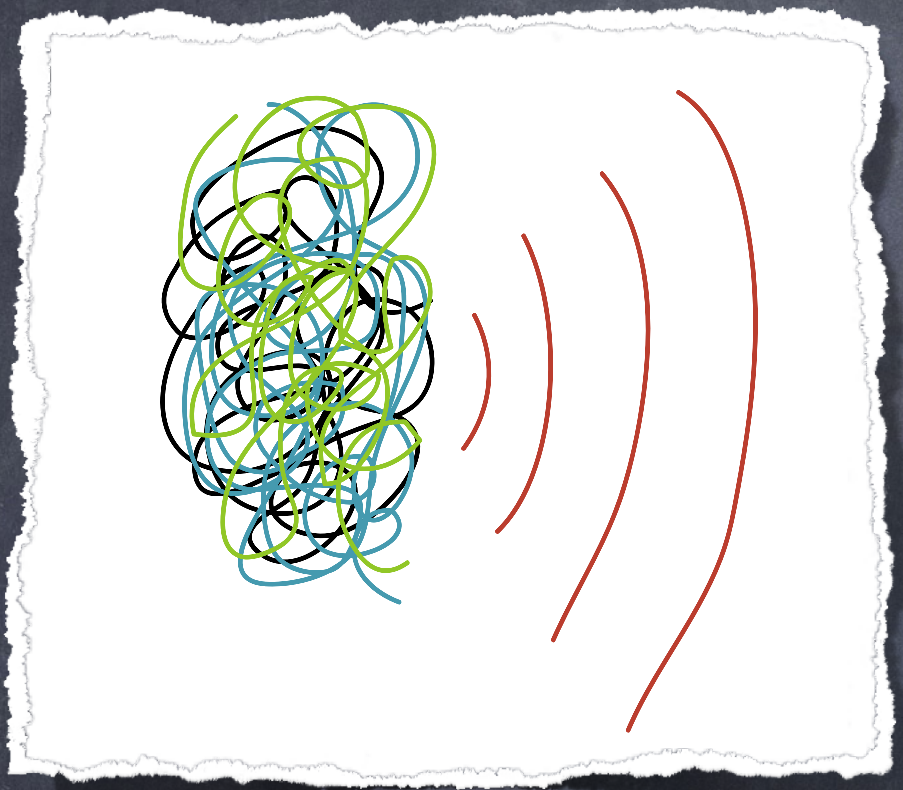
Vortex-sound decomposition

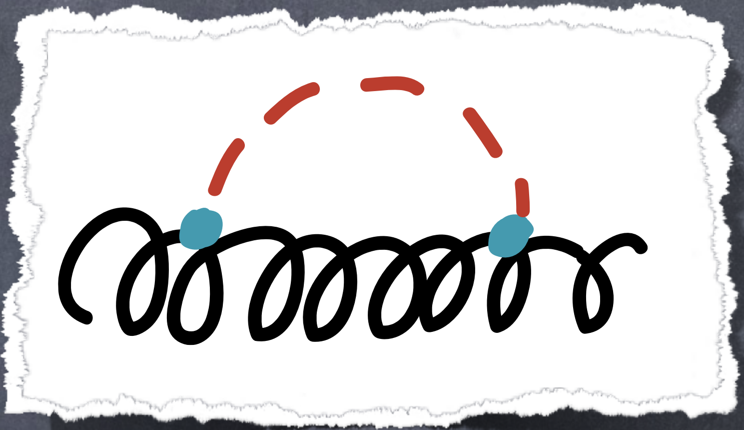


$$\phi^I(\vec{x},t) = \phi_0^I(\vec{x},t) + \underbrace{\delta\phi^I}_{\text{compression}}(\vec{x},t)$$

Expand the action in powers of $\delta\phi$ and v_0/c_s

The sound of turbulence



= Im ()

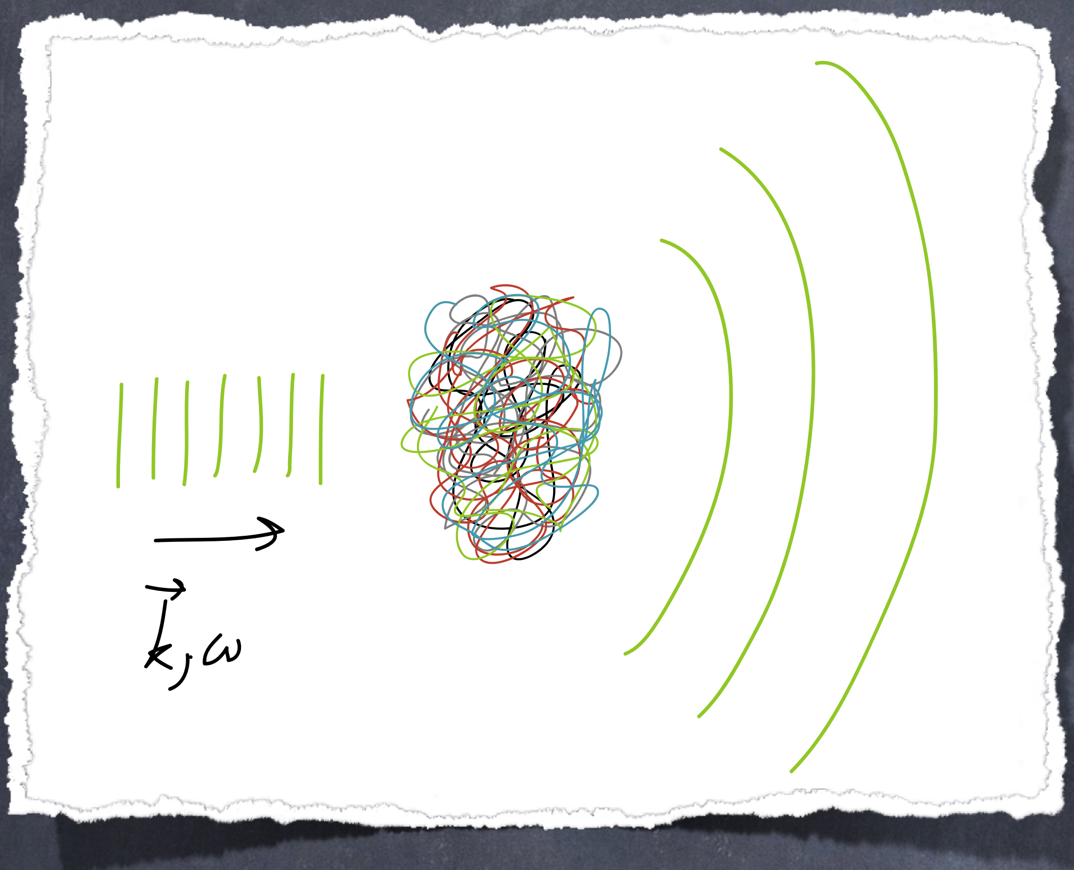
$$P = \frac{\rho + p}{c_s^5} \langle \ddot{Q} \ddot{Q} \rangle$$

$$Q_{ij} \equiv \int d^3x \left(v_i v_j - \frac{c_s^2}{c^2} v^2 \, \delta_{ij} \right)$$

(Lighthill 1954 + relativistic correction)

(similar to Goldberger, Rothstein 2004)

Probing turbulence with sound waves



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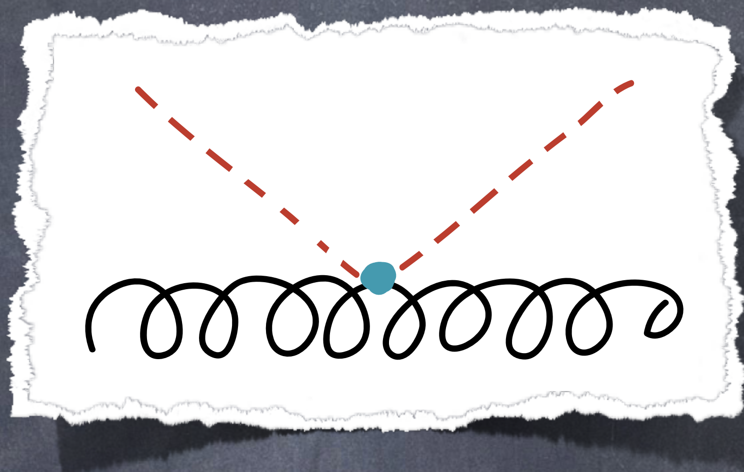
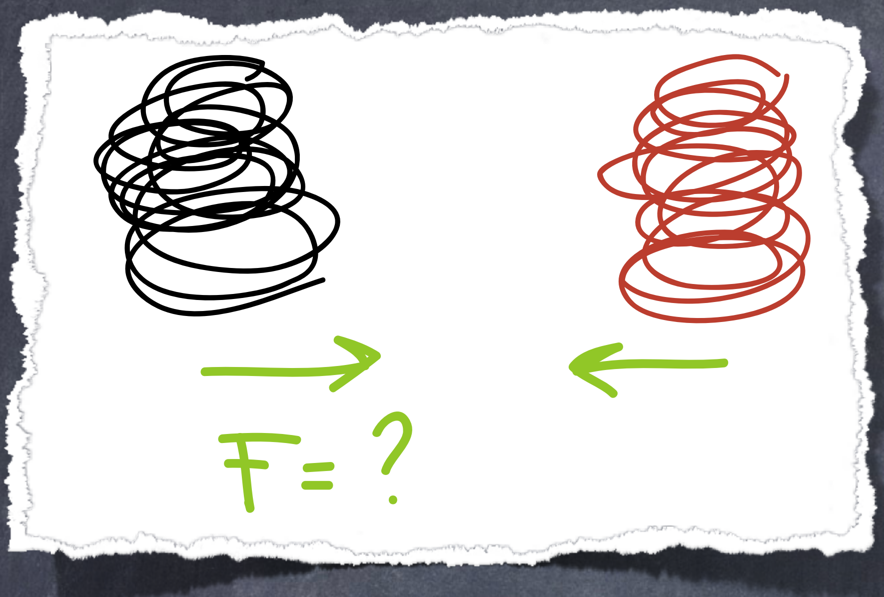


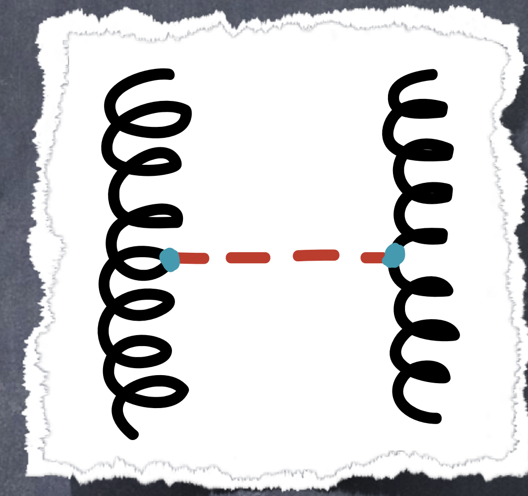
$$\frac{d\sigma}{d\Omega} = \frac{\omega^4}{c_s^6}\left[1 - \frac{c_s^2}{c^2} + \frac{c_s^4}{c^4}\right]\left|\tilde{v}(\Delta\vec{k})\right|^2$$

(Lund, Rojas 1989 + relativistic correction)

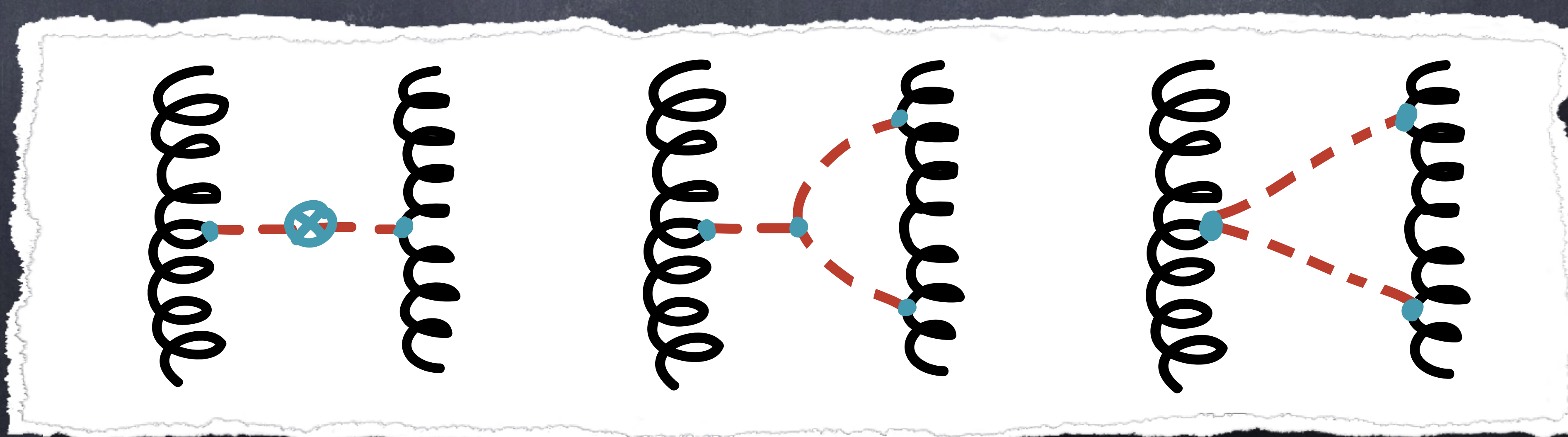
Sound mediated vortex-vortex potential



Leading order



Next to leading order



Long range potential:

$$V \sim \frac{(\rho + p)}{c_s^2} \cdot \frac{q_1 q_2}{r^3} \sim E_{\rm kin} \big(v/c_s\big)^2 (\ell/r)^3$$

$$q \equiv \int_{\rm vortex} d^3x \; v^2$$

Useful? Detectable? Known?

Long range potential:

$$V \sim \frac{(\rho + p)}{c_s^2} \cdot \frac{q_1 q_2}{r^3} \sim E_{\text{kin}} (v/c_s)^2 (\ell/r)^3$$

$$q \equiv \int_{\text{vortex}} d^3x \; v^2$$

Useful? Detectable? Known?

? ? No

(William Irvine, U. of Chicago)

Example 2: String theory for superfluids

Superfluid: same classical eom as fluids, but:

- Spontaneously broken Q ⟹ light Goldstone $\phi(x)$
- Finite density for Q ⟹ $\phi(x) = \mu t + \pi(x)$

  phonon
- Symmetries: Poincaré + shift $\phi(x) \to \phi(x) + a$
- Bulk action: $S = \int d^4x\, P(X) + \ldots$ $\qquad X \equiv (\partial_\mu \phi)^2$

  eq. of state

(Son 2005)

zero T super-fluid vs. ordinary fluid

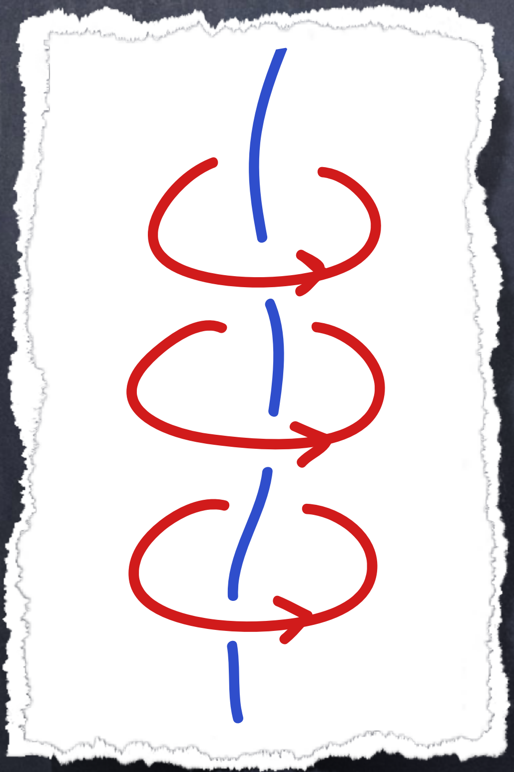
compressional (sound) sector

- Hydrodynamics
- Hydrodynamics

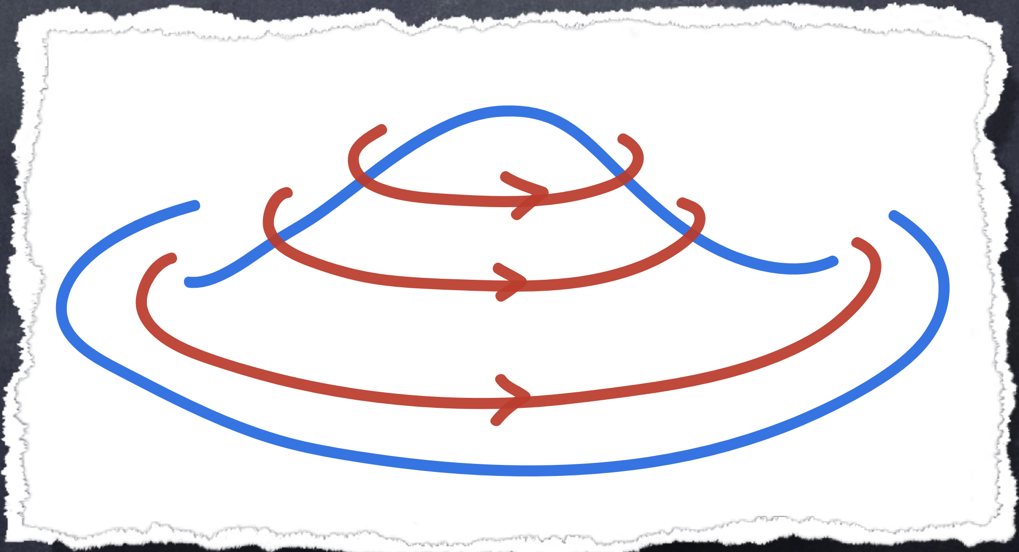
transverse (vortex) sector

- Hard (gapped)
- Soft (gapless)

$$\vec{\nabla} \times \vec{v} = 0$$



$$\vec{\nabla} \times \vec{v} \neq 0$$



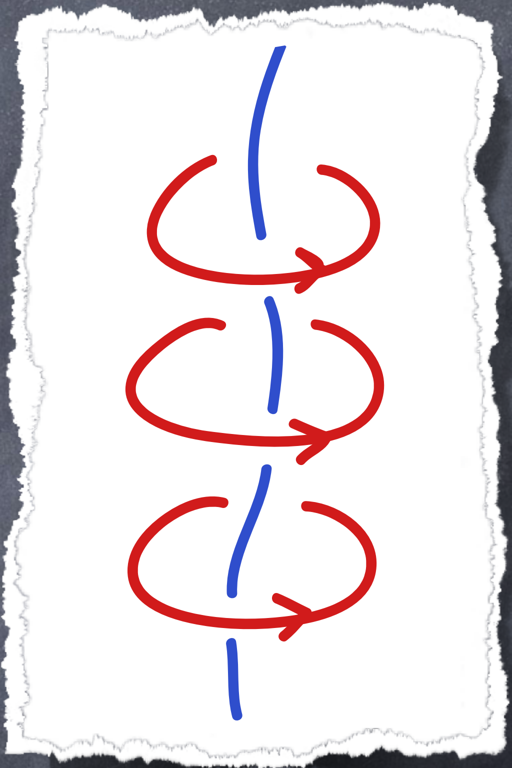
Vortex dynamics (incompressible limit)

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad\qquad \vec{\omega} = \vec{\nabla} \times \vec{v}$$

For vortex lines

$$\Gamma = \oint \vec{v} \cdot d\vec{\ell} \quad \leftrightarrow \quad I$$

$$\vec{v} \quad \leftrightarrow \quad \vec{B}$$



Biot-Savart:

$$\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi} \int \frac{(\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3} \times d\vec{x}\,'$$

1st order EOM!

Unlike $m\vec{a} = \vec{F}_{\rm ext}$

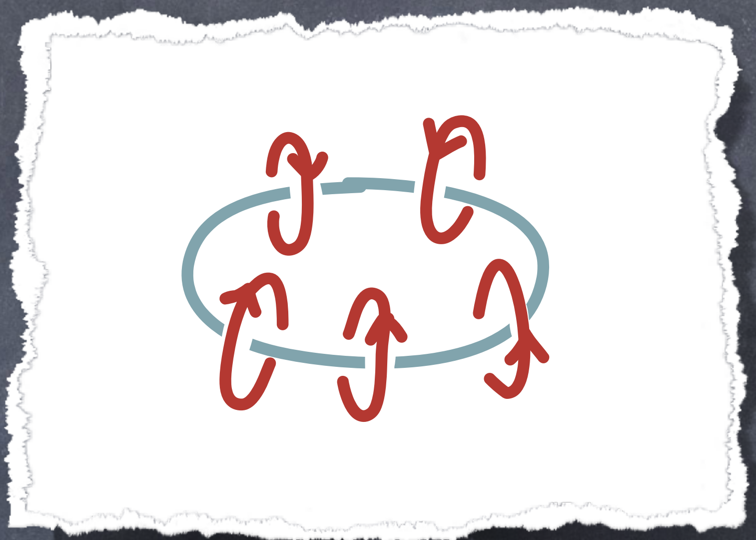
⇨ No room for “forces”

⇨ Instantaneous v determined by geometry

⇨ No free initial condition for v

For vortex rings

$$\vec{v} = \frac{\Gamma}{4\pi R} \log(R/a)\, \hat{n}$$



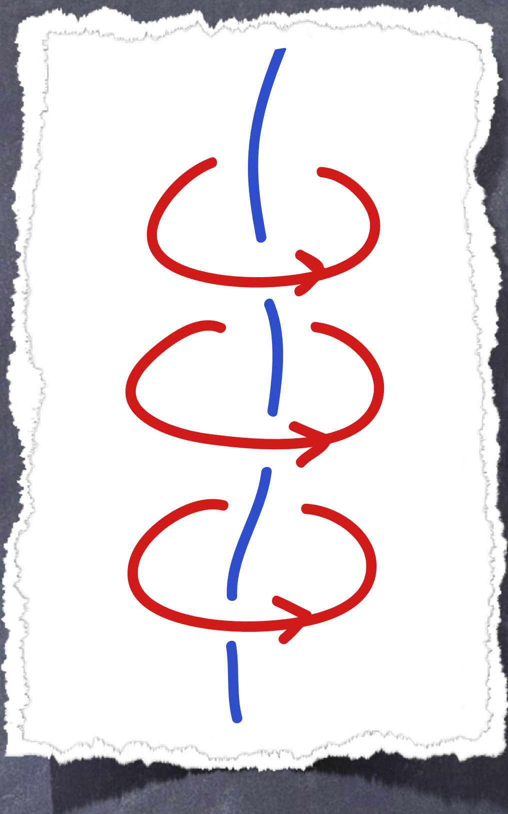
Far away:

$$\vec{v}(\vec{x}) = \vec{B}_{\mathrm{dipole}} \qquad \text{with} \qquad \vec{\mu} = (\pi R^2)\Gamma\, \hat{n}$$

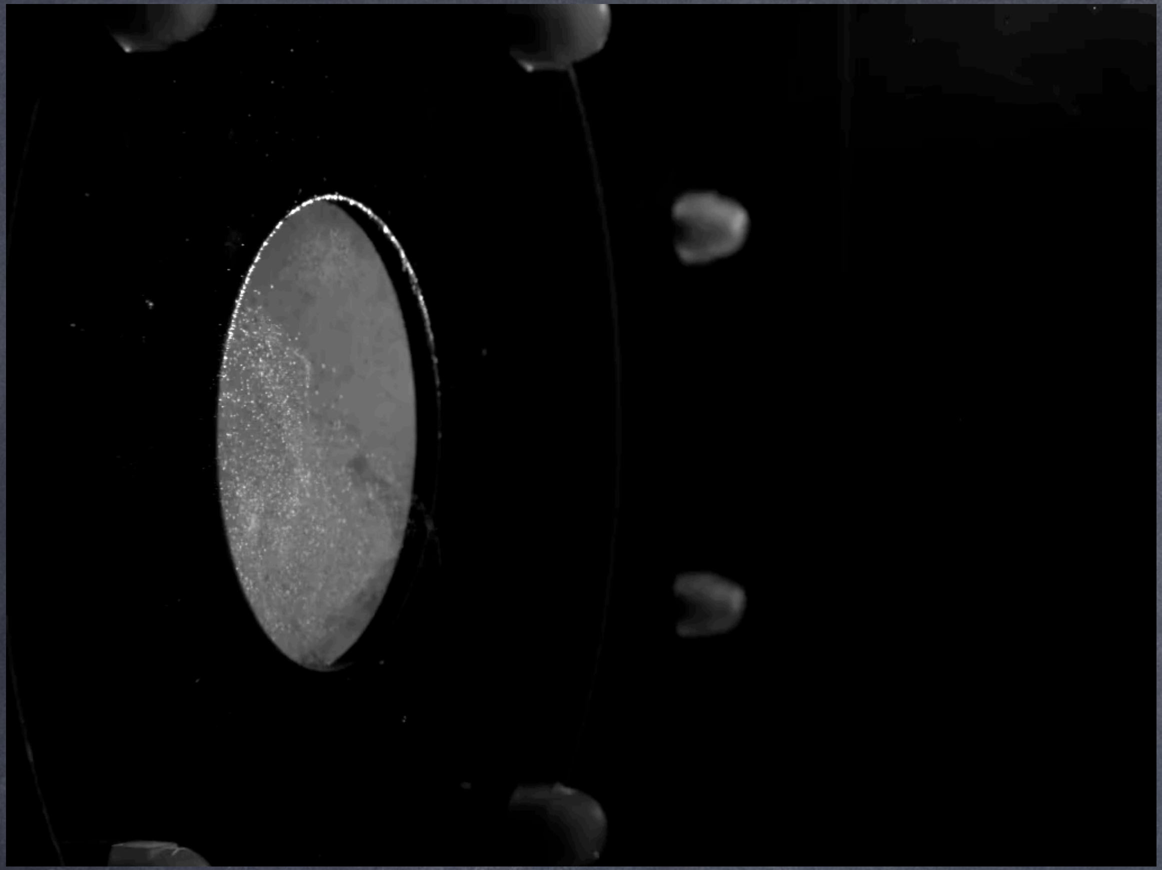
Excitations: Kelvin waves

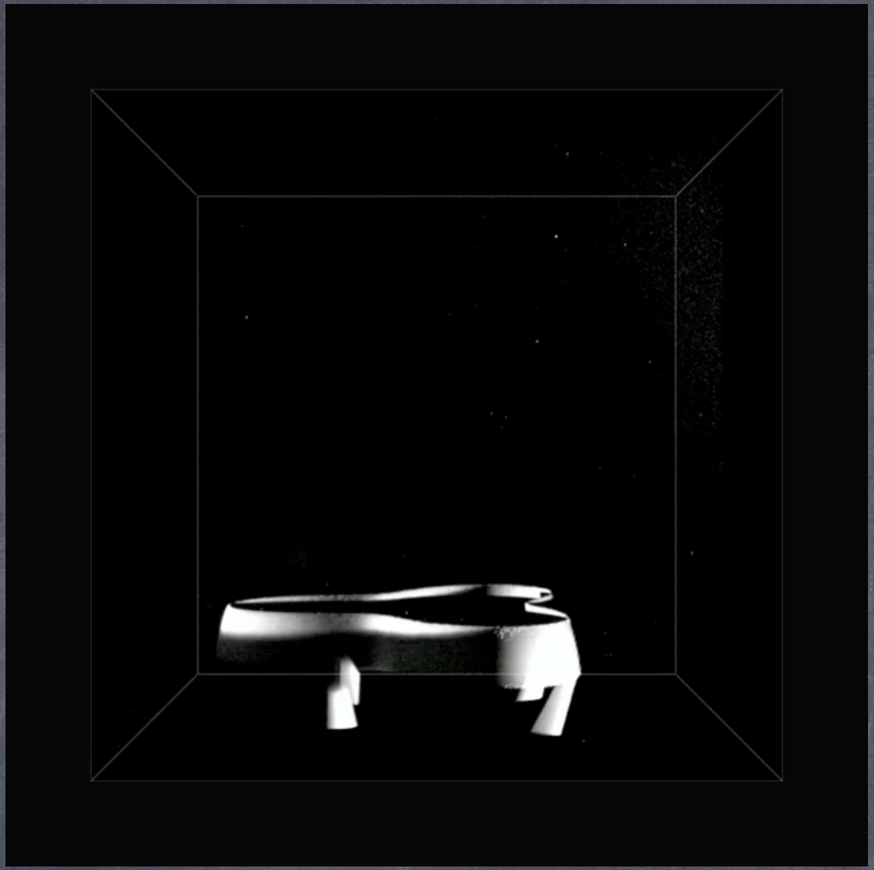
Two modes overall ($\neq 2+2$)

$$\omega_{\pm} = \frac{\Gamma}{2\pi} k^2 \log(1/ka)$$



- fewer modes than 2-derivative eom
- ``non-local" dispersion relation





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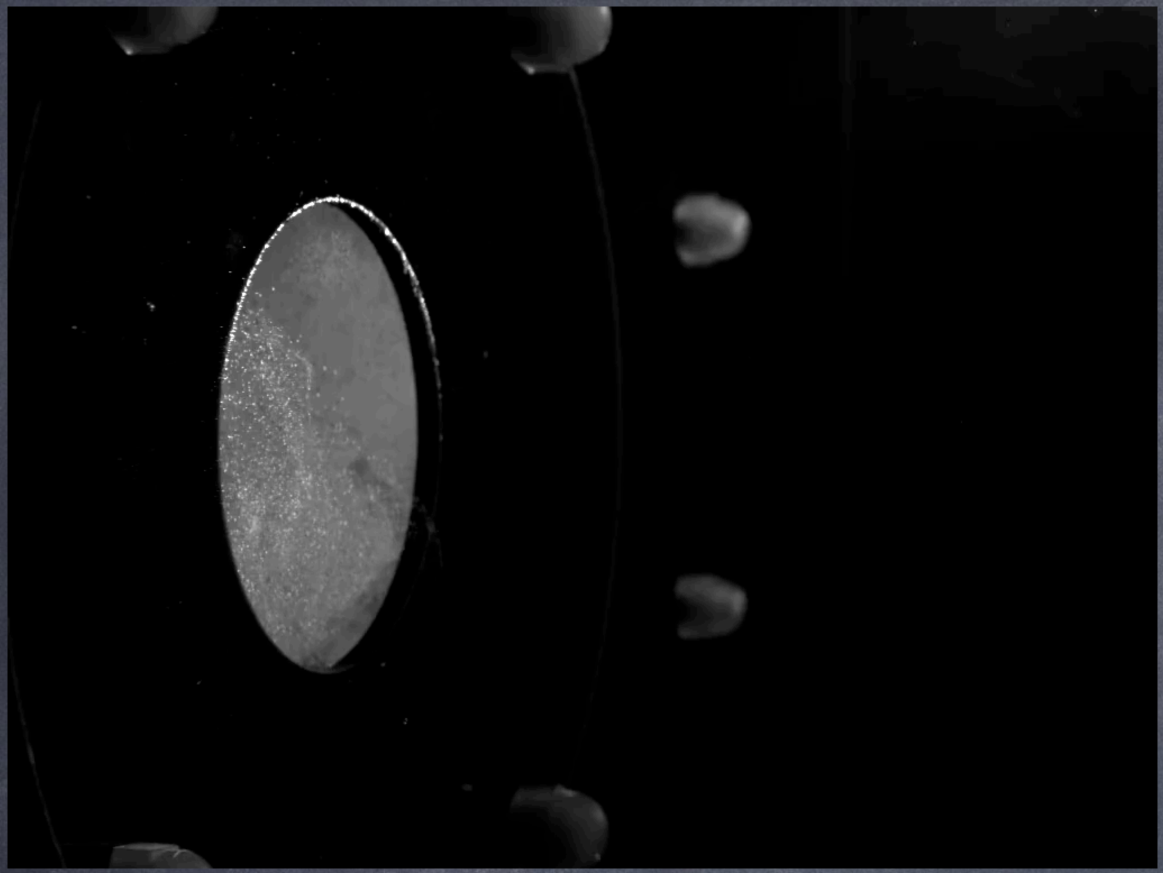
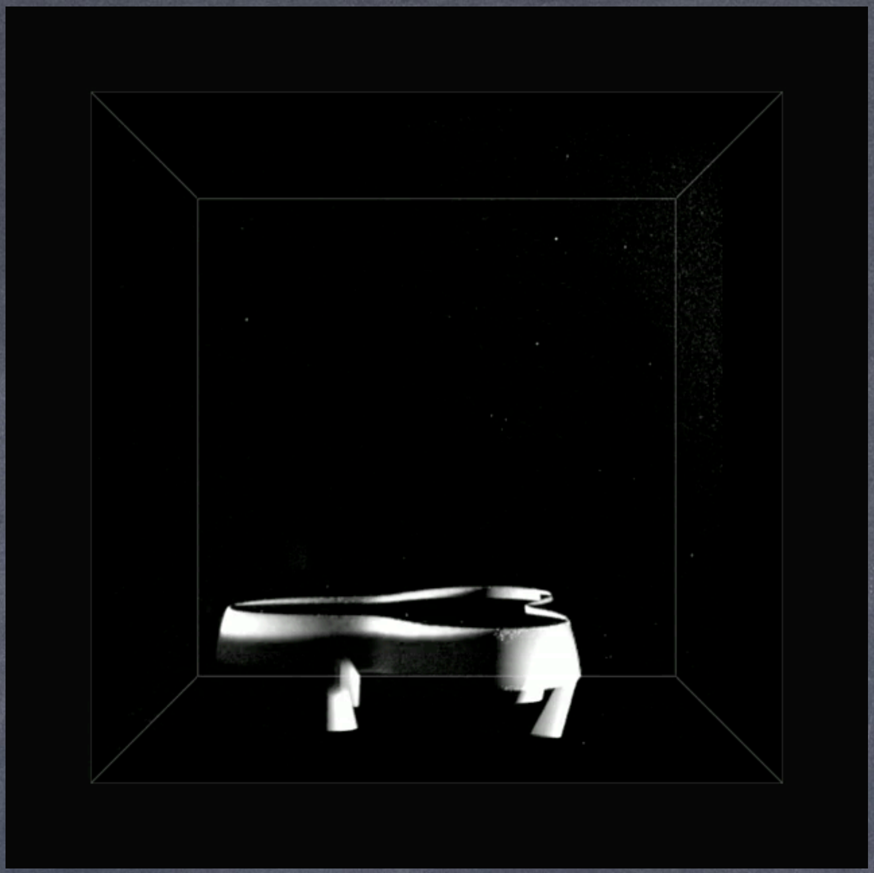

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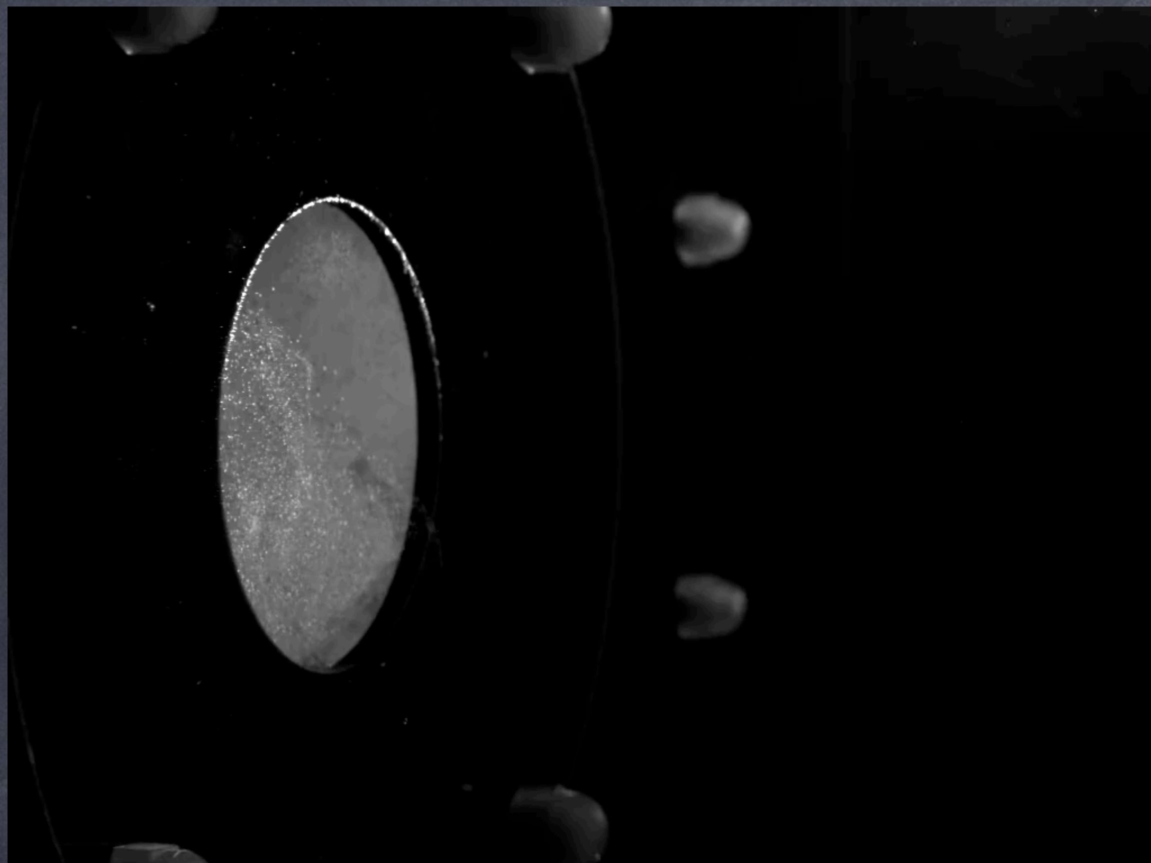

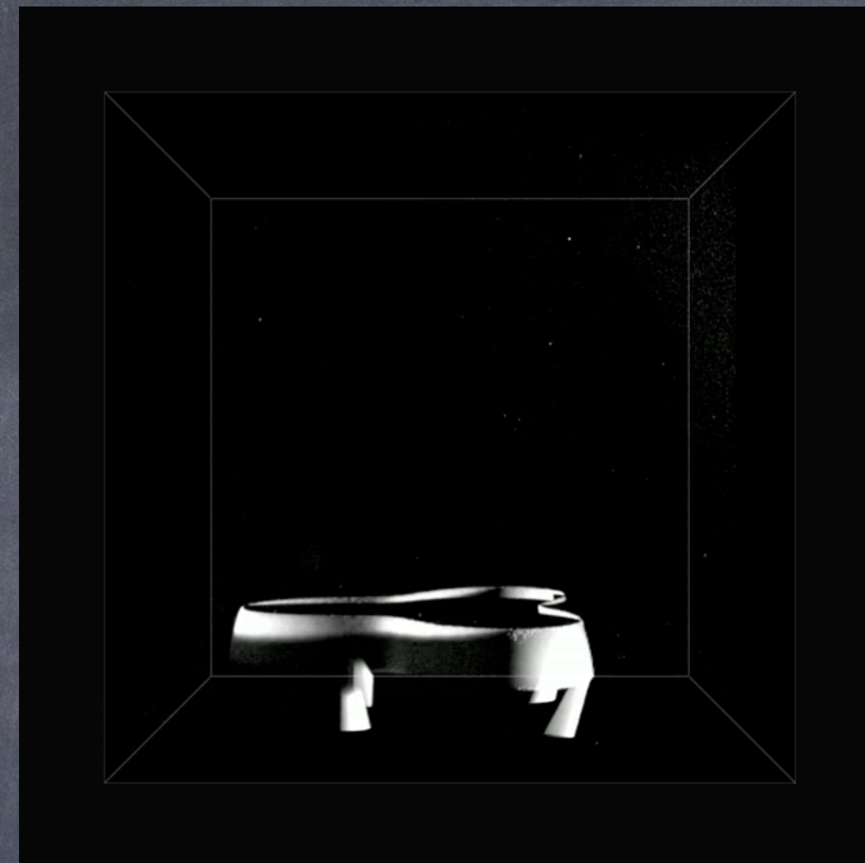

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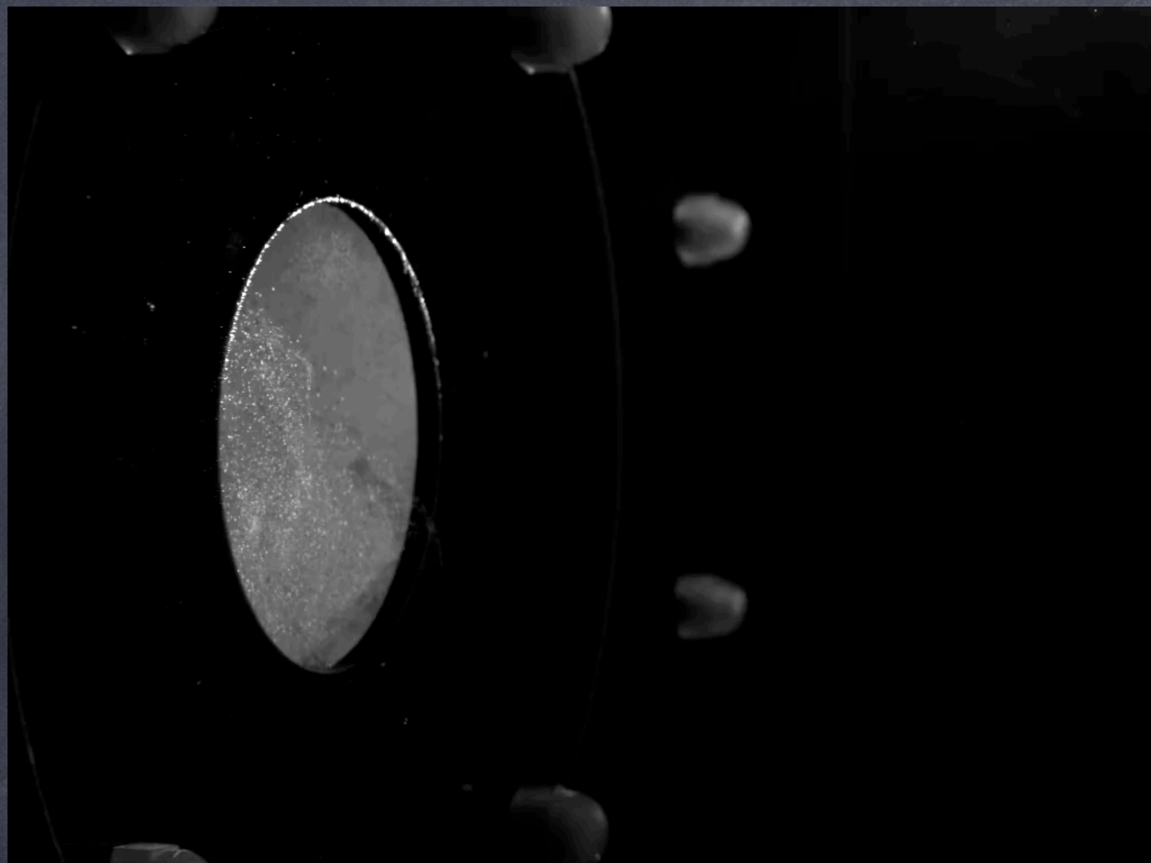

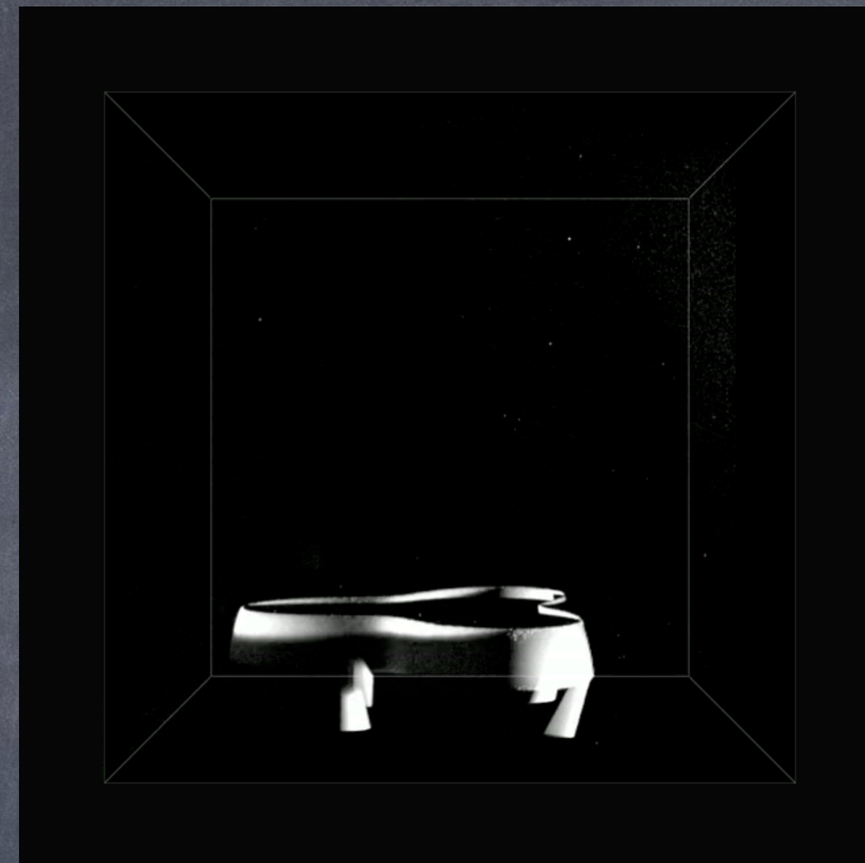

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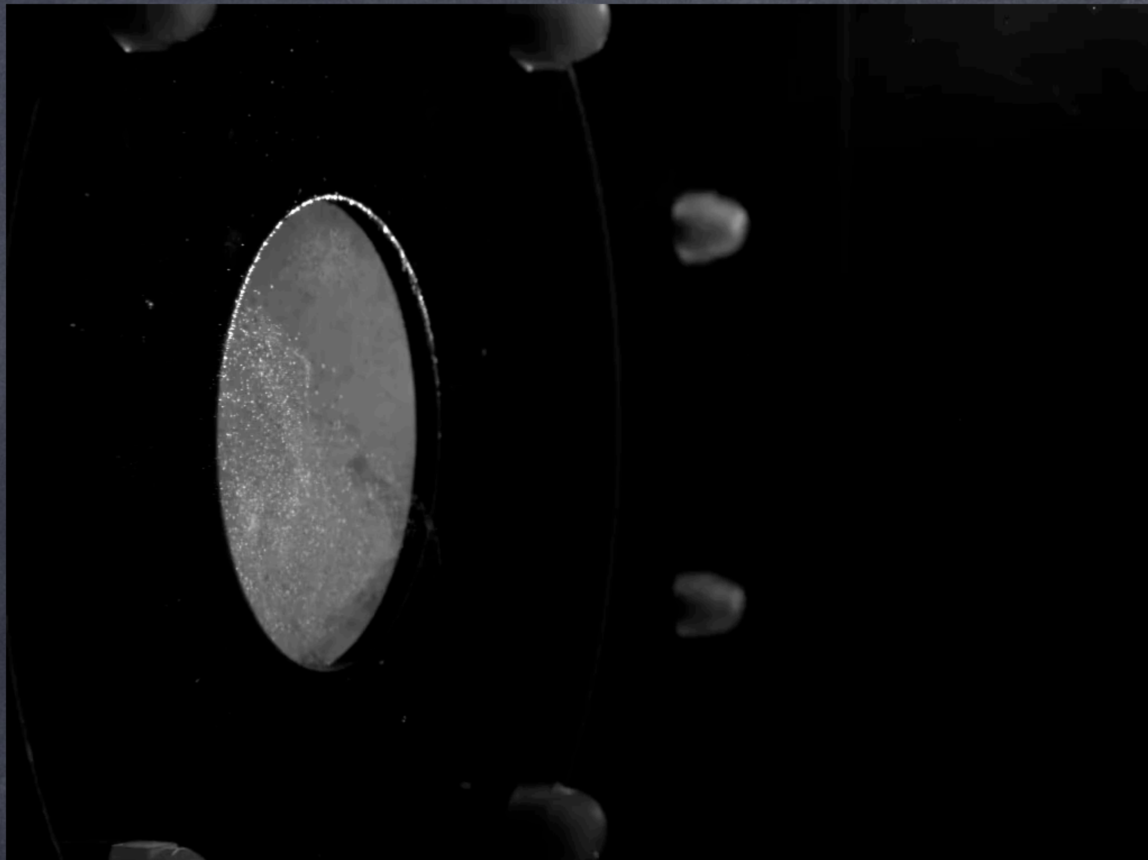

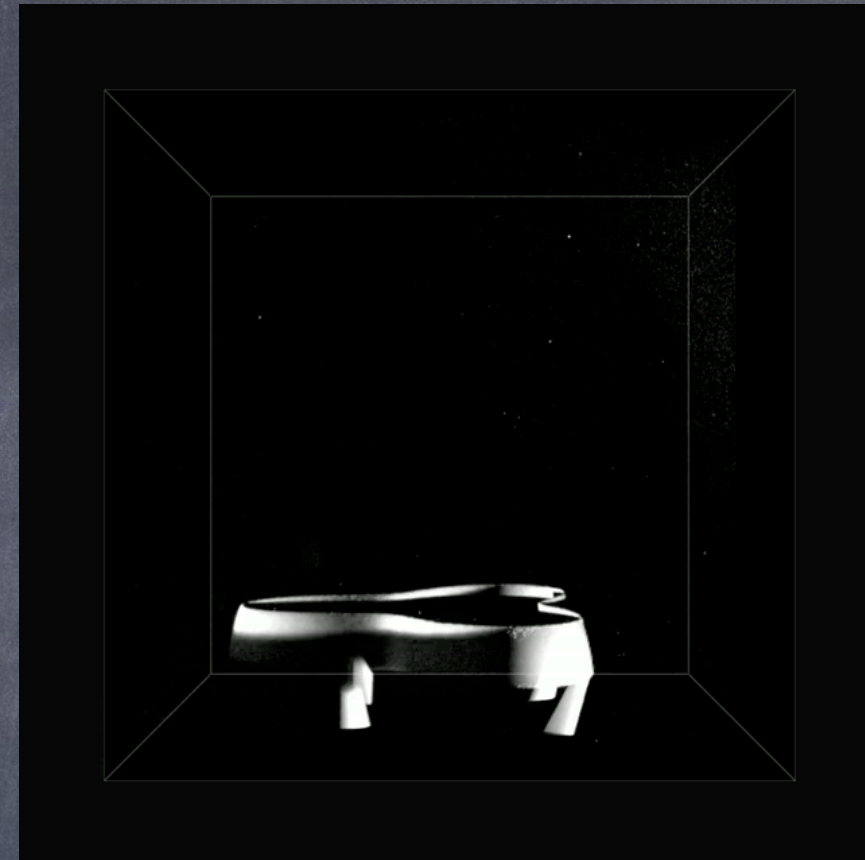

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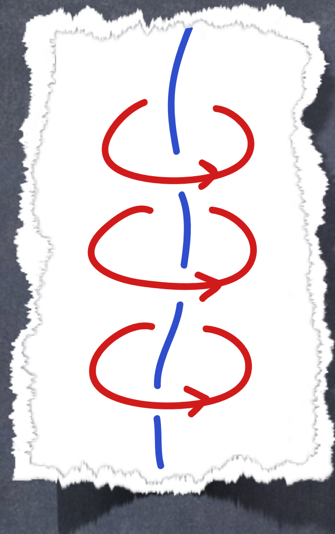
other groups...

other groups...

Coupling with bulk modes

- Problem:



$\phi \to \phi + 2\pi$ ("defect")

- Solution: dual 2-form $\partial_{[\mu}\mathcal{A}_{\nu\rho]} \propto \epsilon_{\mu\nu\rho\alpha}\partial^\alpha\phi$

- Analog of dual EM field for magnetic monopoles

- Equivalent bulk dynamics: $S = \int d^4x\, \rho(Y) + \ldots$

eq. of state

$$Y \equiv (\partial_{[\mu}\mathcal{A}_{\nu\rho]})^2$$

Effective string theory

Bulk: $\mathcal{A}_{[\mu\nu]}$ $\qquad$ $\mathcal{A}_{\mu\nu} \to \mathcal{A}_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

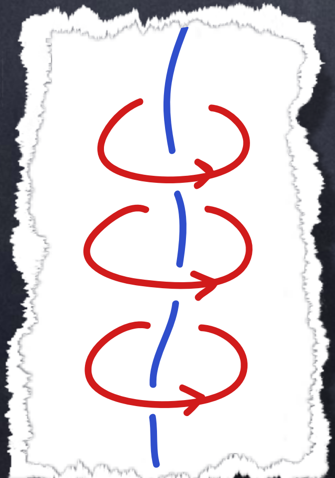
analog of Coulomb V

phonon

$$\mathcal{A}_{00} = 0$$
$$\mathcal{A}_{0i} = n\, A_i(x) \qquad \vec{\nabla}\cdot\vec{A} = 0$$
$$\mathcal{A}_{ij} = n\, \epsilon_{ijk}(x^k + B^k(x)) \qquad \vec{\nabla}\times\vec{B} = 0$$

String: $X^\mu(\tau, \sigma)$ Lorentz 4-vector

world-sheet coordinates



Action

$$S = S_{\rm bulk} + S_{\rm KR} + \ldots$$

Bulk: $\int d^4x\, \rho(Y) + \ldots$ $\qquad Y \equiv (\partial_{[\mu}\mathcal{A}_{\nu\rho]})^2$

Kalb-Ramond: $\lambda \int d\sigma d\tau\, \mathcal{A}_{\mu\nu}\, \partial_\sigma X^\mu \partial_\tau X^\nu$

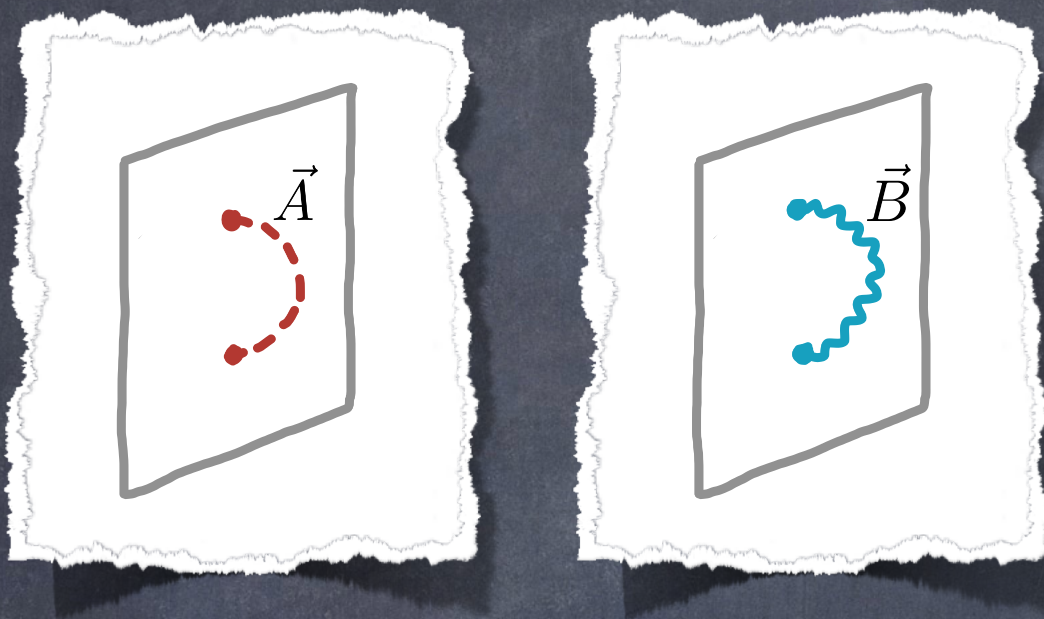
(analog of $q \int A_\mu dx^\mu$)

Perturbation theory: $\mathcal{A}_{\mu\nu} \to \text{background} + \vec{A}(x), \vec{B}(x)$

$$X^\mu \to \text{background} + \vec{\pi}(\tau, \sigma)$$

(Horn, Nicolis, Penco 2015)

Energy of straight string



$$\frac{dE}{dz} = \frac{n^2\lambda^2}{w^2}\log(L\cdot\Lambda)$$

container size

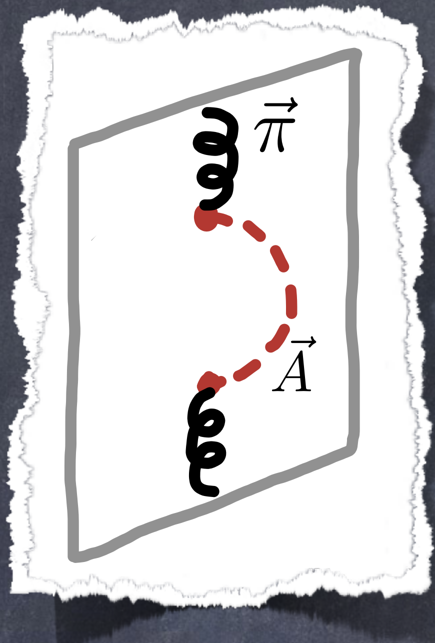
UV cutoff

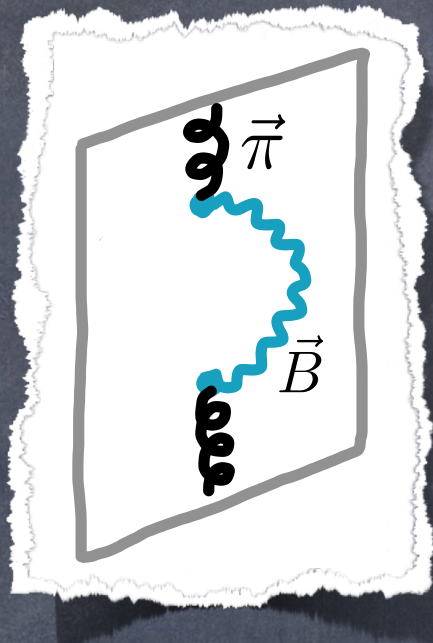
Add ``tension": $S \to S - T\int d\sigma d\tau\,\sqrt{\det\partial_\alpha X^\mu \partial_\beta X_\mu}$

RG running: $\frac{d}{d\log\mu}T(\mu) = -\frac{1}{4\pi}\frac{n^2\lambda^2}{w}$

momentum scale

(nonlinear) Kelvin waves





one time derivative

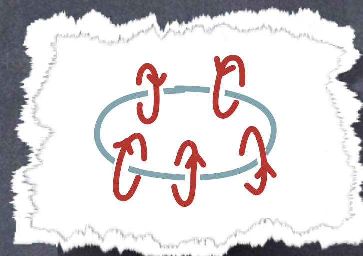
typical momentum

$$S_{\text{eff}}[\vec{\pi}] = \int dt dz \left[n\lambda\, \epsilon_{ab} \pi^a \partial_t \pi^b - T(k)\sqrt{1 + (\partial_z \vec{\pi})^2} \right]$$

⇨ $$\omega_{\pm} = \frac{\Gamma}{4\pi} \log(\mu_0/k)\, k^2$$ (linear)

⇨ "self-pipe" with $\vec{v} = \hat{z}\frac{\Gamma k}{2\pi}$ (nonlinear)

Mechanics of system of vortex rings



$$\vec{\mu} = \Gamma\, \pi R^2 \hat{n}$$

$$\mathcal{L} = \sum_n \left[\vec{\mu}_n \cdot \dot{\vec{x}}_n + \vec{\mu}_n \cdot (\vec{\nabla} \times \vec{A})\right] - \int d^3x \,(\partial_i A_j)^2$$

$$\to \sum_n \left(\vec{\mu}_n \cdot \dot{\vec{x}}_n - \mu_n^{3/2} \log \mu_n\right) - \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Peculiar conservation laws:

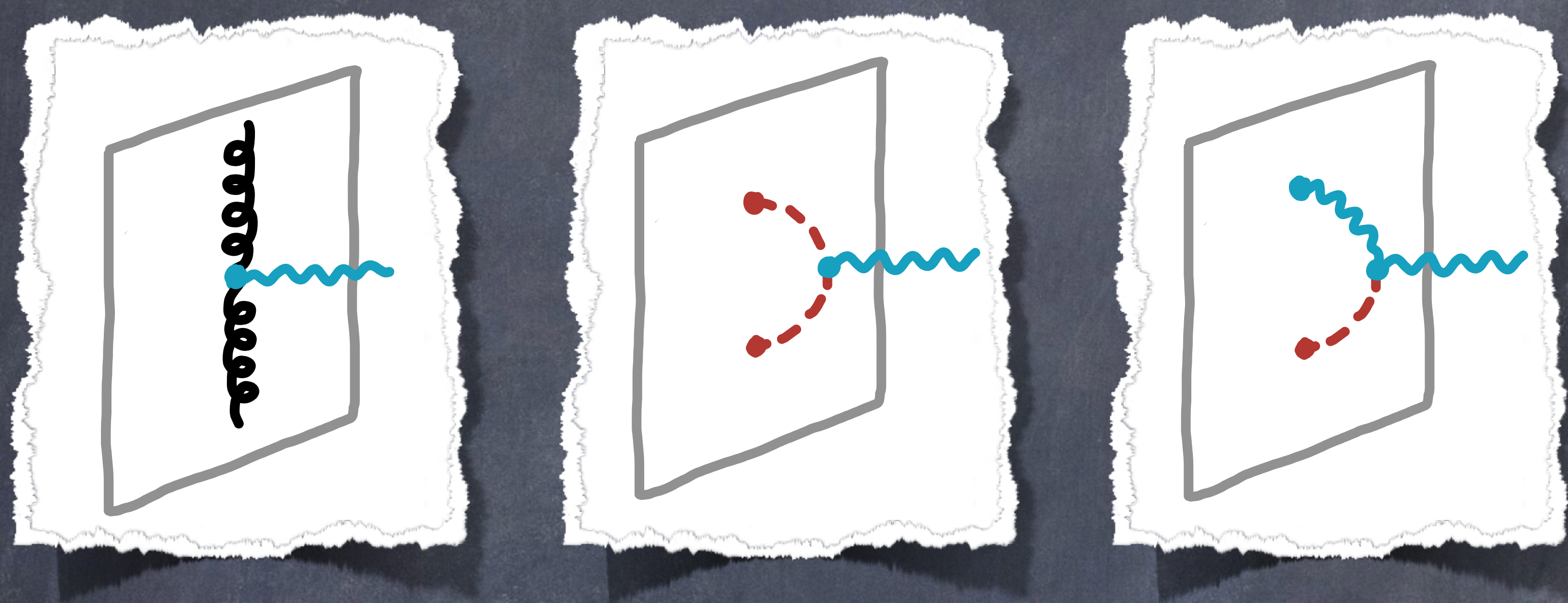
$$\vec{P} = \sum_n \vec{\mu}_n$$

$$\vec{L} = \sum_n \vec{x}_n \times \vec{\mu}_n$$

$$E = \sum_n \mu_n^{3/2} \log \mu_n + \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

(Garcia-Saenz, Mitsou, Nicolis 2018)

Interactions of string modes with sound



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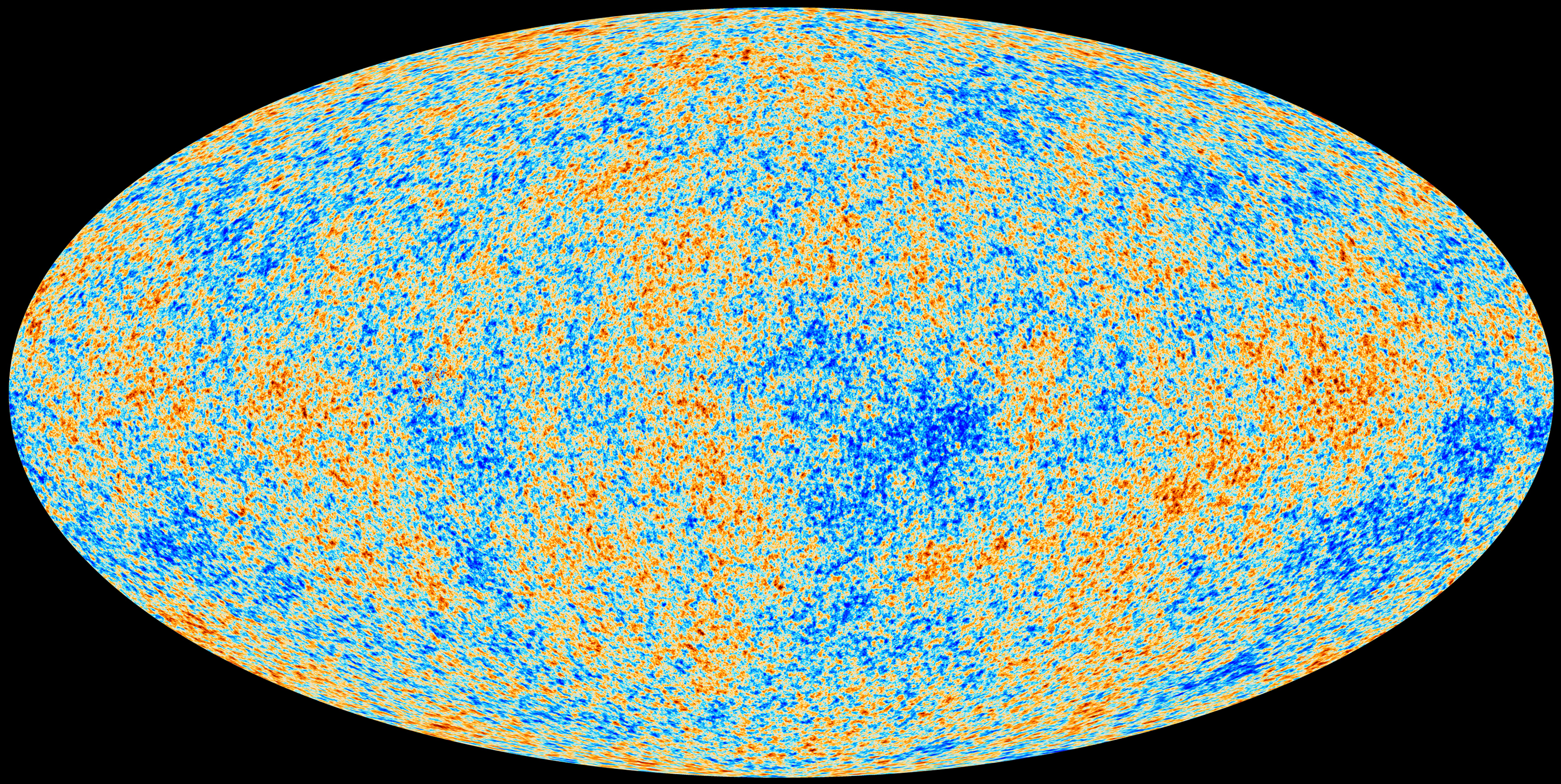
Phonon —> kelvons conversion:

$$\sigma = \frac{1}{16\, w c_s^3}\, \omega^2 q\, \sin^4\theta \sim \sqrt{\omega^5/\log\omega}$$

Example 3: early universe cosmology



$$\delta T/T \sim 10^{-5}$$

Inflation

- The early universe: homogeneous and isotropic

- Usually modeled via $\varphi_a = \varphi_a(t)$

- Time-translations spontaneously broken

  ⇒ Goldstone = adiabatic perturbations (cf. superfluid)

(Creminelli, Luty, Nicolis, Senatore 2006)

- Systematic effective field theory:

$$\langle \delta T(\vec{x}_1) \delta T(\vec{x}_2) \rangle \,, \qquad \langle \delta T(\vec{x}_1) \delta T(\vec{x}_2) \delta T(\vec{x}_3) \rangle \,, \qquad \ldots$$

(Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007)

Alternative: Solid inflation

- t-independent, x-dependent fields: $\varphi_a = \varphi_a(\vec{x})$
- time-translations unbroken
- spatial translations and rotations, broken

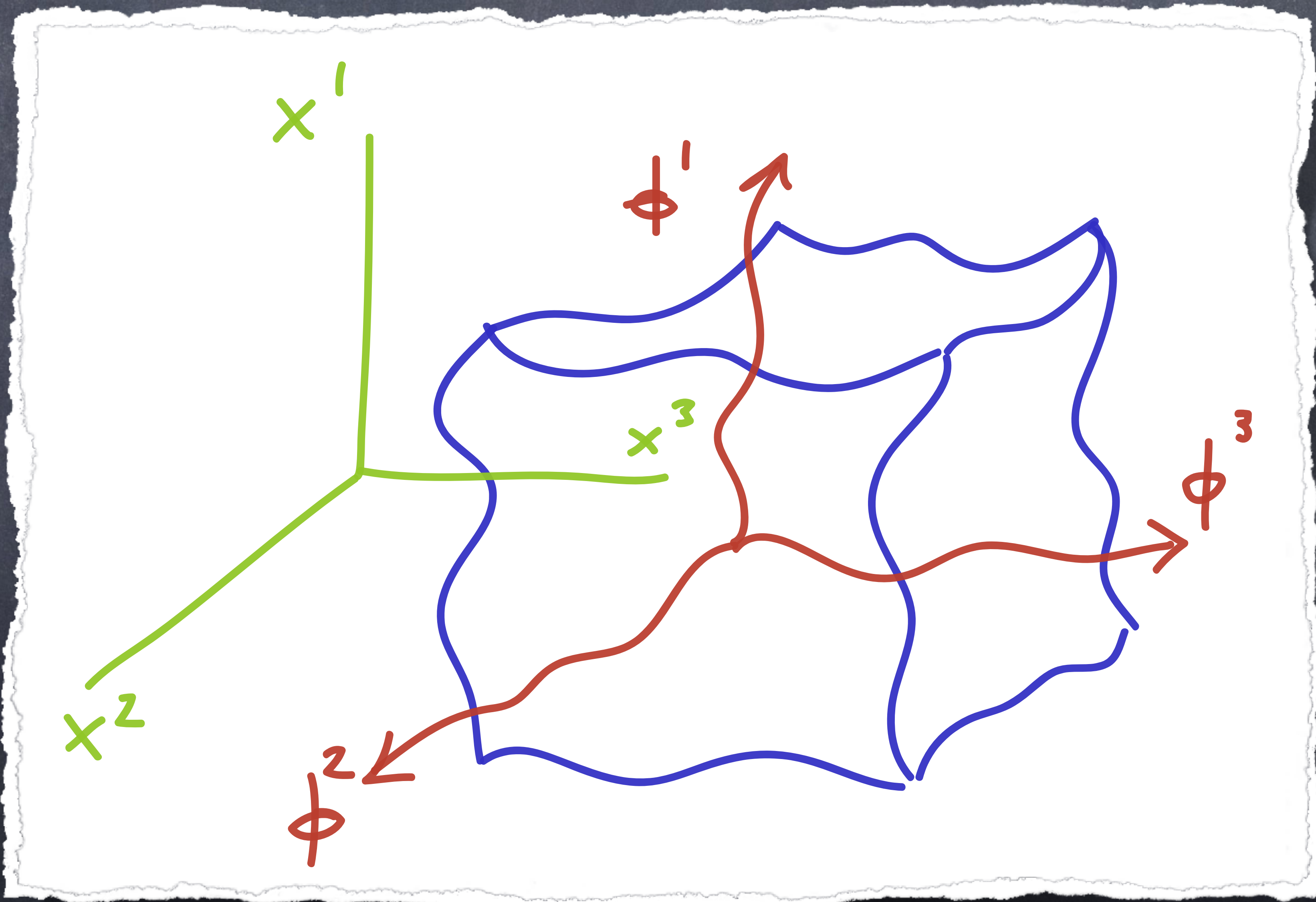
⇨ use EFT for homogeneous, isotropic solid

(Endlich, Nicolis, Wang 2012)

EFT for solids

Dof: volume elements' positions

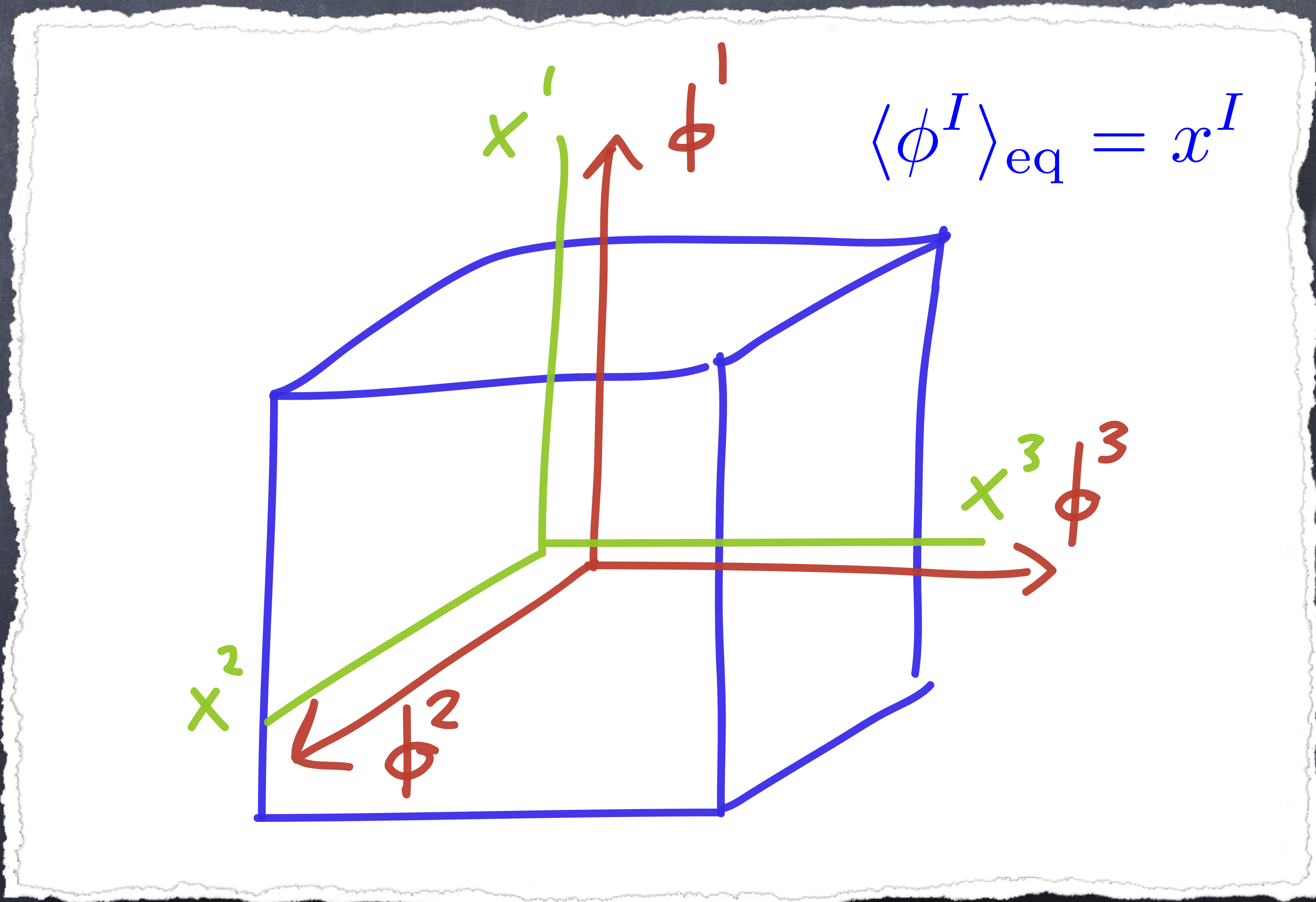
$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$



EFT for solids

Dof: volume elements' positions

$$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$$



Cosmological solid

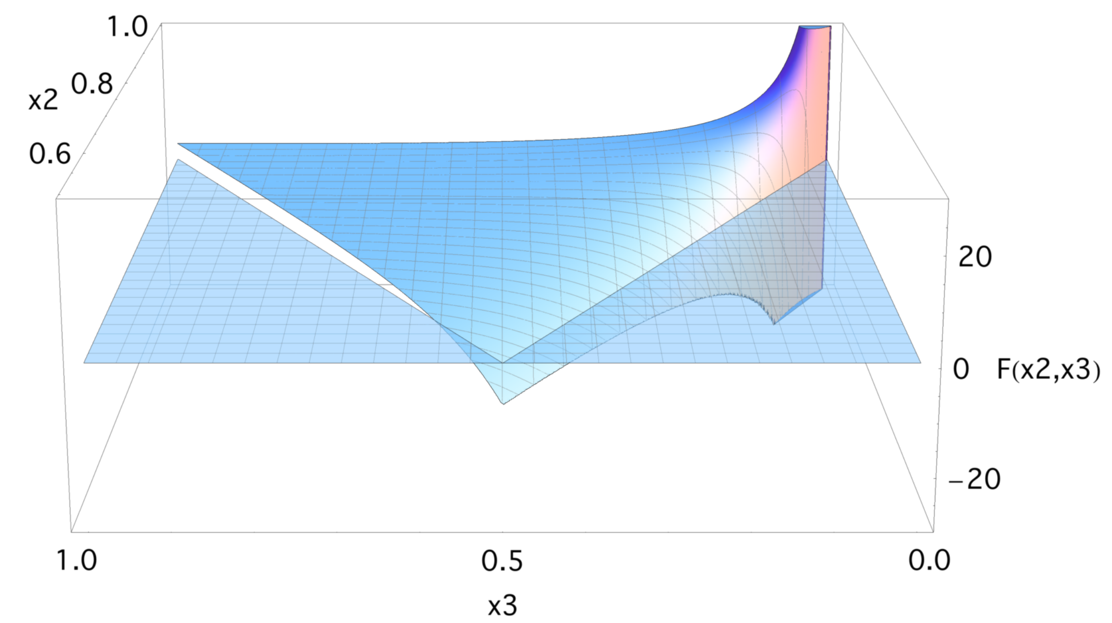
- Three scalar fields: $\phi^I = x^I + \pi^I$
- Internal symmetries: $\phi^I \to \phi^I + a^I$

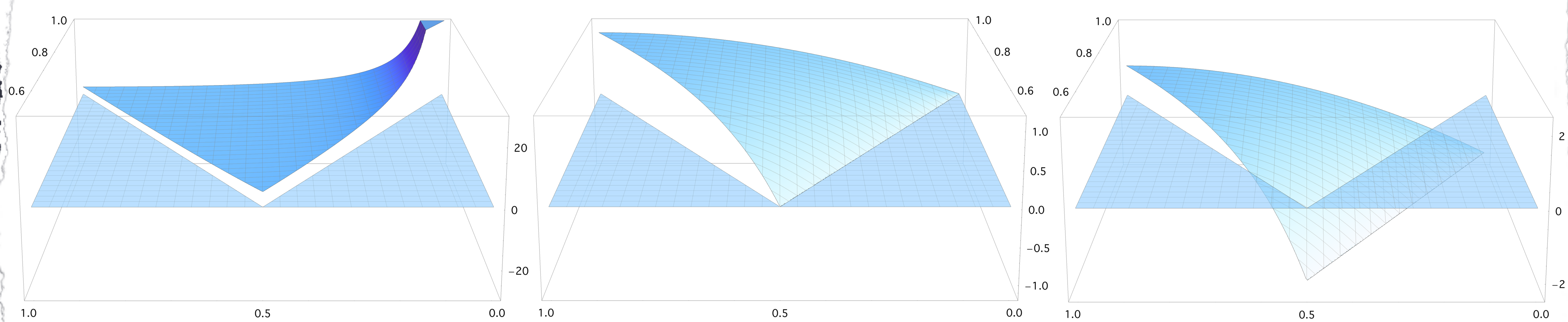
$$\phi^I \to SO(3)\,\phi^I$$

- Action:

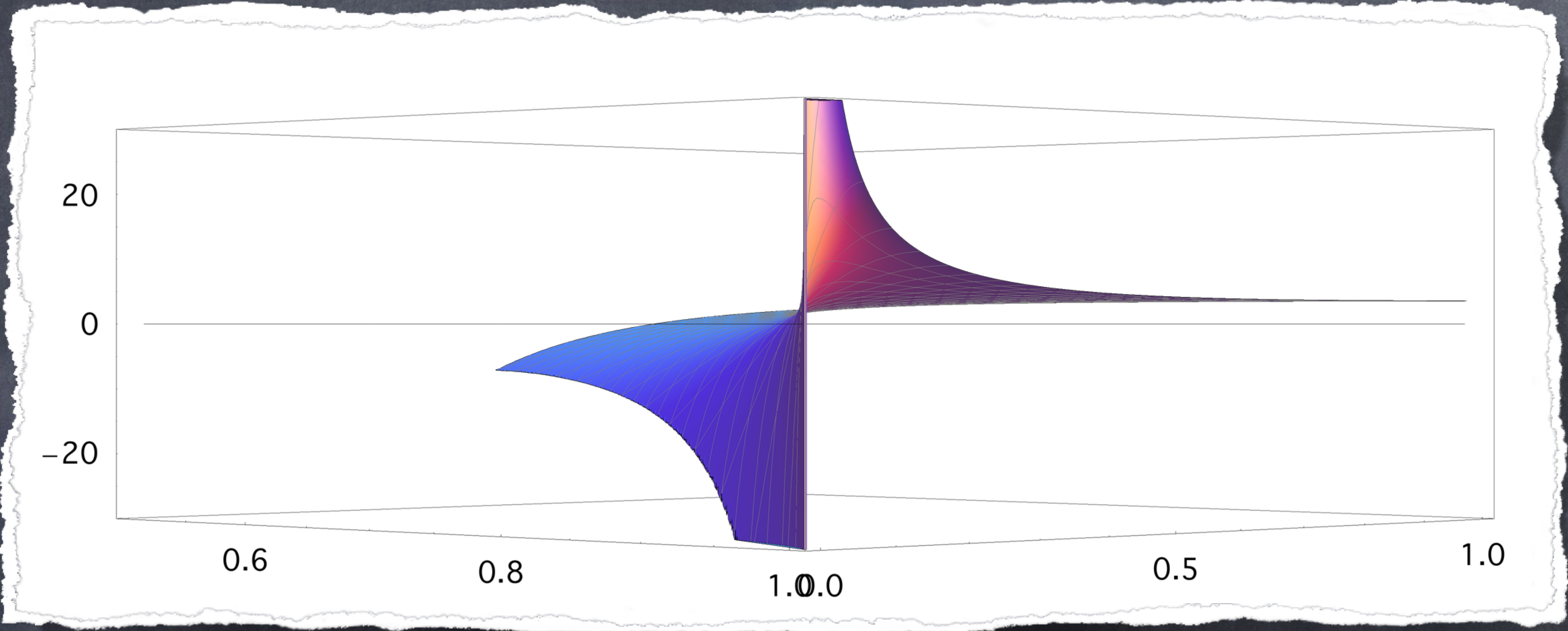
$$S = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G} R + F(\partial_\mu \phi^I \partial^\mu \phi^J) \right]$$

New Goldstone EFT w/ novel predictions





Quadrupolar "squeezed limit"



$$\langle\zeta\zeta\zeta\rangle \to f_{NL} \times \langle\zeta\zeta\rangle\langle\zeta\zeta\rangle \times (1 - 3\cos^2\theta)$$

$$f_{NL} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}$$

2% overlap w/ "local" shape

39% w/ "equilateral"

32% w/ "orthogonal"

Conclusions

- Spontaneously broken symmetries, Goldstone modes, effective field theories:

  Extremely general concepts
  and powerful tools

- Many potential applications in condensed matter, hydrodynamics, and cosmology