Closure Tests of Transverse Momentum Distributions

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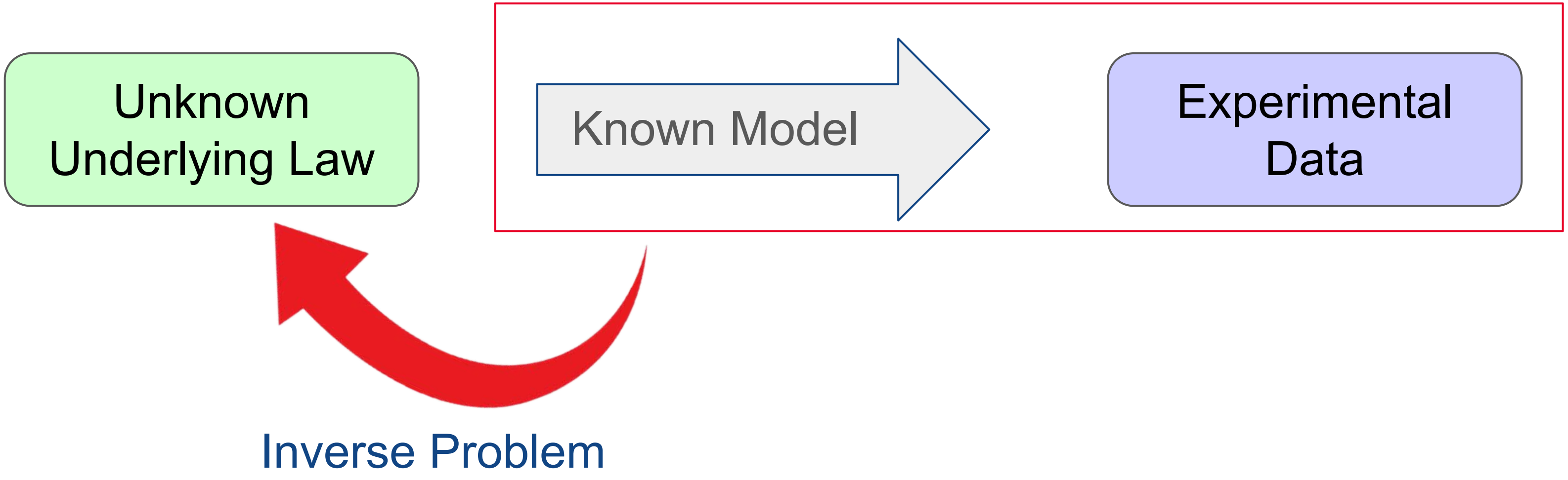
Statistical Analysis of a Framework

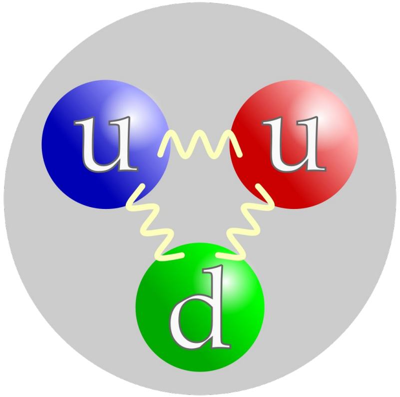
The statistical analysis I performed applies to any framework dealing with inverse problems.



Transverse Momentum Distributions

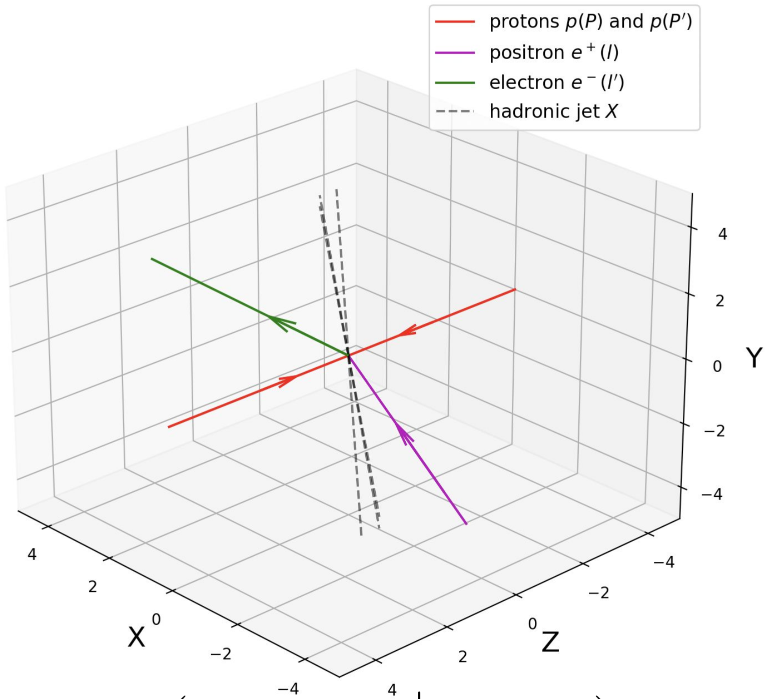
Transverse Momentum Distributions (TMDs) describe the internal structure of hadrons in 3D momentum space.

TMDs can be determined by analyzing available high-precision measurements of hadronic cross sections (e.g., from the Large Hadron Collider).



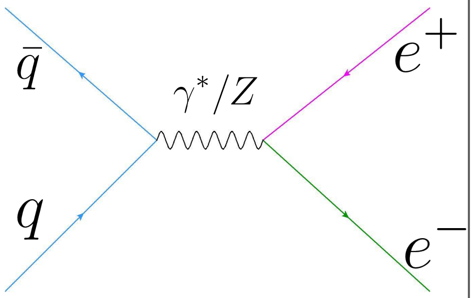
valence quarks of the proton

Factorization Theorems



$$\sigma(p\,p \rightarrow e^+\,e^-\,X)$$

Hard Process:



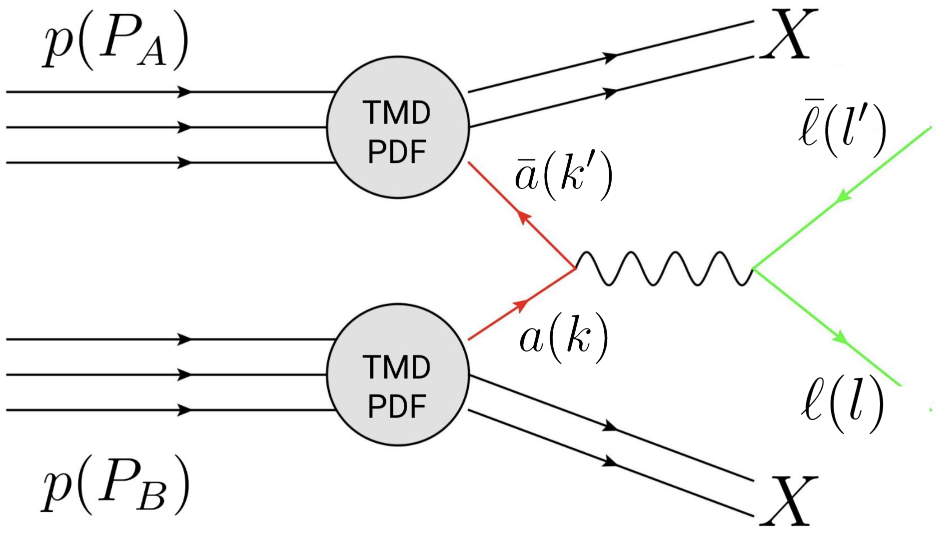
$$\hat{\sigma}(q\,\bar{q} \rightarrow \gamma^*/Z \rightarrow e^-\,e^+)$$

Soft Process:

$$f^a(x, k_T)$$

[Collins, Foundations of Perturbative QCD, 2011]

Drell-Yan Factorization



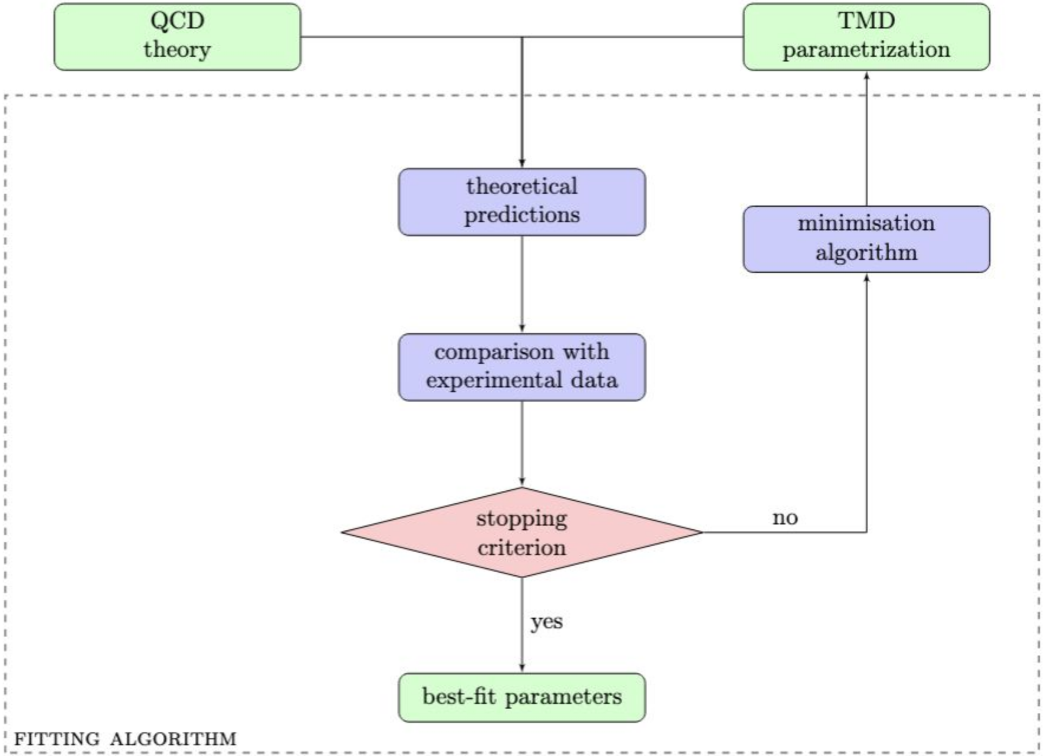
$$\frac{d\sigma}{d|q_T|} \sim H^{DY} \int d^2k_{TA} d^2k_{TB} f_1^a(x, k_{TA}) f_1^{\bar{a}}(x, k_{TB}) \delta^{(2)}(k_{TA} + k_{TB} - q_T)$$

The Fitting Frameworks

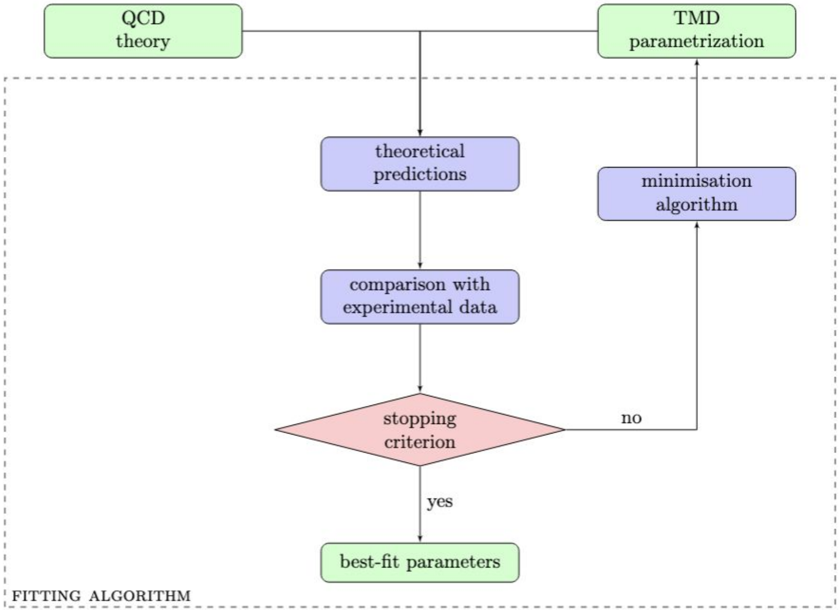
Parametric Regression Techniques

Both analyzed frameworks (PV19 and MAPTMD22) use parametric regression to find the TMD functional forms.

Parametric regression is a general technique that can be used to solve any inverse problem.



Unanswered Questions



How close are the fitted TMDs to the real law?

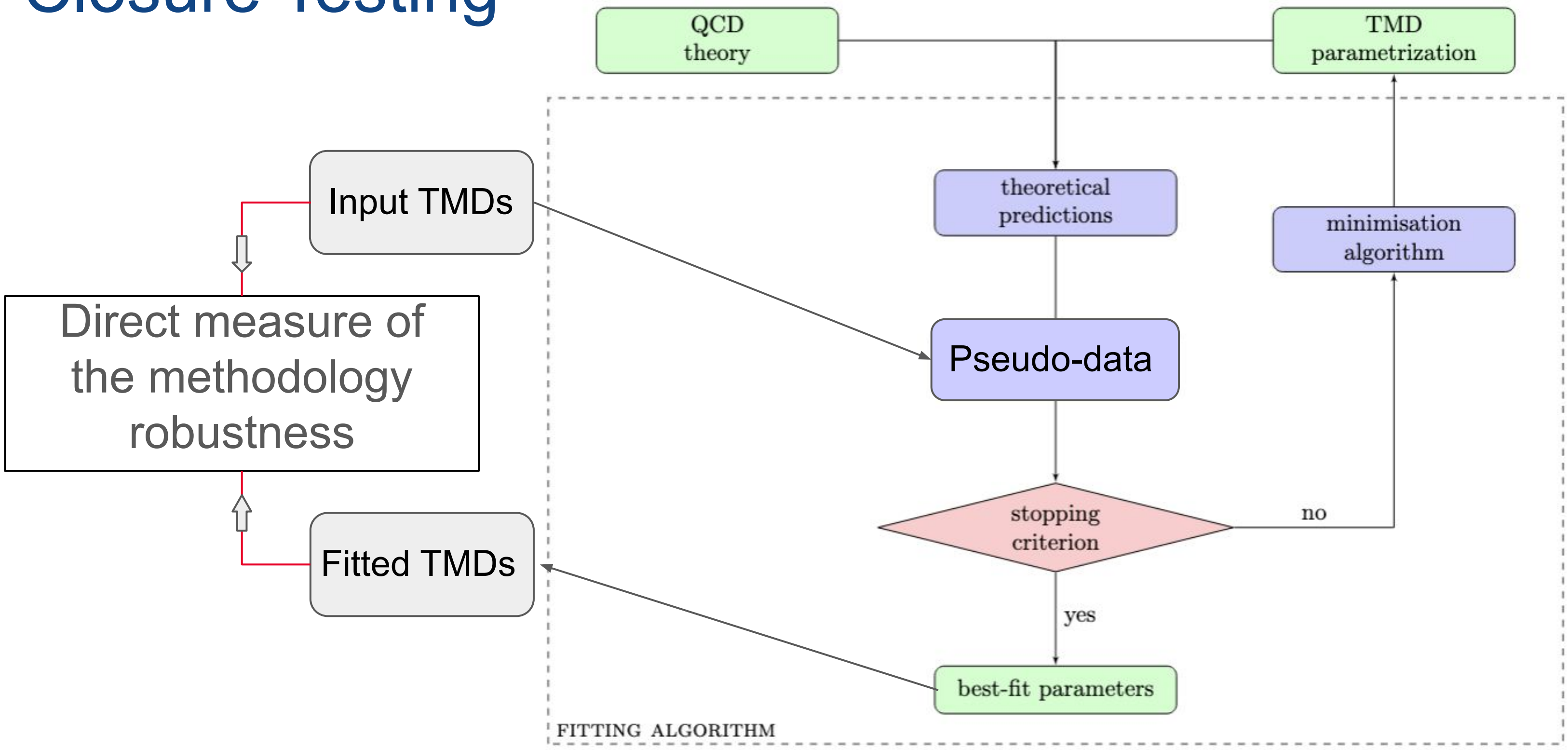
How much of the declared TMD uncertainties derive from the framework?

Are the declared TMD uncertainties statistically faithful?

My thesis took 9 months to address these questions, which are essential for analyzing present and future data (e.g., from the LHC, EIC).

Closure Testing

Closure Testing

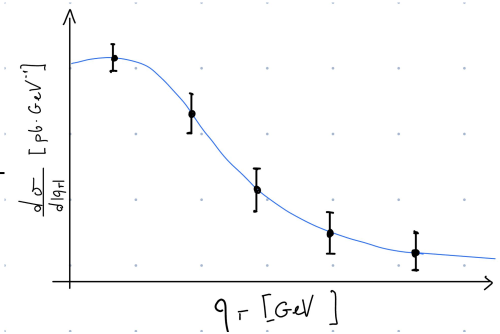


[Del Debbio et al. Eur.Phys.J.C 82 (2022) 4, 330]

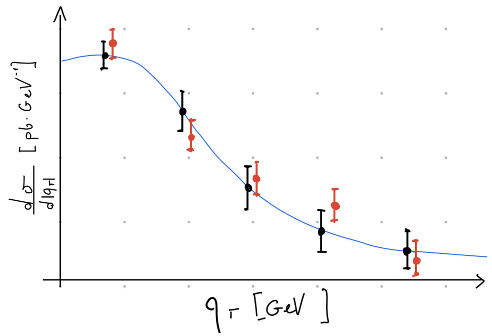
Levels of Fluctuation

Pseudo Data

Level 0 (no fluctuations)

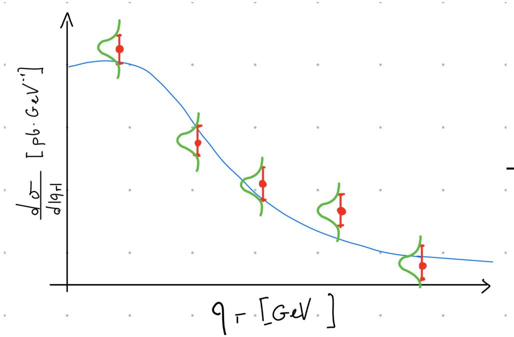


Gaussian fluctuation



Level 1 (L1)

N_{rep} Monte Carlo replicas



Level 2 (L2)

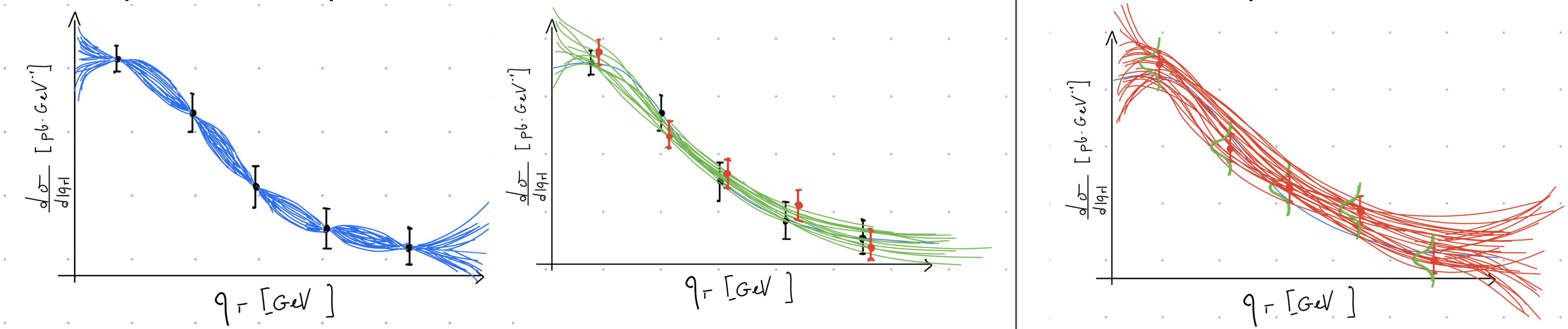
Levels of Uncertainty

Each level of fluctuation introduces a different level of uncertainty. We can fit L0, L1 and L2 data to characterize the uncertainty components.

interpolation/extrapolation

functional

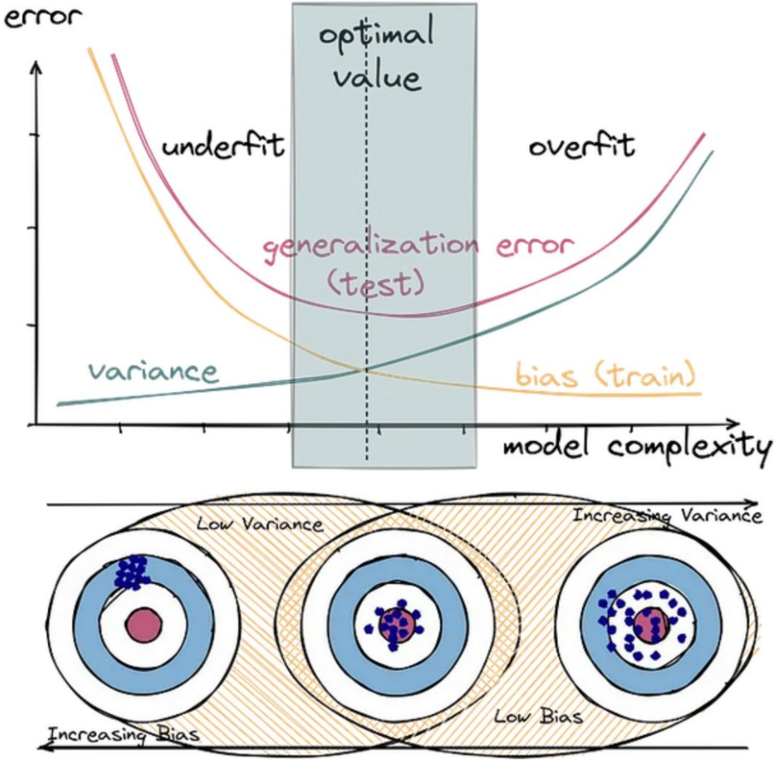
experimental



framework-dependent uncertainties

[Ball et al. JHEP 04 (2015) 040]

Multi Closure Tests



We measure the bias and variance of a large number of fits to assess:

- Framework optimization
- Faithfulness of the declared uncertainties

Results

Some Relevant Results

Some of the results obtained by the closure tests of the MAPTMD22 and PV19 frameworks:

1. L0 Test: The two analyzed frameworks do not generalize well.
2. Uncertainty Faithfulness:

   PV19 ➔ underfitting issue

   MAPTMD22 ➔ the framework is optimized

3. Code Transferability: https://github.com/MapCollaboration/

Thank You

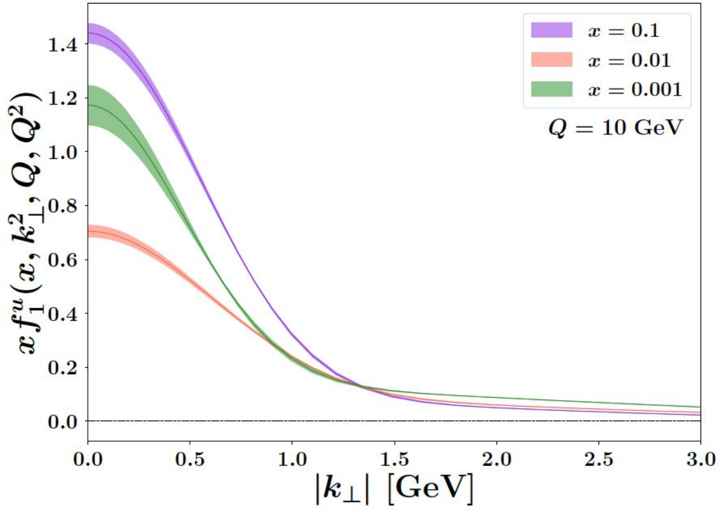
- Backup Slides -

Why are TMDs relevant?

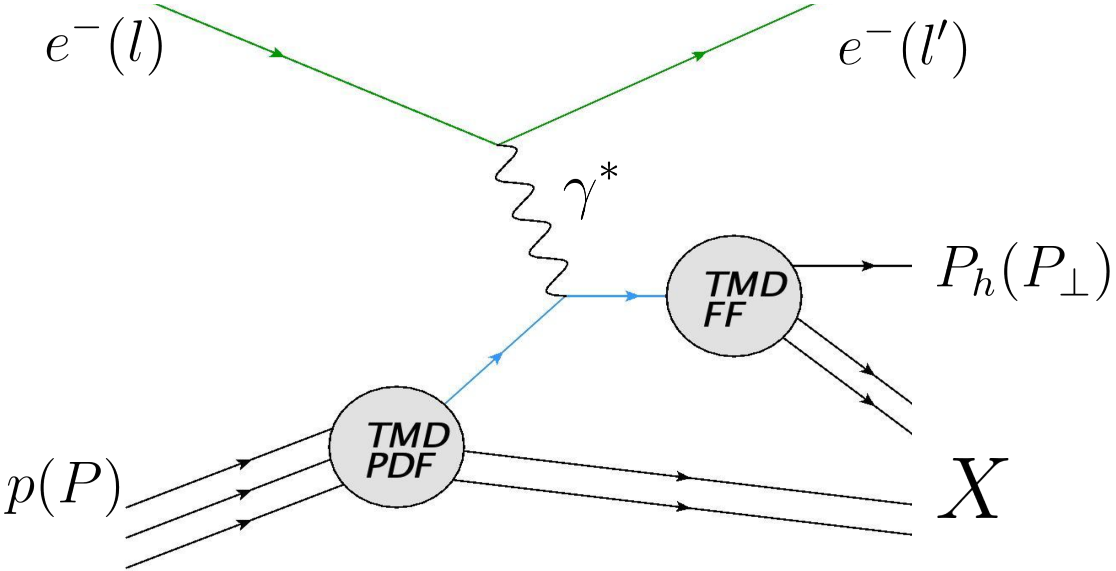
The accurate determination of TMDs is crucial for:

- understanding the dynamical properties of hadrons
- calculating observables, also where no experimental data are available
- testing the Standard Model and search for new physics beyond it

TMDs of the proton at Q=10 GeV

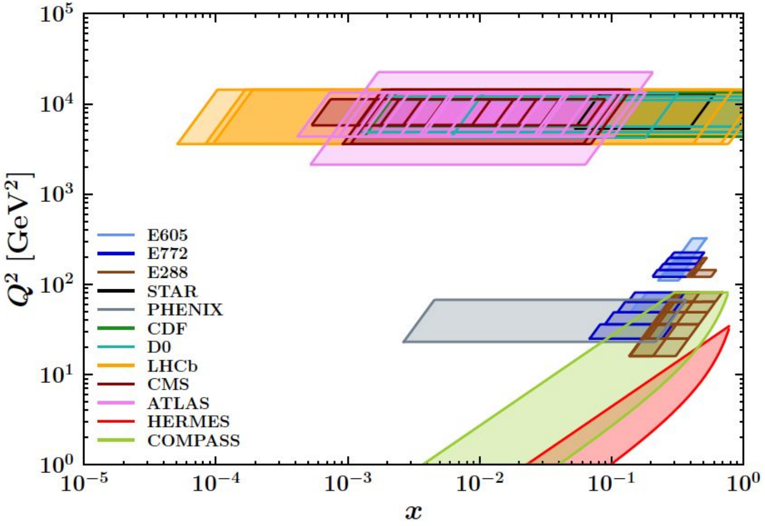


SIDIS Factorization



$$\frac{d\sigma}{d|q_T|} \sim H^{SIDIS} \int d^2k_T \frac{d^2P_\perp}{z^2} f_1^a(x, k_T) D_1^{a\to h}(z, P_\perp)\, \delta^{(2)}(k_T + P_\perp/z + q_T)$$

TMD Determination from Data



[Figure from Bacchetta et al. JHEP 10 (2022) 127]

SIDIS and Drell-Yan data span a large region in $(x\ ,Q^2)$ space

Determining TMDs knowing the factorized cross sections and a set of data is an *inverse problem*

The inverse problem can be solved through parametric regression

Tested Frameworks

PV19

- global fit on Drell-Yan cross sections data
- 353 data points
- TMD PDF defined using 9 parameters
- $\chi^2/N_{\mathrm{dat}} = 1.02$

[Bacchetta et al. JHEP 10 (2022) 127]

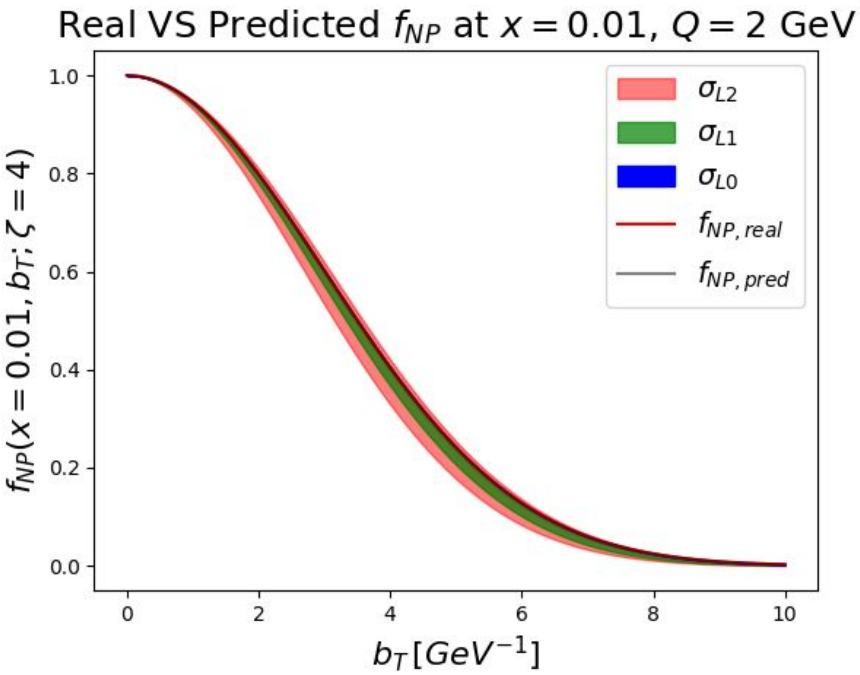
MAPTMD22

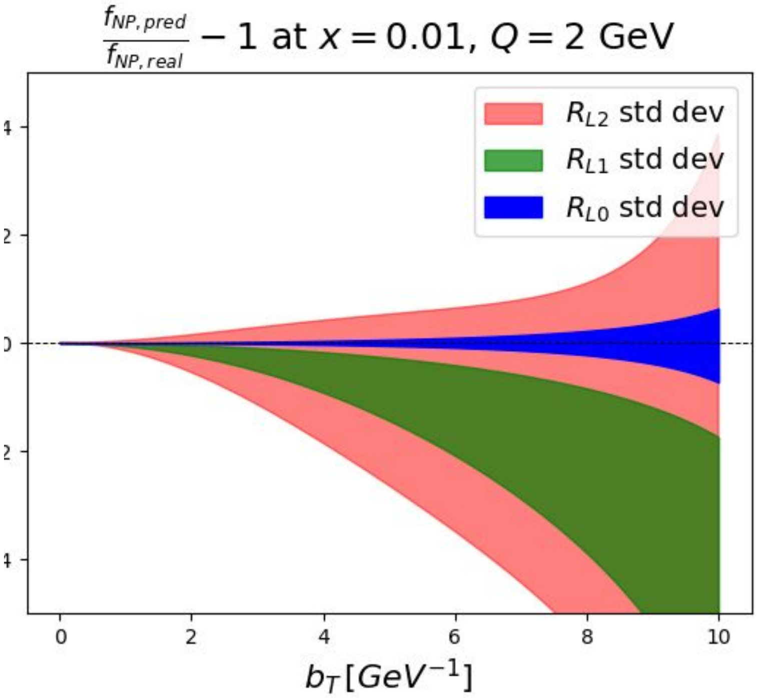
- global fit on Drell-Yan and SIDIS cross sections
- 2031 data points
- TMD PDF defined using 11 parameters
- TMD FF defined using 9 parameters
- $\chi^2/N_{\mathrm{dat}} = 1.06$

[Bacchetta et al. *JHEP* 07 (2020) 117]

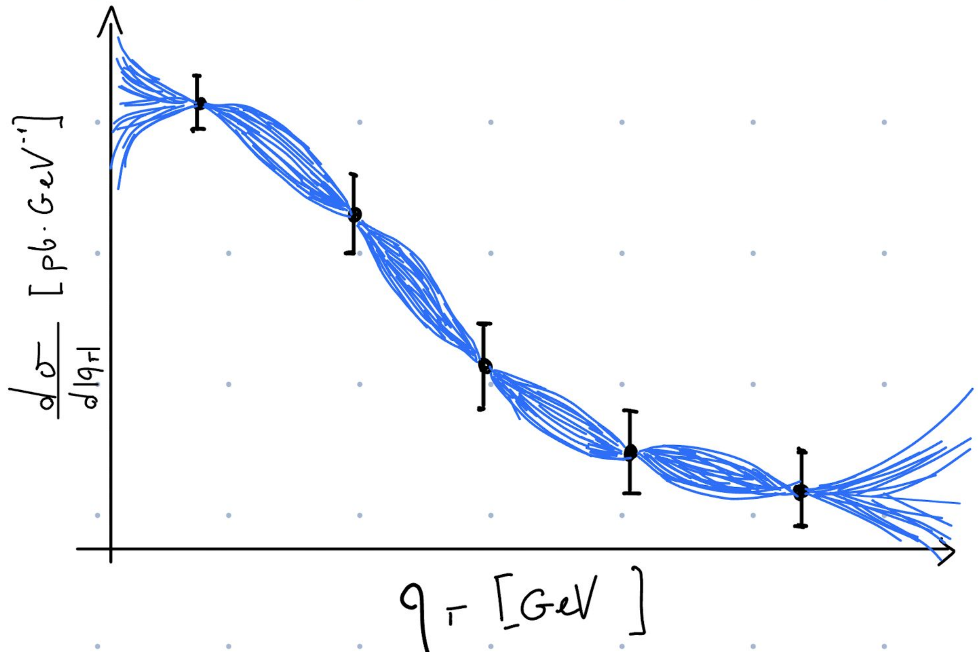
[https://github.com/MapCollaboration]

MAPTMD22 Uncertainties





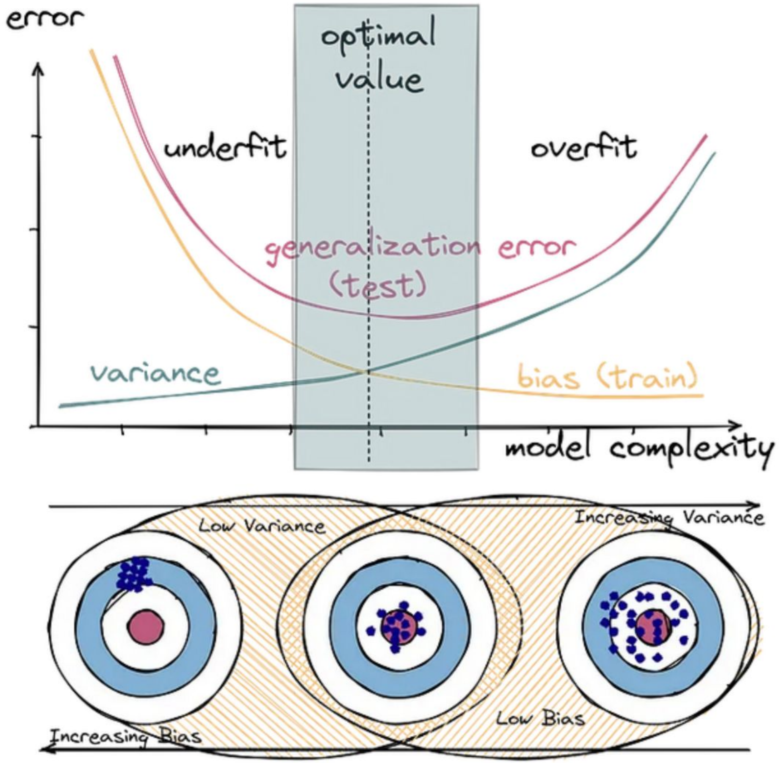
Level 0 Test



From a test on L0 data we can assess two aspects:

- **Frameworks flexibility**: is the framework able to reproduce the solution with $\chi^2 = 0$?
- **Interpolation/extrapolation uncertainty**, given by the finiteness of the dataset

Multi Closure Tests



We produce a large number of fits (50 fit with 100 replicas) to determine:

- Bias-Variance Ratio:

$$\mathcal{R}_{bv} = \sqrt{\frac{\mathbb{E}[bias]}{\mathbb{E}[variance]}} \approx 1$$

- TMD uncertainty faithfulness:

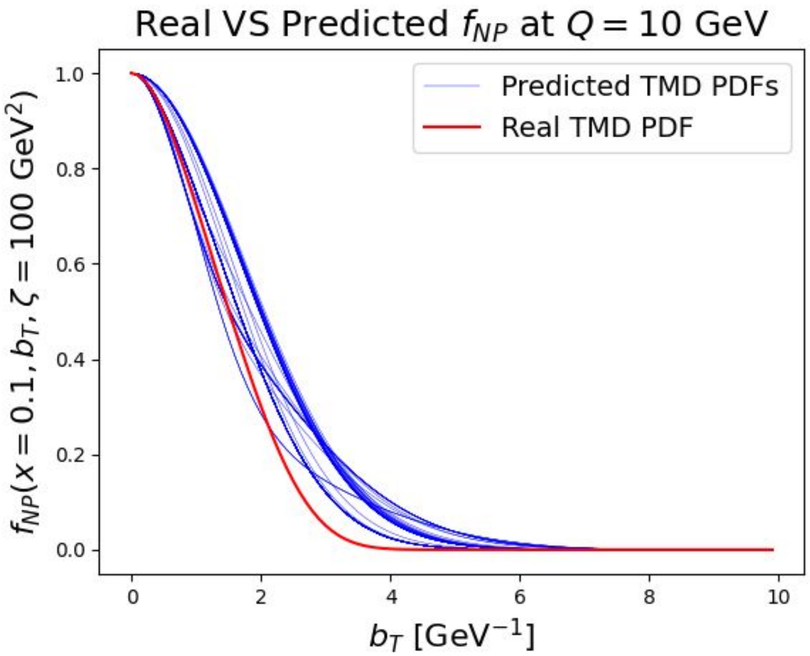
$$\xi_{1\sigma} \approx 0.683$$

Are the Methodologies Flexible?

Test	Mean χ^2	Best MSD	Closure
MAP22oMAP22$_{L0}$	$\mathcal{O}(10^{-14})$	/	yes
PV19oPV19$_{L0}$	$\mathcal{O}(10^{-13})$	/	yes
MAP22oPV19$_{L0}$	0.152	< 0.0015	no
PV19oMAP22$_{L0}$	0.017	< 0.00015	no
MAP22oMIX24$_{L0}$	0.155	< 0.004	no
PV19oMIX24$_{L0}$	$\mathcal{O}(10^{-7})$	/	no

- same input and fitting parametrization
- different input and fitting parameterizations

Are the Methodologies Flexible?

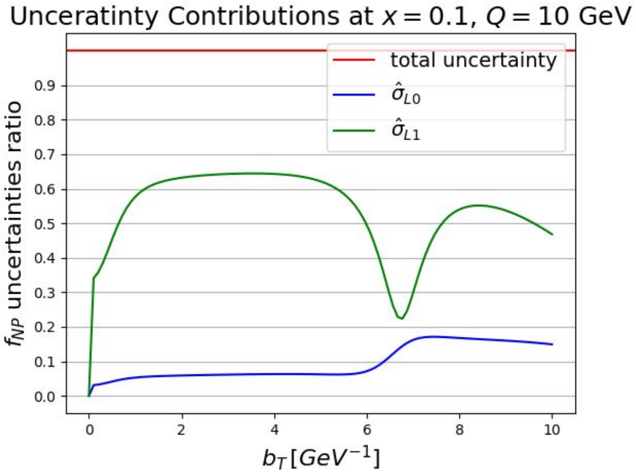


input: PV19, *fit*: MAPTMD22

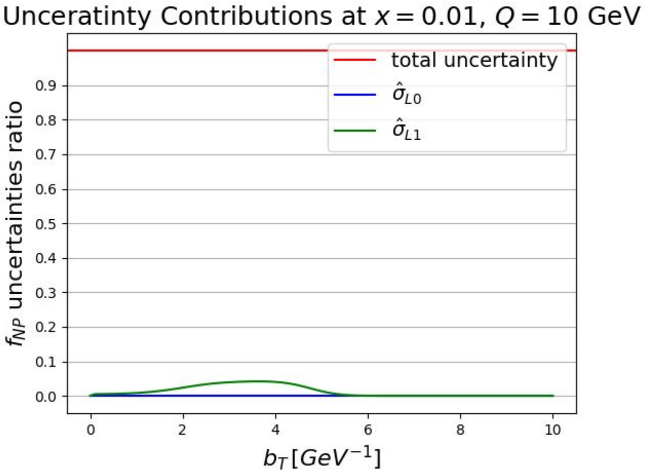
This picture outlines two difficulties:

- Generalization:
  the frameworks cannot generalize well
- Minimizer:
  the minimizer cannot find the best solution in 100% of the cases

Uncertainty Characterization (L0-L1-L2)



MAPTMD22 uncertainty contributions



PV19 uncertainty contributions

More data

X

More precise data

Better predictivity

Bias-Variance Tradeoff

PV19

- Quantile and bias-variance ratio:

$$\mathcal{R}_{bv} = 1.577 \pm 0.068$$
$$\xi_{1\sigma} = 0.486 \pm 0.012$$

- We observe signs of underfitting.
- The TMD uncertainties are underestimated.

MAPTMD22

- Quantile and bias-variance ratio:

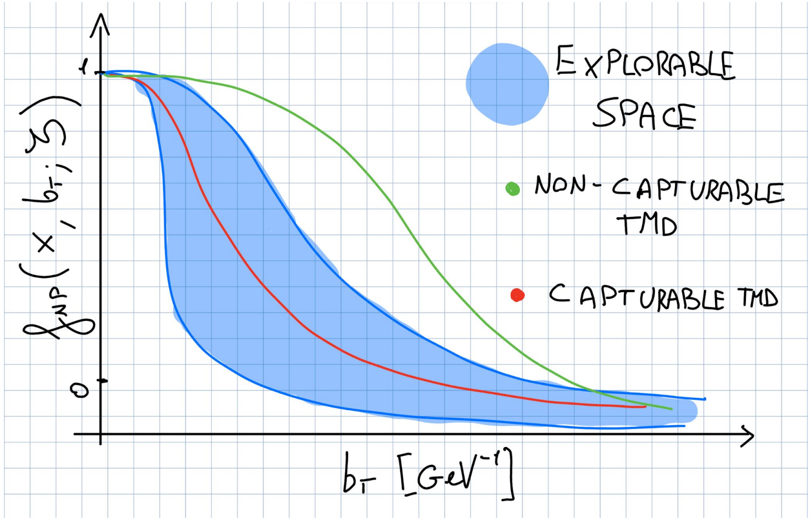
$$\mathcal{R}_{bv} = 0.978 \pm 0.045$$
$$\xi_{1\sigma} = 0.688 \pm 0.010$$

- The model is optimized.
- The uncertainty estimates are statistically faithful.

Level 0 - Flexibility Test

From fits of the L0 datasets, we conclude:

1. The frameworks are reliable under the *assumption* that the real law is contained in the space of functional forms which the parameterizations can explore.
2. A possible solution to the generalization issue is the use of Neural Networks.



Conclusions on Uncertainties

For the cases where input and fit parametrization are the same we discovered:

1. PV19 underfits, and the declared uncertainties should be rescaled to
2. MAPTMD22 is optimized
3. The interpolation/extrapolation uncertainties are subdominant in both frameworks, meaning more precise data should increase the predictivity of the models

Future Applications

The code I developed for performing closure tests of PV19 and MAPTMD22 can be transferred to other present and future methodologies.

The framework optimization and validation is a crucial step to take the maximum out of existing and future experimental data.