Renormalon Contribution to Pseudo- and Quasi-GPDs

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Introduction

- In recent years much progress on lattice calculations for Parton Distribution Functions (PDFs) with popular concepts: pseudo- and quasi-PDF [1]
- Using the same idea for Generalised Parton Distributions (GPDs): introducing pseudo- and quasi-GPD
- Technique of renormalons is used to investigate the structure of power corrections to pseudo- and quasi-GPDs [2]

[1] Braun et al., Phys.Rev.D 99 (2019) 1, 014013
[2] Braun et al., Phys.Rev.D 109 (2024) 7, 074510

Borel Transformation

- Express observable R as series in renormalised coupling α_s

$$R \sim \sum_n r_n \alpha_s^{n+1}$$

- Perturbative coefficients r_n show divergence $\sim n!$ for large orders n
- Borel transformation:

$$B[R](w) = \sum_{n=0}^{\infty} r_n \frac{w^n}{n!}\,, \quad \tilde{R} = \int_0^\infty \mathrm{d}w \; \mathrm{e}^{-w/\alpha_s} \; B[R](w)$$

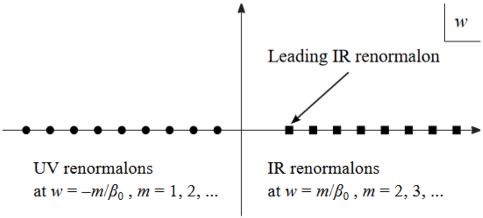
with w: Borel parameter

- Contains information on the divergences of R as its singularities
- *Renormalon*: a singularity of the Borel transform [3]

[3] Beneke, Phys.Rept. 317 (1999) 1-142

Renormalons

- UV and IR renormalons ($\rightarrow$ loop momenta)
- Borel plane: example for Adler function within QCD [4]



- IR renormalons specific for observable, UV renormalons universal for theory
- IR renormalons obstruct definition of Borel integral [5]

[4] Beneke, Phys.Rept. 317 (1999) 1-142
[5] Beneke, Braun, At the frontier of particle physics. Handbook of QCD. Vol. 1-3, 1719-1773

Renormalon Ambiguity

- Deform integration contour, arbitrary choice $\Rightarrow$ ambiguity (exponentially small in α_s), ambiguity in R: *renormalon ambiguity*
- Estimated as

$$\delta R(w_0) = -\pi \frac{1}{\beta_0} \mathrm{e}^{-w_0/(\beta_0 \alpha_s)} \, \mathrm{Res}_{w=w_0} B[R](w)$$

  w_0: position of the singularity
- Indication that exponentially small terms of the same form must be added to expansion of R
  $\rightarrow$ OPE: need nonperturbative power corrections (of at least same order of magnitude) which cancel ambiguity [6]
  $\Rightarrow$ indicate size of power correction [7]
- Estimate rough magnitude and scaling of power corrections to pseudo- and quasi-GPDs

[6] Beneke, Braun, et al., Phys.Lett.B 404 (1997) 315-320
[7] Braun et al., Phys.Rev.D 109 (2024) 7, 074510

Pseudo- and quasi-GPDs

- Quasi-GPD $\mathcal{Q}(x,\xi_v,Pv)$ and pseudo-GPD $\mathcal{P}(x,\xi_v,z^2)$ defined as FT of quasi-Ioffe-time distribution (ITD) $\mathcal{I}$:

$$\mathcal{Q}(x,\xi_v,vP) = (vP)\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{2\pi}\mathrm{e}^{-ixz(vP)}\,\mathcal{I}(z(vP),\xi_v,z^2)$$

$$\mathcal{P}(x,\xi_v,z^2) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{-i\tau x}\,\mathcal{I}(\tau,\xi_v,z^2)$$

- Quasi-ITD:

$$\mathcal{I}(\tau,\xi,z^2) = \int_0^1 \mathrm{d}u\, T_{\mathcal{I}}(u,\tau,\xi,z^2,\mu_F^2)\, I(u\tau,\xi,\mu_F^2)$$

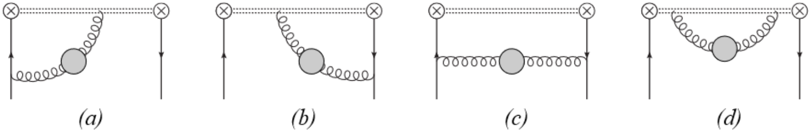
with I the Ioffe-time distribution (position-space GPD)

- Kinematic variables [8]: $\xi_v = \frac{pv-p'v}{pv+p'v}$ quasi-skewness, $P=\frac{p+p'}{2}$ and $\tau = z(vP)$

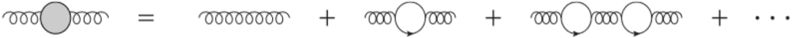
[8] Braun et al., Phys.Rev.D 109 (2024) 7, 074510

Borel transform technique (I)

- To calculate IR renormalon ambiguity for coefficient functions to pseudo- and quasi-GPDs, need an all-order calculation
- Restrict to perturbative series by running coupling effects in one-loop diagrams [9]
- Leading renormalon contribution to $T_{\mathcal{I}}$ given by bubble-chain diagrams



Bubble-chain contribution to gluon propagator [10]:



[9] Braun et al., Nucl.Phys.B 685 (2004) 171-226
[10] Braun et al., Phys.Rev.D 109 (2024) 7, 074510

Borel transform technique (II)

- Computation of Borel transform of full bubble-chain achieved by performing a lowest-order calculation with the Borel representation of the running coupling

$$\beta_0 \frac{\alpha_s(-l^2)}{4\pi} = \int_0^\infty \mathrm{d}w\, \mathrm{e}^{5w/3} \left(\frac{\Lambda^2}{-l^2}\right)^w$$

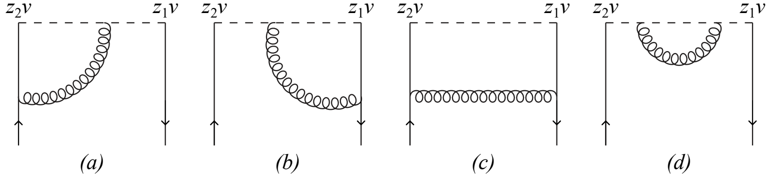
- Leads to a modified expression for the gluon propagator [11]

$$\frac{1}{-l^2 - i\epsilon} \to \frac{(\Lambda^2)^w}{(-l^2 - i\epsilon)^{1+w}}$$

[11] Braun et al., Phys.Rev.D 109 (2024) 7, 074510

Borel transform technique (III)

- Calculate one-loop diagrams



- From regulated sum of these diagrams, obtain the leading $(w = 1)$ renormalon ambiguity for quasi-ITD

$$\mathcal{I}(\xi, \tau, z^2) = I(\xi, \tau) \pm \mathcal{N}\left(\Lambda^2 z^2 |v^2|\right) \delta_R \mathcal{I}(\xi, \tau)$$

with the ambiguity

$$\delta_R \mathcal{I}_{\parallel}(\xi, \tau) = \int_0^1 \mathrm{d}\alpha\, I(\alpha\tau, \xi)\left(\alpha + \bar{\alpha}\ln(\bar{\alpha})\right)\cos(\bar{\alpha}\tau\xi)$$

$$\delta_R \mathcal{I}_{\perp}(\xi, \tau) = \delta_R \mathcal{I}_{\parallel}(\xi, \tau) + \int_0^1 \mathrm{d}\alpha\, I(\alpha\tau, \xi)\alpha\,\frac{\sin(\bar{\alpha}\tau\xi)}{\xi\tau}$$

Power corrections for pseudo- and quasi-GPD

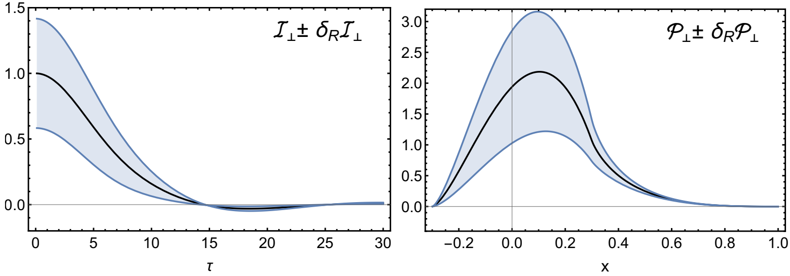
- From renormalon contributions to quasi-ITD, compute power corrections for pseudo- and quasi-GPD as higher-order corrections

$$\mathcal{P}(x,\xi,z^2) = H(x,\xi) \pm \mathcal{N}\left(\Lambda^2 z^2 |v^2|\right) \delta_R \mathcal{P}(x,\xi)$$

$$\mathcal{Q}(x,\xi,(vP)) = H(x,\xi) \pm \mathcal{N}\left(\frac{\Lambda^2 |v^2|}{(vP)^2}\right) \delta_R \mathcal{Q}(x,\xi)$$

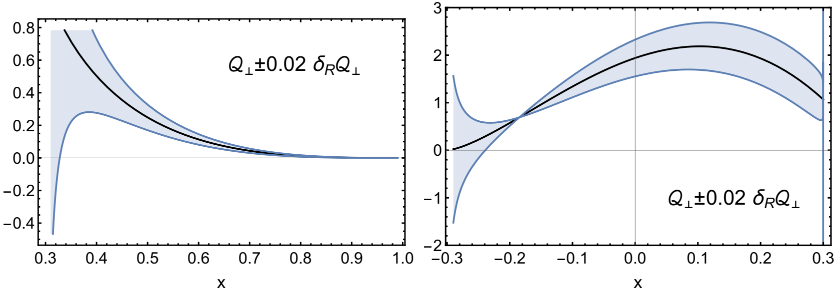
Numerics (I)

- Using a simple model of the valence quark GPD based on the double distribution ansatz [12], visualise the structures for a fixed value of ξ



[12] Belitsky, Radyushkin, Phys.Rept. 418 (2005) 1-387

Numerics (II)



- left: DGLAP region $x > \xi$
- right: ERBL region $-\xi < x < \xi$

Summary

- Computation of Borel transform of quasi-ITD in single bubble-chain approximation, from which the positions of the IR renormalons were identified
- Leading renormalon ambiguity of pseudo- and quasi-GPDs was calculated, predicting the functional dependence of their corresponding power corrections on momentum fraction x and skewness ξ
- Numerical study on the size of power corrections, showing large ambiguities especially in the regions of small x and τ

Explicit results (I)

- Excluding the point $x = \pm\xi$ due to unphysical behaviour
- Pseudo-GPD:

$$\mathcal{P}(x,\xi,z^2) = H(x,\xi) \pm \mathcal{N}\left(\Lambda^2 z^2 |v^2|\right) \delta_R \mathcal{P}(x,\xi)$$

$$\begin{aligned}
2\delta_R \mathcal{P}_{\|}(x,\xi) = {} & \theta(x>\xi)\left\{ \int_x^1 \mathrm{d}y\, \frac{H(y,\xi)}{y-\xi}\Phi\left(\frac{x-\xi}{y-\xi}\right) + \int_x^1 \mathrm{d}y\, \frac{H(y,\xi)}{y+\xi}\Phi\left(\frac{x+\xi}{y+\xi}\right) \right\} \\
& + \theta(-\xi<x<\xi)\left\{ -\int_{-1}^x \mathrm{d}y \frac{H(y,\xi)}{y-\xi}\Phi\left(\frac{x-\xi}{y-\xi}\right) + \int_x^1 \mathrm{d}y \frac{H(y,\xi)}{y+\xi}\Phi\left(\frac{x+\xi}{y+\xi}\right) \right\} \\
& + \theta(x<-\xi)\left\{ -\int_{-1}^x \mathrm{d}y \frac{H(y,\xi)}{y-\xi}\Phi\left(\frac{x-\xi}{y-\xi}\right) - \int_{-1}^x \mathrm{d}y\, \frac{H(y,\xi)}{y+\xi}\Phi\left(\frac{x+\xi}{y+\xi}\right) \right\}
\end{aligned}$$

Explicit results (II)

- Pseudo-GPD:

$$
\begin{aligned}
2\delta_R\mathcal{P}_\perp(x,\xi;z^2) &= 2\delta_R\mathcal{P}_\parallel(x,\xi;z^2) \\
&+ \frac{1}{2\xi}\Bigg\{\theta(x>\xi)\int_x^1 \mathrm{d}y\left[\left(\frac{x+\xi}{y+\xi}\right)^2 - \left(\frac{x-\xi}{y-\xi}\right)^2\right]H(y,\xi) \\
&+ \theta(-\xi<x<\xi)\left[\int_x^1 \mathrm{d}y\left(\frac{x+\xi}{y+\xi}\right)^2 H(y,\xi) + \int_{-1}^x \mathrm{d}y\left(\frac{x-\xi}{y-\xi}\right)^2 H(y,\xi)\right] \\
&+ \theta(x<-\xi)\int_{-1}^x \mathrm{d}y\left[\left(\frac{x-\xi}{y-\xi}\right)^2 - \left(\frac{x+\xi}{y+\xi}\right)^2\right]H(y,\xi)\Bigg\}
\end{aligned}
$$

Explicit results (III)

- Quasi-GPD ansatz:

$$\mathcal{Q}(x,\xi,(vP)) = H(x,\xi) \pm \mathcal{N}\left(\frac{\Lambda^2|v^2|}{(vP)^2}\right)\delta_R\mathcal{Q}(x,\xi)$$

Explicit results (IV)

- Corrections:

$$
\begin{aligned}
&2\delta_R \mathcal{Q}_{\|}(x,\xi) \\
&\quad = \left(1 - \frac{\pi^2}{6}\right) \delta(x-\xi) \left[H'(\xi+0,\xi) - H'(\xi-0,\xi)\right] \\
&\quad + \theta(x>\xi) \left[\frac{H'(x,\xi)}{x-\xi} + \frac{1}{(x-\xi)^2} \int_x^1 \mathrm{d}y \left[\frac{x-\xi}{y-\xi} + \ln\left(1 - \frac{x-\xi}{y-\xi}\right)\right] H'(y,\xi)\right] \\
&\quad + \theta(x<\xi) \left[\frac{H'(x,\xi)}{x-\xi} - \frac{1}{(x-\xi)^2} \int_{-1}^x \mathrm{d}y \left[\frac{x-\xi}{y-\xi} + \ln\left(1 - \frac{x-\xi}{y-\xi}\right)\right] H'(y,\xi)\right] \\
&\quad + \left(1 - \frac{\pi^2}{6}\right) \delta(x+\xi) \left[H'(-\xi-0,\xi) - H'(-\xi+0,\xi)\right] \\
&\quad + \theta(x>-\xi) \left[\frac{H'(x,\xi)}{x+\xi} + \frac{1}{(x+\xi)^2} \int_x^1 \mathrm{d}y \left[\frac{x+\xi}{y+\xi} + \ln\left(1 - \frac{x+\xi}{y+\xi}\right)\right] H'(y,\xi)\right] \\
&\quad + \theta(x<-\xi) \left[\frac{H'(x,\xi)}{x+\xi} - \frac{1}{(x+\xi)^2} \int_{-1}^x \mathrm{d}y \left[\frac{x+\xi}{y+\xi} + \ln\left(1 - \frac{x+\xi}{y+\xi}\right)\right] H'(y,\xi)\right]
\end{aligned}
$$

Explicit results (V)

- Corrections:

$$
\begin{aligned}
2\delta_R \mathcal{Q}_\perp(x,\xi) &= 2\delta_R \mathcal{Q}_\parallel(x,\xi) \\
&\quad + \frac{2}{2\xi}\Bigg\{\theta(x>\xi)\int_x^1 \mathrm{d}y\, \frac{H'(y,\xi)}{y-\xi} - \theta(x<\xi)\int_{-1}^x \mathrm{d}y\, \frac{H'(y,\xi)}{y-\xi} \\
&\quad - \theta(x>-\xi)\int_x^1 \mathrm{d}y\, \frac{H'(y,\xi)}{y+\xi} + \theta(x<-\xi)\int_{-1}^x \mathrm{d}y\, \frac{H'(y,\xi)}{y+\xi}\Bigg\}
\end{aligned}
$$