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Cosmological Correlators at Loop Order

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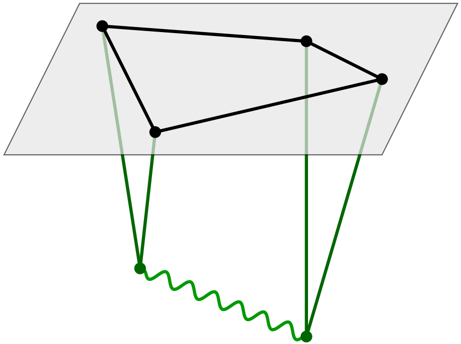
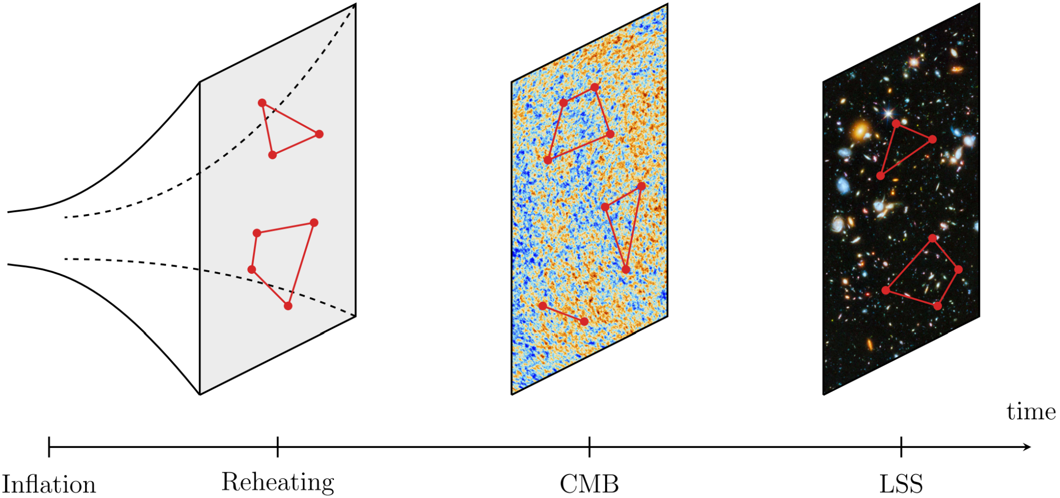


Cosmological Correlators



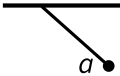
Cosmological Correlators

equal-time in-in correlators in the Schwinger-Keldysh path integral formalism:

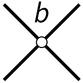
$$\langle \phi^{a_1}(\eta, x_1) \dots \phi^{a_N}(\eta, x_N) \rangle = \int_{\phi_+(\eta_f, x) = \phi_-(\eta_f, x)} \mathcal{D}[\phi_+^a, \phi_-^a] e^{iS[\phi_+] - iS[\phi_-]} \phi_+^{a_1}(\eta, x_1) \dots \phi_+^{a_N}(\eta, x_N)$$

Feynman rules (example: $\mathcal{L}_{\text{int}} \supset \frac{\lambda}{4!} a(\eta)^4 \phi^4$):

$= \mathcal{D}_{ab}^{\nu}(k; \eta_1, \eta_2)$



$= \mathcal{G}_a^{\nu}(k; \eta)$



$= ib\lambda \int_{-\infty}^{0} \mathrm{d}\eta \, a(\eta)^4 \dots$

Cosmological Correlators

Why is this so much harder than 'usual' QFTs in Minkowski space?

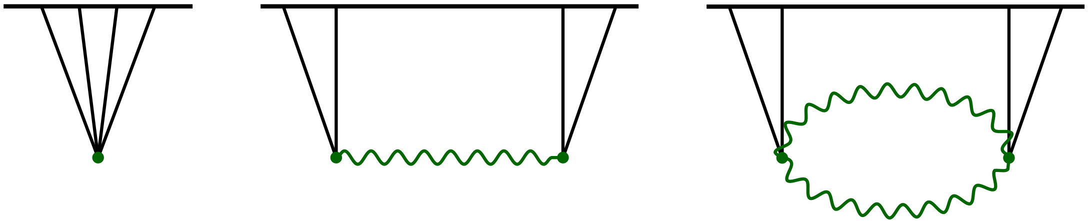
- time-dependent de Sitter background metric: $\mathrm{d}s^2 = \frac{1}{(-\eta)^2}(-\mathrm{d}\eta^2 + \mathrm{d}x^2)$ where $\eta \in (-\infty, 0)$ is the conformal time
- mode functions:

$$u_k(\eta) = -i\frac{\sqrt{\pi}}{2}e^{i\pi\left(\frac{\nu}{2}+\frac{1}{4}\right)}(-\eta)^{d/2}H_\nu^{(1)}(-k\eta)$$

- $\nu = \sqrt{d^2/4 - m^2}$ related to mass
- propagators up to time ordering: $\mathcal{D}_{ab}(k;\eta_1,\eta_2) \sim u_k(\eta_1)u_k^*(\eta_2)$

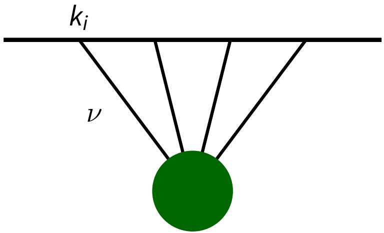
State of the Art

- **conformally coupled scalars** ($\nu = 1/2$): a lot of progress $\rightarrow$ results $>$ 2 loop order, cosmological polytopes, differential equations, ...
- **massless, minimally-coupled scalars** ($\nu = 3/2$): also several loop results $\rightarrow$ study of IR divergences, ...
- arbitrary massive exchanges ($\nu = \sqrt{d^2/4 - m^2}$): some tree-level results + 1 loop bubble

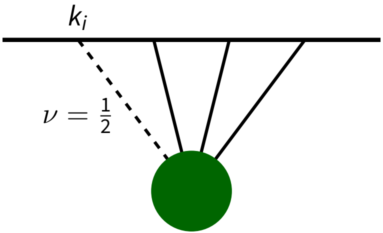


Internal Propagators vs. External Legs

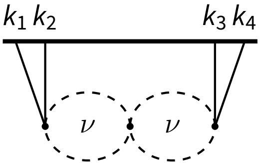
- External legs are not the problem!
- relevant for cosmology: massless scalars
- also: massive bulk-to-boundary propagators related to conformally coupled case by **weight-shifting operators**



$$= \hat{\mathcal{W}}_\nu(k_i)$$



2-Loop Necklace Diagram



$$\sim \int_{-\infty}^{0} \left[\prod_{j=1}^{3} \mathrm{d}\eta_j \, (-\eta_j)^{p_j} \right] e^{iak_{12}\eta_1 + ick_{34}\eta_3} \, \mathcal{Q}^{\nu}_{ab}(k_s; \eta_1, \eta_2) \mathcal{Q}^{\nu}_{bc}(k_s; \eta_2, \eta_3)$$

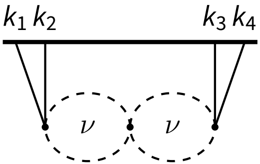
Bubble loop integral:

$$\mathcal{Q}^{\nu}_{ab}(k; \eta_1, \eta_2) \equiv \int \frac{\mathrm{d}^d q}{(2\pi)^d} \mathcal{D}^{\nu}_{ab}(q; \eta_1, \eta_2) \mathcal{D}^{\nu}_{ab}(|k-q|; \eta_2, \eta_3)$$

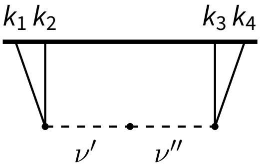
Spectral Representation

Loop integral can be simplified using

$$\mathcal{Q}^{\nu}_{ab}(k;\eta_1,\eta_2) = \int_{-i\infty}^{+i\infty} \mathrm{d}\nu' \, \frac{\nu'}{\pi i} \rho^{dS}_{\nu}(\nu') \mathcal{D}^{\nu'}_{ab}(k;\eta_1,\eta_2)$$



$$= \int_{-i\infty}^{+i\infty} \mathrm{d}\nu' \, \mathrm{d}\nu'' \, \frac{\nu'}{\pi i} \frac{\nu''}{\pi i} \rho^{dS}_{\nu}(\nu') \rho^{dS}_{\nu}(\nu'')$$



Expectation

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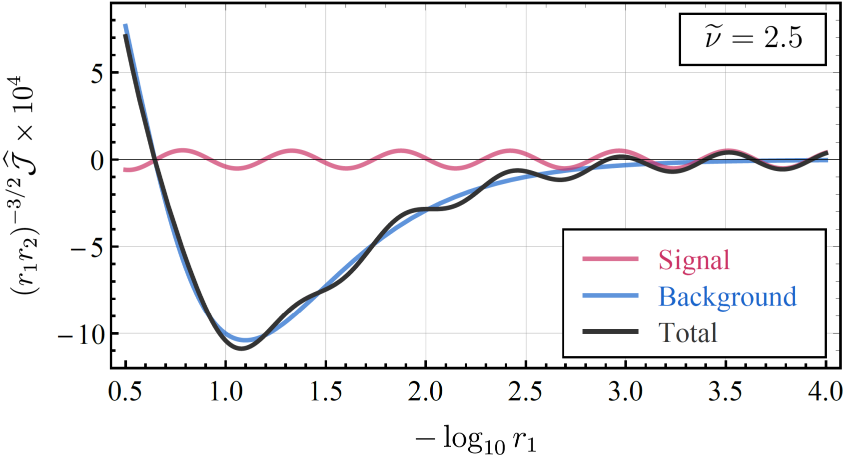

↗ JHEP04(2023)103