AN INTRODUCTION TO THE PHYSICS OF AXIONS

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Outline

Motivation - The Strong CP Problem

The QCD axion

Axions from higher dimensions

Axion Phenomenology

Conclusions

Motivation - The Strong CP Problem

The Strong CP Problem -The Theta term

$$\mathcal{L}'_{\text{QCD}} = -\frac{1}{2}\,\text{Tr}\;G_{\mu\nu}G^{\mu\nu} + \sum_{i}^{N_f} \bar{q}_i \left(i\not{D} - m_i\right) q_i \qquad ,$$

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$$\text{where} \quad \tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}.$$

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Creates a vacuum potential $V_{QCD}(\theta)$ which is minimal for $\bar{\theta} = 0$.

The Strong CP Problem - Measuring θ

We want to measure θ. Which magnitudes *violate* CP symmetry?

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Electric Dipole Moments!

The Theta term - EDM of the neutron

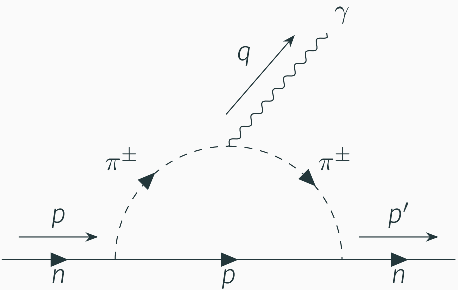

Figure 1: Feynman diagram giving the first order contribution to the neutron eDM.

EDM of the neutron:

From Chiral Perturbation Theory:

$$d_n \approx \bar{\theta} \cdot 2,5 \times 10^{-16} \text{ e cm}$$

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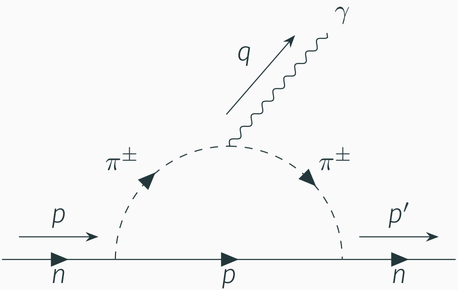

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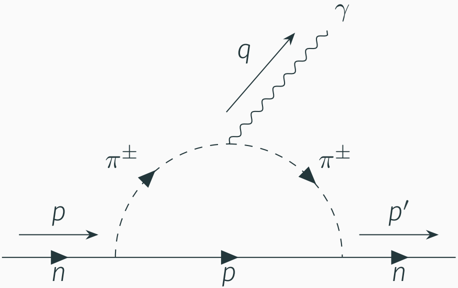

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Conclusion:

$$\bar{\theta} < 10^{-10}$$

NB: $\bar{\theta} = \theta + \theta_u + \theta_d$

The Strong CP Problem

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Why does QCD conserve CP symmetry?

The Strong CP Problem - Possible Solutions

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- Axions!

The QCD axion

The QCD Axion - The Peccei-Quinn Mechanism

Introduce a scalar field a with coupling $\mathcal{L}_{PQ} = \frac{a}{f_a} \frac{g^2}{16\pi^2} \operatorname{Tr} G\tilde{G}$.

$$\mathcal{L}'_\theta = \left(\frac{a}{f_a} + \theta\right) \frac{g^2}{16\pi^2} \operatorname{Tr} G\tilde{G} \implies \bar{\theta}_{eff} = \bar{\theta} + \frac{a}{f_a}$$

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The complete Lagrangian is

$$\mathcal{L}_{PQ} = \frac{1}{2}\partial_\mu a\,\partial^\mu a - \frac{m_a^2}{2}a^2 + \frac{a}{f_a}\frac{g^2}{16\pi^2}\,\mathrm{Tr}\,G\tilde{G}$$

The QCD Axion - The axion

The QCD Axion - The KSVZ model

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KSVZ model

Introduce a heavy quark singlet Q and a complex scalar ϕ with

$$V_{\mathrm{KSVZ}} = -m^2|\phi|^2 + \lambda|\phi|^4 + yQ_L\phi Q_R + \text{h.c.}\,.$$

There is a global $U(1)_{PQ}$: $\quad \phi \to e^{i\alpha}\phi, \quad Q_L \to e^{i\alpha}Q_L$

Break $U(1)_{PQ}$ with a Higgs-like potential. The axion is the Goldstone.

Integrating out the quarks gives the coupling $\mathcal{L}_{PQ} = \dfrac{a}{f_a}\dfrac{g^2}{16\pi^2}\,\mathrm{Tr}\,G\tilde{G}$.

The Axion Quality Problem

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Potential shift :

$$V^n_{PQV}(a) = 2|c| M_P^4 \left(\frac{f_a}{M_P}\right)^n \cos\left(n\frac{a}{f_a} + \varphi\right).$$

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The Strong CP Problem is reintroduced!!!

The Axion Quality Problem - Possible Solution

- Fine tuning of c. – We move the CP problem to the c problem.
- Forbid $\mathcal{L}^n_{PQV}$ with a discrete gauge $\mathbb{Z}_{12}$. - “Unnatural.”
- Guage $U(1)_{PQ}$!

Axions from higher dimensions

Axions from Higher Dimensions - Basics

We introduce a fifth compact spatial dimension with coordinate y.

We introduce a $U(1)$ gauge field $A(x, y)$ which gives the axion once compactified:

$$a(x) = \oint dy\, A_5(x, y) \equiv \int_0^{2\pi R} dy\, A_5(x, y).$$

Then the kinetic term of A gives us

$$\mathcal{L}_{\text{kin}} = \oint dy - \frac{1}{4e^2} F_{MN} F^{MN} = -\frac{1}{8\pi R e^2} \partial_\mu a\, \partial^\mu a \equiv -\frac{1}{2} f^2\, \partial_\mu a\, \partial^\mu a$$

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Straightforward to extend to higher dimensions!

Axions from Higher Dimensions - Coupling to gluons

We now want to couple a to G. Slight difference, now

$$\tilde{G}^{MNP} \equiv (\star_{5D} G)^{MNP} := \frac{1}{3!} \epsilon^{MNPQR} G_{QR}$$

$$\mathcal{L}_{CS} = \frac{1}{8\pi^2} \oint dy\, A_5 \,\mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu 5} = \frac{1}{8\pi^2} a(x) \,\mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu} .$$

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Again, straightforward to extend to higher dimensions!

NB: $A_\mu^{QCD} = A_\mu^{QCD}(x), A_5^{QCD} = A_5^{QCD}(y)$

Axions from Higher Dimensions - Axionic potential

There may also be scalar massive 5D fields ϕ_{5D} coupled to a through $\mathcal{D}_M\phi_{5D}\mathcal{D}^M\phi_{5D}$.

$$V_{\text{axionic}}(a) = V_{QCD}(a) + V_{\text{eff}}(a)$$

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We integrate the ϕ_{5D} out. Semi-clasically:

$$S(\gamma) = \int_\gamma d\tau \left(m_{5D} + iqA_M \frac{dx^M}{d\tau} \right)$$

as $A_5 = \frac{a}{2\pi R}$, using the saddle point approximation,

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$$V_{eff}(a) = \sum_{\nu>0} c_\nu e^{-2\pi R\nu m_{5D}} \cos(q\nu a) \ll V_{QCD}(a)$$

The axion quality problem is exponentially small!

Axions from Higher Dimensions - Other considerations

- This last part is formalised using the Coleman-Weinberg potential. 5D computation available in several papers[1][2].
- We can also extend this to higher dimensions (see my thesis).
- How do we interpret the $U(1)$? What's the conserved charge?

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[1] I. Antoniadis, K. Benakli, et al., "*Finite Higgs mass without Supersymmetry*", New Journal of Physics 3, 10.1088/1367-2630/3/1/320 (2001)

[2] A. Delgado, A. Pomarol, et al., "*Supersymmetry and Electroweak breaking from extra dimensions at the TeV-scale*", Physical Review D - Particles, Fields, Gravitation and Cosmology 60, 10.1103/PhysRevD.60.095008 (1998).

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$$\text{Classical EOM: } \frac{1}{2\pi} \mathrm{d}\left(f^2 \star \mathrm{d}a\right) = \frac{1}{8\pi^2} \operatorname{Tr} G \wedge G,$$

The conserved charge is the instanton number.

$$\text{NB: } \mathrm{d} \operatorname{Tr} G \wedge G = 2 \operatorname{Tr} \mathrm{d}G \wedge G = 2 \operatorname{Tr} \mathcal{D}G \wedge G = 0$$

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Axion Phenomenology

Axion Phenomenology - Other couplings

Axions are not only coupled to QCD:

Other couplings

Axion-Photon coupling:

$$\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

Axion-fermion pseudo-vectorial coupling:

$$g_p \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi.$$

NB: These couplings are model-dependent, and the coupling constants are a function of the parameters of the theory.

Axion Phenomenology - Axion-Like Particles

Axion-Like Particles

ALPs are particles that have the same couplings as the QCD axion, EXCEPT that they are not coupled to QCD. Notably

$$\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

NB: ALPs don't solve the Strong CP Problem.

Axion Phenomenology - Axion-Like Particles II

Properties of ALPs

Very light scalars (Goldstones)

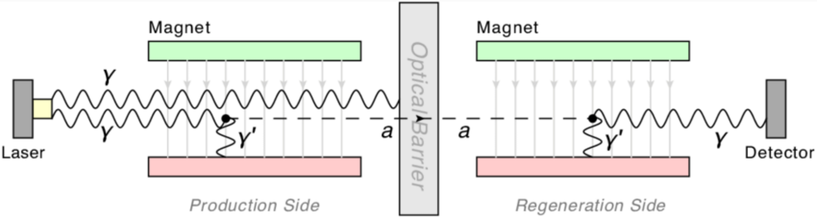
Weakly interacting

Long lived (small coupling constants)

Natural cold dark matter candidate!

Axion Phenomenology - Light-shining-through wall experiment

$$\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} .$$



The concept of the light-shining-through-walls experiments[3]

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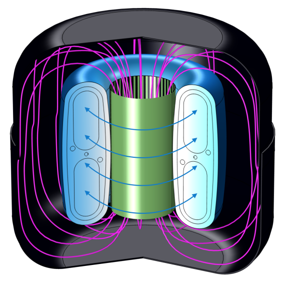
[3] R. Battesti, et al., “*High magnetic fields for fundamental physics*”, Phys. Rept. **765-766**, 1-39, 10.1016/j.physrep.2018.07.005 (2018)

Axion Phenomenology - ABRACADABRA

$$\nabla \times \mathrm{B} = \mathrm{J} + g_{a\gamma\gamma} \frac{\partial a}{\partial t} \mathrm{B}$$



Schematic of the effective axion-induced current (blue), sourced by the magnetic field inside the torus, generating a magnetic field (magenta)[4].

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[4] C. P. Salemi, J. W. Foster, et al., "*The search for low-mass axion dark matter with ABRACADABRA-10cm*", Physical Review Letters 127, 10.1103/PhysRevLett.127.081801 (2021).

Conclusions

Conclusions - Summary

Summary

Axions are strongly theoretically and phenomenologically motivated.

Theoretical ideas (AQP and gauging instantons) naturally leads to higher-dimensional theories (String Theory).

Hard to detect because of the small mass and weakly interacting, but “Cheap” tabletop experiments.

Hot topic.

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