ALARIC - a NLL accurate Parton Shower algorithm

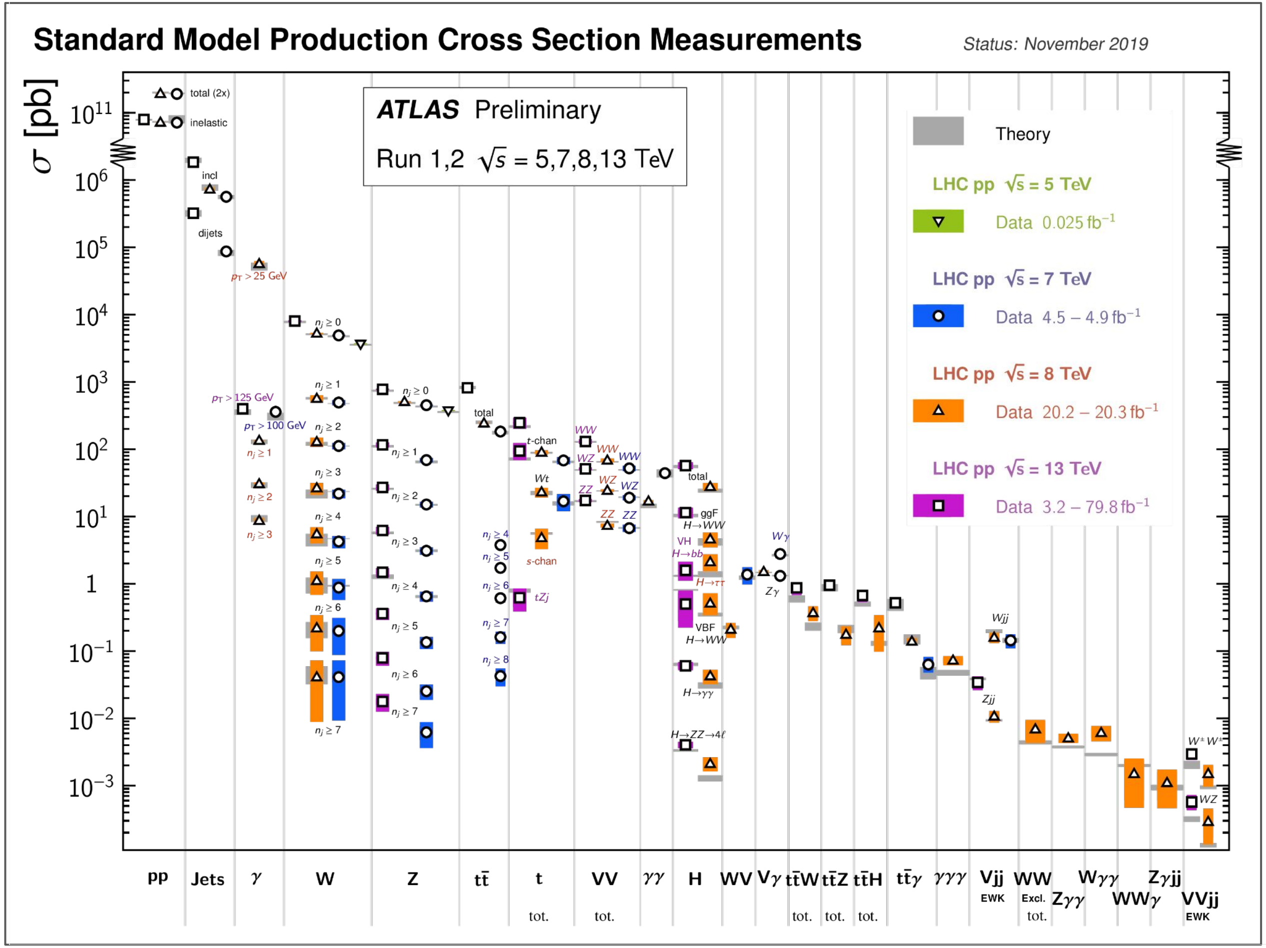
Florian Herren

2208.06057, in collaboration with Stefan Höche, Frank Krauss, Daniel Reichelt & Marek Schönherr

Quest for precision

Measurements and theory predictions reached incredible levels of precision

However, with increasing statistics theoretical uncertainties will become dominant for many processes



https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults

Event Generators

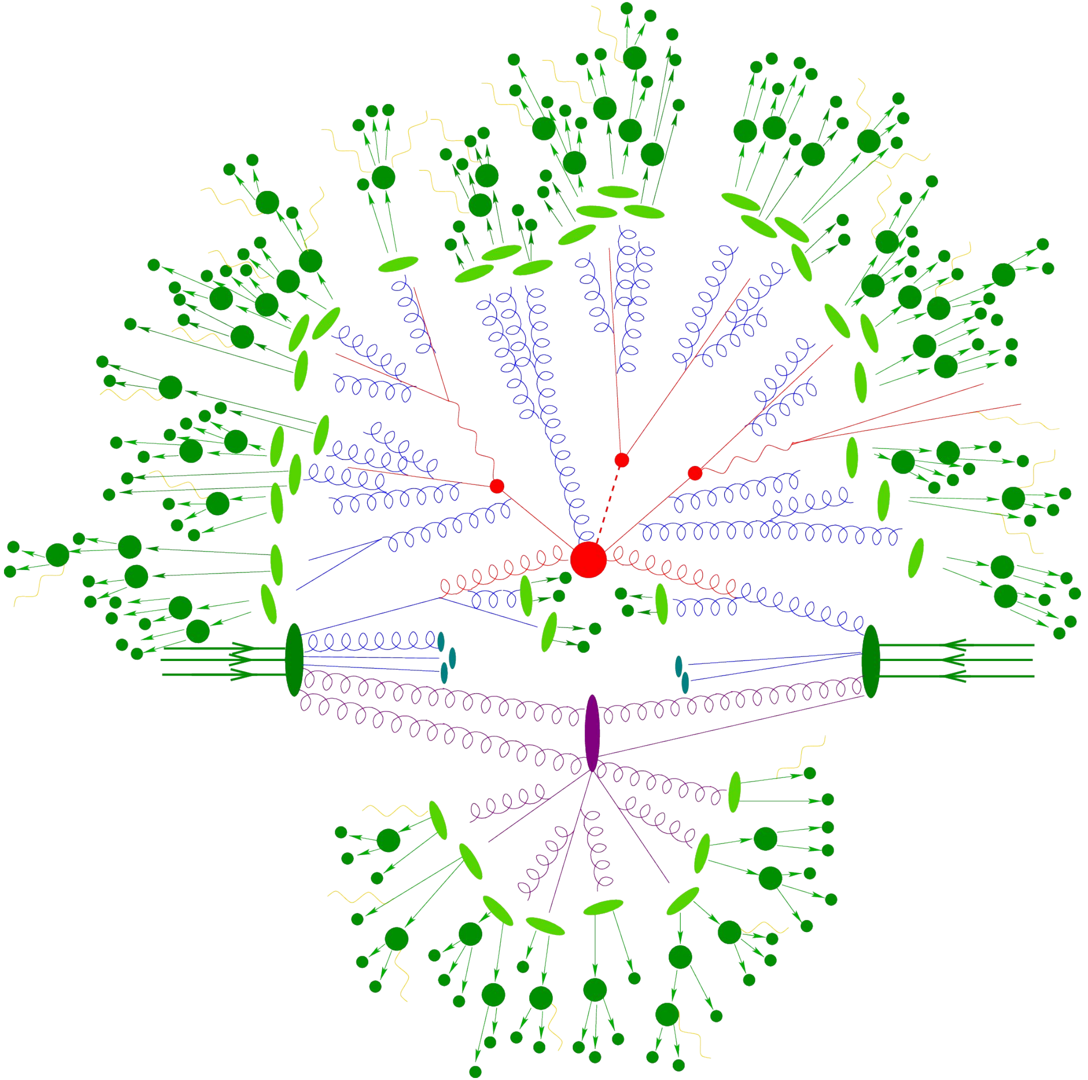
Crucial for precision Collider Physics

Short distance physics:

- Hard Process
- Parton Shower

Long distance physics:

- Underlying Interaction
- Hadronization
- QED FSR
- Hadron Decays



Event Generators

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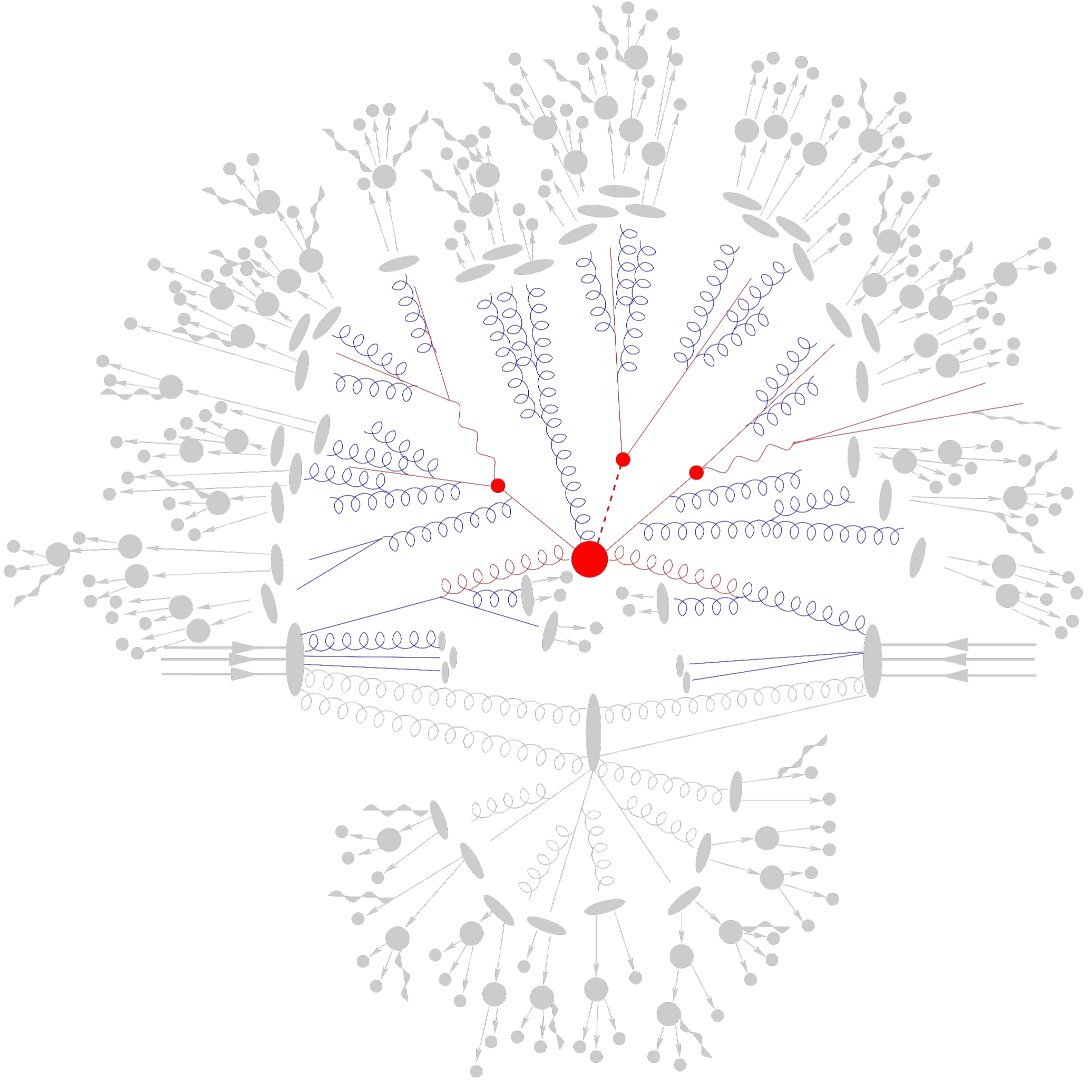


Timeline

Color coherent evolution
Backward evolution

ME corrections
Color dipole formalism
Spin correlations

Color dipole showers

LO merging
NLO matching

Dipole-like showers
Antenna showers
NLO merging

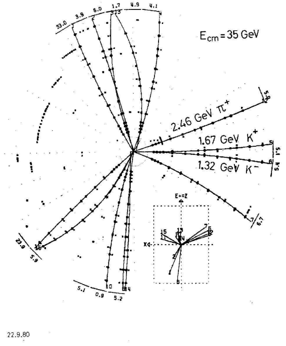
Color ME corrections
NNLO matching

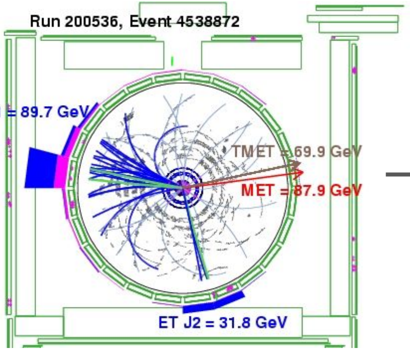
Threshold effects
NLO splitting kernels
Higher-order PS framework

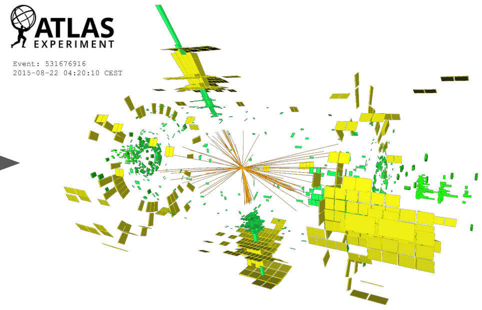
Accuracy quantification
Amplitude evolution

NLL & NGLs in one framework
Efficient 1/Nc corrections

1985 1990 1995 2000 2005 2010 2015 2018 2022

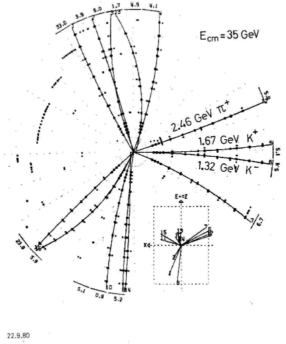


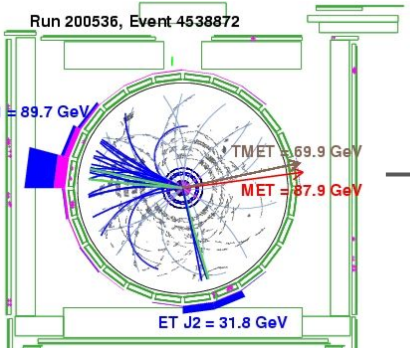


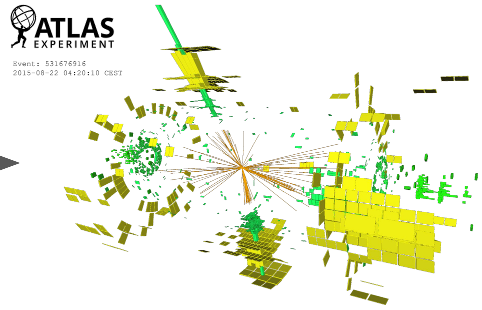


Timeline

- 1985: Color coherent evolution; Backward evolution; ME corrections; Color dipole formalism; Spin correlations
- 1990: Color dipole showers
- 1995
- 2000: LO merging; NLO matching
- 2005: Dipole-like showers; Antenna showers; NLO merging
- 2010: Color ME corrections; NNLO matching
- 2015: Threshold effects; NLO splitting kernels; Higher-order PS framework
- 2018: Accuracy quantification; Amplitude evolution
- 2022: NLL & NGLs in one framework; Efficient 1/Nc corrections

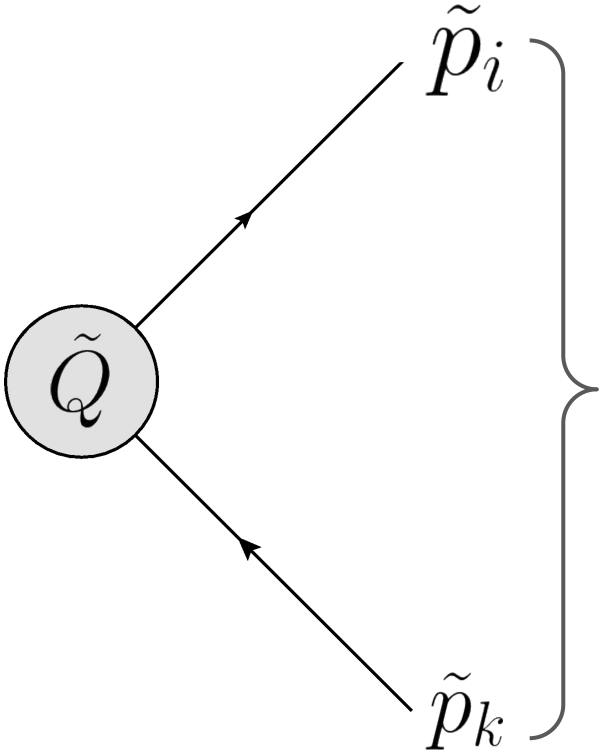






Parton Showers

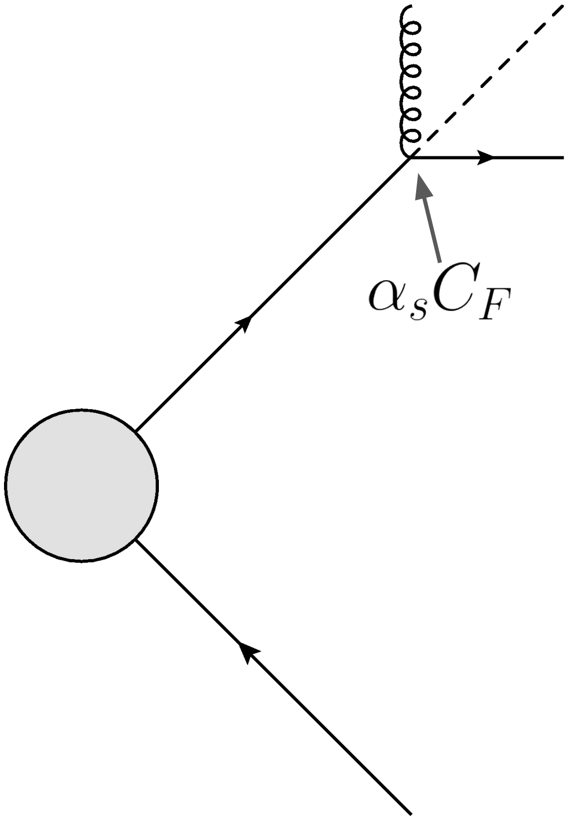
Start with fixed order configuration,
e.g. ee ⟶ qq, qq ⟶ ll, eq ⟶ eq



Color dipole

Parton Showers

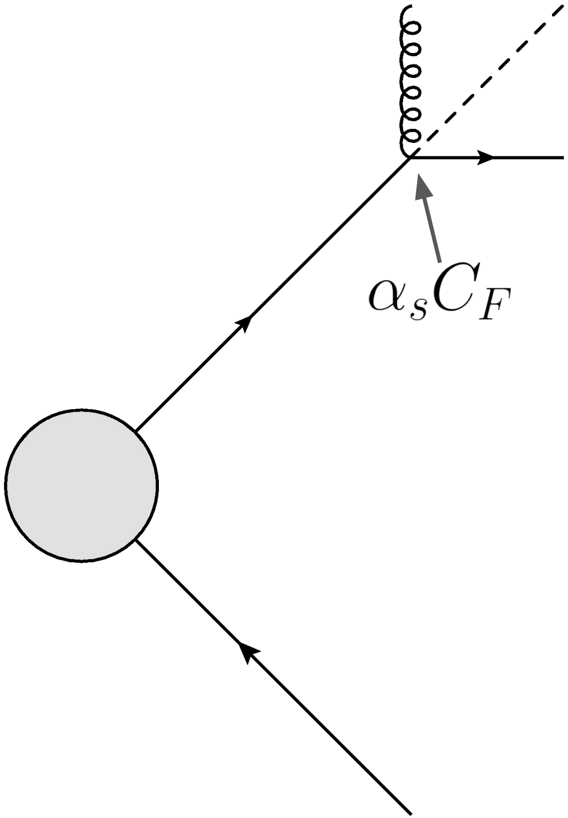
Add one gluon emission



Gluon emission most likely in singular limits!

Parton Showers

Add one gluon emission



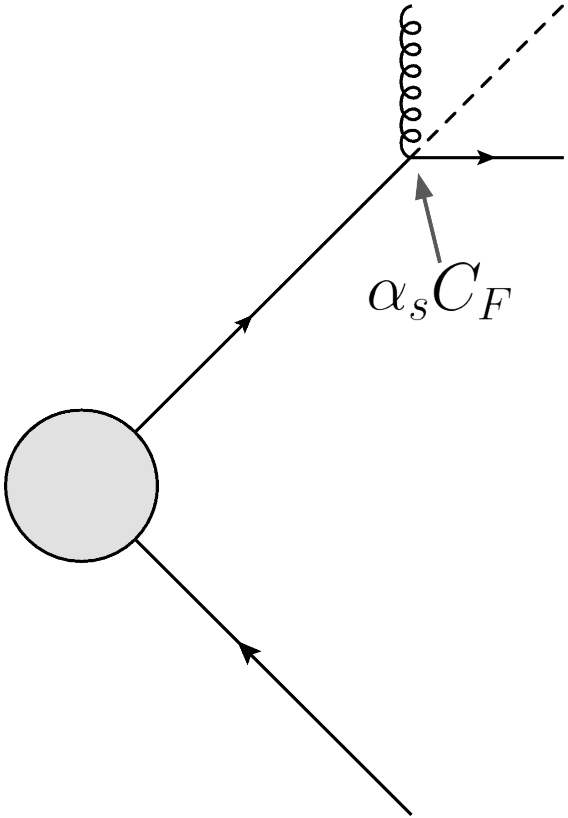
Gluon emission most likely in singular limits!

Gluon soft: $|\mathcal{M}_{qqg}|^2 \approx 2\frac{p_i p_k}{(p_i p_j)(p_j p_k)}|\mathcal{M}_{qq}|^2$

Gluon collinear: $|\mathcal{M}_{qqg}|^2 \approx \frac{P_{qq}(z)}{2(p_i p_j)}|\mathcal{M}_{qq}|^2$

Parton Showers

Add one gluon emission



Gluon emission most likely in singular limits!

Depends on both dipole members

Gluon soft: $|\mathcal{M}_{qqg}|^2 \approx 2\frac{p_i p_k}{(p_i p_j)(p_j p_k)}|\mathcal{M}_{qq}|^2$

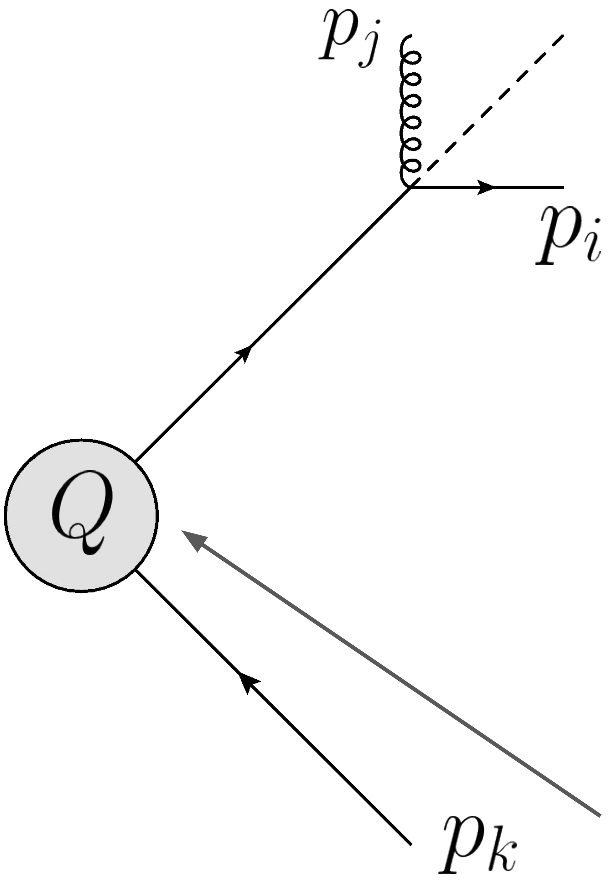
Gluon collinear: $|\mathcal{M}_{qqg}|^2 \approx \frac{P_{qq}(z)}{2(p_i p_j)}|\mathcal{M}_{qq}|^2$

Depends only on radiating parton

Problem #1:
Double counting in iterated limit!

Parton Showers

Add one gluon emission



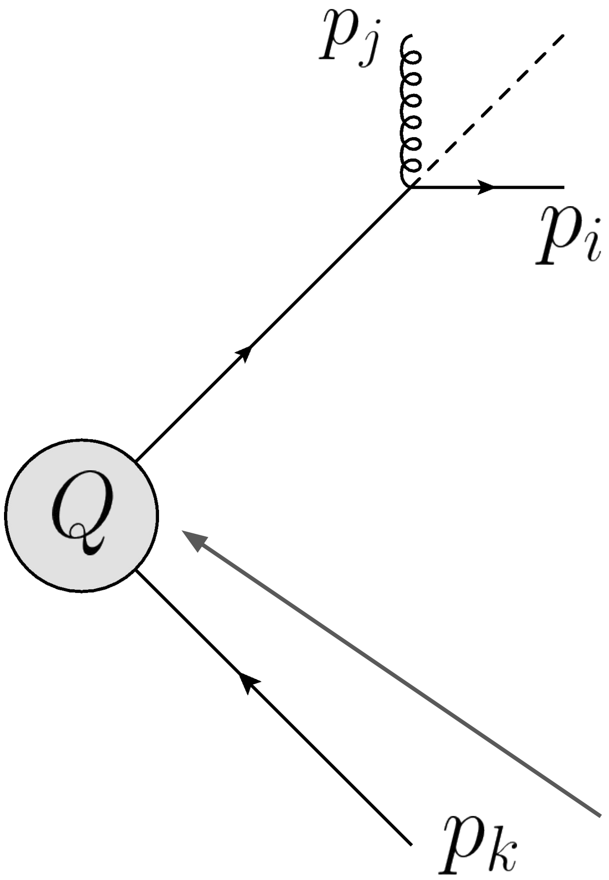
Need momentum mapping between on-shell momenta:

$$\tilde{p}_i,\ \tilde{p}_k \;\Rightarrow\; p_i,\ p_k,\ p_j$$

Also $\tilde{Q}$ may change

Parton Showers

Add one gluon emission



Need momentum mapping between on-shell momenta:

$$\tilde{p}_i, \tilde{p}_k \Rightarrow p_i, p_k, p_j$$

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Problem #2:
Suitable momentum mapping

Conditions:

$$p_i \to z\tilde{p}_i$$

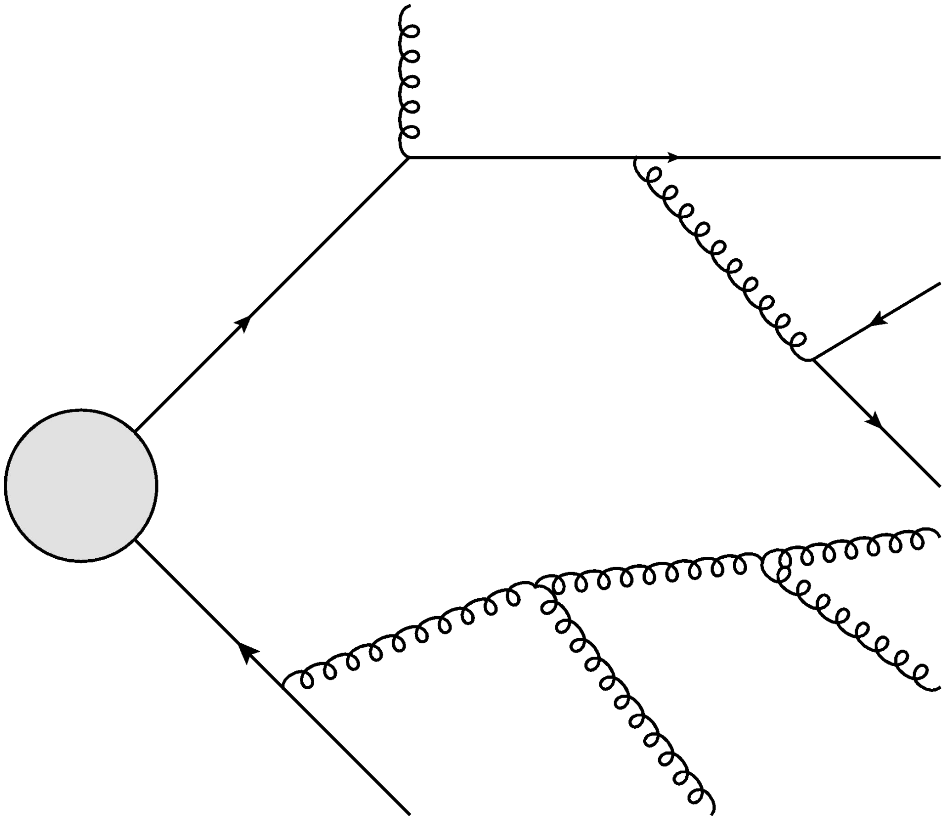
$$p_j \to (1-z)\tilde{p}_i$$

in collinear limit, and

$$Q^2 = \tilde{Q}^2$$

Parton Showers

Repeatedly add emissions



Problem #3:
When do we stop?
→ Evolution variable

Problem #4:
Evolution resums large logarithms, but at which accuracy?

Problem #5:
How do we handle NLO calculations?

NLL Showers

Criteria for NLL accuracy at leading color outlined in:
[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] 2002.11114

Where do the logarithms come from?
(see also [Banfi, Salam, Zanderighi] hep-ph/0407286)

Depends on logarithmic variables of emission pairs:

Energies/Angles	Distinctly different	Comparable
Distinctly different	LL	NLL
Comparable	NLL	NNLL

Shower needs to reproduce the correct tree-level ME squared in these regions

Shower needs to reproduce results of analytic resummation of rIRC observables

Soft Radiation

Factorisation in the soft limit:

$$ {}_n\langle 1,\dots,n|1,\dots,n\rangle_n = -8\pi\alpha_s \sum_{i,k\neq j} {}_{n-1}\langle 1,\dots,\not{j},\dots,n\big|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}\big|1,\dots,\not{j},\dots,n\rangle_{n-1} $$

Eikonal factor:

$$ w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2}\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} $$

Implementing the Eikonal in the collinear limit leads to double-counting of soft singularities

[Marchesini, Webber] *Nucl.Phys.B* 310 (1988) 461-526

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$W_{ik,j}$

Additive matching of singularities:

$$ W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j} $$

$$ \tilde{W}^i_{ik,j} = \frac{1}{2}\left(W_{ik,j} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}} \right) $$

Eikonal factor:

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With p_i on z-axis:

$$\cos\theta_{jk} = \cos\theta^i_j\cos\theta^i_k + \sin\theta^i_j\sin\theta^i_k\cos\phi^i_{jk}$$

Soft Radiation

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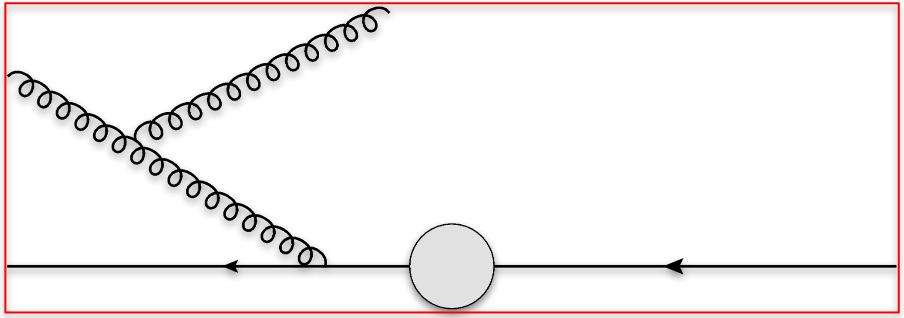
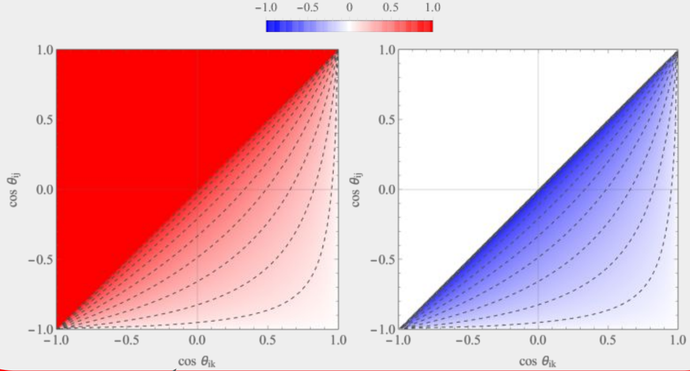


Option 2: Implement radiator differentially



Option 1:
Angular Ordering → Spoils NGLs

Azimuthal average:

$$ \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\tilde{W}^i_{ik,j} = \frac{\theta(\theta_{ik}-\theta_{ij})}{1-\cos\theta_{ij}} $$

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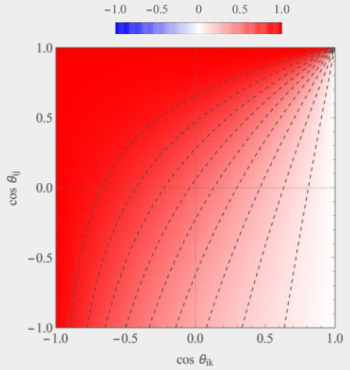
Multiplicative matching of singularities:

$$ W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} $$

$$ \bar{W}^i_{ik,j} = W_{ik,j}\frac{1-\cos\theta_{jk}}{2-\cos\theta_{ij}-\cos\theta_{jk}} $$

[Catani, Seymour] hep-ph/9605323

Implement radiator differentially



Azimuthal average:

$$ \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\bar{W}^i_{ik,j} = \frac{1}{\sqrt{(A^i_{ik,j})^2-(B^i_{ik,j})^2}} $$

$$ A^i_{ij,k} = \frac{2-\cos\theta_{ij}(1+\cos\theta_{ik})}{1-\cos\theta_{ik}} $$

$$ B^i_{ij,k} = \frac{\sqrt{(1-\cos^2\theta_{ij})(1-\cos^2\theta_{ik})}}{1-\cos\theta_{ik}} $$

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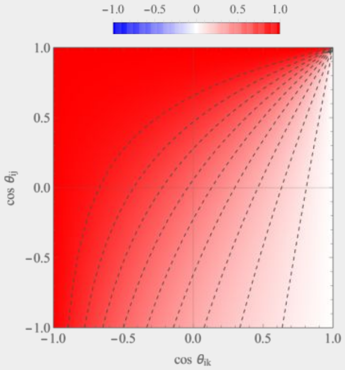


$$ \frac{1}{2p_ip_j}P_{(ij)i}(z) \to \frac{1}{2p_ip_j}P_{(ij)i}(z)+\delta_{(ij)i}\left[\frac{\bar{W}^i_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right] $$

Splitting functions depend on direction of color spectator! N.b.: only leading color

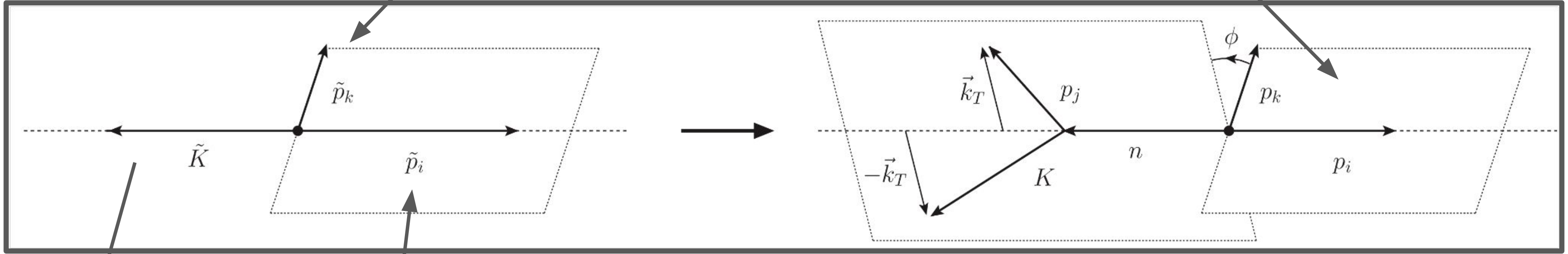
Momentum Mapping

Main Idea:
maintain directions of hard particles exactly

$$p_i = z\tilde{p}_i \qquad z = \frac{p_i n}{(p_i + p_j)n}$$

$$p_k = \tilde{p}_k$$

Color Spectator



Hard system

Emitter

Need to find K and p_j such that:

$$K^2 = \tilde{K}^2 \quad p_j \to (1-z)\tilde{p}_i$$

Shift:
$n = \tilde{K} + (1-z)\tilde{p}_i$

Momentum Mapping

Main Idea:
maintain directions of hard particles exactly

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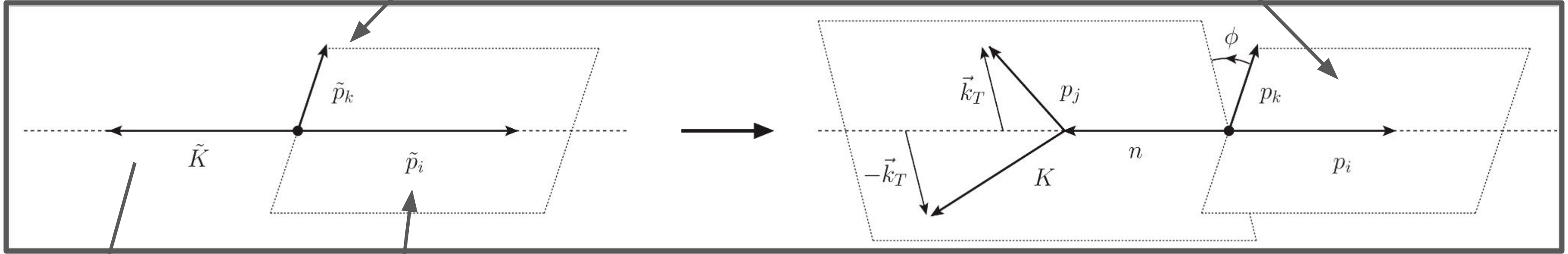
$$p_k = \tilde{p}_k$$

Color Spectator



Hard system

Emitter

$$v = \frac{p_i p_j}{p_i \tilde{K}} \qquad \kappa = \frac{\tilde{K}^2}{2\tilde{p}_i \tilde{K}}$$

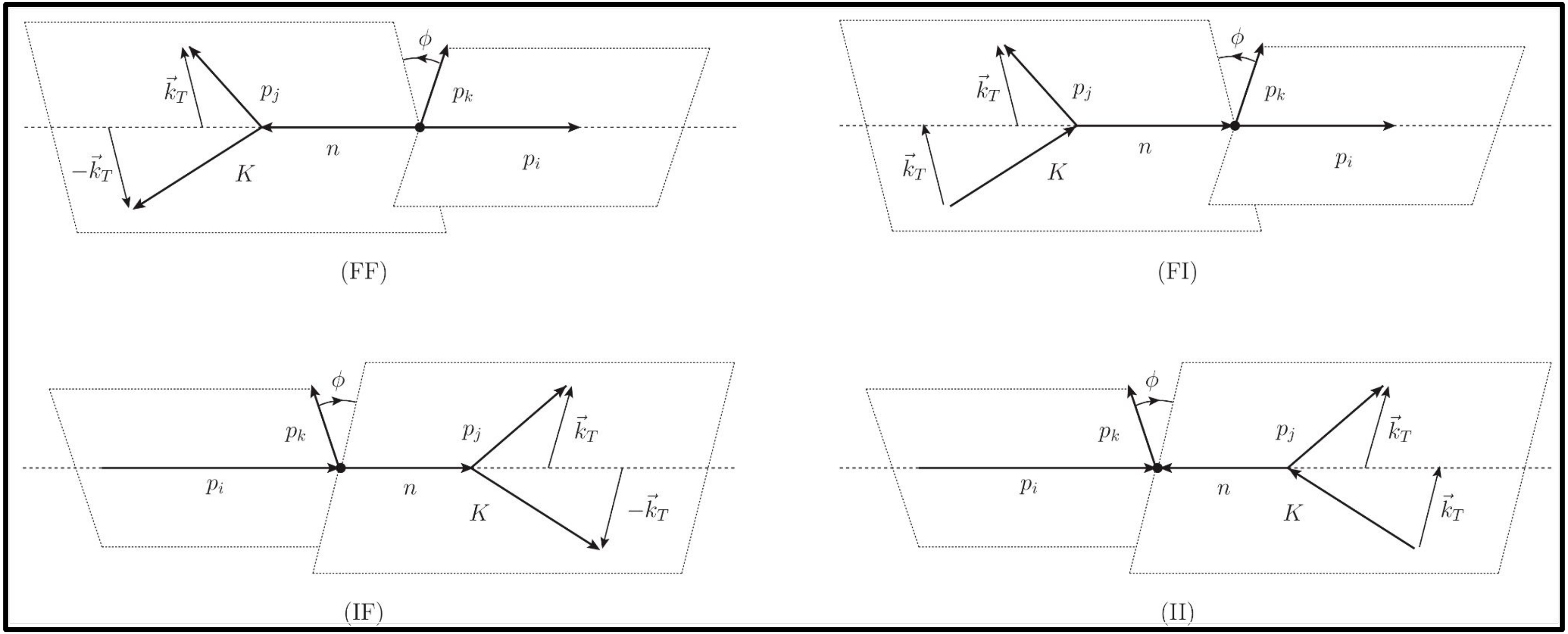
$$p_j = (1-z)\tilde{p}_i + v\left(\tilde{K} - (1 - z + 2\kappa)\tilde{p}_i\right) + k_\perp$$
$$K = \tilde{K} - v\left(\tilde{K} - (1 - z + 2\kappa)\tilde{p}_i\right) - k_\perp$$

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Recoil

Momentum mapping works for initial and final state emitters/spectator
→ e+ e-, pp, DIS, … all treated on same footing



Recoil

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Define

$$X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$
$$= v\big(\tilde{K}^\mu - (1-z+2\kappa)\,\tilde{p}_i^\mu\big) + k_\perp^\mu$$

At most $\mathcal{O}(k_\perp)$ in logarithmically enhanced region

Recoil

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$$\Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right] \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

Recoil

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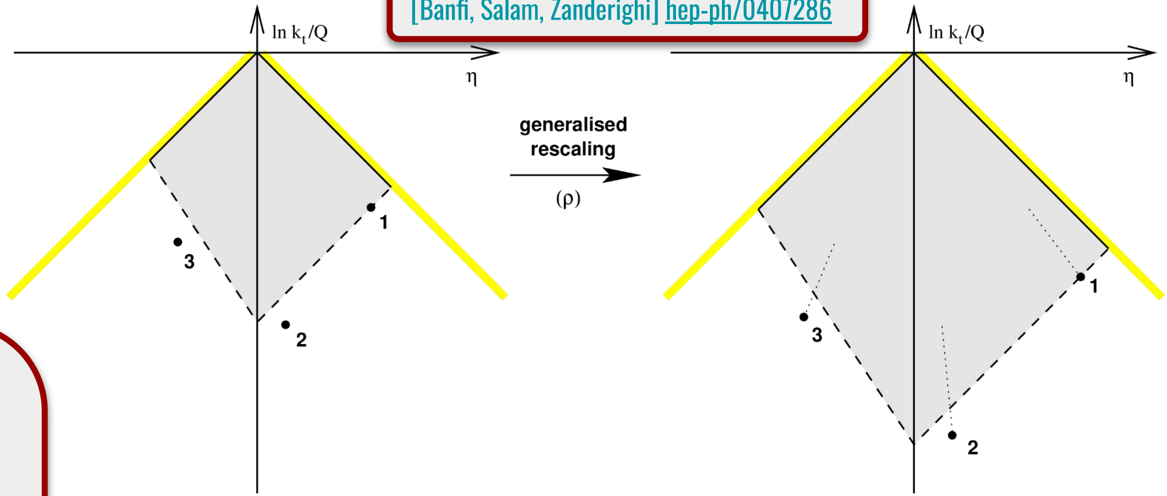
Suppressed by

$$\mathcal{O}(k_\perp/K)$$

$$\Lambda^\mu_\nu \approx g^\mu_\nu + \frac{K_\rho X_\sigma}{K^2}T^{\mu\rho\sigma}_\nu + \mathcal{O}(k_\perp^2)$$

Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286



For one emission kinematic variables in the Lund plane scale like:

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}$$

$$\xi_l = \frac{\eta_l}{\eta_{l,\max}}$$

where a = 1 and b = 0 for Alaric

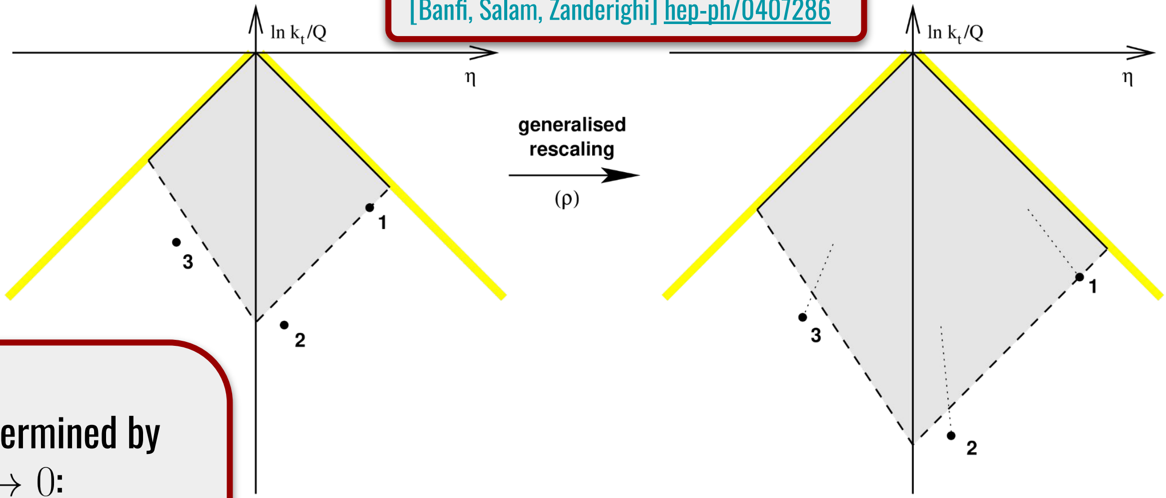
Working in the rest frame of the color dipole, the other momenta scale like:

$$\tilde{p}_l^0 \sim \rho^{1-\xi_l}$$
$$\tilde{p}_l^{1,2} \sim \rho$$
$$\tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

for $\rho \to 0$

Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286



Scaling under an additional emission is determined by the Lorentz transformation in the limit $\rho \to 0$:

$$\Delta p_l^\mu = 2\frac{\tilde{K}X}{\tilde{K}^2}\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\tilde{K}^\mu - \frac{\tilde{p}_l X}{\tilde{K}^2}\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}X^\mu$$

Scaling becomes:

$$\Delta p_l^0 \sim \rho^{1-\xi_l}X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)}\tilde{K}^0 + \rho^{1-\xi_l}X^0 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^0 \sim \rho^{1-\xi_l}$$

$$\Delta p_l^{1,2} \sim \rho^{1-\xi_l}X^{1,2} \sim \rho^{2-\xi_l} \qquad \tilde{p}_l^{1,2} \sim \rho$$

$$\Delta p_l^3 \sim \rho^{1-\xi_l}X^3 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

Evolution

Evolution variable:
$t = 2E_j^2(1-\cos\theta_j^i) = v(1-z)2\tilde{p}_i\tilde{K}$
Corresponds to Lund plane $k_t^2 \to \beta_{\rm PS} = 0$ in the generalized rescaling limit

Rewrite partial radiators:

$$\bar{w}_{ik,j}^i = \frac{\bar{W}_{ik,j}^i}{E_j^2} = \frac{\bar{W}_{ik,j}}{p_i p_j}$$

$$\bar{W}_{ik,j}^i = \frac{z}{1-z}(1-\cos\theta_j^i)\bar{W}_{ik,j}$$

Bounded from above by 2
$\bar{w}_{ik,j}^i \leq 2w_{ik,j}^{(\rm coll)}$

Mild dependence on azimuthal angle

Differential splitting probabilities:

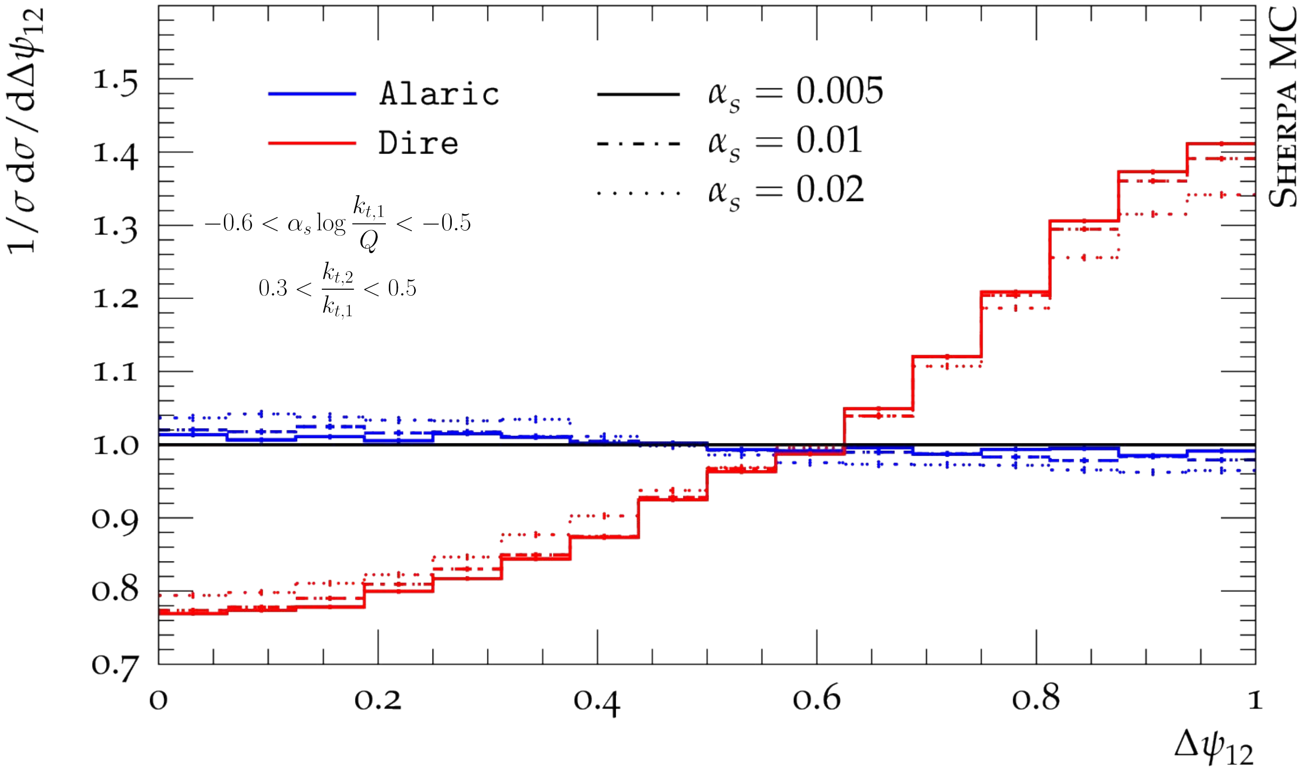
$$\mathrm{d}P_{ik,j}^{i\,(\rm soft)}(t,z,\phi) = \mathrm{d}t\ \mathrm{d}z\ \frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_s}{2\pi t}2C_i\bar{W}_{ik,j}$$

$$\mathrm{d}P_{ik,j}^{i\,(\rm coll)}(t,z,\phi) = \mathrm{d}t\ \mathrm{d}z\ \frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_s}{2\pi t}C_{\tilde{i}i}$$

Soft-subtracted splitting functions

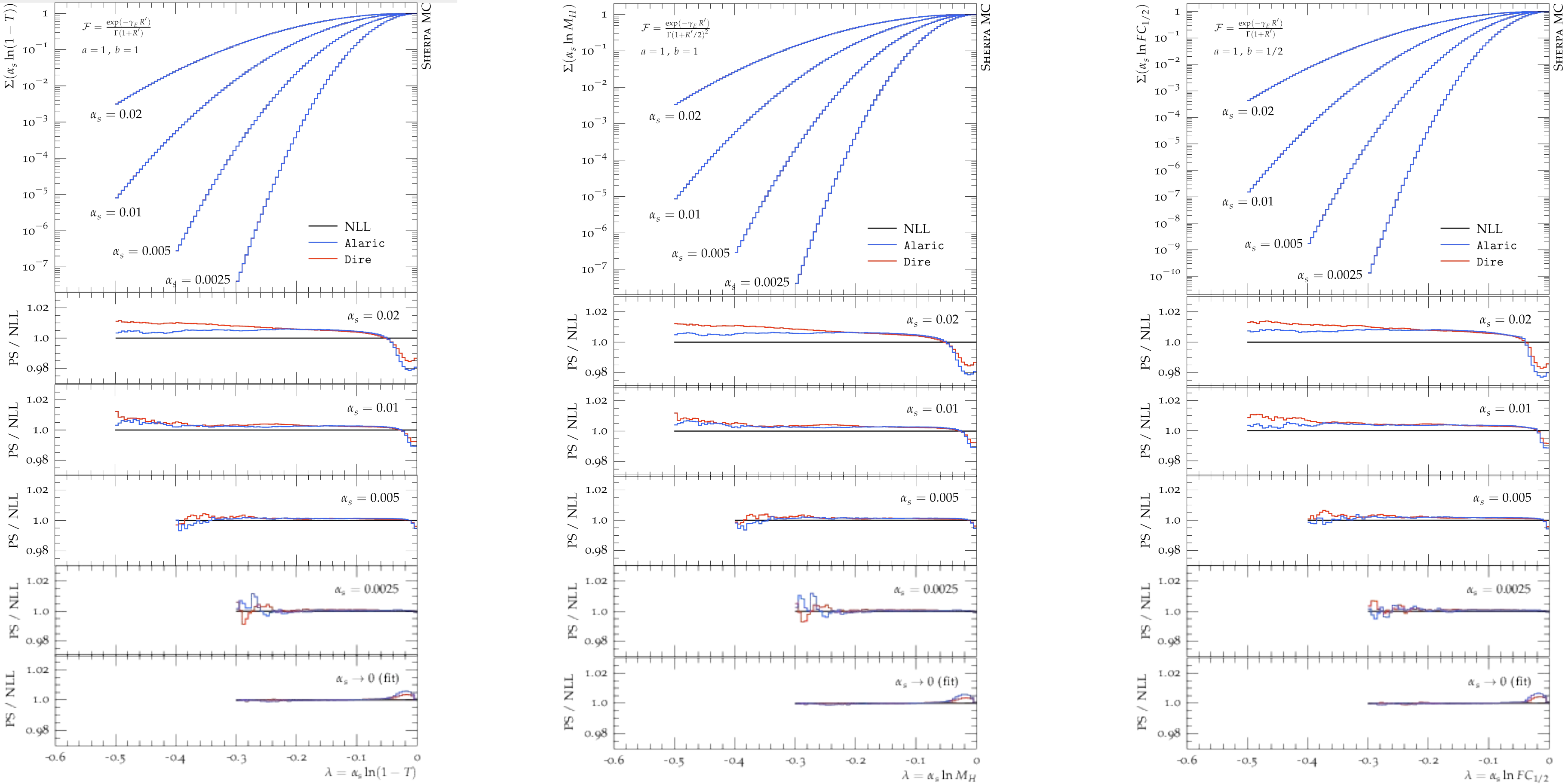
Numerical Tests

Azimuthal angle between two Lund plane declusterings
Tests soft and rapidity separated emissions



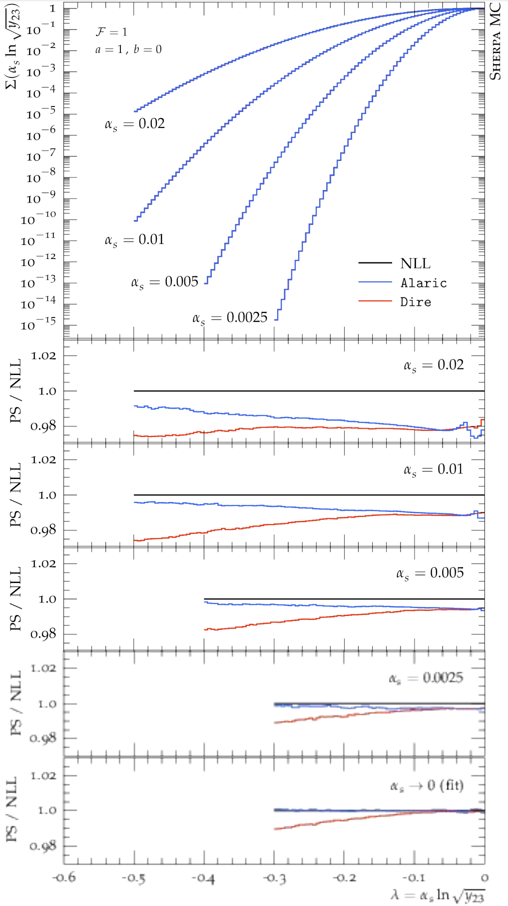
Numerical Tests

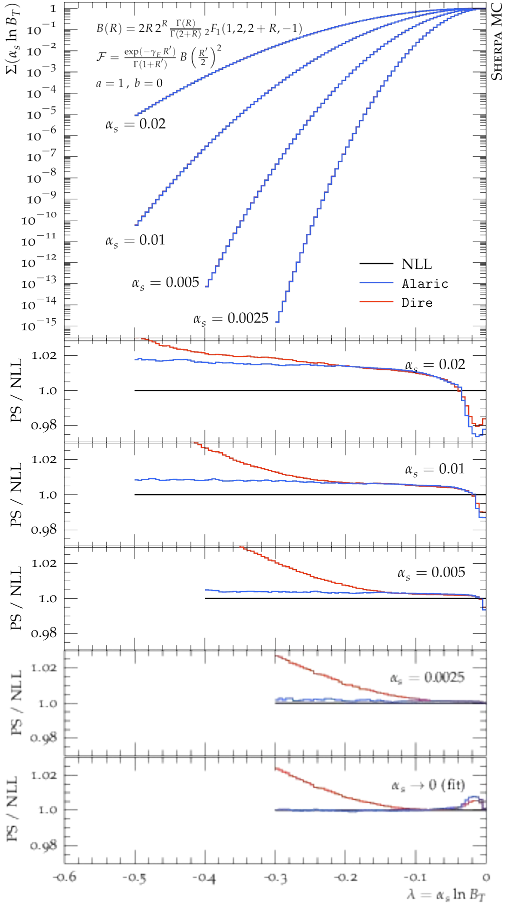
For Thrust, Heavy Jet mass and Fractional Energy Correlators with b = 1,
Alaric is NLL and Dire is indistinguishable from NLL

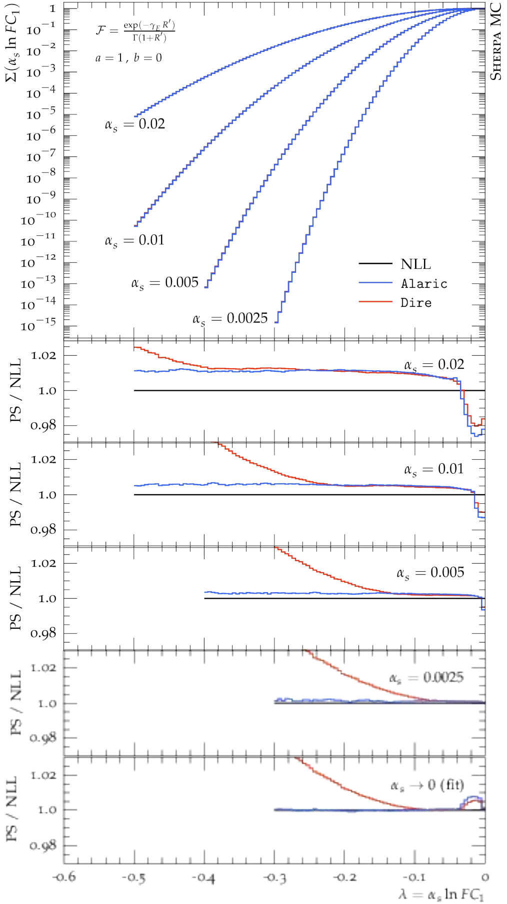


Numerical Tests

For the Two-Jet rate, total Broadening and FC with b = 1
Alaric and Dire differ, here only Alaric is NLL accurate







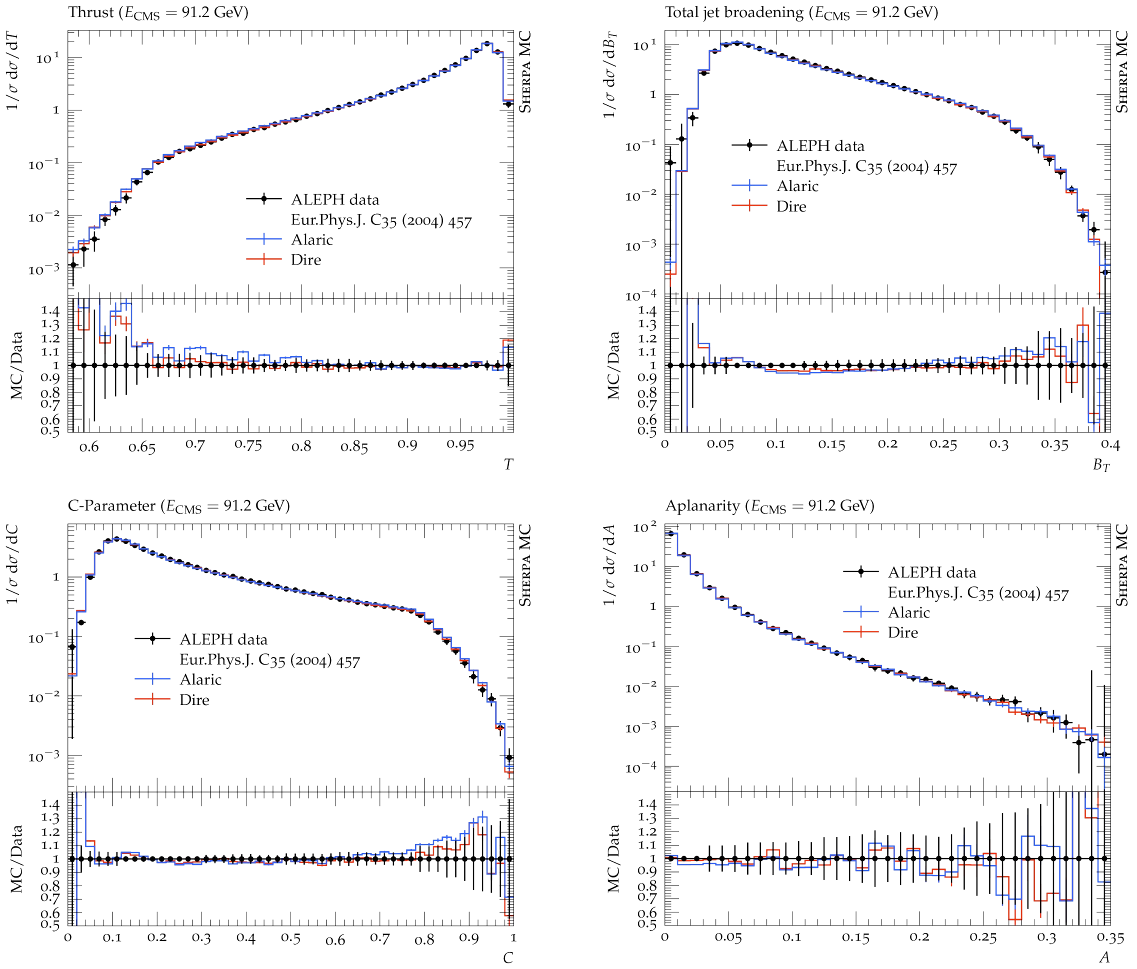
Let’s look at Data

Details:

- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Perturbative region to the right, except for thrust
- Some deviations for Broadening and Aplanarity
- MECs and massive quarks will improve situation



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Details:

- CMW scheme
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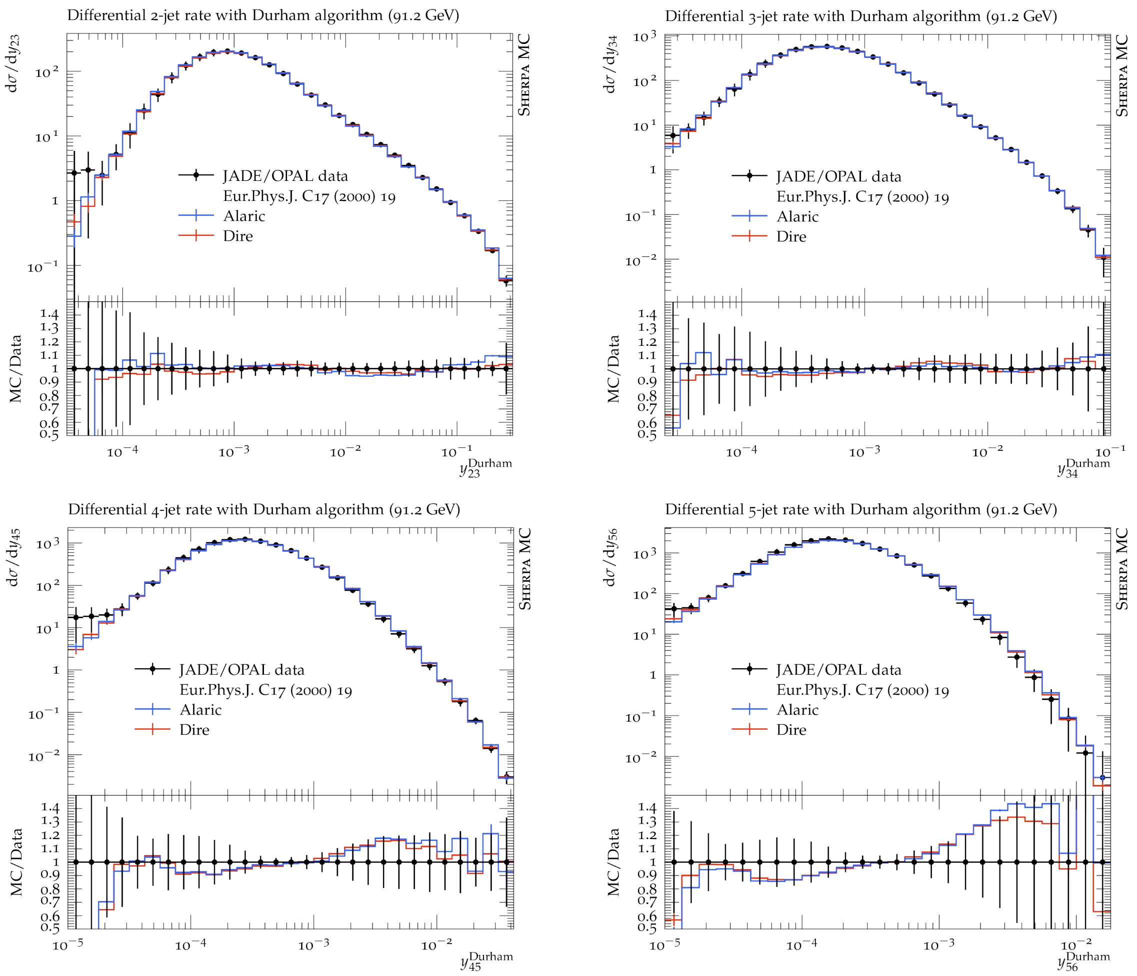
Comments:

- Perturbative region to the right
- b-quark mass corresponds to $y \approx 2.8 \times 10^{-3}$



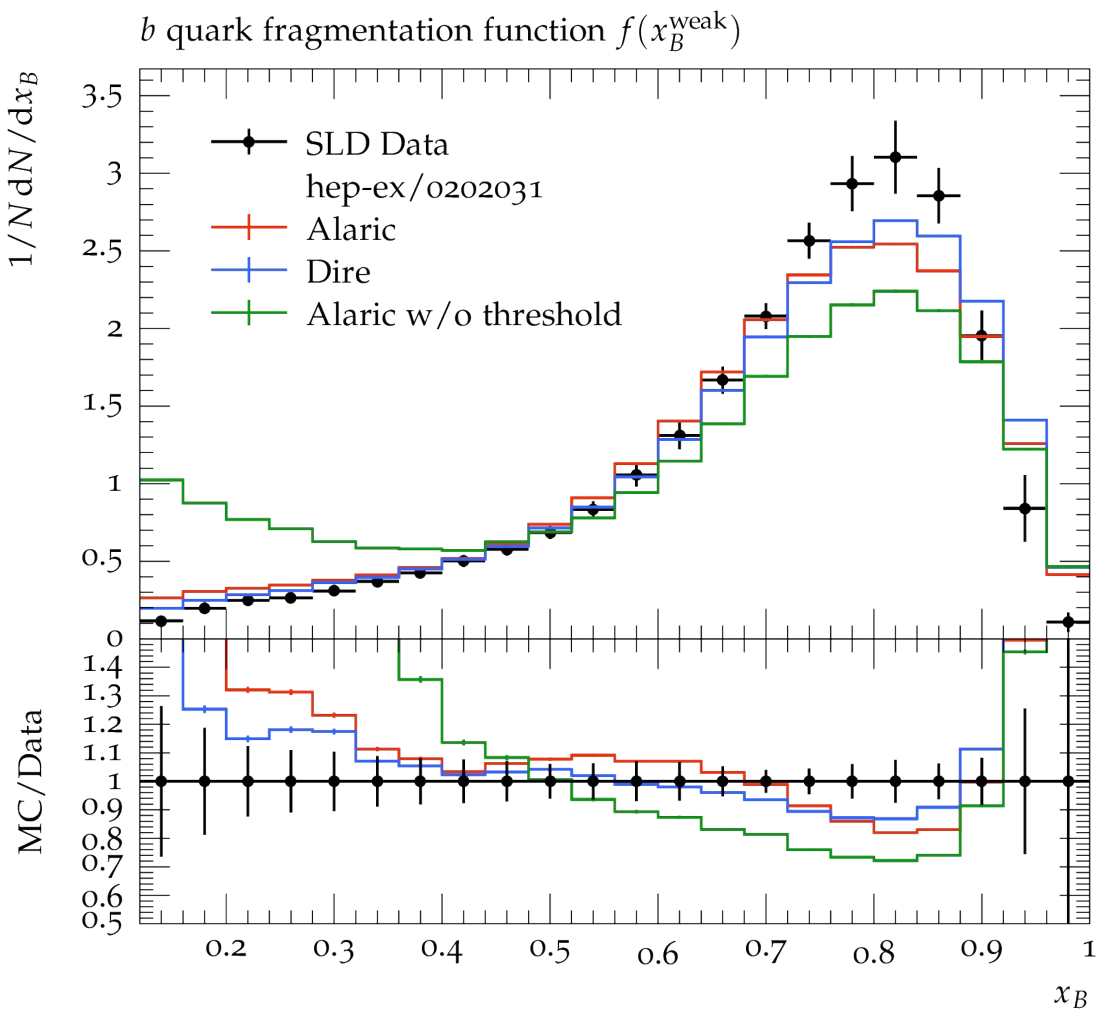
Let's look at Data

Details:

- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Low values of x dominated by $g \to bb$
- Large values of x dominated by $b \to bg$ and hadronization



NLO Matching

If we compare NLO calculation and PS expanded to first order in the strong coupling:

- NLO calculation: contains virtual corrections, one hard, soft or collinear emission
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$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_n \, [B + V] + \int \mathrm{d}\Phi_{n+1} \, R$$

Explicit poles

Divergent in soft and collinear limits

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$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_n \left[B + V + \int \mathrm{d}\Phi_{+1} S\right] + \int \mathrm{d}\Phi_{n+1} \left[R - S\right]$$

Depends on momentum mapping, poles need to be made explicit

Contains Eikonal and splitting functions

Two common schemes:

- [Catani, Seymour] hep-ph/9605323
- [Frixione, Kunszt, Signer] hep-ph/9512328

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In the MC@NLO scheme, the subtraction terms are chosen to be the PS evolution kernels → Need to compute integrated terms with our momentum mapping

[Frixione, Webber] hep-ph/0204244

NLO Matching

Alaric shares many similarities with Catani-Seymour identified particle subtraction
$\rightarrow$ MC@NLO matching straightforward
Follow [Höche, Liebschner, Siegert] 1807.04348

Combined integrated subtraction term for identified parton production with a partonic fragmentation function:

$$\int_{m+1} \mathrm{d}\sigma^S + \int_m \mathrm{d}\sigma^C = \frac{1}{2} \sum_{i=g,q,\bar{q}} \sum_{\tilde{\imath}=1}^{m} \int_0^1 \frac{\mathrm{d}z}{z^{2-2\epsilon}} \int_m \mathrm{d}\sigma^B\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m\right) \otimes \hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})}$$

Insertion operator:

$$\hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})} = \delta(1-z)\mathbf{I}_{\tilde{\imath}i} + \mathbf{P}_{\tilde{\imath}i} + \mathbf{H}_{\tilde{\imath}i}$$

$$\mathbf{I}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; \epsilon) = -\frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left(\frac{4\pi\mu^2}{2p_i p_k}\right)^{\epsilon} \mathcal{V}_{\tilde{\imath}i}(\epsilon)$$

$$\mathbf{P}_{\tilde{\imath}i}\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m; z; \mu_F\right) = \frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \ln \frac{z\mu_F^2}{2p_i p_k} \delta_{\tilde{\imath}i} P_{\tilde{\imath}i}(z)$$

$$\mathbf{H}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left[\tilde{K}^{\tilde{\imath}i}(z) + \bar{K}^{\tilde{\imath}i}(z) + 2P_{\tilde{\imath}i}(z)\ln z + \mathcal{L}^{\tilde{\imath}i}(z; p_i, p_k, n)\right]$$

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Non-trivial integral:

$$\int_0^1 \mathrm{d}z\, \mathbf{H}_{\tilde{i}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{i}}^{m} \frac{\mathbf{T}_{\tilde{i}}\mathbf{T}_k}{\mathbf{T}_{\tilde{i}}^2} \left\{ \mathcal{K}^{\tilde{i}i} + \delta_{\tilde{i}i} \operatorname{Li}_2\left(1 - \frac{2\tilde{p}_i\tilde{p}_k\, \tilde{K}^2}{(\tilde{p}_i\tilde{K})(\tilde{p}_k\tilde{K})}\right) - \int_0^1 \mathrm{d}z\, P_{\mathrm{reg}}^{qq}(z) \ln \frac{n^2 \tilde{p}_i\tilde{p}_k}{2z(\tilde{p}_i n)^2} \right\}$$

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Evaluating the soft counterterm tells us something about efficient scale choices. In our case we obtain:

$$I_{\text{soft}} \propto \left(\frac{\mu^2 (p_k n)}{(p_i p_k)(p_i p_n)}\right)^{\epsilon} \propto \left(\frac{\mu^2}{E_i^2 (1 - \cos\theta_k^i)}\right)^{\epsilon} \propto \left(\frac{\mu^2}{t}\right)^{\epsilon}$$

→ The logarithms resummed by the RG-evolution are large when the soft parton is emitted from a Dipole that originates from a soft or collinear splitting of k → k+i and correspond to the respective $k_\perp$

→ We can minimize the number of explicit higher-order corrections by choosing t as the renormalization scale

Further Developments

<u>Massive Quarks:</u>

- Extension of original Algorithm for massive Quarks in the final state
- Treat quasi-collinear emission separate from soft emission → Splitting & Radiation kinematics
- Momentum mapping extended & MC@NLO matching terms computed
- NLL argumentation similar to massless case

[Assi, Höche] 2307.00728

<u>Hadron colliders:</u>

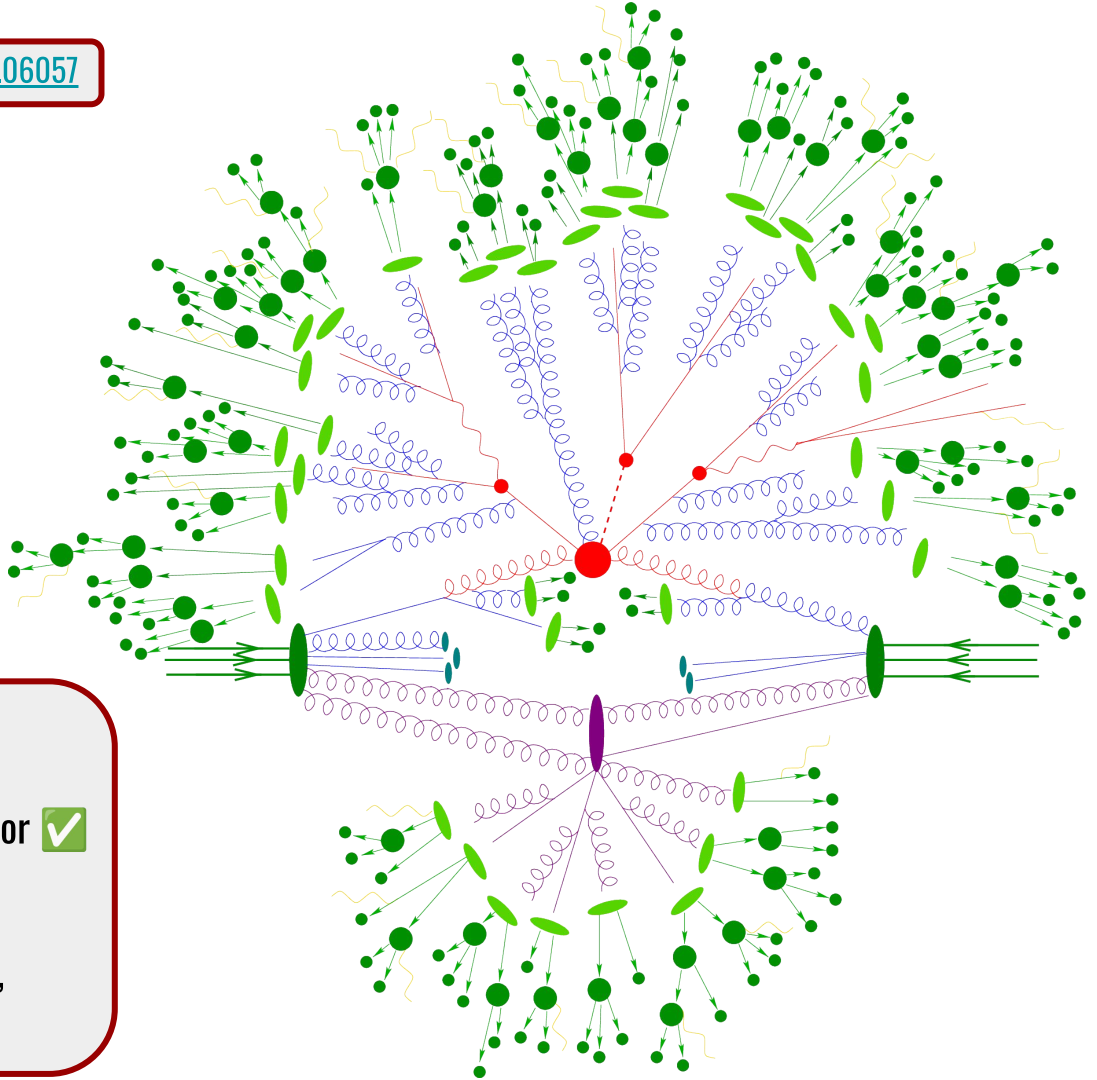
- Publicly available implementation in Sherpa 3 release
- Leading-order multi-jet merging
- Tested on Drell-Yan, Z+jets, inclusive jet production & di-jets
- WIP 1: Implementation of MC@NLO matching
- WIP 2: Multi-jet merging @ NLO

[Höche, Krauss, Reichelt] 2404.14360

Conclusion

2208.06057

- We presented a new NLL accurate Parton Shower Algorithm: Alaric
- First dipole-like algorithm to disentangle colour and kinematics
- Strict positivity of evolution kernels
- Momentum mapping preserves directions of hard partons



Additional developments:

- Initial state emitter and spectator ✅
- Initial state emitter and final state spectator ✅
- Implementation in Sherpa ✅
- Massive quarks ✅

TODO: higher order corrections, spin correlations, subleading colour,...