New insights into N-jettiness computations

TUM/Max-Planck seminar series

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Introduction

Higher-order QCD corrections (at NNLO)

Subtraction methods

Analytically cancel $1/\epsilon^n$ poles by constructing integrable counterterms

Slicing methods

Imposes cuts in some variable to split the phase space. Below the cut a soft-collinear approximation is used

$$\int_0^1 \frac{dx}{x^{1+\nu}} f(0) + \int_0^1 \frac{dx}{x}\left[f(x) - f(0)\right]$$

$$\int_0^\delta \frac{dx}{x^{1+\nu}} f(0) + \int_\delta^1 \frac{dx}{x} f(x)$$

$$\int_0^1 \frac{dx}{x^{1+\nu}} f(x)$$

Higher-order QCD corrections (at NNLO)

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Subtraction methods

Analytically cancel $1/\epsilon^n$ poles by constructing integrable counterterms

- **Antenna subtraction**
  Gehrmann-De Ridder, Gehrmann, Glover - hep-ph/0505111
- **CoLoRFul subtraction**
  Somogyi, Trócsányi, Del Duca - hep-ph/0502226
- **Local analytic sector subtraction**
  Magnea et al. - hep-ph/1806.09570
- **Nested soft-collinear subtraction**
  Caola, Melnikov, Röntsch – hep-ph/1702.0135220
- **Projection-to-Born**
  Cacciari et al. - hep-ph/1506.02660
- **Sector subtraction**
  Czakon - hep-ph/1005.0274, Boughezal et al. - hep-ph/1111.7041

Slicing methods

Imposes cuts in some variable to split the phase space. Below the cut a soft-collinear approximation is used

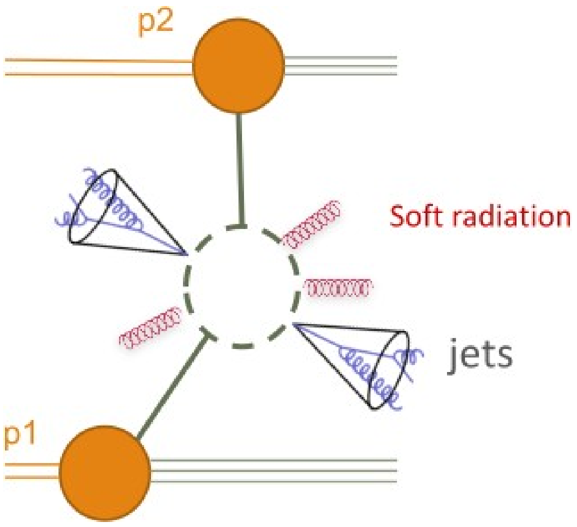
- **q_T-slicing**
  Catani, Grazzini - hep-ph/0703012
- **N-jettiness slicing**
  Boughezal et al. - hep-ph/1504.02131, Gaunt et al. - hep-ph/1505.04794

And many more not included here…

N-jettiness slicing

The N-jettiness variable is defined by

$$\mathcal{T}(\mathcal{R},\mathcal{U}) = \sum_{x\in\mathcal{U}} \min\left\{\frac{2p_x p_{h_1}}{Q_1}, \frac{2p_x p_{h_2}}{Q_2}, \frac{2p_x p_{h_3}}{Q_3}, \ldots\right\}$$



Can be used to perform slicing of the phase space (like in q_T subtraction)

$$\sigma = \int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} + \int_{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}}$$

and, with the factorization theorem from **SCET**, we can reorganize the calculation as

$$\int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} = \int B \otimes B \otimes S \otimes H \otimes \prod_i^N J_i + \mathcal{O}(\mathcal{T}_0)$$

N-jettiness slicing

$$\int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} = \int B \otimes B \otimes S \otimes H \otimes \prod_i^N J_i + \mathcal{O}(\mathcal{T}_0)$$

- The **Beam and Jet functions (B, J_i)** describe initial- and final-state collinear radiation, the **Soft function S** the soft radiation, and the (process dependent) **Hard function H** encodes the virtual corrections
- Small cutoff T_0 required so that power corrections in T_0/Q are under control
- At NNLO, **all ingredients are known**. S was available for 0-, 1- and 2-jettiness, but only recently for generic N-jettiness

(hep-ph/2312.11626, hep-ph/2403.03078)

- At N3LO, S is **only available for zero-jettiness**. Other ingredients are already known

(hep-ph/2409.11042, hep-ph/2412.14001)

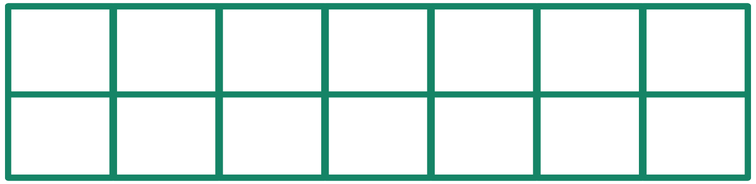
N-jettiness soft function at NNLO

Our soft function calculation

- Previous NNLO calculations based on **decomposing the observable into θ functions** and computing it numerically

(Boughezal et al. - hep-ph/1504.02540, Campbell et al. - hep-ph/1711.09984, Bell et al. - hep-ph/2312.11626)

$$\delta\left[\mathcal{T} - \min(k \cdot p_1, k \cdot p_2)\right] \Rightarrow \left[\theta(k \cdot p_1 - k \cdot p_2) + \theta(k \cdot p_2 - k \cdot p_1)\right]$$

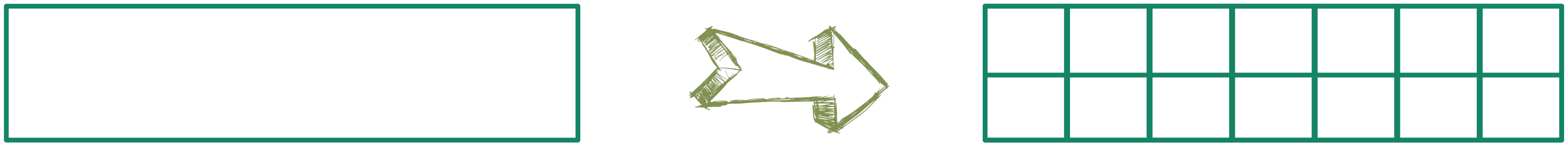


Quickly gets out of hand with number of jets/unresolved particles!!!

Our soft function calculation

- Previous NNLO calculations based on **decomposing the observable into θ functions** and computing it numerically

(Boughezal et al. - hep-ph/1504.02540, Campbell et al. - hep-ph/1711.09984, Bell et al. - hep-ph/2312.11626)



- We use **subtraction methods** to calculate this ingredient of a *slicing method*, showing the **explicit analytical cancellation of divergences** and arriving to a **finite formula** for it. Also, **N is treated genuinely as a parameter!!!**
- We borrow ideas from subtraction schemes to compute ingredients of slicing schemes. We wish to see the general structure, since in principle the soft divergences are not related to the observable

Soft function renormalization

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The **divergent structure** of the soft function is actually **very simple**. It is convenient to work in Laplace space

$$S(u) = \int_0^\infty d\mathcal{T}\; S_{\mathcal{T}}(\mathcal{T}) e^{-u\mathcal{T}}$$

Since the renormalization is multiplicative (with matrices in color space)

$$S = Z\tilde{S}Z^\dagger$$

If we write the expansion in powers of α_s

$$\left(Z_2 = \frac{1}{2}Z_1 Z_1 + Z_{2,r}\right)$$

$$\begin{aligned} Z &= 1 + Z_1 + Z_2, \\ S &= 1 + S_1 + S_2, \\ \tilde{S} &= 1 + \tilde{S}_1 + \tilde{S}_2, \end{aligned}$$

$$\begin{aligned} \tilde{S}_1 &= S_1 - Z_1 - Z_1^\dagger \\ \tilde{S}_2 &= S_2 - Z_2 - Z_2^\dagger + Z_1 Z_1 + Z_1^\dagger Z_1^\dagger - Z_1 S_1 - S_1 Z_1^\dagger + Z_1 Z_1^\dagger \\ &= \frac{1}{2}\tilde{S}_1\tilde{S}_1 + \frac{1}{2}[Z_1, Z_1^\dagger] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{2,r} - Z_{2,r} - Z_{2,r}^\dagger \end{aligned}$$

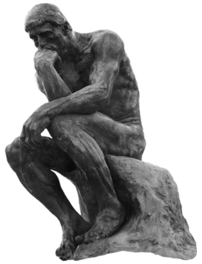
Soft function renormalization

The **divergent structure** of the soft function is actually **very simple**. How simple are this Z functions?

$$Z_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{1}{2\epsilon^2} + \frac{2L_{ij} + i\pi\lambda_{ij}}{2\epsilon} \right)$$

$$Z_{2,r} = a_s^2 \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left(-\frac{3\beta_0}{8\epsilon^3} + \frac{\Gamma_1 - 4\beta_0(2L_{ij} + i\pi\lambda_{ij})}{16\epsilon^2} + \frac{\Gamma_1(2L_{ij} + i\pi\lambda_{ij}) + \gamma_1^S}{8\epsilon} \right)$$

There must be a way to derive a finite representation of the renormalized N-jettiness soft function...



Soft function at NLO

If we take **$Q_i = 2E_i$** with an unresolved gluon *m*, the N-jettiness is given by

$$\mathcal{T}(m) = E_m \psi_m = E_m \min\{\rho_{1m}, \rho_{2m}, \rho_{3m}, \ldots, \rho_{Nm}\} \qquad \rho_{ij} = 1 - \vec{n}_i \cdot \vec{n}_j$$

Then, the soft function is given by

$$S(\mathcal{T}) = -\sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, g_s^2 \underbrace{\int \frac{d\Omega_m^{(d-1)}}{2(2\pi)^{d-1}} \frac{dE_m}{E_m^{1+2\epsilon}} E_m^2}_{\text{Phase space}} \underbrace{\delta(\mathcal{T} - E_m\psi_m)}_{\text{Jettiness constraint}} S_{ij}(m)$$

Eikonal

$$S_{ij}(m) = \frac{1}{E_m^2} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}}$$

We integrate over E_m with delta, only collinear diverengences remain. we use that $\lim_{m||i} \psi_m = \rho_{im}$, so we can rewrite

$$\psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} = \left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right)^{2\epsilon} \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} = \left(1 + 2\epsilon g_{ij,m}^{(2)}\right) \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}}$$

Soft function at NLO

Knowing that ($\eta_{ij} = \rho_{ij}/2$)

$$\left\langle \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} \right\rangle_m = \frac{2\eta_{ij}^{\epsilon}}{\epsilon} K_{ij}^{(2)} = \frac{2\eta_{ij}^{\epsilon}}{\epsilon} \frac{\Gamma(1+\epsilon)^2}{\Gamma(1+2\epsilon)} \, {}_2F_1\left(\epsilon, \epsilon, 1-\epsilon, 1-\eta_{ij}\right),$$

T where <..>$_m$ indicates integration over directions of n_m, in Laplace space we get the following bare soft function

$$S_1 = a_s \left(\mu\bar{u}\right)^{2\epsilon} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon) e^{\epsilon\gamma_E}} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[\frac{\eta_{ij}^{\epsilon}}{\epsilon^2} K_{ij}^{(2)} + \left\langle g_{ij,m}^{(2)} \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} \right\rangle_m \right]$$

By combining S_1 with the renormalization matrices Z^1 and $Z_1^\dagger$, we finally obtain ($L_{ij} = ln\left(\mu\, u\, e^{\gamma_E} \sqrt{\eta_{ij}}\right)$)

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m + \mathcal{O}(\epsilon) \right]$$

Soft function at NNLO

The NNLO contribution to the **bare soft function** is

$$S_2 = S_{2,RR} + S_{2,RV} - a_s \frac{\beta_0}{\epsilon} S_1$$

We further split the double-real contribution into correlated and uncorrelated pieces

$$S_{2,RR,\mathcal{T}} = S_{2,RR,T^4} + S_{2,RR,T^2} = \frac{1}{2} \sum_{(ij),(k,l)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\}\, I_{T^4,ij,kl} - \frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, I_{T^2,ij}$$

The real-virtual contribution reads

$$S_{2,RV,\mathcal{T}} = S_{RV,T^2} + S_{RV,\mathrm{tc}} = \frac{[\alpha_s]\, 2^{-\epsilon}}{\epsilon^2} C_A A_K(\epsilon) \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, I_{RV,ij} + [\alpha_s] \frac{4\pi N_\epsilon}{\epsilon} \sum_{(kij)} \kappa_{ij} F^{kij} I_{kij}$$

where $\kappa_{ij} = \lambda_{ij} - \lambda_{im} - \lambda_{jm}$, with $\lambda_{ij} = 1$ if both *i* and *j* refer to incoming/outgoing partons, and zero otherwise. We have defined $F_{kij} = f_{abc} T^a_k T^b_i T^c_j$, while $A_k(\epsilon)$ and N_ϵ are normalization factors

Soft function at NNLO

TThe calculation of the **renormalized soft function** is organized as follows

$$\tilde{S}_2 = \tilde{S}_2^{\text{uncorr}} + \tilde{S}_2^{\text{corr}} + \tilde{S}_2^{\text{tc}}$$

Where each **finite** piece is the combination of the following contributions

Uncorrelated emission

$$\tilde{S}_2^{\text{uncorr}} = \frac{1}{2}\tilde{S}_1\tilde{S}_1$$

Triple color terms

$$\tilde{S}_2^{\text{tc}} = \frac{1}{2}\left[Z_1, Z_1^\dagger\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{RV,\text{tc}}$$

Correlated emission

$$\tilde{S}_2^{\text{corr}} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^\dagger - \frac{a_s\beta_0}{\epsilon}S_1$$

General strategy at NNLO

Uncorrelated emission

$$\tilde{S}_2^{\text{uncorr}} = \frac{1}{2}\tilde{S}_1\tilde{S}_1$$

Trivially related to iterations of NLO

Triple color terms

$$\tilde{S}_2^{\text{tc}} = \frac{1}{2}\left[Z_1, Z_1^\dagger\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{RV,\text{tc}}$$

Similar to NLO, reuse results from calculation without jettiness-constraint

Correlated emission

$$\tilde{S}_2^{\text{corr}} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^\dagger - \frac{a_s\beta_0}{\epsilon}S_1$$

Use nested soft-collinear subtraction, reuse results from calculation without jettiness-constraint

We already know how to integrate eikonal, we focus on the handling of the jettiness constraint!!!

Uncorrelated emission

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The S_2 contains an iterated contribution of the NLO soft function S_1

$$I_{T^4,ij,kl} = \frac{[\alpha_s]^2}{2} \left\langle \int_0^\infty \frac{dE_m}{E_m^{1+2\epsilon}} \frac{dE_n}{E_n^{1+2\epsilon}} \, \delta(\tau - E_m\psi_m - E_n\psi_n) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_{mn}$$

If we integrate over both energies we can disentangle the jettiness function

$$\int_0^\infty \frac{dE_m}{E_m^{1+2\epsilon}} \frac{dE_n}{E_n^{1+2\epsilon}} \, \delta(\tau - E_m\psi_m - E_n\psi_n) = \frac{\tau^{-1-4\epsilon}}{\Gamma(-4\epsilon)} \frac{\psi_m^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon} \frac{\psi_n^{2}\Gamma(1-2\epsilon)}{2\epsilon}$$

The Laplace transform allows us to identify this iteration

$$S_{2,RR,T^4} = \frac{[\alpha_s]^2}{4} \sum_{(ij),(kl)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\} \left(\frac{u^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon} \right)^2 \left\langle \psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m \left\langle \psi_n^{2\epsilon} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_n = \frac{1}{2} S_1 S_1$$

Triple color terms

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This contribution depends on triple products of color charges

$$\tilde{S}_2^{\mathrm{tc}} = \frac{1}{2}\left[Z_1, Z_1^\dagger\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{RV,\mathrm{tc}}$$

The needed commutators can be computed as shown in (Devoto et al. – hep-ph/2310.17598)

$$\frac{1}{2}[Z_1, Z_1^\dagger] = -\frac{2\pi a_s^2}{\epsilon^2}\sum_{(kij)}\lambda_{kj}L_{ij}F^{kij} = -\frac{\pi a_s^2}{\epsilon^2}\sum_{(kij)}\lambda_{kj}\ln\eta_{ij}\; F^{kij}$$

$$\frac{1}{2}[S_1, Z_1 - Z_1^\dagger] = -\frac{a_s^2\pi(\mu u)^{2\epsilon}}{\epsilon^2}\frac{e^{\gamma_E\epsilon}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\sum_{(kij)}\kappa_{kj}\left\langle \psi_m^{2\epsilon}\frac{\rho_{ki}}{\rho_{km}\rho_{im}}\right\rangle_m F^{kij}$$

And the real-virtual triple-color correlated contribution is

$$S_{RV,\mathrm{tc}} = \frac{a_s^2\pi(\mu\,\bar{u})^{4\epsilon}N_\epsilon 2^{-\epsilon}}{2\epsilon^2}\frac{\Gamma(1-4\epsilon)}{\Gamma^2(1-\epsilon)e^{2\gamma_E\epsilon}}\sum_{(kij)}\kappa_{kj}\left\langle \psi_m^{4\epsilon}\frac{\rho_{ki}}{\rho_{km}\rho_{im}}\left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}}\right)^{\epsilon}\right\rangle_m F^{kij}$$

Triple color terms

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We can just follow the NLO case

$$\left\langle \psi_m^{2\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \right\rangle_m = \left\langle (1 + 2\epsilon g_{ki,m}^{(2)}) \frac{\rho_{ki}^{1-2\epsilon}}{\rho_{km}^{1-2\epsilon}\rho_{im}^{1-2\epsilon}} \right\rangle_m$$

$$\left\langle \psi_m^{4\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m = \left\langle \left(1 + 4\epsilon g_{ki,m}^{(4)}\right) \frac{\rho_{ki}^{1-4\epsilon}}{\rho_{km}^{1-4\epsilon}\rho_{im}^{1-4\epsilon}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m$$

What about the rest of the finite part? The idea is to use the integral of

$$\left\langle \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m$$

which was calculated in Devoto et al. - hep-ph/2310.17598, and use it to extract the result

Correlated emission

The calculation of the correlated terms are the main bulk of the calculation

$$\tilde{S}_2^{\mathrm{corr}} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^{\dagger} - \frac{a_s \beta_0}{\epsilon} S_1$$

Renormalization terms do not require integration, and the real-virtual one is

$$S_{RV,T^2} \propto -\frac{[\alpha_s]^2}{\epsilon^3} C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left\langle \psi_m^{4\epsilon} \left(\frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right)^{1+\epsilon} \right\rangle_m$$

(Born-like)

The first term, that involves the correlated emission eikonal, is the one that requires attention

$$S_{2,RR,T^2,\tau} = -\frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, I_{ij,\tau} = -\frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, \frac{g_s^4}{2} \int [dp_m][dp_n] \, \delta\left(\tau - E_m \psi_m - E_n \psi_n\right) \tilde{S}_{ij}^{gg}(m,n)$$

(There is also an analogous and simpler quark contribution, but we focus on the gluon case)

Correlated emission

We perform a **nested-subtraction of all divergent limits**

$$I_{ij} = (1 - S_\omega) I_{ij} + S_\omega I_{ij}$$

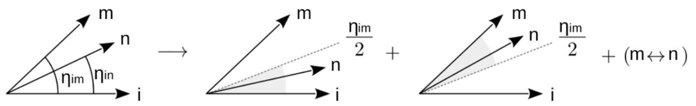
Subtract strongly-ordered 2-soft limit ($E_n = \omega\, E_m$)

$$(1 - S_\omega) I_{ij}^{dc} + (1 - S_\omega) I_{ij}^{tc}$$

Introduce partitions to separate double and triple collinear divergences

Introduce sectors to disentangle collinear divergences and subtract

$$\left\{ \theta^{bd} C_{mn} + \left(1 - \theta^{bd} C_{mn}\right) \left[C_{imn} + (1 - C_{imn}) \right] \right\} (1 - S_\omega) I_{ij}^{tc}$$



Correlated emission

We perform a **nested-subtraction of all diverg**

Energy ordering: $\psi_{mn} = \psi_m + \omega \psi_n$

$$I_{ij} = (1 - S_\omega) I_{ij} + S_\omega I_{ij}$$

Subtract strongly-ordered 2-soft limit ($E_n = \omega\, E_m$)

$$(1 - S_\omega) I_{ij}^{dc} + (1 - S_\omega$$

te double

es

Due to the symmetry of the problem, we can extend the ω integration to infinity and do it:

$$\psi_{mn}^{4\epsilon} \to \psi_m^{2\epsilon} \psi_n^{2\epsilon}$$

Introduce sectors to disentangle collinear divergences and subtract

$$\{\theta^{bd} C_{mn} + (1 - \theta^{bd} C_{mn})\, [C_{imn} + (1 - C_{imn})]\}\, (1 - S_\omega) I_{ij}^{tc}$$

Correlated emission

We perform a **nested-subtraction of all diver**

Energy ordering:

$$\psi_{mn} = \psi_m + \omega\psi_n$$

$$I_{ij} = (1 - S_\omega) I_{ij} + S_\omega I_{ij}$$

Subtract strongly-ordered 2-soft limit ($E_n = \omega E_m$)

$$(1 - S_\omega) I_{ij}^{dc} + (1 - S ...) I^{tc}$$

Introduce partitions to separate double ... s

Goes like 1/ϵ in prefactor, we can expand jettiness function:

$$\psi_{mn}^{4\epsilon} \to 1 + 4\epsilon \log\left(\psi_{mn}\right)$$

Introduce sectors to disentangle collinear divergences and subtract

$$\{\theta^{bd} C_{mn} + (1 - \theta^{bd} C_m ...) I_{ij}^{tc}$$

Correlated emission

We perform a **nested-subtraction of all diverg**

Energy ordering: $\psi_{mn} = \psi_m + \omega\psi_n$

$$I_{ij} = (1 - S_\omega)I_{ij} + S_\omega I_{ij}$$

Subtract strongly-ordered 2-soft limit ($E_n = \omega\, E_m$)

Double collinear: like NLO

$$\psi_{mn}^4 \to \psi_m^4$$

$(1 -$

Triple collinear: jettiness takes particular value

$$\psi_{mn}^4 \to (\rho_{im} + \omega\rho_{in})^{4\epsilon}$$

e partitions to separate double e collinear divergences

Introduce sectors to disentangle collinear divergences and subtract

$$\{\theta^{bd}C_{mn} + (1 - \theta^{bd}C_{mn})\,[C_{imn} + (1 - C_{imn})]\}\,(1 - S_\omega)I_{ij}^{tc}$$

Finite! Just integrate numerically!

The final result

- The NLO result was

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle L_{ij,m}^{\psi} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m \right]$$

where $L_{ij} = ln\ (\mu\, u\, e^{\gamma_E} \sqrt{\eta_{ij}}\ )$ and $L_{ij,m}^{\psi} = \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right)$

- The NNLO one is

$$\tilde{S}_2 = \frac{1}{2}\tilde{S}_1^2 + a_s^2 C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, G_{ij} + a_s^2\, n_f\, T_R \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, Q_{ij} + a_s^2 \pi \sum_{(kij)} F^{kij}\, \kappa_{kj} G_{kij}^{\mathrm{triple}}$$

where G_{ij}, Q_{ij} and $G_{kij}^{\mathrm{triple}}$ are **finite** functions with analytical terms along with a **low number numerical four-dimensional integrations** over one- and two-particle phase space

The final result

They look like this

$$
\begin{aligned}
G_{kij}^{\rm triple} = \Bigg[& \frac{8}{3} L_{ki}^3 + 4 L_{ki} \left(\mathrm{Li}_2(1-\eta_{ki}) + \frac{\pi^2}{12} + \left\langle \frac{\rho_{ki}}{\rho_{i\mathfrak{m}}\rho_{k\mathfrak{m}}} \ln\left(\frac{\psi_{\mathfrak{m}}\rho_{ki}}{\rho_{k\mathfrak{m}}\rho_{i\mathfrak{m}}} \right) \right\rangle_{\mathfrak{m}} \right) \\
& + 2\mathrm{Li}_3(1-\eta_{ki}) - 6\mathrm{Li}_3(\eta_{ki}) + \mathrm{Li}_2(1-\eta_{ki})(2\ln 2 - 8\ln(\eta_{ki})) - \ln^3(\eta_{ki}) \\
& - 3\ln(1-\eta_{ki})\ln^2(\eta_{ki}) - \ln 2 \ln^2(\eta_{ki}) + \frac{\pi^2}{6}\ln(\eta_{ki}) - \bar{G}_{r,\rm fin}^{ikj} - 2W_{kij} \\
& + 2 \left\langle \frac{\rho_{ki}}{\rho_{i\mathfrak{m}}\rho_{k\mathfrak{m}}} \ln\left(\frac{\psi_{\mathfrak{m}}\rho_{ki}}{\rho_{k\mathfrak{m}}\rho_{i\mathfrak{m}}} \right) \ln\left(\frac{\psi_{\mathfrak{m}}\rho_{i\mathfrak{m}}\rho_{kj}}{\rho_{j\mathfrak{m}}\rho_{ki}^2} \right) \right\rangle_{\mathfrak{m}} + \mathcal{O}(\epsilon) \Bigg].
\end{aligned}
$$

We note that the function L_{ki} is given in Eq. (4.19), the function W_{kij} reads

$$
W_{kij} = \left\langle \frac{\rho_{ki}}{\rho_{i\mathfrak{m}}\rho_{k\mathfrak{m}}} \ln\frac{\rho_{kj}}{\rho_{j\mathfrak{m}}} \ln\frac{\rho_{ki}}{\rho_{i\mathfrak{m}}\rho_{k\mathfrak{m}}} + \frac{1}{\rho_{i\mathfrak{m}}} \ln\rho_{i\mathfrak{m}} \ln\frac{\rho_{kj}}{\rho_{ij}} \right\rangle_{\mathfrak{m}},
$$

and the function $\bar{G}_{r,\rm fin}^{ikj}$ can be found in Eq. (H.16) of Ref. [38].

$$
\begin{aligned}
G_{ij} = & \frac{22}{9} L_{ij}^3 + \left(\frac{67}{9} - \frac{\pi^2}{3} \right) L_{ij}^2 + L_{ij} \left(\frac{11}{3} \left\langle L_{ij,\mathfrak{m}}^{\psi} \frac{\rho_{ij}}{\rho_{i\mathfrak{m}}\rho_{j\mathfrak{m}}} \right\rangle_{\mathfrak{m}} + \frac{11}{3}\mathrm{Li}_2(1-\eta_{ij}) \right. \\
& \left. + \frac{202}{27} - 7\zeta_3 \right) + \left\langle \frac{\rho_{ij}}{\rho_{i\mathfrak{m}}\rho_{j\mathfrak{m}}} \left(\left(L_{ij,\mathfrak{m}}^{\psi} \right)^2 \left(\frac{11}{6} - \ln\left(\frac{\eta_{ij}}{\eta_{i\mathfrak{m}}\eta_{j\mathfrak{m}}} \right) \right) \right. \right. \\
& + L_{ij,\mathfrak{m}}^{\psi} \left(2\ln^2\left(\frac{\eta_{ij}}{\eta_{i\mathfrak{m}}\eta_{j\mathfrak{m}}} \right) + \ln\left(\frac{\eta_{ij}}{\eta_{i\mathfrak{m}}\eta_{j\mathfrak{m}}} \right) \left(-\frac{11}{3} + \ln(\eta_{i\mathfrak{m}}\eta_{j\mathfrak{m}}) \right) + \frac{137}{18} - \frac{\pi^2}{2} \right. \\
& \left. \left. \left. - \frac{1}{2}\ln^2\left(\frac{\eta_{i\mathfrak{m}}}{\eta_{j\mathfrak{m}}} \right) + \mathrm{Li}_2(1-\eta_{ij}) + \frac{11}{3}\ln 2 - \frac{11}{6}\ln(\eta_{i\mathfrak{m}}\eta_{j\mathfrak{m}}) - \frac{(\rho_{i\mathfrak{m}}+\rho_{j\mathfrak{m}})}{3\rho_{ij}} \right) \right) \right\rangle_{\mathfrak{m}}
\end{aligned}
$$

...

$$
\begin{aligned}
& + 8\mathrm{Li}_4\left(\frac{1}{2}\right) - \frac{11}{9}\zeta_3 - \frac{11}{80}\pi^4 + \frac{937}{432}\pi^2 + \frac{403}{162} \\
& - \left\langle \bar{C}_{\mathfrak{mn}} \ln\left(\frac{\psi_m}{\rho_{im}} \right) \ln\left(\frac{\psi_n}{\rho_{jn}} \right) \frac{\rho_{ij}}{\rho_{\mathfrak{mn}}\rho_{i\mathfrak{m}}\rho_{j\mathfrak{n}}} \right\rangle_{\mathfrak{mn}} + \frac{1}{2} \left\langle L_{ij,\mathfrak{m}}^{\psi} \frac{\rho_{ij}}{\rho_{i\mathfrak{m}}\rho_{j\mathfrak{m}}} \right\rangle_{\mathfrak{m}}^2 \\
& + \frac{1}{2} \sum_{x\in\{i,j\}} \int_0^1 \frac{\mathrm{d}\omega}{\omega} \left\langle (1-\theta^{b+d} C_{\mathfrak{mn}}) \left[\mathrm{d}\Omega_{\mathfrak{mn}} \right] \bar{C}_{x\mathfrak{mn}}\, w^{x\mathfrak{m},x\mathfrak{n}} \ln\psi_{\mathfrak{mn}}\, \bar{S}_\omega \left[\omega^2 \tilde{S}_{ij}(\mathfrak{m},\mathfrak{n}) \right] \right\rangle_{\mathfrak{mn}} \\
& + \frac{1}{2} \int_0^1 \frac{\mathrm{d}\omega}{\omega} \left\langle (w^{i\mathfrak{m},j\mathfrak{n}} + w^{j\mathfrak{m},i\mathfrak{n}}) \ln\psi_{\mathfrak{mn}}\, \bar{S}_\omega \left[\omega^2 \tilde{S}_{ij}(\mathfrak{m},\mathfrak{n}) \right] \right\rangle_{\mathfrak{mn}}.
\end{aligned}
$$

G_{ij}, Q_{ij} and $G_{kij}^{\rm triple}$ are **finite** functions with analytical terms along with a **low number numerical four-dimensional integrations** over one- and two-particle phase space

Numerical checks

Numerical checks

Karlsruhe Institute of Technology

We compared our results with

Bell, Dehnadi, Mohrmann, Rahn, arXiv hep-ph/2312.11626

We focus in the “new” 3-jettiness case, with two back-to-back beams. The five directions are

$$n_1 = (0, 0, 1), \quad n_2 = (0, 0, -1), \quad n_3 = (\sin\theta_{13}, 0, \cos\theta_{13}),$$

$$n_4 = (\sin\theta_{14}\cos\phi_4, \sin\theta_{14}\sin\phi_4, \cos\theta_{14}), \quad n_5 = (\sin\theta_{15}\cos\phi_5, \sin\theta_{15}\sin\phi_5, \cos\theta_{15})$$

in the following phase-space point

$$\theta_{13} = \frac{3\pi}{10}, \quad \theta_{14} = \frac{6\pi}{10}, \quad \theta_{15} = \frac{9\pi}{10}, \quad \phi_4 = \frac{3\pi}{5}, \quad \phi_5 = \frac{6\pi}{5}$$

Numerical checks

Karlsruhe Institute of Technology

Dipole configurations

Dipoles	Gluons		Quarks	
	G_{ij}^{nl}	Bell et al.	Q_{ij}^{nl}	Bell et al.
12	116.20 ± 0.01	116.20 ± 0.16	-36.249 ± 0.001	-36.244 ± 0.009
13	38.13 ± 0.03	37.63 ± 0.03	-21.717 ± 0.007	-21.732 ± 0.005
14	63.63 ± 0.01	63.66 ± 0.06	-25.189 ± 0.003	-25.192 ± 0.006
15	107.17 ± 0.01	106.99 ± 0.12	-35.268 ± 0.001	-35.256 ± 0.009
23	97.11 ± 0.01	96.97 ± 0.10	-32.875 ± 0.002	-32.872 ± 0.008
24	67.36 ± 0.02	67.51 ± 0.08	-26.821 ± 0.003	-26.815 ± 0.007
25	30.87 ± 0.03	30.73 ± 0.04	-21.561 ± 0.009	-21.561 ± 0.005
34	69.43 ± 0.01	69.24 ± 0.07	-25.854 ± 0.002	-25.861 ± 0.006
35	106.13 ± 0.02	105.97 ± 0.13	-34.799 ± 0.002	-34.796 ± 0.008
45	74.45 ± 0.02	74.36 ± 0.09	-28.247 ± 0.004	-28.251 ± 0.007

Tripole sums

	$\tilde{c}_{\text{tripoles}}$	Bell et al.
$\tilde{c}_{\text{tripoles}}^{(2,124)}$	-683.25 ± 0.01	-683.23 ± 0.04
$\tilde{c}_{\text{tripoles}}^{(2,125)}$	-2203.3 ± 0.2	-2203.5 ± 0.1
$\tilde{c}_{\text{tripoles}}^{(2,145)}$	-6.324 ± 0.004	-6.325 ± 0.04
$\tilde{c}_{\text{tripoles}}^{(2,245)}$	-0.837 ± 0.008	-0.830 ± 0.039

The tripole sums correspond to the four independent color structures as specified in hep-ph/2312.11626

N-jettiness subleading power corrections

Power corrections in N-jettiness slicing

$$\int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} = \int B \otimes B \otimes S \otimes H \otimes \prod_i^N J_i + \mathcal{O}(\mathcal{T}_0)$$

- Slicing methods suffer from **instabilities** due to large cancellations between contributions if the slicing parameter (cutoff) is not sufficiently small
- Can improve this by including more terms in the computation of the singular contribution
- Power-suppressed terms, particularly **subleading** ones, were studied in recent years, mostly at NLO

(hep-ph/1802.00456, hep-ph/1807.10764, hep-ph/1907.12213, hep-ph/1905.08741)

But we aim for a general approach valid for an arbitrary of process!!!

Power corrections to color-singlet production

In the process with $f_a(p_a) + f_b(p_b) \longrightarrow X(P_X)$, at NLO we consider emission of a gluon k

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} = \mathcal{N} \int \underbrace{[\mathrm{d}\tilde{P}_X]_m [\mathrm{d}k] \delta(p_a + p_b - k - \tilde{P}_X) \delta(\mathcal{T} - \mathcal{T}_0(p_a, p_b, k))}\, \mathcal{O}(\tilde{P}_X) \underbrace{\sum_{\mathrm{col,pol}} |\mathcal{M}|^2 (p_a, p_b, k, \tilde{P}_X)}$$

Power corrections primarily require the expansion in T of two building blocks:

Phase space

Dependence on T can be expressed in a process independent manner

Matrix element

Process independent Next-to-soft corrections from **LBK** theorem

No analogous theorem for collinear radiation!

Power corrections to color-singlet production

- The expansion controlled by gluon transverse momentum $k_\perp$. The jettiness (*T*) constraint forces the gluon energy or the $k_\perp$ to be *O(T)*:

- **If $k_\perp$ is *O*(1), the gluon energy is *O(T)* and the expansion is the soft expansion**
- **If $k_\perp$ is *O(T/Q)*, then the angle is small and we expand in $k_\perp$ (collinear expansion)**

Zero-jettiness

$$\mathcal{T}(p_a, p_b, k) = \min\left(\frac{2p_a k}{Q}, \frac{2p_b k}{Q}\right)$$

The two distinct integration regions - soft and collinear - are associated with two "branches" of the cross section with respect to *T*:

$$\frac{d\Sigma}{d\mathcal{T}} \sim \mathcal{T}^{-1-2\epsilon} f_s(\mathcal{T}) + \mathcal{T}^{-1-\epsilon} f_c(\mathcal{T})$$

The soft contribution

- For the **phase space:** use mapping that absorbs the gluon *k* into the colorless final state (hep-ph/1910.01024)

$$P^{\mu}_{ab} = \lambda^{-1}\Lambda^{\mu}_{\nu}(P^{\nu}_{ab} - k^{\nu}), \qquad \lambda = \sqrt{1 - \frac{2P_{ab}\cdot k}{P^2_{ab}}} \approx 1 - \frac{P_{ab}\cdot k}{P^2_{ab}} + \mathcal{O}(k^2)$$

- For the **matrix element:** we can use the **LBK** theorem to get the subleading terms

We arrive to the general expression

$$\frac{\mathrm{d}\sigma^{(s)}}{\mathrm{d}\mathcal{T}} = \mathcal{N}\int [\mathrm{d}\Phi_m(p_a,p_b,P_X)] \Bigg\{ \mathcal{O}(P_X)\Bigg[I_1 - \kappa_m I_2 - I_2 \sum_{i\in L_f} p_i^{\mu}\frac{\partial}{\partial p_i^{\mu}}\Bigg] |\mathcal{M}|^2(p_a,p_b,P_X) - I_2 |\mathcal{M}|^2(p_a,p_b,P_X)\sum_{i=1}^{m} p_i^{\mu}\frac{\partial}{\partial p_i^{\mu}}\mathcal{O}(P_X)\Bigg\}$$

The soft contribution

We arrive to the general expression in terms of the LO cross section:

$$\frac{\mathrm{d}\sigma^{(s)}}{\mathrm{d}\mathcal{T}} = \mathcal{N}\int [\mathrm{d}\Phi_m(p_a,p_b,P_X)] \Bigg\{ \mathcal{O}(P_X) \Bigg[I_1 - \kappa_m I_2 - I_2 \sum_{i\in L_f} p_i^\mu \frac{\partial}{\partial p_i^\mu} \Bigg] |\mathcal{M}|^2(p_a,p_b,P_X) - I_2 |\mathcal{M}|^2(p_a,p_b,P_X) \sum_{i=1}^{m} p_i^\mu \frac{\partial}{\partial p_i^\mu} \mathcal{O}(P_X) \Bigg\}$$

Where the integrals are

$$I_1 = [\alpha_s] \left(\frac{Q}{\sqrt{s}}\right)^{-2\epsilon} \frac{4}{\epsilon\, \mathcal{T}^{1+2\epsilon}}, \quad I_2 = [\alpha_s] \left(\frac{Q\mathcal{T}}{\sqrt{s}}\right)^{-2\epsilon} \frac{4Q}{s} \left(\frac{1}{2\epsilon} - \frac{1}{2} - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2)\right), \quad \kappa_m = m(d-2) - d$$

The collinear contribution

- For **phase space**: we use a mapping that absorbs the transverse momentum of *k* into the colorless final state through a Lorentz Transformation

$$k^\mu = (1-x)p_a^\mu + \tilde{k}^\mu \quad \Rightarrow \quad P_X^\mu = \Lambda^\mu_\nu \left(\tilde{P}_X^\nu + \tilde{k}^\nu\right)$$

Since $\tilde{k} \sim \sqrt{(\mathcal{T})}$, we need *second order expansion of boost and matrix element*!

- For **matrix element:** we have **no analogous theorem (yet) available** to get the subleading terms! First, we can use the fact that it is Lorentz invariant

$$\sum_{\text{pol}} |M(p_a, p_b, k, \Lambda^{-1}(P_x))|^2 = \sum_{\text{pol}} |M(\Lambda p_a, \Lambda p_b, \Lambda k, P_x)|^2$$

The collinear contribution

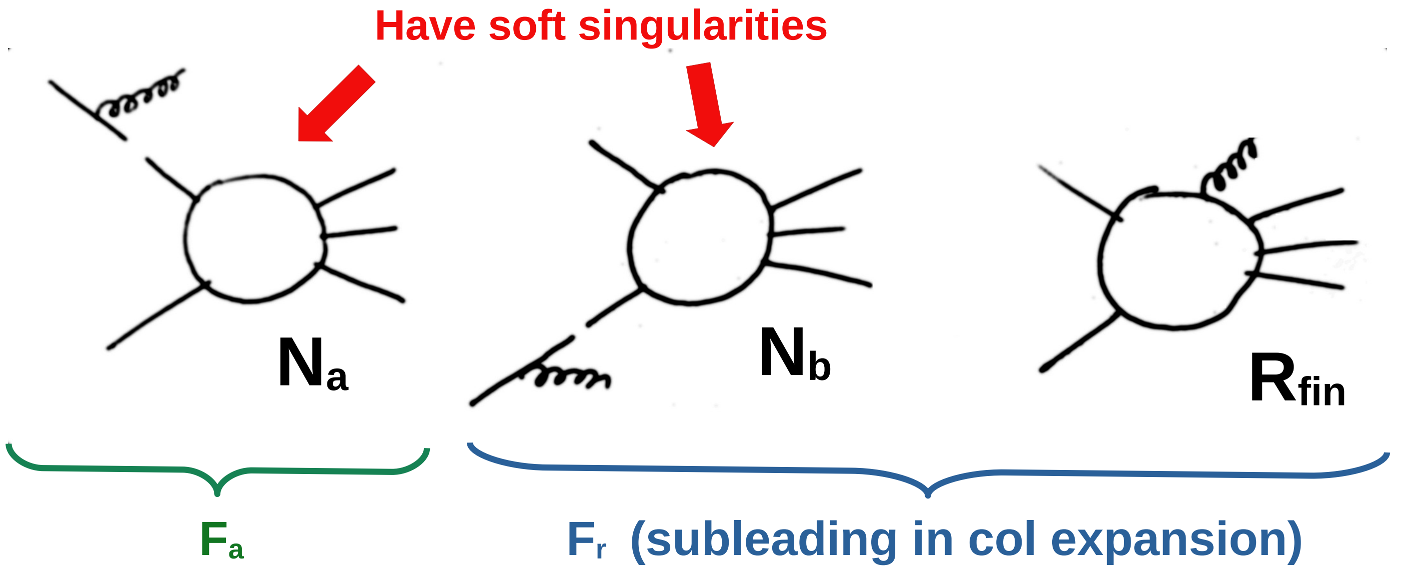
LP collinear expansion given by AP splitting kernels, but no much information beyond that. *We need a way of getting subleading terms in a way we can isolate soft poles*

$$\mathcal{M} = -g_s T^a \epsilon^\nu \bar{v}_b \left[N_a \frac{(\hat{p}_a - \hat{k})\gamma^\nu}{(-2p_a \cdot k)} + \gamma^\nu \frac{(\hat{p}_b - \hat{k})}{2p_b \cdot k} N_b + R^\nu_{\text{fin}}(p_b, p_a, k; Q_X) \right] u_a$$

Shut up and calculate

$$\sum |\mathcal{M}|^2 = F_{aa} + F_{ar} + F_{rr}$$

$$\rho^{(a)}_{\mu\nu} = -g_{\mu\nu} + \frac{k_\mu p_{b,\nu} + p_{b,\mu} k_\nu}{k \cdot p_b}$$



The collinear contribution

The problem is that some terms require expansion, e.g.

$$\frac{2P_{qq}(x)}{2p_a \cdot k}\mathrm{Tr}\left[N_a\, \hat{p}_a\, N_a^+ \hat{p}_b\right]$$

This means we have to derivate **N_a, N_b, R_{fin}**. Some of them can be rewritten in terms of derivatives of the Born (like in LBK)

$$\left(\frac{1}{x}\partial_{a\mu} - \partial_{b\mu}\right)|\mathcal{M}(xp_a, p_b, P_X)|^2$$

But in general we need to calulate traces of these Green's functions and their derivatives. But we can calculate in a systematic way using **Berends-Giele like recursion relations**

The collinear contribution

E.g.

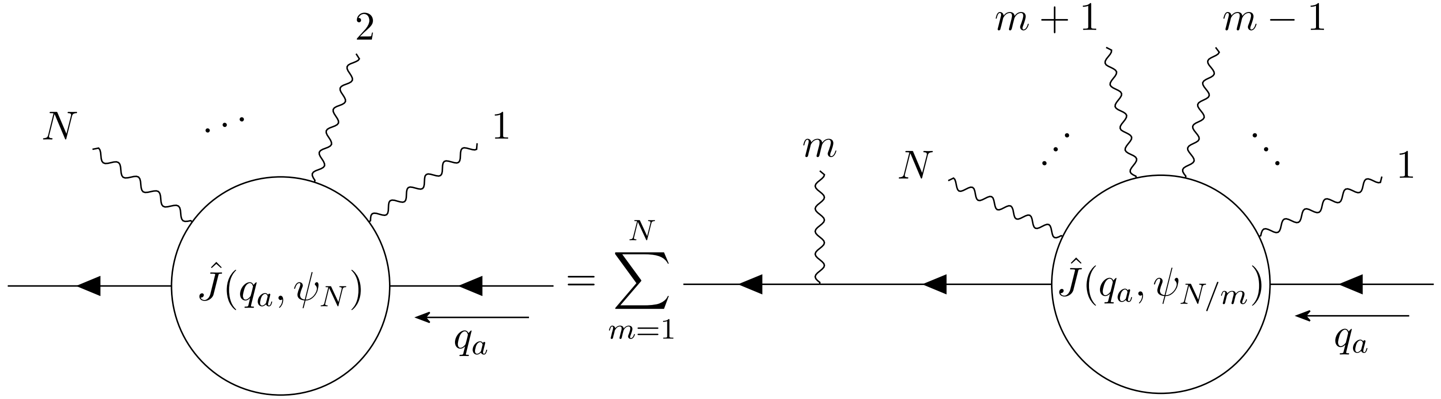
Collinear contribution

$$
\begin{aligned}
C^{k}_{3a} = & \frac{g_{\perp}^{\mu\nu}}{2}\mathrm{Tr}\left[\left(-N_a^{(1),\mu}x\hat{p}_a + N_a^{(0)}\gamma^{\mu}\right)\left(R_{\text{fin}}^{(0),\nu,+} - N_b^{(0),+}\frac{\hat{p}_a\gamma^{\nu}}{s}\right)\hat{p}_b\right] \\
& + g_{\perp}^{\mu\nu}\mathrm{Tr}\left[N_a^{(0)}x\hat{p}_a\left(R_{\text{fin}}^{(1),\nu\mu,+} - N_b^{(1),\mu,+}\frac{\hat{p}_a\gamma^{\nu}}{2xs}\right)\hat{p}_b\right] + \frac{g_{\perp}^{\mu\nu}}{2}\mathrm{Tr}\left[N_a^{(0)}x\hat{p}_a R_{\text{fin}}^{(0),\nu,+}\gamma^{\mu}\right] \\
& - \frac{1}{s}\mathrm{Tr}\left[N_a^{(0)}\hat{p}_a N_b^{(0),+}(x\hat{p}_a + \hat{p}_b)\right] + \text{c.c.}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{C}^{\mathrm{NLP},a} = & -2\int \mathrm{d}\Phi_m\, |\mathcal{M}(p_b, p_a, P_X)|^2\, \mathcal{O}(P_X) + \int \mathrm{d}x\, \mathrm{d}\Phi_m^{xa}\Bigg\{\frac{\bar{P}_{qq}(x)}{x}\Bigg[W_a(x) \\
& + \frac{s}{4}(1-x)g_{\perp}^{\rho\alpha}\left(D_{\rho}^{xa,b}\,|\mathcal{M}|^2(p_b, xp_a, \ldots) - 2\mathrm{Tr}\left[N_a\gamma_{\rho}N_a^{+}\hat{p}_b\right]\right) b_{a\alpha}^{\mu\nu} L_{\mu\nu} \\
& + |\mathcal{M}|^2(p_b, xp_a, \ldots)\, l_a^{\mu\nu}(x) L_{\mu\nu} - \frac{s\,(1-x)}{4}|\mathcal{M}|^2(p_b, xp_a, \ldots) t_a^{\mu\mu_1,\nu\nu_1} L_{\mu\mu_1} L_{\nu\nu_1}\Bigg] \\
& - \frac{1}{(1-x)_+}\left(\kappa_m + 2p_a^{\mu}\frac{\partial}{\partial p_{a,\mu}} + \left(g^{\rho\sigma} + \omega_{ab}^{\rho\sigma}\right) L_{\rho\sigma}\right)|\mathcal{M}|^2(p_b, xp_a, \ldots) \\
& - \frac{2p_b^{\nu}}{(1-x)_+}\left(\mathrm{Tr}\left[N_a\hat{p}_a R_{\nu}^{\text{fin},+}\hat{p}_b\right] + \text{c.c.}\right) + F_{\text{fin},a} \\
& + \frac{s}{4}(1-x)g_{\perp}^{\alpha\beta}\Bigg[-2\mathrm{Tr}\left[N_a\gamma_{\beta}N_a^{+}\hat{p}_b\right] \\
& + \mathrm{Tr}\left[N_a\gamma_{\beta}\gamma_{\rho}\hat{p}_a\left(R_{\text{fin}}^{\rho,+} + \frac{N_b^{+}(\hat{p}_b - (1-x)\hat{p}_a)\gamma^{\rho}}{(1-x)s}\right)\hat{p}_b\right] + \text{c.c.} \\
& + \frac{2x}{1-x}\mathrm{Tr}\left[N_a\hat{p}_a\left(R_{\beta}^{\text{fin},+} - \frac{N_b^{+}\hat{p}_a\gamma_{\beta}}{s}\right)\hat{p}_b\right] + \text{c.c.}\Bigg] b_{a\alpha}^{\mu\nu} L_{\mu\nu}\Bigg\}\, \mathcal{O}(P_X)
\end{aligned}
$$

Recursion in N-photon production

We can calculate a quark current in a recursive way



$$\hat{J}(q_a, \psi_N) = \frac{i}{\hat{q}_a - \sum \hat{p}_n} \sum_{m=1}^{N} (ie_q \hat{\epsilon}_m)\, \hat{J}(q_a, \psi_{N/m})$$

Then:

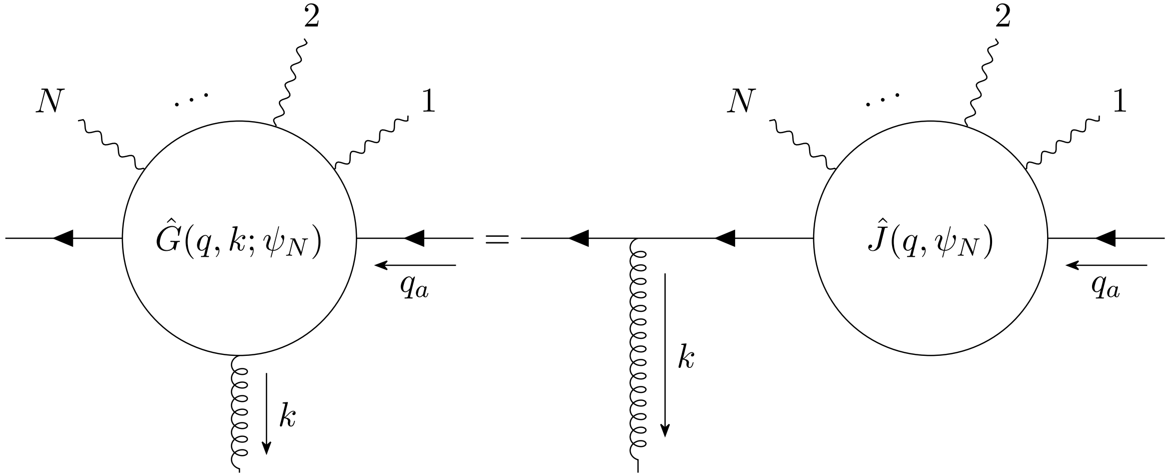
$$N_{a,b} = \sum_{m=1}^{N} (ie_q \hat{\epsilon}_m)\, \hat{J}\left(q_{a,b}, \psi_{N/m}\right)$$

$$q_a = x p_a, \qquad q_b = p_a$$

Recursion in N-photon production

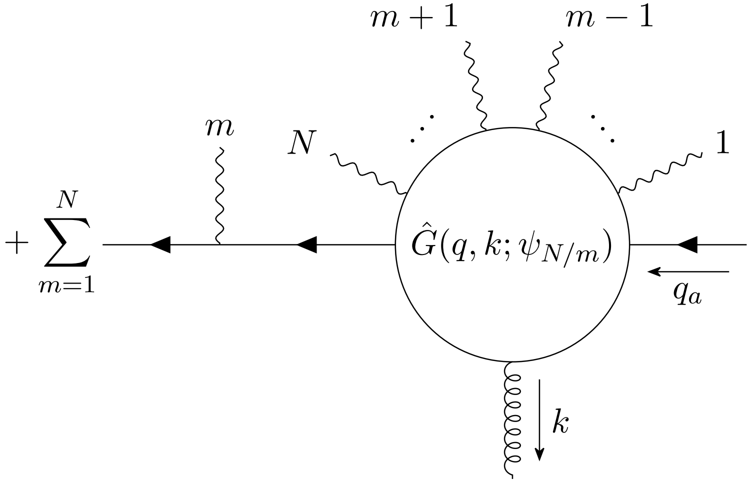
For R_{fin}

$$R^{\nu}_{\text{fin}}(q,k,\psi_N) = \sum_{m=1}^{N} (ie_q\hat{\epsilon}_m)\,\hat{G}^{\nu}\left(q,k,\psi_{N/m}\right)$$



Where the recursion relation for G also involves J:

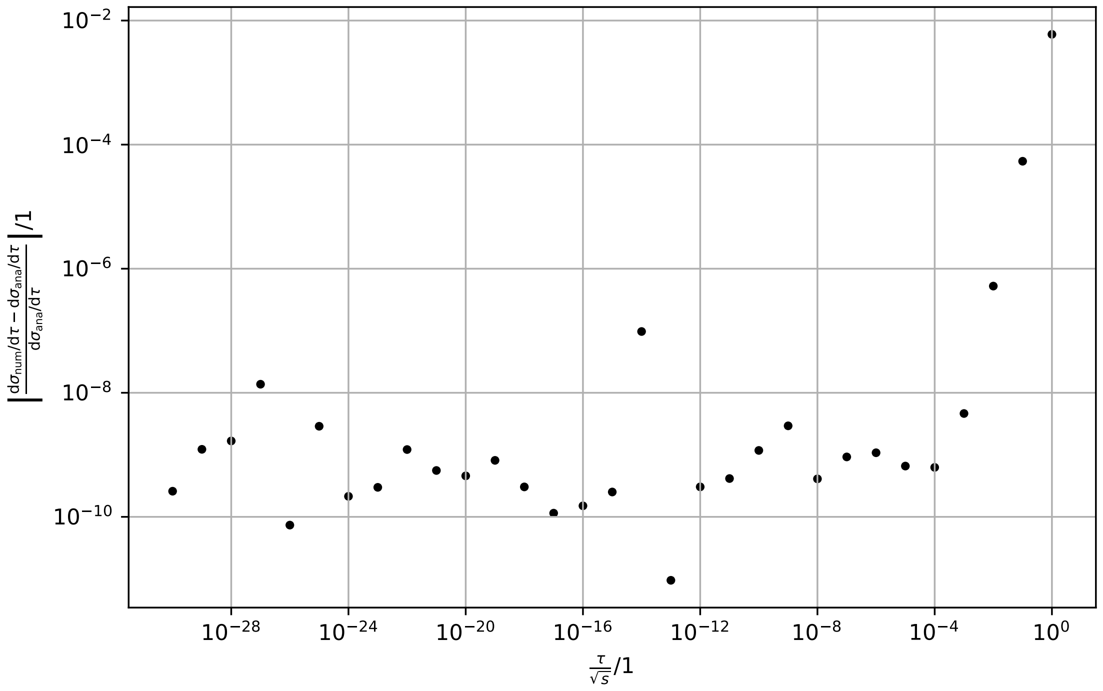
$$\hat{G}^{\nu}(q,k;\psi_N) = \frac{i}{\hat{q}-\hat{k}-\sum\hat{p}_n}\left[i\gamma^{\nu}\hat{J}(q,\psi_N) + \sum_{m=1}^{N}(ie_q\hat{\epsilon}_m)\,\hat{G}^{\nu}\left(q,k;\psi_{N/m}\right)\right]$$



Examples: Drell-Yan

- In these simple cases we can compare with naive expansion of NLO matrix element.
- We can also do some numerical checks:

coefficient	fit	analytic
LP, LL	−4.740 740 718	−4.740 740 741
LP, NLL	13.741 118 266	13.741 118 217
NLP, LL	0.000 179 950	0.000 000 000
NLP, NLL	−1.071 083 950	−1.072 546 919



$$\mathcal{O}(p_1, p_2) = \theta\left((p_1 + p_2)^2 - s_0\right) \qquad s_0 = 0.1, \quad Q = 0.1, \quad s = 1$$

Did the same for 2γ, and now we apply it to 4γ+ to show the potential of the approach

Conclusions

In conclusion...

- ✓ We calculated the N-jettiness soft function, demonstrating **analytical cancellation** of poles
- ✓ Derived a **simple representation** for finite jettiness-dependent remainder, allowing for *faster implementations*. In agreement with other calculations
- ✓ Showcased the benefits of using subtraction-inspired methods to derive building blocks of slicing methods

- ✓ We built a **process-independent framework** for subleading power corrections in a generic color-singlet production case
- ✓ Next-to-soft corrections easily obtained from LBK theorem, but next-to-collinear term is a complicated business


Thank you!!!