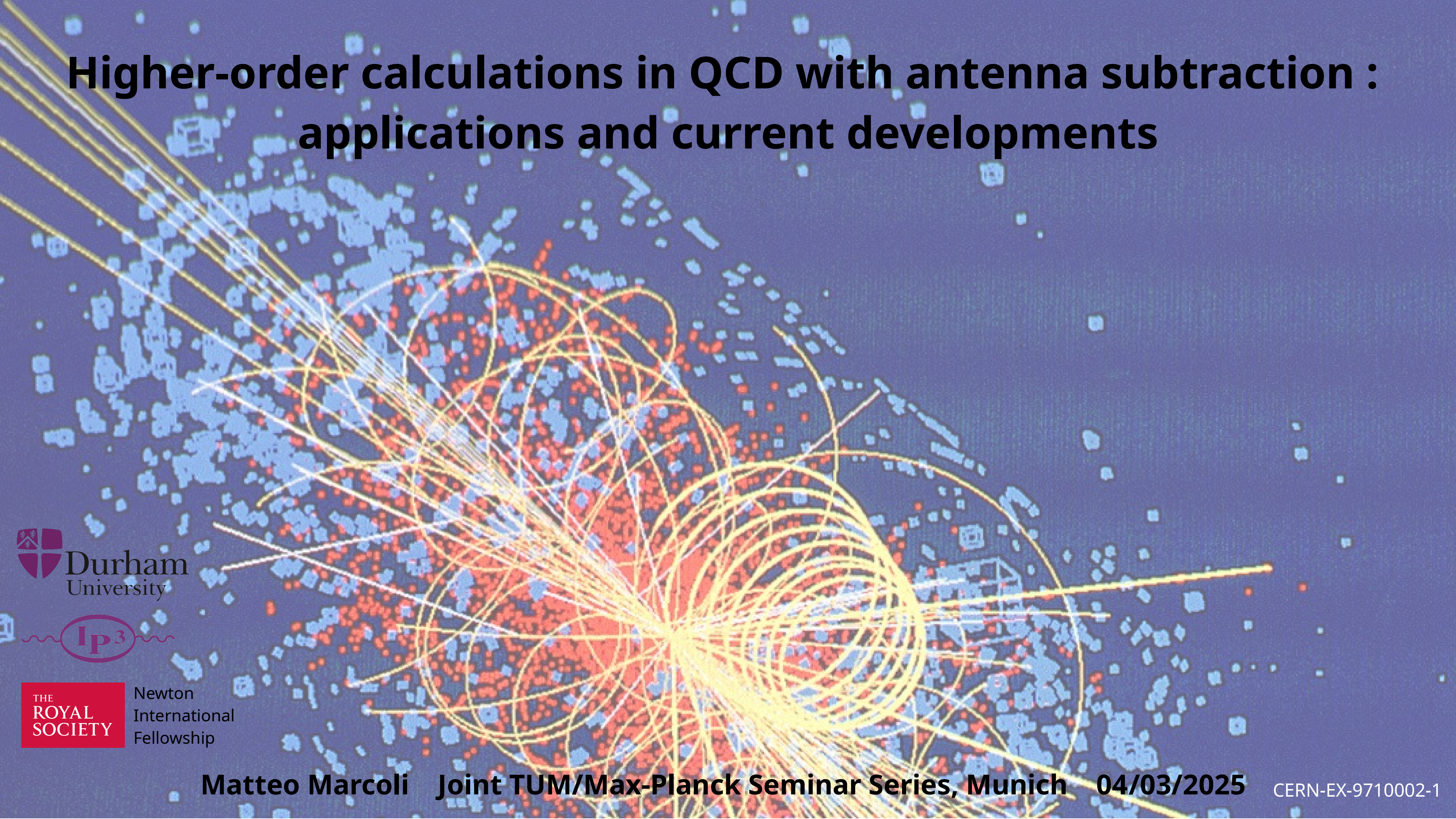


Higher-order calculations in QCD with antenna subtraction : applications and current developments

Durham University

IP3

THE ROYAL SOCIETY Newton International Fellowship

Matteo Marcoli Joint TUM/Max-Planck Seminar Series, Munich 04/03/2025

CERN-EX-9710002-1

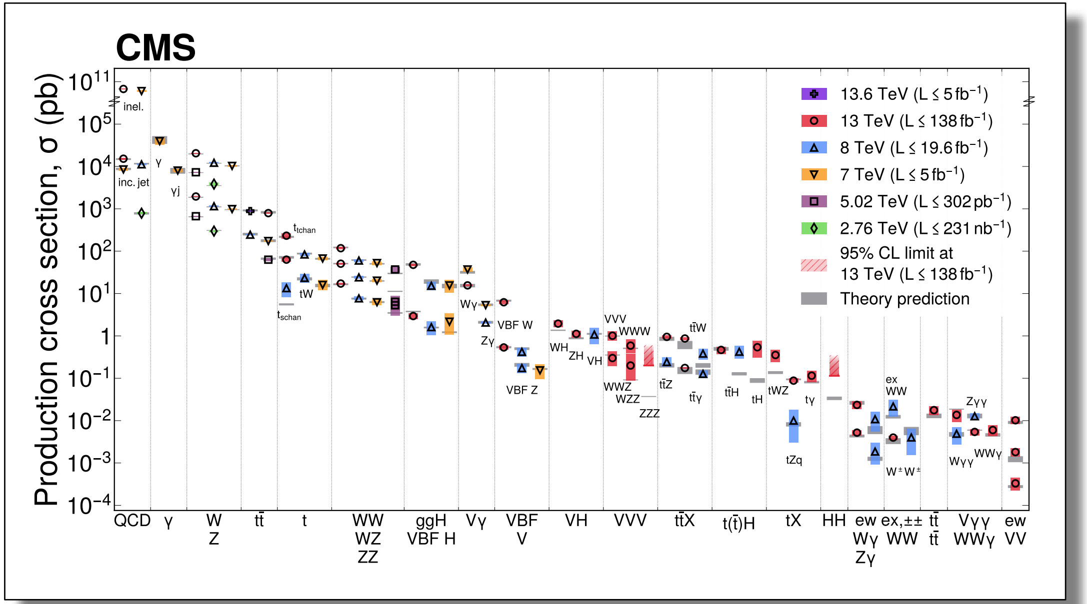
Overview

- Introduction
- Antenna subtraction: $pp \to \gamma\gamma + j$ @NNLO
- Generalized antenna functions
- Summary and outlook

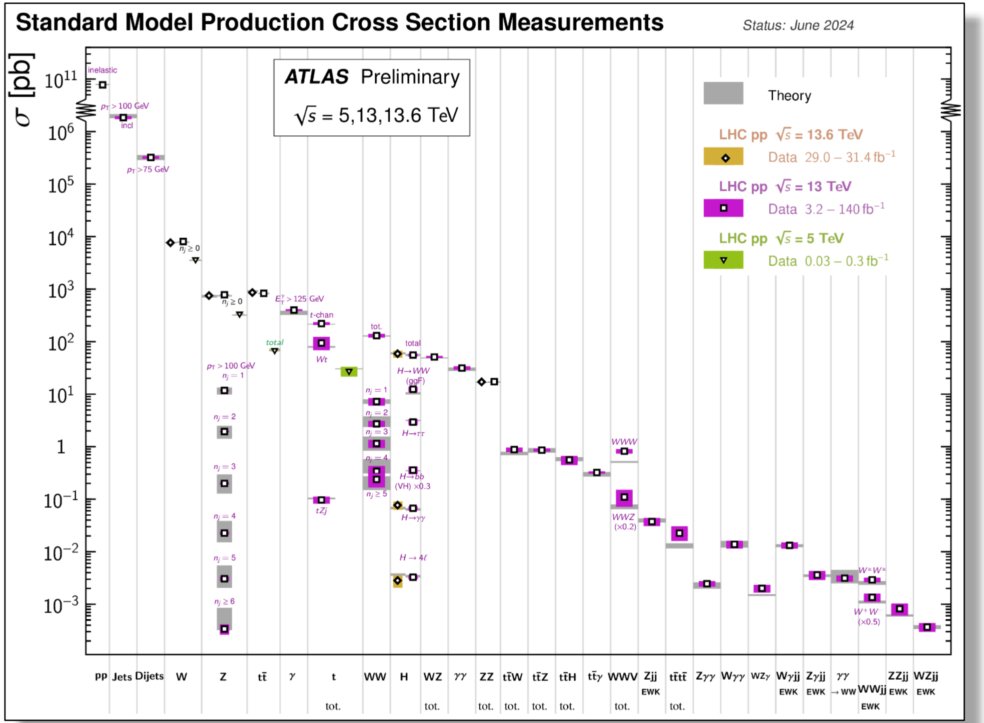
INTRODUCTION

Precision Phenomenology for Collider Physics

Precise theoretical predictions are crucial to **probe the Standard Model** and search for **new physics**.



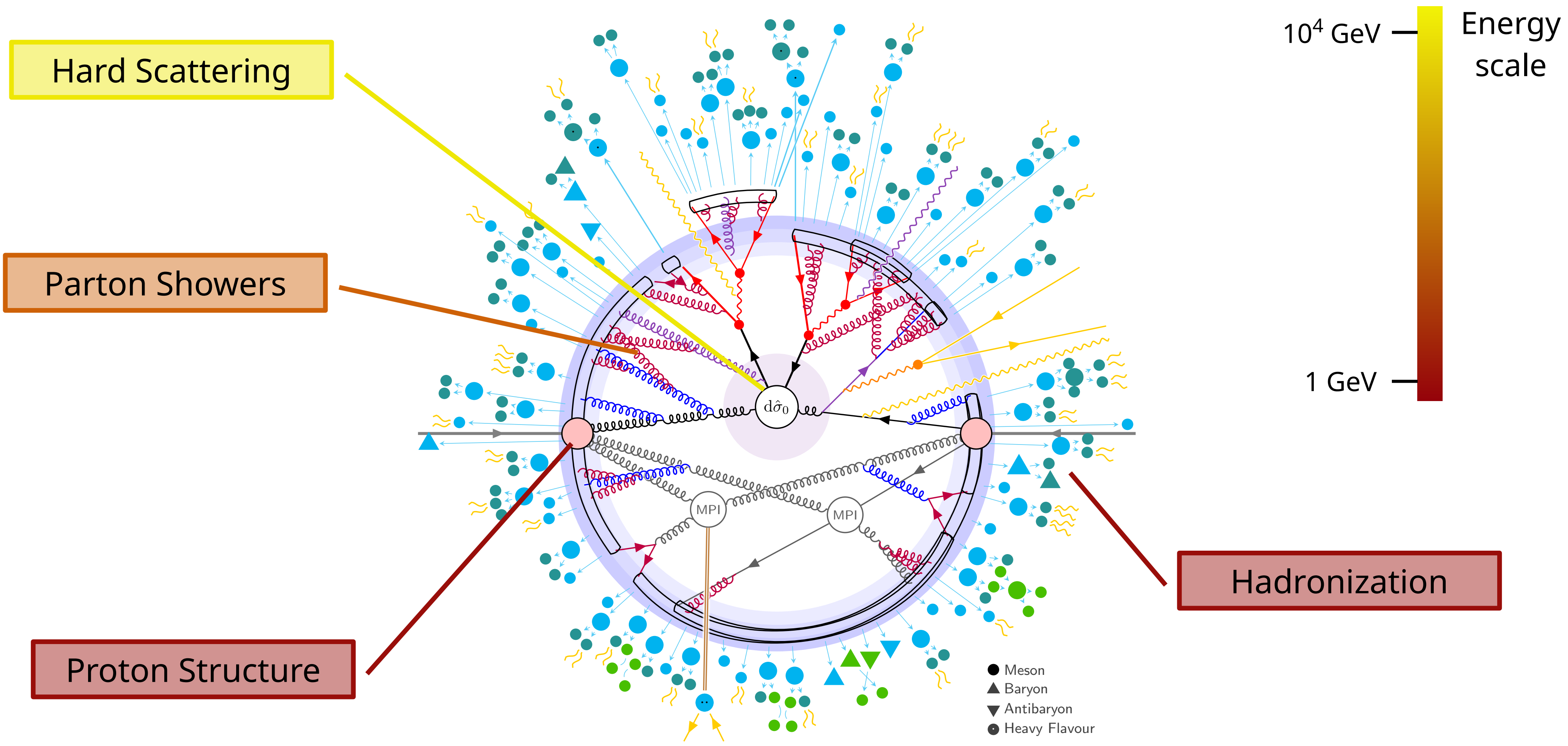
[CMS-SMP-23-004]



[ATL-PHYS-PUB-2024-011]

New data from the LHC and future colliders demand **improvement** and **automation** of precision calculations.

Particle collision: a tale of many scales



[Pythia manual]

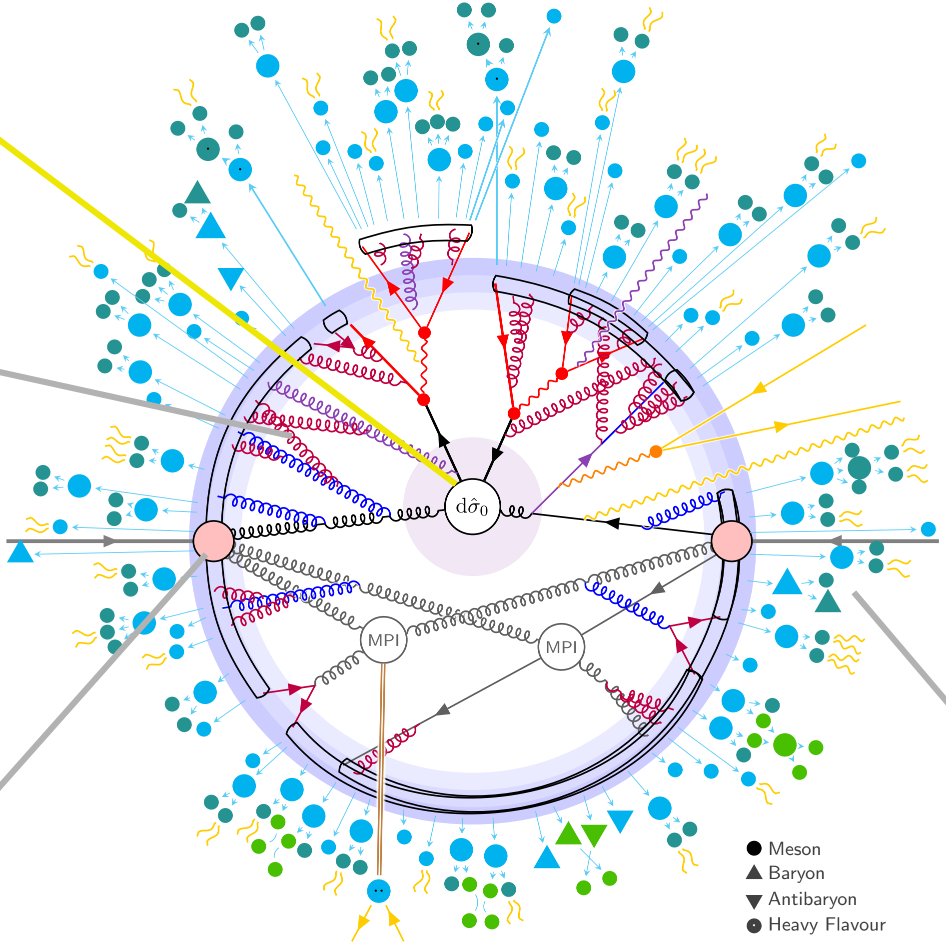
Particle collision: a tale of many scales

in this talk:

Hard Scattering

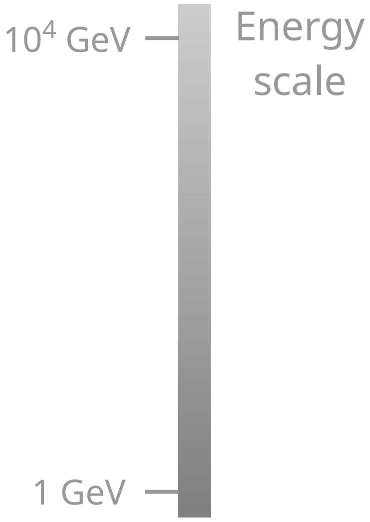
Parton Showers

Proton Structure

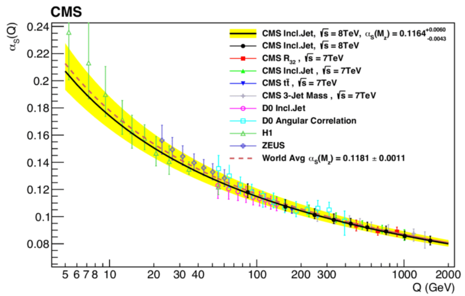


[Pythia manual]

Hadronization



Fixed-order calculations in QCD



High energy: $\boldsymbol{\alpha_s < 1}$, **perturbative regime** of QCD

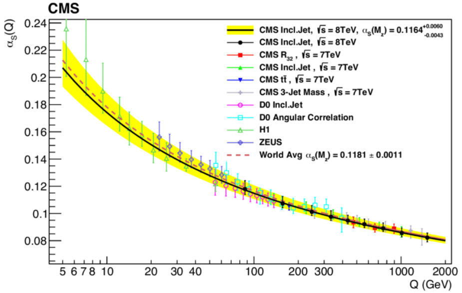
$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \dots$$

Leading Order (LO)	Next-to-Leading Order (NLO)	Next-to-Next-to Leading Order (NNLO)	Next-to-Next-to-Next-to Leading Order (N^3LO)
	O(10%) - O(100%)	O(1%) - O(10%)	≤ O(1%)

\>> accuracy

\>>> complexity, manpower, computational cost

Fixed-order calculations in QCD



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state-of-the-art of FO calculations

Leading Order (LO)	Next-to-Leading Order (NLO)	Next-to-Next-to Leading Order (NNLO)	Next-to-Next-to-Next-to Leading Order (N^3LO)
	O(10%) - O(100%)	O(1%) - O(10%)	≤ O(1%)

>> accuracy

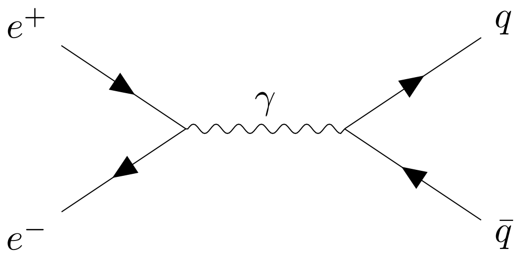
>>> complexity, manpower, computational cost

A simple example: $e^+e^- \to$ jets

$\Phi_n \equiv$ n-particle phase space

$M_n^\ell \equiv$ n-parton ℓ-loop matrix element

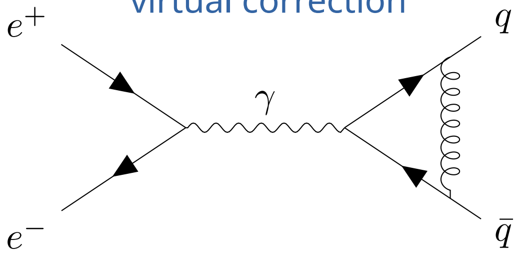
LO:



$$\frac{1}{2s}\int \Phi_2\, M_2^0 = \sigma_0$$

NLO:

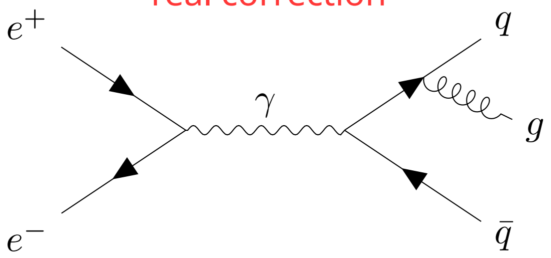
(renormalized) virtual correction



dim. reg. $d = 4 - 2\epsilon$

$$\frac{1}{2s}\int \Phi_2\, M_2^1 = \frac{\alpha_s C_F}{\pi}\sigma_0\left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} + \text{finite terms}\right)$$

real correction



$$\frac{1}{2s}\int \Phi_3\, M_3^0 = \frac{\alpha_s C_F}{\pi}\sigma_0 \int_0^1 \frac{\mathrm{d}y}{y^{1+\epsilon}}\,\frac{\mathrm{d}z}{z^{1+\epsilon}}(1-y)^{-2\epsilon}(1-z)^{-\epsilon}\left[(1-y)(1-z+z^2) - \tfrac{1}{2}(z^2+y^2+y^2z^2)\right]$$

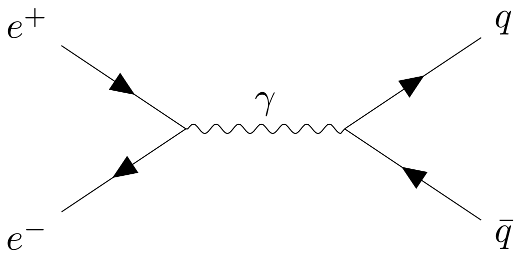
$$= \frac{\alpha_s C_F}{\pi}\sigma_0\left(+\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \text{finite terms}\right)$$

A simple example: $e^+e^- \to$ jets

$\Phi_n \equiv$ n-particle phase space

$M_n^\ell \equiv$ n-parton ℓ-loop matrix element

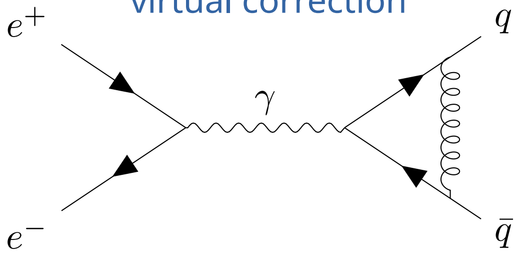
LO:



$$\frac{1}{2s}\int \Phi_2\, M_2^0 = \sigma_0$$

NLO:

(renormalized) virtual correction



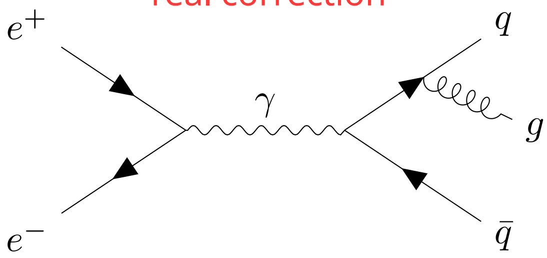
dim. reg. $d = 4 - 2\epsilon$

$$\frac{1}{2s}\int \Phi_2\, M_2^1 = \frac{\alpha_s C_F}{\pi}\sigma_0 \left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} + \text{finite terms} \right)$$

infrared singularities

the sum is finite!

real correction



$$\frac{1}{2s}\int \Phi_3\, M_3^0 = \frac{\alpha_s C_F}{\pi}\sigma_0 \int_0^1 \frac{\mathrm{d}y}{y^{1+\epsilon}} \frac{\mathrm{d}z}{z^{1+\epsilon}} (1-y)^{-2\epsilon}(1-z)^{-\epsilon} \left[(1-y)(1-z+z^2) - \tfrac{1}{2}(z^2+y^2+y^2z^2)\right]$$

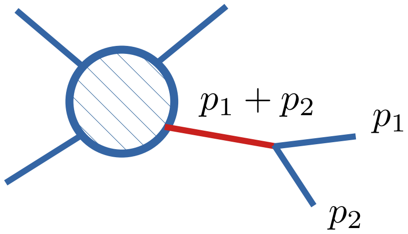
$$= \frac{\alpha_s C_F}{\pi}\sigma_0 \left(+\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \text{finite terms} \right)$$

Infrared divergences arise when **massless** propagators go **on-shell**

in loop integrals, explicit singularities

$$M_n^1 = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + \dots$$

after phase-space integration, implicit singularities



propagator $\sim \frac{1}{(p_1+p_2)^2} = \frac{1}{2E_1E_2(1-\cos\theta_{12})}$ diverges when

soft limit $E_{1,2} \to 0$

collinear limit $\theta_{12} \to 0$

Fortunately, QCD has a **universal behaviour** in IR limits!

- IR-singularities of loop amplitudes:

$$|A^1\rangle = I^1\,|A^0\rangle + |A^1_{\rm fin}\rangle\ , \quad |A^2\rangle = I^1\,|A^1\rangle + I^2\,|A^0\rangle + |A^2_{\rm fin}\rangle$$

[Catani '98] [Bern,De Freitas,Dixon '03] [Gardi,Magnea '09] [Becher,Neubert '09]

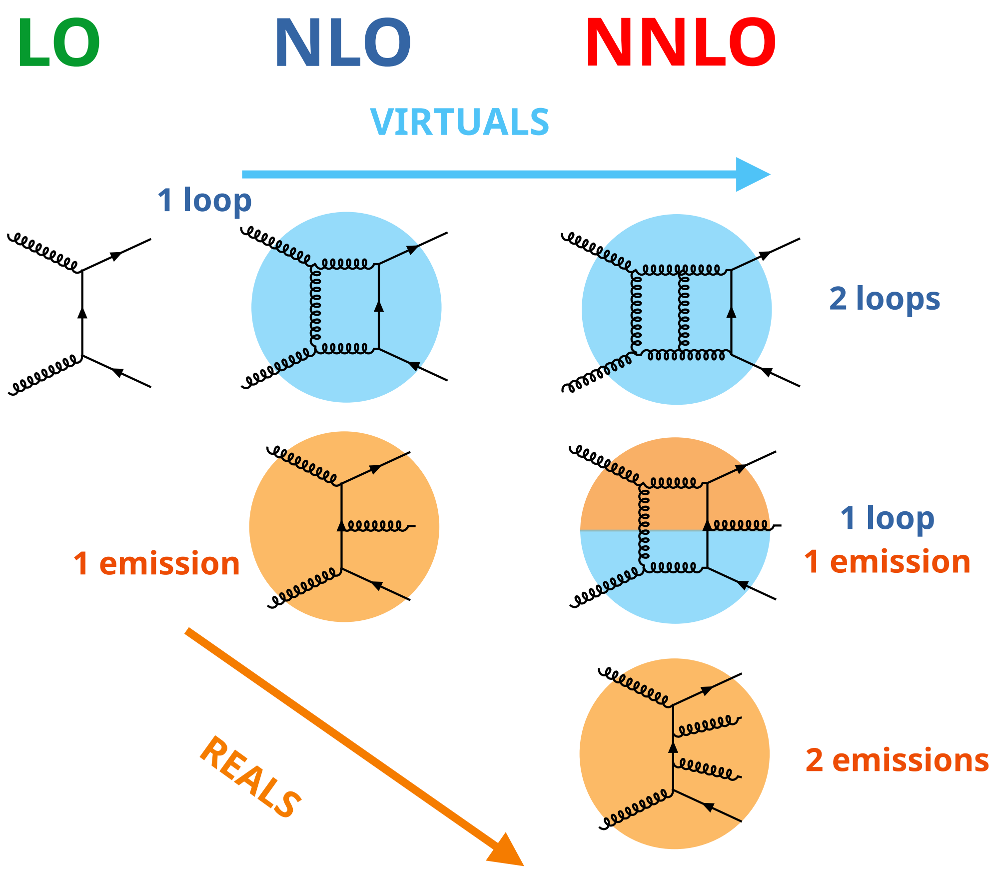
- factorization of scattering amplitudes in soft and collinear limits:

$$|A(q,p_1,\dots,p_n)\rangle \sim \sum_{i=1}^n \boldsymbol{T}_i \frac{p_i^\mu}{p_i\cdot q}\,|A(p_1,\dots,p_n)\rangle$$

$$|A(\dots,p_i,p_j,\dots)|^2 \sim \frac{2}{s_{ij}} P_{I\leftarrow ij}(z) |A(\dots,p_I,\dots)|^2$$

[Altarelli,Parisi '77] [Ellis,Marchesini,Webber '87] [Berends,Giele '89] [Campbell,Glover '98] [Catani,Grazzini '00]

Infrared divergences



The cancellation of IR singularity for **IR-safe** (sufficiently inclusive) observables is guaranteed

[Bloch,Nordsieck 1937]	abelian (QED)
[Kinoshita 1962] [Lee,Nauenberg 1964]	non-abelian (QCD)

Why can't we directly compute the sum of virtuals and reals? ***

- fully differential over different phase-spaces;
- no analytical control (PDFs, cuts, …)
- numerical integration

Infrared divergences need to be properly **regularized** and **subtracted**.

This is done within **subtraction** or **slicing schemes.**

***: Loop Tree Duality – based subtraction

NLO calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \ldots$$

Next-to-Leading Order (NLO)

😐 Hard;

✓ Fully general approaches;

✓ Automated

✗ Not accurate enough;

General techniques:

- Dipole subtraction; [Catani,Seymour '96]
- FKS subtraction; [Frixione,Kunszt,Signer '96]

+

Automation of one-loop amplitudes:

- Recola;
- OpenLoops;

Public tools implementing NLO calculations:

- MadGraph5;
- Sherpa;
- Herwig;
- POWHEG BOX;
- …

+ parton showers

NNLO calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \dots$$

Next-to-Next-to Leading Order (NNLO)

☹ Harder;

✓ Computed for all 2→2 processes, and recently some 2→3;

Thanks also to 2-loop 5-point amplitudes

✗ No fully general approaches;

✗ Not automated;

Several proposed/implemented approaches:

- Antenna subtraction; [Gehrmann,Gehrmann-De Ridder,Glover '05] [Currie,Glover,Wells '13]
- CoLoRFul subtraction; [Del Duca,Duhr,Kardos,Somogyi,Szor,Trocsanyi,Tulipant '16]
- qT-slicing; [Catani,Grazzini '07]
- Sector-improved residue subtraction; [Czakon '10] [Czakon,Heymes '14]
- N-jettiness slicing; [Gaunt,Stahlhofen,Tackmann,Walsh '14]
- Projection-to-Born; [Cacciari,Dreyer,Karlberg,Salam,Zanderighi '18]
- Local analytic sector subtraction; [Magnea,Maina,Pelliccioli, Signorile-Signorile,Torrielli,Uccirati '17]
- Nested soft-collinear subtraction; [Caola,Melnikov,Rontsch '17]

Public tools: MCFM, MATRIX

Non-public tools: NNLOJET, STRIPPER, ...

NNLO calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \ldots$$

Next-to-Next-to Leading Order (NNLO)

☹ Harder;

✓ Computed for all 2→2 processes, and recently some 2→3;

Thanks also to 2-loop 5-point amplitudes

≈ Towards fully general approaches;

≈ Towards automation;

Several proposed/implemented approaches:

- Antenna subtraction; [Chen,Gehrmann,Glover,Huss,MM '22] gluons-only [Gehrmann,Glover,MM '23] general
- CoLoRFul subtraction; [Del Duca,Duhr,Fekeshazy,Guadagni,Mukherjee, Somogyi,Tramontano,Van Thurenhout '24]
- qT-slicing; colour-singlet production and decay
- Sector-improved residue subtraction; [Czakon,Mitov,Poncelet et al. '21,'22,'23]
- N-jettiness slicing; NNLO correction for several 2→3 processes
- Projection-to-Born; final-state radiation only
- Local analytic sector subtraction; [Bertolotti,Magnea,Pelliccioli,Ratti, Signorile-Signorile,Torrielli,Uccirati '22]
- Nested soft-collinear subtraction; [Devoto,Melnikov,Rontsch, Signorile-Signorile,Tagliabue '23] gluons-only

Public tools: MCFM, MATRIX

Non-public tools: NNLOJET, STRIPPER, ...

ANTENNA SUBTRACTION

Local subtraction at NLO

$$\int_n \equiv$$ Integration over an n-particle phase space

Partonic cross section at NLO:

$$\mathrm{d}\sigma_{NLO} = \int_n \mathrm{d}\sigma^V + \int_{n+1} \mathrm{d}\sigma^R$$

infrared divergent infrared divergent

Subtraction at NLO:

virtual subtraction term

real subtraction term

$$\mathrm{d}\sigma_{NLO} = \int_n [\mathrm{d}\sigma^V - \mathrm{d}\sigma^T] + \int_{n+1} [\mathrm{d}\sigma^R - \mathrm{d}\sigma^S]$$

with $\mathrm{d}\sigma^T = -\int_1 \mathrm{d}\sigma^S$ to recover the original result.

Local subtraction at NNLO

$$\int_n \equiv$$ Integration over an n-particle phase space

Partonic cross section at NNLO:

$$\mathrm{d}\sigma_{NNLO} = \int_n \mathrm{d}\sigma^{VV} + \int_{n+1} \mathrm{d}\sigma^{RV} + \int_{n+2} \mathrm{d}\sigma^{RR}$$

infrared divergent — infrared divergent — infrared divergent

Subtraction at NNLO:

$$\mathrm{d}\sigma_{NNLO} = \int_n [\mathrm{d}\sigma^{VV} - \mathrm{d}\sigma^{U}] + \int_{n+1} [\mathrm{d}\sigma^{RV} - \mathrm{d}\sigma^{T}] + \int_{n+2} [\mathrm{d}\sigma^{RR} - \mathrm{d}\sigma^{S}]$$

double-virtual subtraction term

real-virtual subtraction term

double-real subtraction term

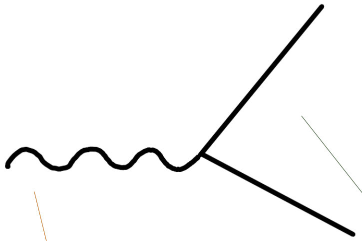
with:

$$\mathrm{d}\sigma^{S} = \mathrm{d}\sigma^{S,1} + \mathrm{d}\sigma^{S,2} \qquad \mathrm{d}\sigma^{T} = \mathrm{d}\sigma^{VS} - \int_1 \mathrm{d}\sigma^{S,1} \qquad \mathrm{d}\sigma^{U} = -\int_1 \mathrm{d}\sigma^{VS} - \int_2 \mathrm{d}\sigma^{S,2}$$

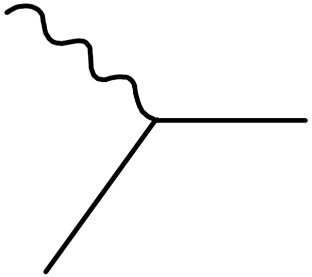
Antenna idea: use matrix elements to fix matrix elements

The divergent behaviour of QCD matrix elements is universal → Let's use **simple** QCD matrix elements to capture it!

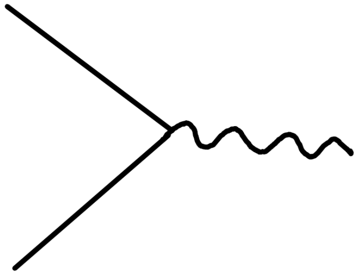
colour-singlet decay



DIS-like kinematics



colour-singlet production



colour singlet

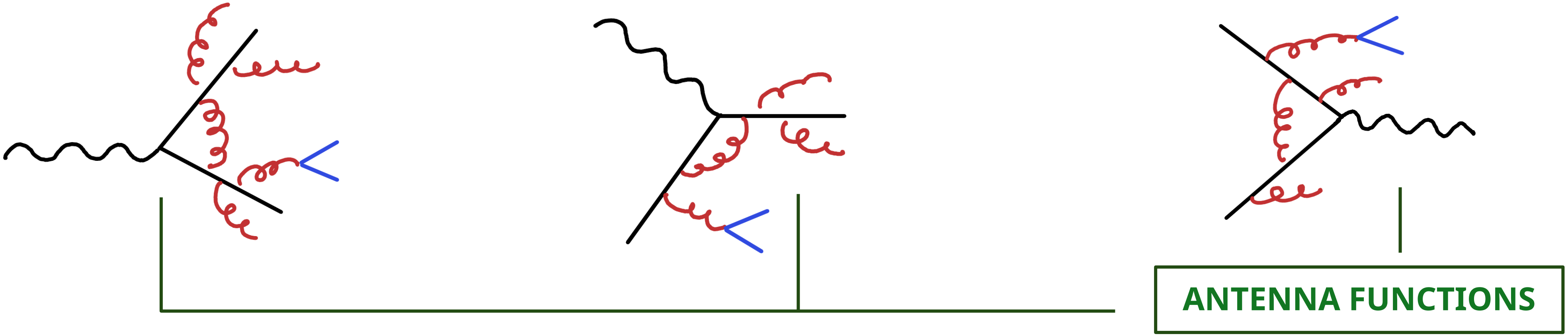
partons (quarks or gluons)

All three configurations needed for hadronic processes (FS and IS radiation)

Main idea: use matrix elements to fix matrix elements

The divergent behaviour of QCD matrix elements is universal → Let's use simple QCD matrix elements to capture it!

colour-singlet decay | DIS-like kinematics | colour-singlet production



These (colour-ordered) matrix elements can be used to construct subtraction terms!

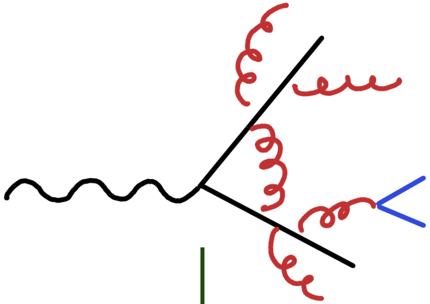
[Gehrmann-De Ridder,Gehrmann,Glover '03,'04,'05]

The two original partons constitute the **colour dipole** (antenna) emitting radiation.

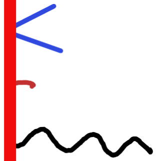
Main idea: use matrix elements to fix matrix elements

The divergent behaviour of QCD matrix elements is universal → Let's use simple QCD matrix elements to capture it!

colour-singlet decay



roduction



decomposition of QCD amplitudes in colour space:

$$|\mathcal{A}_n^\ell(\{p\}_n)\rangle = \sum_{c\in I^\ell} C_{n,c}^\ell \, A_{n,c}^\ell(\{p\}_n)$$

colour basis

colour-ordered partial amplitude

squared **colour-ordered** amplitude at tree-level

$$M_{n,c}^0(\{p\}_n) = |A_{n,c}^0(\{p\}_n)|^2$$

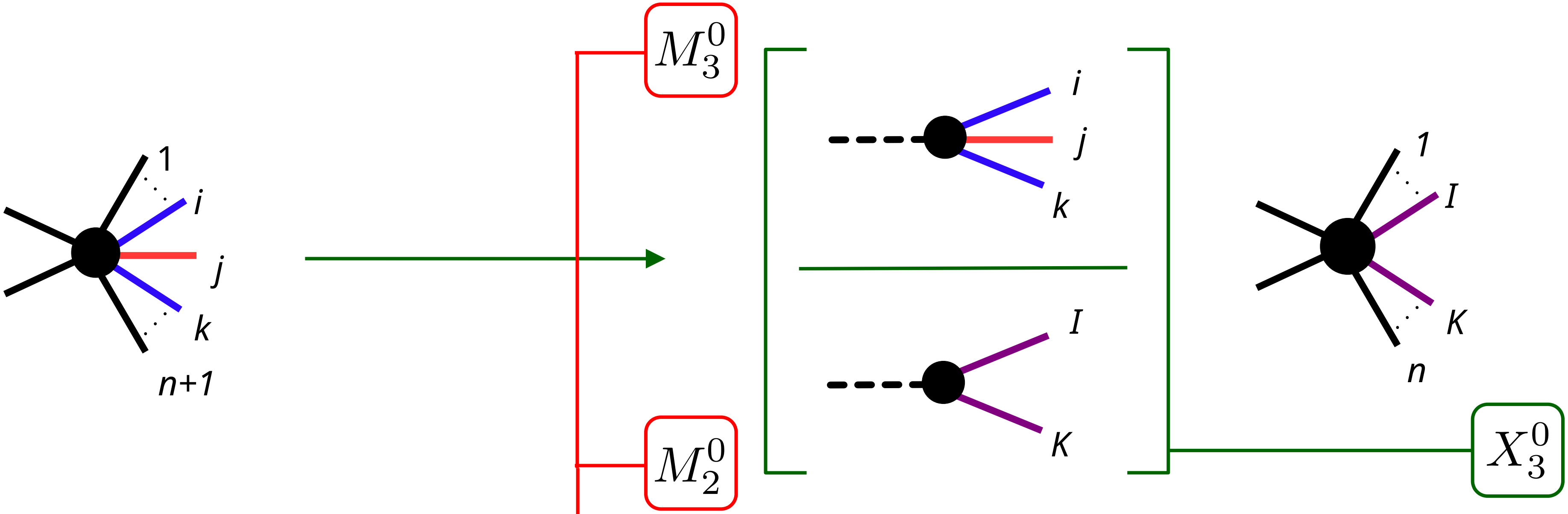
UNCTIONS

These (colour-ordered) matrix e
can be used to construct subtra

tute the
colour dipole (antenna) emitting radiation.

[Gehrmann-De Ridder,Gehrmann,Glover '03,'04,'05]

NLO: three-parton tree-level antenna functions



$$X_3^0 = \frac{M_3^0}{M_2^0}$$

$$\mathcal{X}_3^0 \propto \int \mathrm{d}\Phi_3 \, X_3^0$$

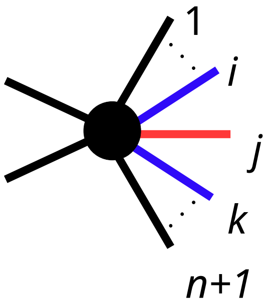
Integrated antenna function

Matrix elements for **colour singlet** decay.

Encapsulates the divergent behaviour when parton **j** becomes **soft** or **collinear** to **i**, **k**.

Antenna subtraction at NLO

Real

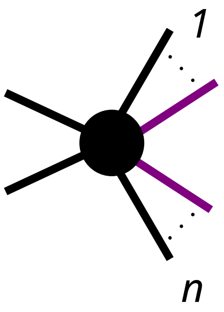


parton j soft or collinear to i or k

factorization properties of QCD

3-parton tree antenna

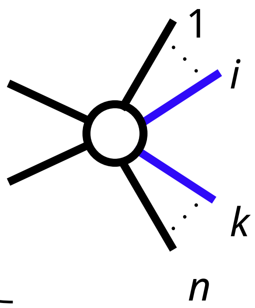
$$X_3^0(i,j,k)\cdot$$



Antenna function

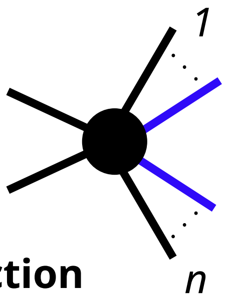
Analytical integration over PS of the unresolved radiation

Virtual

Poles 

= Poles

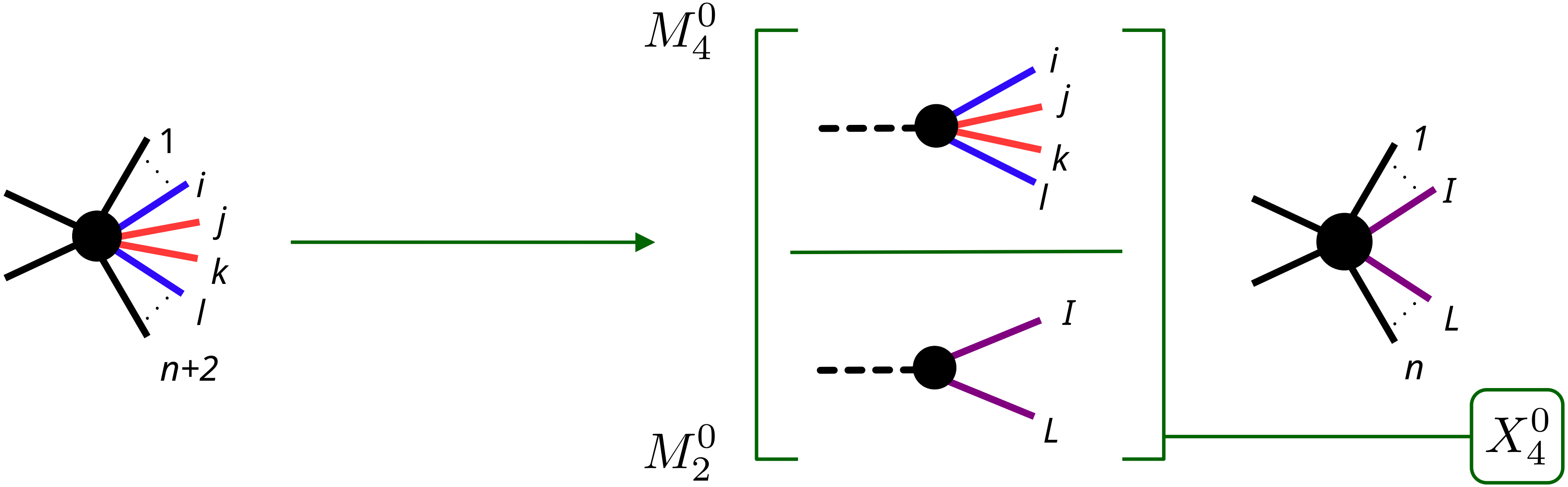
$$\mathcal{X}_3^0(s_{ik})\cdot$$



Integrated antenna function

[Gehrmann-De Ridder,Gehrmann,Glover ‘05] [Currie,Glover,Wells ‘13]

NNLO: four-parton tree-level antenna functions

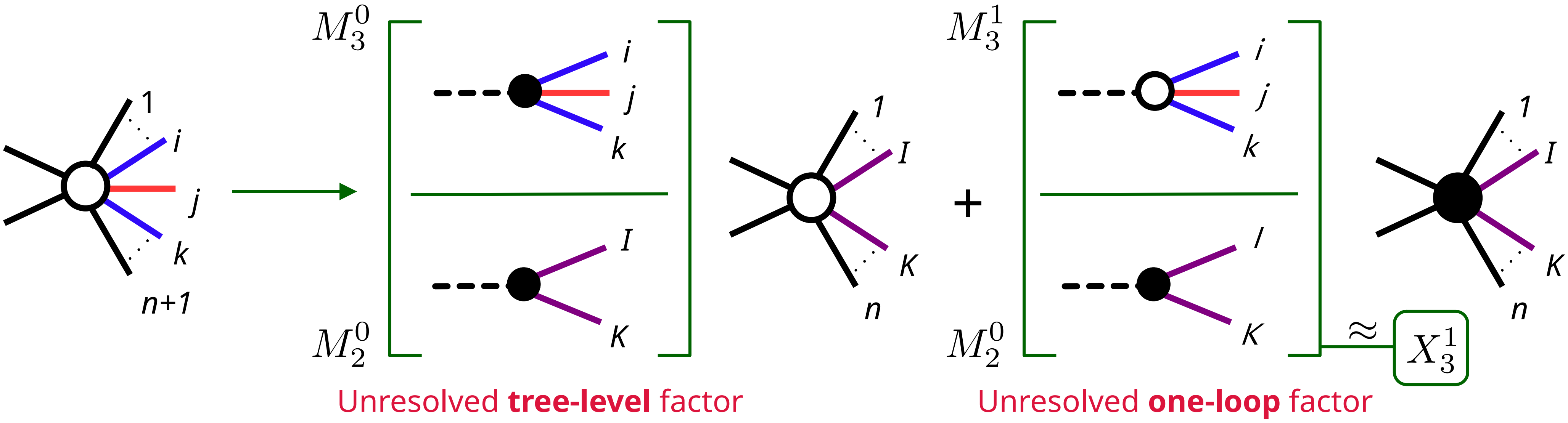


$$X_4^0 = \frac{M_4^0}{M_2^0}$$

$$\mathcal{X}_4^0 \propto \int \mathrm{d}\Phi_4 \, X_4^0$$

Four-parton tree-level antenna functions are extracted analogously to the three-parton ones

NNLO: three-parton one-loop antenna functions



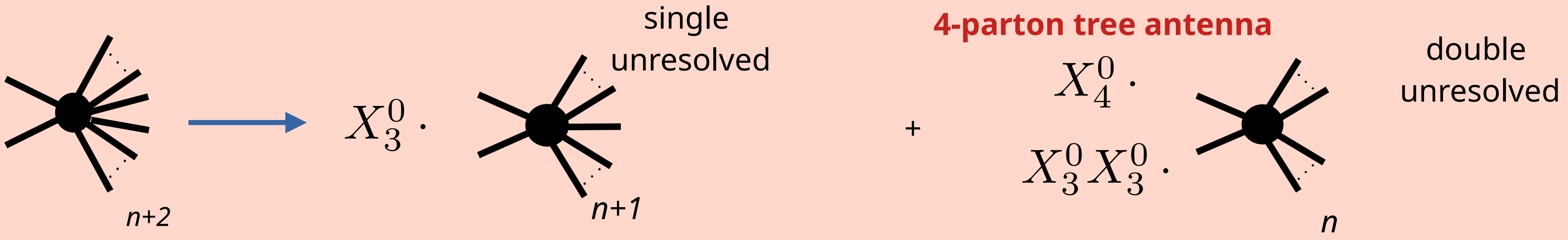
Three-parton one-loop antenna defined removing from the one-loop decay matrix element the unresolved tree-level configuration:

$$X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_1^2}{M_2^0}, \qquad \mathcal{X}_3^1 \propto \int \mathrm{d}\Phi_3 \, X_3^1$$

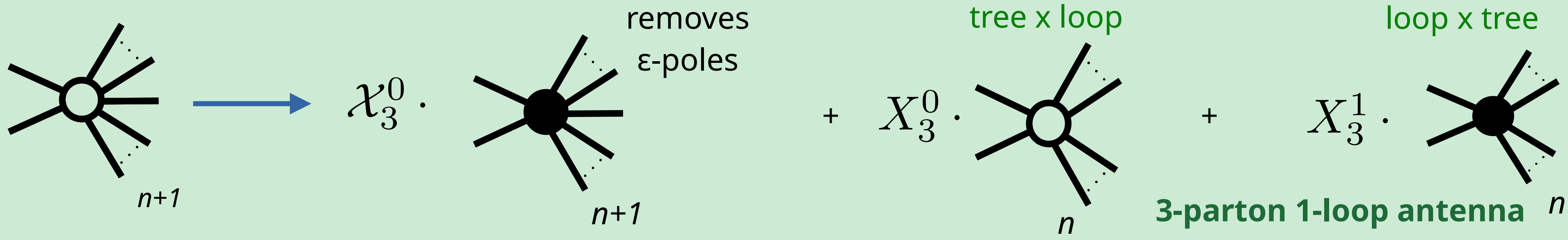
Antenna subtraction at NNLO

[Gehrmann-De Ridder,Gehrmann,Glover '05] [Currie,Glover,Wells '13]

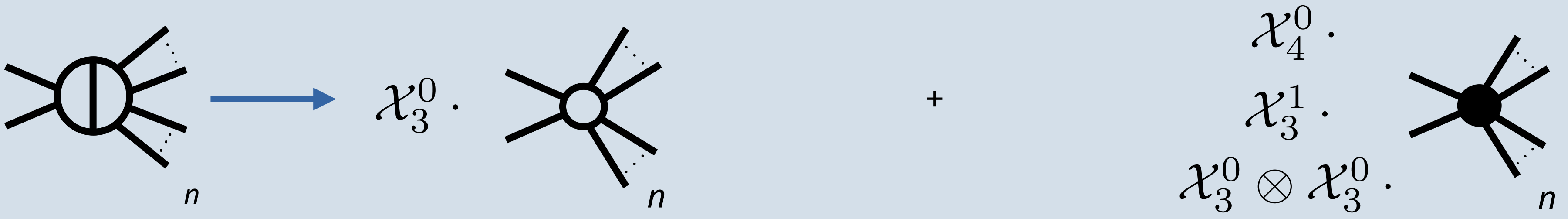
RR:



RV:



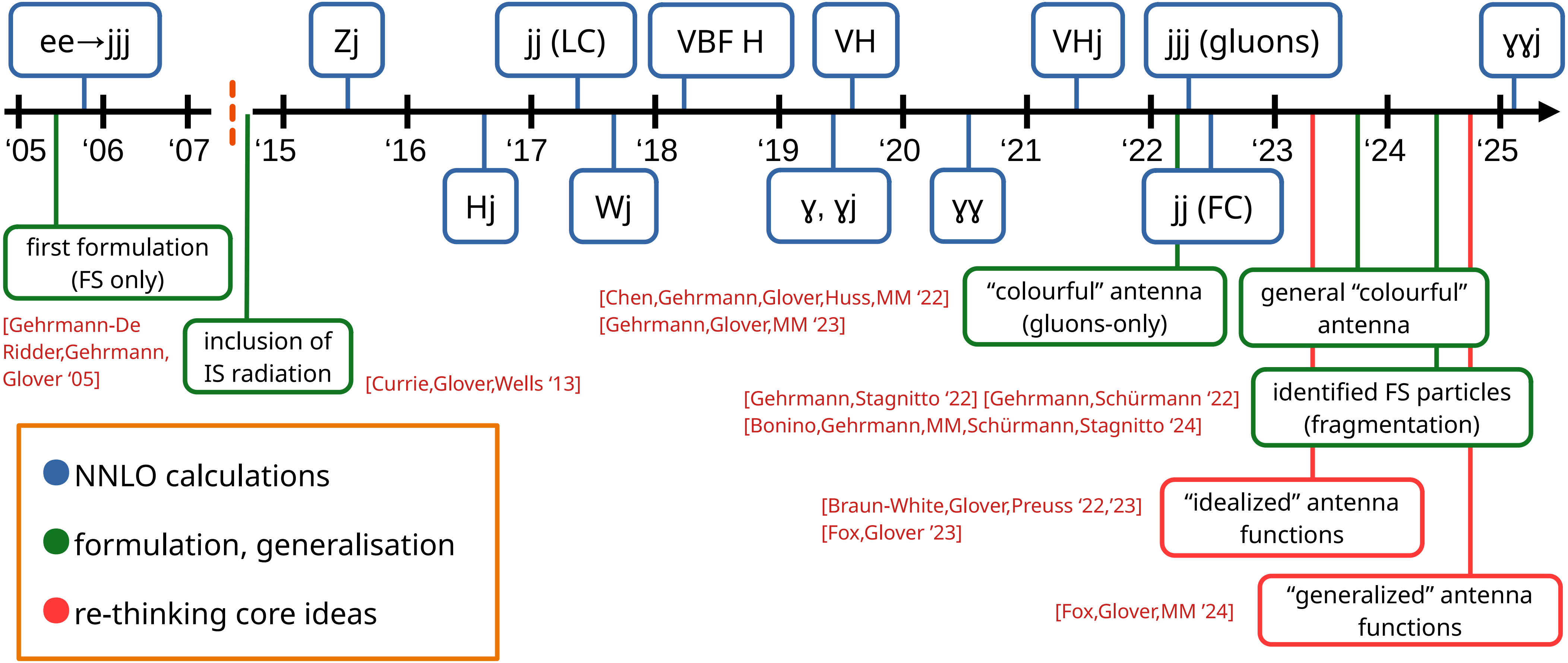
VV:



History of antenna subtraction

[Braun-White, Chen, Cruz-Martinez, Fox, Garcia-Rodriguez, Gauld, Gehrmann, Gehrmann-De Ridder, Glover, Hoefer, Huss, Jaquier, Majer, MM, Mo, Morgan, Schuermann, Stagnitto, Pires, Walker, Withehead]

Successfully applied at NNLO to a variety of processes within the ***NNLOJET*** Monte Carlo framework



A cutting-edge application

Diphoton production at hadron colliders:

- background for H→γγ and BSM signals;
- perturbative QCD;
- systematics of photon isolation;

ATLAS analysis at 13 TeV:

Fiducial cuts: [ATLAS 2107.09330]

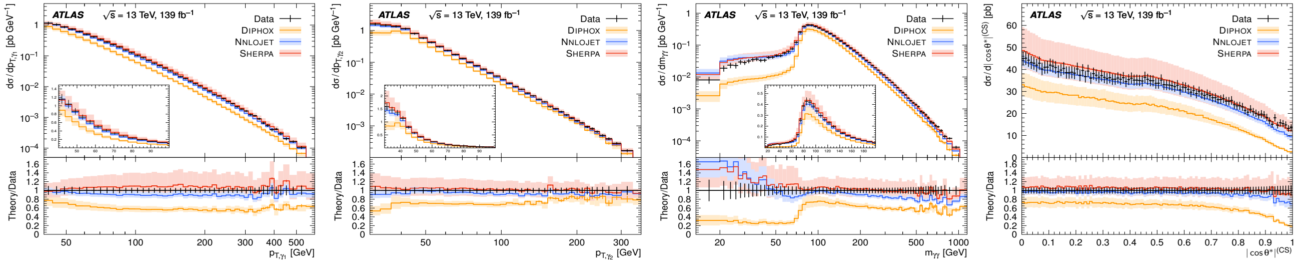
$$p_{T,\gamma_1} > 40\ \text{GeV}, \quad p_{T,\gamma_2} > 30\ \text{GeV},$$
$$|\eta_\gamma| \in (0, 1.37) \cup (1.52, 2.37), \quad \Delta R_{\gamma\gamma} > 0.4$$

Photon isolation:

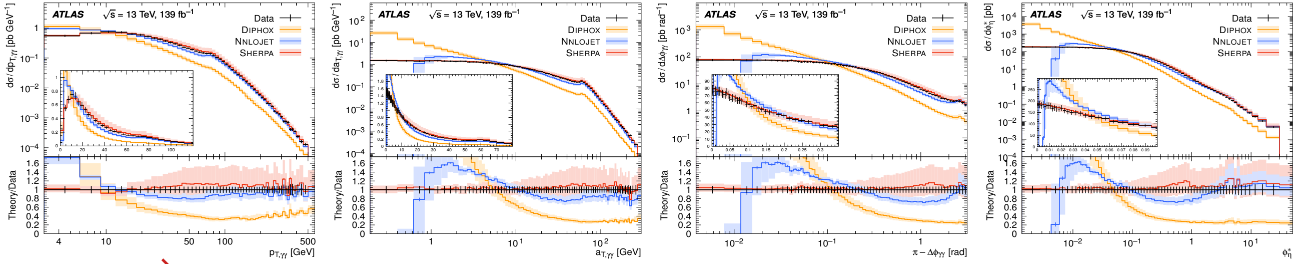
$$(\mathrm{R}, \epsilon_{T,\gamma}) = (0.2, 0.09)$$

Compared with SHERPA, NNLOJET, DIPHOX

Non-trivial for back-to-back photons: $p_{T,\gamma_1},\ p_{T,\gamma_2},\ m_{\gamma\gamma},\ |\cos\theta_{CS}|$



Vanishing for back-to-back photons: $p_{T,\gamma\gamma},\ a_{T,\gamma\gamma},\ \phi_{\text{acop}},\ \phi^*_\eta$



only NLO-accurate!

event shapes:

hadron collider thrust

$$a_T = 2 \cdot \frac{|p_{x,\gamma_1} p_{y,\gamma_2} - p_{y,\gamma_1} p_{x,\gamma_2}|}{|\vec{p}_{T,\gamma_1} - \vec{p}_{T,\gamma_2}|}$$

decorrelation angle

$$\phi_{\text{acop}} = \pi - \Delta\phi_{\gamma\gamma}$$

acoplanarity

$$\phi^*_\eta = \tan\frac{\pi - \Delta\phi_{\gamma\gamma}}{2}\sqrt{1 - \tanh^2(\Delta\eta_{\gamma\gamma}/2)}$$

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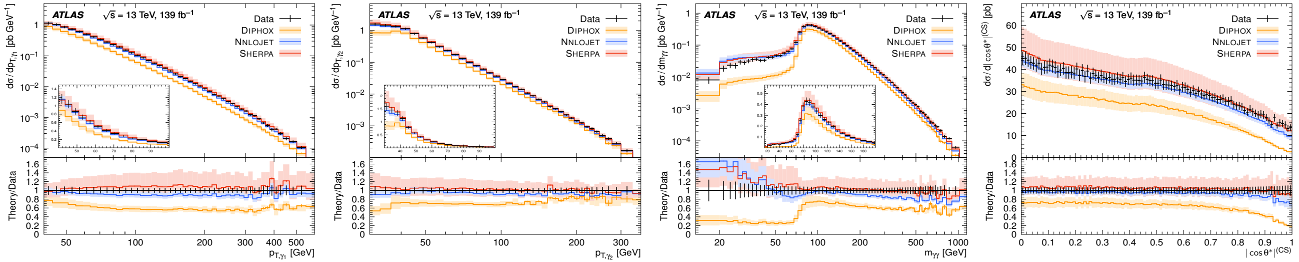
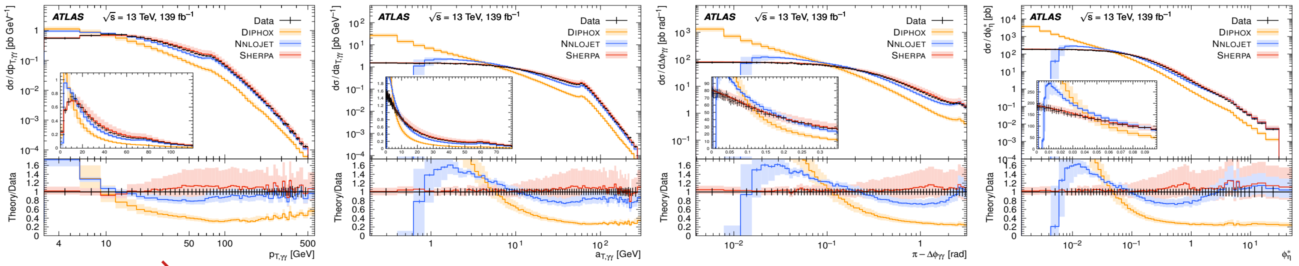


Vanishing for back-to-back photons: $p_{T,\gamma\gamma}$, $a_{T,\gamma\gamma}$, ϕ_{acop}, ϕ^*_η



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pp→γγ+jet @NNLO: [Buccioni,Chen,Feng,Gehrmann,Huss,MM ‘25]

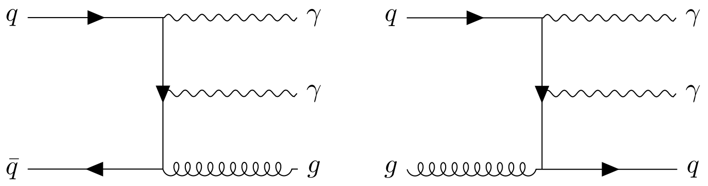
- NNLO-accurate diphoton at non-zero p_T;
- 2→3 process;
- step towards inclusive diphoton production at N³LO;

Calculation

Previous calculation with STRIPPER with leading-colour 2-loop f.r. [Chawdry,Czakon,Mitov,Poncelet '21]

We include of **new ingredients** and extend to a **more inclusive phase-space**

LO: $q\bar{q}$- and qg-initiated channels $\mathcal{O}(\alpha_s)$



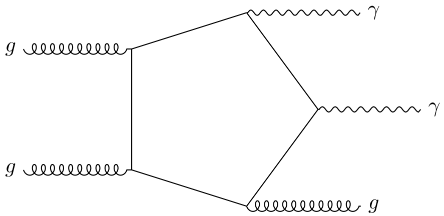
NLO: $\mathcal{O}(\alpha_s^2)$

NNLO: $\mathcal{O}(\alpha_s^3)$ [Buccioni,Lang,Lindert, Maierhoefer,Pozzorini,Zhang,Zoller '19]

- RV and RR**: OpenLoops**
- VV: inclusion of **full-colour 2-loop 5-point** finite reminder. **NEW**

[Agarwal,Buccioni,von Manteuffel,Tancredi '21]

At NNLO, gg-initiated loop-induced channel opens up:



$$\propto \left(\sum_i q_i^2\right)^2$$

$\mathcal{O}(\alpha_s^3)$

PDF-enhanced

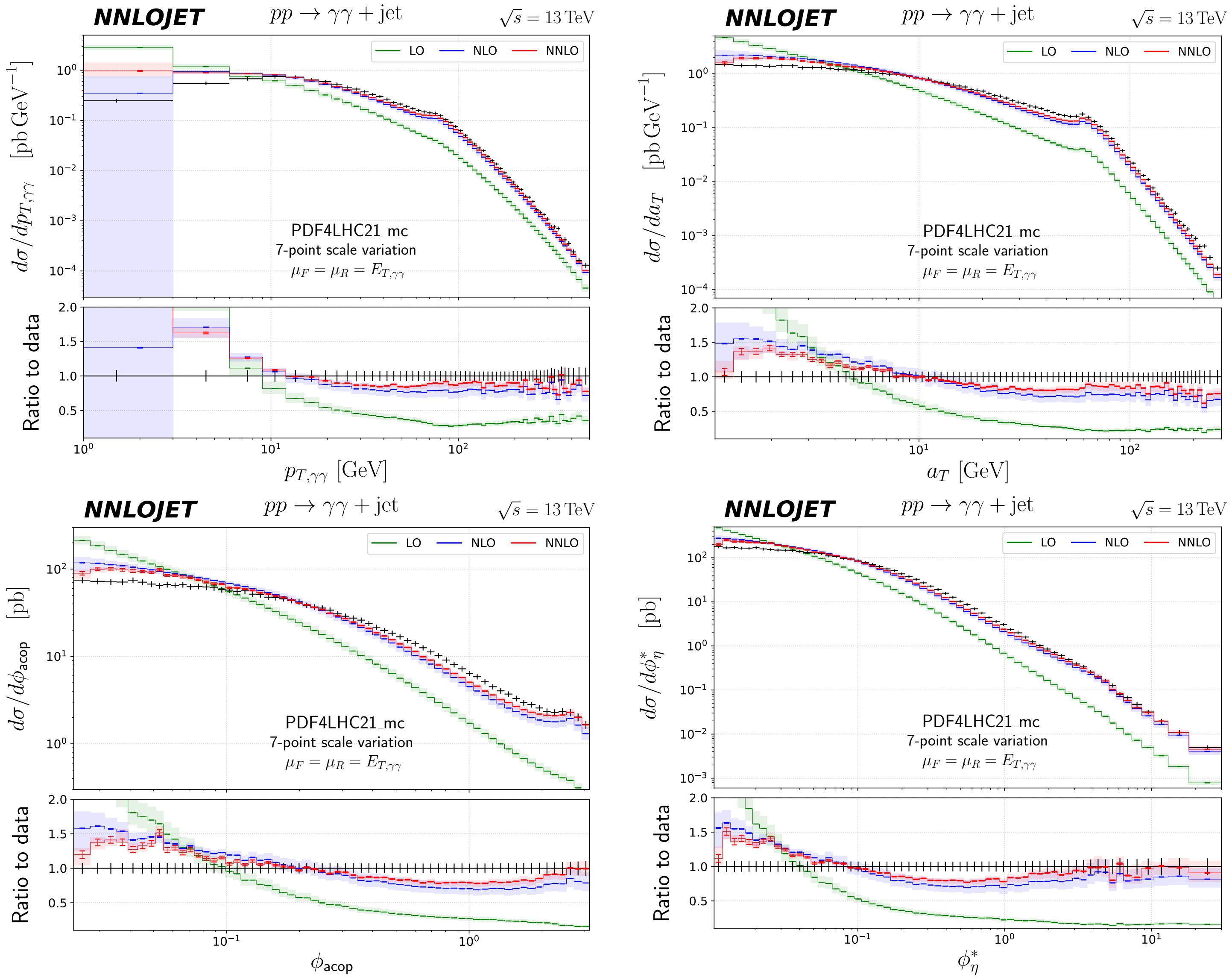
NLO corrections can be sizeable and help with theory uncertainties **NEW**

$\mathcal{O}(\alpha_s^4)$

[Badger,Gehrmann,MM,Moodie '21]
gg-initiated channels only at NLO

- Reals: 6-point 1-loop squared [OpenLoops]
- Virtuals: 5-point (2-loop X 1-loop)

[Badger,Brønnum-Hansen,Chicherin,Gerhmann, Hartanto,Henn,MM,Moodie,Peraro,Zoia '21]
[Agarwal,Buccioni,von Manteuffel,Tancredi '21]



[Buccioni,Chen,Feng,Gehrmann,Huss,MM '25]

- NNLO = O(α_s^3) terms
- Hybrid isolation [Siegert '16]
  Smooth cone $(R_d, \epsilon_d, n) = (0.1, 0.15, 1)$
  [Frixione '98]

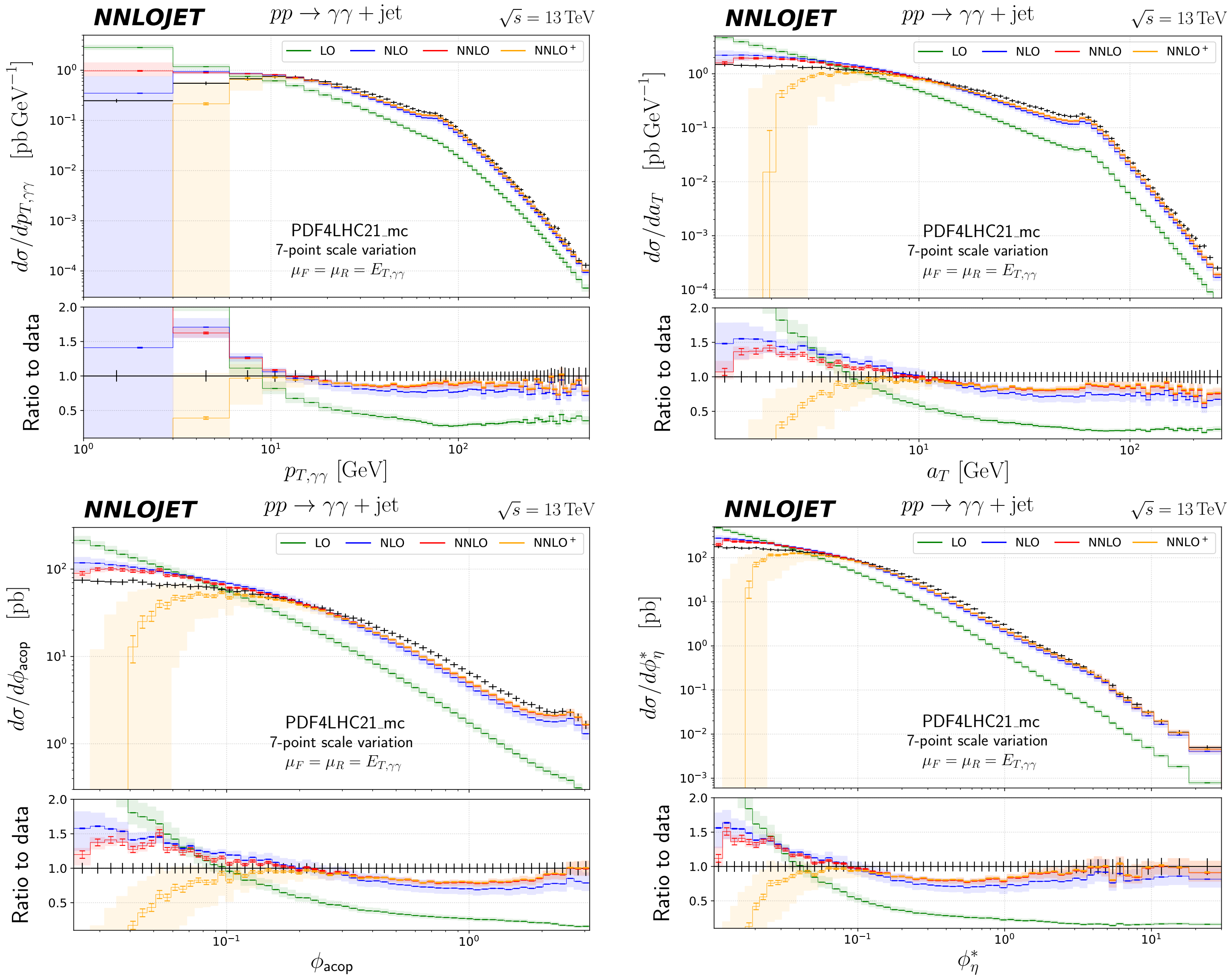
✅ NNLO improves agreement with data, within NLO bands

🟨 still systematic undershooting of data (isolation, PS, …)

✅ good **perturbative** convergence even close to back-to-back limit

✅ good **numerical** stability even close to back-to-back limit

✅ event shapes offer much better resolution



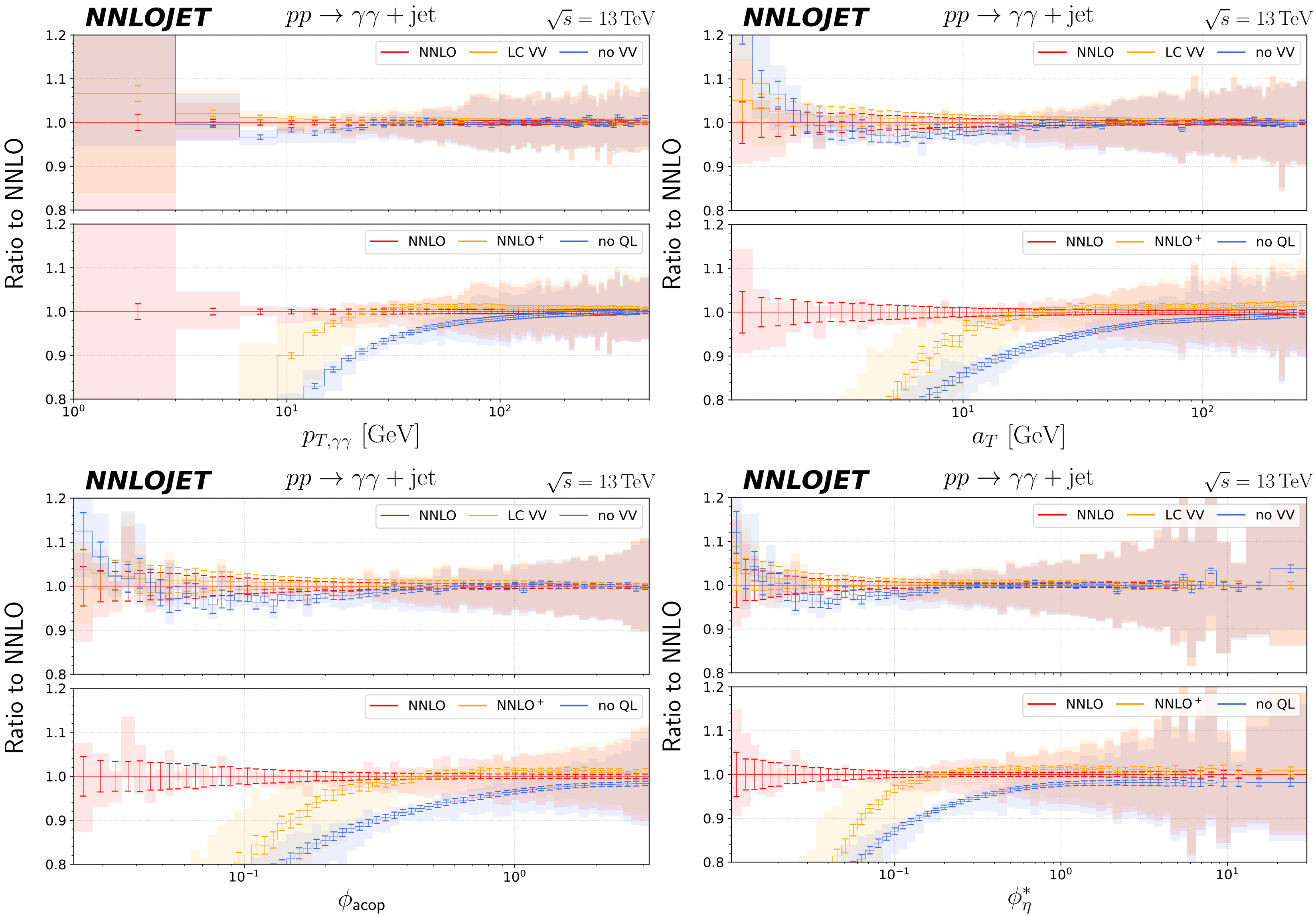
[Buccioni,Chen,Feng,Gehrmann,Huss,MM '25]

- NNLO$^+$ = O(α_s^3) + NLO loop-induced O(α_s^4);

✅ negligible at high p_T (and event shapes)

✅ small intermediate range where NLO loop-induced helps with uncertainty

🟨 large logarithms kicking not so close to back-to-back limit



[Buccioni,Chen,Feng, Gehrmann,Huss,MM '25]

no VV: no 2-loop finite reminder

LC VV: only leading-colour 2-loop finite reminder

no QL: no loop-induced process

NNLO$^+$: with NLO correction to loop-induced process

- fair to truncate 2-loop f.r. at LC: SLC ≤ 0.5%
- LO loop-induced necessary, NLO corrections quickly give large logs ($O(\alpha_s^4)$)

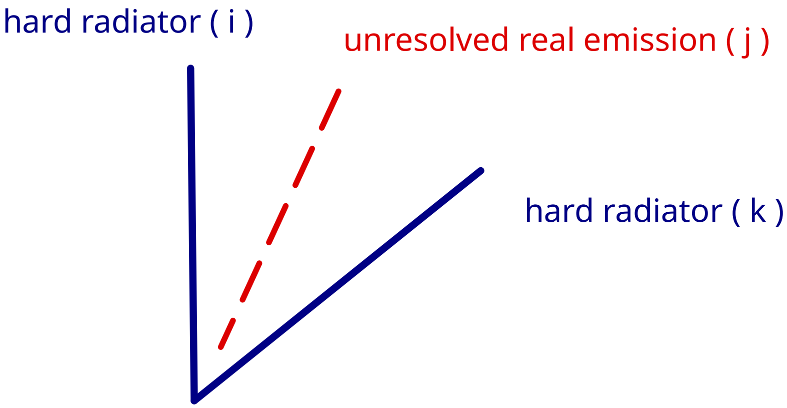
GENERALIZED ANTENNA FUNCTIONS

Generalized antenna functions [Fox,Glover,MM ‘24]

Antenna functions capture IR emissions between **two** hard radiators (antenna configuration)

What **emission topologies** can they describe? (final-state radiation only)

NLO: one unresolved emission → only one possible topology



colour-ordered squared amplitude for real emision

$$M^0_{n+1}(\ldots, i, j, k, \ldots)$$

IR limits ↓

$$X_3^0(i^h, j, k^h)\, M_n^0(\ldots, (\widetilde{ij}), (\widetilde{jk}), \ldots)$$

3→2 momentum mapping:
- momentum conservation
- on-shellness
- IR limits

[Kosower ‘02]

antenna function

Born-level squared amplitude

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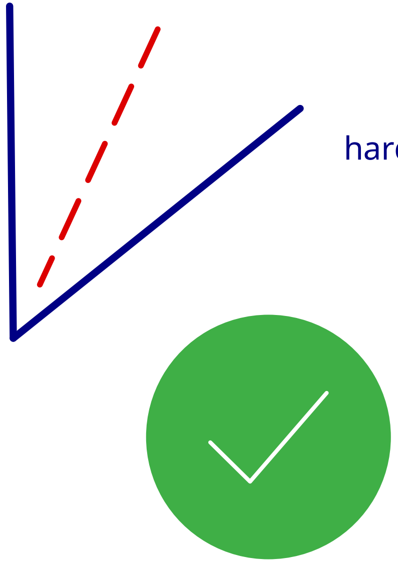

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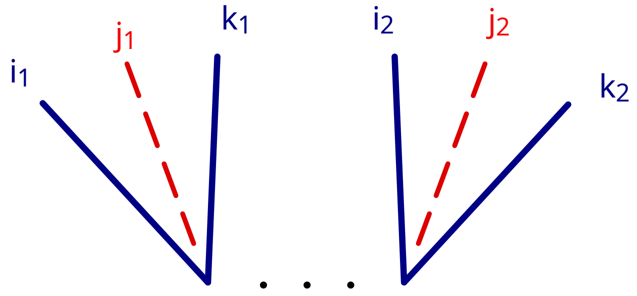

$$M^0_{n+2}(\dots,i_1,j_1,k_1,\dots,i_2,j_2,k_2,\dots)$$

fully **iterated** structure

$$X^0_3(i_1^h,j_1,k_1^h)X^0_3(i_2^h,j_2,k_2^h)M^0_n(\dots,(\widetilde{i_1j_1}),(\widetilde{j_1k_1}),\dots,(\widetilde{i_2j_2}),(\widetilde{j_2k_2}),\dots)$$

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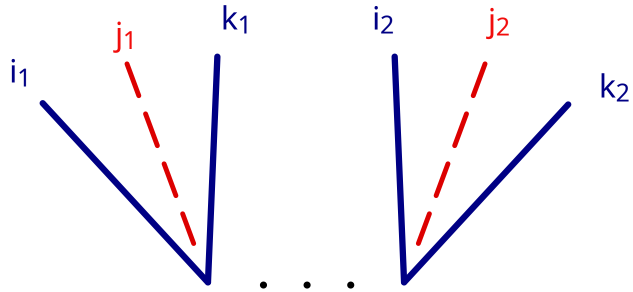

$$M^0_{n+2}(\ldots, i_1, j_1, k_1, \ldots, i_2, j_2, k_2, \ldots)$$

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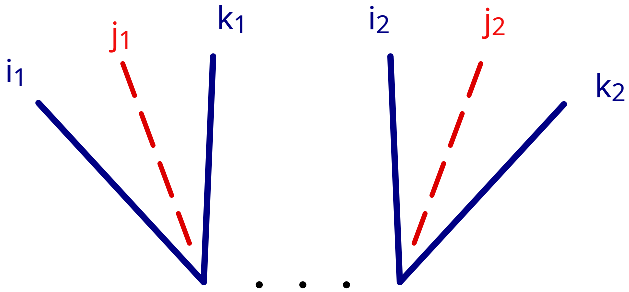

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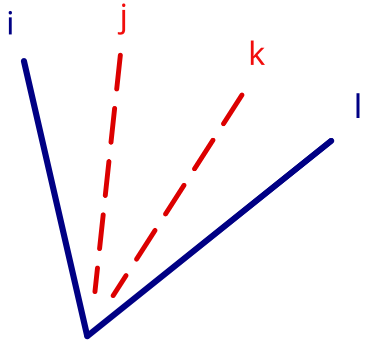
colour-connected emissions: both hard radiators shared



4→2 momentum mapping

$$M^0_{n+2}(\ldots, i, j, k, l, \ldots)$$

$$X_4^0(i^h, j, k, l^h)\, M_n^0(\ldots, (\widetilde{ijk}), (\widetilde{jkl}), \ldots)$$
$$-X_3^0(i^h, j, k^h)\, X_3^0((\widetilde{ij})^h, (\widetilde{jk}), l^h)\, M_n^0(\ldots, (\widetilde{(\widetilde{ij})(\widetilde{jk})}), (\widetilde{(\widetilde{jk})l}), \ldots)$$
$$-X_3^0(l^h, k, j^h)\, X_3^0((\widetilde{lk})^h, (\widetilde{kj}), i^h)\, M_n^0(\ldots, (\widetilde{i(\widetilde{jk})}), (\widetilde{(\widetilde{jk})(\widetilde{kl})}), \ldots)$$

necessary to avoid over-counting of single-unresolved behaviour

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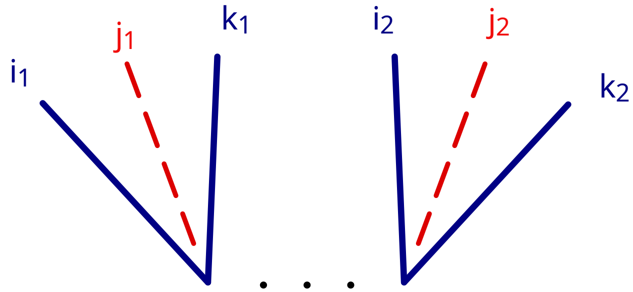

$$M_{n+2}^0(\ldots,i_1,j_1,k_1,\ldots,i_2,j_2,k_2,\ldots)$$

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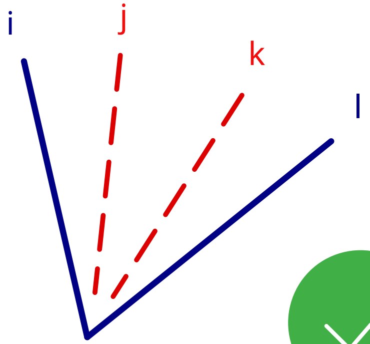

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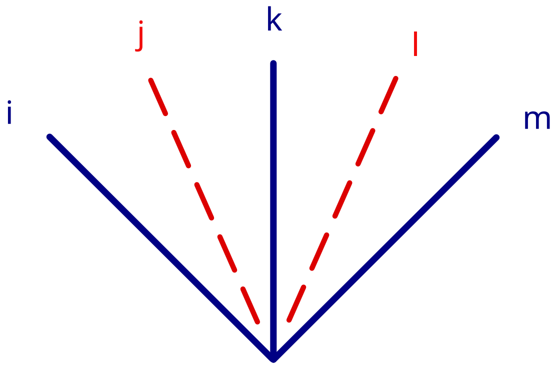
There is more … **almost colour-connected** emissions: only one shared hard radiator

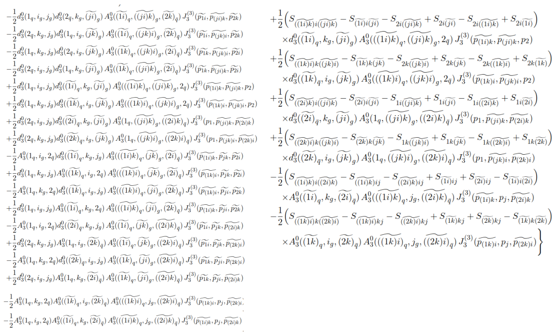


NOT fully iterated: the two emissions "feel" each other through the recoil on the shared radiator

traditional antenna functions can be used,
but a **very complicated** sequence of iterated structures is needed, plus **Large-Angle-Soft-Terms**

[Gehrmann De-Ridder,Gehrmann,Glover,Heinrich '07]
[Weinzierl '08] [Currie,Glover,Wells '13]

most complicated, inefficient and **ugly** sector of antenna subtraction



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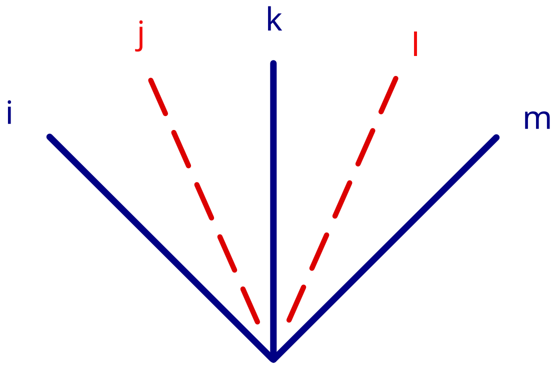

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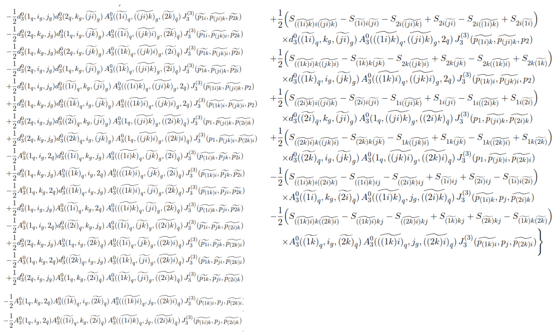

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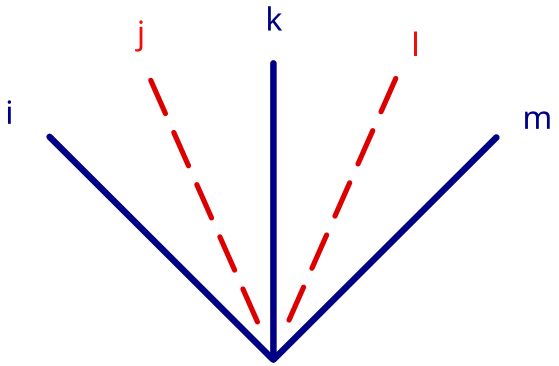

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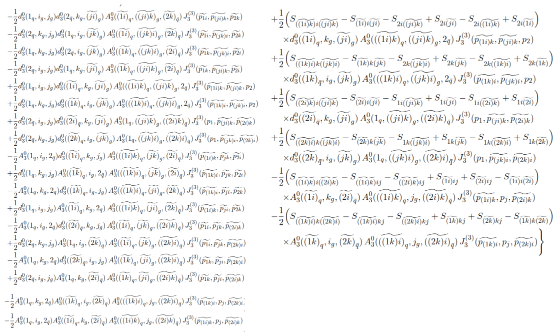


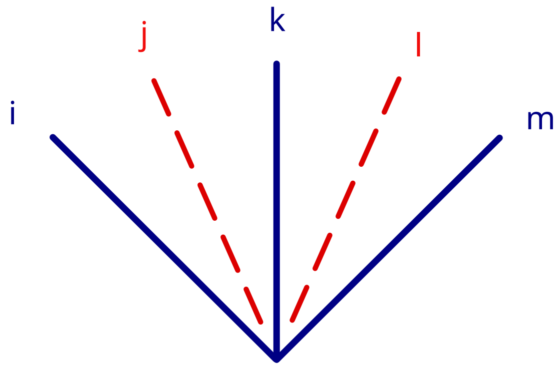
Ideally we want:

$$M^0_{n+2}(\ldots, i, j, k, l, m \ldots)$$

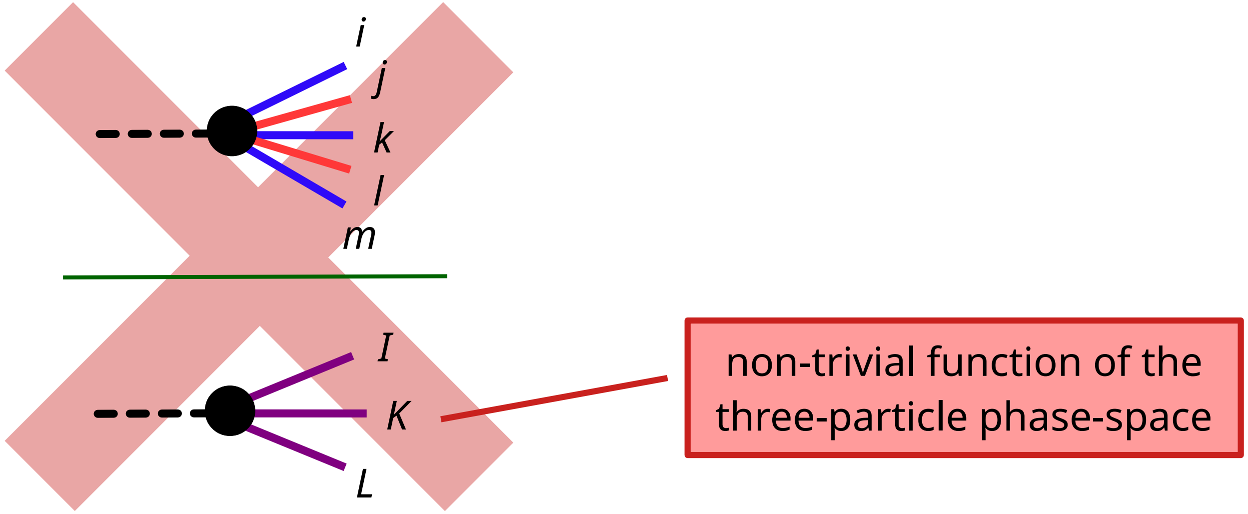
5→3 momentum mapping

$$X^0_{5,3}(i^h, j, k^h, l, m^h) M^0_n(\ldots, (ijk), (ijklm), (klm), \ldots)$$
$$-X^0_3(i^h, j, k^h) X^0_3((jk)^h, l, m^h) M^0_n(\ldots, (ij), ((jk)l), (lm), \ldots)$$
$$-X^0_3(m^h, l, k^h) X^0_3((lk)^h, j, i^h) M^0_n(\ldots, (ij), (j(kl)), (lm), \ldots)$$

Generalized antenna functions [Fox,Glover,MM '24]



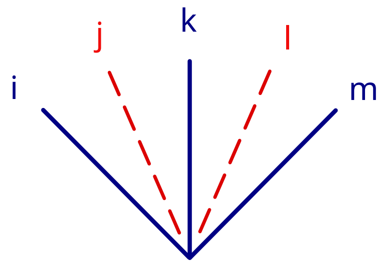
Not possible with matrix element-based antenna functions 😢



Generalized antenna functions [Fox,Glover,MM ‘24]

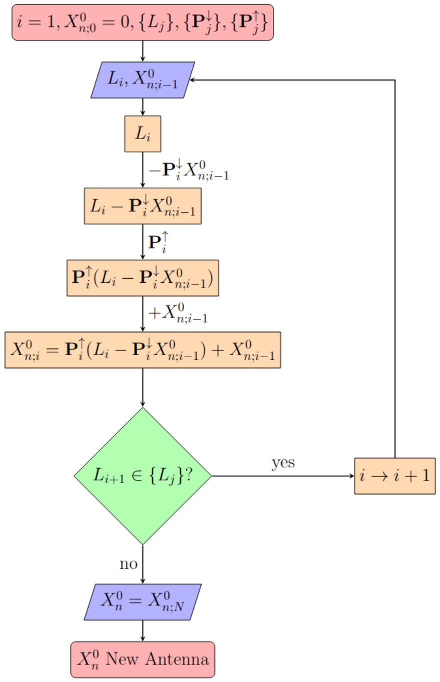
[Braun-White,Nigel,Preuss ‘22,’23]

With the **idealized antenna algorithm**, it is possible to construct antenna functions with **more than two hard radiators: generalized antenna functions**



Idealized antenna algorithm:

- no more matrix element
- build antenna function from **a set of target IR limits**
- **arbitrary number** of hard radiators and emissions



[Braun-White,Nigel,Preuss ‘22]

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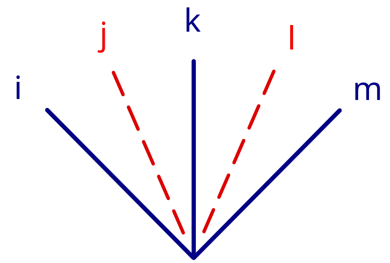

Problem: 5→3 mapping?

3 hard radiators → non-trivial reduced phase space → more complicated analytical **integration**

→ more **constrained** form of the mapping

Generalized antenna functions [Fox,Glover,MM ‘24]

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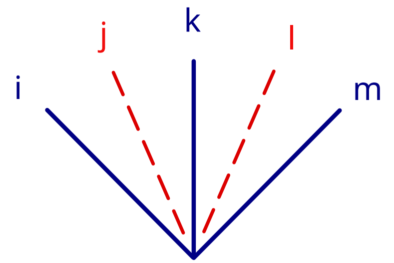

Solution: **iterated dipole mapping**

$$\mathrm{map}_{5\to3}: \quad \begin{aligned} p_I &= p_i + p_j - \frac{s_{ij}}{s_{ik}+s_{jk}} p_k \\ p_K &= \left(1 + \frac{s_{ij}}{s_{ik}+s_{jk}} + \frac{s_{lm}}{s_{lk}+s_{mk}}\right) p_k \\ p_M &= p_l + p_m - \frac{s_{lm}}{s_{lk}+s_{mk}} p_k \end{aligned}$$

- ✅ momentum conservation
- ✅ on-shellness condition
- ✅ IR-limits
- ✴️ easy analytical integration

$$p_i + p_j + p_k + p_l + p_m = p_I + p_K + p_M\,, \quad p_I^2 = p_K^2 = p_M^2 = 0$$

Generalized antenna functions [Fox,Glover,MM ‘24]

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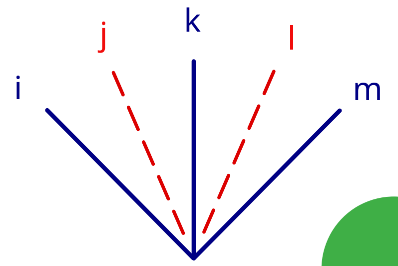


final-state radiation only, extension to IS in progress

Solution: **iterated dipole mapping**

$$\text{map}_{5\to 3}: \quad \begin{aligned} p_I &= p_i + p_j - \frac{s_{ij}}{s_{ik}+s_{jk}} p_k \\ p_K &= \left(1 + \frac{s_{ij}}{s_{ik}+s_{jk}} + \frac{s_{lm}}{s_{lk}+s_{mk}}\right) p_k \\ p_M &= p_l + p_m - \frac{s_{lm}}{s_{lk}+s_{mk}} p_k \end{aligned}$$

- ✅ momentum conservation
- ✅ on-shellness condition
- ✅ IR-limits
- ✴️ easy analytical integration

$$p_i + p_j + p_k + p_l + p_m = p_I + p_K + p_M\,, \quad p_I^2 = p_K^2 = p_M^2 = 0$$

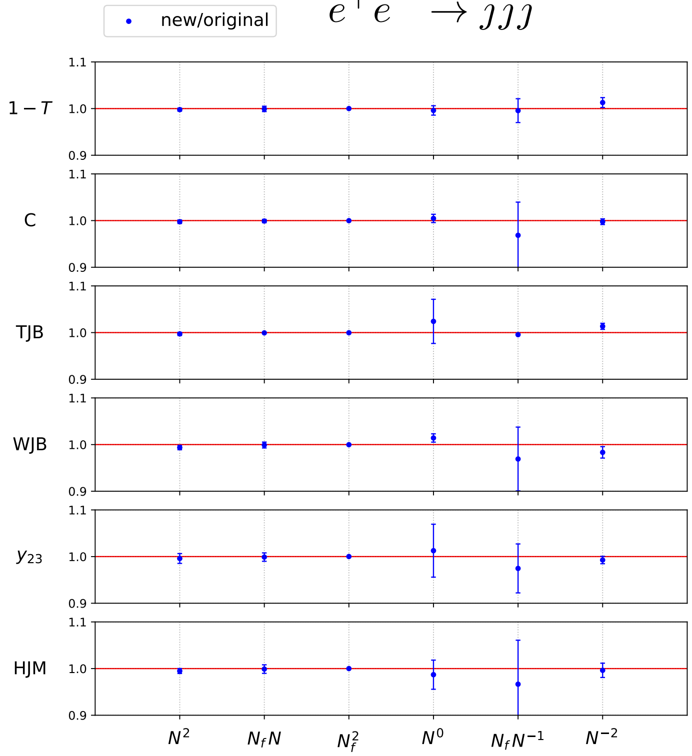
Generalized antenna functions: applications

NNLO correction to event shapes in e⁺e⁻ annihilation:

- perfect agreement with original method
- up to 10x faster [Fox,Glover,MM '24]

$e^+e^- \to jjj$

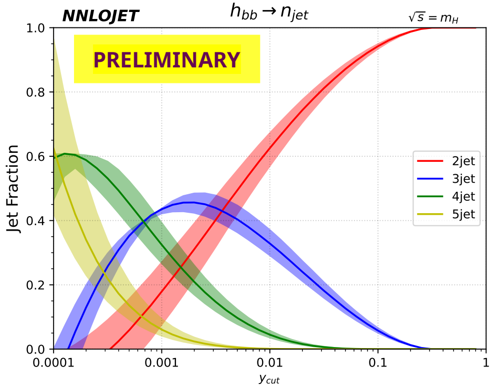
event shapes



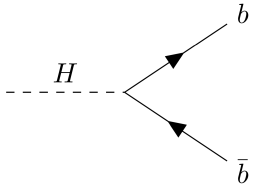
individual colour factors

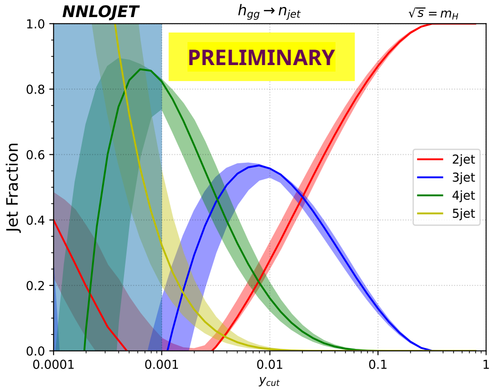
Hadronic Higgs decays: $H \to jjj$

- differences between H→bb and H→gg
- jet rates at order α_s^3 (3jet @NNLO, 2jet @N³LO)

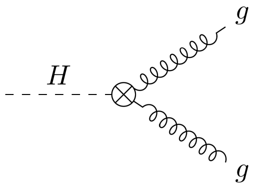


Higgs decay to bottom quarks via Yukawa interaction:





Higgs decay to gluons via effective vertex ($m_t \to \infty$):



SUMMARY AND CONCLUSIONS

Precision calculations are necessary to keep probing the SM and looking for New Physics at colliders. Frontier: generalisation and **automation of NNLO calculations.**

Antenna subtraction has been quite successful at NNLO. Can be used in cutting-edge scenarios and there is ongoing work for its generalisation.

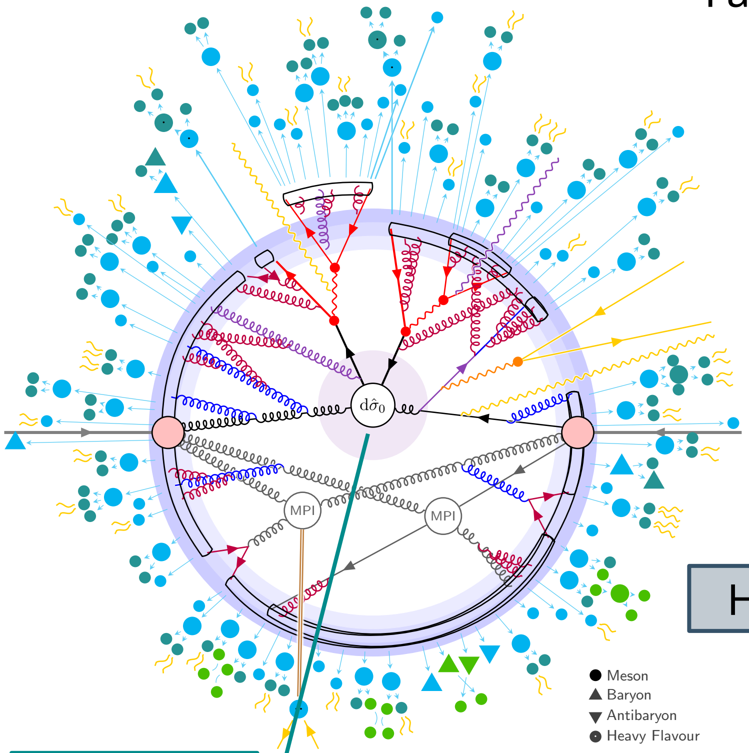
Recently core ingredients have been upgraded: **idealized** and **generalized** antenna functions. More **elegant and efficient** formulation.

Outlook: applications to high-multiplicity processes at hadron and lepton colliders ($pp\to jjj$, $pp\to Vjj$, $e^+e^-\to jjjj$), extension to N^3LO.

Thank you for your attention!

BACKUP SLIDES

Particle collisions



Hard scattering

[Pythia manual]

Factorization theorem for hadronic collisions:

PDFs

low energy, long distance

$$d\sigma_{pp\to X} = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 f_1(x_1,\alpha_s(\mu_R),\mu_F) f_2(x_2,\alpha_s(\mu_R),\mu_F)$$

$$\times d\hat{\sigma}_{ab\to X}(x_1,x_2,\alpha(\mu_R),\mu_R,\mu_F) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^p$$

high energy, short distance

Hadronic cross section

Partonic cross section

N^3LO calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \dots$$

Next-to-Next-to-Next-to Leading Order (N3LO)

☹ Much harder;

✓ Inclusive and differential predictions for simple 2→1 processes;

✗ Way far from generalization/ automation;

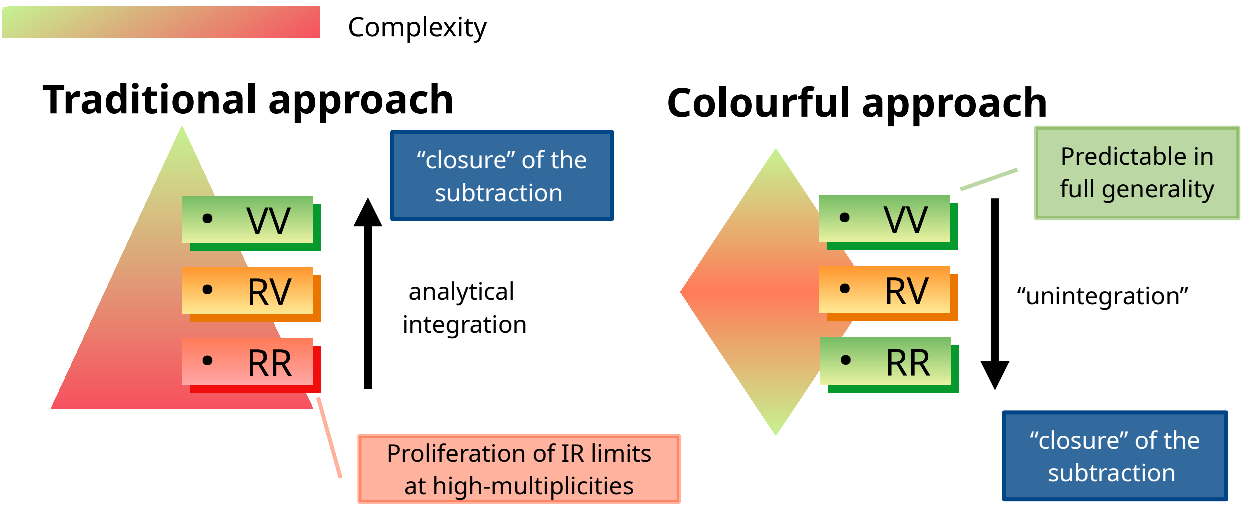
Very few techniques available, applied to specific processes:

- Projection to Born;
- Slicing (qT, 0-jettiness);

Colourful antenna subtraction

Ultimate goal: combine **generalized antenna functions** with the **colourful antenna subtraction** method

[Chen,Gehrmann,Glover,Huss,MM '22]
[Gehrmann,Glover,MM '23]



$$\mathrm{d}\sigma^{T} = \mathcal{N}_{V}\int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n\, J_n^n(\Phi_n)\,\left[2\,\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle\right]$$

one loop

$$\mathrm{d}\sigma^{U} = \mathcal{N}_{VV}\int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n J_n^n(\Phi_n)$$
$$\times 2\Big\{\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^1\rangle + \langle A_{n+2}^1|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle - \frac{\beta_0}{\epsilon}\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle$$
$$-\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}\otimes\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle + \langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(2)}|A_{n+2}^0\rangle\Big\}$$

Exploit **universal** IR singularity structure in virtual corrections to **systematically** construct real-radiation counterterms

two loops

Mapping (in)dependence

[Fox,Glover,MM '24]

Let's consider:

- $\mathbf{n_p}$ momenta **{p}** involved in an unresolved configuration
- $\mathbf{n_q}$ spectator momenta **{q}**

mapping to absorb the recoil of unresolved radiation:

$$\{p\} \rightarrow \{\tilde{p}\}$$

generic subtraction term

$$\mathrm{d}\sigma^S \propto \int \mathrm{d}PS_{n_p+n_q}(\{p\},\{q\})\, X(\{p\})\, M(\{\tilde{p}\},\{q\})\, J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\},\{q\})$$

full phase-space

unresolved factor (antenna function)

resolved matrix element

measurement function

selects n_{jets} jets
applies fiducial cuts

The mapping is chosen to induce a **factorization of the phase space**

$$\mathrm{d}PS_{n_p+n_q}(\{p\},\{q\}) = \mathrm{d}PS_X(\{p\}/\{\tilde{p}\})\, \mathrm{d}PS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\},\{q\})$$

unresolved phase-space

resolved phase-space: the measurement function acts on it

$$\mathrm{d}\sigma^S \propto \int \mathrm{d}PS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\},\{q\}) \left[\int X(\{p\})\mathrm{d}PS_X(\{p\}/\{\tilde{p}\})\right] M(\{\tilde{p}\},\{q\})\, J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\},\{q\})$$

$$= \int \mathrm{d}PS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\},\{q\})\, \mathcal{X}(\{\tilde{p}\})\, M(\{\tilde{p}\},\{q\})\, J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\},\{q\})$$

integrated unresolved factor

two mappings are **equivalent** if the yield the same $\mathcal{X}(\{\tilde{p}\})$

Mapping (in)dependence [Fox,Glover,MM '24]

two hard radiators: $n_{\widetilde{p}} = 2\,,\qquad \{p_1,\ldots,p_{n_p}\} \to \{\widetilde{p}_A,\widetilde{p}_B\}$

$$s_{1\ldots n_p} \equiv (p_1 + \cdots + p_{n_p})^2 = (\widetilde{p}_A + \widetilde{p}_B)^2 \equiv s_{AB}$$

$$\mathcal{X}\left(\{\widetilde{p}_A,\widetilde{p}_B\}\right) = C(\epsilon)(s_{AB})^{\alpha}$$

any momentum-conserving mapping gives same result

three hard radiators: $n_{\widetilde{p}} = 3\,,\qquad \{p_1,\ldots,p_{n_p}\} \to \{\widetilde{p}_A,\widetilde{p}_B,\widetilde{p}_C\}$

$$s_{1\ldots n_p} \equiv (p_1 + \cdots + p_{n_p})^2 = (\widetilde{p}_A + \widetilde{p}_B + \widetilde{p}_C)^2 \equiv s_{ABC}$$

$$\mathcal{X}\left(\{\widetilde{p}_A,\widetilde{p}_B,\widetilde{p}_C\}\right) = \sum_i C_i(\epsilon)(s_{AB})^{\alpha_i}(s_{AC})^{\beta_i}(s_{BC})^{\gamma_i} + \ldots$$

many "unfixed" scales, different result for different mappings

Local subtraction at N³LO

Partonic cross section at N³LO:

$$\mathrm{d}\sigma_{N^3LO} = \int_n \mathrm{d}\sigma^{VVV} + \int_{n+1} \mathrm{d}\sigma^{RVV} + \int_{n+2} \mathrm{d}\sigma^{RRV} + \int_{n+2} \mathrm{d}\sigma^{RR3}$$

infrared divergent   infrared divergent   infrared divergent   infrared divergent

Subtraction at NNLO:

$$\mathrm{d}\sigma_{NNLO} = \int_n \left[\mathrm{d}\sigma^{VVV} - \mathrm{d}\sigma^{W}\right] + \int_n \left[\mathrm{d}\sigma^{RVV} - \mathrm{d}\sigma^{U}\right] + \int_{n+1} \left[\mathrm{d}\sigma^{RRV} - \mathrm{d}\sigma^{T}\right] + \int_{n+2} \left[\mathrm{d}\sigma^{RRR} - \mathrm{d}\sigma^{S}\right]$$

triple-virtual subtraction term

double-virtual real subtraction term

double-real-virtual subtraction term

triple-real subtraction term

with:

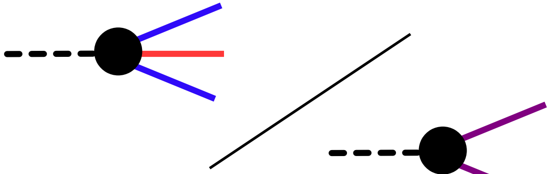
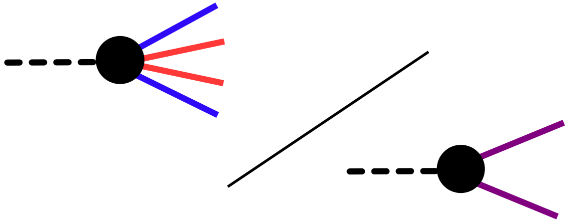
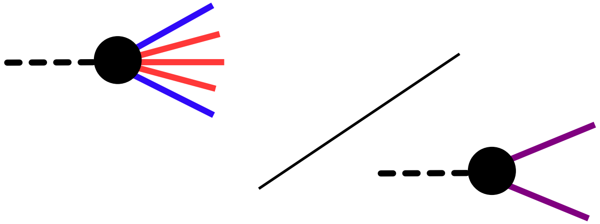
$$\mathrm{d}\sigma^{S} = \mathrm{d}\sigma^{S,1} + \mathrm{d}\sigma^{S,2} + \mathrm{d}\sigma^{S,3}$$

$$\mathrm{d}\sigma^{T} = \mathrm{d}\sigma^{VS,1} + \mathrm{d}\sigma^{VS,2} - \int_1 \mathrm{d}\sigma^{S,1}$$

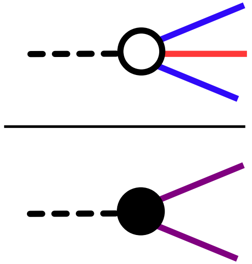
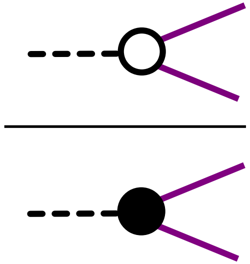
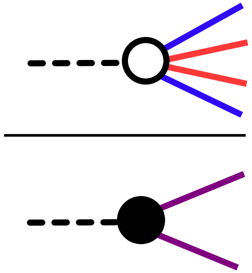
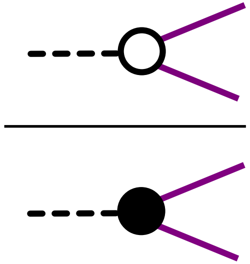
$$\mathrm{d}\sigma^{U} = \mathrm{d}\sigma^{VVS} - \int_1 \mathrm{d}\sigma^{VS,1} - \int_2 \mathrm{d}\sigma^{S,2}$$

$$\mathrm{d}\sigma^{W} = -\int_1 \mathrm{d}\sigma^{VVS} - \int_2 \mathrm{d}\sigma^{VS,2} - \int_3 \mathrm{d}\sigma^{S,3}$$

Tree-level antenna functions

		Antenna	Integrated antenna
NLO:		$X_3^0 = \frac{M_3^0}{M_2^0}$	$\mathcal{X}_3^0 \propto \int \mathrm{d}\Phi_3 \, X_3^0$
NNLO:		$X_4^0 = \frac{M_4^0}{M_2^0}$	$\mathcal{X}_4^0 \propto \int \mathrm{d}\Phi_4 \, X_4^0$
N³LO:		$X_5^0 = \frac{M_5^0}{M_2^0}$	$\mathcal{X}_5^0 \propto \int \mathrm{d}\Phi_5 \, X_5^0$

One-loop antenna functions

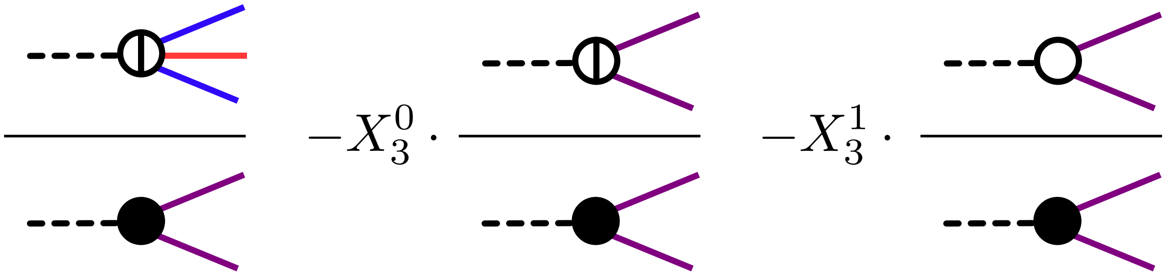
		Antenna	Integrated antenna
NLO:	❌		
NNLO:	 $-X_3^0 \cdot$ 	$X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_3^1 \propto \int \mathrm{d}\Phi_3 \, X_3^1$
N³LO:	 $-X_4^0 \cdot$ 	$X_4^1 = \frac{M_4^1}{M_2^0} - X_4^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_4^1 \propto \int \mathrm{d}\Phi_4 \, X_4^1$

Two-loop antenna functions

NLO: ❌

NNLO: ❌

N^{3}LO:



Antenna $$X_3^2 = \frac{M_3^2}{M_2^0} - X_3^0 \frac{M_2^2}{M_2^0} - X_3^1 \frac{M_2^1}{M_2^0}$$

Integrated antenna $$\mathcal{X}_3^2 = \int \mathrm{d}\Phi_3 X_3^2$$

Analytic integration

Integration of **renormalized matrix elements** for colour-singlet decay over the **fully inclusive phase space**:

$$\int \mathrm{d}\Phi_5 M_5^0, \quad \int \mathrm{d}\Phi_4 M_4^1, \quad \int \mathrm{d}\Phi_3 M_3^2, \quad \int \mathrm{d}\Phi_2 M_2^3$$

[Jakubcik,MM,Stagnitto '22]
[Chen,Jakubcik,MM,Stagnitto '23]

Two-parton three-loop, for validation:

$$\int \mathrm{d}\Phi_5 M_5^0 + \int \mathrm{d}\Phi_4 M_4^1 + \int \mathrm{d}\Phi_3 M_3^2 + \int \mathrm{d}\Phi_2 M_2^3 =$$ finite N^{3}LO inclusive XS

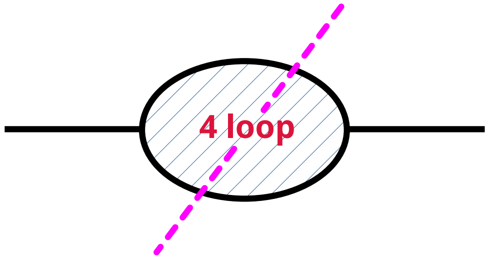
Master integrals from

[Gituliar,Magerya,Pikelner '18]
[Magerya,Pikelner '19]

Reverse unitarity: $$2\pi i\delta^+(p^2) \to \frac{1}{p^2 - i0} - \frac{1}{p^2 + i0}$$

[Cutkosky '60]
[Anastasiou, Melnikov '02,'03]

- Phase space and (genuine) loop integrals addressed simultaneously;
- **Systematic treatment of all four layers** within a common framework;



Application: jet production at lepton colliders

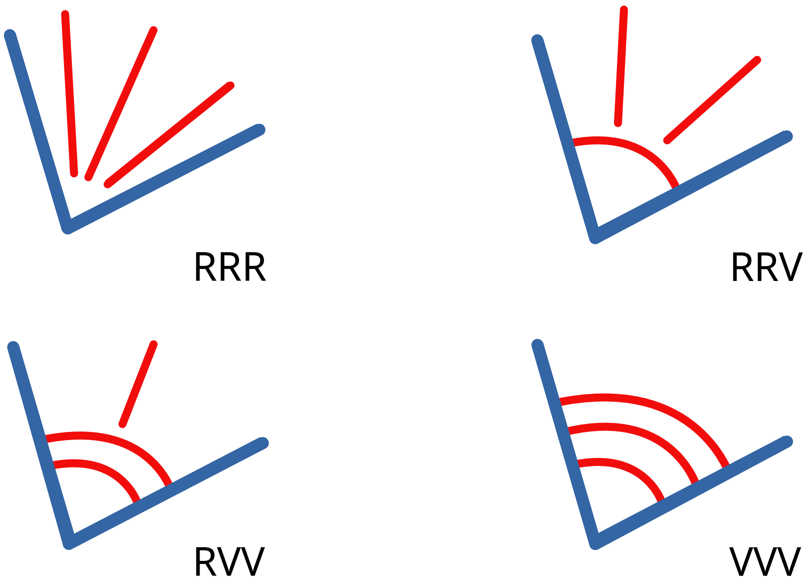
Motivation: you have to start somewhere

Simplifications:

- only **q-$\bar{\text{q}}$** N^3LO antenna functions;
- only **dipole-like correlations** at N3LO (two hard legs);

Goals:

- definition of **N3LO antenna functions**;
- removal of double- and single-unresolved limits;



Two-jet production rate computed at N^3LO in [Gerhrmann De-Ridder,Gehrmann,Glover,Heinrich '08]

Interesting to compute: **forward-backward asymmetry**, sensitive to the weak mixing angle

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

$$\sigma_B = \int_{-1}^0 \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

angle between beam and **flavoured jet** axis

NNLO study in [Altarelli,Lampe '93] [Ravindran,van Nerveen '98] [Catani,Seymour '98] [Weinzierl '06]