Towards a String Worldsheet Description of Flat Space Holography

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Outline

- Motivations
- Background
- Results
- Outlook

Motivations

In a nutshell: my research revolves around

Scattering Amplitudes,

and

String Theory,

with a view towards (Flat-Space) **Holography**.

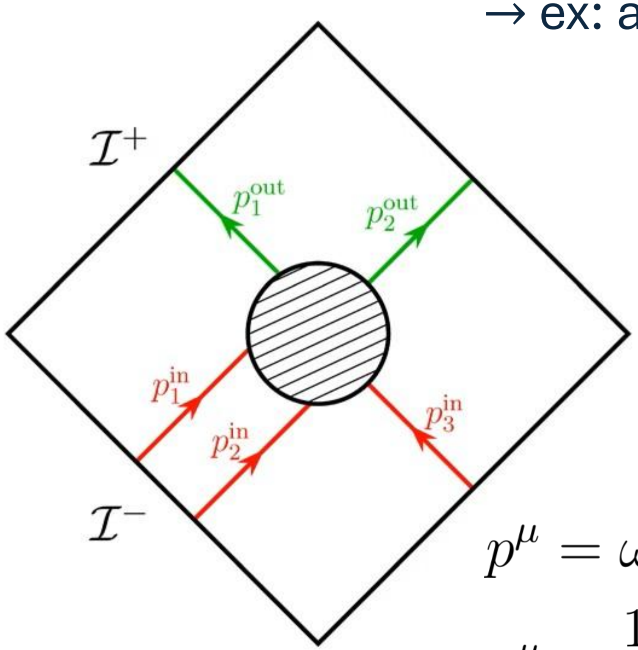
Scattering Amplitudes and Flat Space Holography

- Celestial Amplitudes: Dual formulation of D=4 amplitudes in 2D CFT.
- Carrollian Amplitudes: Dual formulation of D=4 amplitudes in 3D CFT.

Bottom-up approach: constrain S-matrix in the IR.

→ ex: asymptotic symmetries (BMS), soft theorems, memory effects.

[Bondi, van der Burg, Metzner ‘62]
[Strominger ‘17] [XK ‘23] [Donnay ‘23] (reviews)



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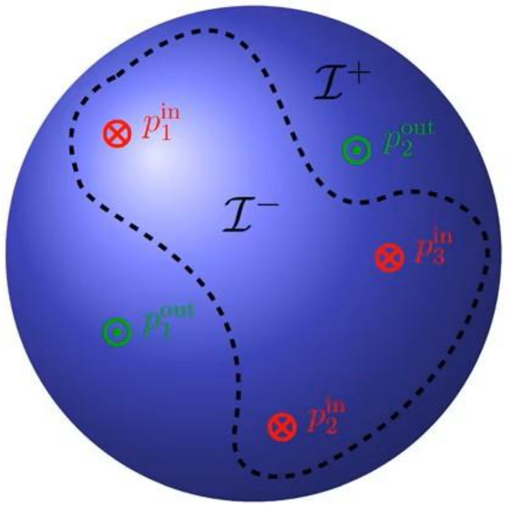


$$p^\mu = \omega q^\mu$$

$$q^\mu = \frac{1}{2}(1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$$

→ The Carrollian and celestial perspectives are related by integral transforms.

[Donnay, Fiorucci, Herfray, Ruzziconi ’22 & ‘23]

Celestial Holography

Mellin transform

$$\varphi^{\pm}_{\Delta}(x^{\mu}; z, \bar{z}) = \int_0^{\infty} d\omega\, \omega^{\Delta-1} e^{\pm i\omega q\cdot x - \epsilon\omega}$$
$$= \frac{(\mp i)^{\Delta}\Gamma(\Delta)}{[-x\cdot q(z,\bar{z}) \mp i\epsilon]^{\Delta}}$$

Recast S-matrix into boost eigenstates:

[Pasterski, Shao '17] [Pasterski, Shao, Strominger '17]

Celestial primaries are conformal primaries of 2D CFT with scaling dimension

$$\Delta = 1 + i\lambda,\ \lambda \in \mathbb{R}$$

[de Boer, Solodukhin '03]

Amplitudes:

$$\mathcal{A}_{\rm cel} = \left(\prod_{n=1}^{4}\int_0^{\infty} d\omega_n\, \omega_n^{i\lambda_n}\right)\mathcal{A}$$

energy is integrated out

[Pasterski, Shao, Strominger '17]
[Schreiber, Volovitch, Zlotnikov '17]

CCFT is an exotic CFT: amplitudes have distributional support

Conformal multiplets, celestial OPE

[Strominger '17] [Pasterski '21] [Raclariu '21] (reviews)

Why String Theory?

- Features of amplitudes derived from string theory (ex: double copy, BCJ amplitude relations)
  [Kawai, Lewellen, Tye '86] [Bern, Carasco, Johansson '08] [Stieberger '09] [Stieberger '24]

- UV: soft behaviour of string amplitudes, EFT for higher spin theory
  [Gross, Mende '88] [Gross, Manes '89]

- String worldsheet is a 2D CFT: connections to CCFT?
  [Stieberger, Taylor '18] [Jiang '22]

- Top-down construction, cf. AdS/CFT

> "What is one example of a top-down construction of a 2d celestial dual for a string compactification to 4d?"
>
> Prof. Andrew Strominger (Harvard), *Strings 2024*

Background

We are currently elaborating on a **string worldsheet** connection to flat space holography, as first pointed out in

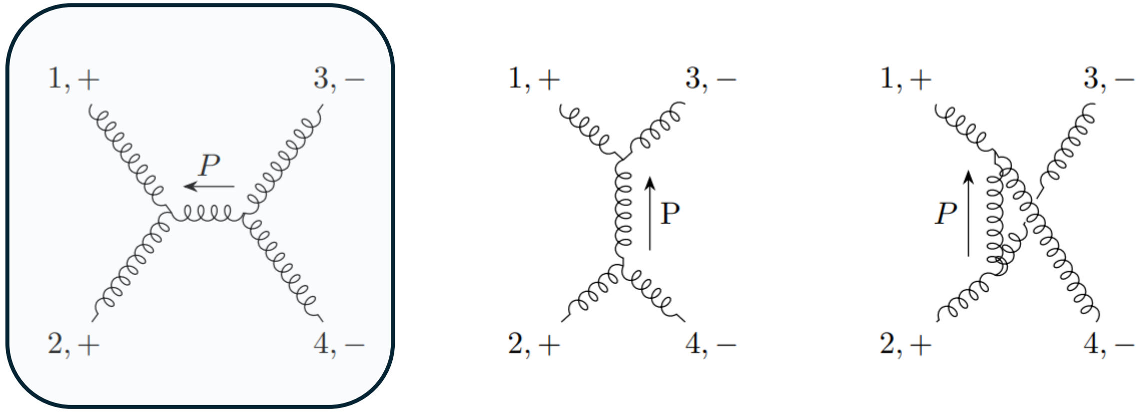
S. Stieberger and T. R. Taylor,

“Strings on Celestial Sphere”

(Nuclear Physics B, 2018)

Four-Point Tree-Level Amplitudes

- Yang-Mills:



partial amplitudes, no canonical trace factor

s > 0, u < 0, so poles due to massive string excitations appear in the s-channel only

- Type I Superstring: $$\mathcal{A}_{\text{string}}(1,2,3,4) = \frac{\Gamma(1-s)\Gamma(1-u)}{\Gamma(1+t)} A_{\text{YM}}(1,2,3,4)$$

$l_s = \sqrt{\alpha'}$

$T = \frac{1}{2\pi\alpha'}$

Mellin transform

Veneziano amplitude

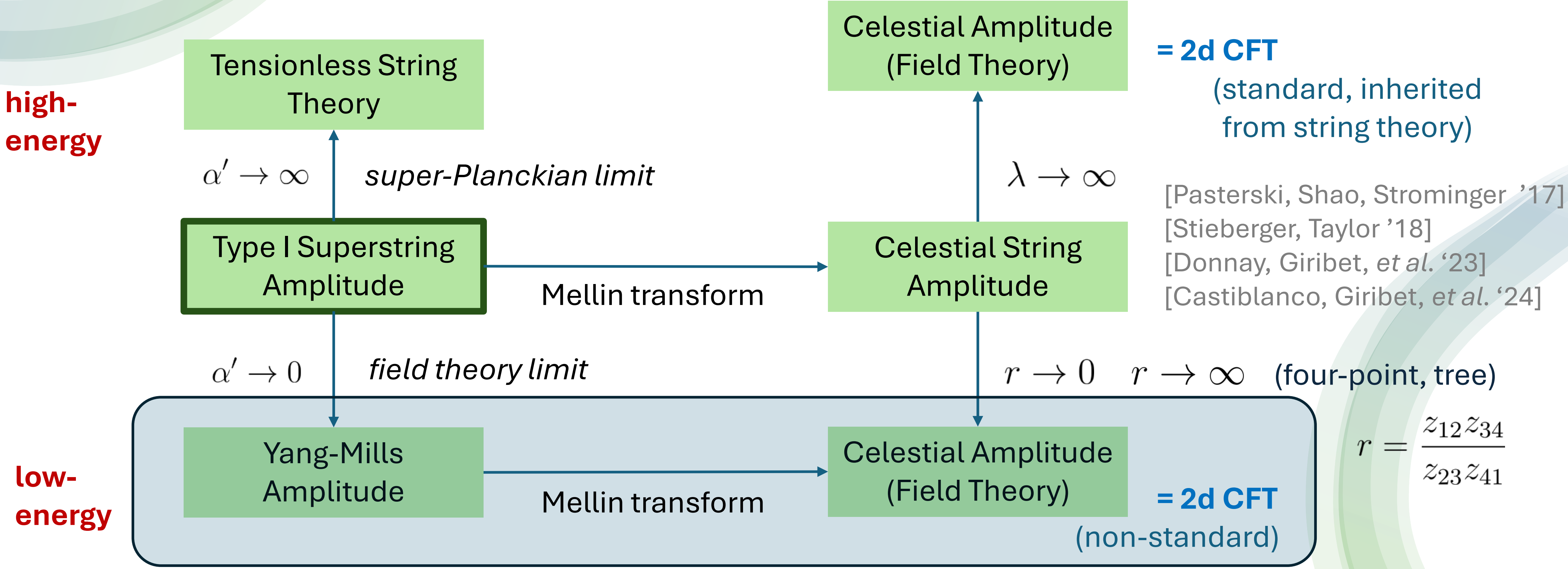
$$s = \alpha'(p_1+p_2)^2,\ t = \alpha'(p_1+p_3)^2,\ u = \alpha'(p_2+p_3)^2$$

- Celestial string 4-pt amplitudes:
  - UV-softness renders Mellin integrals finite: celestial string amplitudes are well-defined
  - Simple overall dependence on α': $(\alpha')^\beta$ (tree-level)
  - Low-energy and high-energy limits also recovered.

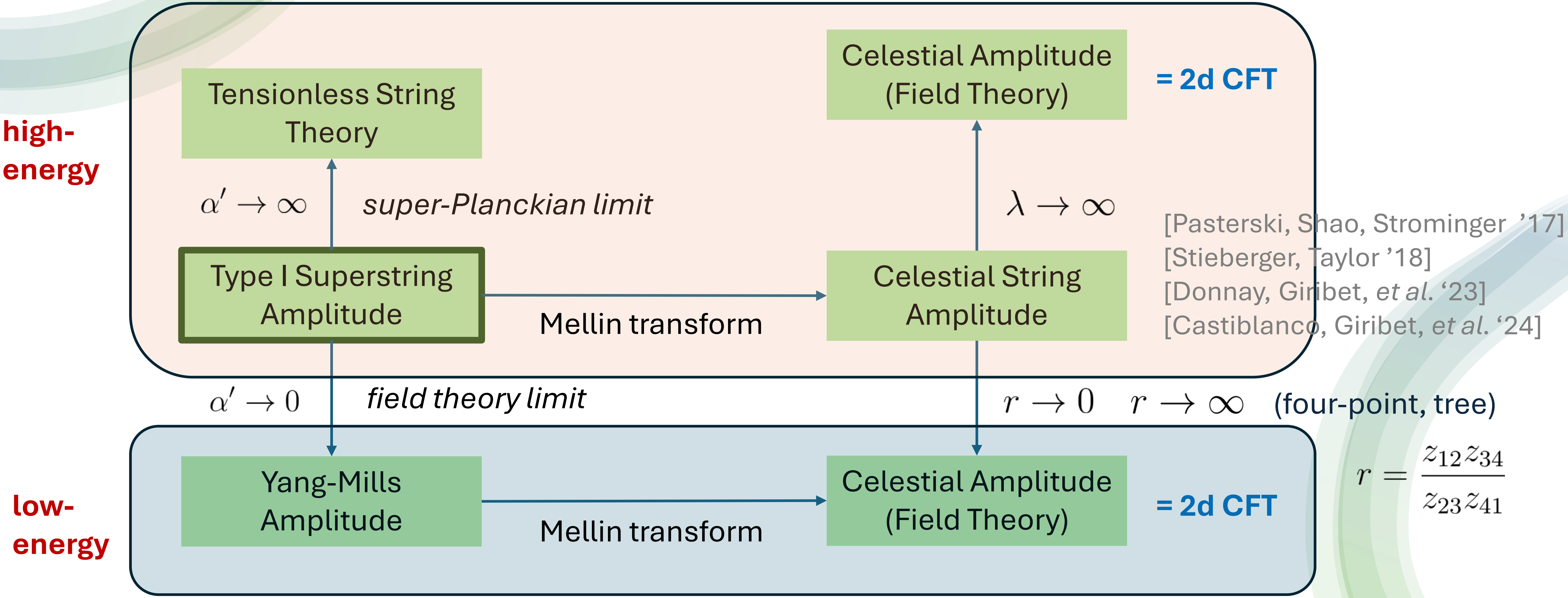
[Pasterski, Shao, Strominger ’17]
[Stieberger, Taylor ’18]

$$\beta = -\frac{i\lambda}{2}$$

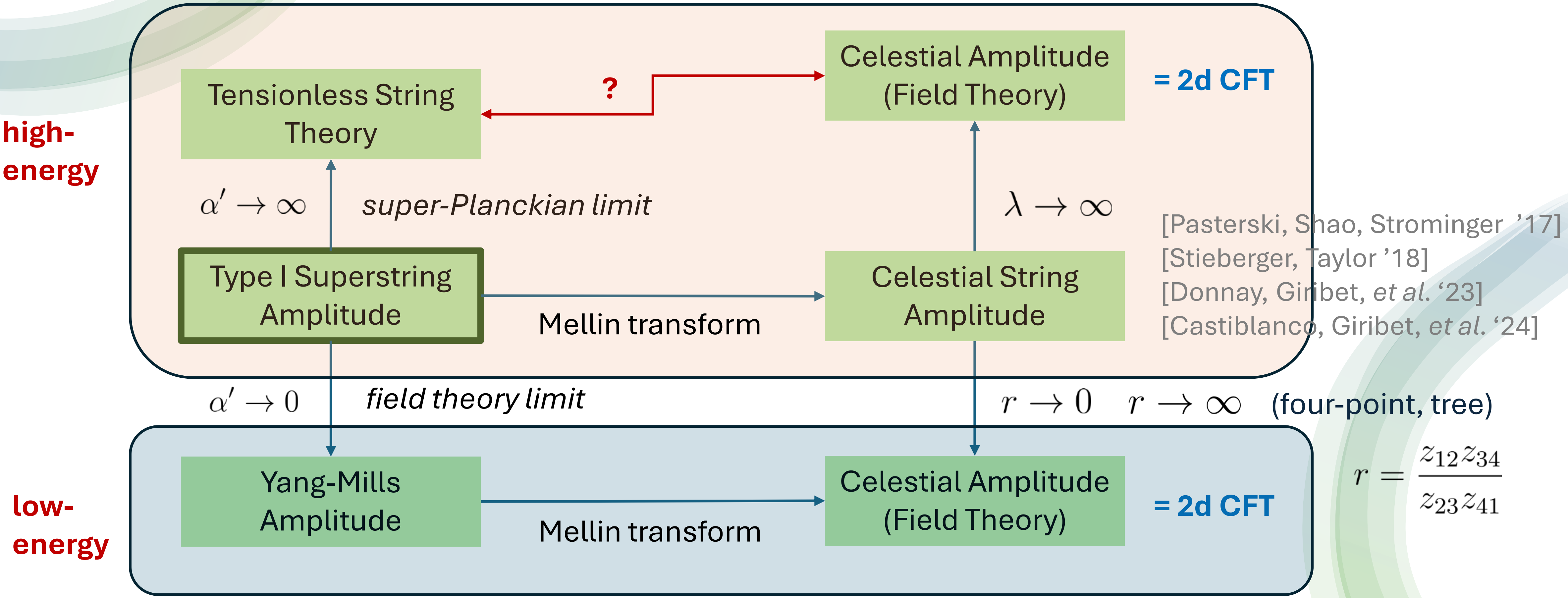
Representations of String Amplitudes



Representations of String Amplitudes



Representations of String Amplitudes



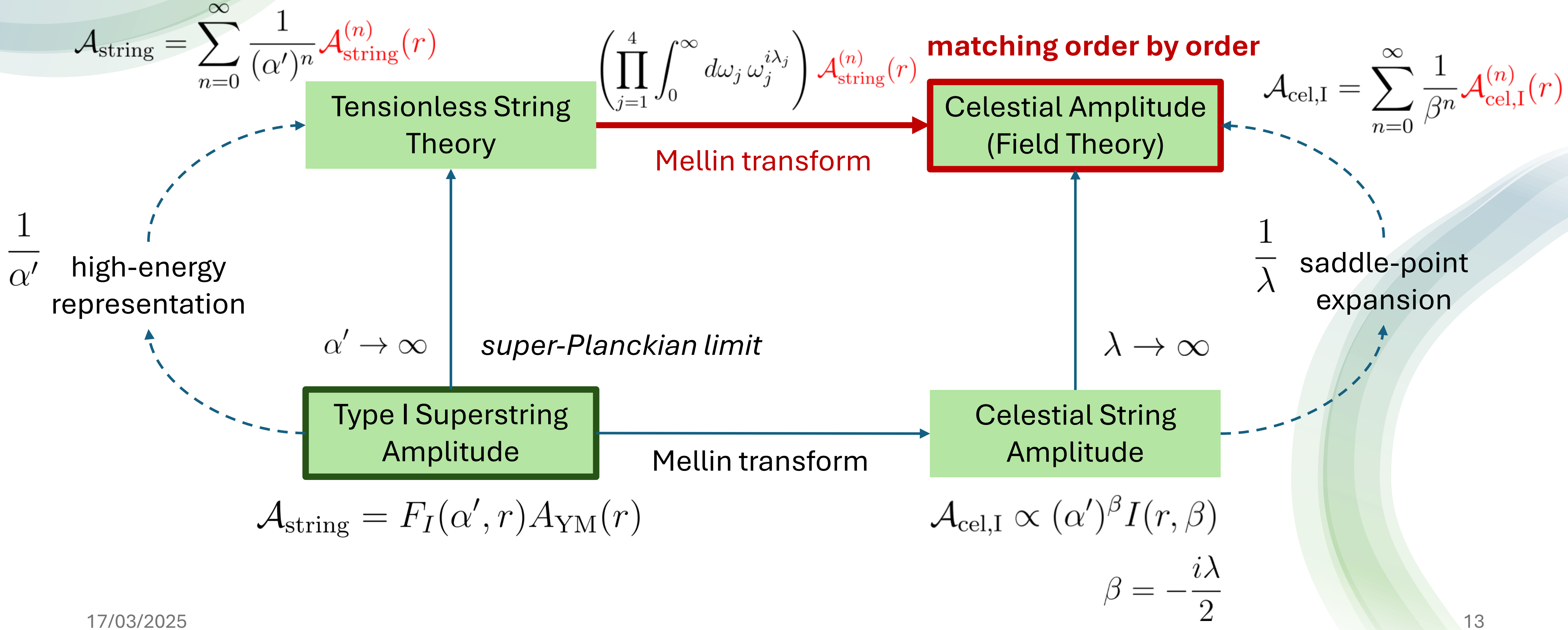
Results

We find:

- An **exact matching** of the $1/\alpha'$ and $1/\lambda$ expansions,
- **at all subleading orders** in the quantum fluctuations around the classical solution.
- The string **worldsheet** becomes **celestial**.

High-Energy String Theory and the Celestial Sphere

[XK, Stieberger, *to appear*]



This is highly non-trivial as the Mellin transform **mixes IR and UV physics**.

It hints a **<u>dual description</u>** of high-energy string theory as the large (conformal) energy expansion on the celestial sphere:

- soft modes = higher-spin states
- celestial sphere = string worldsheet

Allows for the interpretation of the CCFT as a 2d free worldsheet CFT.

Outlook

Prospects for Future Research

→ Generalizations: higher-point, beyond tree-level [Castiblanco, Giribet, Marin, Rojas ‘24] [Donnay, Giribet, Gonzalez, Puhm, Rojas ’23]

→ New perspective to understand tensionless strings

Relation to field theory (higher-spin) & double copy? [Schlotterer, Stieberger ‘13] [Stieberger ‘24]

Tensionless strings exhibit BMS symmetry.

cf. ambitwistor construction. [Bagchi ‘13] [Bagchi, Chakraborty, *et al*. ‘24] [Casali, Tourkine ‘16] [Casali, Herfray, Tourkine ‘17]

→ Implications for the study of amplitudes in AdS

[Alday, Chester, Hansen, Zhong ‘24] [Alday, Hansen, Nocchi ‘24]

→ What about Carrollian strings? Flat-space limit of AdS/CFT? [Stieberger, Taylor, Zhu ‘24]

→ Low-energy vs. high-energy representations: [Mizera ‘18] [Mazloumi, Stieberger ‘23]

twisted intersection theory mixes $\alpha' \leftrightarrow 1/\alpha'$

Thank you!

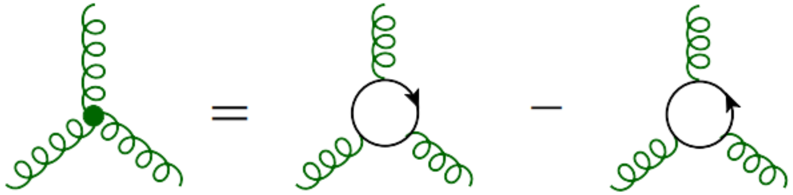
Backup Slides

Yang-Mills and Color-Ordering

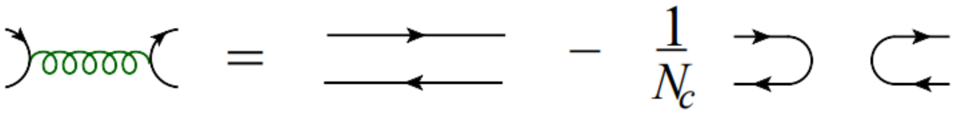
[Elvang, Huang '14] [Dixon '13]

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - i\sqrt{2}g\partial^\mu A^\nu A_\nu A_\mu + \frac{g^2}{4}A^\mu A^\nu A_\nu A_\mu\right)$$

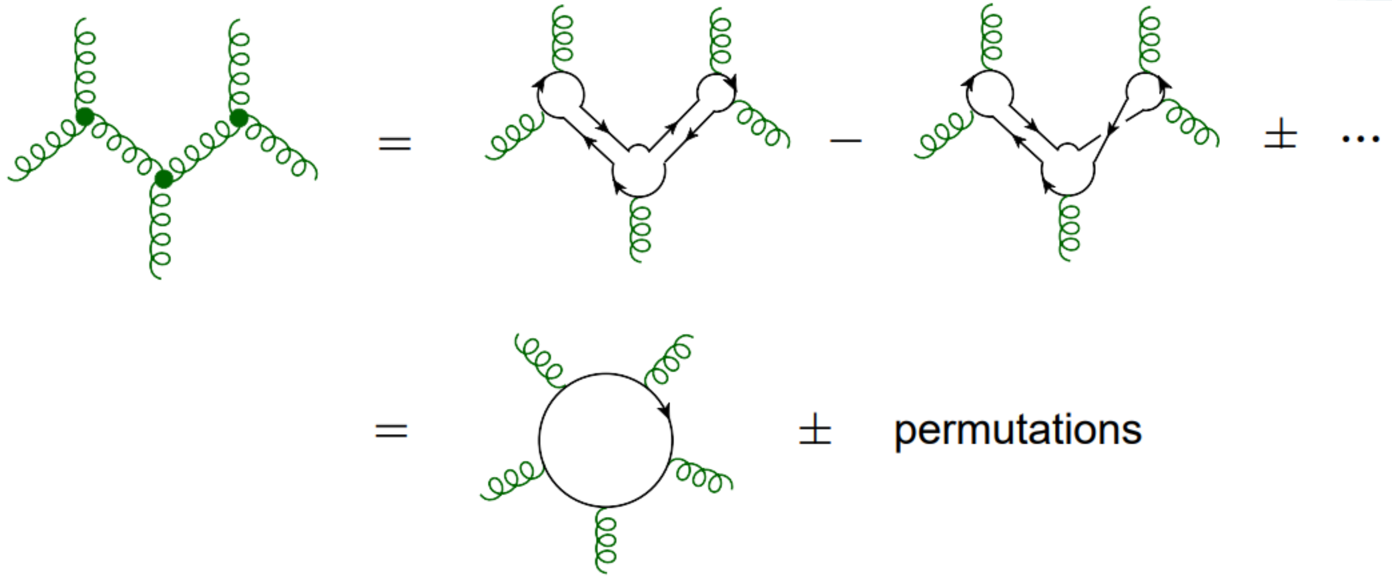
YM Lagrangian
Gervais-Neveu gauge



$$\tilde{f}^{abc} = \mathrm{Tr}(T^a T^b T^c) - \mathrm{Tr}(T^a T^c T^b)$$



$$(T^a)_{i_1}^{\bar{j}_1}(T^a)_{i_2}^{\bar{j}_2} = \delta_{i_1}^{\bar{j}_2}\delta_{i_2}^{\bar{j}_1} - \frac{1}{N_c}\delta_{i_1}^{\bar{j}_1}\delta_{i_2}^{\bar{j}_2}$$



$$A_n^{\text{full,tree}} = g^{n-2}\sum_{\text{perms } \sigma} A_n[1\sigma(2 \ldots n)]\mathrm{Tr}(T^{a_1}T^{\sigma(a_2 \ldots}T^{a_n)})$$

4pt Gluon Yang-Mills Amplitudes

$$\mathcal{A}_{\text{YM}}(-,-,+,+) = r\frac{z_{12}\bar{z}_{34}}{\bar{z}_{12}z_{34}}\delta^4(\omega_1 q_1 + \omega_2 q_2 - \omega_3 q_3 - \omega_4 q_4)$$

$$\mathcal{A}_{\text{cel,YM}} = 8\pi\delta(r-\bar{r})\delta\left(\sum_{n=1}^{4}\lambda_n\right)\left(\prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j}\bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}\right) r^{\frac{5}{3}}(r-1)^{\frac{2}{3}}$$

$$h_1 = \frac{i\lambda_1}{2}, \quad h_2 = \frac{i\lambda_2}{2}, \quad h_3 = 1+\frac{i\lambda_3}{2}, \quad h_4 = 1+\frac{i\lambda_4}{2}$$

$$\bar{h}_1 = 1+\frac{i\lambda_1}{2}, \quad \bar{h}_2 = 1+\frac{i\lambda_2}{2}, \quad \bar{h}_3 = \frac{i\lambda_3}{2}, \quad \bar{h}_4 = \frac{i\lambda_4}{2}$$

$$J_i = h_i - \bar{h}_i$$

$$\Delta_i = h_i + \bar{h}_i$$

The Infrared Triangle of Gravity

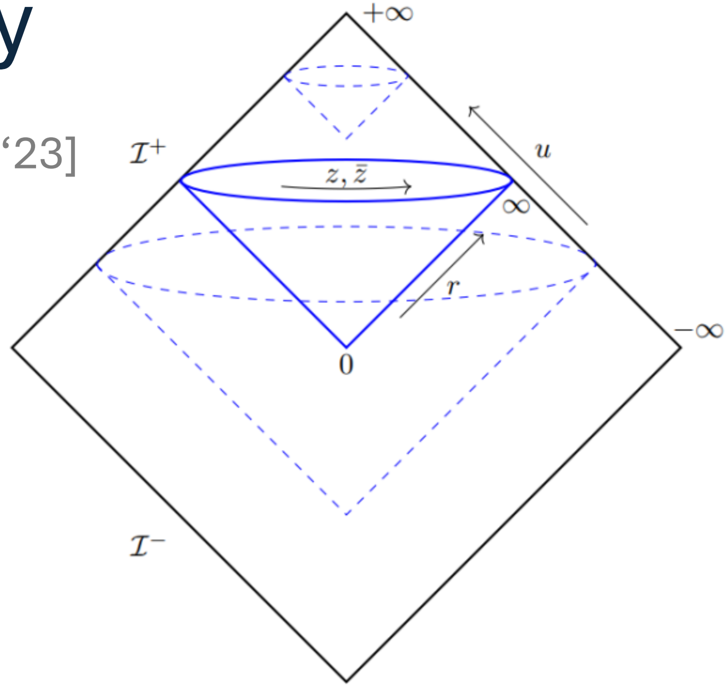
[Strominger ‘17] [XK ‘23]

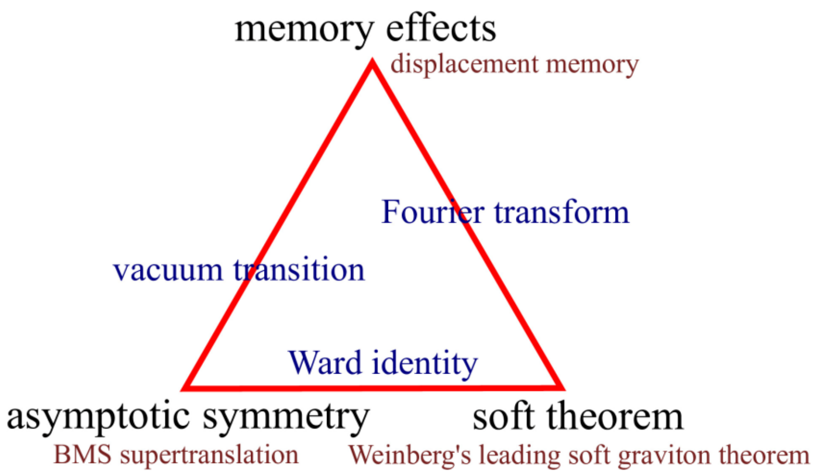
Bondi gauge

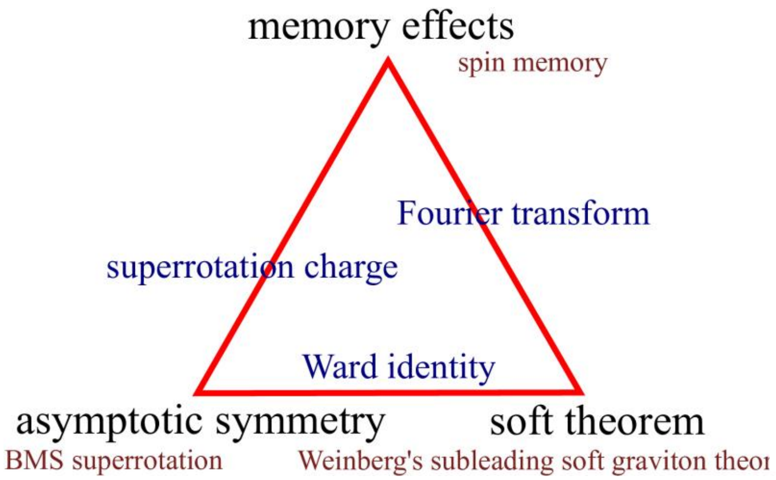
$$g_{rr} = g_{rA} = 0 \quad \text{and} \quad \partial_r \det\left(\frac{g_{AB}}{r^2}\right) = 0.$$

Retarded Bondi-Sachs metric

$$ds^2 = -U du^2 - 2e^{2\beta} du dr + g_{AB}\left(dx^A + \frac{1}{2}U^A du\right)\left(dx^B + \frac{1}{2}U^B du\right).$$





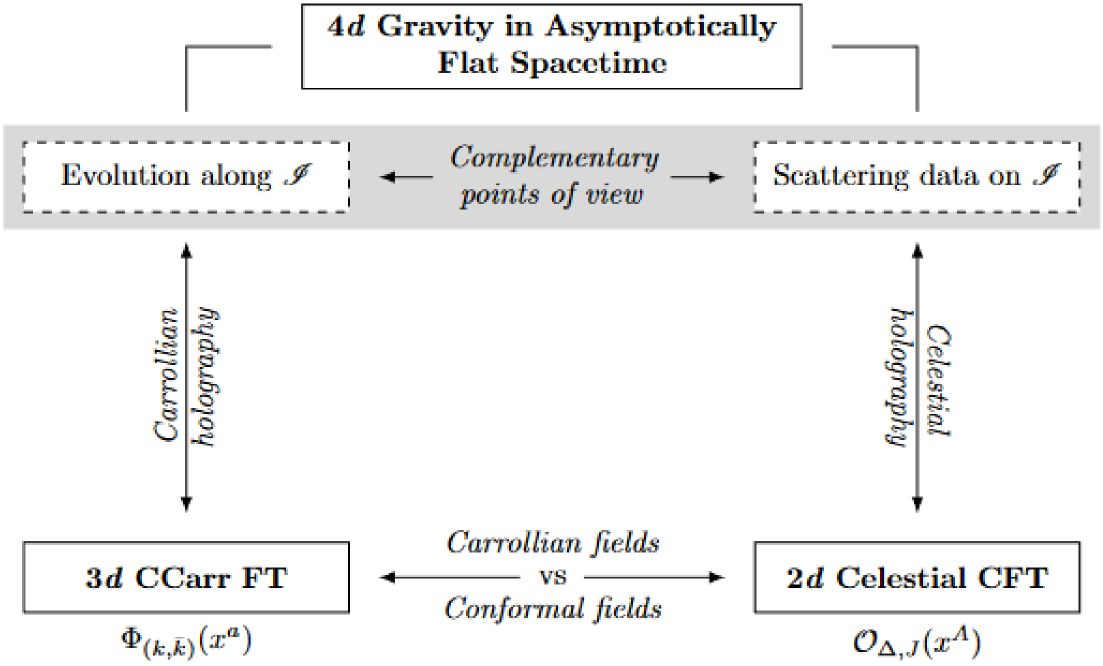


→ Construction of an operator whose insertion in the 4d tree-level quantum gravity S-matrix obeys Virasoro-Ward identities of 2d CFT stress tensor

[Kapec, Mitra *et al*. ‘17]

Carrollian Holography

[Donnay, Fiorucci, Herfray, Ruzziconi ‘22]



dual field theory assumed to live on AFS 3D null boundary (= Carrollian manifold)

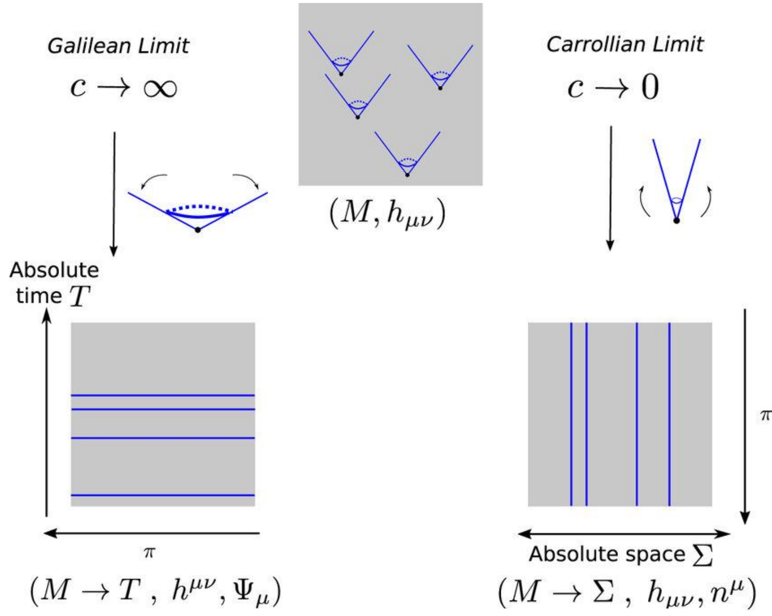
[Lévy-Leblond ‘65]

CCarr ≅ BMS

[Duval, Gibbons, Horvathy ‘14]

Galilean vs. Carrollian limit

[Herfray ‘22]



$$C_n\left(\{u_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \ldots, \{u_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right) = \prod_{i=1}^{n}\left(\int_0^{+\infty}\frac{d\omega_i}{2\pi}e^{i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \ldots, \{\omega_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right).$$

4pt Gluon Type I Celestial String Amplitude

$$\mathcal{A}_{\text{cel,I}} = 4(\alpha')^\beta \delta(r-\bar{r}) \left(\prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j} \bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j} \right) r^{\frac{5-\beta}{3}} (r-1)^{\frac{2-\beta}{3}} I(r,\beta)$$

$$I(r,\beta) \equiv \frac{1}{2} \int_0^\infty dw\, w^{-\beta-1} F_{\text{I}}(rw,-w)$$

analytic continuation

$a < 0$

$$I(r,\beta) \equiv \frac{\Gamma(1-\beta)}{2} (-a)^{-\beta} \int_0^1 dx f(x) e^{\beta g(x)}$$

$$g(x) \equiv \ln[-\ln x + a \ln(1-x)]. \qquad f(x) \equiv \frac{e^{-g(x)}}{x} \qquad a = \frac{1}{r}$$

Double Copy in String Theory

Tree-level four open superstring amplitude (canonical color ordering):

$$\mathcal{A}(1,2,3,4) = \frac{\Gamma(1-s)\Gamma(1-u)}{\Gamma(1+t)} A_{\text{YM}}(1,2,3,4)$$

Tree-level four closed superstring amplitude:

$$\mathcal{M} = \pi \frac{su}{t} \frac{\Gamma(-s)\Gamma(-u)\Gamma(-t)}{\Gamma(s)\Gamma(u)\Gamma(t)} A_{\text{YM}}(1,2,3,4)\tilde{A}_{\text{YM}}(1,2,3,4)$$

Small α' Representation

Tree-level four open string amplitude (canonical color ordering):

[Schlotterer, Stieberger '13]

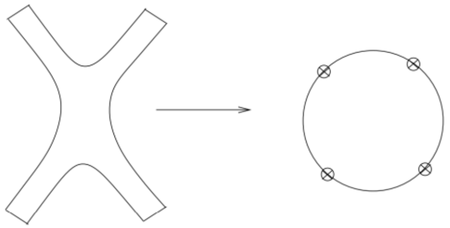
$$\mathcal{A}_0(1,2,3,4) = \exp\left\{\sum_{n=1}^{\infty}\frac{\zeta(2n)}{(2n)}\left(s^{2n}+u^{2n}-t^{2n}\right)\right\}$$
$$\times \exp\left\{\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{(2k+1)}\left(s^{2k+1}+u^{2k+1}+t^{2k+1}\right)\right\} A_{\mathrm{YM}}(1,2,3,4)$$

Tree-level four closed superstring amplitude:

$$\mathcal{M}_0 = \pi\frac{su}{t}\exp\left\{2\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{(2k+1)}\left(s^{2k+1}+u^{2k+1}+t^{2k+1}\right)\right\} A_{\mathrm{YM}}(1,2,3,4)\tilde{A}_{\mathrm{YM}}(1,2,3,4).$$

Double copy: $\mathcal{M}_0 = \pi s\,\mathrm{sv}\,\mathcal{A}_0(1,2,3,4)\times\tilde{A}_{\mathrm{YM}}(1,2,4,3),$

Classical Solution at High Energies



$$\mathcal{A} \sim \int \mathcal{D}g\mathcal{D}X \exp\left(-\frac{1}{4\pi\alpha'}\int d\zeta_1 d\zeta_2 \sqrt{g} g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu\right)\prod_{i=1}^{4} V_i(z_i)$$

$$V_j(z_j) \sim \int dz_j \sqrt{g} e^{ip_j\cdot X(z_j)}$$

Integrated over the boundary of the disk

$$\mathcal{A} \sim \frac{g}{\mathrm{Vol}(SL(2,\mathbb{R}))}\delta^{26}\left(\sum_i p_i\right)\int \mathcal{D}X \prod_{i=1}^{4} dz_i \prod_{j<l}^{4} |z_j - z_j|^{2\alpha' p_j\cdot p_l}$$

$$\sim g\int_0^1 dz|z|^{2\alpha' p_1\cdot p_2}|1-z|^{2\alpha' p_2\cdot p_3} \sim g\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' u)}{\Gamma(1+\alpha' t)}$$

+ other channels

$z_1 = 0$

$z_2 = z$

$z_3 = 1$

$z_4 \to \infty$

Stirling: $\mathcal{A} \sim (2\pi\alpha')^{1/2}\left(\frac{su}{t}\right)^{1/2} e^{-\alpha' s\ln(\alpha' s) - \alpha' u \ln(\alpha' u) - \alpha' t \ln(\alpha' t)}(-1)^{-\frac{1}{2}-\alpha' s - \alpha' u}$

(s channel)

From the POV of spacetime: $X_c^\mu(\zeta) = -i\sum_k p_k^\mu \ln\left(1 - \frac{\zeta}{z_k}\right)$

(classical solution)

$p_i^2 = 0$

Dominates path integral at high energies