Introduction	Formalism	Results	An example

## String Corrections To Black Holes In Five Dimensions

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Introduction	Formalism	Results	An example
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Introduction			

• General supersymmetric solution of a  $\mathcal{N}=$  2, D= 5 model with term

 $A \wedge \operatorname{tr}(R \wedge R)$ 

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Introduction			

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• Interpretation: SUGRA as non-renormalizable effective theory

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Introduction			

• General supersymmetric solution of a  $\mathcal{N}=2,\ D=5$  model with term

$$A \wedge \operatorname{tr}(R \wedge R)$$

- Interpretation: SUGRA as non-renormalizable effective theory
- Motivation: e.g.  $S = k \log \Omega$ Bekenstein-Hawking  $S = \frac{A}{4}$ Wald  $S \propto \int_{H} d^{d-2}x \sqrt{|h|} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon^{\mu\nu} \epsilon^{\rho\sigma}$

[Castro et al. String theory effects on five-dimensional black-hole physics]

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Introduction			

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Bekenstein-Hawking 
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[Castro et al. String theory effects on five-dimensional black-hole physics]

• Another motivation: AdS/CFT

gravity side	SU(N) side
higher orders in curvature	1/N effects

[Cremonini et al. Black holes in five-dimensional gauged supergravity with higher derivatives]

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$\mathcal{N}=$ 2, $D=$ 5 sup	ergravity with vec	tor multiplets	

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- Pure gravity
  - bosons:  $e_{\mu}^{a}$
  - fermions:  $\psi^i_\mu$
- $n_V + 1$  gauge fields  $G = U(1)^{n_V + 1}$ 
  - bosons:  $A^I_{\mu}$ ,  $M^I$
  - fermions:  $\Omega^{li}$

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  - fermions:  $\Omega^{li}$
- Bosonic action:  $S = rac{1}{16\pi G}\int d^5x \sqrt{|g|} \mathcal{L}$

$$\mathcal{L} = -R - G_{IJ} \left( \frac{1}{2} F_{\mu\nu}^{I} F^{J\mu\nu} - \nabla_{\mu} M^{I} \nabla^{\mu} M^{J} \right) + \frac{1}{24\sqrt{|g|}} c_{IJK} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu}^{I} F_{\rho\sigma}^{J} A_{\tau}^{K}$$

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$\mathcal{N}=2,~D=5$ sup	pergravity with vec	tor multiplets	

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• Very special geometry constraint & graviphoton

$$\frac{1}{3!}c_{IJK}M^{I}M^{J}M^{K} = 1 \qquad A_{gr.ph.} \propto c_{IJK}M^{I}M^{J}A^{K}$$

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$\mathcal{N}=2, D=5$ su	pergravity with vec	tor multiplets	

- Pure gravity
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• Motivation: dimensional reduction from D = 11 on Calabi-Yau

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Off-shell formalism	ı		

 $\bullet~$  String theory calculations  $\rightarrow~$  supersymmetry considerations

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Off-shell formalism	า		

- $\bullet~$  String theory calculations  $\rightarrow~$  supersymmetry considerations
- $\bullet$  Disentangle SUSY algebra and EOM's  $\rightarrow$  off-shell formalism

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Introduction	Formalism	Results	An example
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Off-shell formalisr	n		

- $\bullet~$  String theory calculations  $\rightarrow~$  supersymmetry considerations
- $\bullet$  Disentangle SUSY algebra and EOM's  $\rightarrow$  off-shell formalism

• Drawback  $\rightarrow$  auxiliary fields  $D, v_{\mu\nu}, \chi^i$ 

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Off-shell formalism	า		

- $\bullet~$  String theory calculations  $\rightarrow~$  supersymmetry considerations
- $\bullet$  Disentangle SUSY algebra and EOM's  $\rightarrow$  off-shell formalism
- Drawback  $\rightarrow$  auxiliary fields  $D, v_{\mu\nu}, \chi^i$
- Resulting lagrangian:  $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

$$\begin{split} \mathcal{L}_{0} &= \frac{1}{2}D(\mathcal{N}-1) - \frac{1}{4}(\mathcal{N}+3)R + (3\mathcal{N}+1)v^{2} + 2\mathcal{N}_{I}v \cdot F^{I} + \\ &+ \mathcal{N}_{IJ}\left(\frac{1}{4}F^{I} \cdot F^{J} - \frac{1}{2}\partial M^{I} \cdot \partial M^{J}\right) + \\ &+ \frac{1}{24}\frac{1}{\sqrt{|g|}}c_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}A^{I}_{\mu}F^{J}_{\nu\rho}F^{K}_{\sigma\tau} \\ \mathcal{L}_{1} &= \frac{c_{2I}}{24} \cdot \left\{\frac{1}{16}\epsilon^{abcde}A^{I}_{a}C_{bcfg}C_{de}^{\ fg} + \text{supersymmetric completion} \right. \end{split}$$

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Off-shell formalism	า		

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•  $\mathcal{L}_1$  encodes one-loop string corrections

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Modified	very special geometry	constraint	

• Let us focus on *D*-terms:

$$\mathcal{L} = \frac{1}{2}D\left(\mathcal{N} - 1\right) + \frac{c_{2I}}{24} \cdot \frac{1}{6}D\left(\frac{1}{2}M'D + v \cdot F'\right) + \dots$$
$$\mathcal{N} := \frac{1}{3!}c_{IJK}M'M^{J}M^{K}$$

Modified ver	v special geometr	v constraint	
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Introduction	Formalism	Results	An example

• Let us focus on *D*-terms:

$$\mathcal{L} = \frac{1}{2}D\left(\mathcal{N} - 1\right) + \frac{c_{2I}}{24} \cdot \frac{1}{6}D\left(\frac{1}{2}M^{I}D + v \cdot F^{I}\right) + \dots$$
$$\mathcal{N} := \frac{1}{3!}c_{IJK}M^{I}M^{J}M^{K}$$

• Equation of motion of D field

$$\mathcal{N} = 1 - \frac{c_{2l}}{24} \cdot \frac{1}{3} \left( M^{\prime} D + v \cdot F^{\prime} \right)$$

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BPS conditions			

some fraction of supersymmetry  $\Leftrightarrow \exists \epsilon(x) \neq 0$  such that is preserved

$$\delta\psi_{\mu} \equiv \left\{\nabla_{\mu} + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}v^{ab}\gamma_{\mu}\gamma_{ab}\right\}\epsilon = 0$$
  
$$\delta\chi \equiv \left\{D - 2\gamma^{c}\gamma^{ab}\nabla_{a}v_{bc} - 2\gamma^{a}\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(v\cdot\gamma)^{2}\right\}\epsilon = 0$$
  
$$\delta\Omega' \equiv \left\{-\frac{1}{4}F_{ab}'\gamma^{ab} - \frac{1}{2}\gamma^{\mu}\partial_{\mu}M' - \frac{1}{3}M'v^{ab}\gamma_{ab}\right\}\epsilon = 0$$

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Spinorial geometry	, techniques		

• Killing spinor equation:  $\nabla \epsilon + \mathcal{O} \epsilon = 0$  or  $\mathcal{O} \epsilon = 0$ 



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Spinorial geometry	techniques		

- Killing spinor equation:  $\nabla \epsilon + \mathcal{O} \epsilon = 0$  or  $\mathcal{O} \epsilon = 0$
- Systematic study:
  - find the orbits of Spin(1,4) on the spinor module  $\Delta$
  - etermine a standard representative for each orbit
  - **(**) exploit invariance under LLT's to cast  $\epsilon$  in standard form:

$$\exp\left\{\frac{1}{4}\lambda^{ab}(x)\gamma_{ab}\right\}\epsilon(x) = \epsilon^{\mathrm{std}}(x)$$

choose a basis for spinor module: Δ = span(λ<sub>1</sub>, λ<sub>2</sub>, λ<sub>3</sub>, λ<sub>4</sub>)
 write down Killing equation in components:

$$0 = \left(\nabla \epsilon^{\mathsf{std}} +\right) \mathcal{O} \epsilon^{\mathsf{std}} = \sum_{i=1}^{4} C_i \lambda_i \quad \Rightarrow \quad C_i = 0$$

study geometrical properties of 0-,1-,2-forms constructed out of the Killing spinor

Orbits and k-forms		
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$$\begin{array}{l} \text{Standard representatives } (\psi = \sum_{i=1}^{4} C_i \lambda_i \neq 0):\\ q(\psi) := |C_1|^2 + |C_2|^2 - |C_3|^2 - |C_4|^2\\ \psi \sim \begin{cases} \sqrt{q(\psi)} \ \lambda_1 & \text{if } q(\psi) > 0\\ \lambda_1 + \lambda_3 & \text{if } q(\psi) = 0\\ \sqrt{-q(\psi)} \ \lambda_3 & \text{if } q(\psi) < 0 \end{cases}$$

Orbits and	<i>k</i> -forms		
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Introduction	Formalism	Results	An example

• Standard representatives 
$$(\psi = \sum_{i=1}^{4} C_i \lambda_i \neq 0)$$
:  
 $q(\psi) := |C_1|^2 + |C_2|^2 - |C_3|^2 - |C_4|^2$   
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• Killing spinors to be studied

$$\epsilon^{\mathrm{std}}(x) = e^{\phi(x)}\lambda_1 \;, \;\; \epsilon(x)^{\mathrm{std}} = e^{\phi(x)}\lambda_3 \;, \;\; \epsilon(x)^{\mathrm{std}} = \lambda_1 + \lambda_3$$

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Orbits and k-	forms		
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• Standard representatives 
$$(\psi = \sum_{i=1}^{4} C_i \lambda_i \neq 0)$$
:  

$$a(\psi) := |C_i|^2 + |C_i|^2 - |C_i|^2 - |C_i|^2$$

$$q(\psi) := |\mathcal{C}_1|^2 + |\mathcal{C}_2|^2 - |\mathcal{C}_3|^2 - |\mathcal{C}_4|$$
 $\psi \sim egin{cases} \sqrt{q(\psi)} \ \lambda_1 & ext{if } q(\psi) > 0 \ \lambda_1 + \lambda_3 & ext{if } q(\psi) = 0 \ \sqrt{-q(\psi)} \ \lambda_3 & ext{if } q(\psi) < 0 \end{cases}$ 

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• 0-,1-,2-forms out of spinor bilinears

$$\Omega^{ij}_{(0)} = \overline{\epsilon}^i \epsilon^j , \quad \Omega^{ij}_{(1)} = \overline{\epsilon}^i \gamma_\mu \epsilon^j \, dx^\mu , \quad \Omega^{ij}_{(2)} = \frac{1}{2} \, \overline{\epsilon}^i \gamma_{\mu\nu} \epsilon^j \, dx^\mu \wedge dx^
u$$

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Orbits and k	r-forms		
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• Standard representatives 
$$(\psi = \sum_{i=1}^{4} C_i \lambda_i \neq 0)$$
:

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• Killing spinors to be studied

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u$$

• real parametrization

$$i\Omega_{(0)}^{ij} = \epsilon^{ij}S$$
,  $i\Omega_{(1)}^{ij} = \epsilon^{ij}V$ ,  $[\Omega_{(2)}]_{j}^{i} = \sum_{k=1}^{3} X^{(k)}[\sigma^{(k)}]_{j}^{i}$ 

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Killing spinor  $e^{\text{std}} = e^{\phi} 1$ :

•  $V^{\mu}$  is Killing and timelike  $\Rightarrow V^{\mu} = (\partial/\partial t)^{\mu}$ 

• 
$$ds^2 = e^{4\phi}(dt + \omega)^2 - e^{-2\phi}ds_B^2$$

• *B* is hyper-Kähler with self-dual curvature:

$$\begin{split} \hat{\nabla}\hat{J}^{(k)} &= 0 \qquad \qquad \hat{J}^{(k)} \circ \hat{J}^{(\ell)} &= -\delta^{k\ell}\mathbb{I} + \epsilon^{k\ell m}\hat{J}^{(m)} \\ \hat{\ast}\hat{\Omega}^{i}{}_{j} &= +\hat{\Omega}^{i}{}_{j} \qquad \text{remark: } \operatorname{Ric} = 0 \end{split}$$

• 
$$v, D, F_{0i}^{I}$$
 are given in terms of  $\phi, \omega$ 



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• 
$$v, D, F_{0i}'$$
 are given in terms of  $\phi, \omega$ 

Killing spinor  $\epsilon^{\text{std}} = e^{\phi} e_1$ : as above, although

- opposite self-duality
- v, D,  $F_{0i}^{\prime}$  are given by different functions of  $\phi, \omega$

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BPS conditions for	$\epsilon^{std} = 1 + e_1$		

Killing spinor  $\epsilon^{\text{std}} = 1 + e_1$ ;

- $V^{\mu}$  is Killing and lightlike  $\Rightarrow$   $V^{\mu} = (\partial/\partial v)^{\mu}$
- 2-forms  $X^{(k)}$  are closed  $\Rightarrow dX^{(k)} = du \wedge dx^k$

• 
$$ds^2 = e^U(\mathcal{F}du^2 + 2dudv) - e^{-2U}(d\vec{x} + \vec{a}du)^2$$

- v, D are given in terms of  $U, \vec{a}$
- Bianchi identity

$$dF' = 0 \implies \begin{cases} M' = e^{U}H' & \text{with} & \vec{\nabla}^2 H' = 0\\ F' & \text{is fixed modulo a gradient in } \mathbb{R}^3 \end{cases}$$

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Killing Spinor Iden	tities		

• BPS conditions necessary but not sufficient



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Killing Spinor Ider	itities		

- BPS conditions necessary but not sufficient
- KSI: off-shell constaints among bosonic EOM's

$$\sum_{b} \left. \frac{\delta \mathcal{L}}{\delta \phi_{b}} \frac{\delta (\delta_{\epsilon} \phi_{b})}{\delta \phi_{f}} \right|_{\text{fermions}=0} \equiv 0 \quad \text{if } \epsilon \text{ is Killing}$$

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Killing Spinor Id	entities		

- BPS conditions necessary but not sufficient
- KSI: off-shell constaints among bosonic EOM's

$$\sum_{b} \left. \frac{\delta \mathcal{L}}{\delta \phi_{b}} \frac{\delta (\delta_{\epsilon} \phi_{b})}{\delta \phi_{f}} \right|_{\text{fermions}=0} \equiv 0 \quad \text{if } \epsilon \text{ is Killing}$$

• Example: if 
$$\epsilon^{\mathsf{std}} = e^{\phi} 1$$

$$\mathcal{E}(A)_I^0 - \mathcal{E}(M)_I = 0$$
  

$$\mathcal{E}(A)_I^i = 0$$
  

$$\nabla^i \mathcal{E}(D) - \frac{1}{4} \mathcal{E}(v)^{0i} - \mathcal{E}(D)v^{0i} = 0$$
  

$$\frac{1}{4} \mathcal{E}(v)^{(-)ij} + \mathcal{E}(D)v^{(-)ij} = 0$$
  

$$\mathcal{E}(g)_{00} = \mathcal{E}(g)_{0i} = \mathcal{E}(g)_{ij} = 0 \quad (*)$$

 $(*) \equiv$  other bosons on-shell

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• Ansatz: 
$$\vec{a} = 0$$
 ,  $\mathcal{F} = 0$  ,  $F'_{+i} = 0$ ;

- Ansatz: no *u*-dependence; radial  $\vec{x}$ -dependence
- Modified very special geometry constraint

$$e^{-3U} = \frac{1}{3!} c_{IJK} H^{I} H^{J} H^{K} + \frac{c_{2I}}{24} \cdot \left[ \frac{1}{2} U' H^{I'} + H^{I} \frac{1}{r^{2}} \left( r^{2} U' \right)^{\prime} \right]$$

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Example <sup>.</sup>	magnetic attractor ( $\epsilon^{s}$	$t^{td} = 1 + e_1$	

• Ansatz: 
$$\vec{a} = 0$$
 ,  $\mathcal{F} = 0$  ,  $F_{+i}^{I} = 0$ ;

- Ansatz: no *u*-dependence; radial  $\vec{x}$ -dependence
- Modified very special geometry constraint

$$e^{-3U} = \frac{1}{3!} c_{IJK} H^{I} H^{J} H^{K} + \frac{c_{2I}}{24} \cdot \left[ \frac{1}{2} U^{\prime} H^{I\prime} + H^{\prime} \frac{1}{r^{2}} \left( r^{2} U^{\prime} \right)^{\prime} \right]$$

- Magnetic attractor:  $H^{I} = \frac{p^{I}}{2r}$
- Geometry:

 $AdS_3 \times S^2$  with  $\ell_A = 2\ell_S \Rightarrow$  maximal supersymmetry

- Only non-vanishing components:  $F'_{\theta\phi} \Rightarrow$  magnetic field
- Radius and charges

$$2\ell_S = \frac{1}{3!}c_{IJK}p^Ip^Jp^K + \frac{c_{2I}}{24}\cdot 2p^K$$

 $\begin{array}{ll} \mbox{magnetic attractor} &\equiv \mbox{exact solution approximating near-horizon geometry of a} \\ & \mbox{magnetically chaged black string} \end{array}$ 

Example:	"small"	magnetic string		
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$$2\ell_{\mathcal{S}} = \frac{1}{3!} c_{IJK} p^{I} p^{J} p^{K} + \frac{c_{2I}}{24} \cdot 2p^{I}$$

• "Small" string 
$$\rightarrow c_{IJK} p^I p^J p^K = 0$$

•  $c_{2I} = 0 \Rightarrow \ell_S = 0 \rightarrow \text{degenerate horizon} \rightarrow \text{naked singularity}$ 

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[Castro et al. String theory effects on five-dimensional black-hole physics]Corrections smooth out singularity!

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Conclusions &	Perspectives		

## • Analysis of the most general supersymmetric solution:

- off-shell formalism VS on-shell formalism
- spinorial geometry techniques VS Fierz identities
- supersymmetry constraints are independent of dynamics

• can corrections smooth all singularities?

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Conclusions &	Perspectives		

- Analysis of the most general supersymmetric solution:
  - off-shell formalism VS on-shell formalism
  - spinorial geometry techniques VS Fierz identities
  - supersymmetry constraints are independent of dynamics

- can corrections smooth all singularities?
- Possible developments:
  - $\bullet\,$  multiple Killing spinors  $\rightarrow\,$  classification of solutions
  - computation of entropy corrections
  - gauged supergravity, AdS/CFT