Drell-Yan lepton-pair production as a precision laboratory

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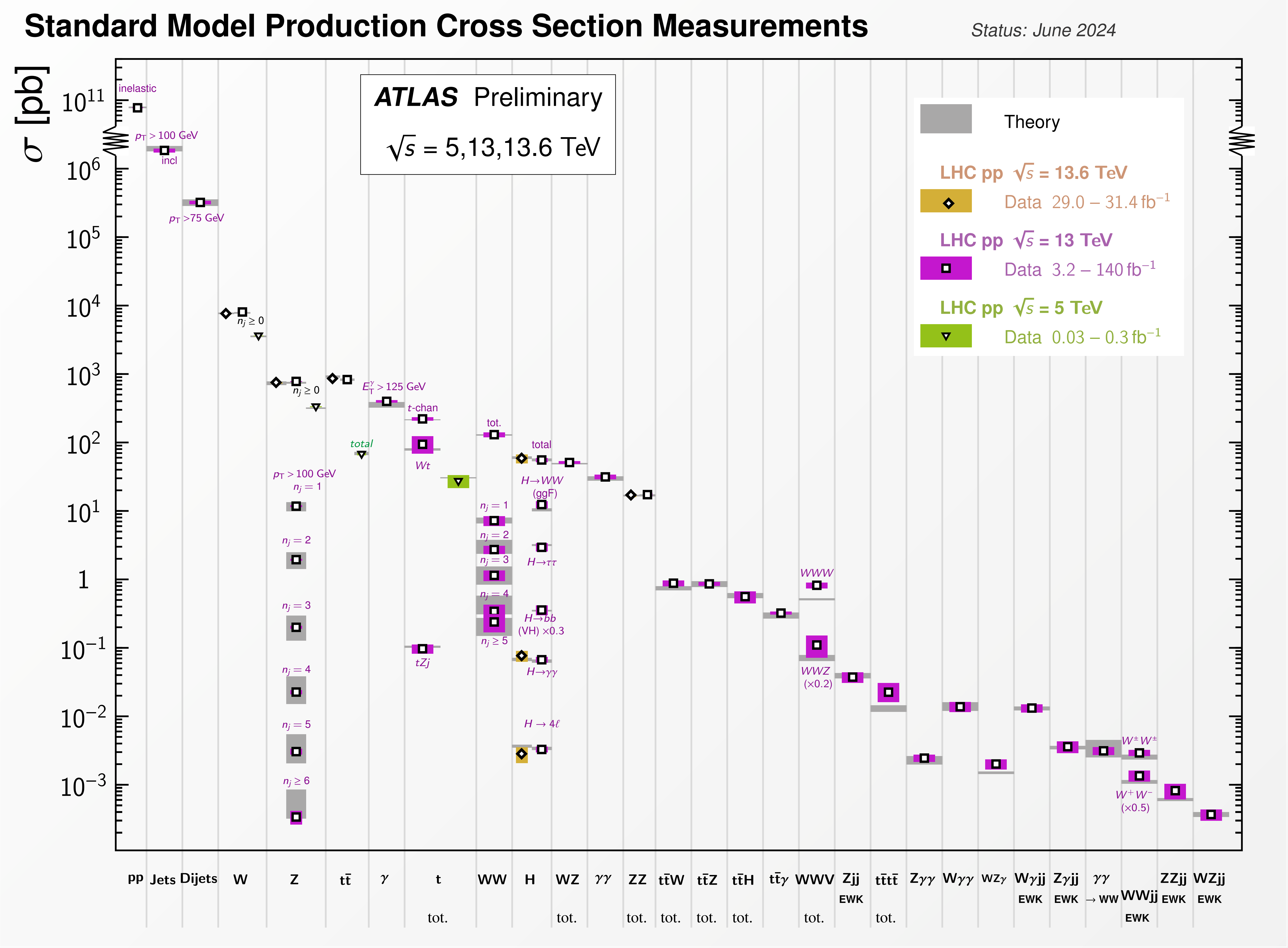
J'étais affecté du mal aigu de la précision



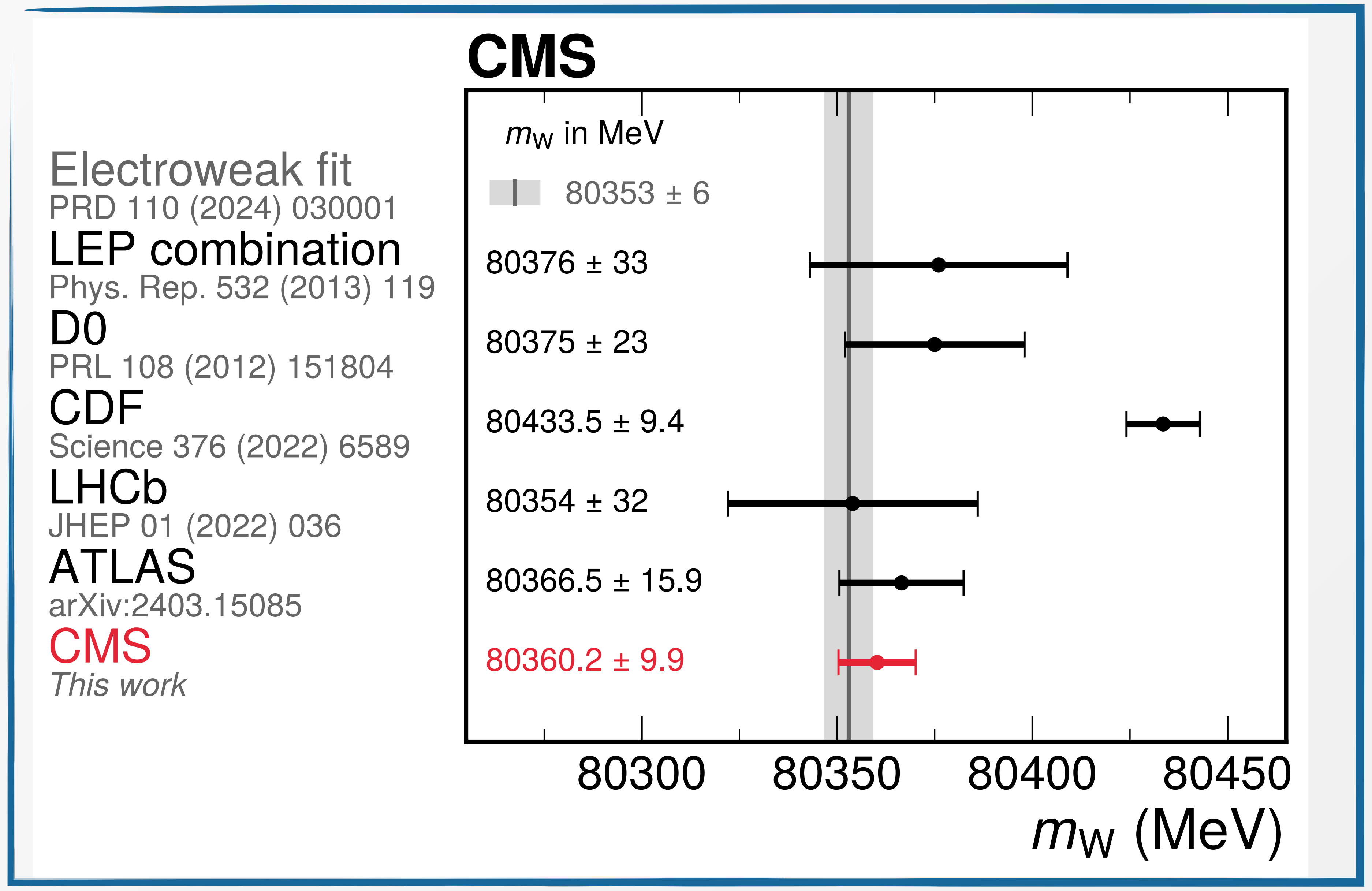
Vast amount of processes being thoroughly tested at the LHC

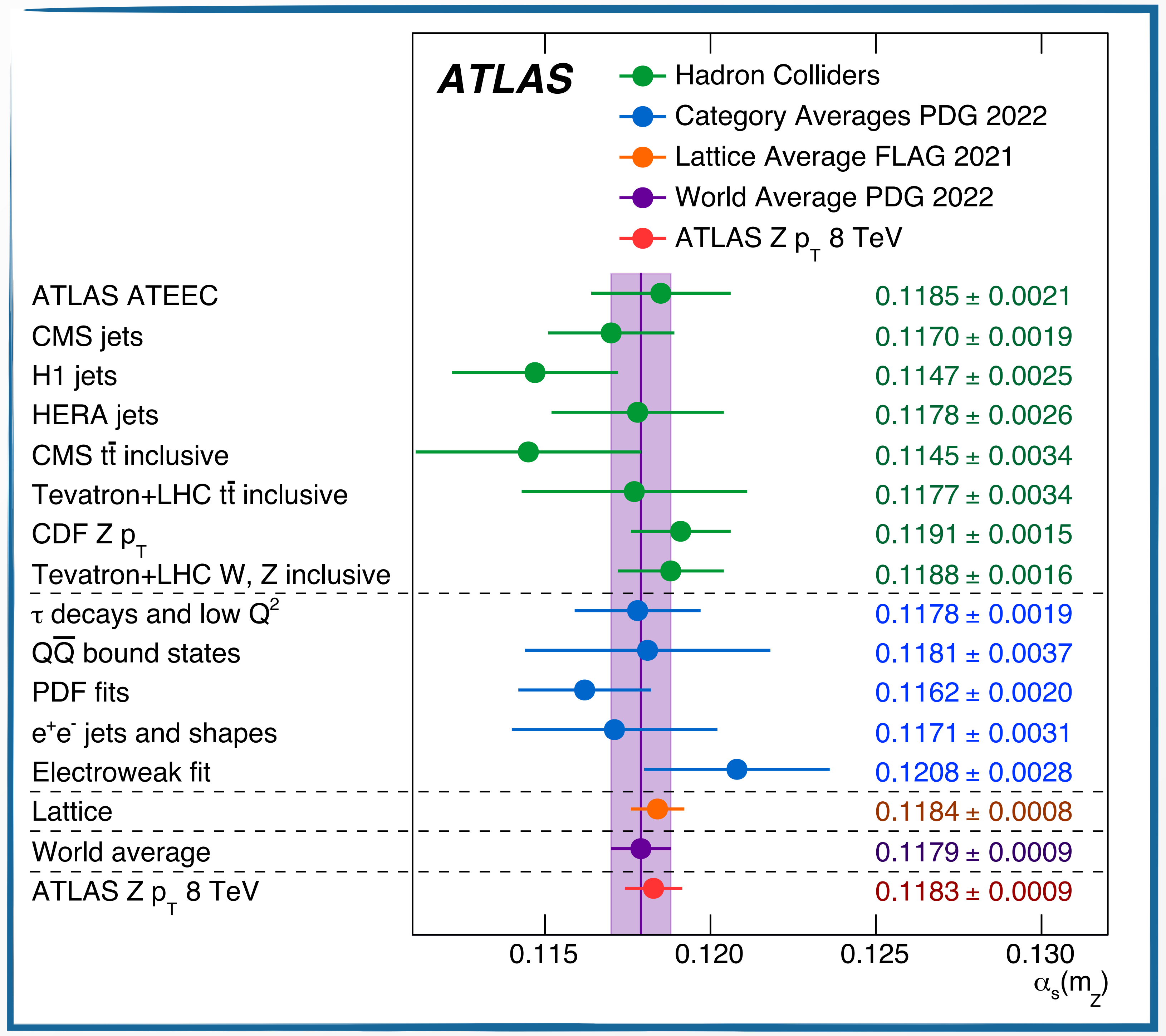
Agreement between data and accurate theoretical predictions across many orders of magnitude

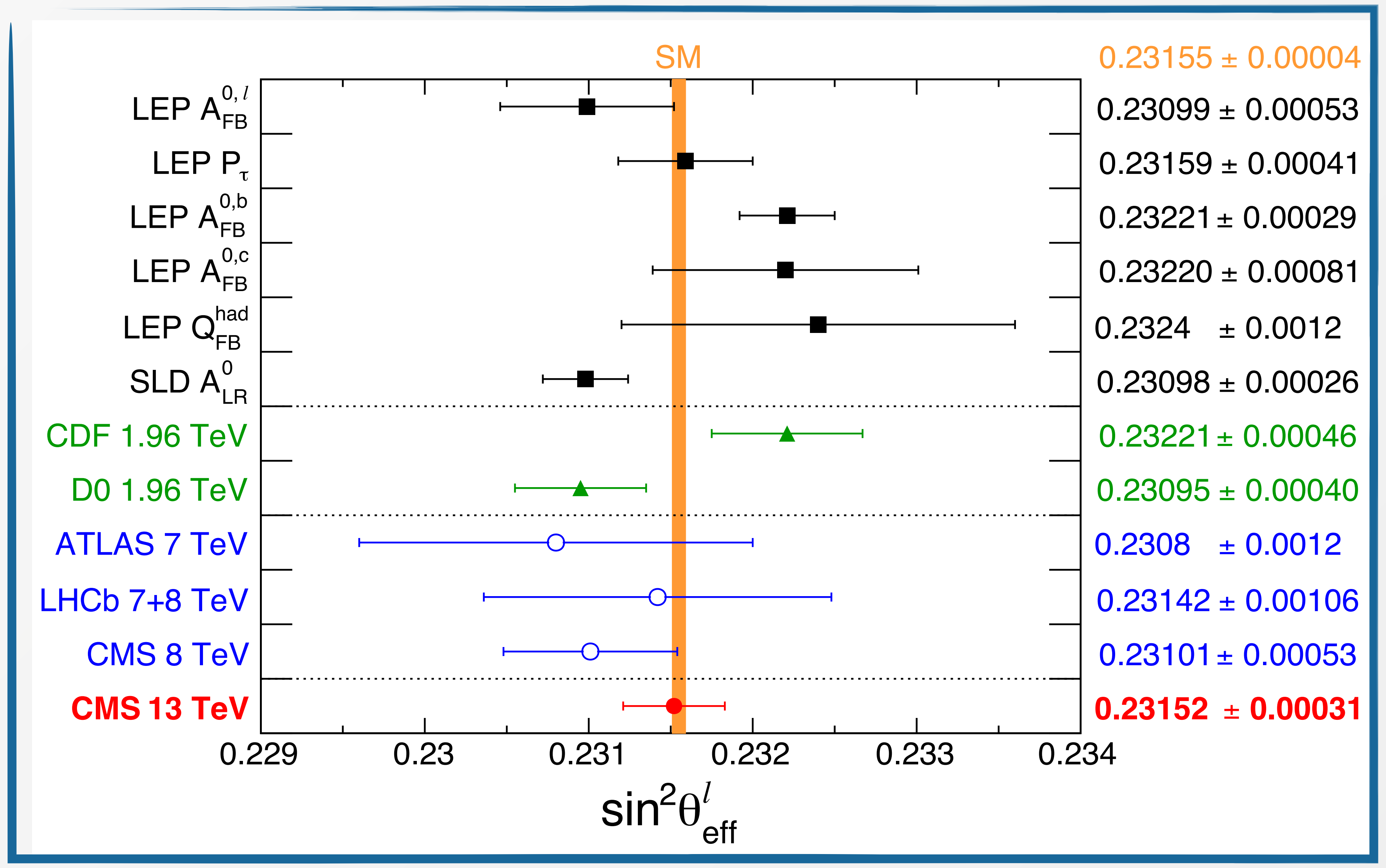
LHC now firmly established as a **precision machine**

J'étais affecté du mal aigu de la précision

Three of the **most precise measurements** of fundamental SM parameters have been performed at the LHC in the last couple of years







Dilepton production plays a central role in the LHC precision programme

Drell-Yan production and precision

NC DY lepton-pair **invariant mass** → Z properties (Z resonance) and $\sin^2\theta_{\text{eff}}^{\ell\ell}$ ($A_{FB}(m_{\ell\ell})$)

CC DY lepton-pair **transverse mass** → m_W (W resonance)

NC/CC DY charged lepton p_T^{ℓ} → m_Z (Z resonance), m_W (W resonance)

NC DY lepton pair $p_T^{\ell\ell}$ → α_s (low $p_T^{\ell\ell}$)

+ important PDF constraints using multi-differential distributions (rapidity, transverse momentum…)
+ Constraints on BSM models

Drell-Yan production and precision

NC DY lepton-pair **invariant mass** → Z properties (Z resonance) and $\sin^2\theta_{\text{eff}}^{\ell\ell}$ ($A_{FB}(m_{\ell\ell})$)

CC DY lepton-pair **transverse mass** → m_W (W resonance)

Reliable predictions through fixed-order perturbation theory

Drell-Yan production and precision

NC/CC DY charged lepton p_T^ℓ → m_Z (Z resonance), m_W (W resonance)

NC DY lepton pair $p_T^{\ell\ell}$ → α_s (low $p_T^{\ell\ell}$)

All-order resummation needed for meaningful predictions due to sensitivity to QCD radiation

Drell-Yan production and precision

Theoretical understanding of fixed-order and all-order structure of QCD/EW radiation in Drell-Yan process is crucial

Precision physics at the LHC: setting a limit on infinite error

collinear factorisation

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under control

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$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \ldots \qquad \alpha_s \sim 0.1$$

$$+ \hat{\sigma}^{(0,1)}_{ab} + \ldots \qquad \alpha \sim 0.01$$

$$+ \hat{\sigma}^{(1,1)}_{ab} + \ldots \qquad \alpha\alpha_s \sim 0.001$$

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$\mathcal{O}(1)$ accuracy (order of magnitude)

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$\mathcal{O}(10-20\%)$
accuracy

Precision physics at the LHC: setting a limit on infinite error

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$\mathcal{O}(5-10\%)$
accuracy

Precision physics at the LHC: setting a limit on infinite error

collinear factorisation

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$\mathcal{O}(1-5\%)$
accuracy

The purest and most thoughtful minds are those which love colour the most

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \ldots$$
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- **QCD** corrections by and large **dominant**

NNLO differential cross sections

[Anastasiou, Dixon, Melnikov, Petriello (2003)], [Melnikov, Petriello (2006)] [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] [Catani, Ferrera, Grazzini (2010)]

N^{3}LO inclusive cross sections and di-lepton rapidity distribution

[Duhr, Dulat, Mistlberger (2020)] [Chen, Gehrmann, Glover, Huss, Yang, and Zhu (2021)] [Duhr, Mistlberger (2021)]

N^{3}LO fiducial cross sections and distributions

[Camarda, Cieri, Ferrera (2021)], [Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli (2022)] [Chen, Gehrmann, Glover, Huss, Yang, and Zhu (2022)], [Neumann, Campbell (2022) and (2023)] [Billis, Michel, Tackmann (2024)]

Drell-Yan: NNLO QCD

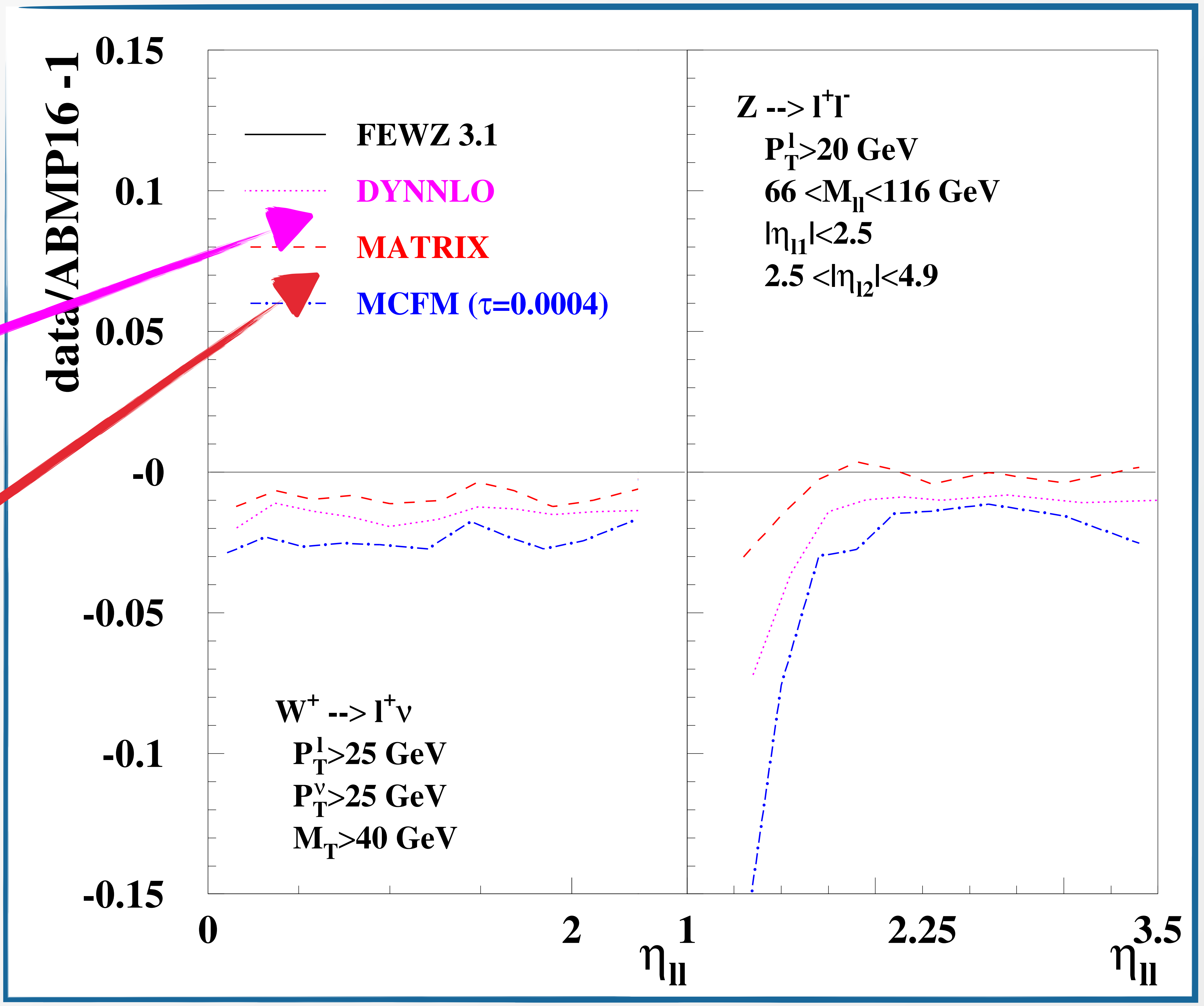
Reliability of state-of-art predictions is crucial. Several public codes available reaching fully differential NNLO QCD accuracy.

- Local subtraction: FEWZ, NNLOJET
- Slicing: DYTURBO, MATRIX (q_T slicing), MCFM (0-jettines, q_T slicing,)

Slicing methods suffer in the presence of symmetric / **asymmetric cuts on the leptons**: **percent-level** differences when compared to results obtained with **local subtractions** may be present due to **linear power corrections** in the slicing variable

$r_{\rm cut} \sim 0.01$

$r_{\rm cut} \sim 0.0005 - 0.001$



[Alekhin, Kardos, Moch, Trócsányi ‘21]

Solutions:

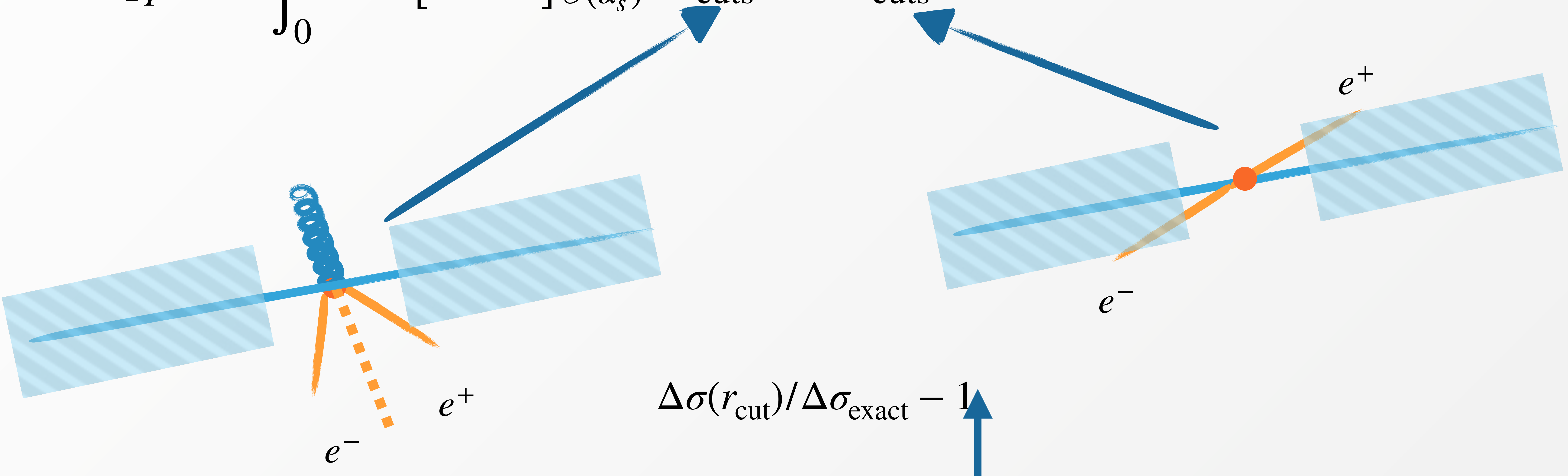
- Transverse momentum recoil for q_T slicing
  [Buonocore, Kallweit, LR, Wiesemann’21] [Camarda, Cieri, Ferrera ’21]
- Projection to Born for jettiness-slicing
  [Vita 2401.03017][Campbell et al 2408.05265]

Linear power corrections

Linear power corrections in q_T have a **purely kinematical origin** and can **be predicted by factorisation**

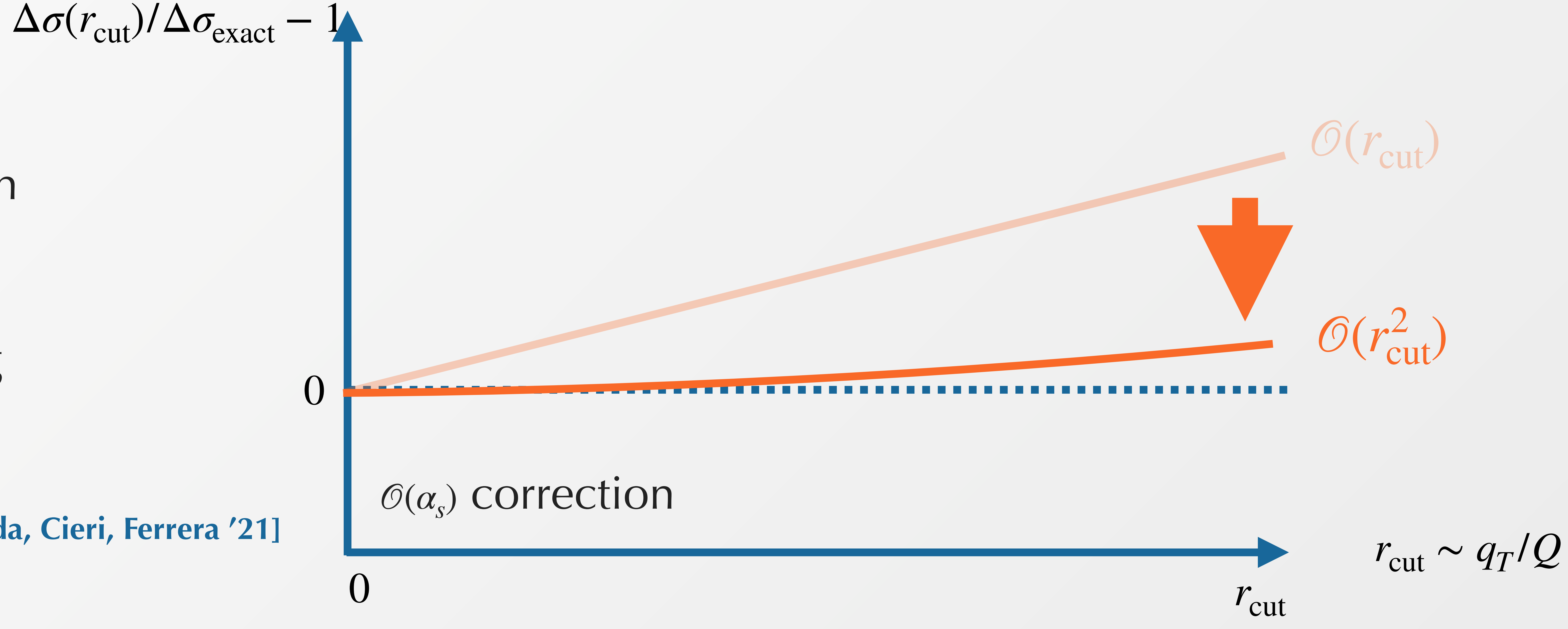
[Ebert, Michel, Stewart, Tackmann '20]

$$\Delta\sigma^{\mathrm{linPCs}}(q_T^{\mathrm{cut}}) = \int_0^{r_{\mathrm{cut}}} dr' \left[d\sigma^{\mathrm{sing}}\right]_{\mathcal{O}(\alpha_s^k)} \left(\Theta^{\mathrm{recoil}}_{\mathrm{cuts}} - \Theta^{\mathrm{Born}}_{\mathrm{cuts}}\right)$$



Resorting to the recoil prescription allows for the inclusion of all missing fiducial linear power corrections below q_T^{cut}, improving dramatically the efficiency of the non-local subtraction

[Buonocore, Kallweit, LR, Wiesemann'21] [Camarda, Cieri, Ferrera '21]



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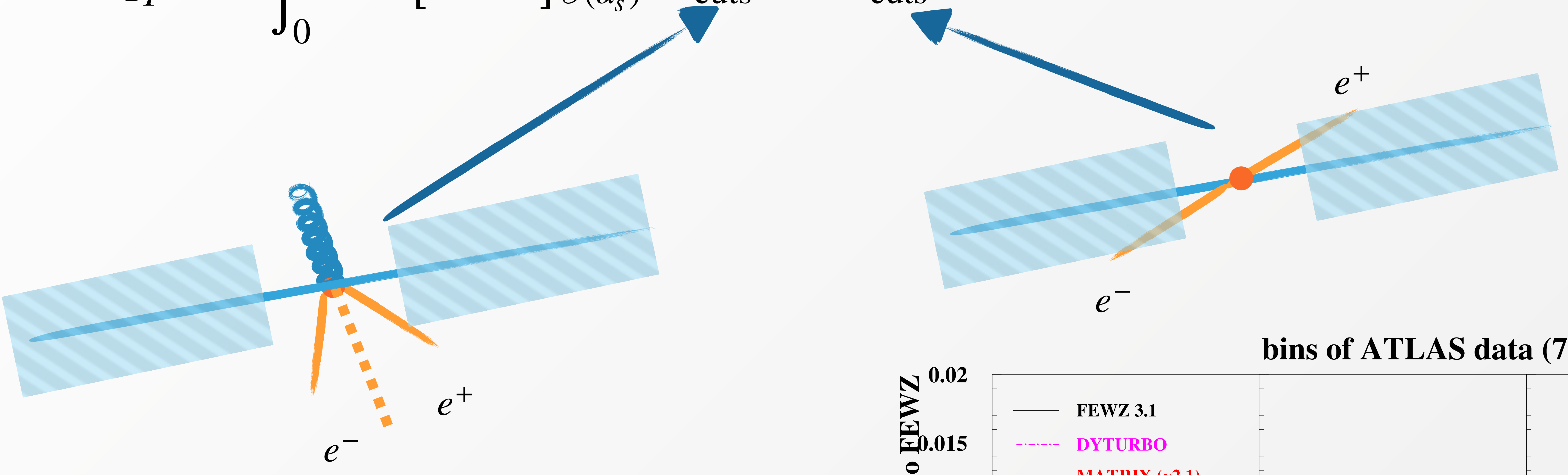
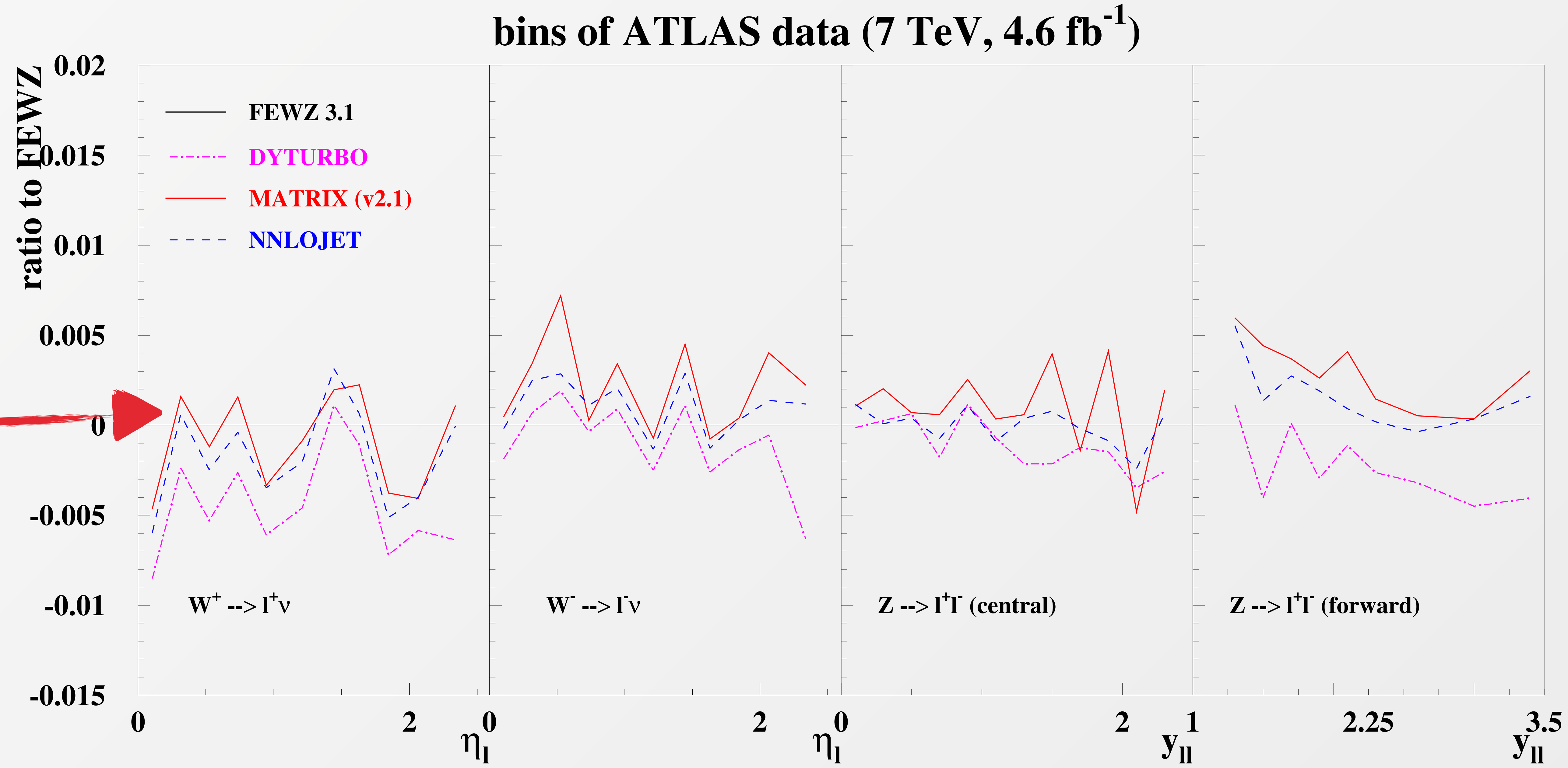


Excellent agreement between available public codes once improved slicing is used



Linear power corrections and perturbative instability

Problems related to symmetric / asymmetric cuts have been known since a long time

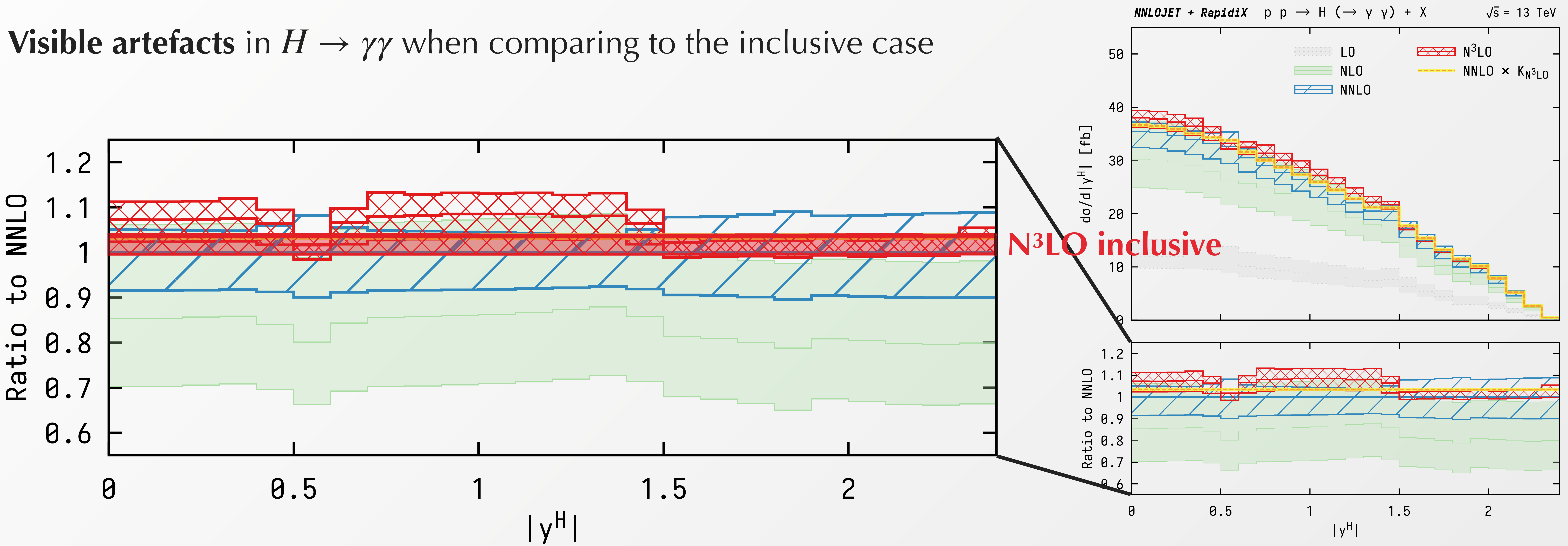
Perturbative instability induced by sensitivity to soft radiation in configurations close to the back-to-back limit

[Klasen, Kramen '96][Harris, Owen '97][Frixione, Ridolfi '97]

Linear sensitivity of the acceptance at small q_T leads to a (alternating sign) **factorial growth**

[Salam, Slade '21]

Visible artefacts in $H \to \gamma\gamma$ when comparing to the inclusive case



[Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, '21]

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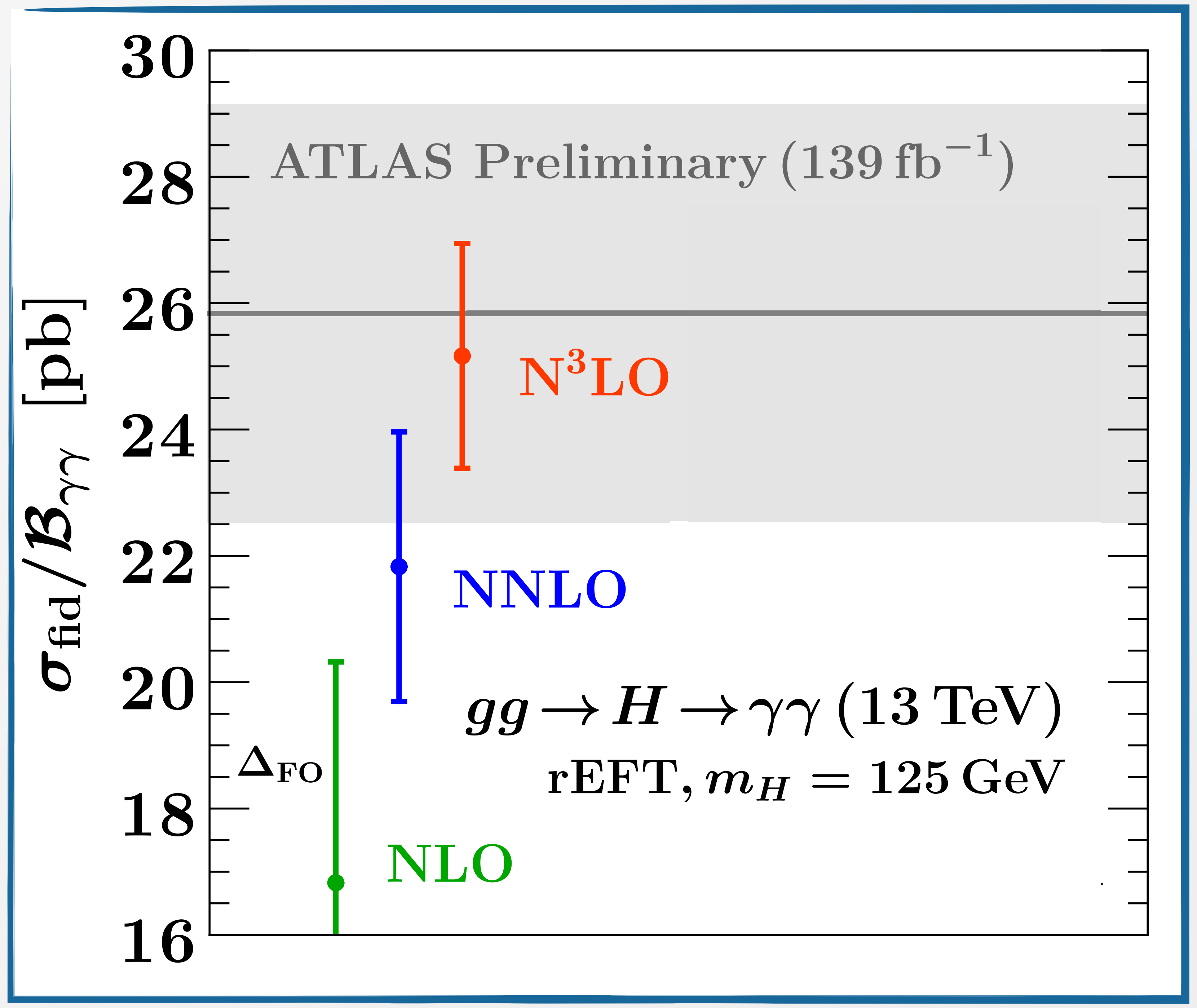
Solution 1: Linear fiducial power corrections can be resummed at all orders in perturbation theory

$$\sigma_{\rm incl}^{\rm FO} = 13.80\,[1 + 1.291 + 0.783 + 0.299]\,\text{pb}$$

$$\begin{aligned}\sigma_{\rm fid}^{\rm FO}/\mathcal{B}_{\gamma\gamma} = 6.928\,[1 &+ (1.300 + 0.129_{\rm fpc}) \\ &+ (0.784 - 0.061_{\rm fpc}) \\ &+ (0.331 + 0.150_{\rm fpc})]\,\text{pb}\,.\end{aligned}$$



[Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann '21]

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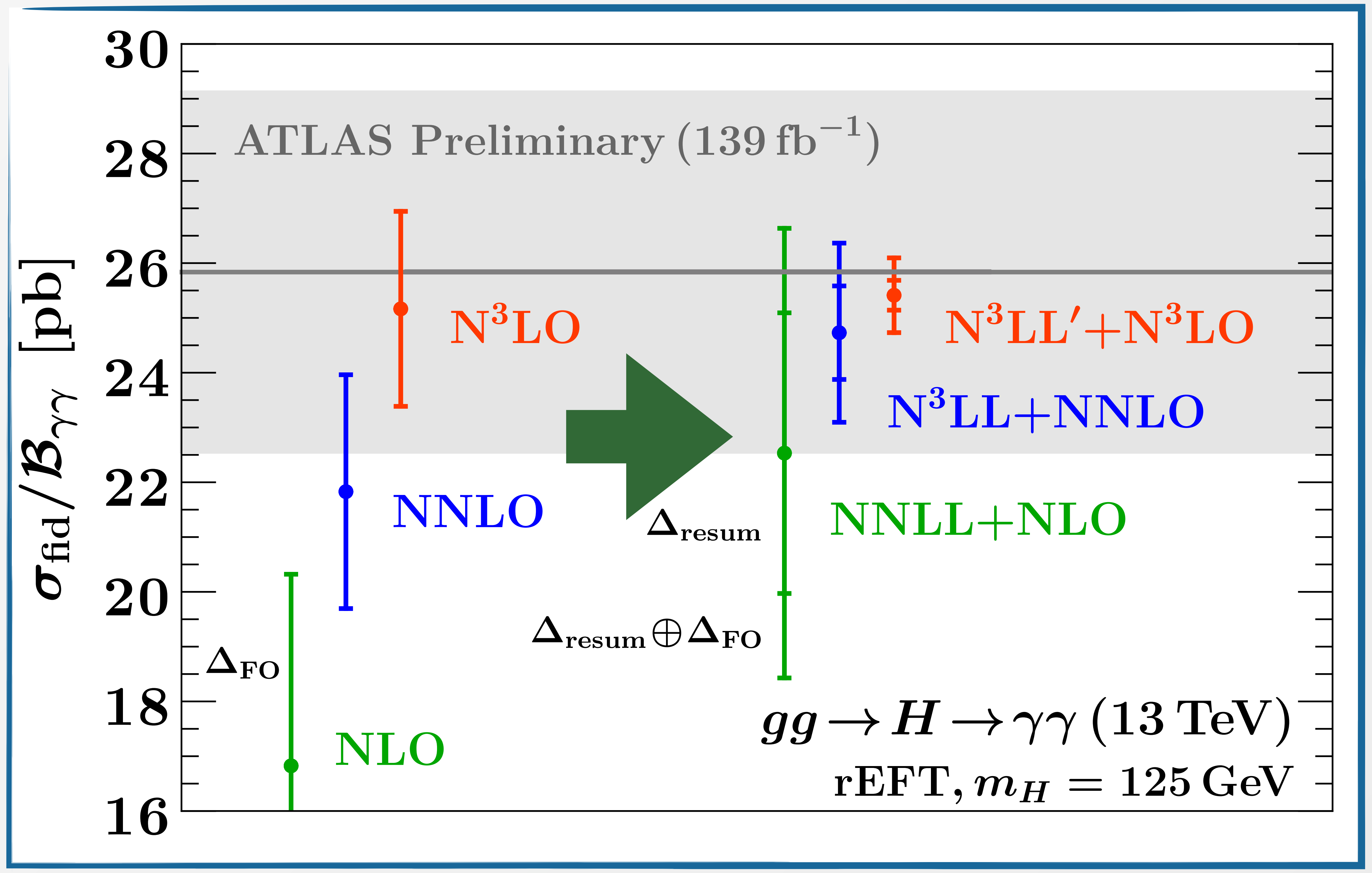

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Linear sensitivity of the acceptance at small q_T leads to a (alternating sign) **factorial growth**

Solution 1: Linear fiducial power corrections can be resummed at all orders in perturbation theory

Solution 2: Resorting to alternative definition of cuts can resolve the issue of linear fiducial power corrections altogether

[Salam, Slade '21]

Symmetric $p_T^{\ell_1}, p_T^{\ell_2} > p_T^{\rm cut}$

Asymmetric $\begin{cases} p_T^{\ell_1} > p_T^{\rm cut} + \Delta \\ p_T^{\ell_2} > p_T^{\rm cut} \end{cases}$

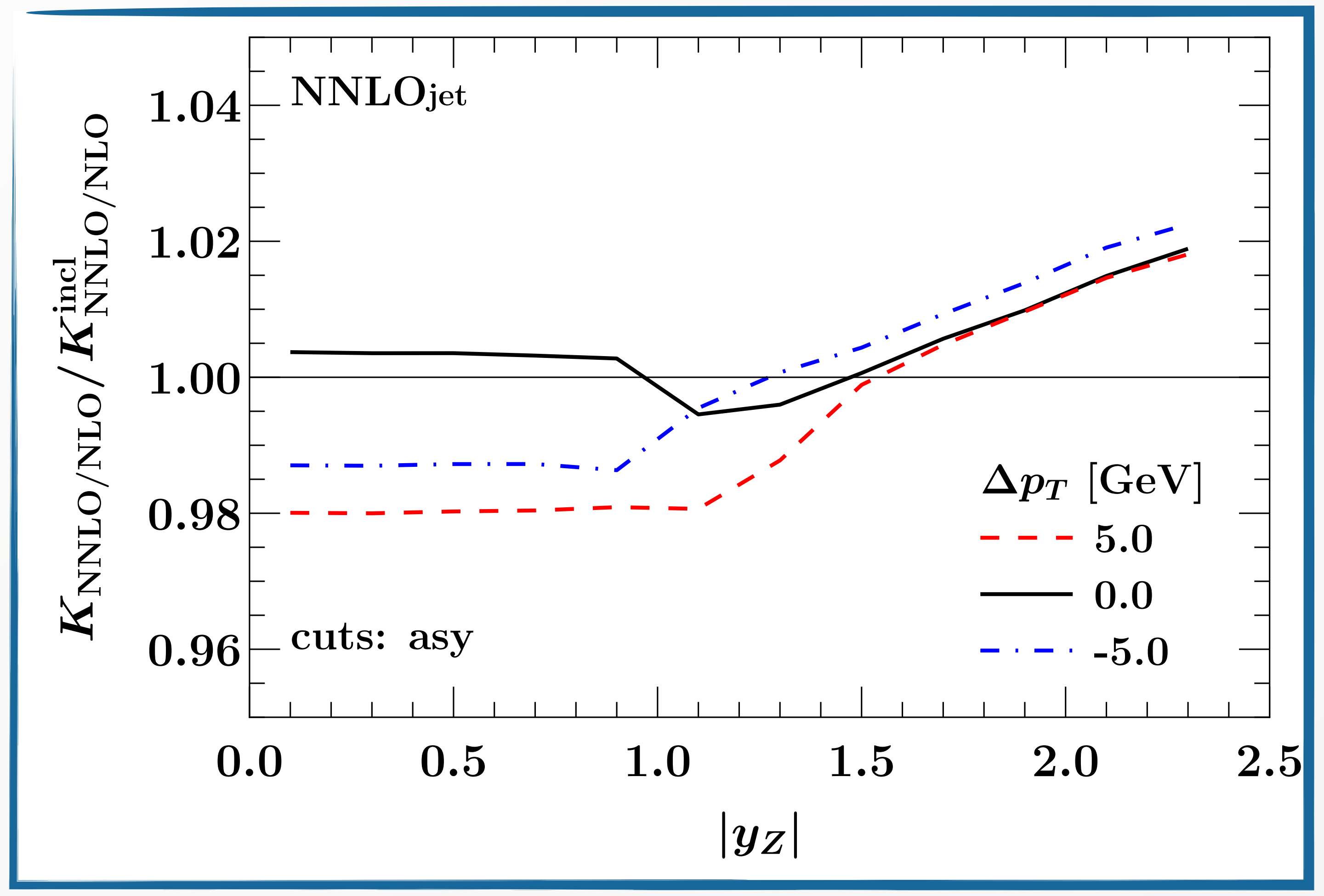
Product $\begin{cases} \sqrt{p_T^{\ell_1} p_T^{\ell_2}} > p_T^{\rm cut} + \Delta \\ p_T^{\ell_2} > p_T^{\rm cut} \end{cases}$

Linear power corrections and Drell-Yan@NNLO

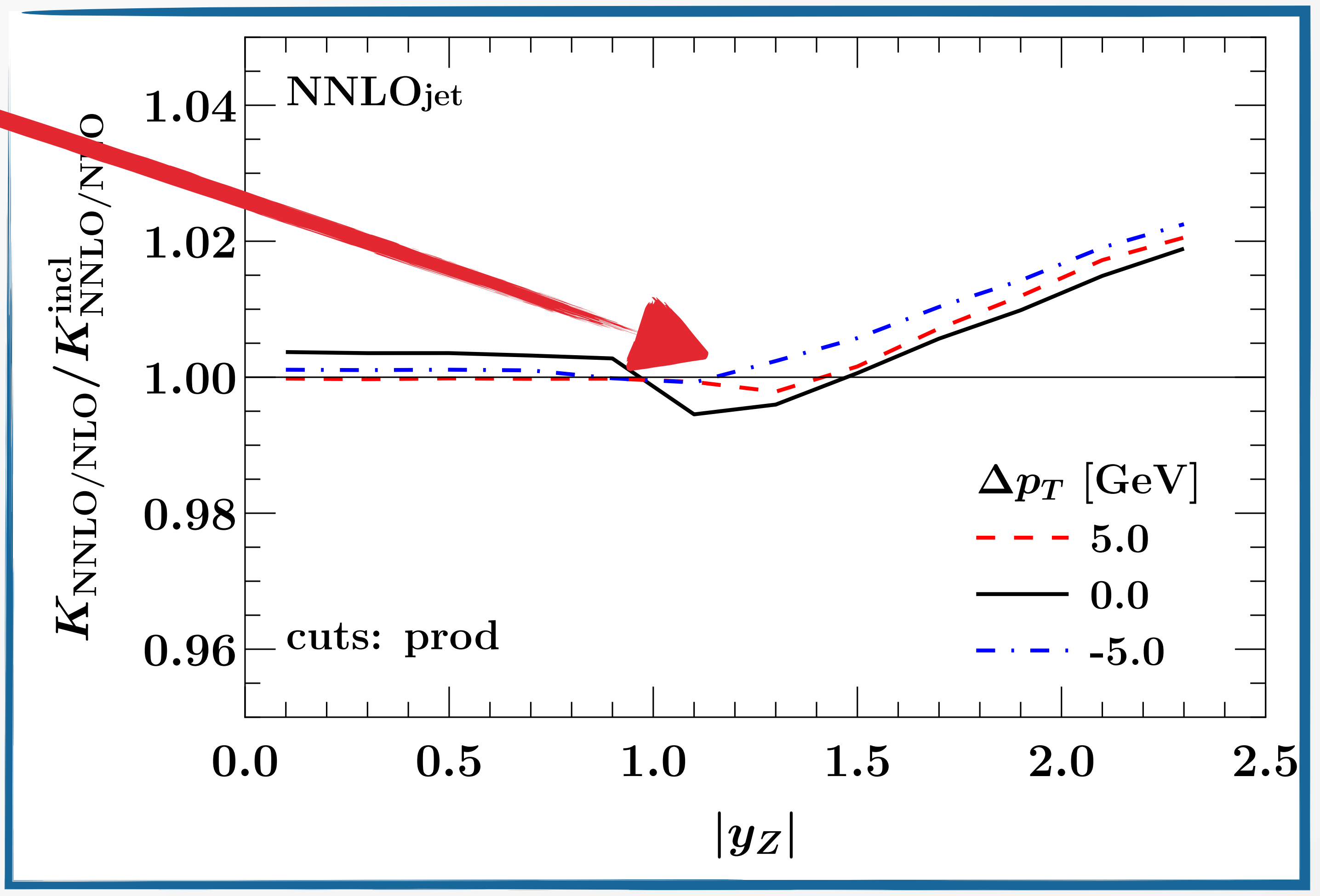
[Alekhin, …, LR et al 2405.19714]

(A)Symmetric cuts: impact on DY rapidity distribution

Double ratio of NNLO/NLO K-factors fiducial/inclusive cuts



Better and smoother behaviour of product cuts, although overall smaller effects with respect to the $H \to \gamma\gamma$ case



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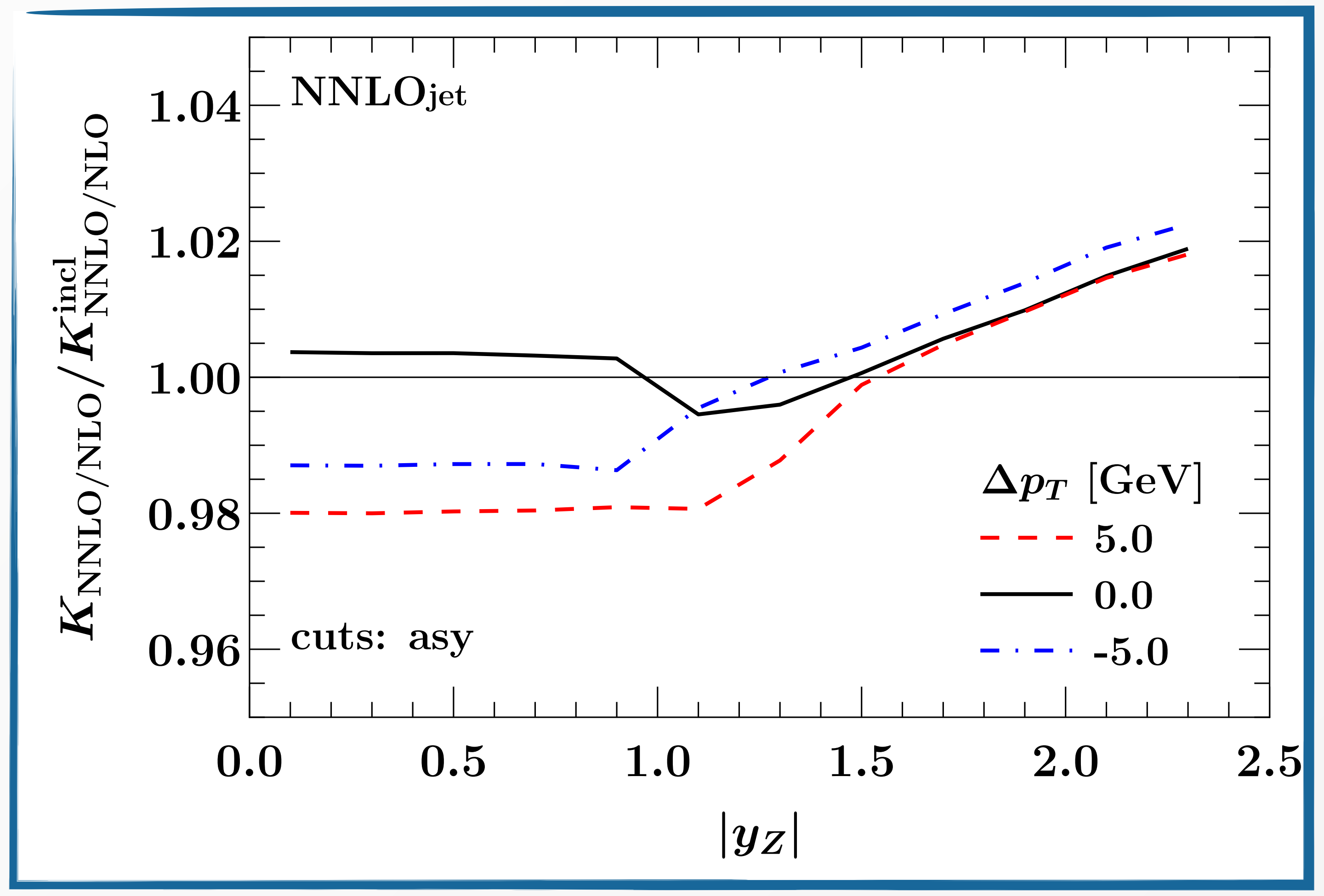
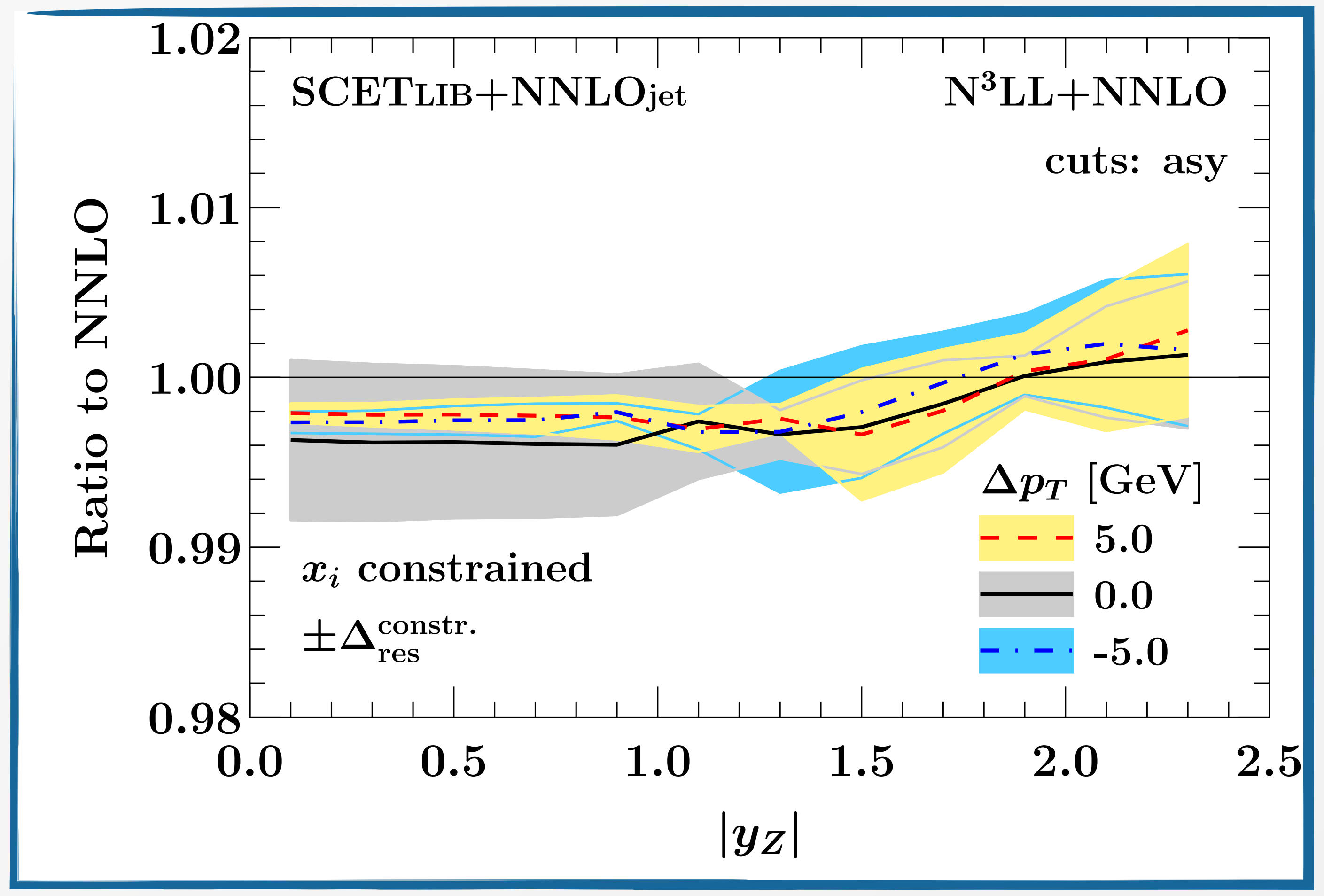
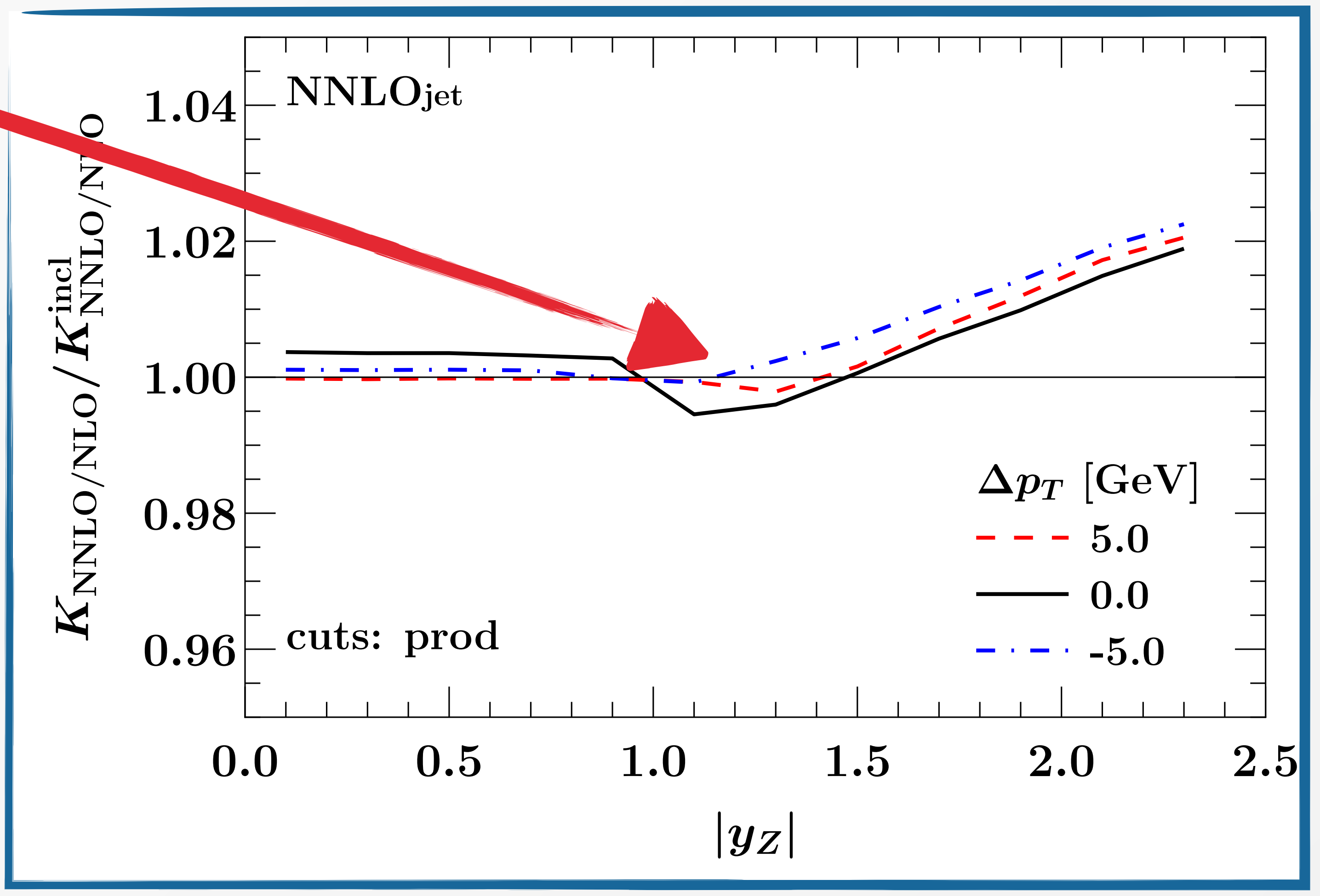
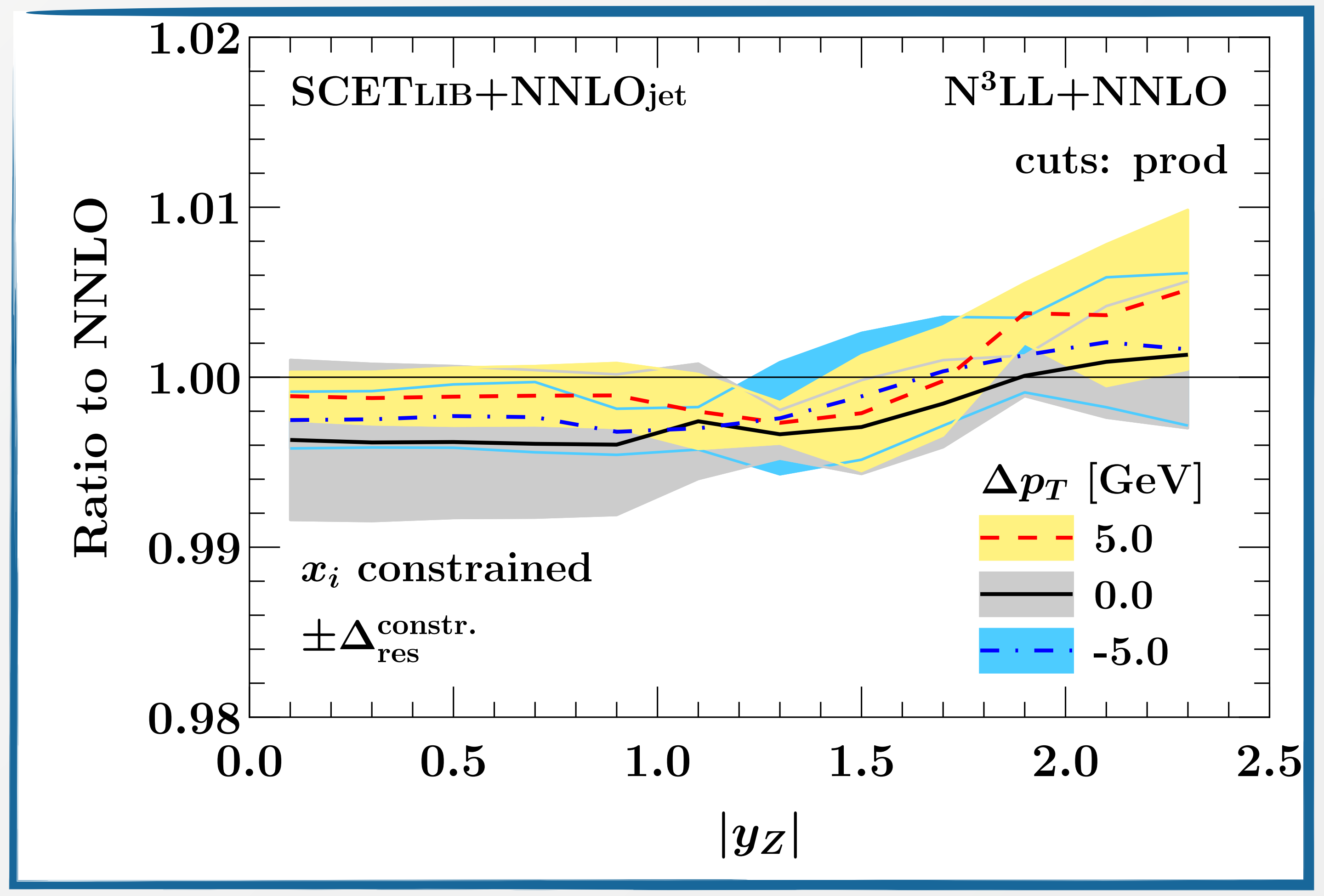








Impact of all-order resummation of linear power corrections studied at N³LL′ with SCETLIB and RadISH

Effects below 0.5% with respect to the NNLO prediction

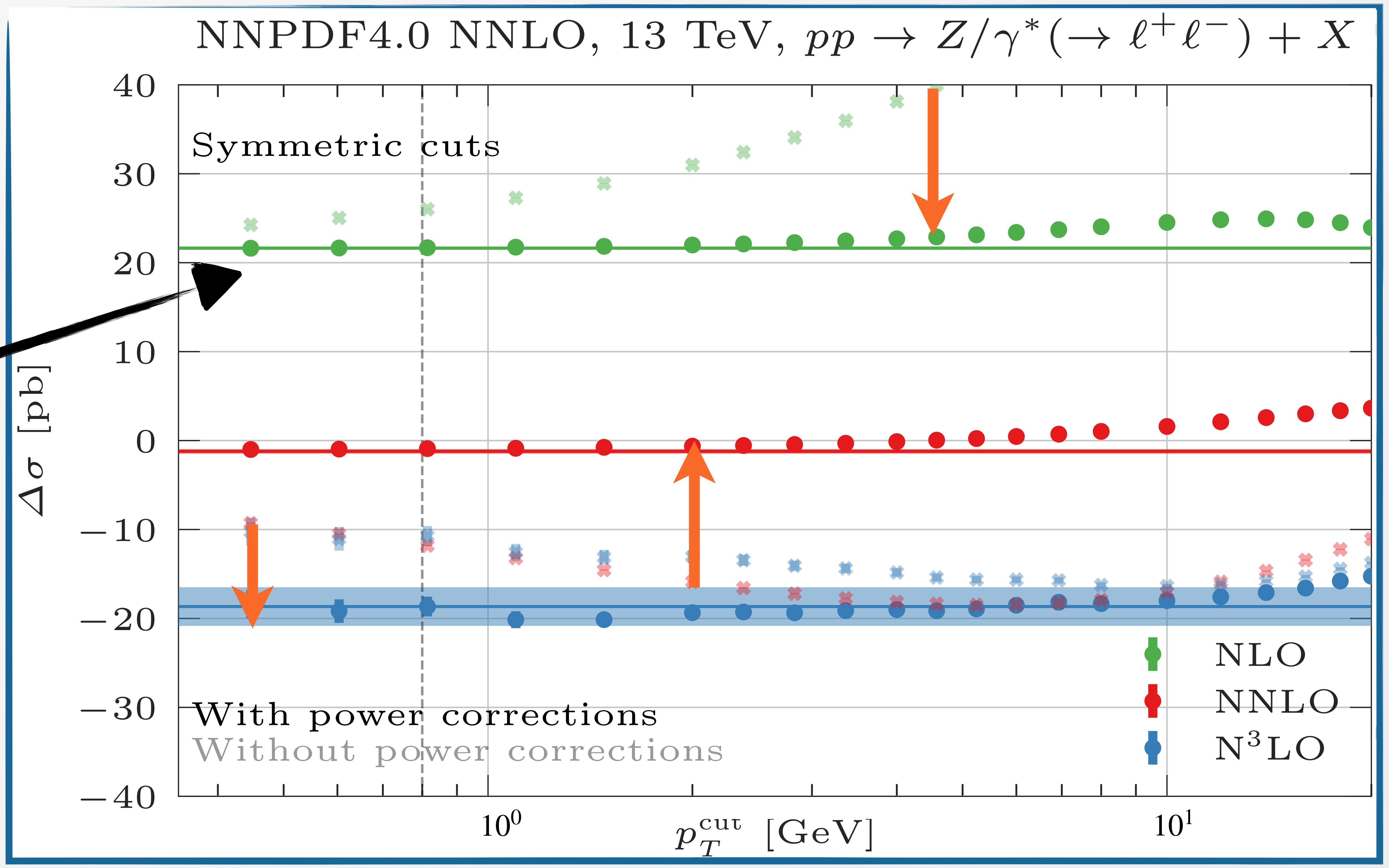
Impact of the choice of cuts **relatively minor**, although in perspective the underlying ambiguity may be a insurmountable issue with **legacy data**

Linear power corrections and Drell-Yan@N³LO

$$d\sigma_V^{\mathrm{N^3LO}} \equiv \mathscr{H}_V^{\mathrm{N^3LO}} \otimes d\sigma_V^{\mathrm{LO}} + \left(d\sigma_{V+\mathrm{jet}}^{\mathrm{NNLO}} - \left[d\sigma_V^{\mathrm{N^3LL}} \right]_{\mathcal{O}(\alpha_s^k)} \right) \Theta(p_T > p_T^{\mathrm{cut}}) + \mathcal{O}((p_T^{\mathrm{cut}}/M)^n)$$

ATLAS fiducial region $\quad p_T^{\ell^\pm} > 27\,\mathrm{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$

- When using symmetric cuts, mandatory to include missing linear **power corrections** to reach a **precise control of the N^{k}LO correction** down to small values of p_T^{cut}
- Plateau at small p_T^{cut} indicates the desired independence of the slicing parameter



[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

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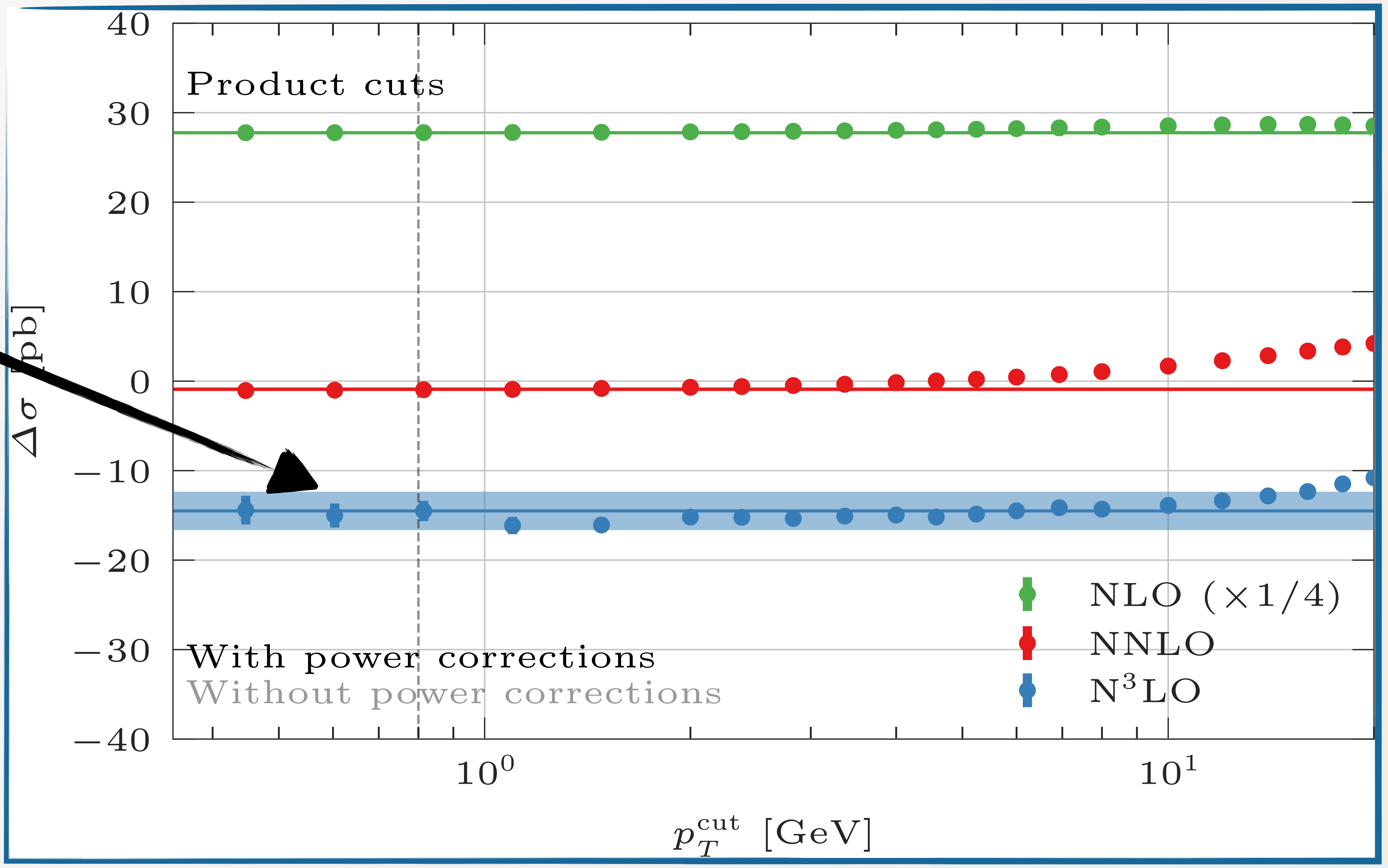
Product cuts
[Salam, Slade '21]

$$\sqrt{|\vec{p}_T^{\,\ell^+}|\,|\vec{p}_T^{\,\ell^-}|} > 27\,\mathrm{GeV} \qquad \min\{|\vec{p}_T^{\,\ell^\pm}|\} > 20\,\mathrm{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$$

- **Alternative set of cuts** which does not suffer from linear power corrections
- Improved convergence, result independent of the recoil procedure



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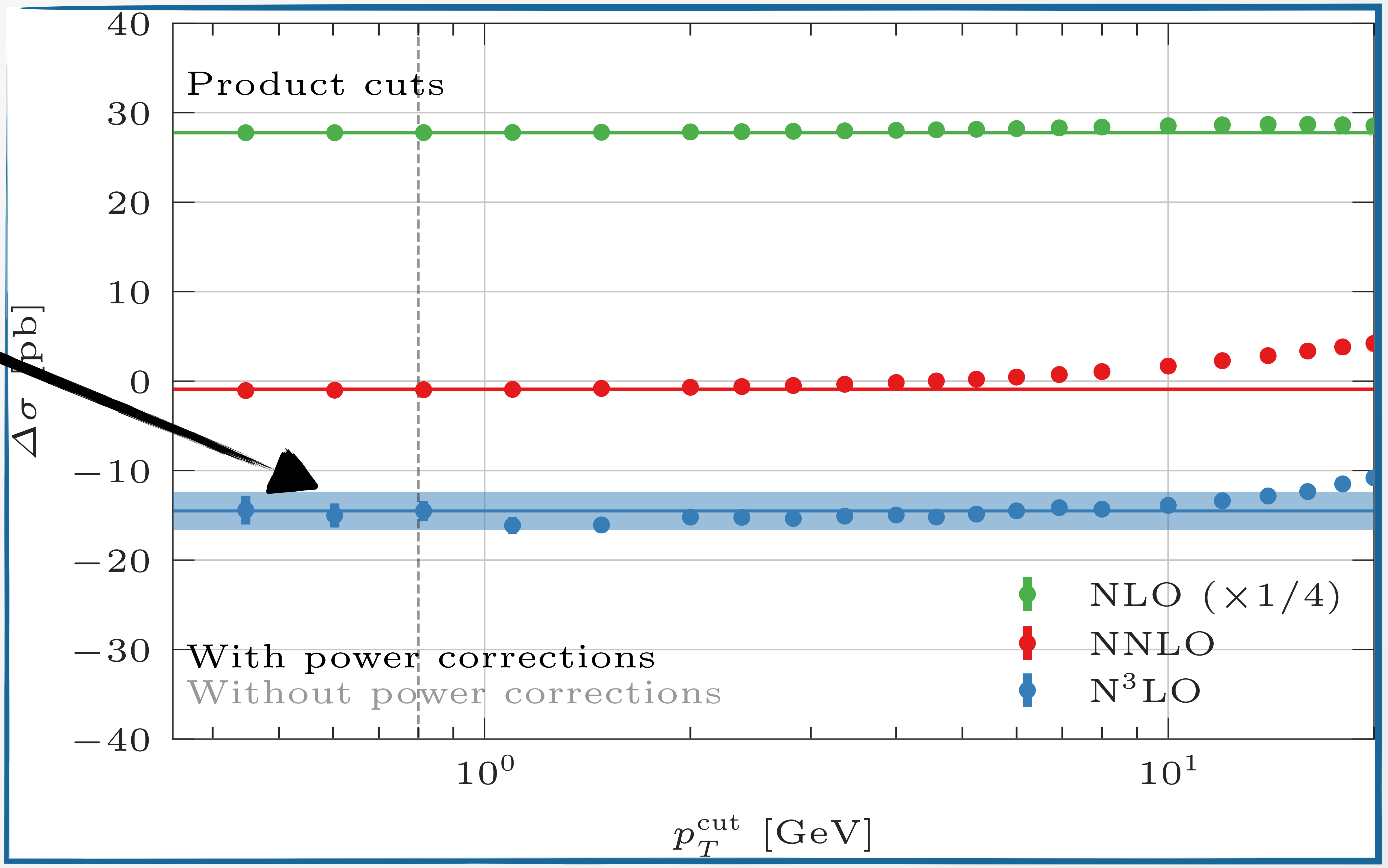
- **Alternative set of cuts** which does not suffer from linear power corrections
- Improved convergence, result independent of the recoil procedure
- Exquisite control on the **fixed order component** (from NNLOJET) allows to push to low values of the slicing parameter p_T^{cut}
- Computation extremely demanding computationally in the NNLO V+j component: $\mathcal{O}$(several M) CPU hours

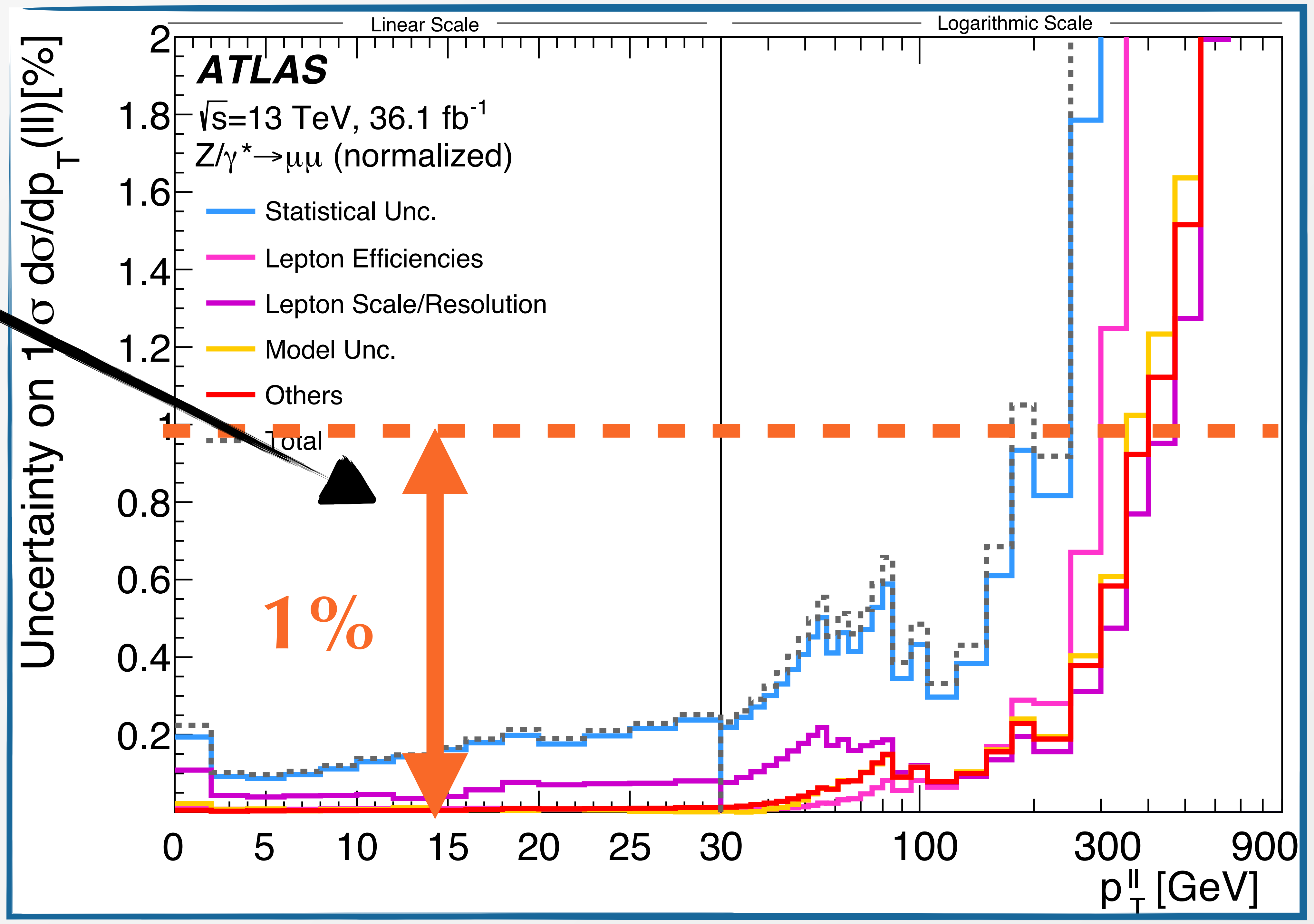


[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

Drell-Yan: transverse observables

Kinematic distributions which involve the production of a lepton pair in association with QCD radiation play a special role, as they are sensitive to accompanying hadronic activity **only through kinematic recoil**

Measurement of transverse and angular observables often lead to very small experimental uncertainties



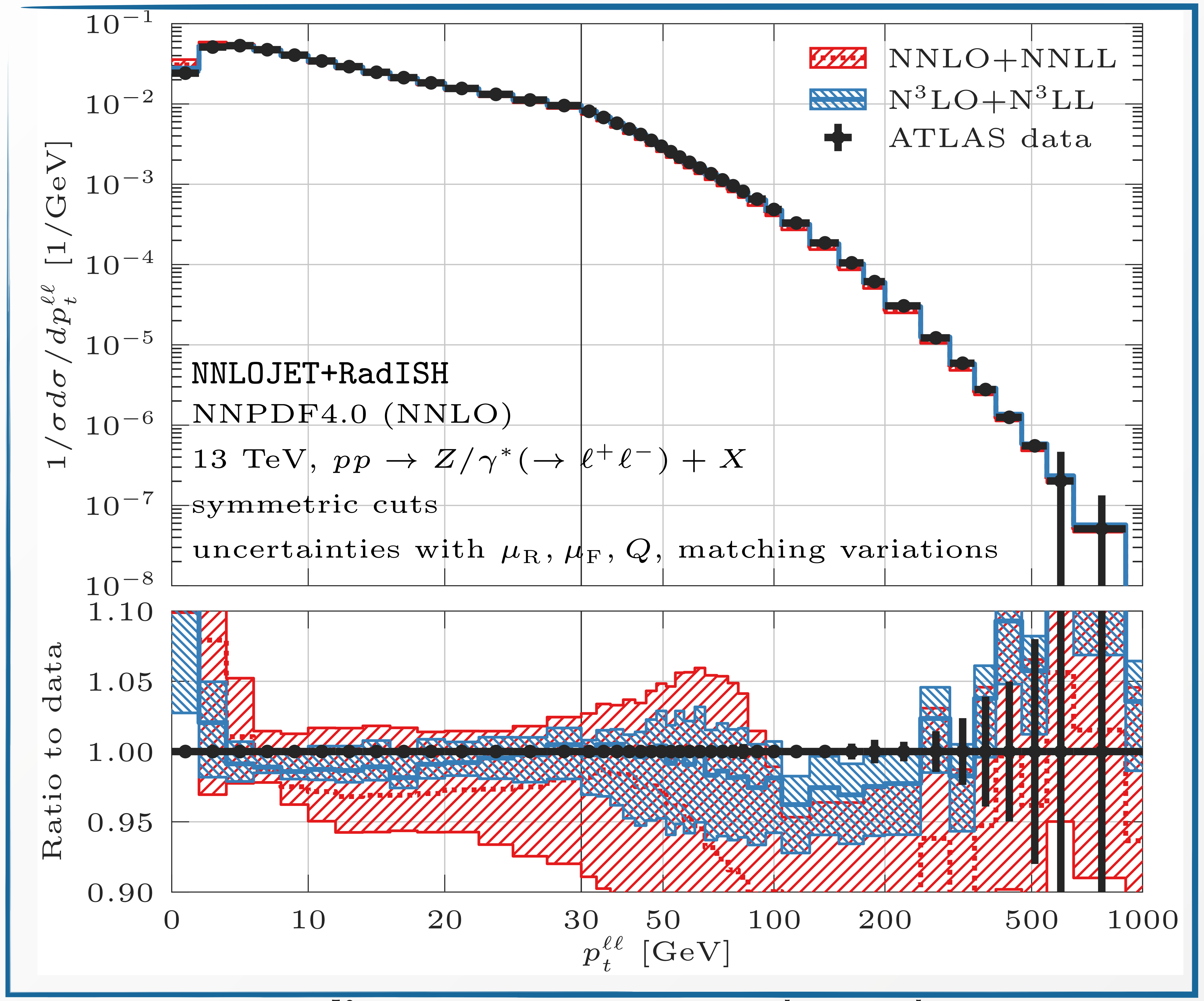
[ATLAS 2019]

Fixed-order perturbative description breaks in the $p_T \to 0$ limit, due to the appearance of large logarithms of $p_T/m_{\ell\ell}$, which must be resummed lest they spoil the perturbative convergence

Resummation for DY production performed within a variety of formalisms (**direct QCD in *b* or momentum space**, **SCET**, **TMD**) with high-logarithmic accuracy N^{3}LL′ ($\alpha_S^n \ln^{n-2} q_T/M$ and $\alpha_S^n \ln^{2n-6} q_T/M$). N^{4}LL ingredients partially available

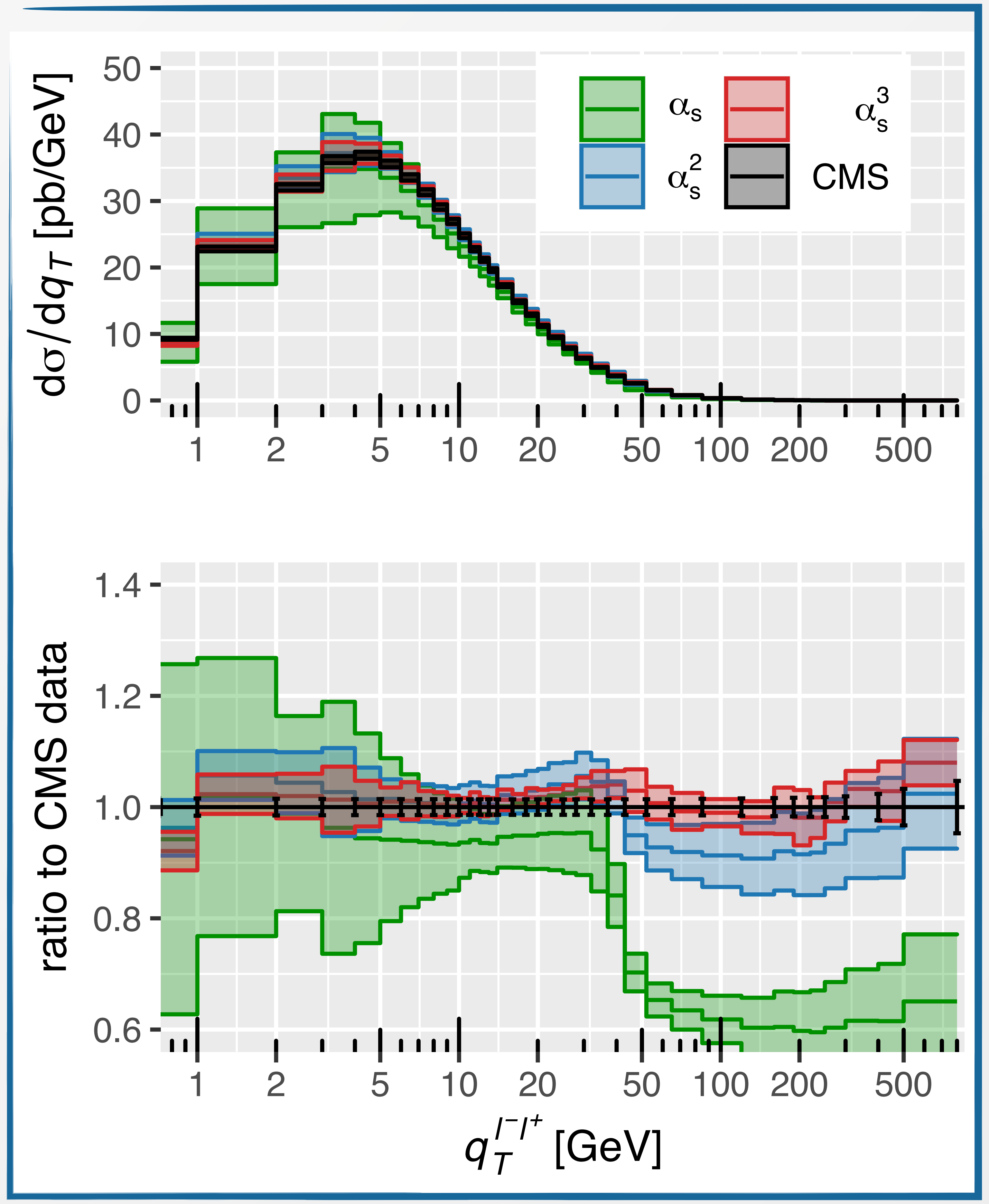
Drell-Yan: precise description of the transverse momentum spectra

State-of-the-art predictions achieve N^{3}LL′/aN^{4}LL+N^{3}LO accuracy



direct-space approach (RadISH)

[Chen, Gehrman, Glover, Huss, Monni, Re, LR, Torrielli 2022]



SCET formalism (Cute-MCFM)

[Neumann, Campbell 2022]

Excellent description of experimental data, with **residual scale uncertainties at the few % level**

W and Z production: understanding correlations

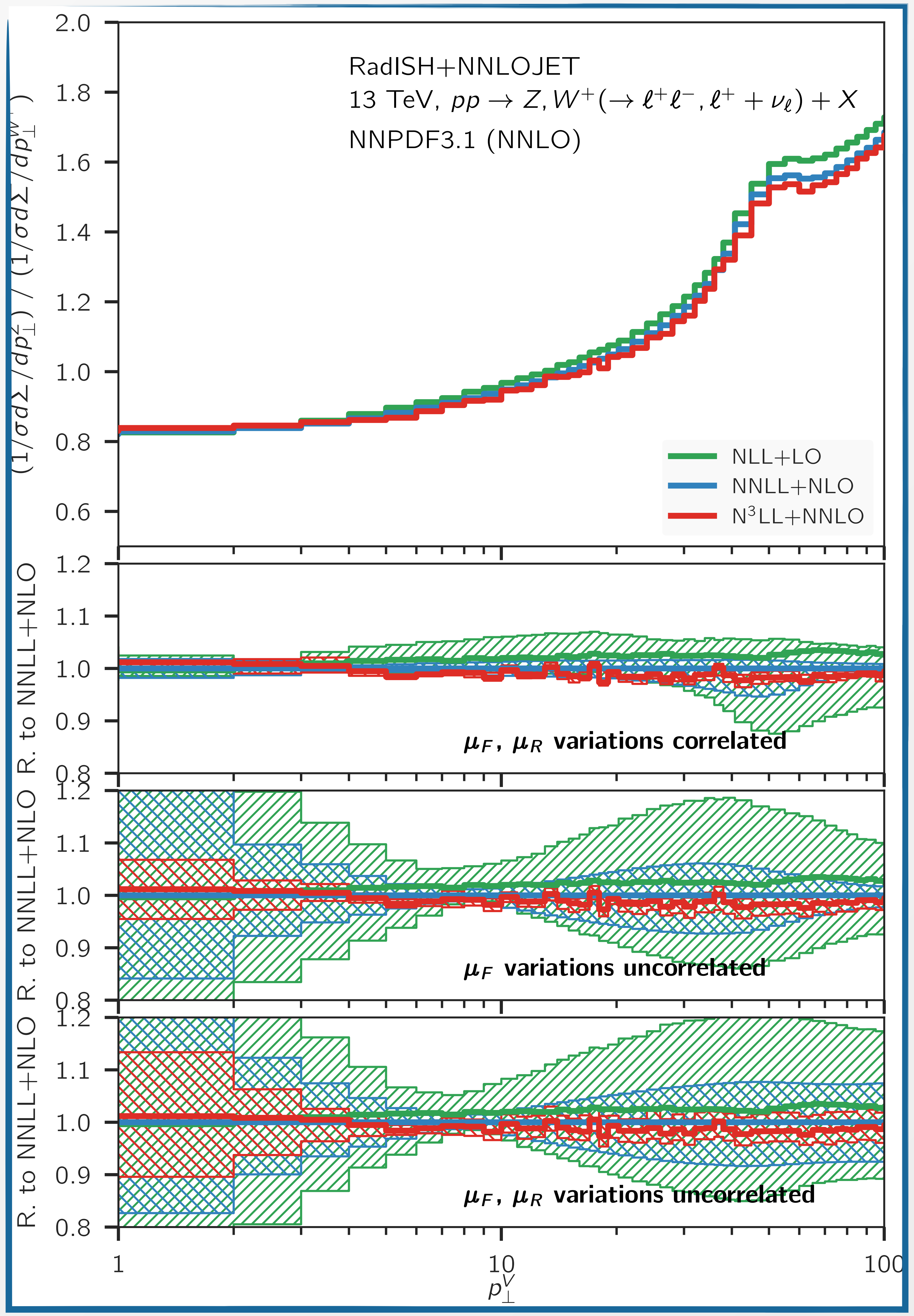
Precise data on q_T^Z spectrum can be employed in measurement of m_W only indirectly, by modelling the differences between Z and W production processes

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{q_T^W} \sim \frac{1}{\sigma^Z_{\rm data}}\frac{d\sigma^Z_{\rm data}}{q_T^Z}\,\frac{\frac{1}{\sigma^W_{\rm theory}}\frac{d\sigma^W_{\rm theory}}{q_T^W}}{\frac{1}{\sigma^Z_{\rm theory}}\frac{d\sigma^Z_{\rm theory}}{q_\perp^Z}}$$

e.g. m_W determination by ATLAS

Z and W production share a similar pattern of QCD radiative corrections, but a **precise understanding of the correlation** between the two processes is crucial to propagate consistently the information

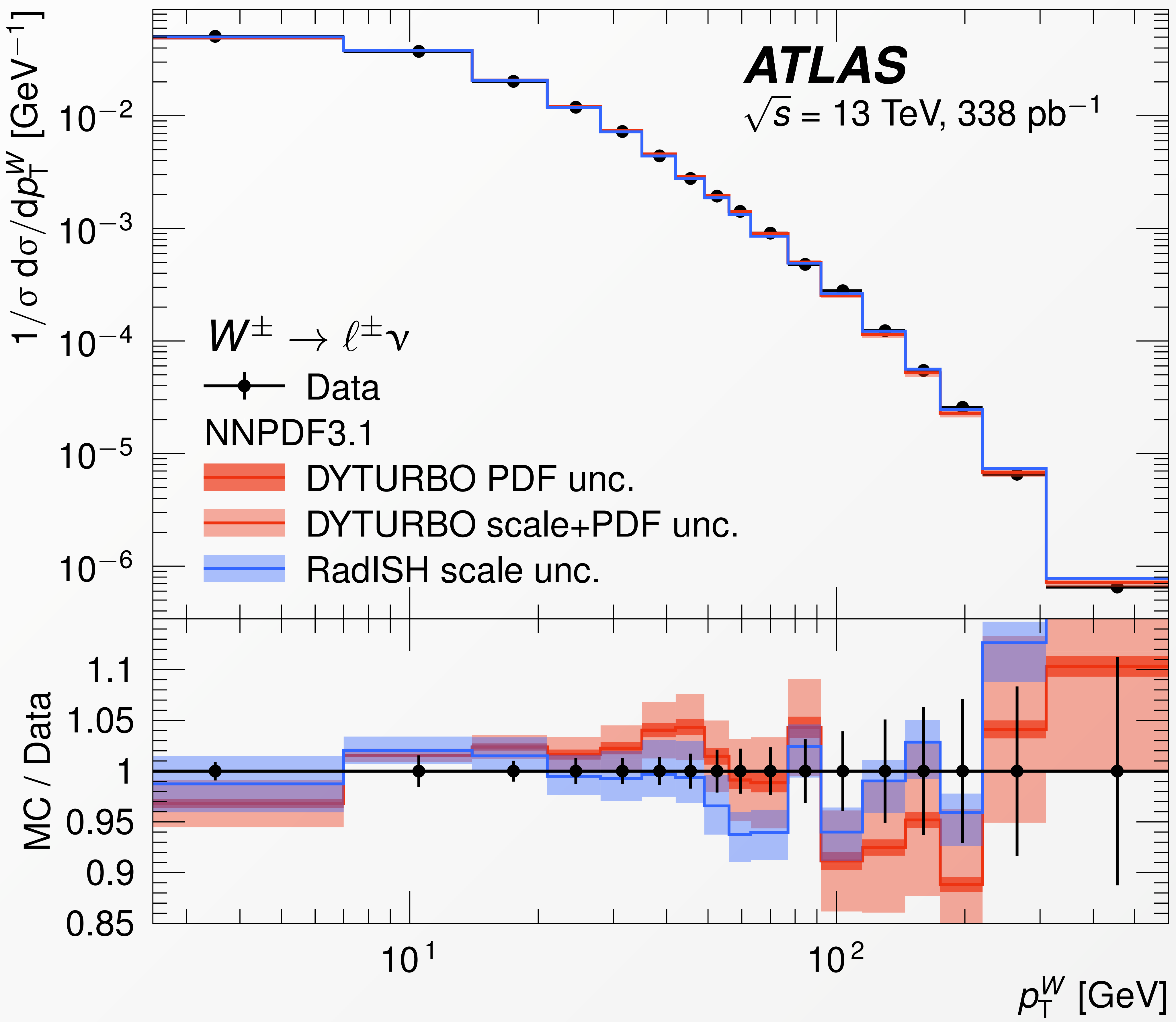
Alternative uncertainty estimate: each resummation order only depends on a few semi-universal parameters: treat them as **theory nuisance parameters** [Tackmann, 2411.18606]



[Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker '19]

Transverse momentum in W production

Direct measurement of W transverse momentum would provide a direct way to test W/Z modelling and reduce the related uncertainties in a measurement of m_W



Low-pileup runs in recent ATLAS measurement show remarkable agreement with N^{3}LL+N^{3}LO (RadISH+NNLOJET) and NNLL+NNLO (DYTURBO) predictions

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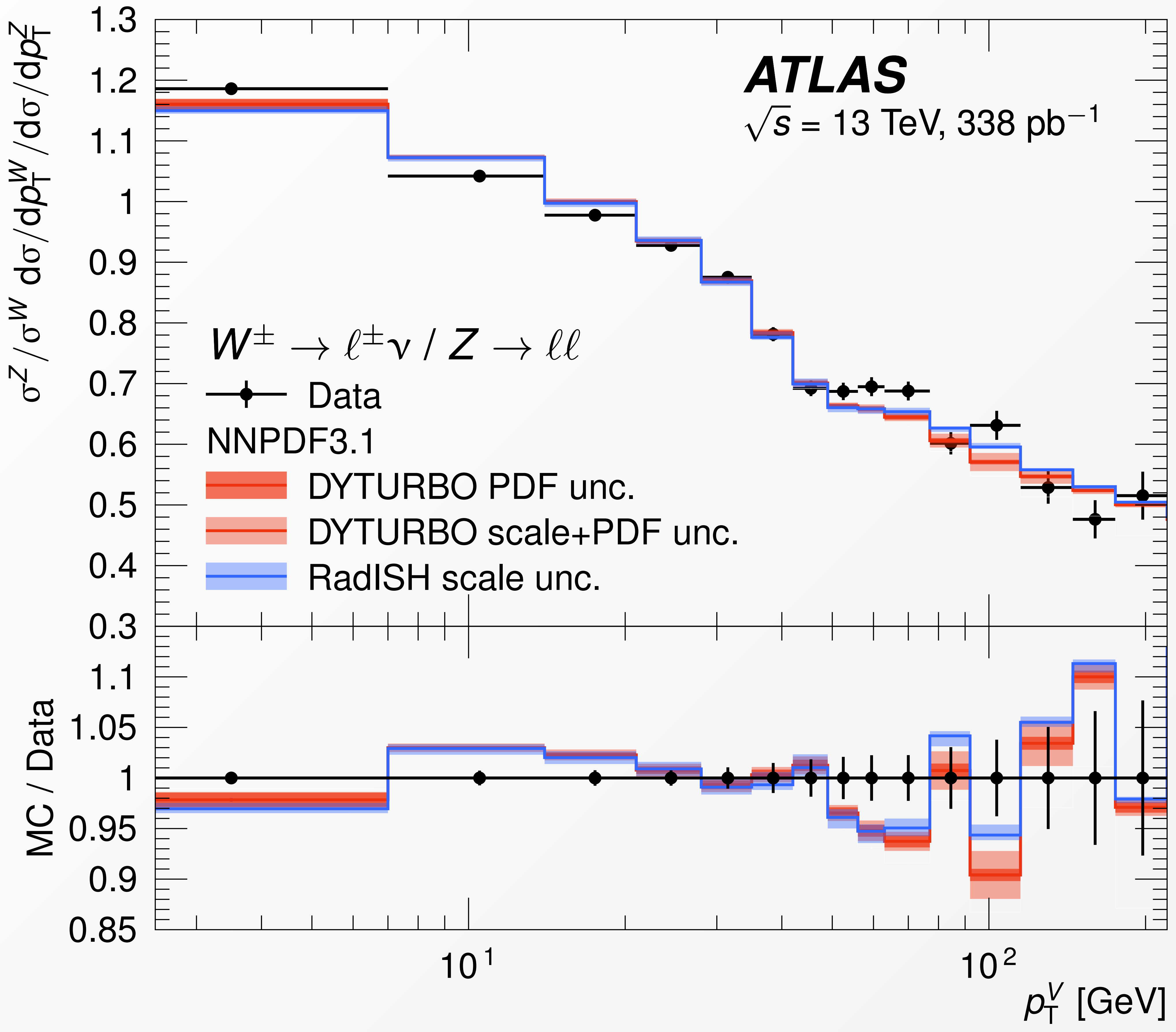

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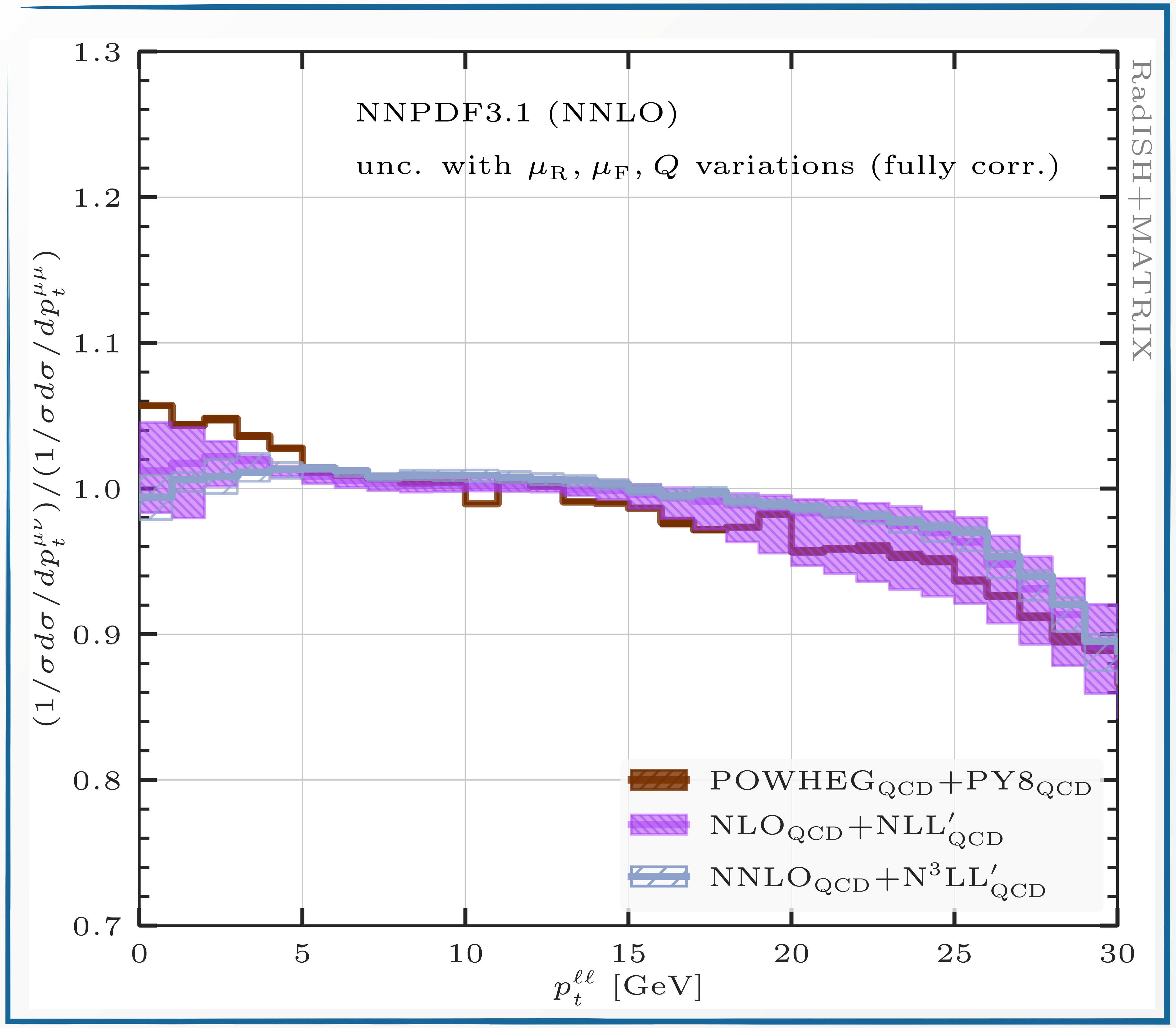

[Buonocore, LR, Torrielli '24]

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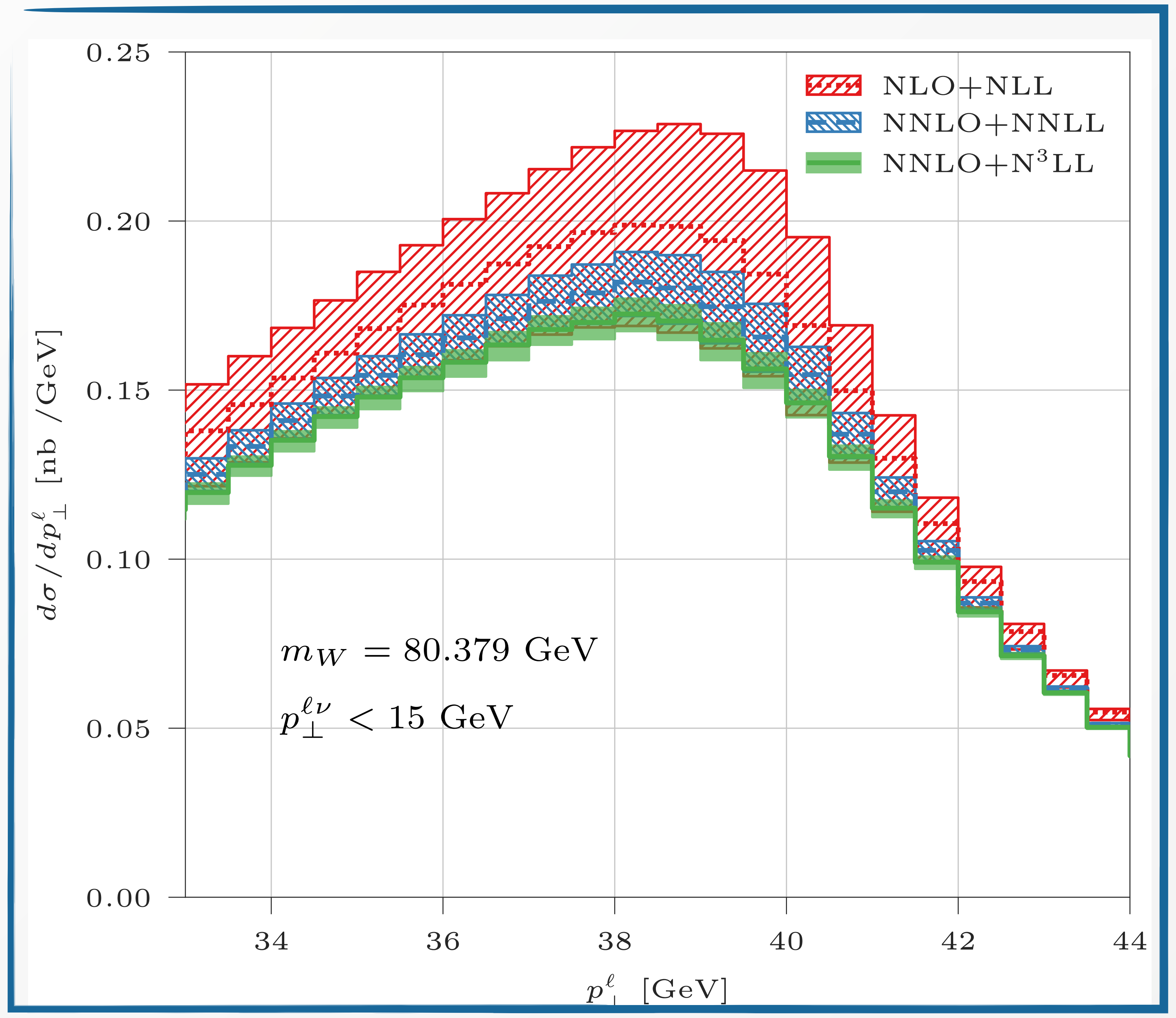
Tuned MC predictions (POWHEG+PY8) display the same level of discrepancy and are relatively insensitive to choice of tune, intrinsic k_T, MPI and hadronisation effects

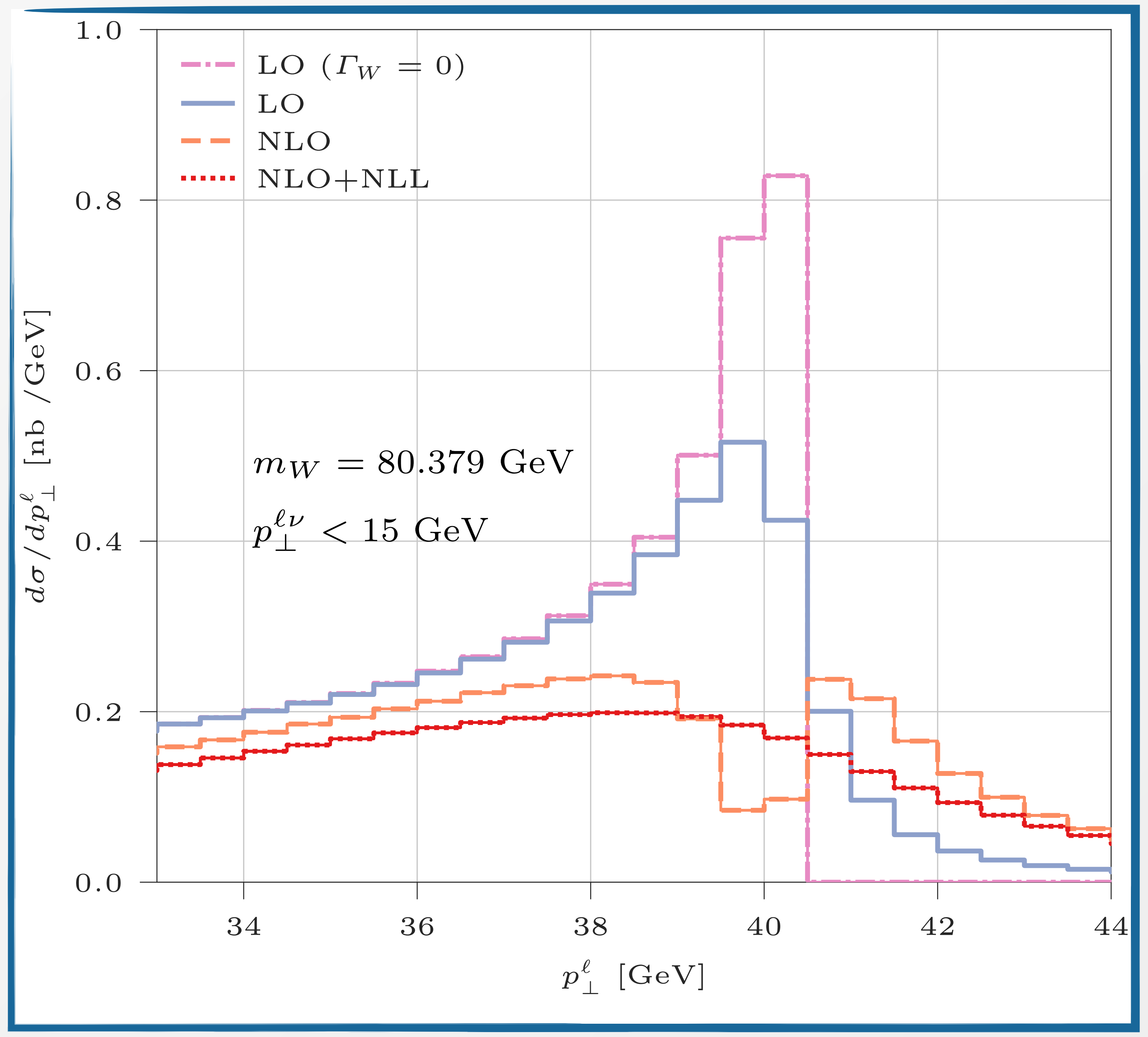
Hints towards a perturbative origin of this discrepancy

Fiducial distributions and transverse momentum resummation

- Transverse momentum resummation affects observables sensitive to **soft gluon emission** as the lepton transverse momentum in Drell-Yan
  [Balázs, Yuan '97] [Catani, de Florian, Ferrera, Grazzini '15]
- Leptonic transverse momentum is a particularly relevant observable due to its importance in the **extraction of the *W* mass**
- Inclusion of resummation effects necessary to cure (integrable) divergences due to the presence of a **Sudakov shoulder** at $m_{\ell\ell}/2$
  [Catani, Webber '97]



[LR, P. Torrielli, A. Vicini '23]



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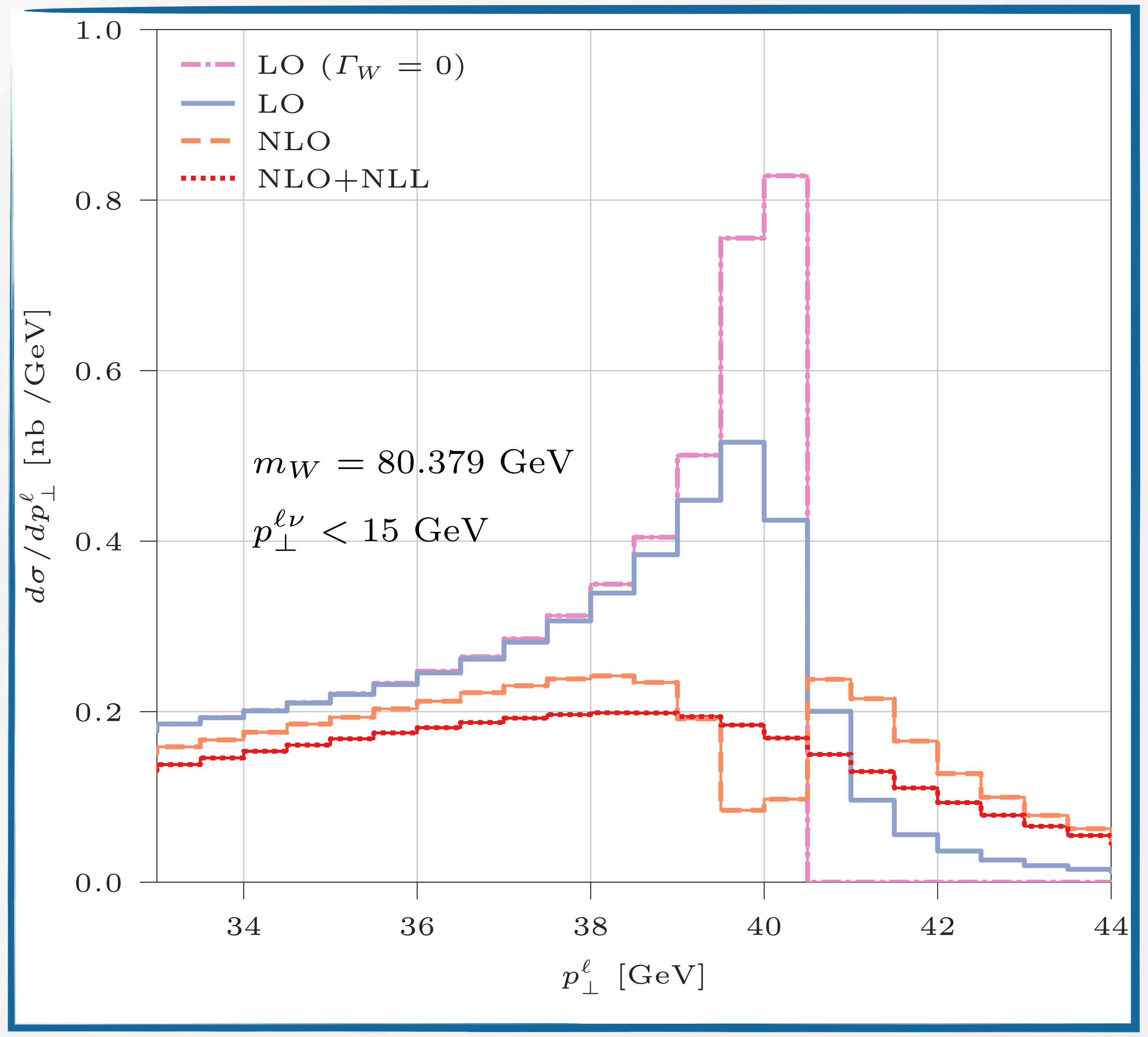
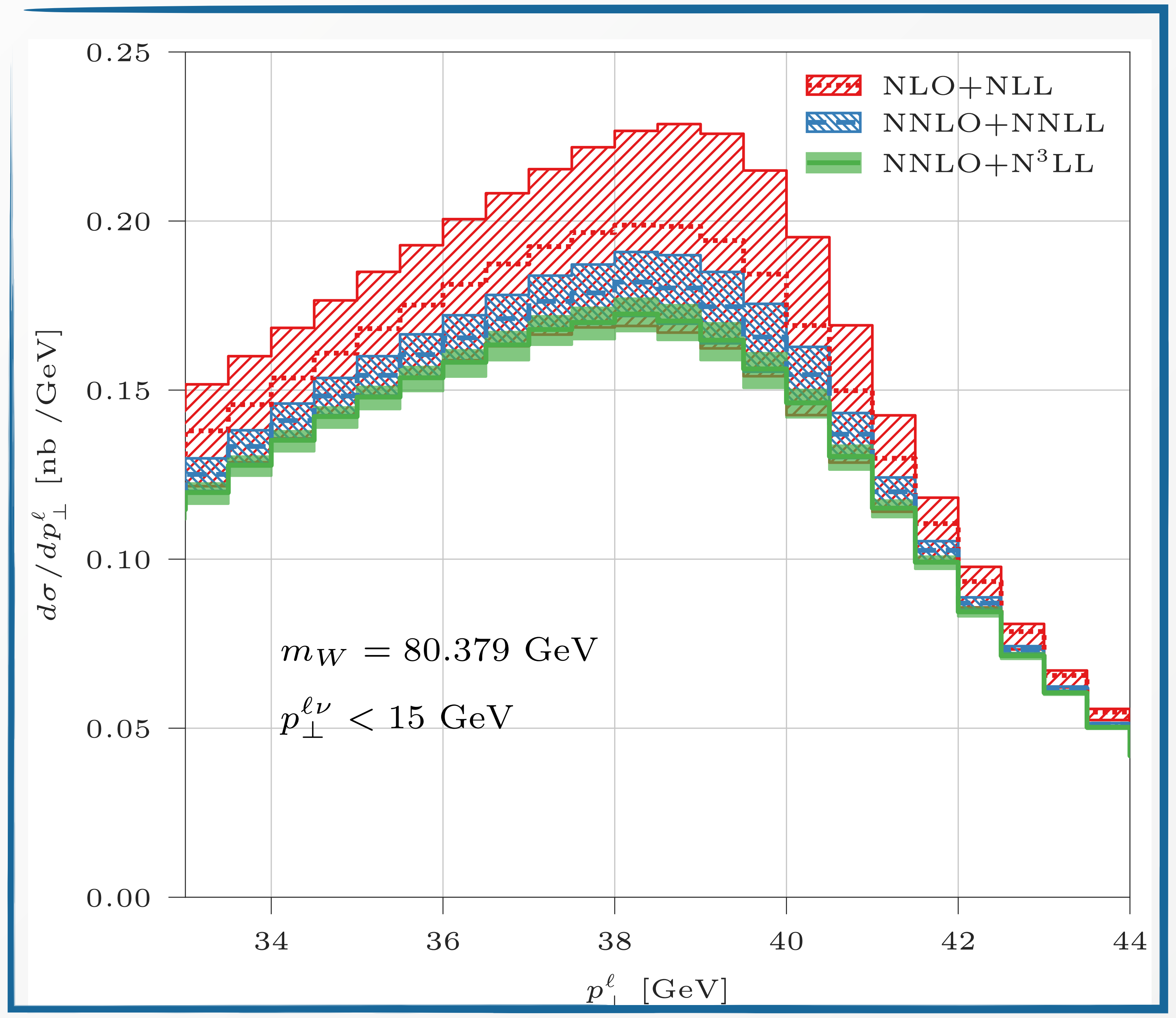




[LR, P. Torrielli, A. Vicini '23]

EW corrections become relevant for correct shape

Drell-Yan: NLO EW

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \hat{\sigma}_{ab}^{(1,0)} + \hat{\sigma}_{ab}^{(2,0)} + \hat{\sigma}_{ab}^{(3,0)} + \ldots$$
$$+ \hat{\sigma}_{ab}^{(0,1)} + \ldots$$
$$+ \hat{\sigma}_{ab}^{(1,1)} + \ldots$$

- **NLO EW** corrections

known since long time ago
[S. Dittmaier and M. Kramer (2002)], [Baur,Wackeroth (2004)], [Baur, Brein, Hollik, Schappacher, Wackeroth (2002)], [Zykunov (2006,2007)]

automatised and readily available in different generators
[Les Houches 2017, 1803.07977]

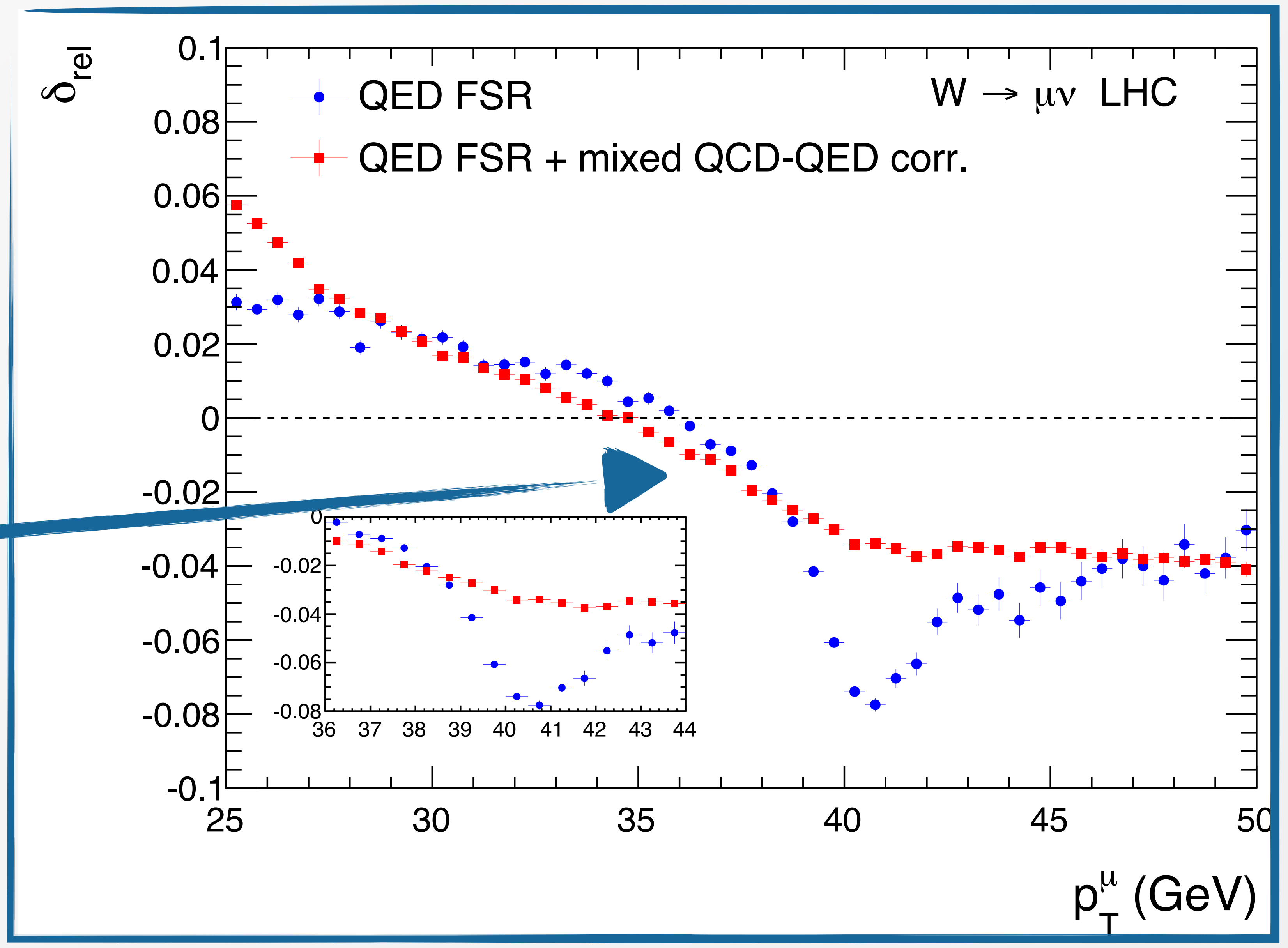
Impact of QED corrections and interplay with QCD

Both p_T^ℓ and m_T features large radiative corrections due to **QED final state radiation** at the **jacobian peak**

The precise shape of p_T^ℓ at the Jacobian peak determined by the **interplay of QCD and QED corrections**

Data/Theory comparisons made at the level of **pure QCD models**

Large FSR QED removed relying on MC modelling (PHOTOS) **[Barberio, van Eijk ,Was '91][Golonka, Was, '06]**



[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

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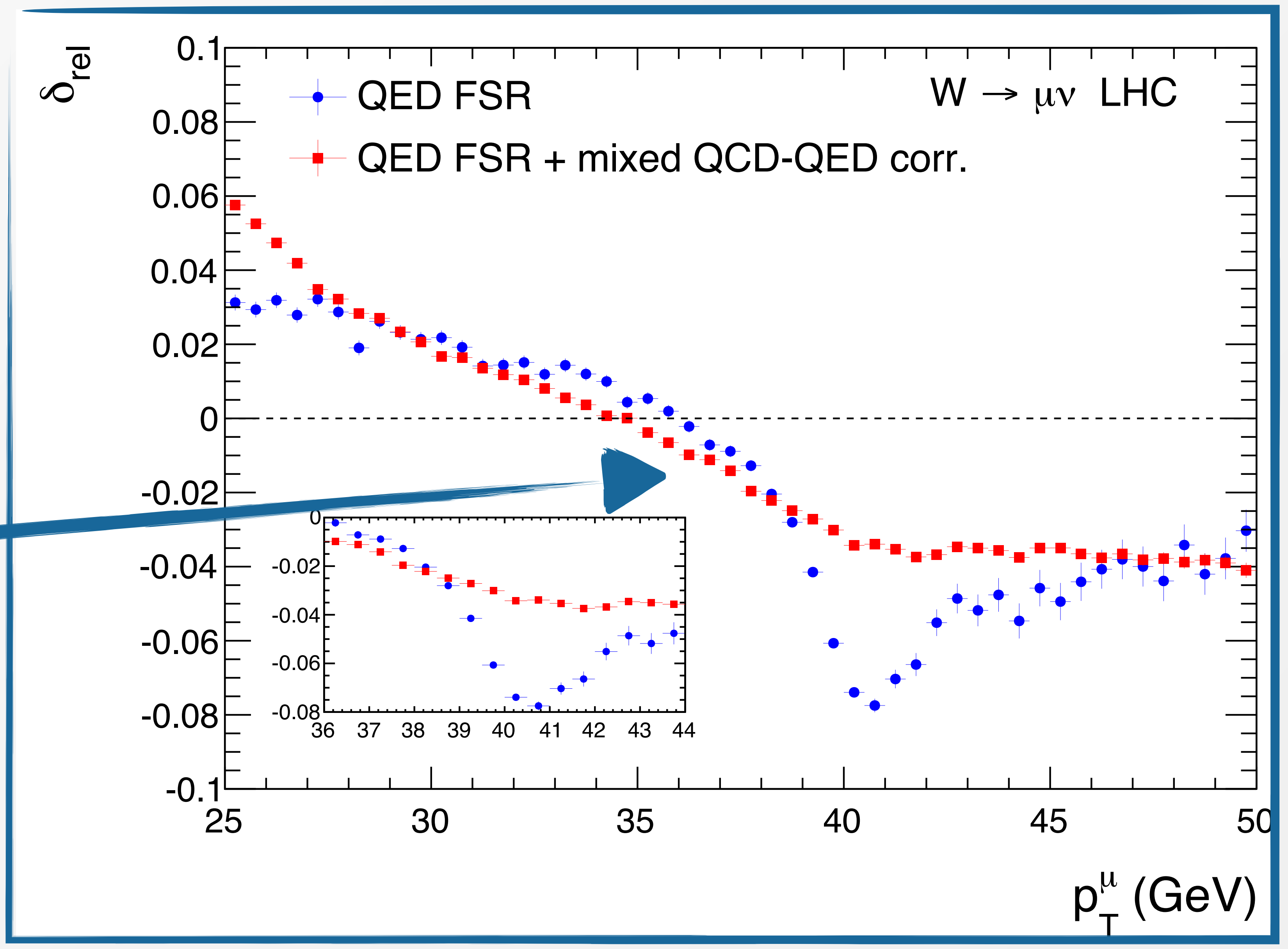

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Data/Theory comparisons made at the level of **pure QCD models**

Large FSR QED removed relying on MC modelling (PHOTOS) [Barberio, van Eijk ,Was '91][Golonka, Was, '06]

Overall good description of the main QED effects; however, impact of full EW effects and the interplay with QCD corrections not transparent (assumption of **complete factorization**)

Mixed QCD×EW corrections

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \ldots$$

$$+\hat{\sigma}^{(0,1)}_{ab} + \ldots$$

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Neutral current DY NNLO QCDxEW

[Bonciani, Buonocore, Grazzini, Kallweit,Rana, Tramontano, Vicini '21] [Armadillo, Bonciani, Devoto, Rana, Vicini '22] [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile '22][Armadillo, Bonciani, Buonocore, Devoto, Grazzini, Kallweit, Rana, Vicini '24]

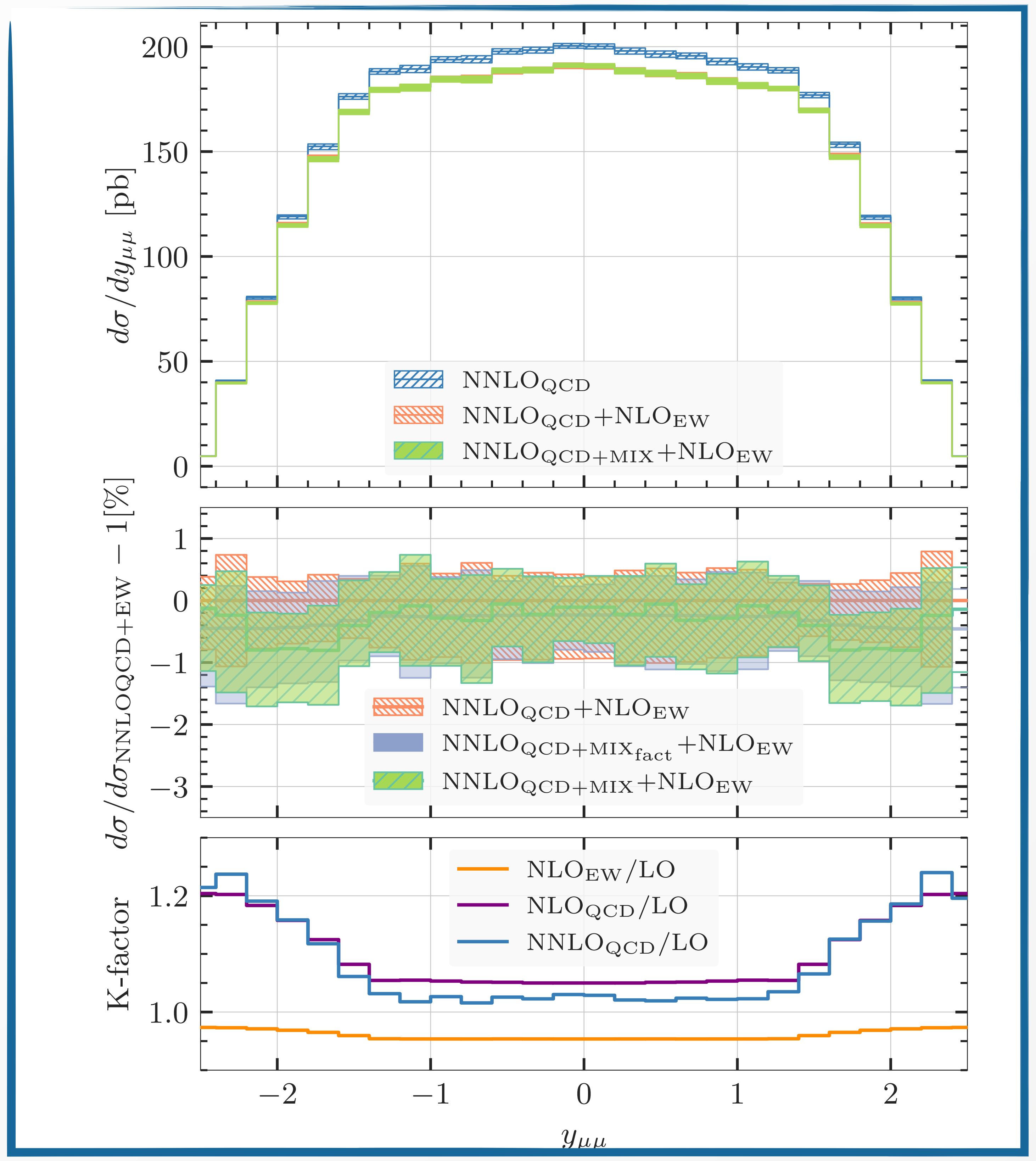
Charged-current DY (2-loop amplitude QCDxEW)

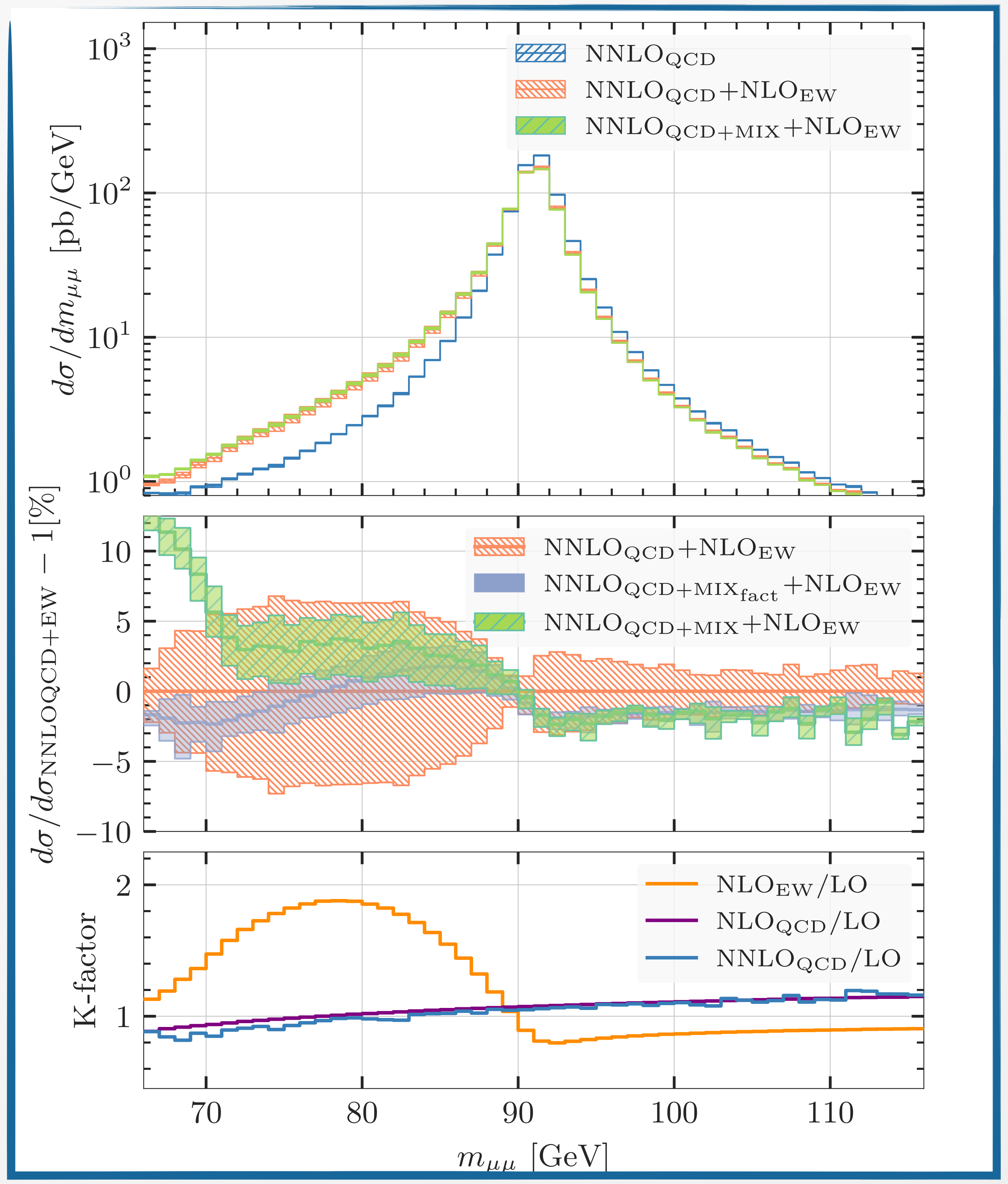
[Armadillo, Bonciani, Devoto, Rana, Vicini '24]

+ Results in pole approx

[Dittmaier, Huss, and Schwinn (2014,2015)] [Dittmaier, Huss, and Schwarz (2024)]

Mixed QCD×EW corrections: results at NNLO





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Small but non-trivial distortion of the rapidity distribution (**absent in the naive factorised approximation**); impact on PDF fits

Factorised approximation fails below the Z resonance; impact on $\sin^2\theta_{\text{eff}}^{\ell\ell}$ extraction

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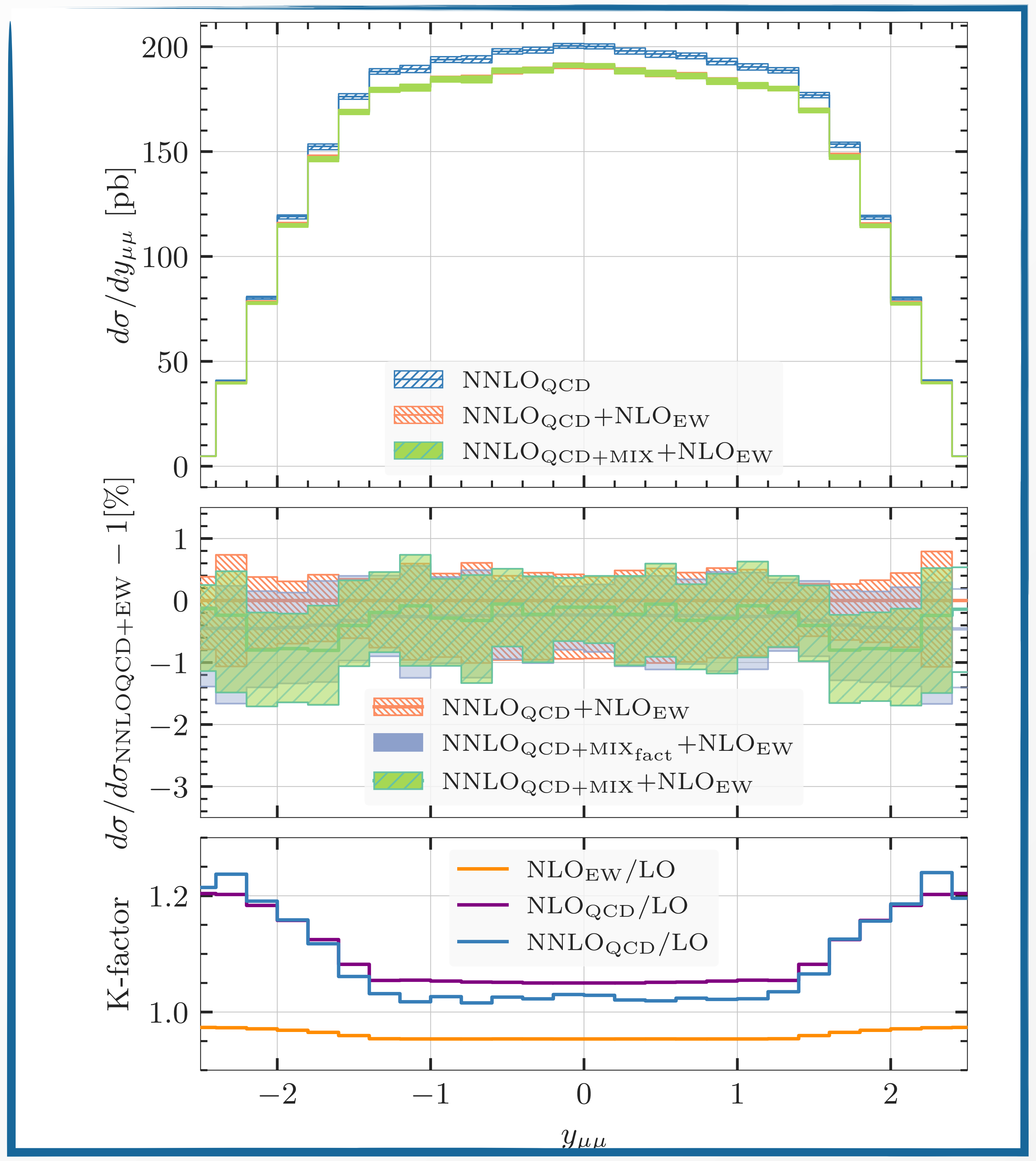

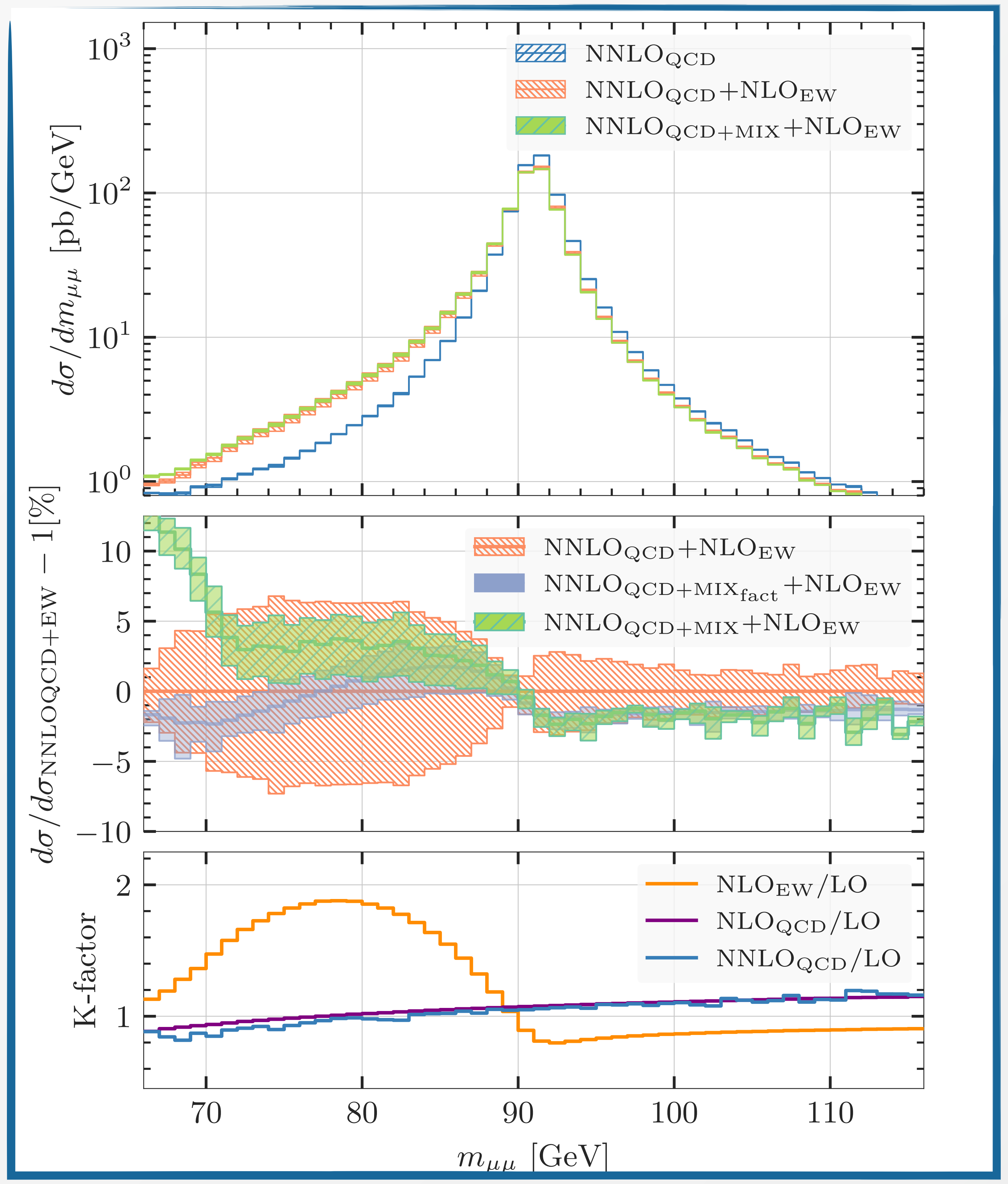

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How about recoil-sensitive observables?

Intermezzo: direct space approach to transverse momentum resummation

Direct-space resummation in the RadISH formalism is based on a physical picture in which hard particles incoming to a primary scattering coherently radiate an ensemble of soft and collinear partons

[Monni, Re, Torrielli 2016, Bizon, Monni, Re, LR, Torrielli 2017]

$$\frac{d\sigma^{(sing)}(q_T)}{d\Phi_B} = \int dq_T \frac{d\sigma^{\rm sing}}{dq_T d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \mathcal{L}(k_{t1}) e^{-R(k_{t1})} \mathcal{F}(q_T, \Phi_B, k_{t1})$$

$$\mathcal{L}(k_{t1}) = \sum_{c\bar{c}} |\mathcal{M}_B|^2_{c\bar{c}} \sum_{i,j} [C_{ci} \otimes f_i(k_{t1})](x_1)[C_{\bar{c}j} \otimes f_j(k_{t1})](x_2) H$$

Collinear and **hard** functions

$$R(k_{t1}) = \int_{k_{t1}}^{m_{\ell\ell}} \frac{dq}{q} [A(\alpha_s(q)) \ln \frac{m_{\ell\ell}}{q} + B(\alpha_s(q^2))]$$

Universal **Sudakov radiator:** exponentiation of soft-collinear emissions, accounts for the tower of $\alpha_s^m \alpha^n ln^{m+n} q_T/M$ terms

Logarithmic accuracy defined in terms of $L = \ln(k_{t,1}/m_{\ell\ell})$

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Goal: combine higher-order QCD resummation with the resummation of leading EW and mixed QCD-EW effects for *bare* muons

Result: **flexible "analytic" resummation tool**, including **matching to available fixed-order results**

Resummation: combined radiator at NLL

QCD radiator

$$R_{\text{NLL}}^{\text{QCD}}(k_{t1}) = \int_{k_{t1}}^{m_{\ell\ell}} \frac{dq}{q}[A(\alpha_s(q^2))\ln\frac{m_{\ell\ell}}{q} + B(\alpha_s(q^2))]$$

QED radiator encoding ISR can be obtained by abelianisation of the previous result

$$R_{\text{NLL}}^{\text{QED}}(k_{t1}) = \int_{k_{t1}}^{m_{\ell\ell}} \frac{dq}{q}[A'(\alpha_s(q^2))\ln\frac{m_{\ell\ell}}{q} + B'(\alpha_s(q^2))]$$

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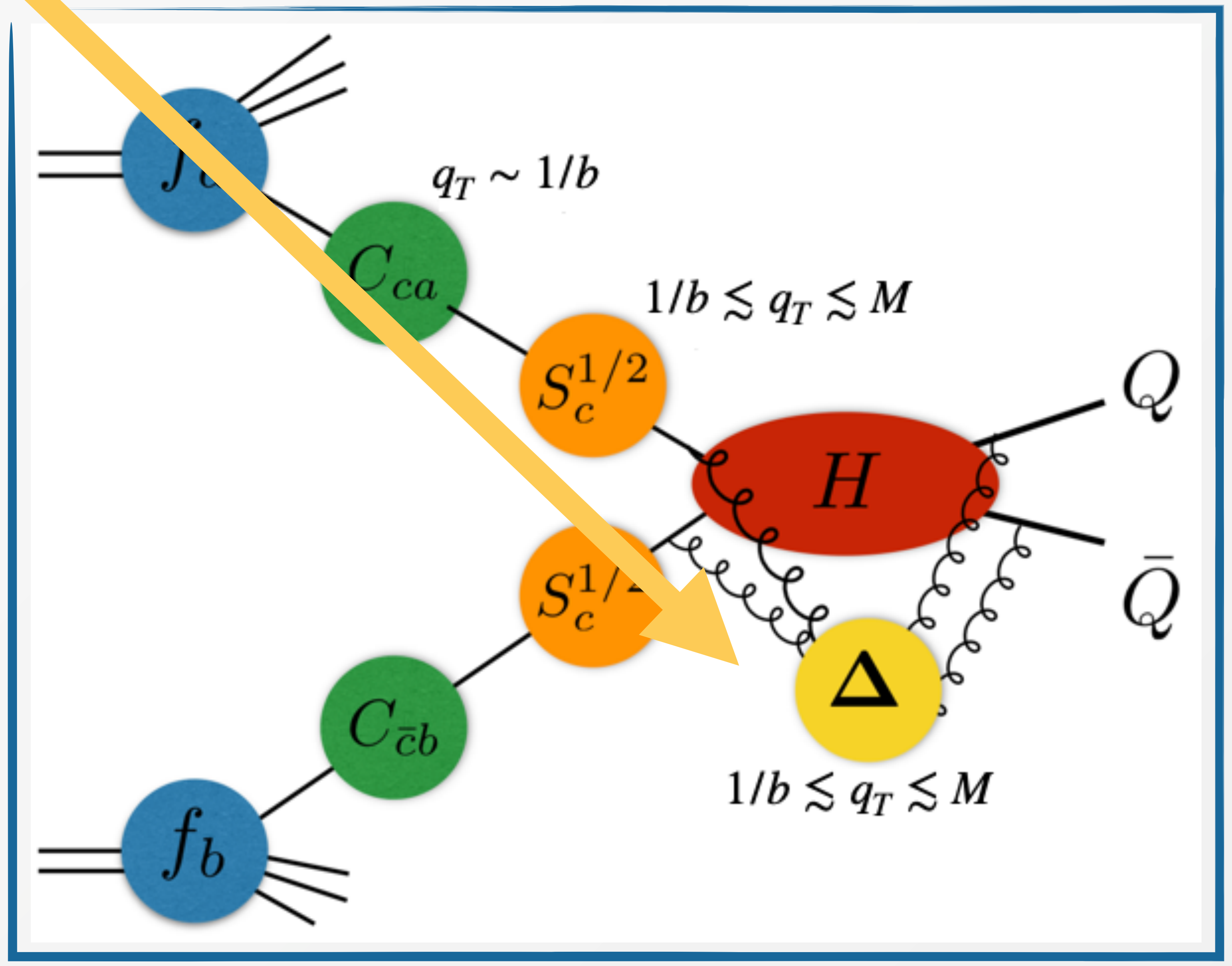
In presence of **charged, massive final state particles** (e.g. $pp \to \mu^+\mu^-, pp \to \mu^+\nu_\mu$) one needs to take into account the effect of additional QED **soft wide-angle radiation** (analogue to resummation for heavy quark pairs) [Catani, Grazzini, Torre '14]



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$$= \int_{k_{t1}}^{m_{\ell\ell}} \frac{dq}{q}[A'(\alpha_s(q^2))\ln\frac{m_{\ell\ell}}{q} + \tilde{B}'(\alpha_s(q^2))]$$

Resummation: combined radiator at NLL

$$R_{\rm NLL}^{\rm QED}(k_{t1}) = \int_{k_{t1}}^{m_{\ell\ell}} \frac{dq}{q}[A'(\alpha_s(q^2))\ln\frac{m_{\ell\ell}}{q} + B'(\alpha_s(q^2))]$$

Finally, to construct the combined QCD and QED radiator, one has to include the mixed QCD-QED contributions to the running of the QCD and QED couplings

$$\frac{d\ln\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s(\mu^2)) \longrightarrow \frac{d\ln\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s(\mu^2), \alpha(\mu^2))$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2)) \longrightarrow \frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2), \alpha_s(\mu^2))$$

Resummation: combined radiator at NLL

Final form of the combined QCD and QED radiator at NLL (see also [Cieri, Ferrera, Sborlini, 2018][Autieri, Cieri, Ferrera, Sborlini, 2023])

$$R_{\mathrm{NLL}}^{\mathrm{QCD+QED}}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) - Lg_1'(\alpha L) - g_2'(\alpha L) - g_{1,1}(\alpha_s L, \alpha L) - g_{1,1}'(\alpha L, \alpha_s L)$$

At this accuracy, the same form can be used in direct space formulation

$$\sigma(q_T) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R^{\mathrm{QCD+QED}}(v_1)} R'\left(v_1\right) d\mathcal{Z} \Theta\left(q_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

$$d\mathcal{Z} = \epsilon^{R'(k_{t,1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right)$$

Formula now describes an ensemble of gluons and photons recoiling against a colourless (possibly charged) system

Resummation: direct-space formulation at NLL

Final formula at NLL, fully differential on Born variables, now including also the effect of hard-collinear radiation

$$\frac{d\sigma(q_T)}{d\Phi_B} = \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L (e^{-R(v_1)} \mathscr{L}_{\mathrm{NLL}}(v_1)) R'\left(v_1\right) d\mathscr{Z} \Theta\left(q_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

$$\mathscr{L}_{\mathrm{NLL}}(v_1) = \sum_{c,c'} \frac{d|\mathscr{M}_B|^2_{cc'}}{d\Phi_B} f_c(v_1, x_1) f_{c'}(v_1, x_2)$$

Formula can be straightforwardly promoted at 'prime' accuracy by including the hard-virtual and hard-collinear terms*

$$\mathscr{L}_{\mathrm{NLL}'}(v_1) = \sum_{c,c'} \frac{d|\mathscr{M}_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 f_i(v_1, x_1) f_j(v_1, x_2)$$
$$\times \left\{ \delta_{ci}\delta_{c'j}\delta(1-z_1)\delta(1-z_2)\left[1 + \frac{\alpha_s}{2\pi}H_1(\mu_R) + \frac{\alpha}{2\pi}\tilde{H}'_1(\mu_R)\right]\right.$$
$$\left. + \left[\frac{\alpha_s}{2\pi}C_{ci,1}\delta(1-z_2)\delta_{c'j} + \frac{\alpha}{2\pi}C'_{ci,1}\delta(1-z_2)\delta_{c'j} + \{z_1, c, i \leftrightarrow z_2, c', j\}\right]\right\}$$

*EW term also includes a contribution from **soft wide-angle radiation**

Resummation: mixed QCDxQED corrections [Buonocore, LR, Torrielli '24]

$$\frac{d\sigma(q_T)}{d\Phi_B} = \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L (e^{-R(v_1)} \mathscr{L}_{\mathrm{NLL}'}(v_1)) R'(v_1)\, d\mathscr{Z} \Theta \left(q_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|) \right)$$

Provided that relevant mixed QCDxQED corrections are included in the evolution of the PDFs (which is the case for e.g. NNPDF31luxQED PDFs) the above formula already contains the bulk of the mixed $\mathcal{O}(\alpha\alpha_s)$ corrections

The expansion of the above formula at order $\mathcal{O}(\alpha\alpha_s)$ matches the fixed-order result at small q_T with the exception of a single logarithmically enhanced term, whose contribution is included in the (NNLL) coefficient B_{11}, which can be obtained by direct abelianisation of the NNLL QCD coefficient B_2

By exponentiating the B_{11} term the NLL radiator can be promoted to

$$\tilde{R}_{\mathrm{NLL}}^{\mathrm{QCD+QED}}(L) = R_{\mathrm{NLL}}^{\mathrm{QCD+QED}}(L) + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} B_{11} L$$

Such that the formula above also predicts all the $\mathcal{O}(\alpha\alpha_s)$ logarithmically-enhanced terms

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$$\frac{d\sigma(q_T)}{d\Phi_B} = \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L \left(e^{-R(v_1)} \mathscr{L}_{\mathrm{NLL}'}(v_1)\right) R'(v_1)\, d\mathcal{Z}\, \Theta\left(q_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Finally, the luminosity can be upgraded to contain also the hard-collinear mixed terms entering at $\mathcal{O}(\alpha\alpha_s)$

$$\mathscr{L}_{\mathrm{NLL}'}(v_1) \to \mathscr{L}_{\mathrm{NLL}'}(v_1) + \sum_{c,c'} \frac{d|\mathscr{M}_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 f_i(v_1, x_1) f_j(v_1, x_2)$$
$$\times \left\{ \delta_{ci}\delta_{c'j}\delta(1-z_1)\delta(1-z_2) \left[\frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}\tilde{H}_{11}(\mu_R)\right] + \left[\frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}\, C_{ci,11}\delta(1-z_2)\delta_{c'j} + \{z_1, c, i \leftrightarrow z_2, c', j\}\right]\right\}$$

Allows for a **consistent matching** at $\mathcal{O}(\alpha\alpha_s)$

Resummation: direct-space formulation beyond NLL

The RadISH formalism can be extended beyond NLL accuracy by including consistently higher towers of logarithmic terms

Formula at $N^3LL'_{QCD}$ accuracy

$N^3LL'_{QCD}+NLL'_{QED}$ resummation can be straightforwardly achieved by modifying the contribution entering at NLL

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathscr{L}_{\mathrm{N^3LL'}}(k_{t1})\right)\int d\mathcal{Z}\,\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\\
&+\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathscr{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL'}}(k_{t1}) - \beta_0\frac{\alpha_s^3(k_{t1})}{\pi^2}\left(\hat{P}^{(0)}\otimes\hat{C}^{(1)}+\hat{C}^{(1)}\otimes\hat{P}^{(0)}\right)\otimes\mathscr{L}_{\mathrm{NLL}}(k_{(t1})+\frac{\alpha_s^3(k_{t1})}{\pi^2}2\beta_0\ln\frac{1}{\zeta_s}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL}}(k_{(t1})\\
&+\frac{\alpha_s^3(k_{t1})}{2\pi^2}\left(\hat{P}^{(0)}\otimes\hat{P}^{(1)}+\hat{P}^{(1)}\otimes\hat{P}^{(0)}\right)\otimes\mathscr{L}_{\mathrm{NLL}}(k_{(t1})\Bigg\}\times\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\Big\}\\
&+\frac{1}{2}\int\frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\Bigg\{\mathscr{L}_{\mathrm{NLL}}(k_{t1})\big(R''(k_{t1})\big)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathscr{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL}}(k_{t1})+\frac{\alpha_s^2(k_{t1})}{\pi^2}\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)R''(k_{t1})\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL}}(k_{(t1}) - \ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}}(R''(k_{t1})^2\partial_L\mathscr{L}_{\mathrm{NLL}}(k_{(t1})\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^3}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathscr{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\times\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1})\right) -\\
&\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s2})\right)+\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-7}\frac{1}{v}\right)
\end{aligned}
$$

[Re, LR, Torrielli '21]

Validation

We expand the resummation formula and we compute (N)NLO predictions via q_T subtraction

$$d\sigma_X^{\mathrm{N^kLO}} \equiv \mathscr{H}_{\mathrm{X}}^{\mathrm{N^kLO}} \otimes d\sigma_{\mathrm{X}}^{\mathrm{LO}} + \left[d\sigma_{\mathrm{X+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_X^{\mathrm{N^kLL}} \right]_{\mathcal{O}(\alpha_s^k)} \right]_{q_T > q_t^{\mathrm{cut}}} + \mathcal{O}((q_T^{\mathrm{cut}}/M)^n)$$

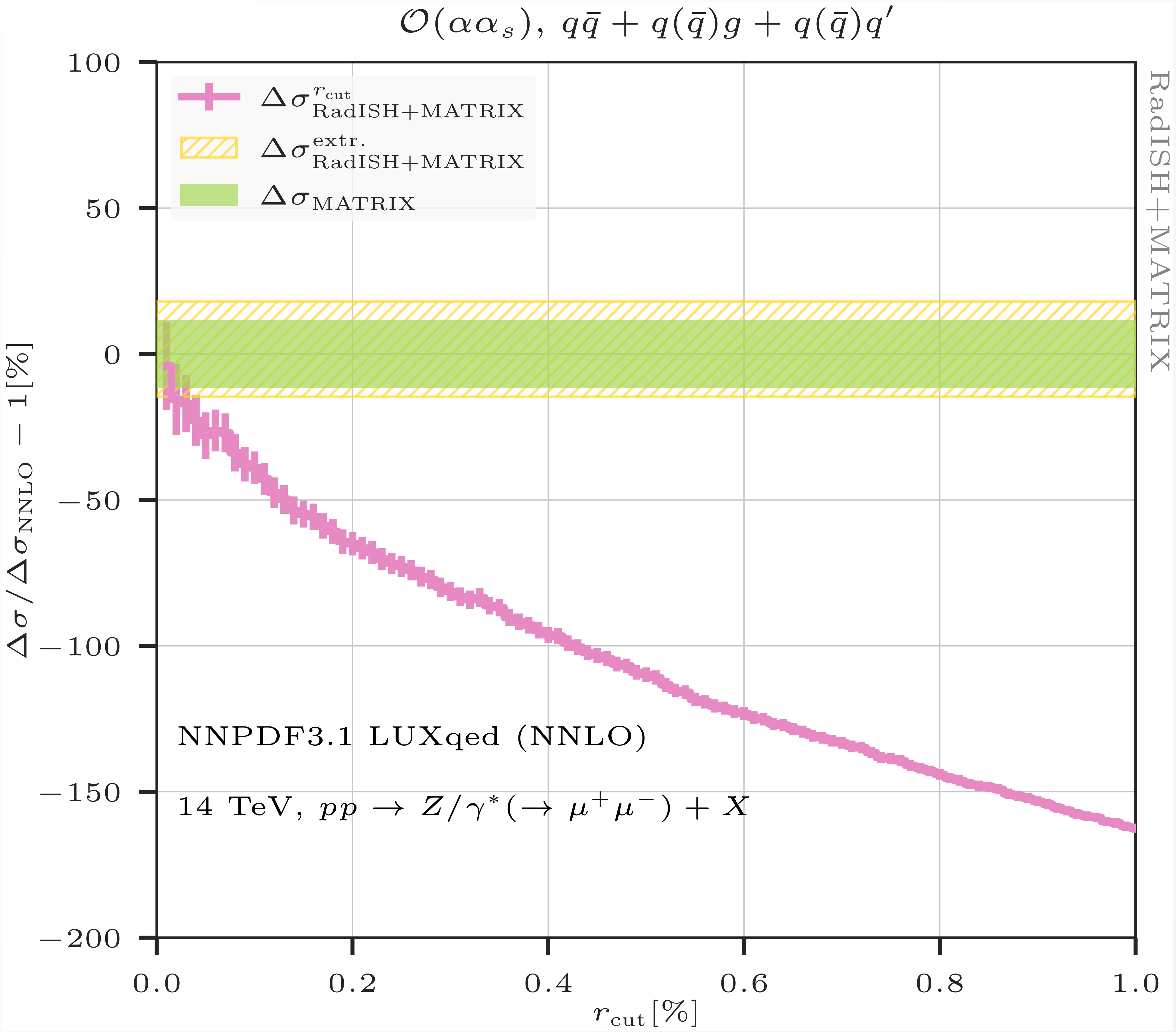
Comparison against independent (N)NLO computation in MATRIX guarantees that resummation coefficients + hard-virtual / soft-wide angle terms are correctly implemented

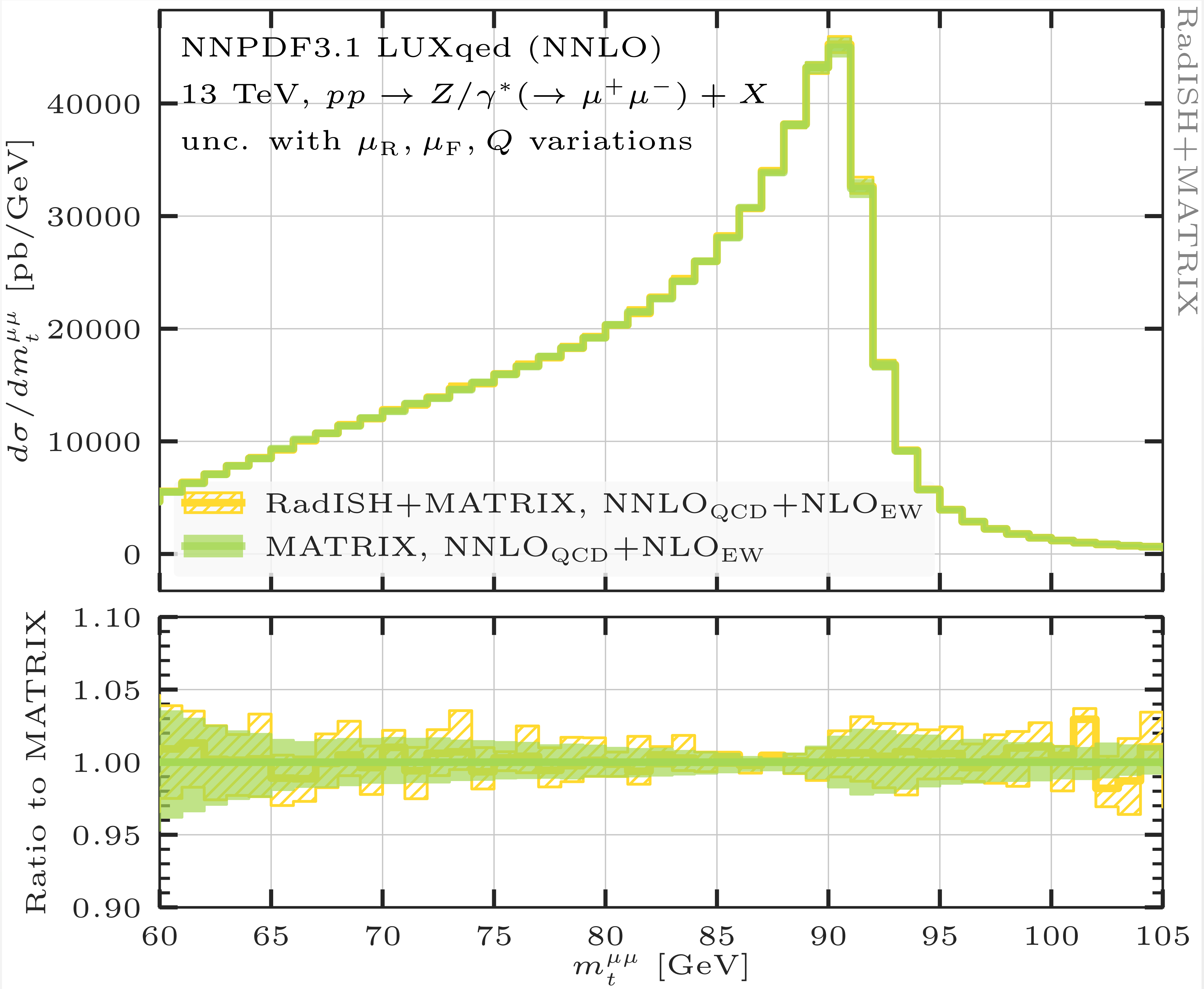
NB: **above cut part** in RadISH predictions always computed with MATRIX

Validation performed for all individual channels up to $\mathcal{O}(\alpha_s \alpha)$; photon-induced contributions in NC DY implemented only up to $\mathcal{O}(\alpha)$

Percent-level control of $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha_s \alpha)$ corrections at the level of the total cross-section within fiducial cuts in the setups considered

Validation: fiducial predictions for Z production



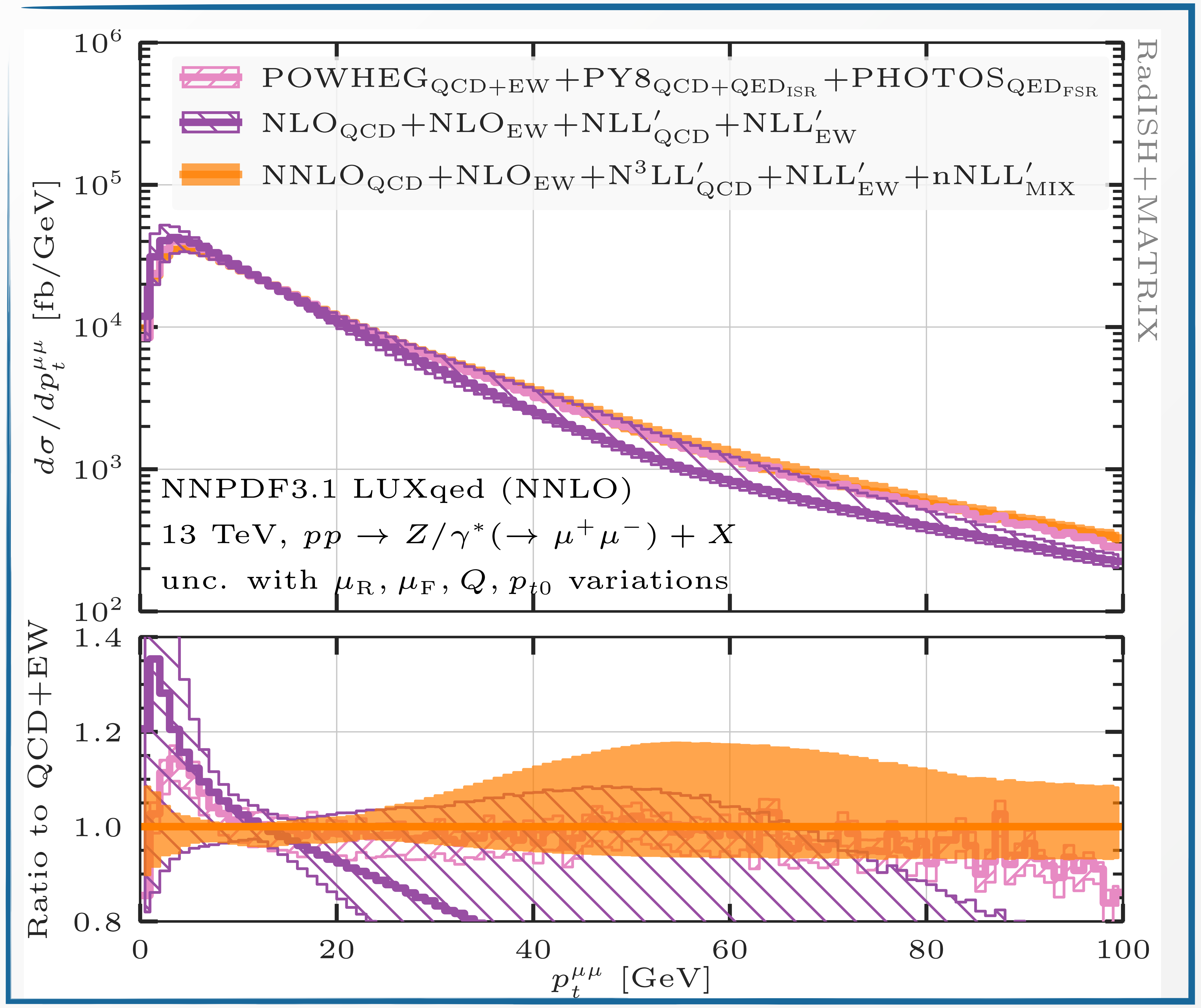


Percent-level agreement at the level of differential distributions

W and Z production: the role of EW corrections

QED and mixed QCD-EW correction patterns in W and Z production differ due to the **different number of charged legs** in NC and CC Drell-Yan production

LL QED and (factorizable) QCD/EW corrections are typically estimated by interfacing QCD Monte Carlo programs with dedicated QED shower programs, such as PHOTOS



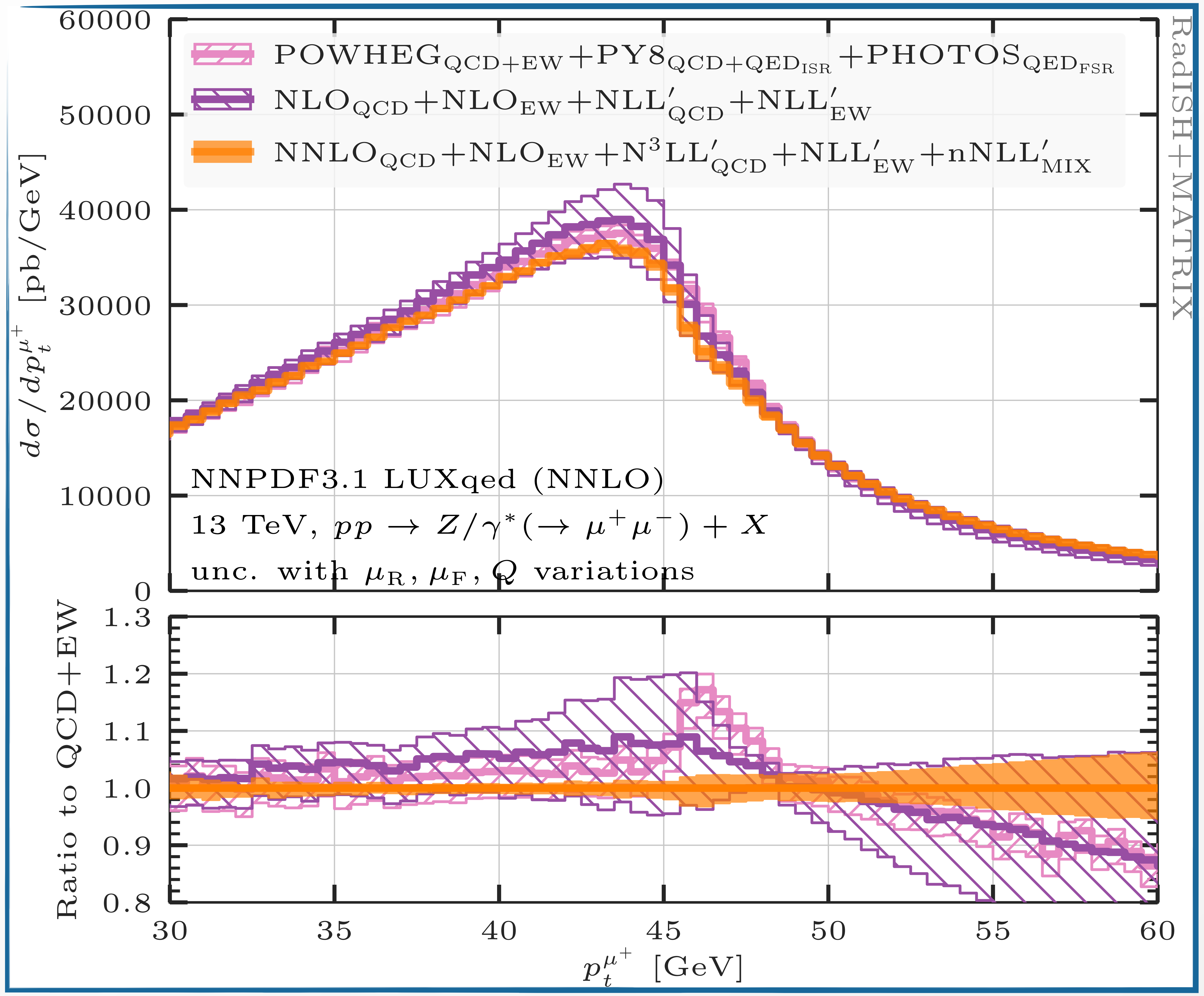
NLL$'_{\text{EW}}$+nNLL$'_{\text{MIX}}$ (including non-factorisable contributions) resummation **available for the first time at the level of bare muons**, allowing for a level of flexibility comparable to that of dedicated EW MC generators

Availability of such a tool allows to compare QED showers to predictions with higher formal accuracy

Alternative assessment of robustness of QED FSR treatment in current analyses

Impact on recoil-sensitive observables

$p_T^{\mu^\pm} > 27$ GeV, $|y_\mu| < 2.5$, 66 GeV $< m_{\mu\mu} < 116$ GeV



No matching at $\mathcal{O}(\alpha_s\alpha)$

Differences between 'best' predictions and results with lower formal accuracy obtained with approximate corrections in parton showers based on a factorised approach
[Barze' et al (2012,2013)], [Calame et al (2017)]

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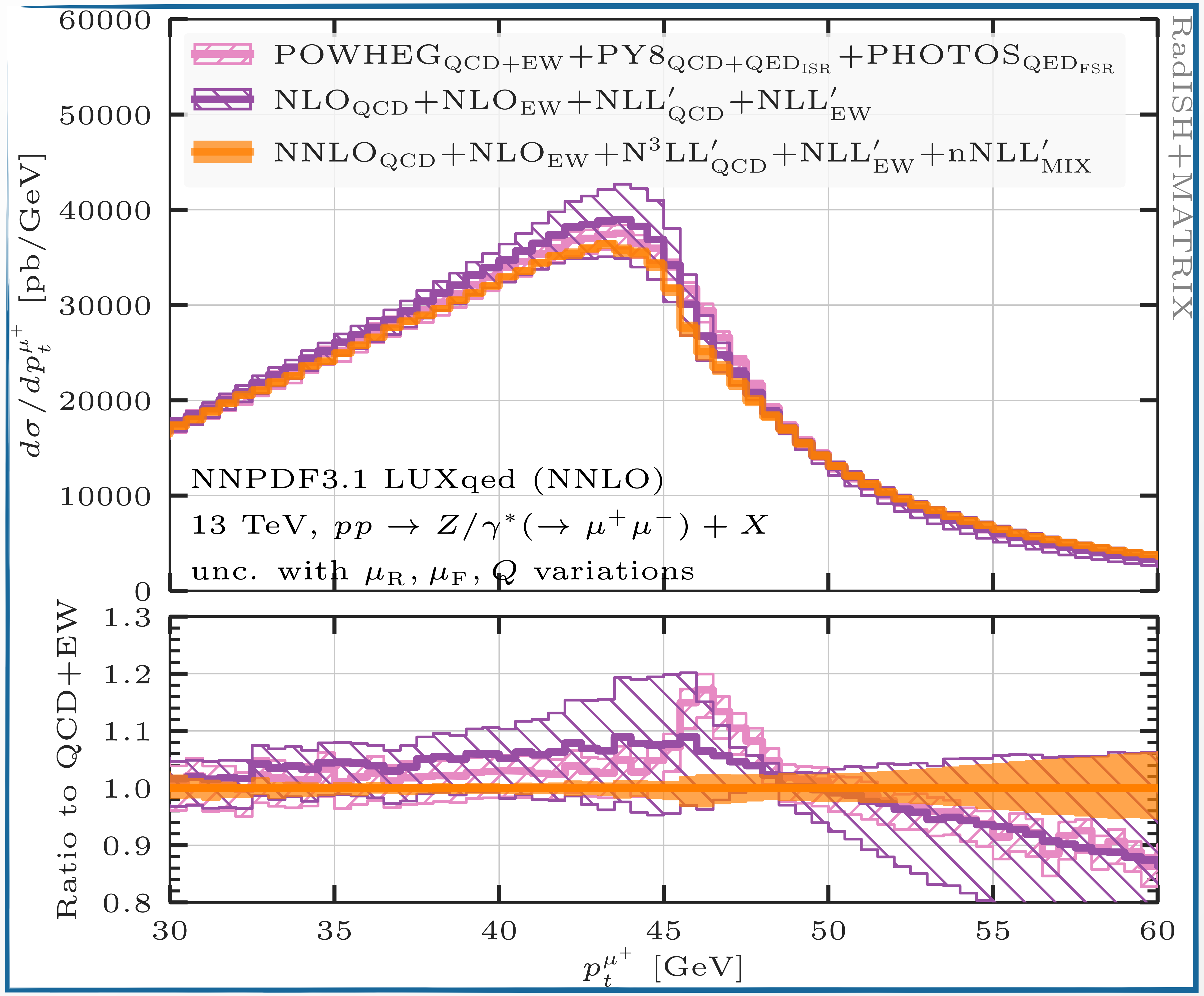

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However

Resummation of large logarithms of the lepton mass associated with fiducial cuts is missing
Fixed-order terms retrieved upon **matching**.

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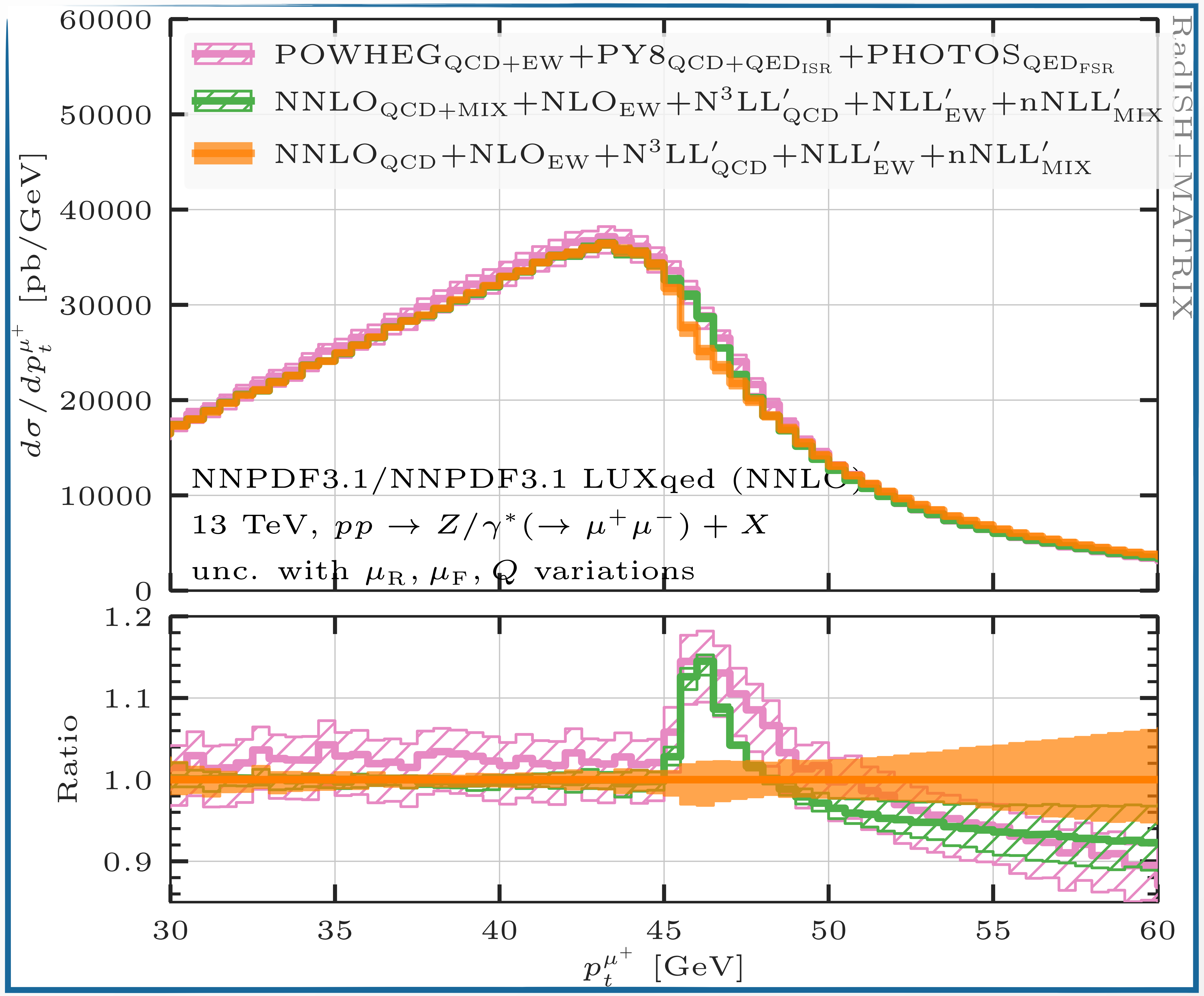


With matching at $\mathcal{O}(\alpha_s\alpha)$

Differences between 'best' predictions and results with lower formal accuracy obtained with approximate corrections in parton showers based on a factorised approach [Barze' et al (2012,2013)], [Calame et al (2017)]

However

Resummation of large logarithms of the lepton mass associated with fiducial cuts is missing Fixed-order terms retrieved upon **matching**.

$\mathcal{O}(\alpha_s\alpha)$ matching mandatory: it induces large corrections

The situation can be mitigated by an **improved treatment of the quasi-collinear** photon emission region (**WIP**)

Summary

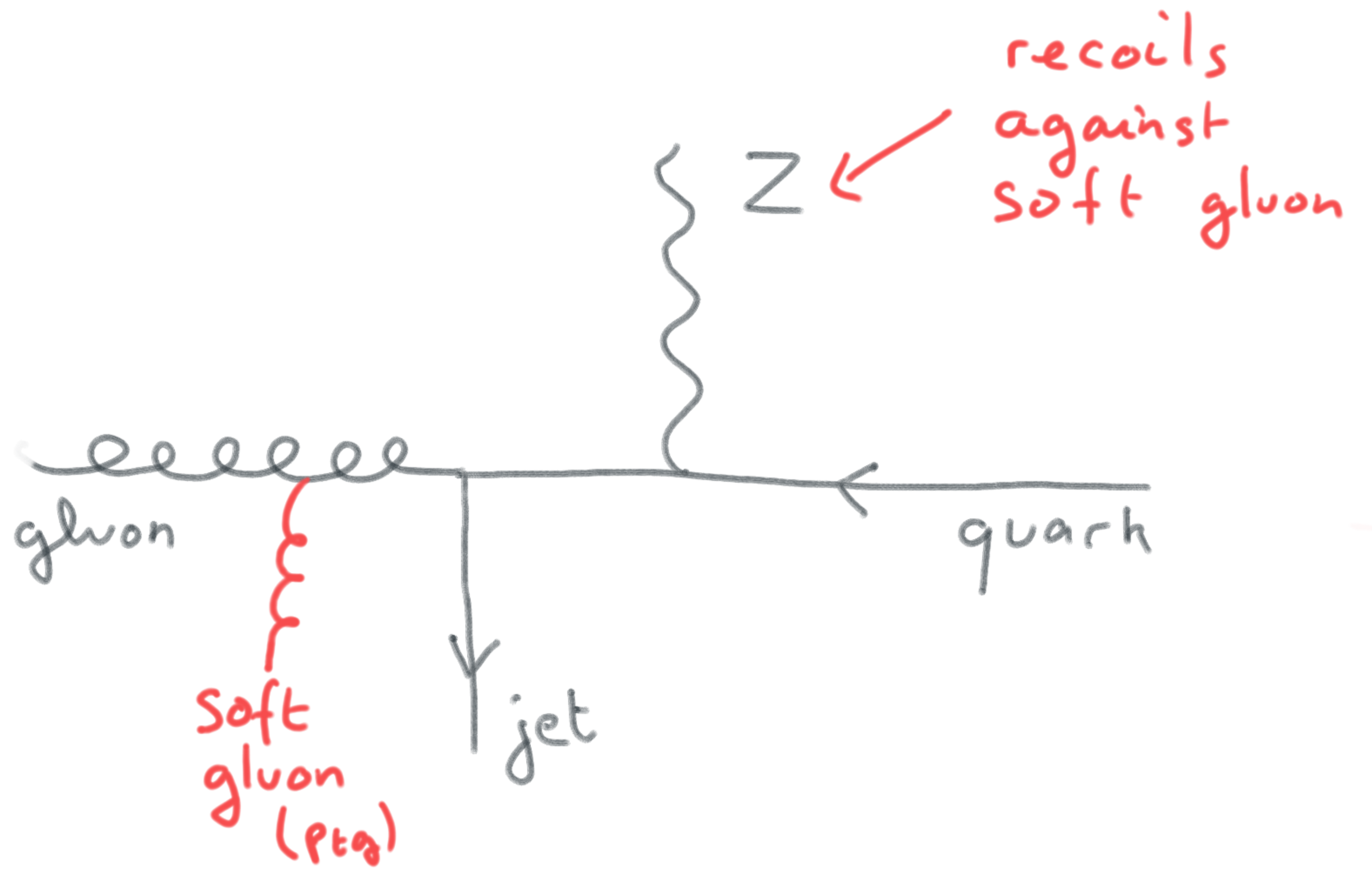
- Modelling of **theoretical uncertainties** crucial for EW precision programme at the LHC
- High-accuracy fixed order predictions, supplemented with resummation for observable sensitive to soft/collinear radiation, needed to treat the acute disease of precision which afflicts us
- Perturbative QCD predictions have reached a **remarkable level of accuracy**
- **Interplay with QED/mixed QCD/EW predictions mandatory** for a successful precision programme, alongside comprehension of NP physics, PDF uncertainty, including MHOU (not discussed in this seminar)
- Availability of **analytic tools** allows us to compare parton showers showers to predictions with higher formal accuracy
- Monte Carlo tunes for sub-percent precision must be handled with care. Availability of accurate perturbative calculation may provide insight on tuning parameters to avoid unphysical correlations

Backup

Non perturbative corrections and q_T^Z

Collinear factorization valid up to power corrections $\mathcal{O}(\Lambda^n_{\mathrm{QCD}}/Q^n)$

In principle, easy to imagine mechanisms for linear power corrections, which would be a disaster for precision programme at the LHC



[G.P. Salam]

Linear term could be generated when integrating over soft d.o.f. which is not azimuthally symmetric

Luckily, for q_T this does not happen!

[Ravasio, Limatola, Nason 2021]
[Caola, Ravasio, Limatola, Melnikov, Nason 2022]

No linear power corrections affect the transverse momentum spectrum

Treatment of non-perturbative corrections

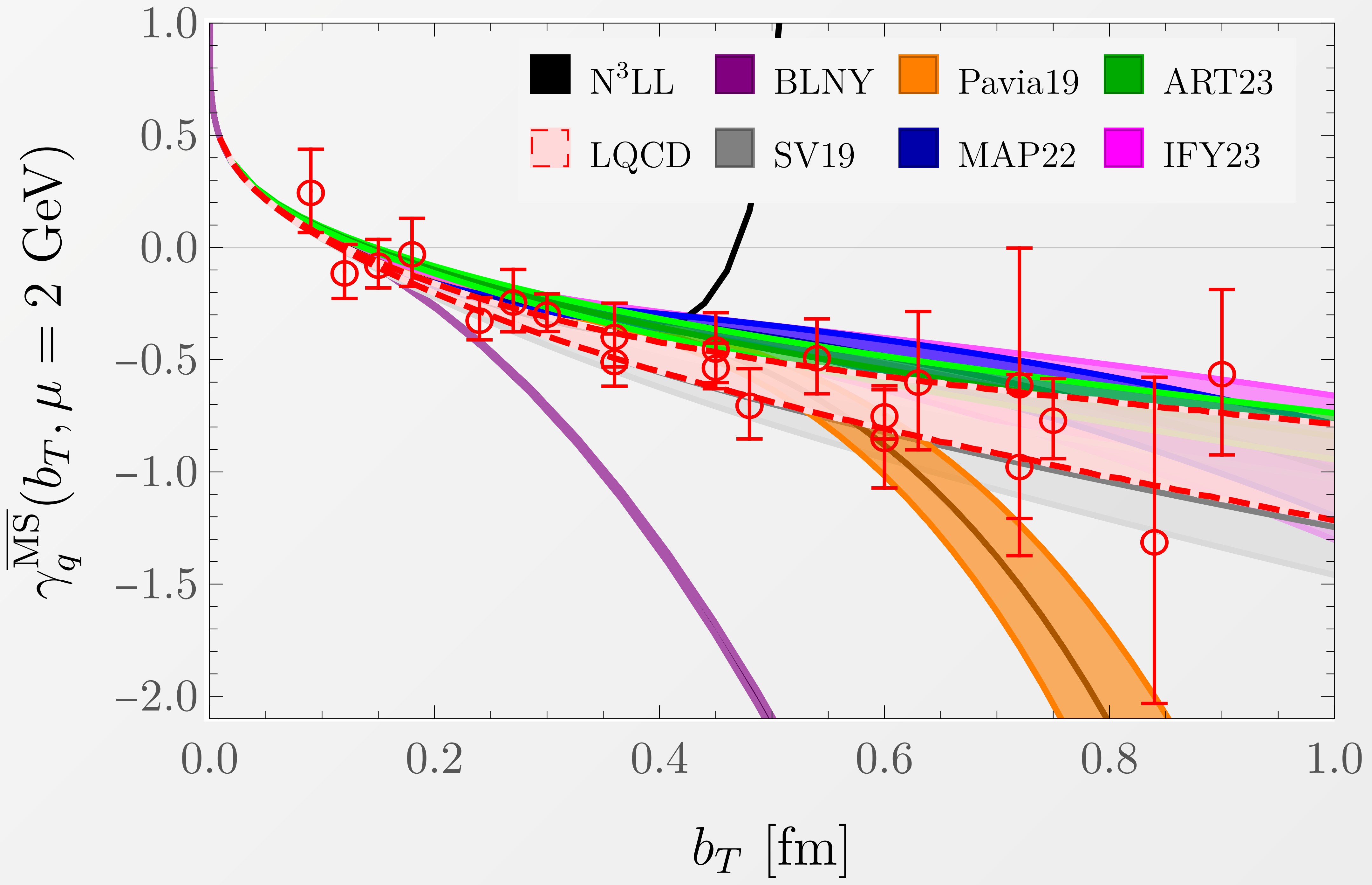
Nevertheless, NP corrections **can be sizeable** in the first q_T bins. Often supplemented by introducing a non-perturbative correction **determined from data**

e.g. in TMD factorisation $$\tilde{f}_c^{\mathrm{TMD}}(x_1, b, \mu, \zeta) = \tilde{f}_c^{\mathrm{NP}}(x_1, b, \mu)\tilde{f}_c^{\mathrm{TMD}}(x_1, b^*, \mu, \zeta)$$

Properties of $\tilde{f}_c^{\mathrm{NP}}(x_1, b, \mu)$ determined by TMD factorisation; function is not universal, as it depends on the **strategy used to regularise the Landau pole**

Extraction from data of the non-perturbative component to the Collins-Soper kernel can be compared with recent **lattice QCD computation**

Progress in lattice computations opens the door for future first-principles QCD predictions of the CS kernel and to possible combination with fits to data

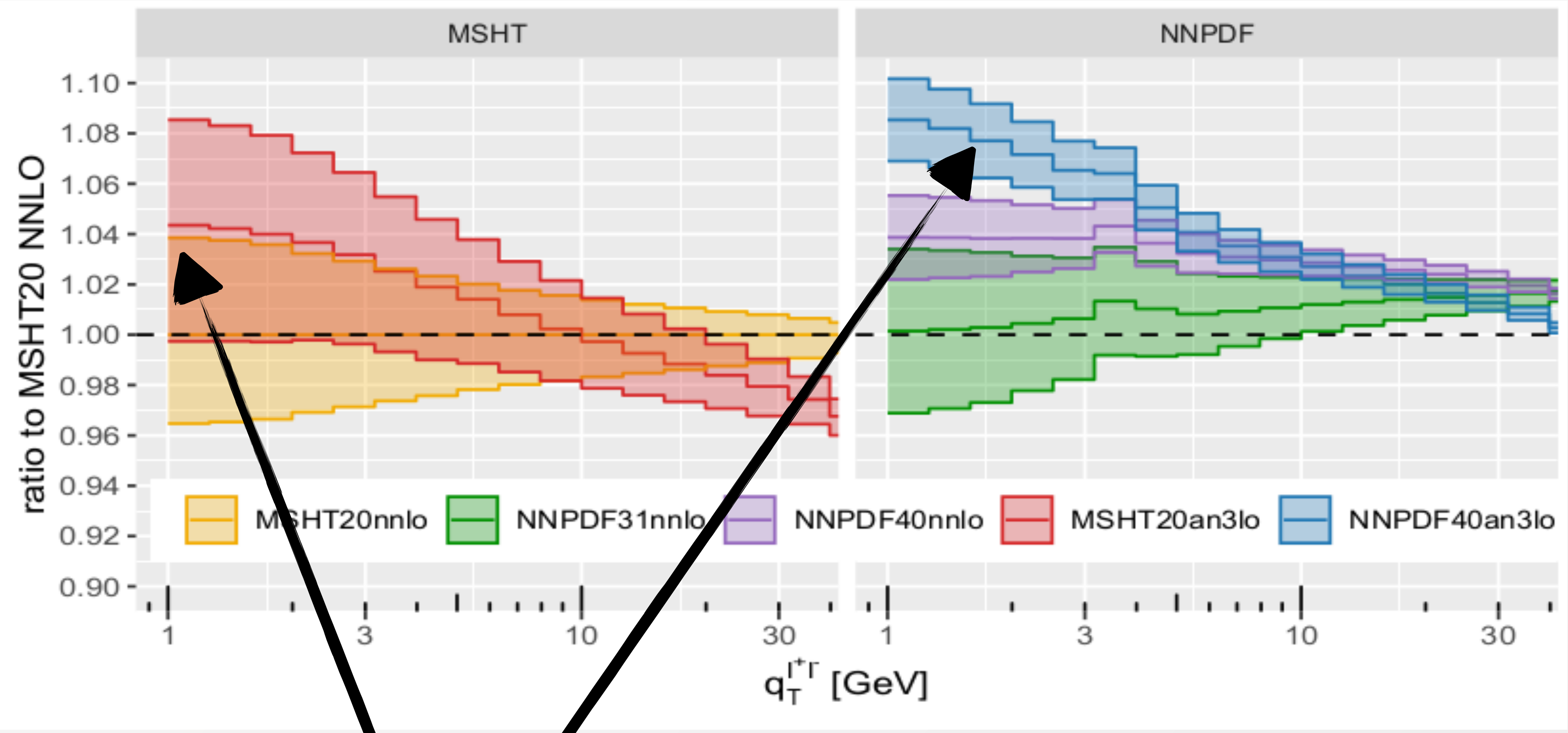


[Avkhadiev, Shanahan, Wagman, Zhao 2024]

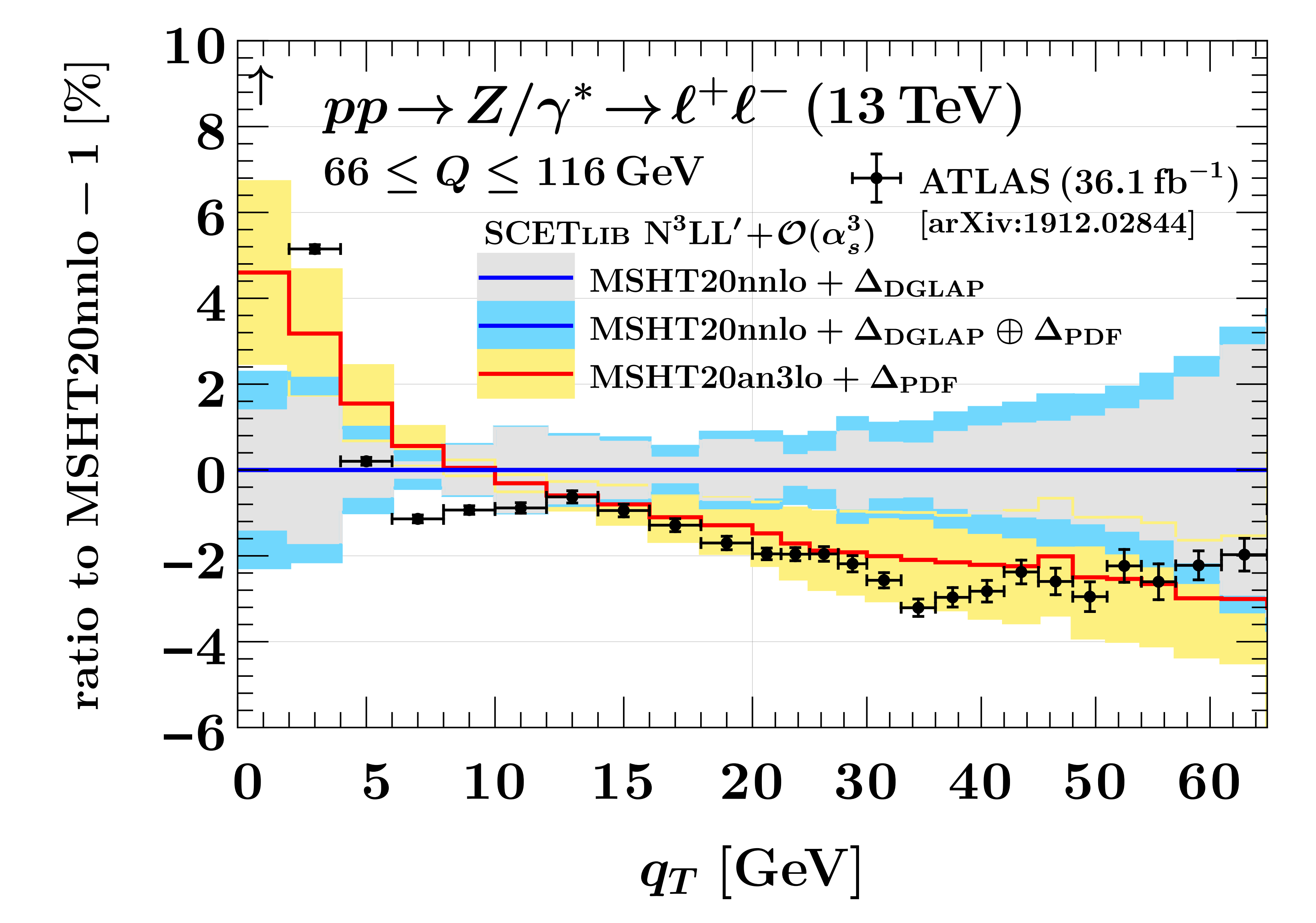
The role of PDFs

Non negligible differences in absolute value between different groups (NNPDF, MHST)

Discrepancy explained by fitted (NNPDF) vs. perturbative (MSHT) charm and different value of the charm mass, still state-of-the-art PDFs set **can differ at the few % level**



[Neumann @ Loops and Legs 2024]



[Michel @ EW WG 2022]

aN^{3}LO PDFs from MSHT or NNPDF have a similar impact in shape on the Z q_T spectrum. Substantial differences can impact the agreement with the experimental data

Precision programme requires a deeper understanding of PDF/N^{3}LO DGLAP role for such a crucial observable

EW corrections: ratio q_T^W / q_T^Z

Comparison with PWG_EW+PY8+PHOTOS, PWG_QCD+PY8+PHOTOS and $\mathrm{NLL}'_{\mathrm{QCD}} + \mathrm{NLO}_{\mathrm{QCD}} + \mathrm{NLL}'_{\mathrm{EW}} + \mathrm{NLO}_{\mathrm{EW}}$

- Nice perturbative stability and robustness against shower tuning
- Better agreement of "simpler" PWG_QCD+PY8+PHOTOS to RadISH, residual difference similar to pure QCD case
- PWG_EW+PY8+PHOTOS result deviates significantly from our best prediction