Universität Münster

institut für theoretische physik

Among the Rarest: Theoretical Insights into Four Top-Quark Production at the LHC

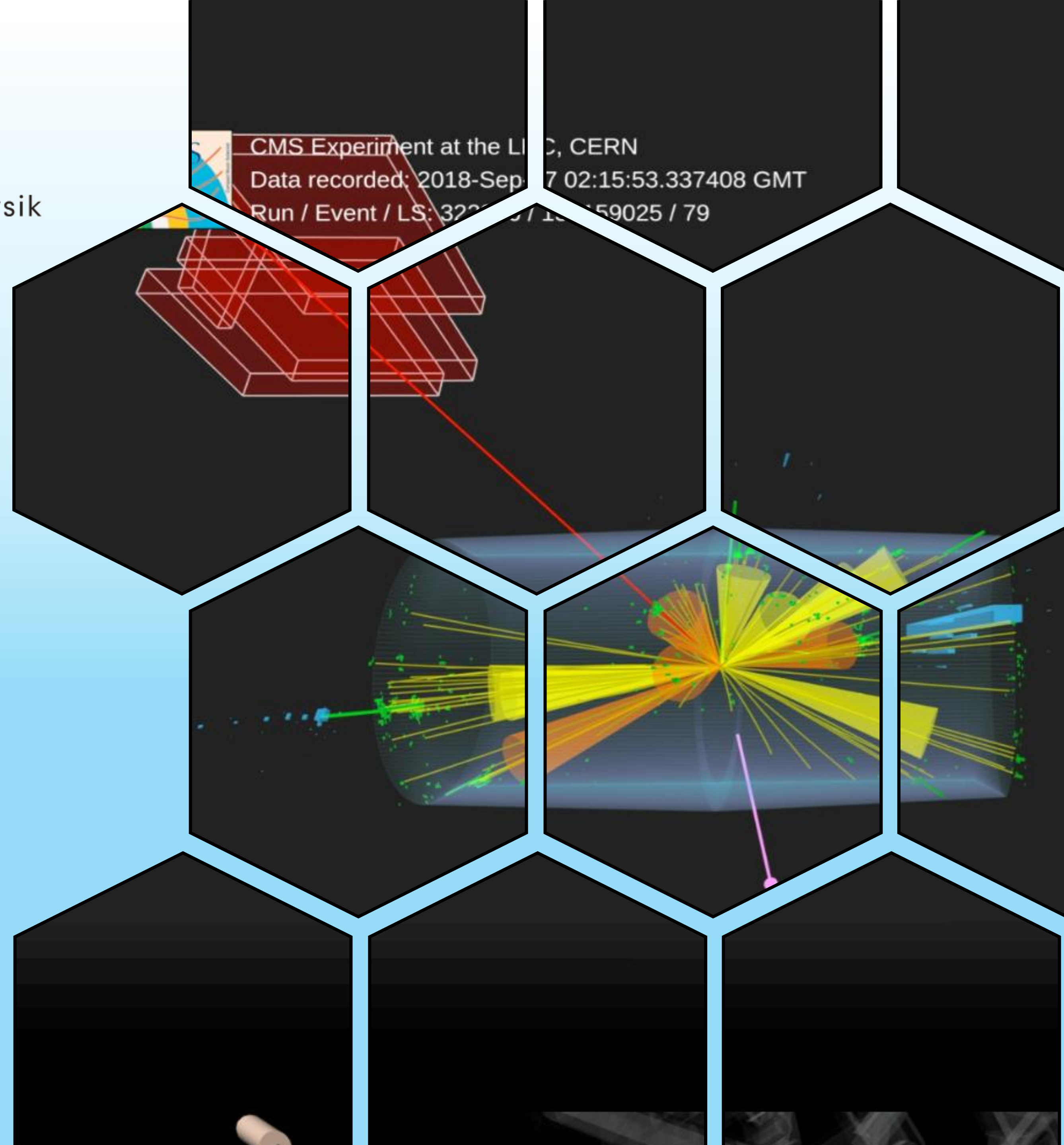
Based on: **2505.10381**

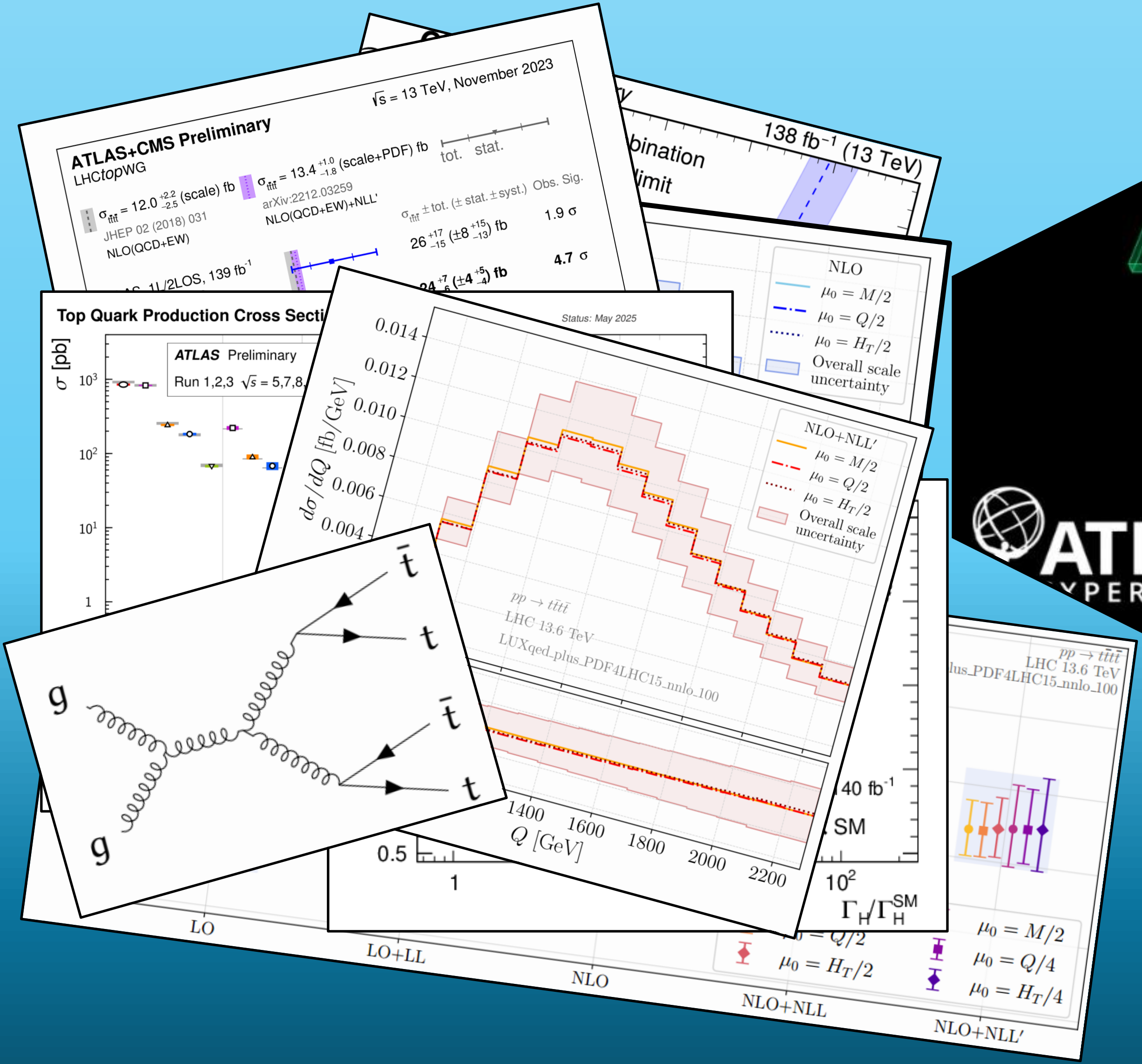
Michele Lupattelli

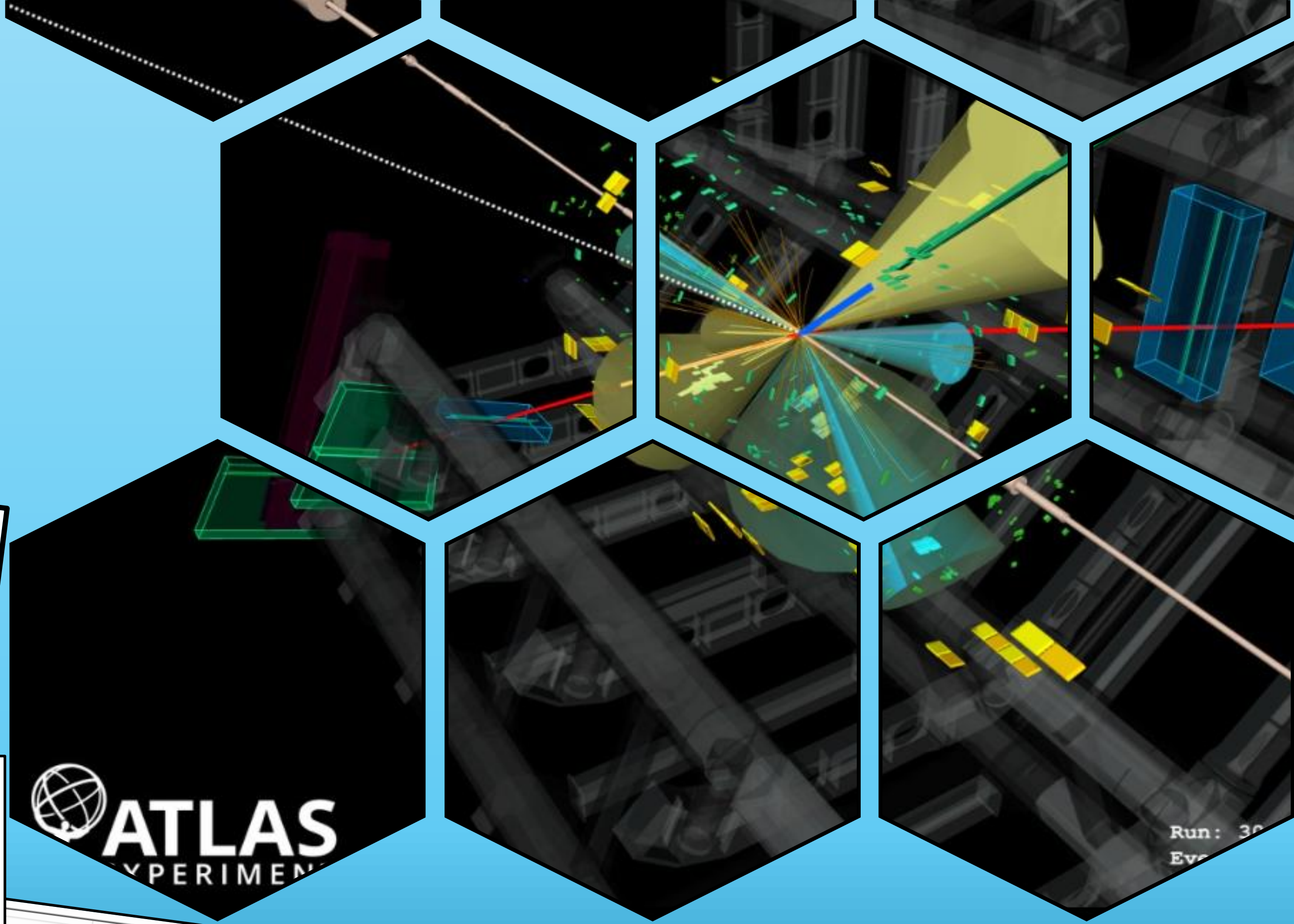
in collaboration with

M. van Beekveld, A. Kulesza, T. Saracco

Garching, 24 June 2025




ATLAS+CMS Preliminary
LHCtopWG
√s = 13 TeV, November 2023
tot. stat.
Top Quark Production Cross Section
ATLAS Preliminary
Run 1,2,3
Status: May 2025
138 fb⁻¹ (13 TeV)
NLO
Overall scale uncertainty
NLO+NLL′
LHC 13.6 TeV
LUXqed_plus_PDF4LHC15_nnlo_100
Q [GeV]
g
g
t
t̄
LO
LO+LL
NLO
NLO+NNL
NLO+NLL′


ATLAS
EXPERIMENT

1

MOTIVATIONS

I discuss the properties of the top quark and motivate the importance of studying the four top-quark production process.

2

QCD CALCULATIONS

I shortly discuss the structure of QCD calculations.

3

SOFT GLUON RESUMMATION

I introduce the framework of soft gluon resummation and go into some general technical details and some specific of the calculations presented here.

4

RESULTS

I present and discuss the results of the calculations, showing predictions for the total cross section and the invariant-mass distribution.

5

SUMMARY AND CONCLUSIONS

I summarize the presentation, draw the conclusion and give an outlook.

1

MOTIVATIONS

t

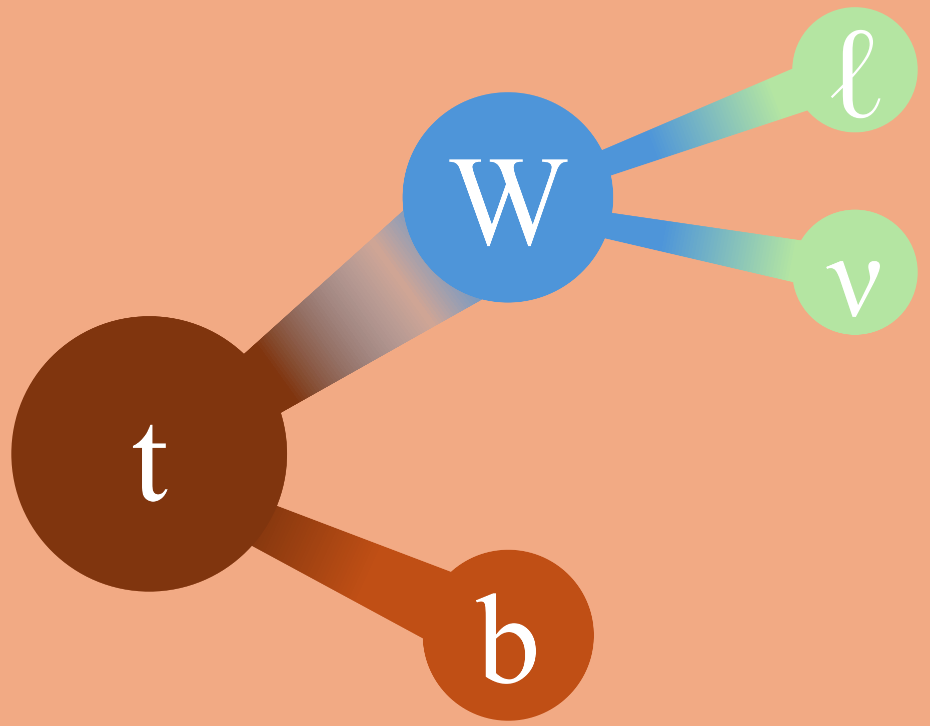
$m = 173.3$ GeV

$\Gamma = 1.42$ GeV

$\tau = 5\times10^{-25}$ s

$q/e = ⅔$

$s = ½$



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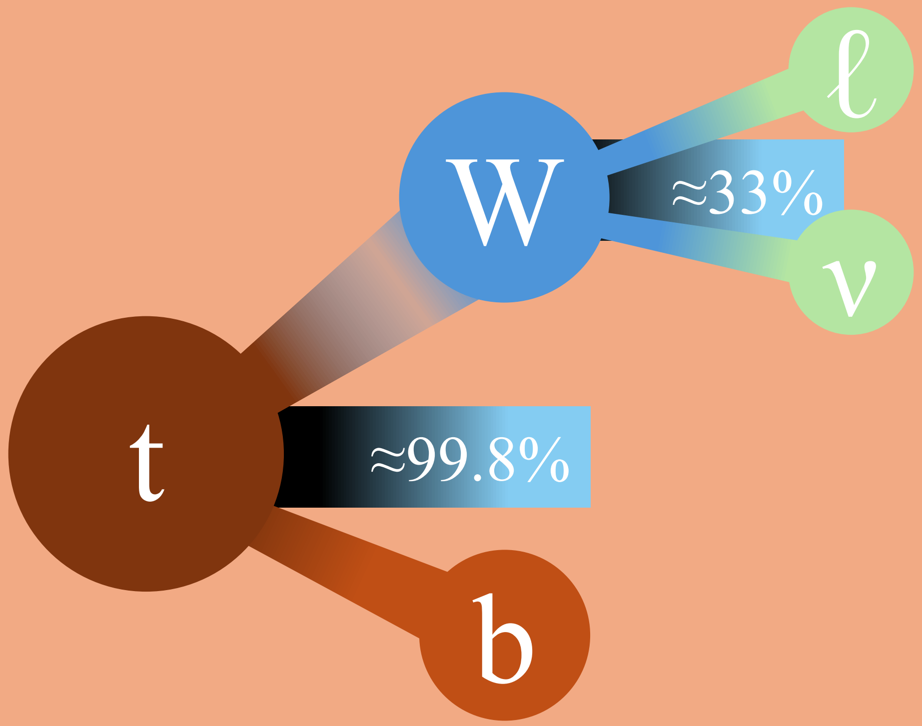

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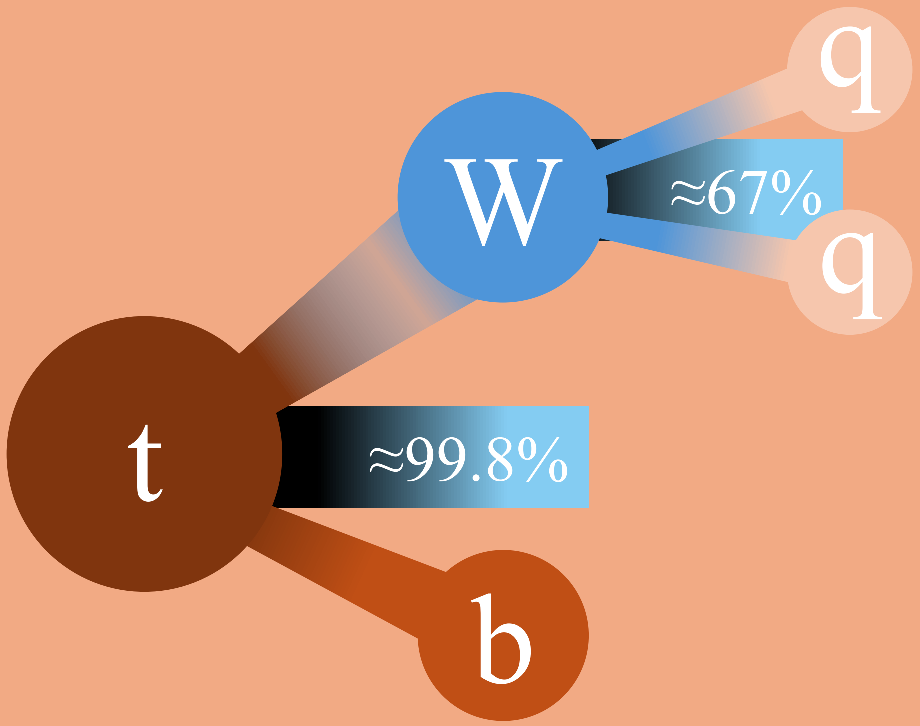

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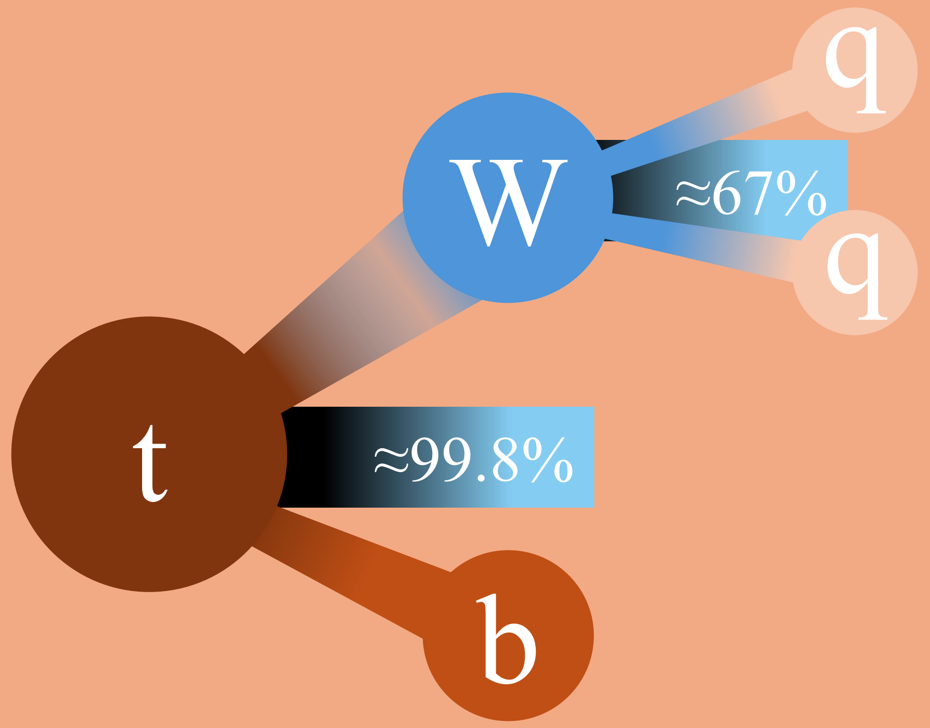

It is a very special particle!

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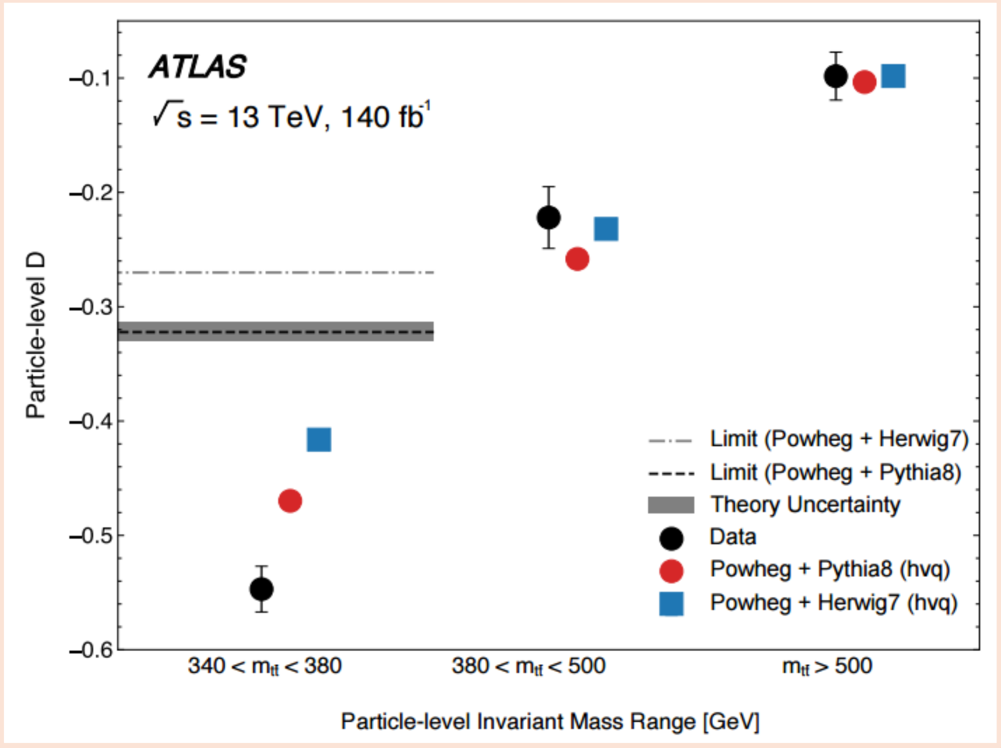

$\tau_h = 1/\Lambda_{QCD} = 10^{-24}$ s → Cannot form bound states

$\tau_{sd} = m/\Lambda_{QCD}^2 = 10^{-21}$ s → Spin correlations carried by decay products (Quantum Entanglement)

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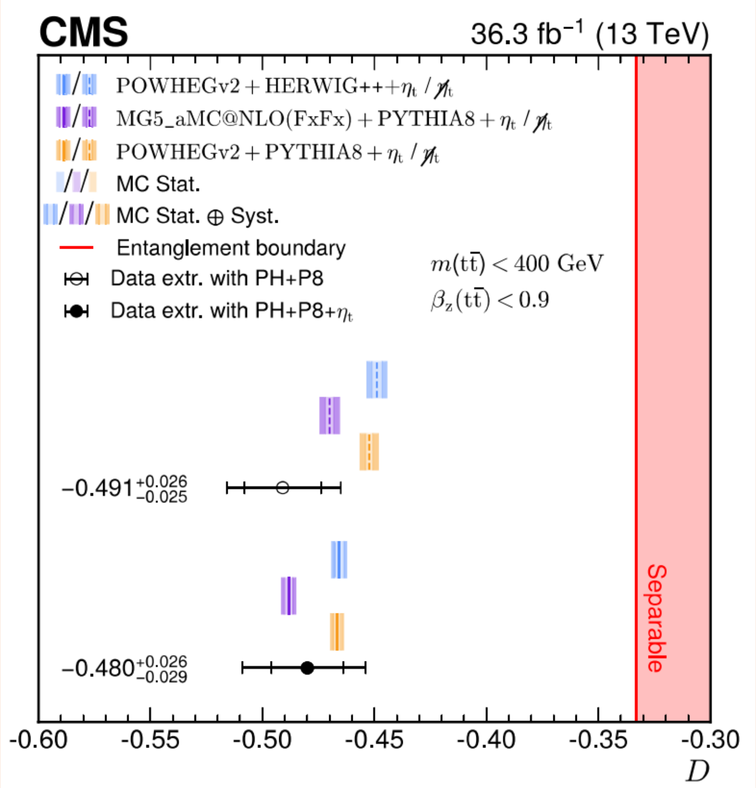

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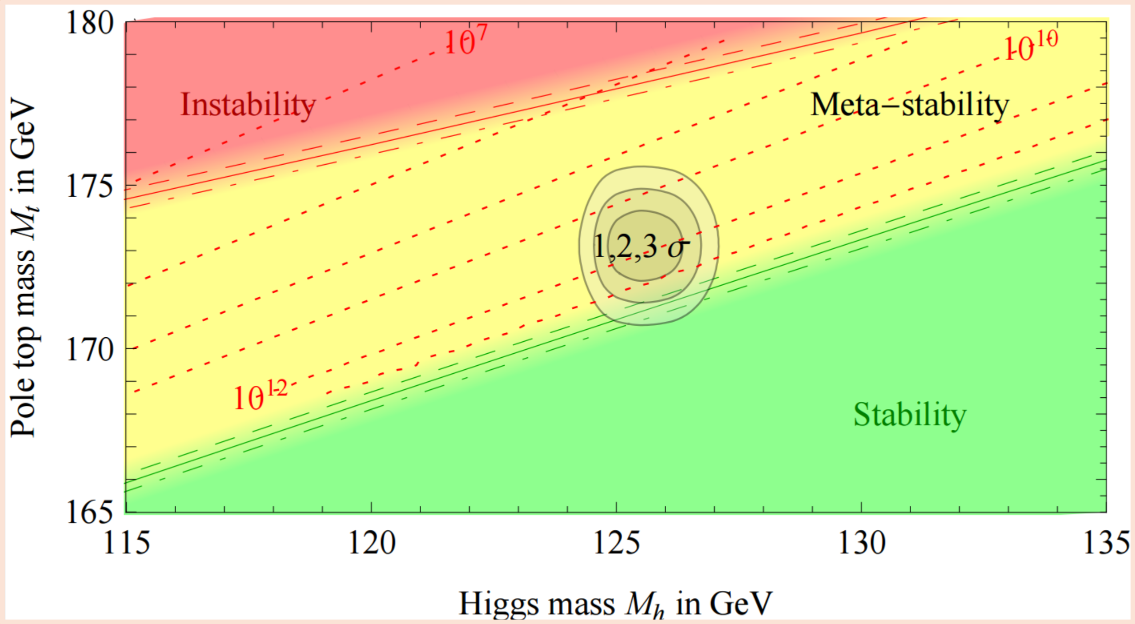
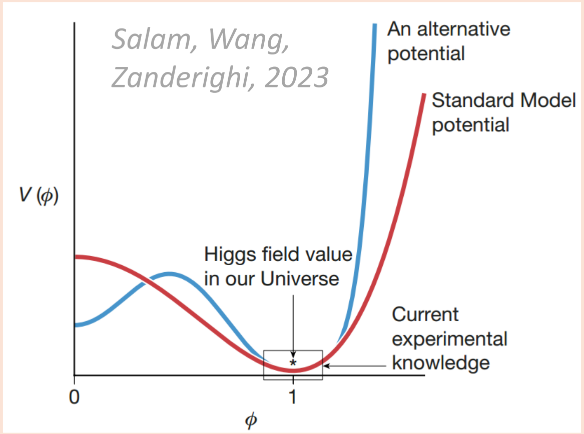
Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 2012



Top-Yukawa coupling $Y_t = m_t\sqrt{2}/v \approx 1$

→ Impact on the mass of the Higgs boson and on the stability of the electroweak vacuum



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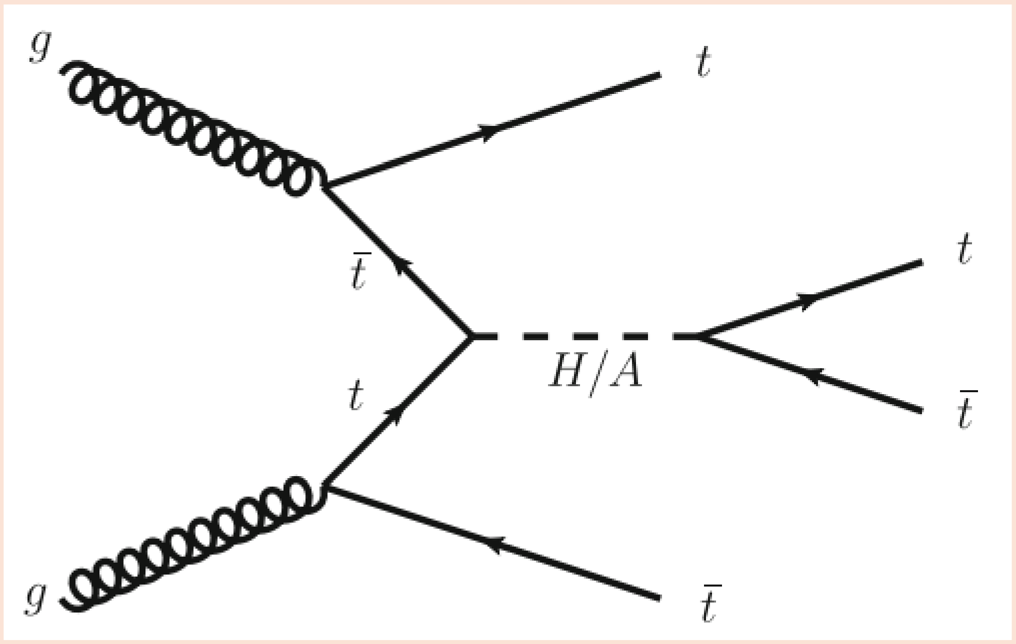
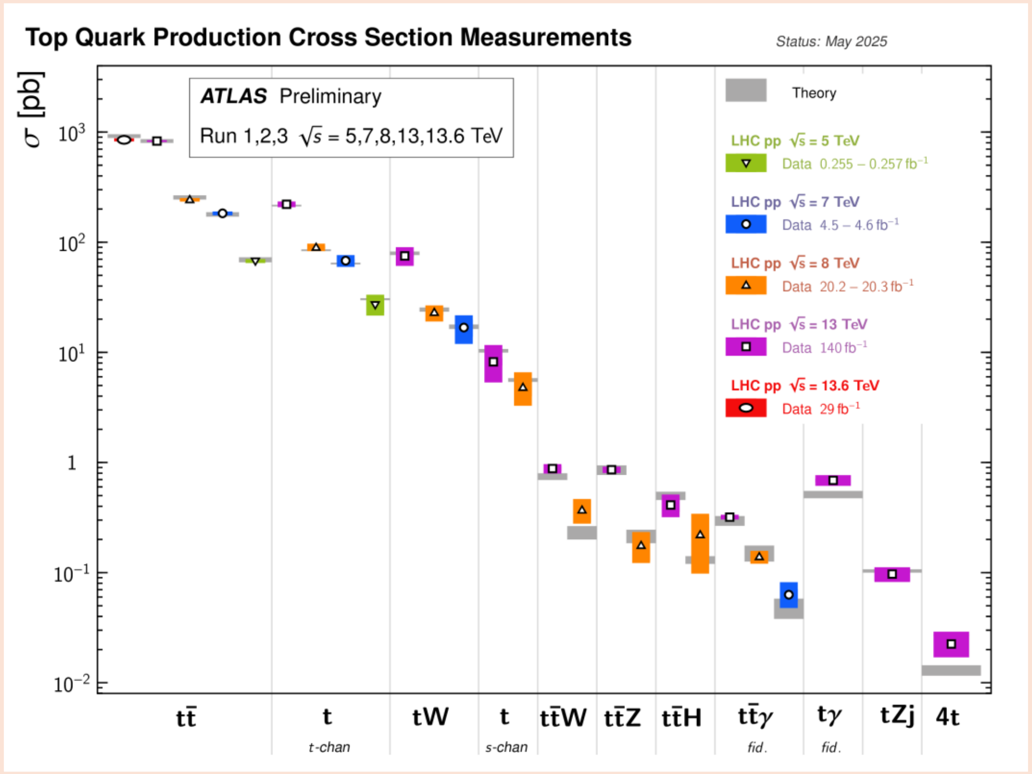
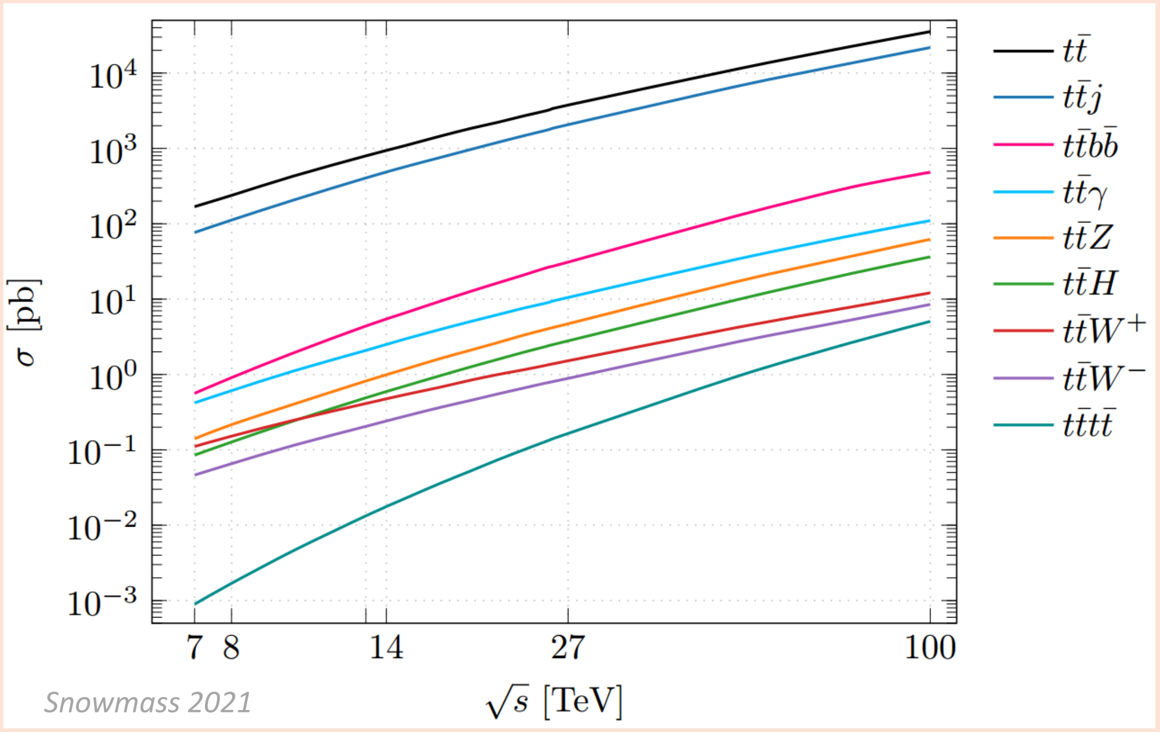
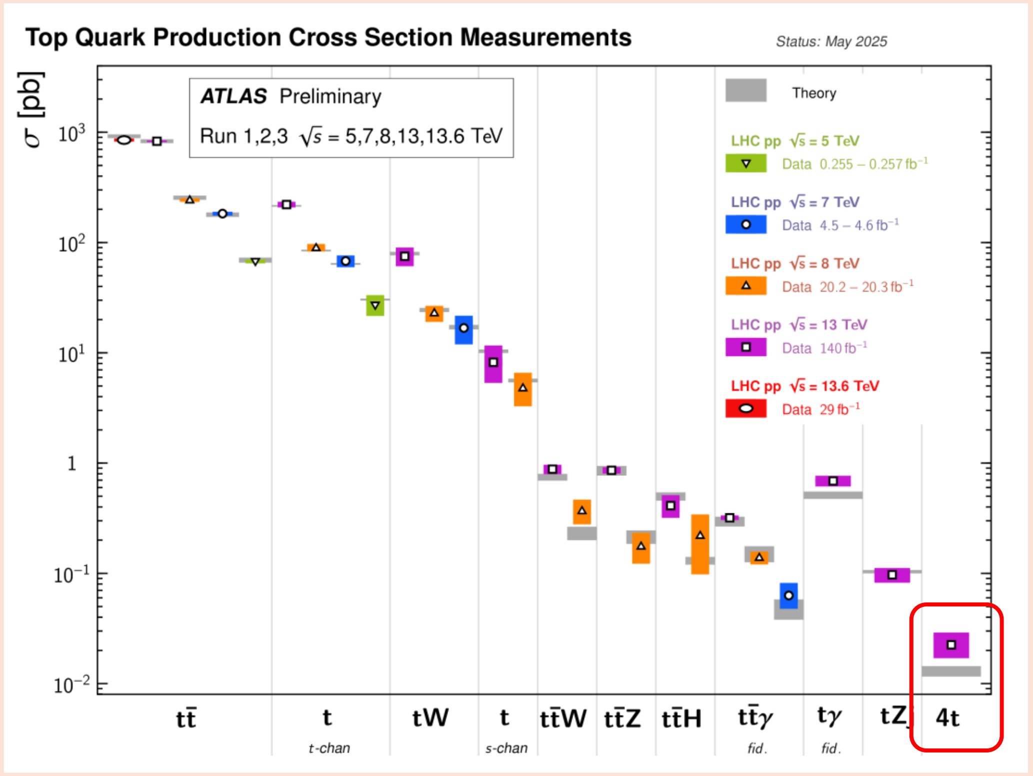


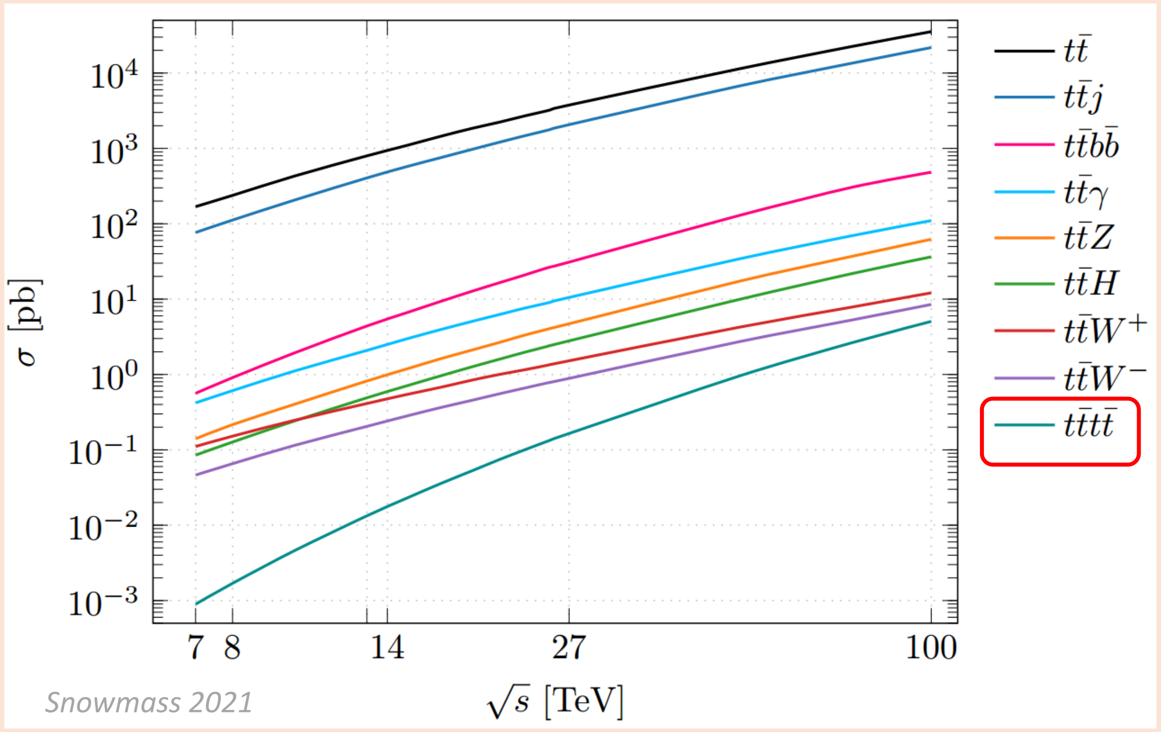
Heavy BSM particles can decay into $t\bar{t}$.
Rare processes (such as $t\bar{t}t\bar{t}$ production) more sensitive to these effects.





The production of four top quarks in proton-proton collisions is one of the rarest processes of the Standard Model.





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Eur. Phys. J. C (2023) 83:496
https://doi.org/10.1140/epjc/s10052-023-11573-0

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Experimental Physics

Observation of four-top-quark production in the multilepton final state with the ATLAS detector

ATLAS Collaboration*

CERN, 1211 Geneva 23, Switzerland

Received: 29 March 2023 / Accepted: 2 May 2023 / Published online: 12 June 2023
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Letter

Check for updates

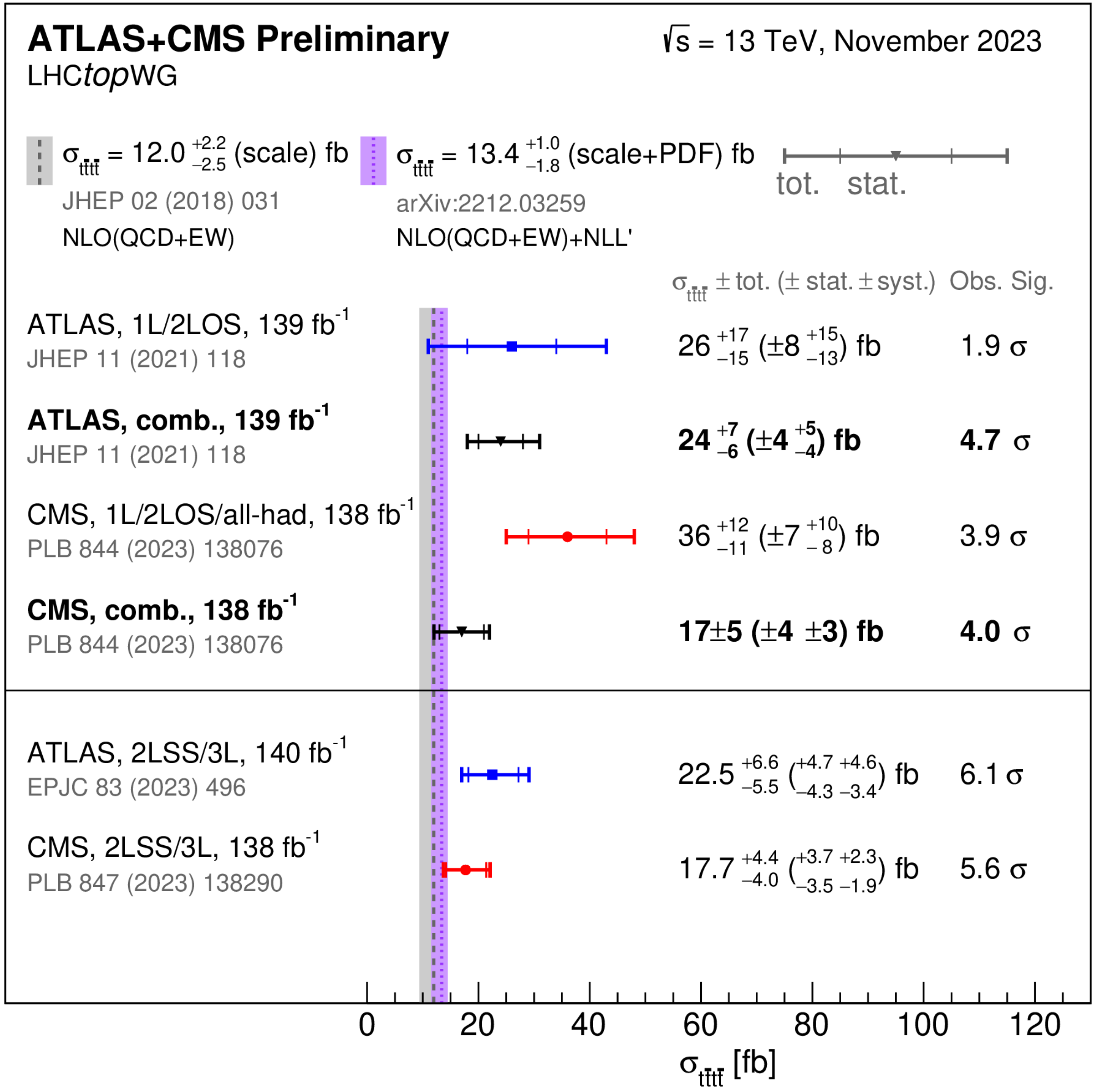
Observation of four top quark production in proton-proton collisions at $\sqrt{s} = 13\,\text{TeV}$

The CMS Collaboration *

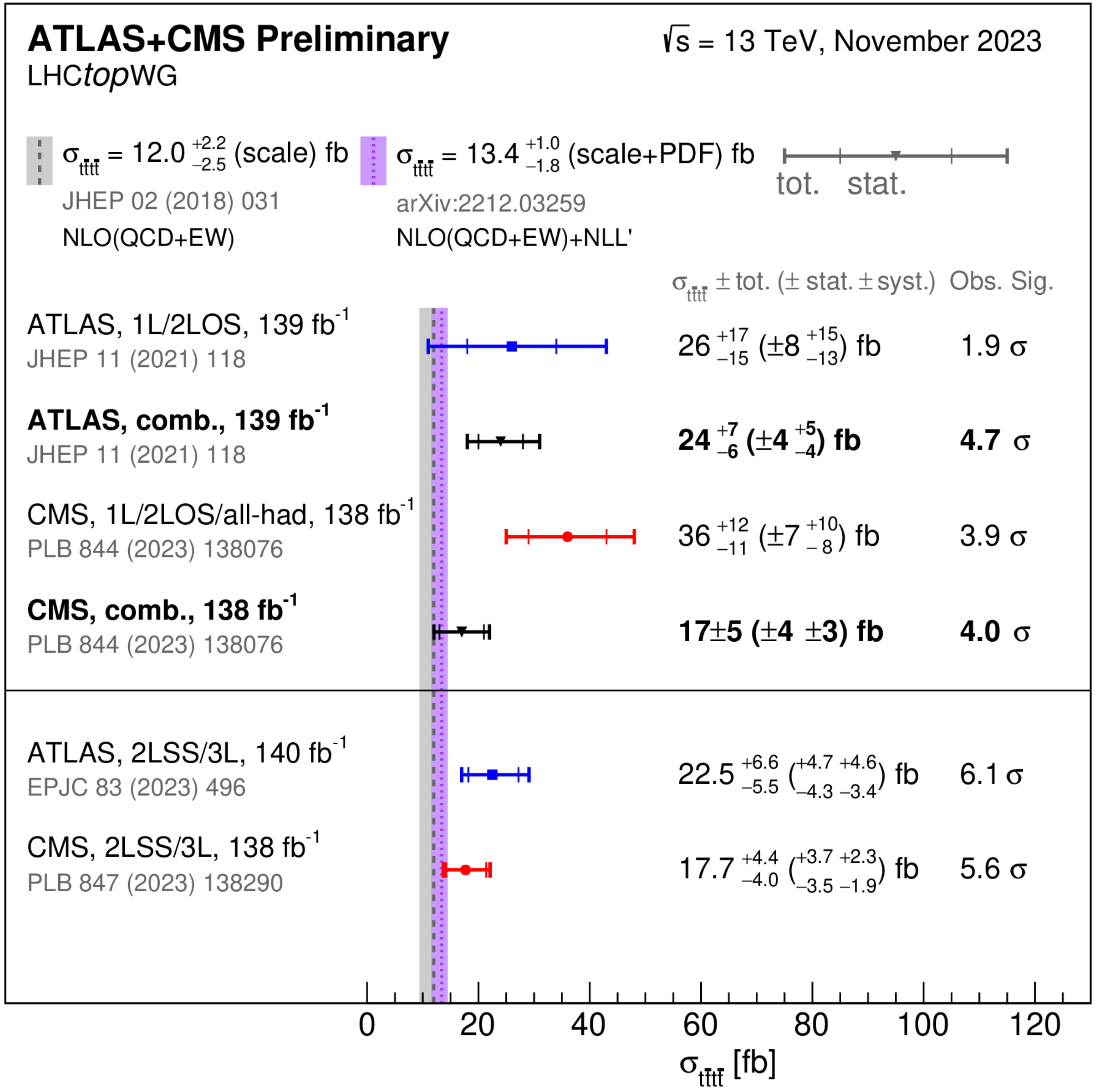
CERN, Geneva, Switzerland

The production of four top quarks in proton-proton collisions is one of the rarest processes of the Standard Model.

Observed for the first time in 2023 at the LHC.

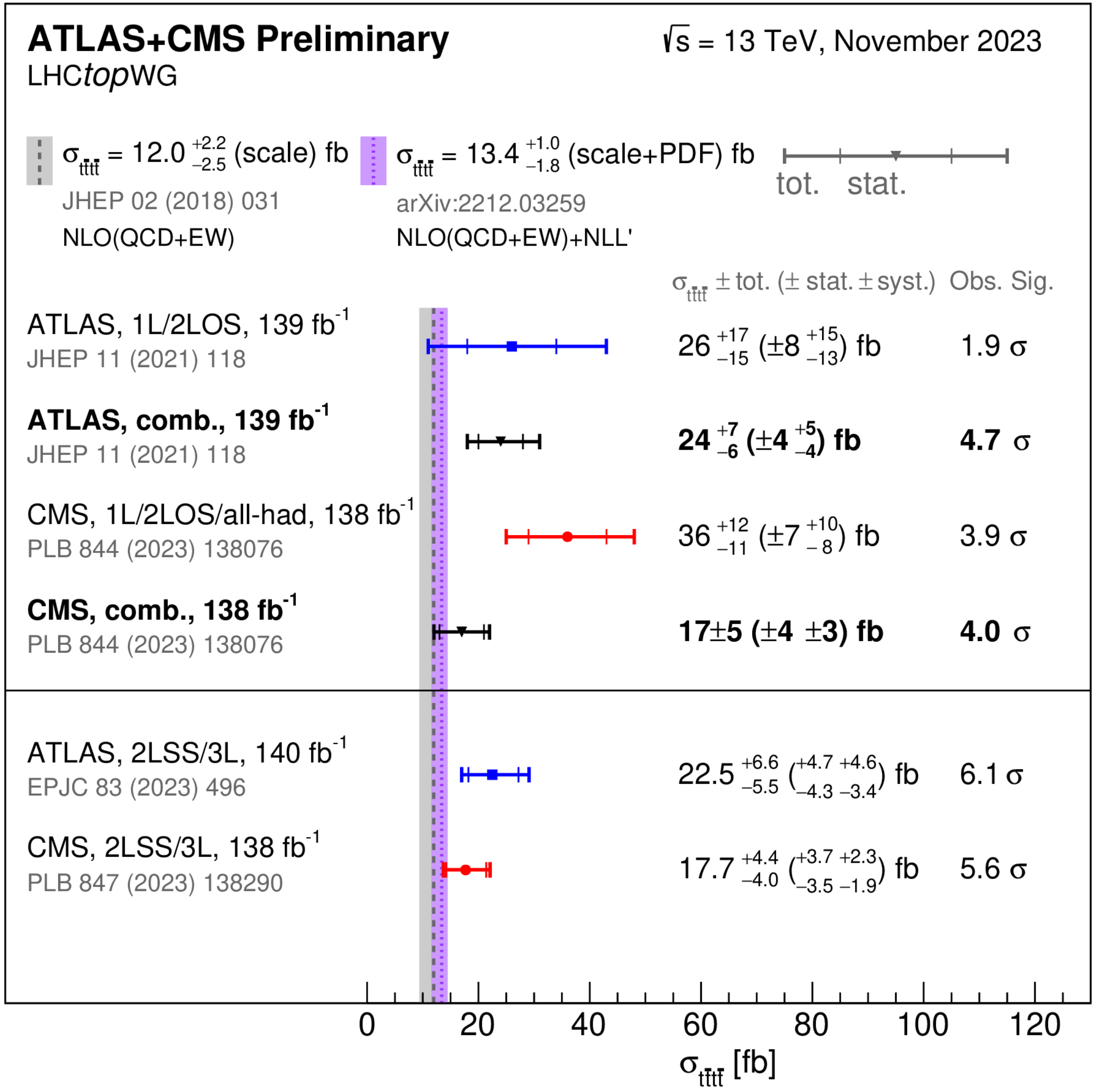


ATLAS+CMS Preliminary
LHCtopWG
√s = 13 TeV, November 2023
σ tttt = 12.0 +2.2 −2.5 (scale) fb
JHEP 02 (2018) 031
NLO(QCD+EW)
σ tttt = 13.4 +1.0 −1.8 (scale+PDF) fb
arXiv:2212.03259
NLO(QCD+EW)+NLL'
tot. stat.
σ tttt ± tot. (± stat. ± syst.) Obs. Sig.
ATLAS, 1L/2LOS, 139 fb-1
JHEP 11 (2021) 118
26 +17 −15 (±8 +15 −13) fb 1.9 σ
ATLAS, comb., 139 fb-1
JHEP 11 (2021) 118
24 +7 −6 (±4 +5 −4) fb 4.7 σ
CMS, 1L/2LOS/all-had, 138 fb-1
PLB 844 (2023) 138076
36 +12 −11 (±7 +10 −8) fb 3.9 σ
CMS, comb., 138 fb-1
PLB 844 (2023) 138076
17±5 (±4 ±3) fb 4.0 σ
ATLAS, 2LSS/3L, 140 fb-1
EPJC 83 (2023) 496
22.5 +6.6 −5.5 (+4.7 −4.3 +4.6 −3.4) fb 6.1 σ
CMS, 2LSS/3L, 138 fb-1
PLB 847 (2023) 138290
17.7 +4.4 −4.0 (+3.7 −3.5 +2.3 −1.9) fb 5.6 σ
0 20 40 60 80 100 120
σ tttt [fb]



Consistent with Standard Model predictions:

- ATLAS: 1.8, 1.7 standard deviations;
- CMS: 1.3, 1.1 standard deviations;



Consistent with Standard Model predictions:

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- CMS: 1.3, 1.1 standard deviations;

HL-LHC → reduction of experimental uncertainties.

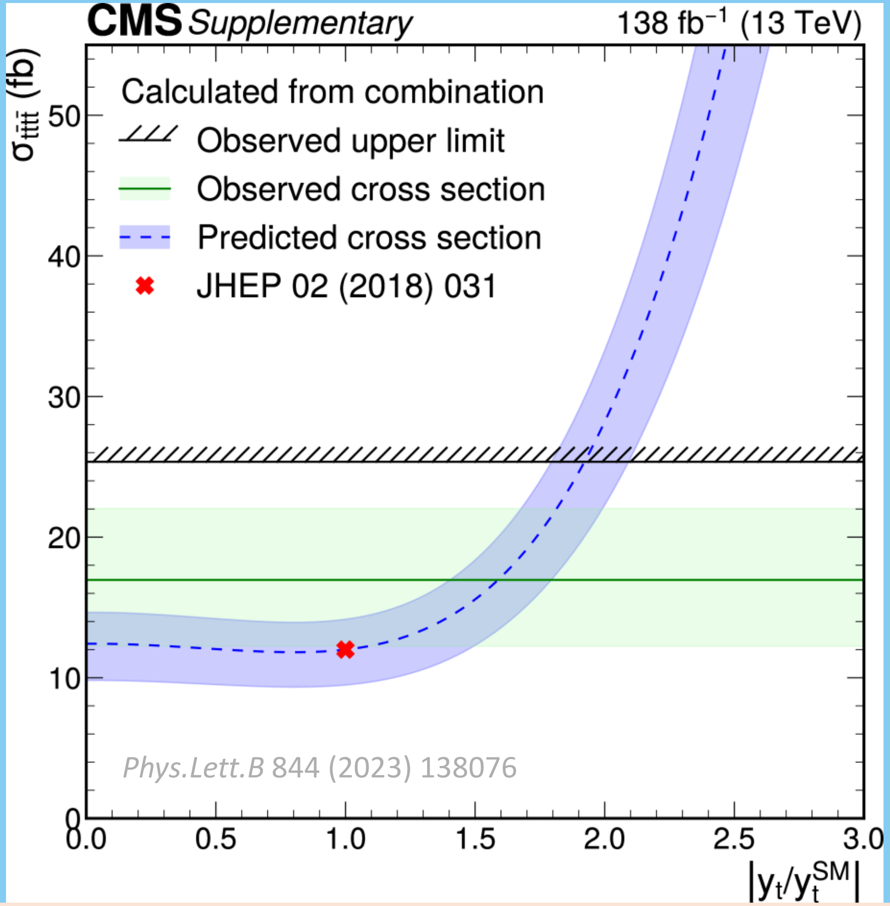
Accuracy of theoretical predictions must improve as well.

Sensitive to top-Yukawa coupling

Can be used to constrain width of Higgs boson

Can hide new physics (BSM)

Can constrain operators in EFT



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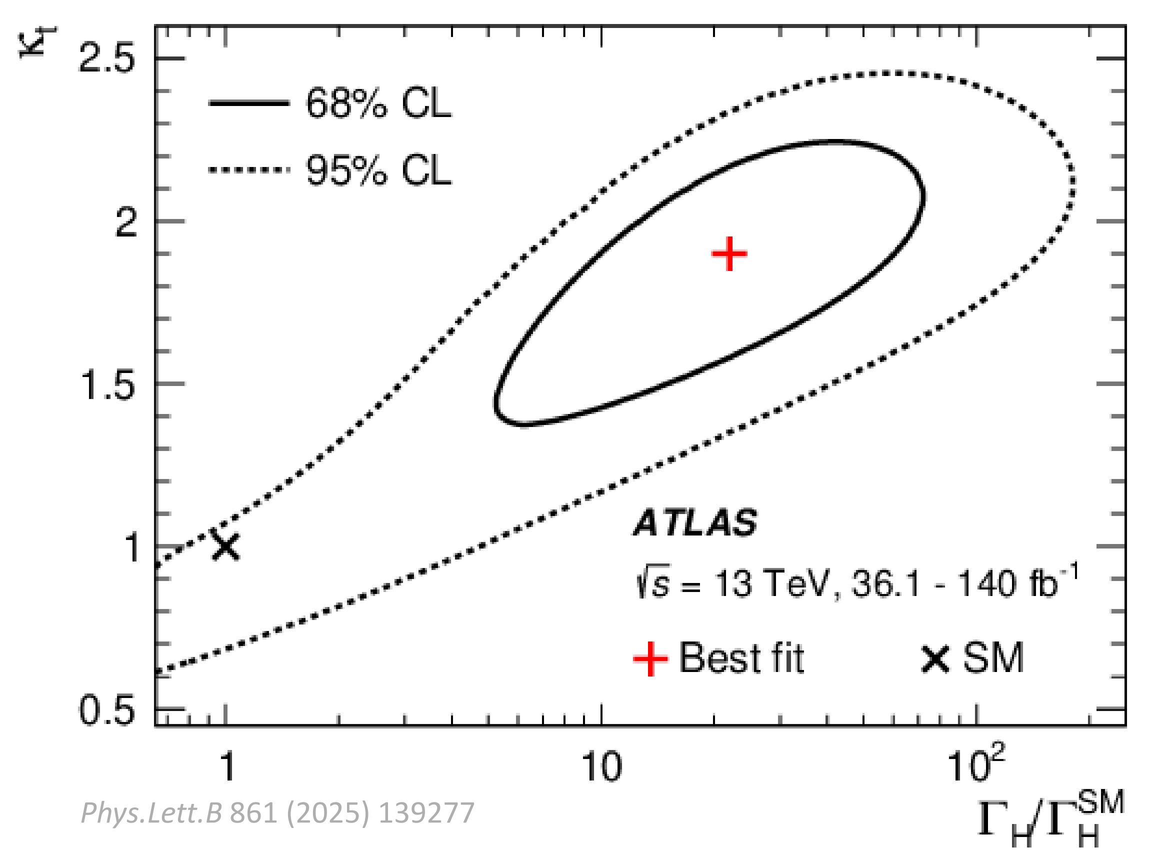

Phys.Lett.B 861 (2025) 139277

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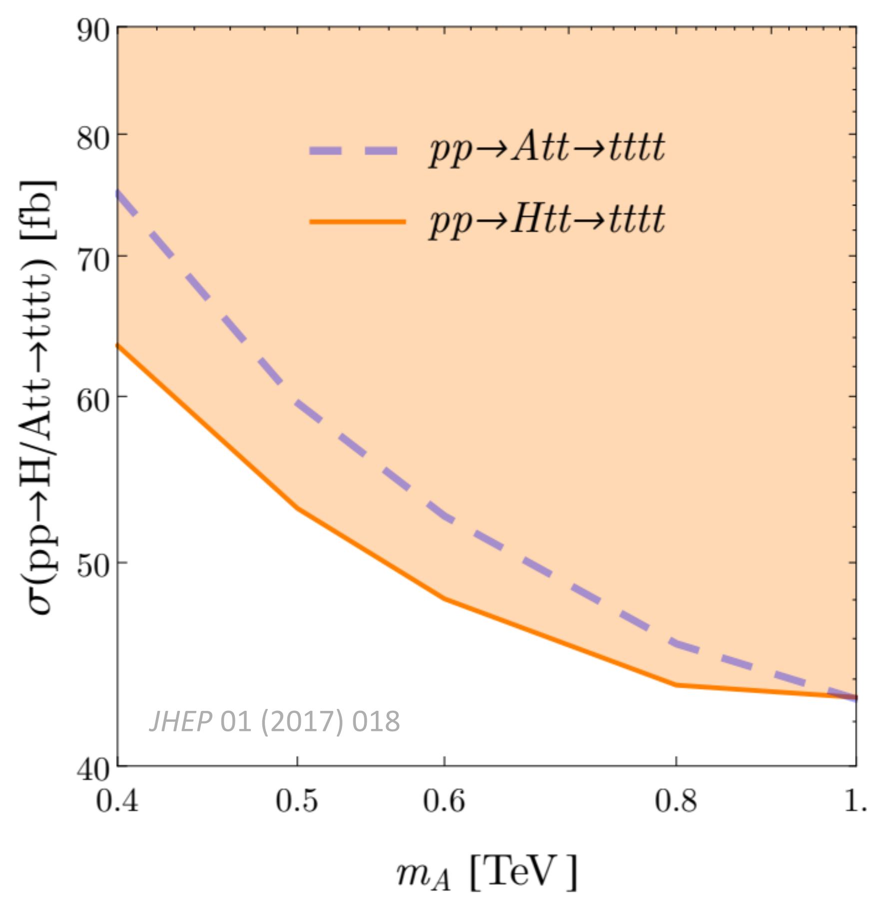

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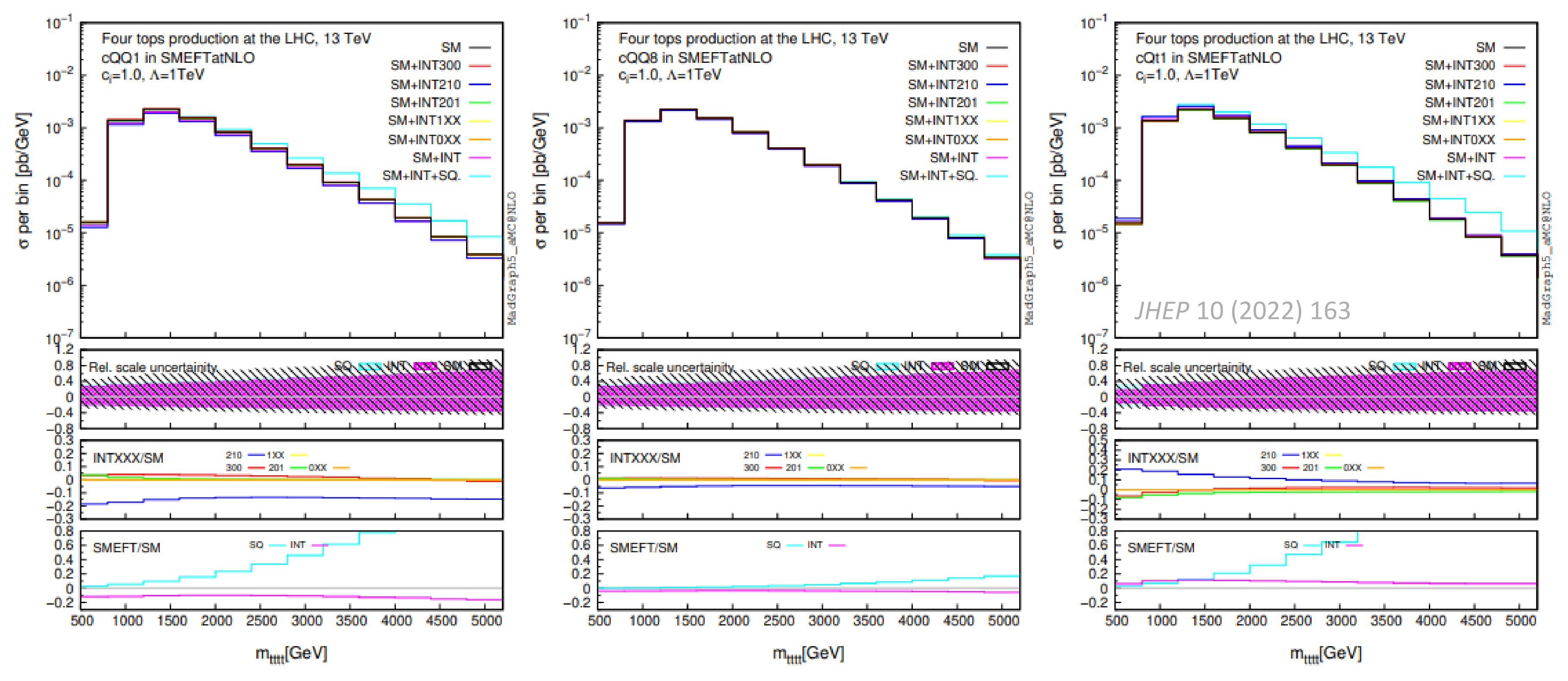

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State-of-the-art $t\bar{t}t\bar{t}$ theory

- First calculations of NLO QCD corrections in *[Bevilacqua, Worek ’12]*
- Matched with parton shower and studied in aMC@NLO *[Alwall et al. ’14][Maltoni, Pagani, Tsinikos ’15]*
- Full set of EW corrections added in *[Frederix, Pagani, Zaro ’17]*
- Spin correlations in LO top quark decays within the framework of Powheg Box *[Jezo, Krauss ’21]*
- Effect of soft-gluon corrections at NLO+NLL’ in the absolute-mass threshold formalism studied for the first time in *[van Beekveld, Kulesza, Moreno Valero ’22]*
- Spin correlations in NLO top quark decays using NWA *[Bevilacqua, Worek ’24]*

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- Spin correlations in NLO top quark decays using NWA *[Bevilacqua, Worek '24]*
- Effect of soft-gluon corrections at NLO+NLL' in the **invariant-mass threshold formalism** *[presented today]*

2

QCD CALCULATIONS

Cross section in hadron-hadron collisions

$$\sigma_{h_1h_2\to X}(S) = \sum_{a,b} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \, f_{a/h_1}(x_1,\mu_F) f_{b/h_2}(x_2,\mu_F) \int \mathrm{d}\hat{s} \, \delta(\hat{s} - x_1x_2S) \, \hat{\sigma}_{ab\to X}(\hat{s},\mu_R,\mu_F)$$

Cross section in hadron-hadron collisions

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Partonic cross section

$$\hat{\sigma}_{ab}(\hat{s},\mu_R,\mu_F) = \alpha_s^n(\mu_R)\left[\hat{\sigma}_{ab}^{(0)}(\hat{s}) + \frac{\alpha_s(\mu_R)}{\pi}\hat{\sigma}_{ab}^{(1)}(\hat{s},\mu_R,\mu_F) + \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^2\hat{\sigma}_{ab}^{(2)}(\hat{s},\mu_R,\mu_F) + \ldots\right]$$

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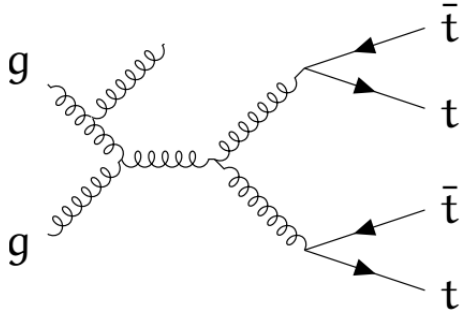
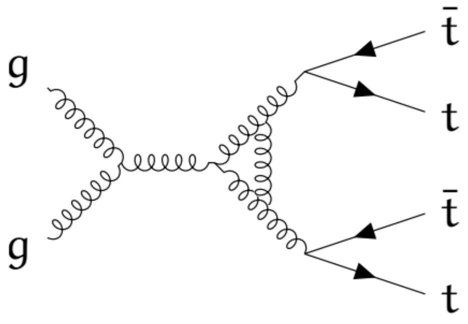
NLO: real corrections



NLO: virtual corrections



Cancellation of IR divergences between reals and virtuals (**KLN theorem**). Consider $d\sigma/dQ$. Terms proportional to

$$\alpha_s^n\left[\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}}\right]_+ , \quad m=0,1$$

survive, where

$$\hat{\rho} = \frac{Q^2}{s} \qquad s=(p_a+p_b)^2 \qquad Q^2 = p_X^2$$

They provide important contributions to the cross section in the limit $\hat{\rho}\to 1$.

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The more terms we include, the more precise the calculation:

… and the more complicated the calculation

Estimate of missing higher order terms (theoretical uncertainty)

When feasible, calculation of NNLO, N3LO, etc…

When not feasible, use approximations (e.g. soft gluon resummation)

Example: $gg \to H$

J.S. Hoff, 2015

Order	Channel	Number of diagrams (fermionic loops)		
LO	gg	#1		
NLO	$q\bar{q}$	#1		
	qg	#1		
	gg_{virt}	#10		
	gg_{real}	#38		
	Σ	#50		
NNLO	qq'	#1		
	qq	#2		
	$q\bar{q}$	#84	= #81	+#3 n_l
	qg	#124	= #122	+#2 n_l
	gg_{virt}	#294	= #252	+#42 n_l
	gg_{real}	#2458	= #2293	+#165 n_l
	Σ	#2964	= #2752	+#212 n_l

Partonic cross section

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Scale variation

$$\left(\frac{\mu_R}{\mu_0},\frac{\mu_F}{\mu_0}\right) \in \{(0.5,0.5),(0.5,1),(1,0.5),(1,1),(1,2),(2,1),(2,2)\}.$$

Partonic cross section

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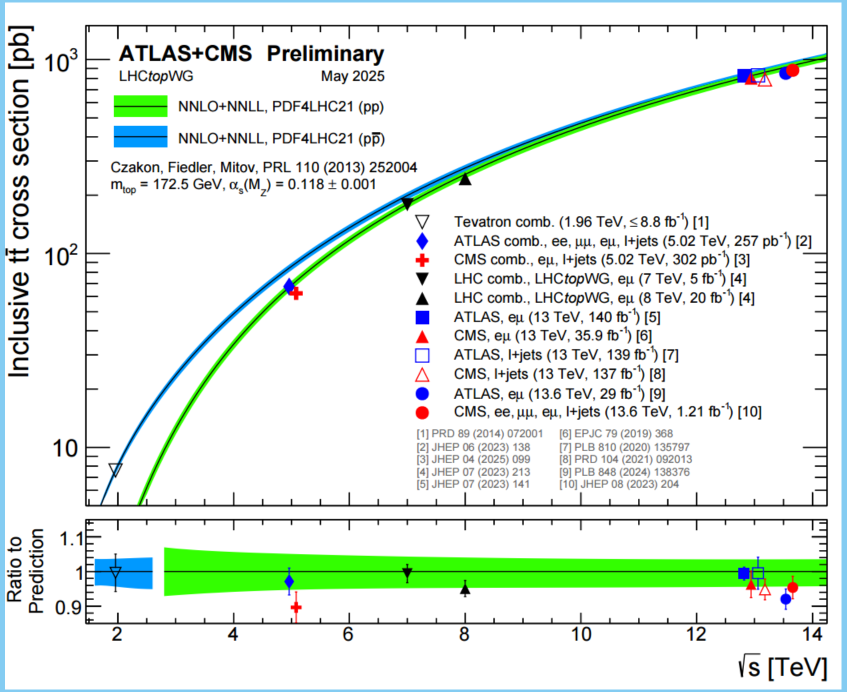

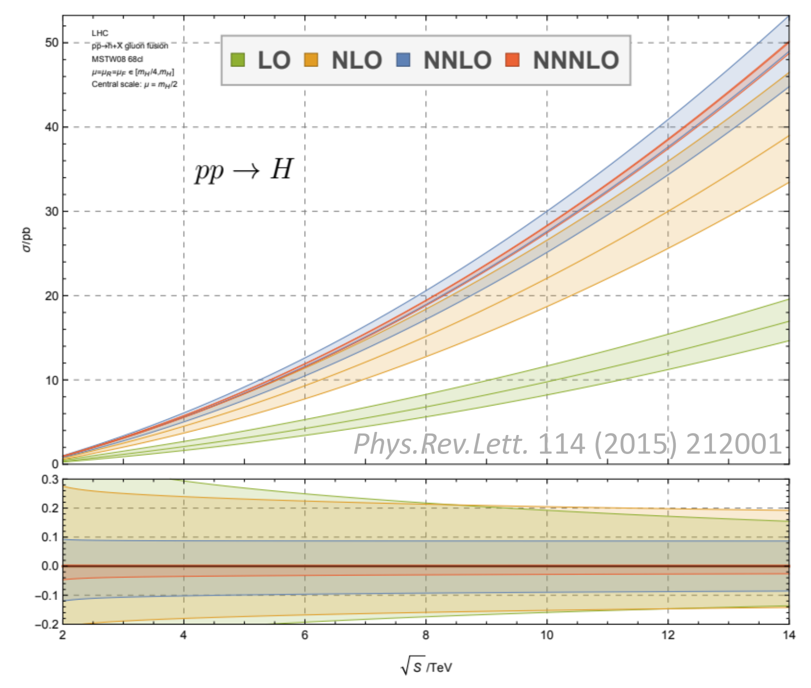

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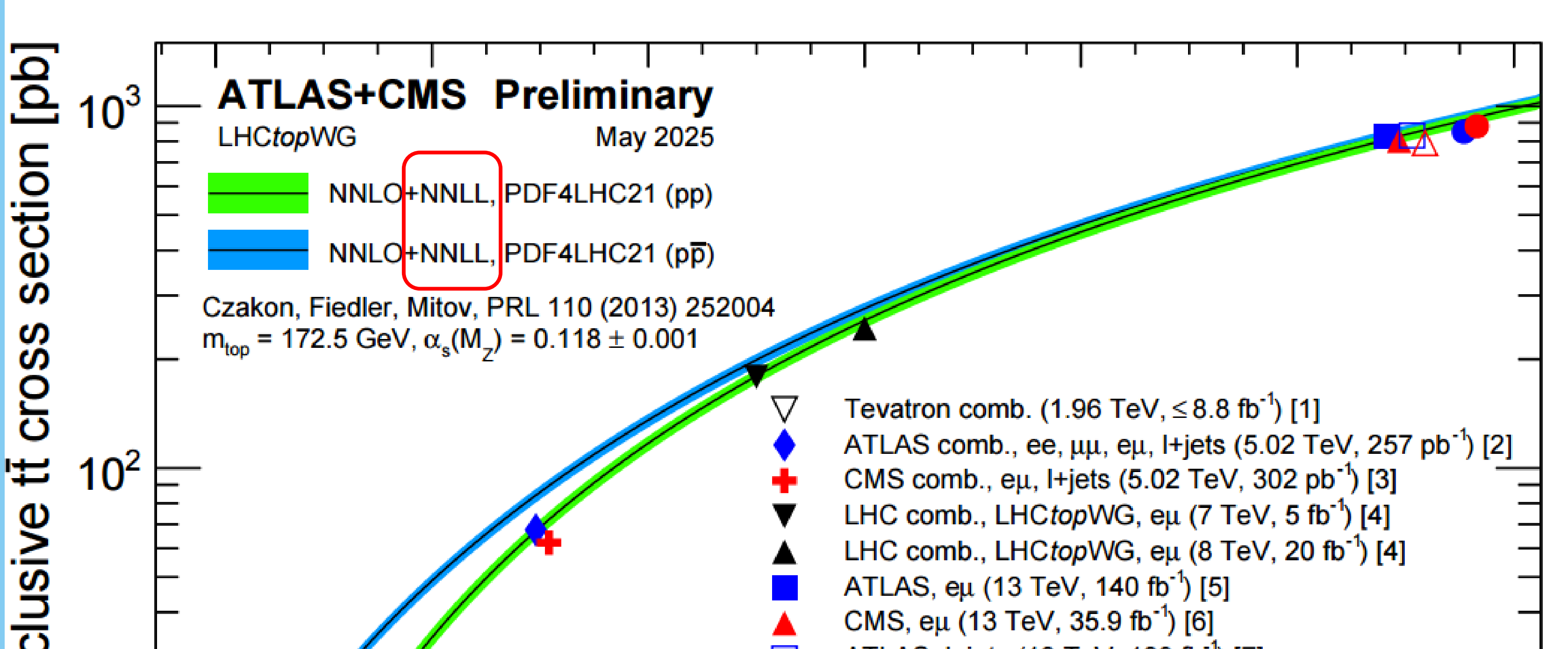

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The more terms we include, the more precise the calculation:

… and the more complicated the calculation

Estimate of missing high…

When feasible, calculation of…

When not feasible, use approximations

at NLO

Cancellation of IR divergences between reals and virtuals (**KLN theorem**). Consider $d\sigma/dQ$. Terms proportional to

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The more terms we include, the more precise the calculation:

… and the more

Estimate of missing

When feasible, calculation of

When not feasible, use approximations (e.g. soft gluon resummation)

at NLO

valid to all orders

Cancellation of IR divergences between reals and virtuals (**KLN theorem**). Consider $d\sigma/dQ$. Terms proportional to

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They provide important contributions to the cross section in the limit $\hat{\rho} \to 1$.

Partonic cross section

$$\hat{\sigma}_{ab}(\hat{s},\mu_R,\mu_F) = \alpha_s^n(\mu_R)\left[\hat{\sigma}_{ab}^{(0)}(\hat{s}) + \frac{\alpha_s(\mu_R)}{\pi}\hat{\sigma}_{ab}^{(1)}(\hat{s},\mu_R,\mu_F) + \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^2 \hat{\sigma}_{ab}^{(2)}(\hat{s},\mu_R,\mu_F) + \ldots\right]$$

The more terms we include, the more precise the calculation:

... and the more complicated the calculation

Estimate of missing higher order terms (theoretical uncertainty)

When feasible, calculation of NNLO, N3LO, etc...

When not feasible, use approximations (e.g. soft gluon resummation)

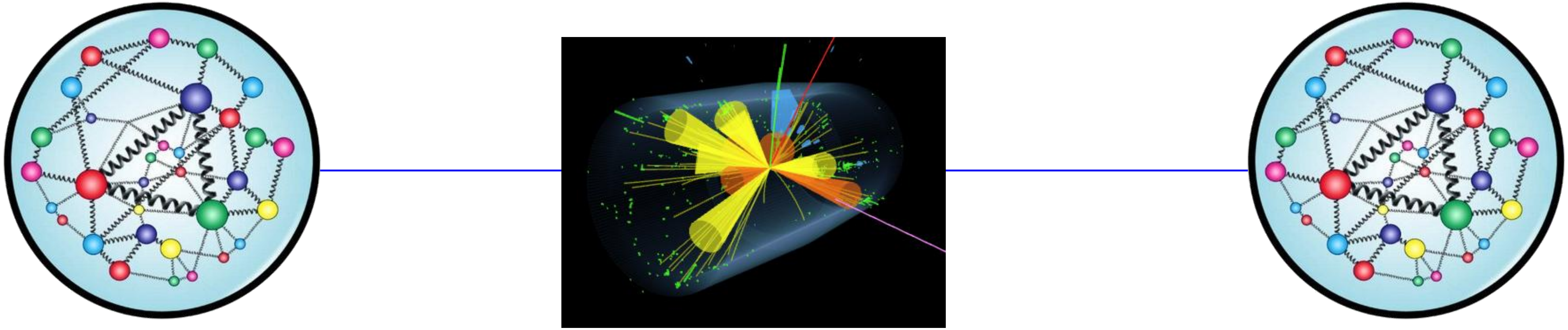
valid to all orders

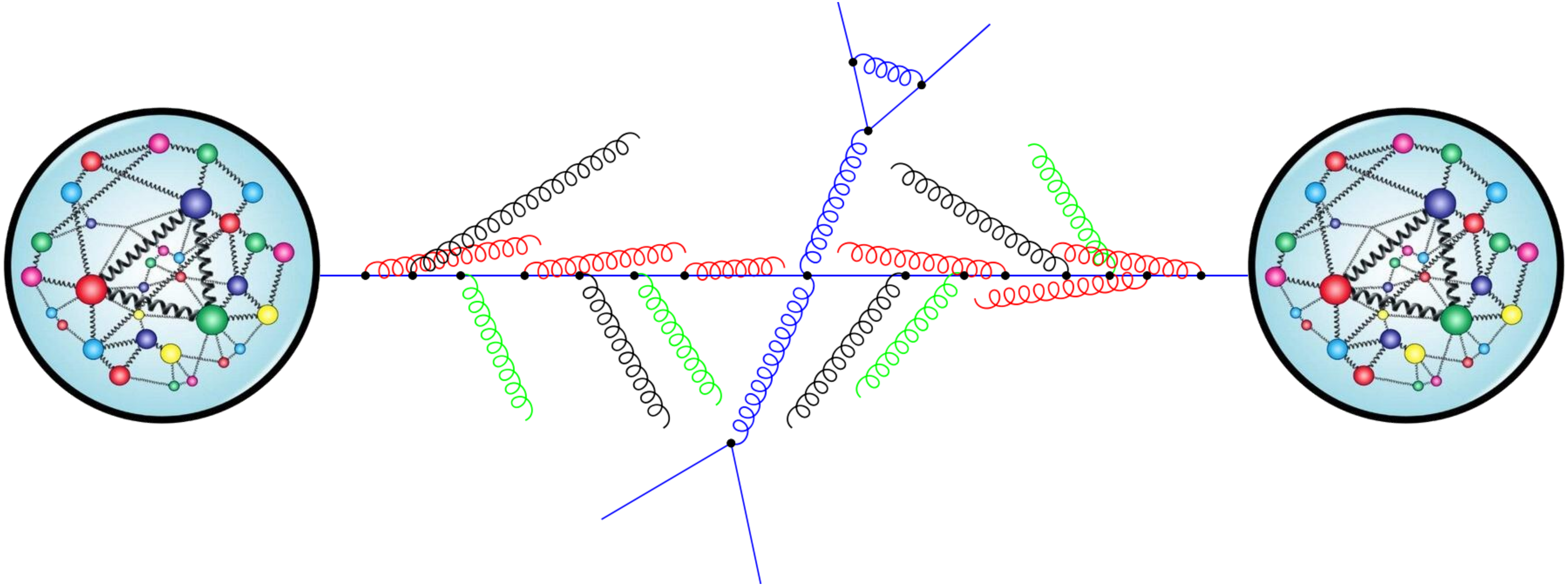
We consider in the soft gluon emission limit $\hat{\rho} \to 1$ all-orders contributions proportional to

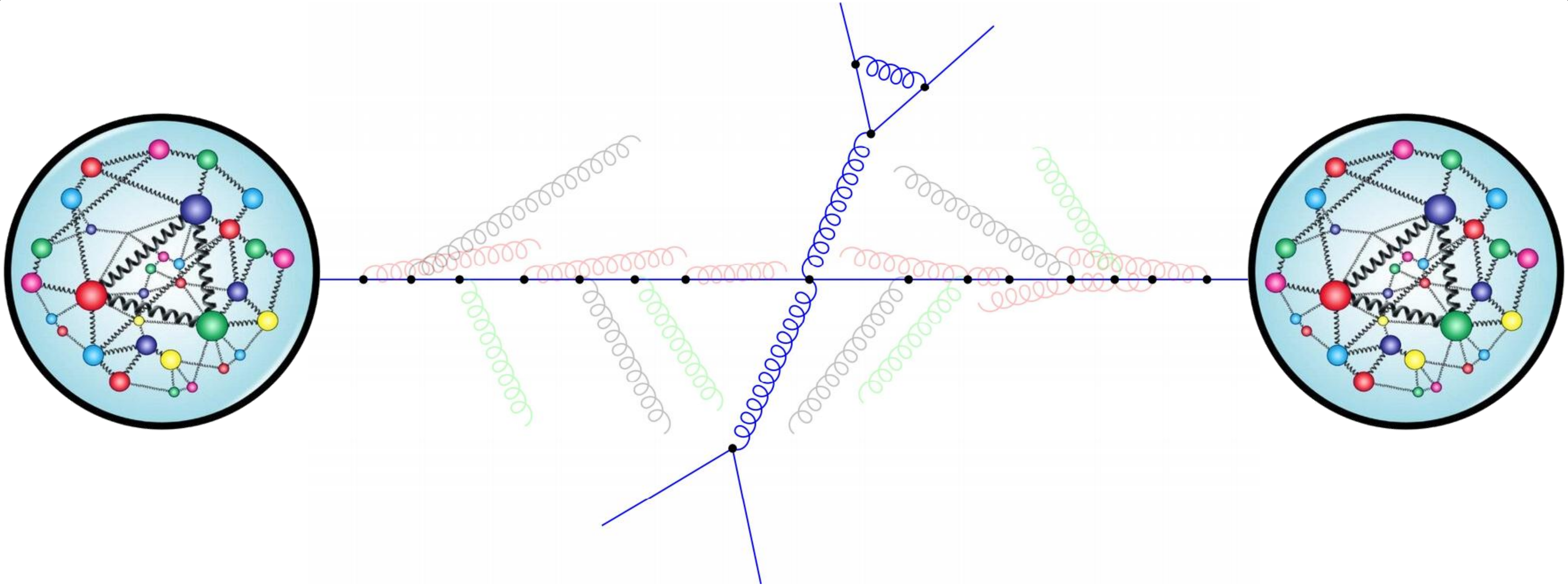
$$\alpha_s^n \left[\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}}\right]_+, \quad m \leq 2n-1$$

3

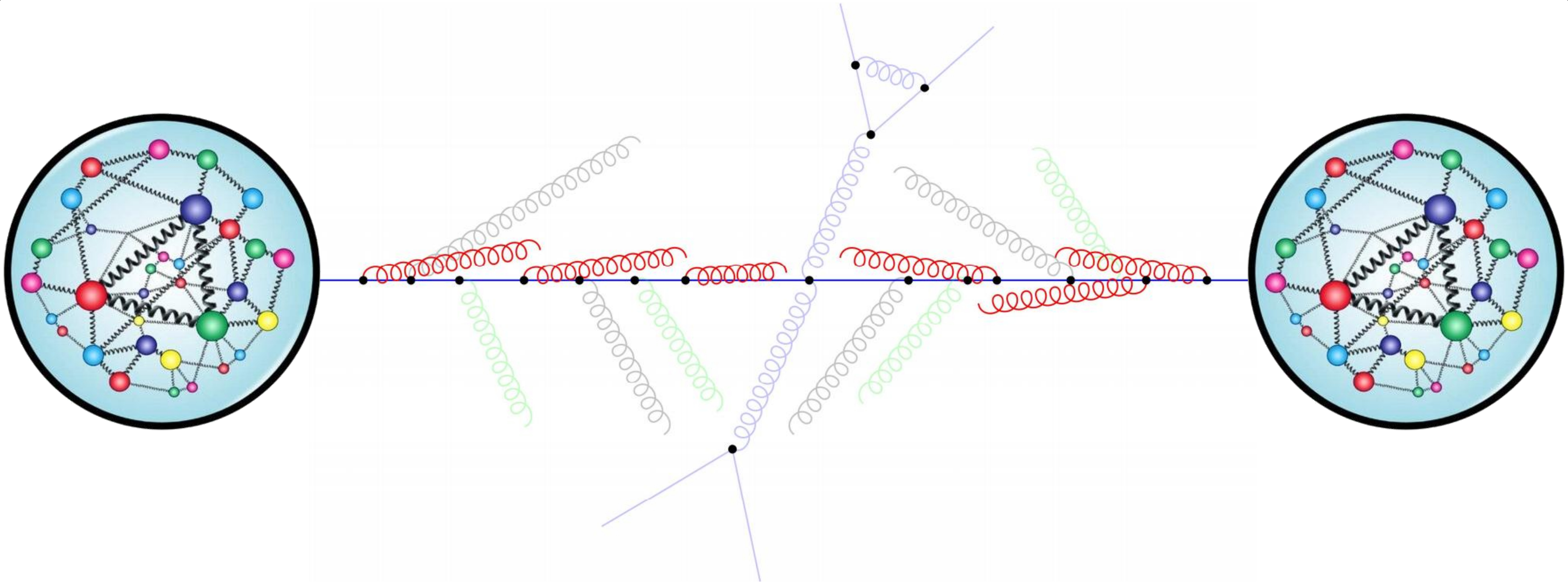
SOFT GLUON RESUMMATION





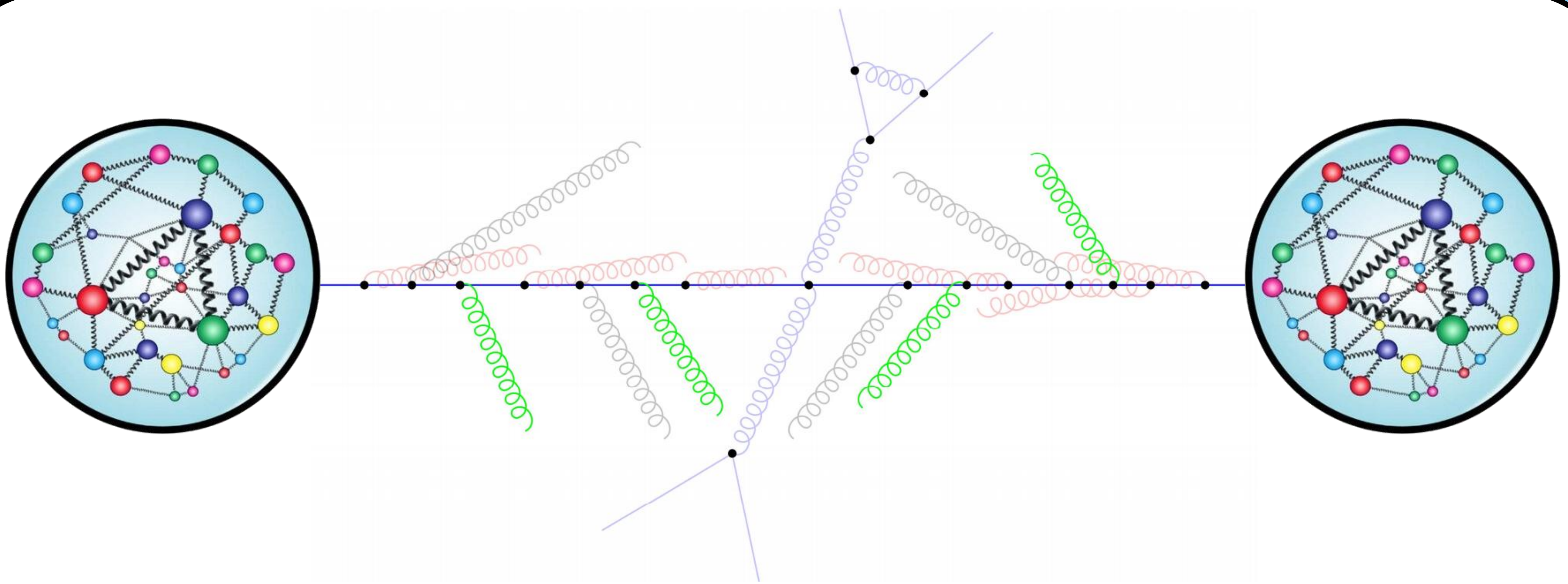


HARD PROCESS



HARD PROCESS

COLLINEAR
EMISSIONS



HARD PROCESS

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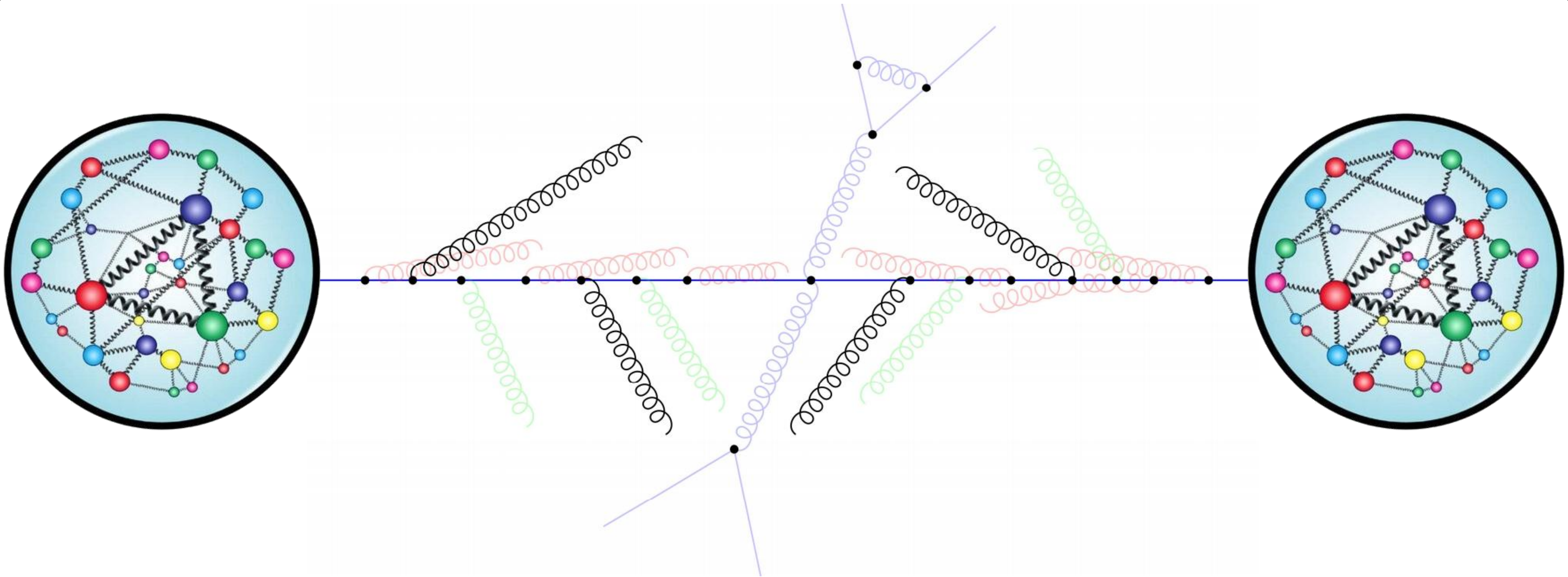
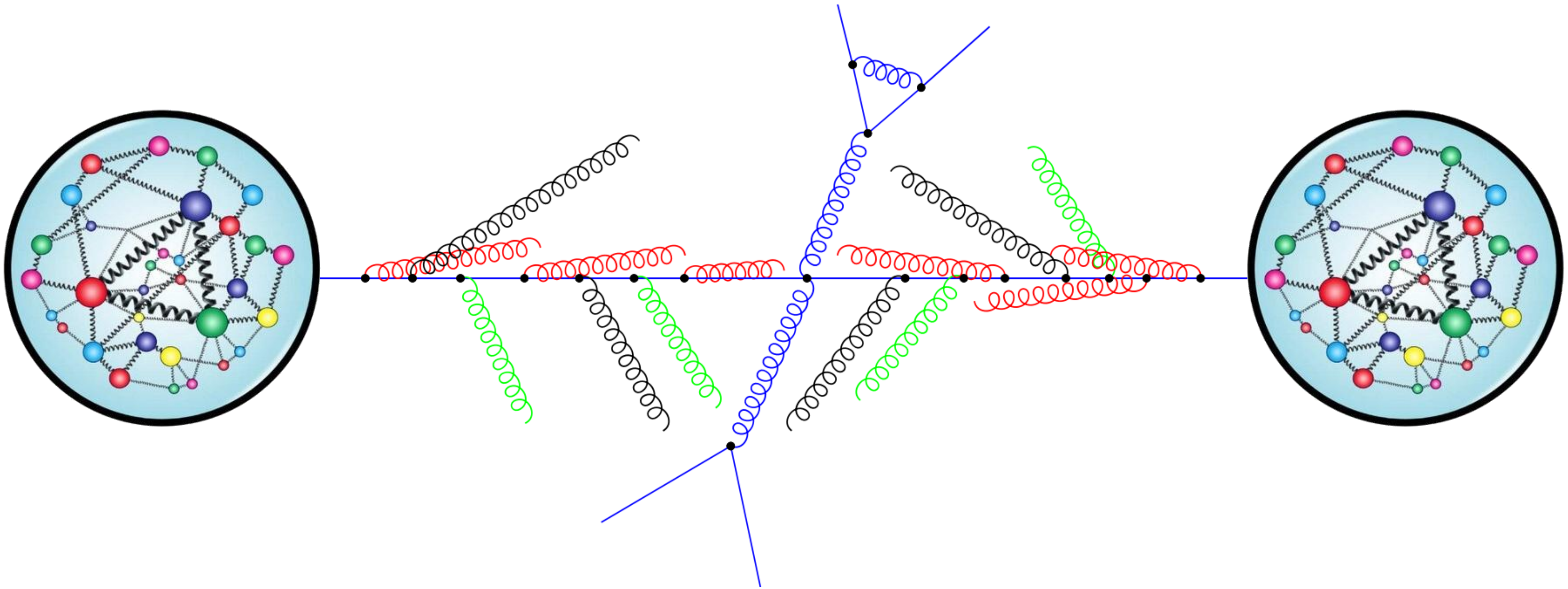


HARD PROCESS

COLLINEAR EMISSIONS

SOFT EMISSIONS

ADDITIONAL HARD EMISSIONS



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SOFT
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ADDITIONAL
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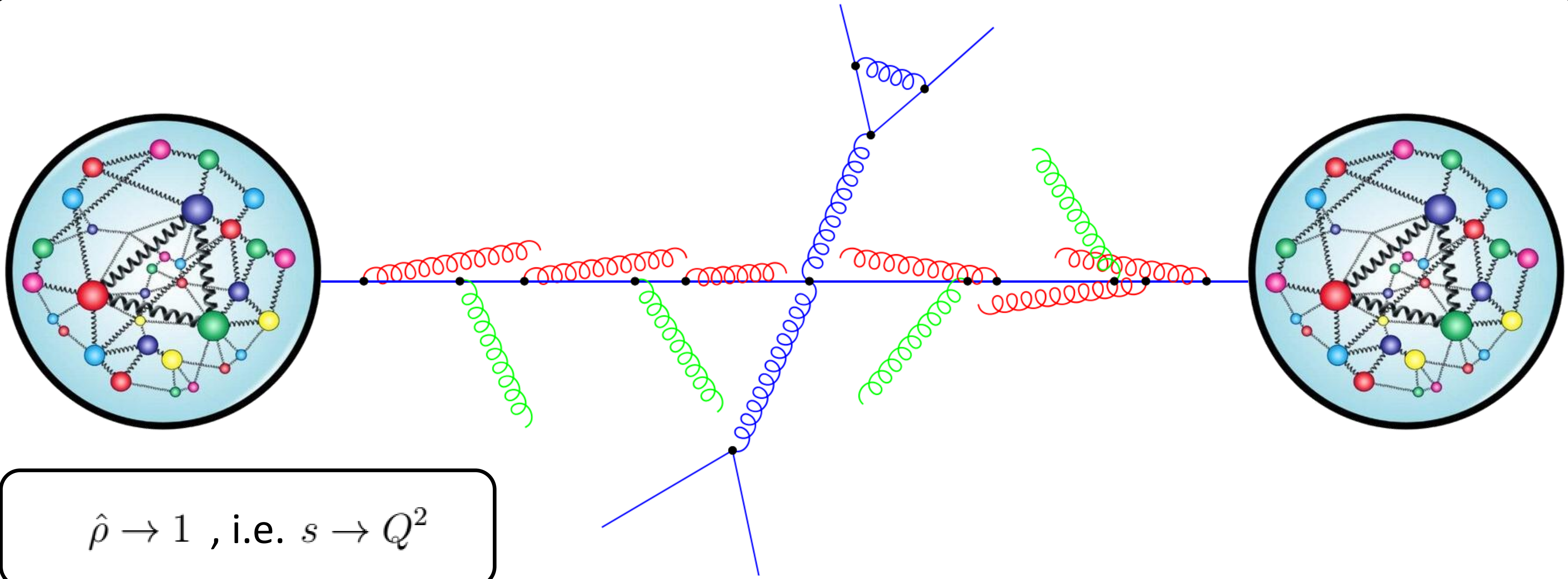


$\hat{\rho} \to 1$, i.e. $s \to Q^2$

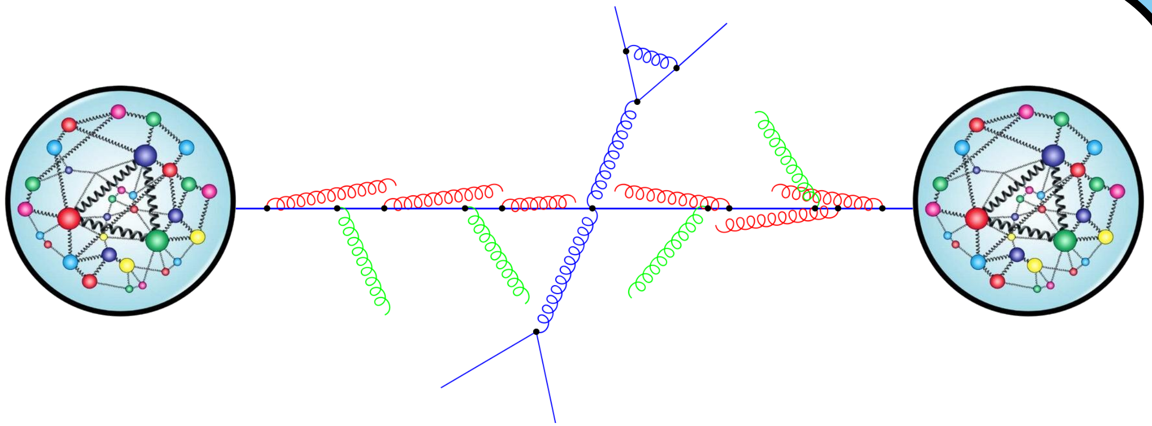
HARD PROCESS

COLLINEAR EMISSIONS

SOFT EMISSIONS

$$\mathcal{A}_{m+n} = \mathcal{A}_m \prod_{i=1}^{n} g_s t_i F_{\text{soft},i}$$

$$F_{\text{soft},i} = \frac{p \cdot \varepsilon}{p \cdot q}$$



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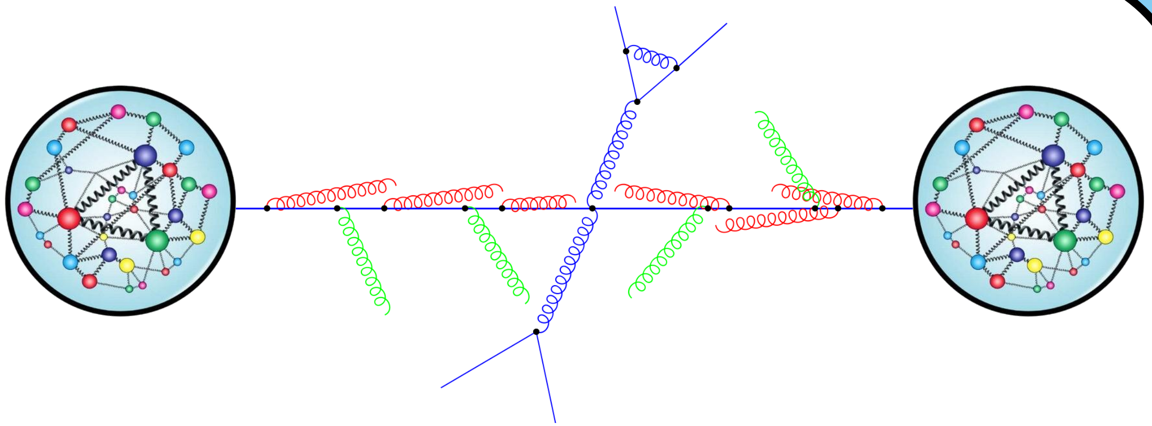

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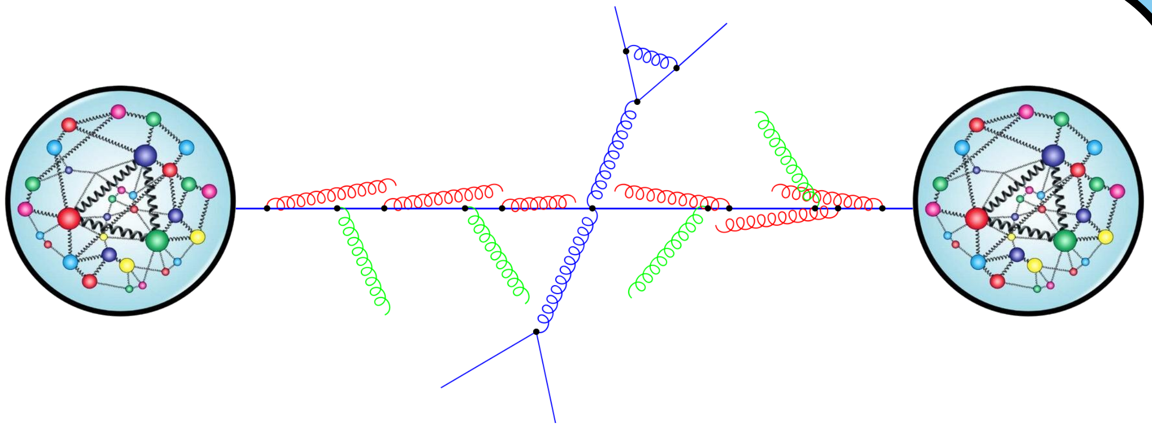

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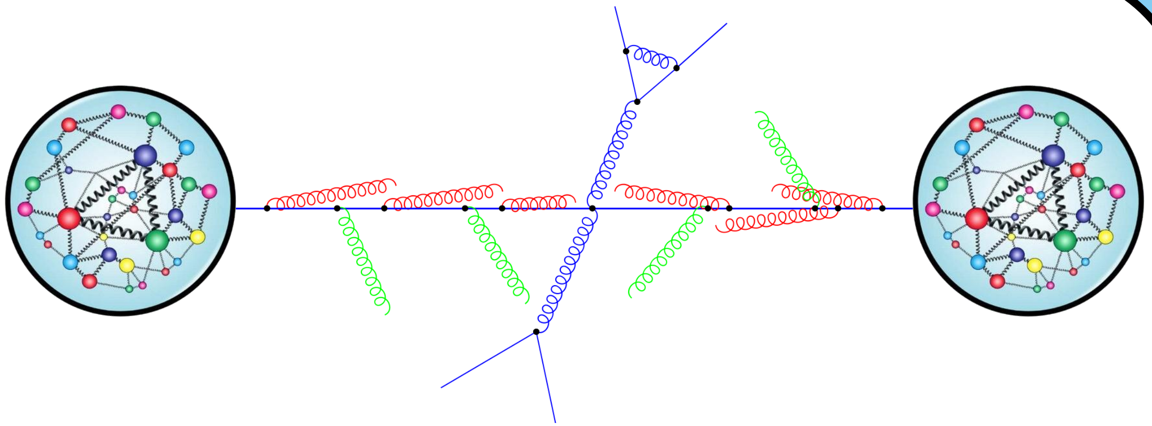

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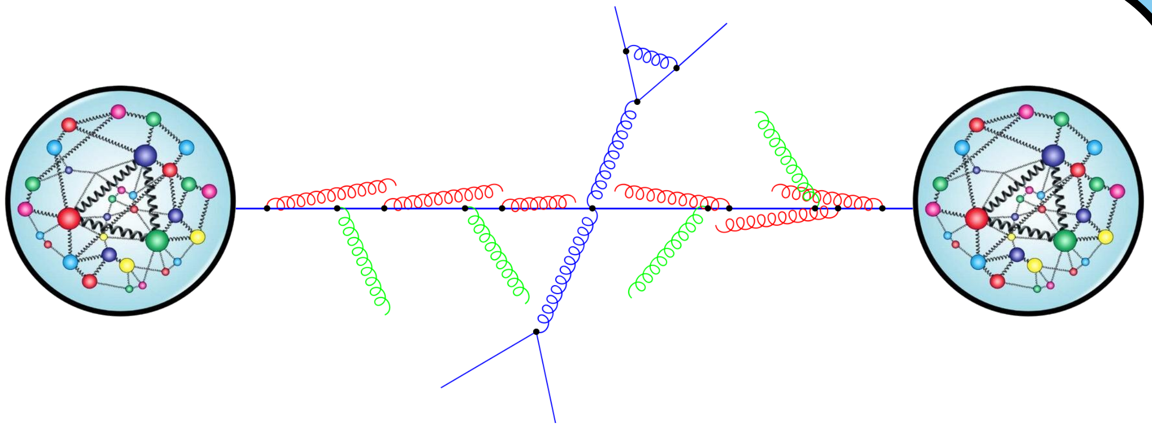
Dirac delta prevents factorization of phase space

$$\int \mathrm{dPS}_{m+n} = \int \mathrm{dPS}_m \left[\prod_{i=1}^{n} \mathrm{d}\Phi_i(z_i)\right] \times \delta\left(\prod_{i=1}^{n}(1-z_i) - \hat{\rho}\right)$$

$$\hat{\rho} = \frac{Q^2}{s} \qquad z_i = \frac{2k_{i,0}}{\sqrt{s}}$$



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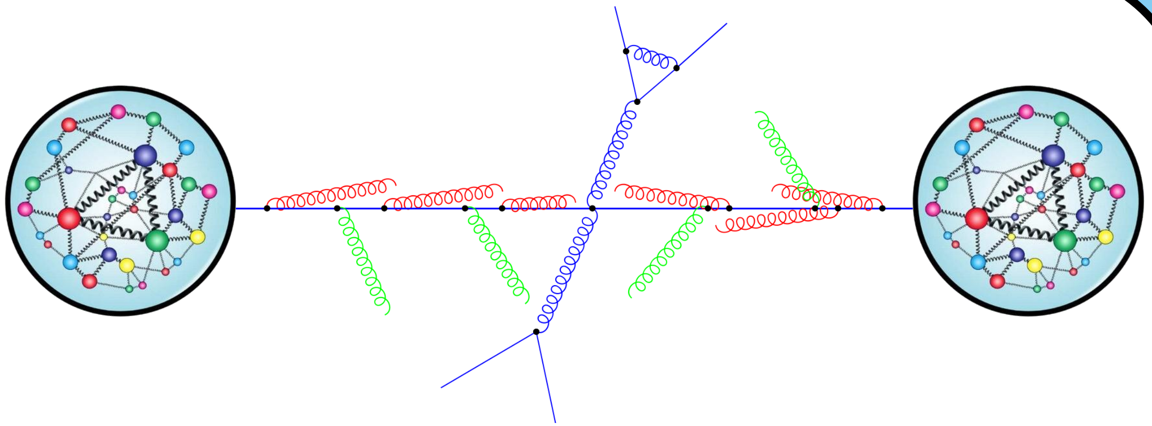

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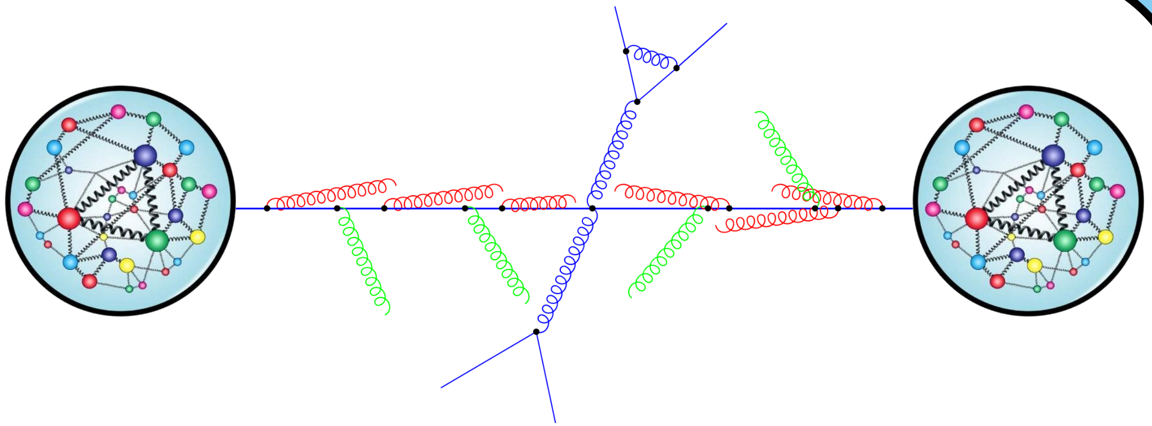
$$\int_0^1 \mathrm{d}\hat{\rho}\, \hat{\rho}^{N-1} \delta\left(\prod_{i=1}^{n}(1-z_i) - \hat{\rho}\right) = \prod_{i=1}^{n}(1-z_i)^{N-1}$$



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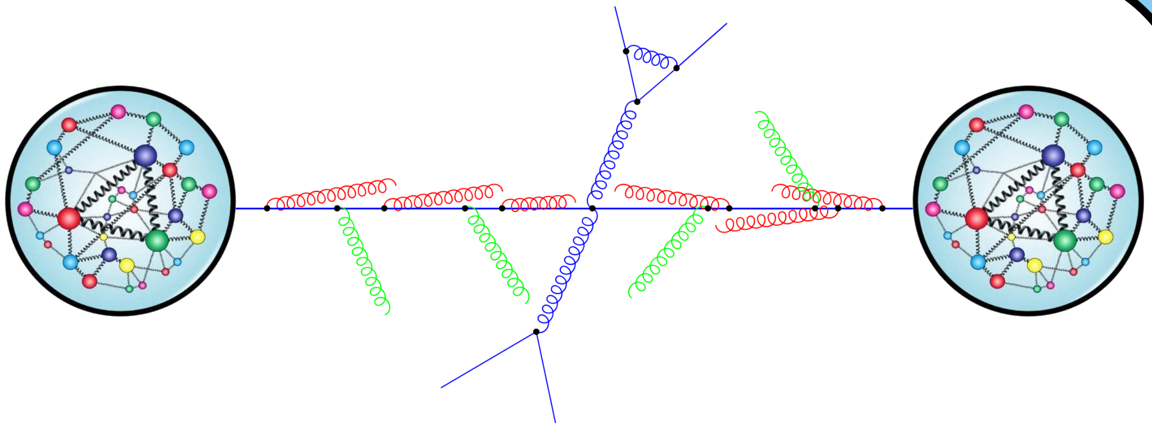

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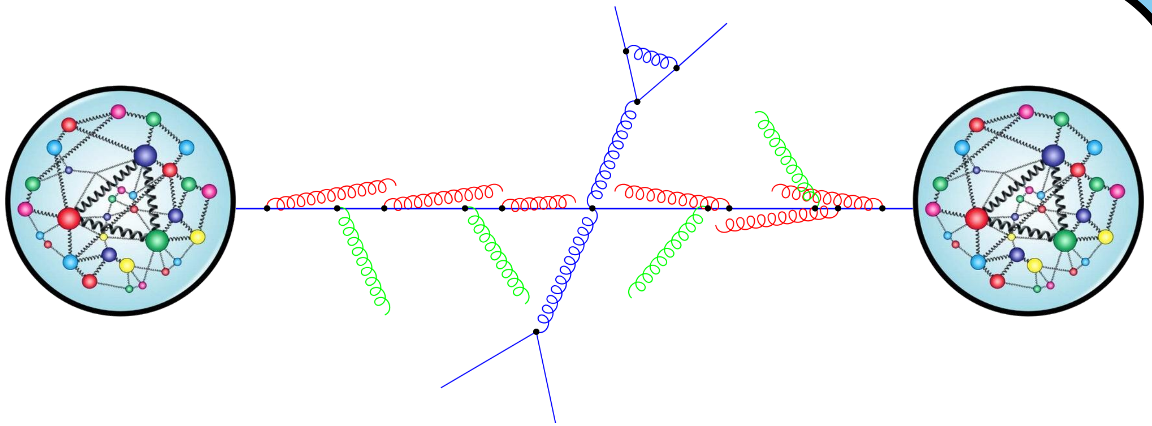
$$\frac{d\tilde{\sigma}_{pp\to t\bar{t}t\bar{t}}(N)}{dQ} = \int_0^1 d\rho\, \rho^{N-1} \frac{d\sigma_{pp\to t\bar{t}t\bar{t}}(\rho)}{dQ} \qquad \rho = \frac{Q^2}{S}$$



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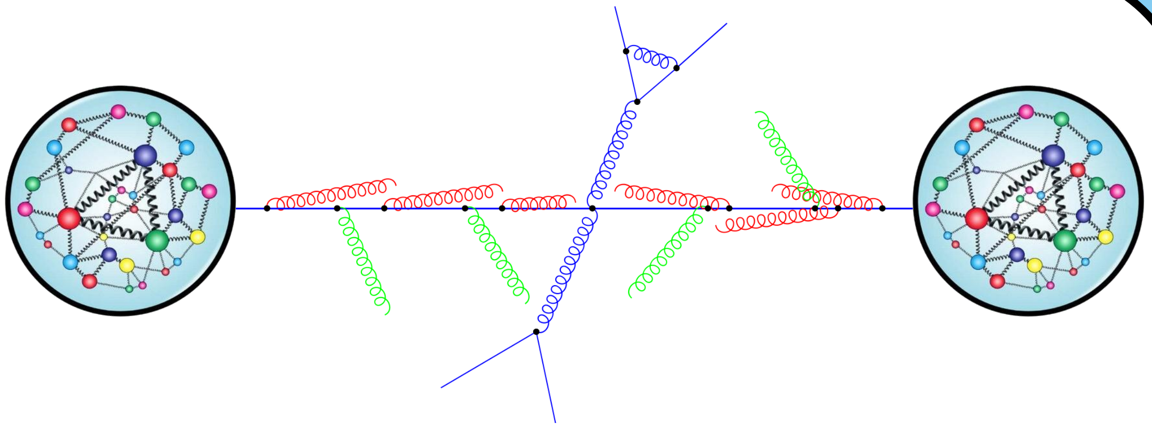

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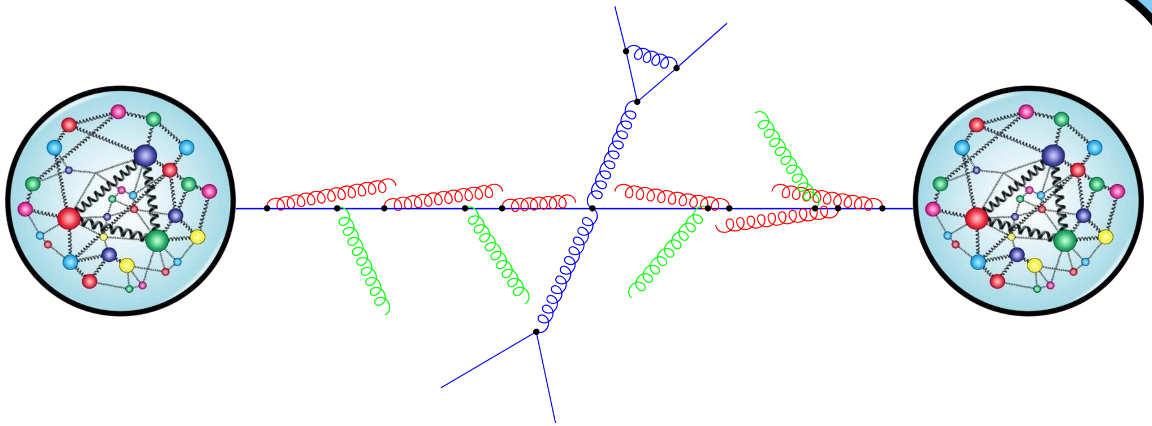

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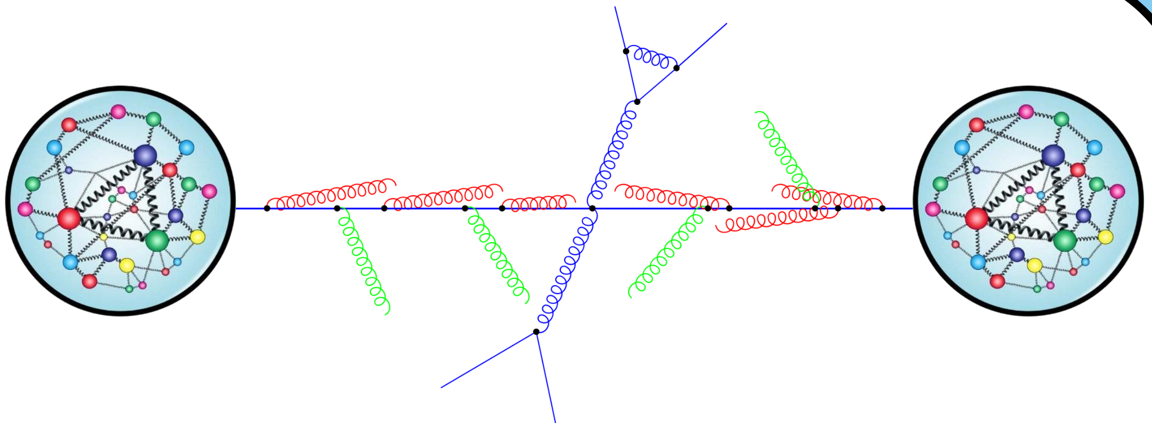

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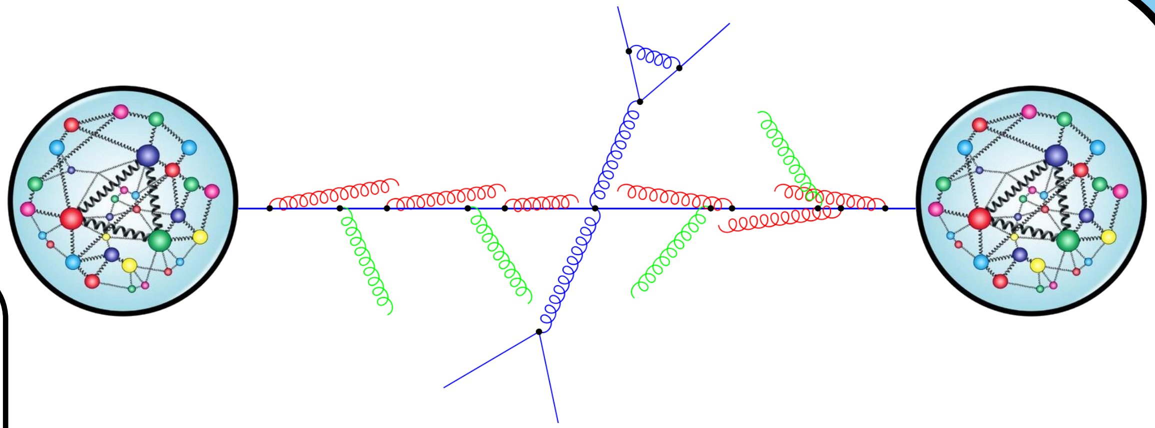
$$\frac{d\tilde{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N)}{dQ} = \text{Tr}\left[\bar{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}(N+1)\,\mathbf{H}_{ij\to t\bar{t}t\bar{t}}\right]\Delta_i(N+1)\Delta_j(N+1)$$



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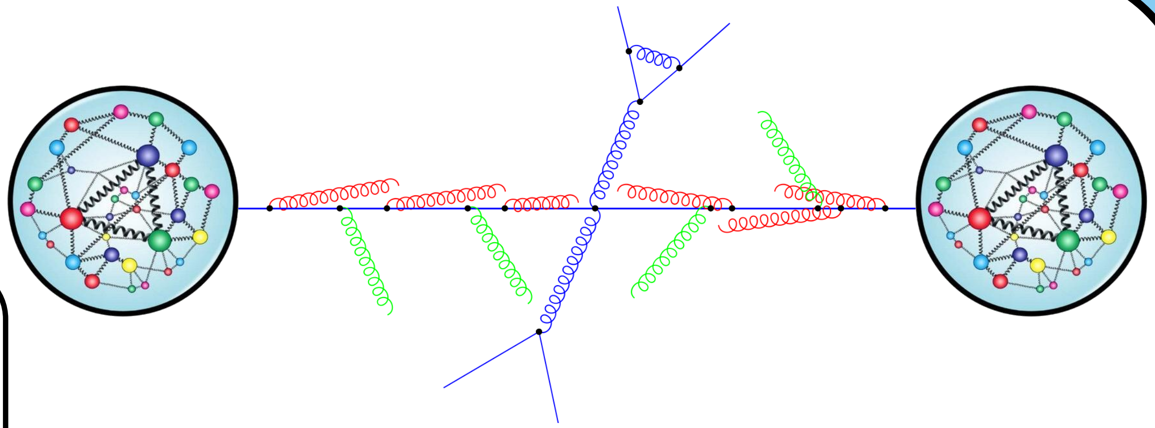

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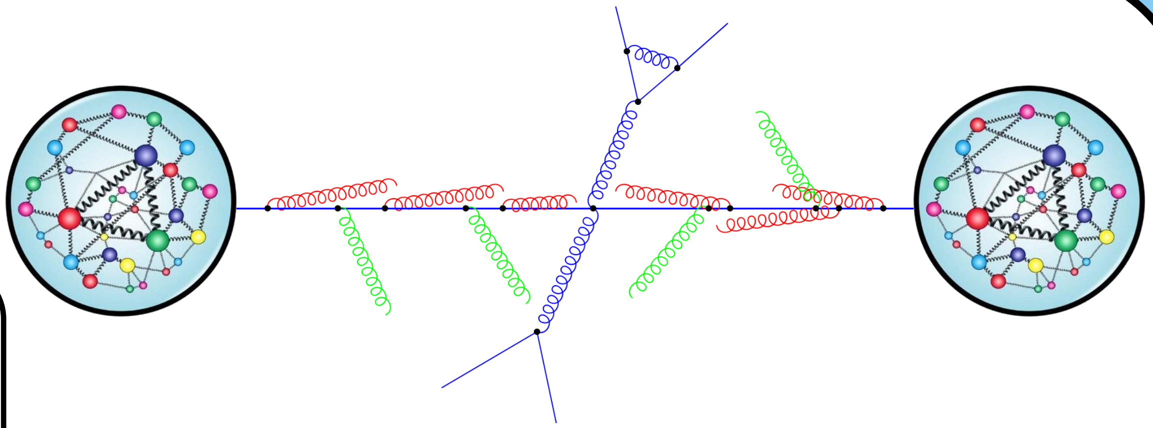

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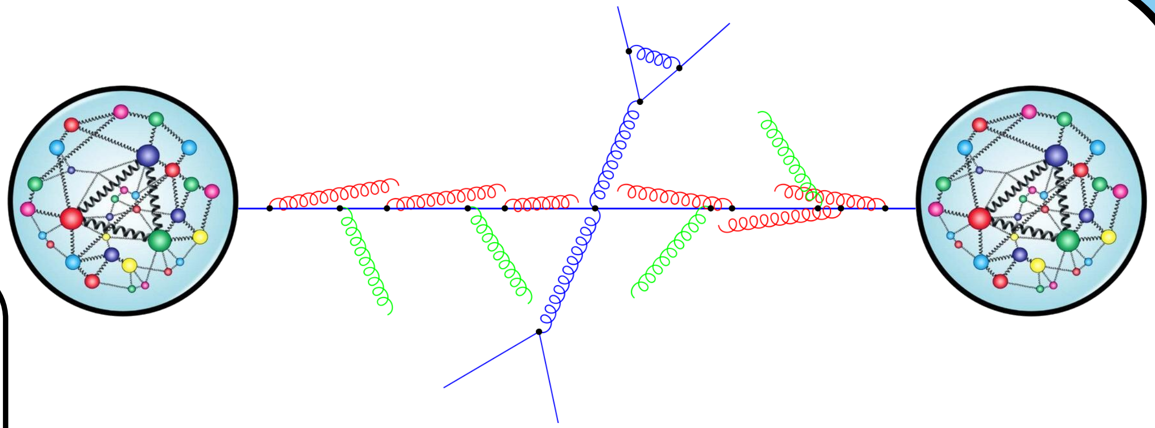
Factorization chieved in Mellin space → **Mellin space** → $N \to \infty$ isolates the limit $\hat{\rho} \to 1$ → Hard and soft parts described by matrices in color space → Hard and soft functions free of collinear div. → jet functions Δ_i → Hard function free of logarithmic enhancements

$$\frac{d\tilde{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N)}{dQ} = \text{Tr}\left[\bar{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}(N+1)\ \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\right]\ \Delta_i(N+1)\Delta_j(N+1)$$



HARD FUNCTION

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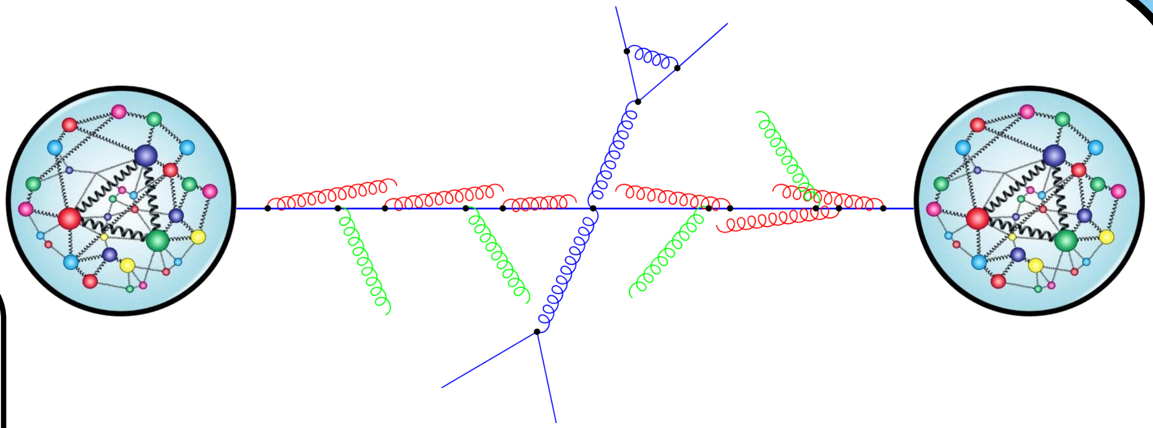

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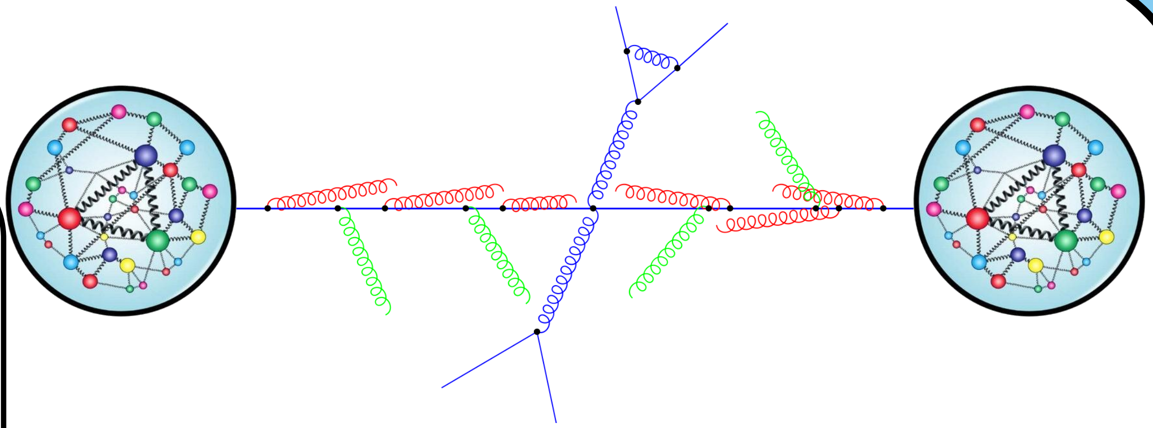
Go back to momentum space

Inverse Mellin transform:

$$\frac{d\sigma^{\text{res}}_{pp\to t\bar{t}t\bar{t}}(\rho)}{dQ} = \sum_{i,j} \int_{\mathcal{C}} \frac{dN}{2\pi i} \rho^{-N} f_i(N+1) f_j(N+1) \frac{d\tilde{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N)}{dQ}$$

Minimal Prescription method (Nucl. Phys. B 478 (1996) 273)



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How are all the orders included?

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Renormalization group equations

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$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tilde{\sigma}_{ij}(N) = 0$$

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$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathbf{S} = -\mathbf{\Gamma}^{\dagger}\mathbf{S} - \mathbf{S}\mathbf{\Gamma} \xrightarrow{\text{solution}} \mathbf{S} = \bar{\mathbf{U}}\tilde{\mathbf{S}}\mathbf{U}$$

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$$\mathbf{S} = \bar{\mathbf{U}} \tilde{\mathbf{S}} \mathbf{U}$$

$$\bar{N} = N e^{\gamma_E}$$

$$\mathbf{S} = \mathbf{S}(\mu)$$

$$\tilde{\mathbf{S}} = \mathbf{S}\left(\frac{Q}{\bar{N}}\right)$$

$$\mathbf{U} = \mathcal{P} \exp\left[\frac{1}{2}\int_{\mu^2}^{Q^2/\bar{N}^2} \frac{\mathrm{d}\mu^2}{\mu^2} \mathbf{\Gamma}\left(\mu^2, \alpha_s(\mu^2)\right)\right]$$

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$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tilde{\sigma}_{ij}(N) = 0$$

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$$\Delta_i(N) = \exp\left\{ \sum_{k=1} \alpha_s^{k-2} g_k \left(\alpha_s \log N \right) \right\}$$

$$\mathbf{U} = \mathcal{P} \exp\left[\frac{1}{2} \int_{\mu^2} \frac{\mathrm{d}\mu^2}{\mu^2} \mathbf{\Gamma}\left(\mu^2, \alpha_s(\mu^2)\right) \right]$$

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The resummation exponential reads:

$$\exp\left\{\sum_{n=1}^{\infty}\alpha_s^n\sum_{m=1}^{n+1}G_{nm}\log^m N\right\}$$

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LL:	$\alpha_s^n \log^{n+1} N$
NLL:	$\alpha_s^n \log^n N$
NNLL:	$\alpha_s^{n+1} \log^n N$
$\mathrm{N^3LL}$:	$\alpha_s^{n+2} \log^n N$
$\mathrm{N^jLL}$:	$\alpha_s^{n+j-1} \log^n N$

What about the individual objects?

Hard function:

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s}{4\pi}\mathbf{H}^{(1)} + \ldots$$

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LL, NLL NNLL

What about the individual objects?

Soft function:

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s}{4\pi}\tilde{\mathbf{S}}^{(1)} + \ldots$$

What about the individual objects?

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$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s}{4\pi}\tilde{\mathbf{S}}^{(1)} + \ldots$$

LL, NLL NNLL

What about the individual objects?

Soft anomalous dimension:

$$\mathbf{\Gamma} = \frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{\Gamma}^{(2)} + \ldots$$

What about the individual objects?

Soft anomalous dimension:

$$\mathbf{\Gamma} = \frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{\Gamma}^{(2)} + \dots$$

NLL NNLL

What about the individual objects?

Jet functions:

$$\Delta_i(N) = \exp\left\{\sum_{k=1} \alpha_s^{k-2} g_k\left(\alpha_s \log N\right)\right\}$$

g_1 LL, g_2 NLL, g_3 NNLL

NLL' accuracy

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s}{4\pi}\mathbf{H}^{(1)}$$

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s}{4\pi}\tilde{\mathbf{S}}^{(1)}$$

$$\mathbf{\Gamma} = \frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)}$$

$$\Delta_i = \exp\left\{g_1 \log N + g_2\right\}$$

Improves NLL with NLO hard and soft functions.

$H^{(1)}$ includes one-loop virtual corrections and accounts for $O(\alpha_s)$ $\log N$-independent contributions not captured by the NLL jet functions.

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Matching to NLO: **NLO+NLL'**

$$\mathrm{d}\sigma^{\mathrm{f.o.+res}} = \mathrm{d}\sigma^{\mathrm{f.o.}} + \left[\mathrm{d}\sigma^{\mathrm{res}} - \mathrm{d}\sigma^{\mathrm{res}}|_{\mathcal{O}(\alpha_s^n)}\right]$$

NLO obtained with MG5_aMC@NLO *(JHEP 07 (2014) 079 - JHEP 07 (2018) 185)*

Additional details

- Absolute-mass threshold resummation $\hat{\rho} = (4m_t)^2/s$ (Phys. Rev. Lett. 131 (2023) 211901)
  - soft-gluon corrections to σ from region where final state produced almost at rest.
- Invariant-mass threshold resummation $\hat{\rho} = Q^2/s$ (this work)
  - soft-gluon corrections to σ for all the invariant-mass configurations of final state.
  - effect of soft gluon corrections on the **invariant-mass distribution** of final state.

Choice of threshold variable

Additional details

Soft radiation sensitive to overall color structure of hard process

$\Longrightarrow$ H and S are matrices in colour space:

- $q\bar{q}$ channel: 6-dimensional colour space
- gg channel: 14-dimensional colour space

Color decomposed amplitudes extracted from custom version of OpenLoops *(Eur. Phys. J. C 79 (2019) 866)*

Color decomposition of amplitudes

Additional details

$$\mathbf{U} = \mathcal{P} \exp\left[\frac{1}{2}\int_{\mu^2}^{Q^2/\bar{N}^2} \frac{\mathrm{d}\mu^2}{\mu^2}\mathbf{\Gamma}\left(\mu^2, \alpha_s(\mu^2)\right)\right]$$

Diagonalization necessary to get rid of path-ordering operator.

It needs to be performed for every phase-space point in IMT.

Diagonalization soft anomalous dimension

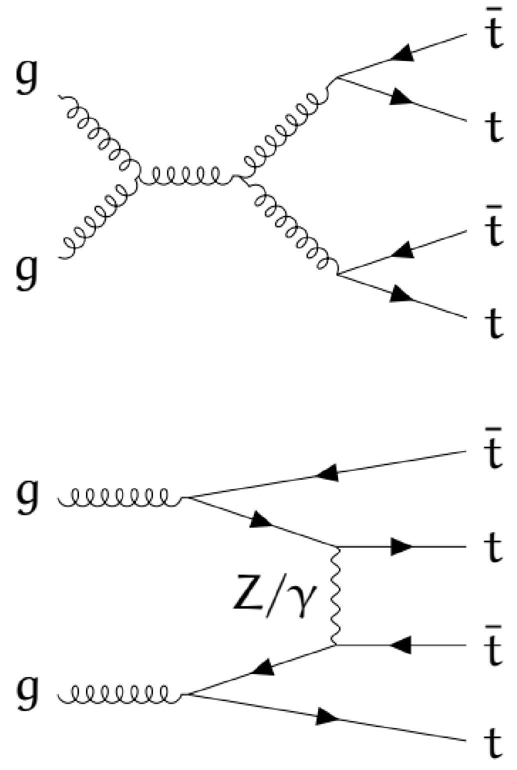
4

RESULTS

Default setup

Default setup

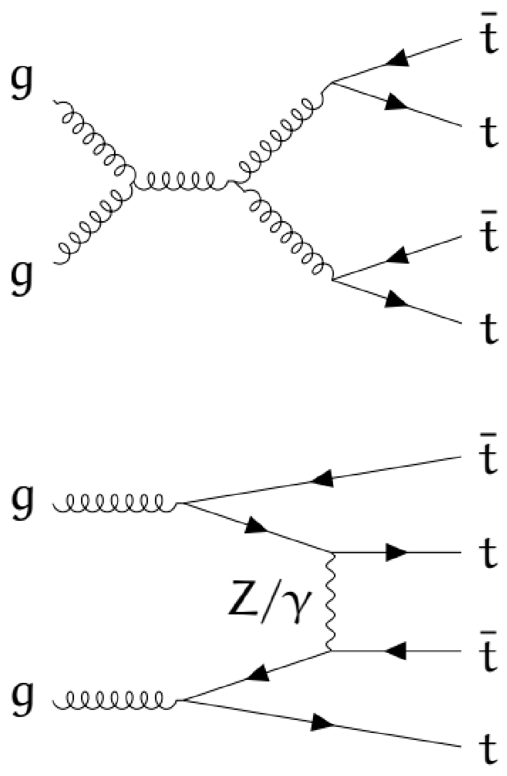
Accuracy: NLO+NLL’ (NLO = NLO QCD+EW)



Default setup

Accuracy: NLO+NLL’ (NLO = NLO QCD+EW)

PDF: LUXqed_plus_PDF4LHC15_nnlo_100



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PDF: LUXqed_plus_PDF4LHC15_nnlo_100

LHC centre-of-mass energy: 13.6 TeV

Default setup

Accuracy: NLO+NLL’ (NLO = NLO QCD+EW)

PDF: LUXqed_plus_PDF4LHC15_nnlo_100

LHC centre-of-mass energy: 13.6 TeV

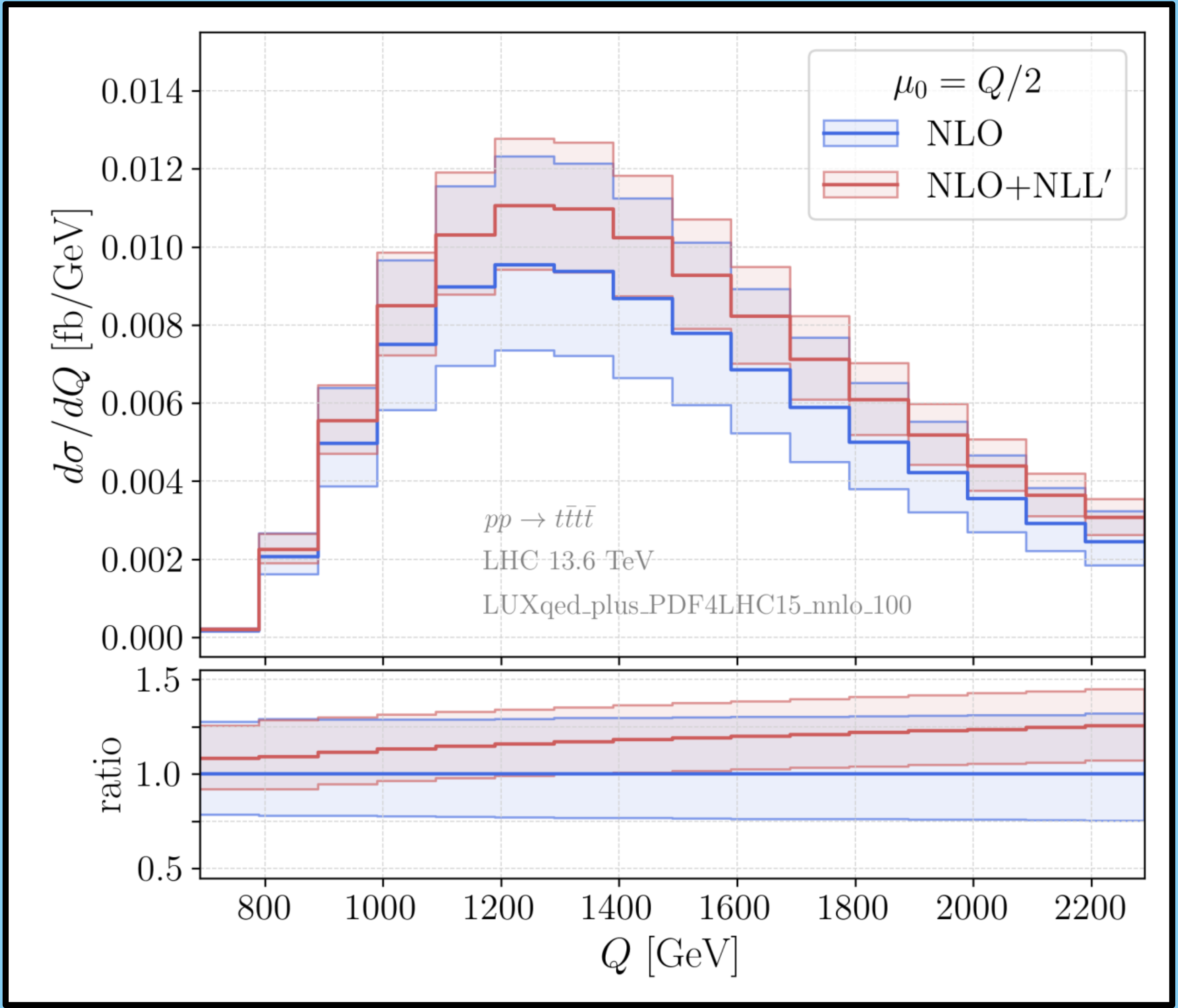
Scale choice: $\mu_R = \mu_F = \mu_0$, with $\mu_0 = Q/2, M/2, H_T/2$

Q invariant mass $t\bar{t}t\bar{t}$

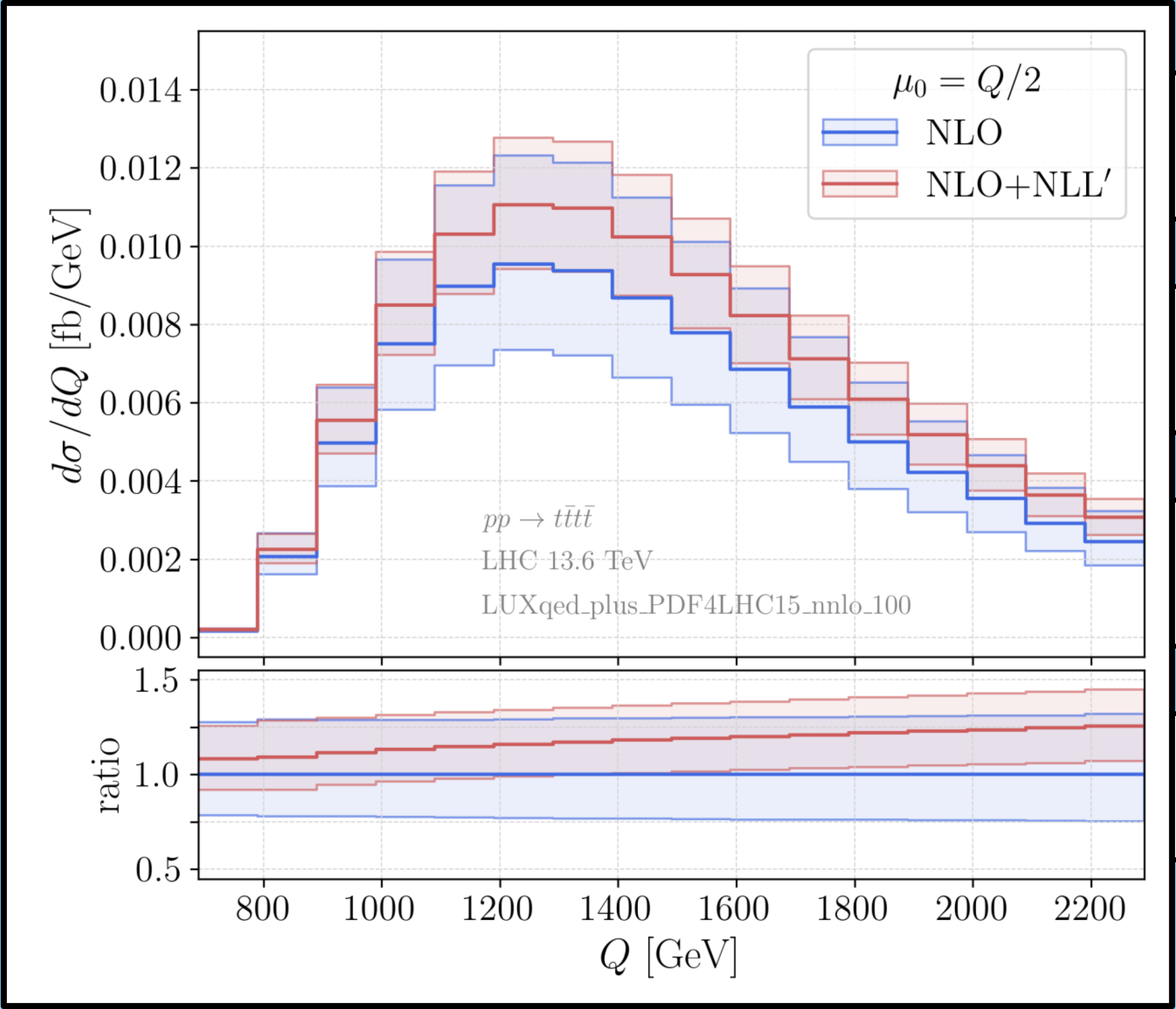
$$M = 4m_t$$

$$H_T = \sum_{i=1}^{4} \sqrt{m_t^2 + p_{T,i}^2}$$

Invariant-mass distribution



Invariant-mass distribution



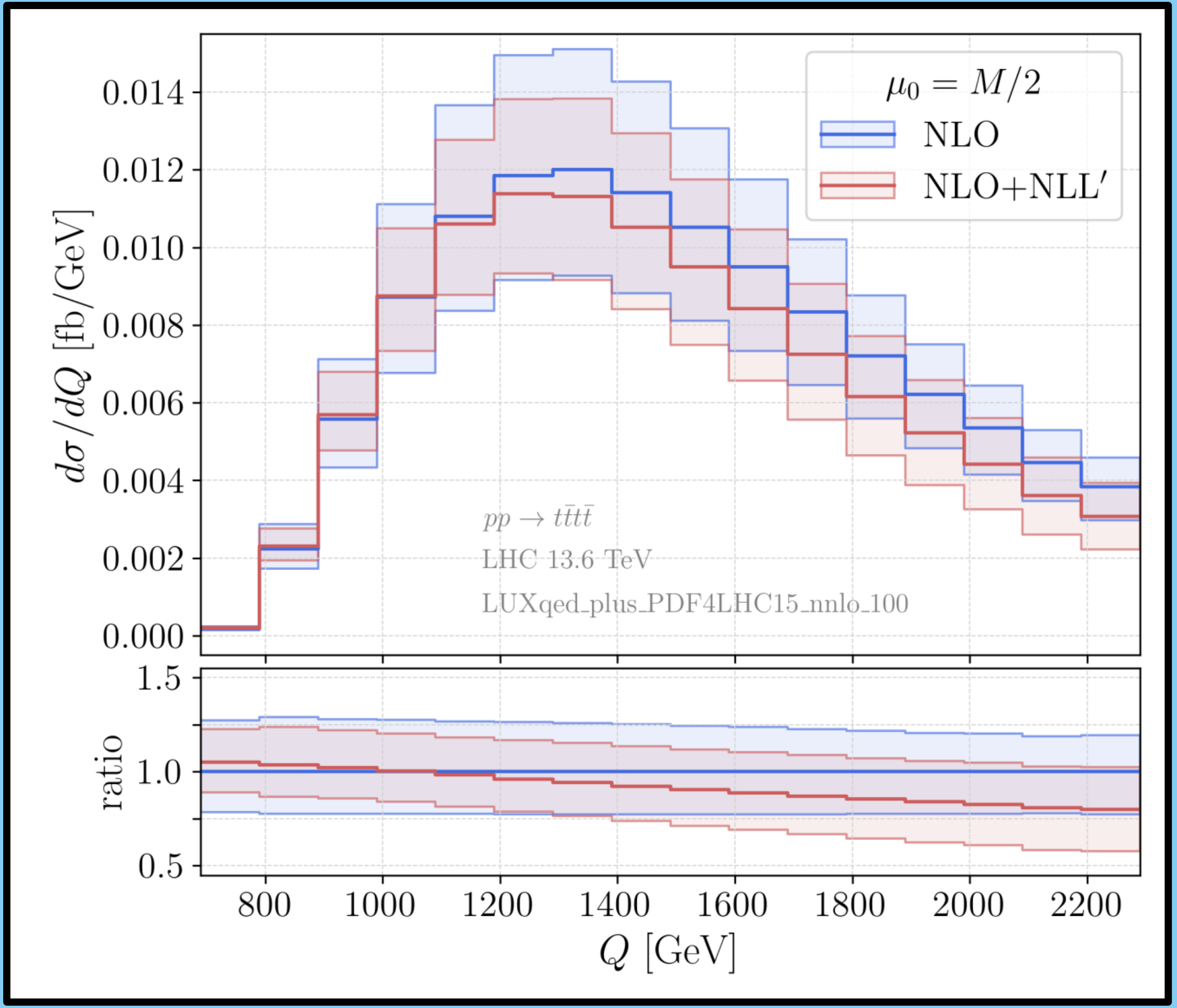
Change in shape substantial

NLL’ corrections vary in range [8%,25%]

NLL’ corrections increasingly positive

Scale uncertainty substantially reduced

Invariant-mass distribution



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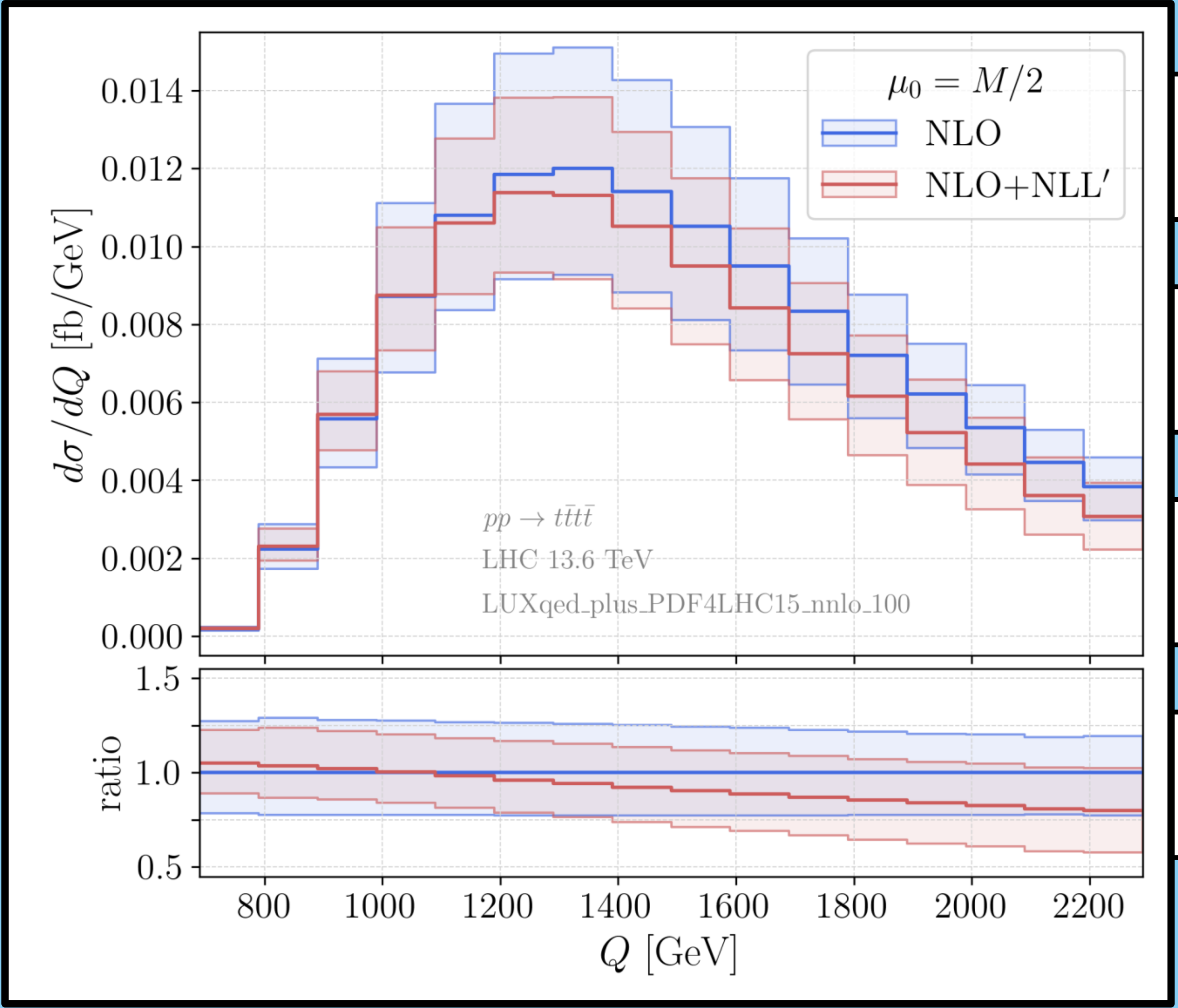


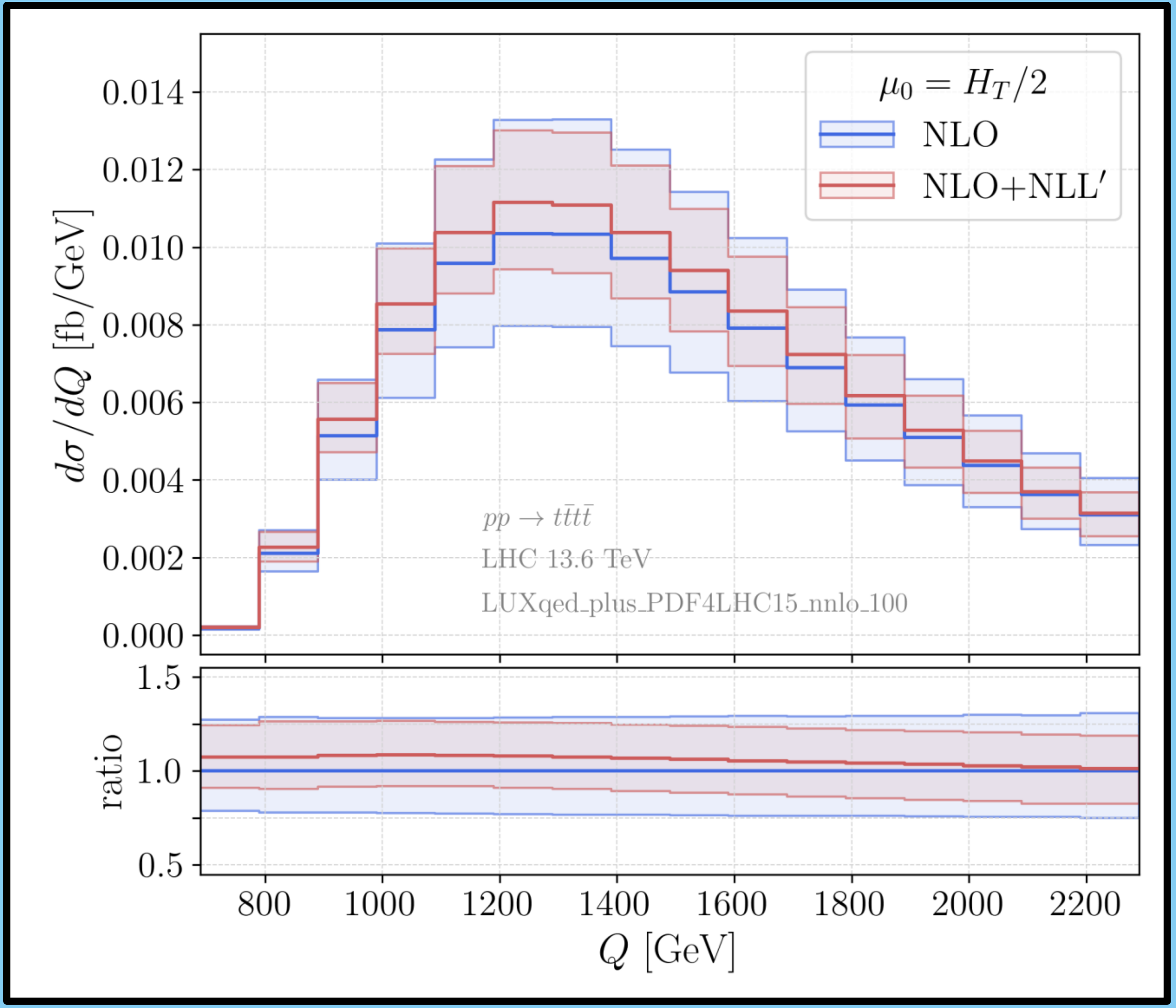
Change in shape substantial

NLL’ corrections vary in range [5%,-20%]

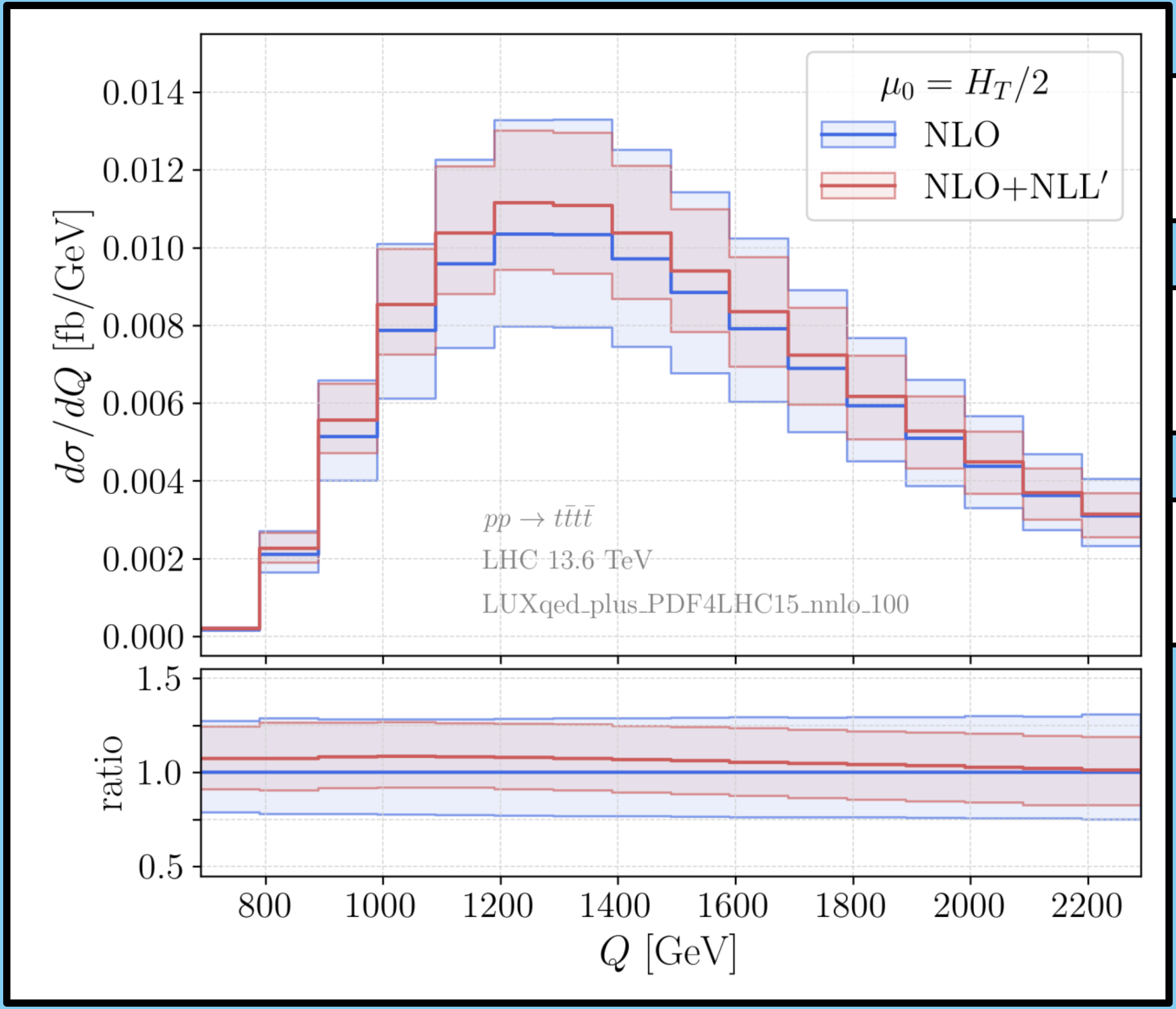
NLL’ corrections start off positive, change sign and then increasingly negative

Scale uncertainty substantially reduced

Invariant-mass distribution



Invariant-mass distribution

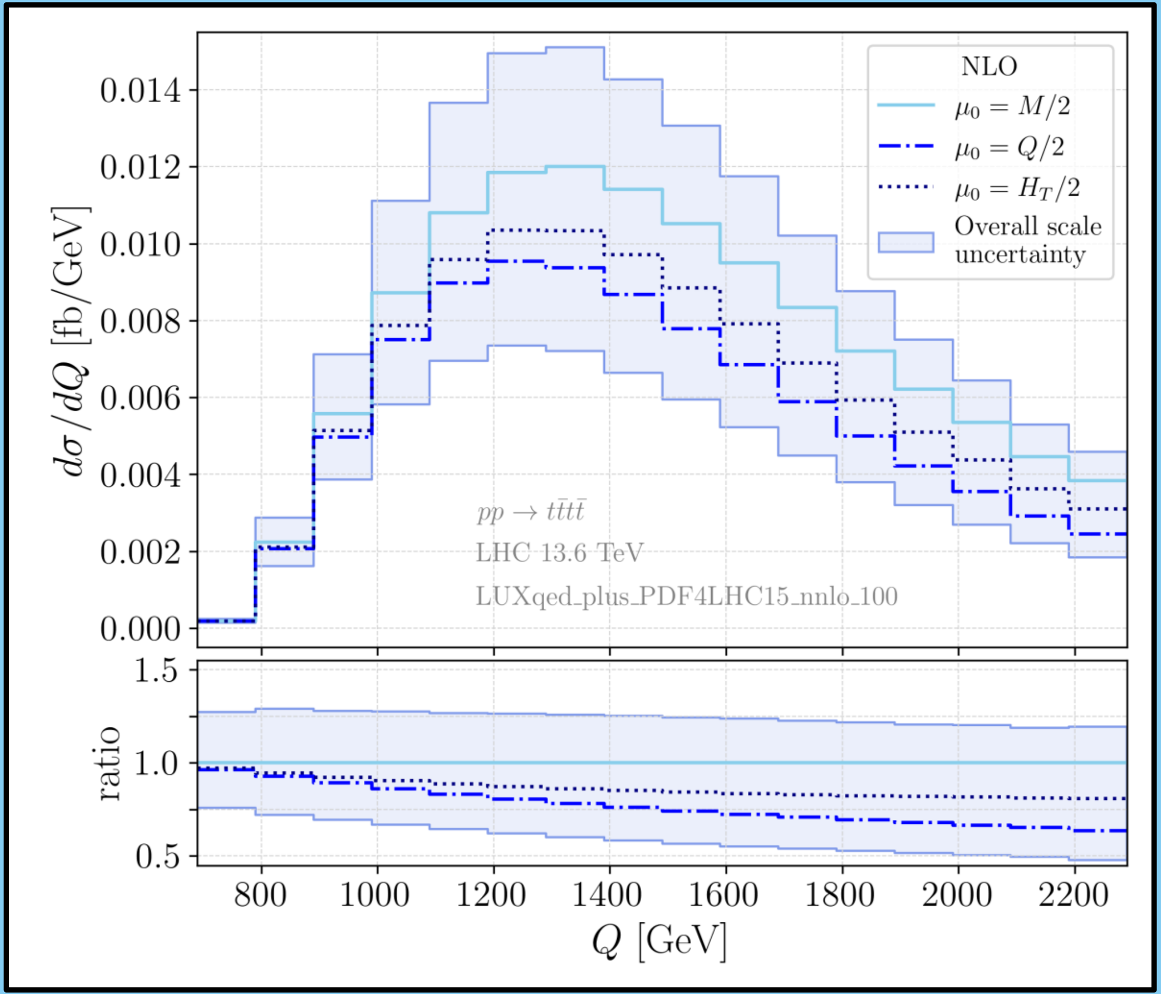


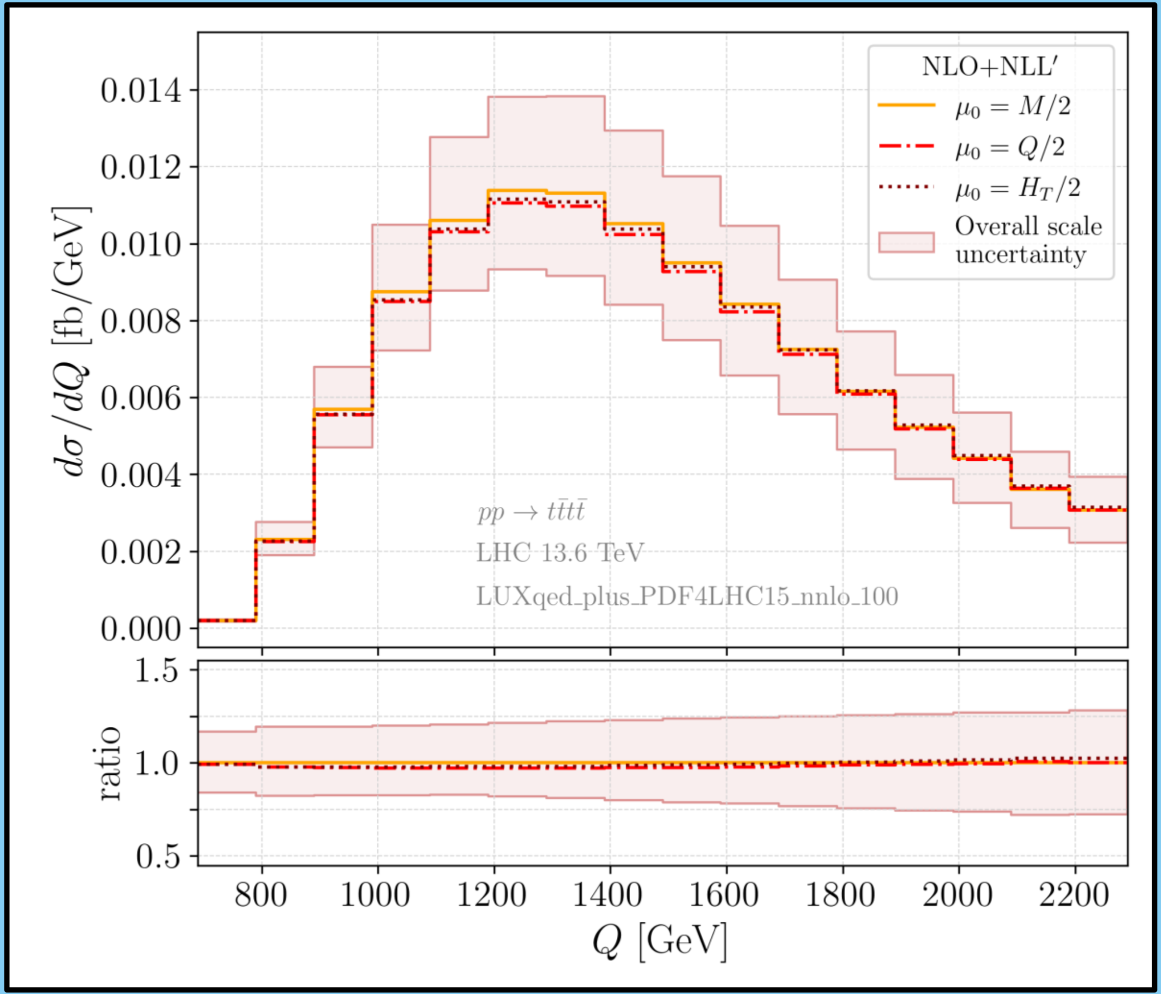
Smaller shape modification

NLL’ corrections vary in range [1%,8%]

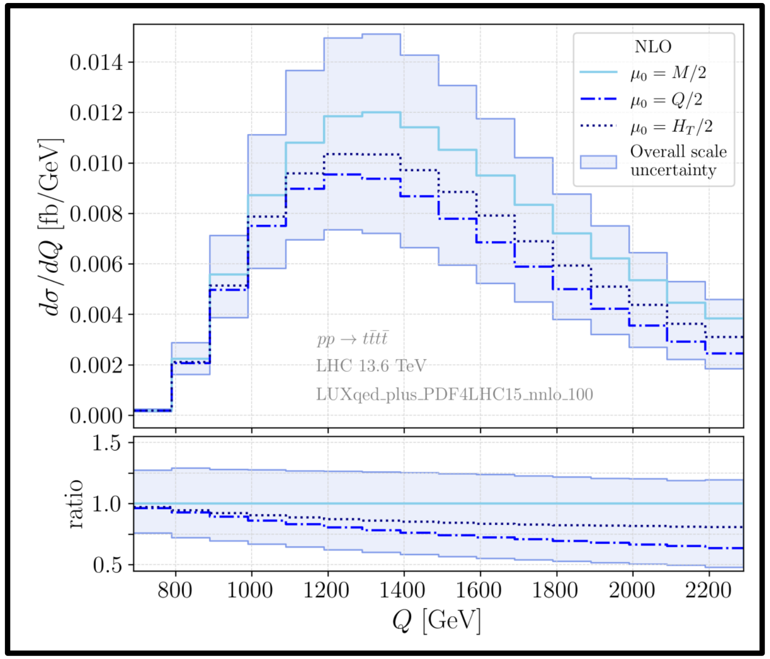
Scale uncertainty substantially reduced

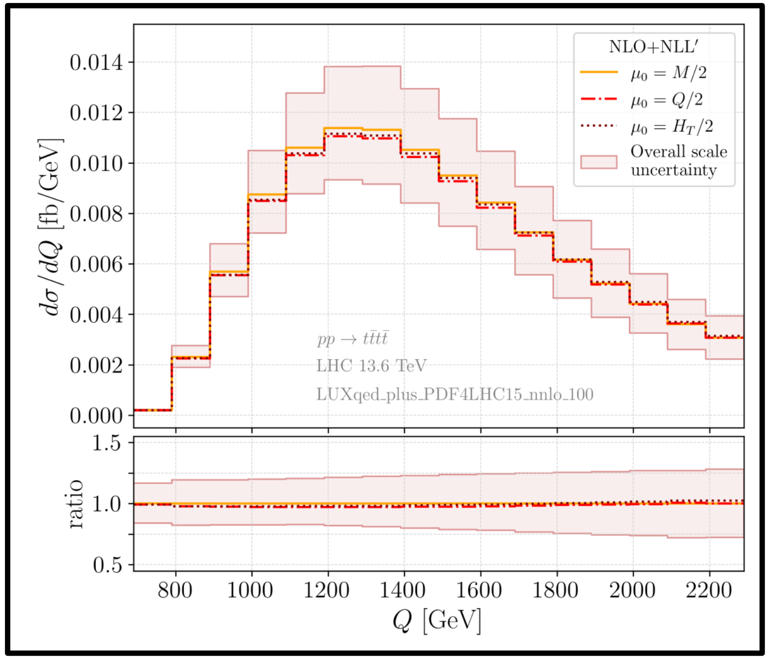
Invariant-mass distribution





Invariant-mass distribution





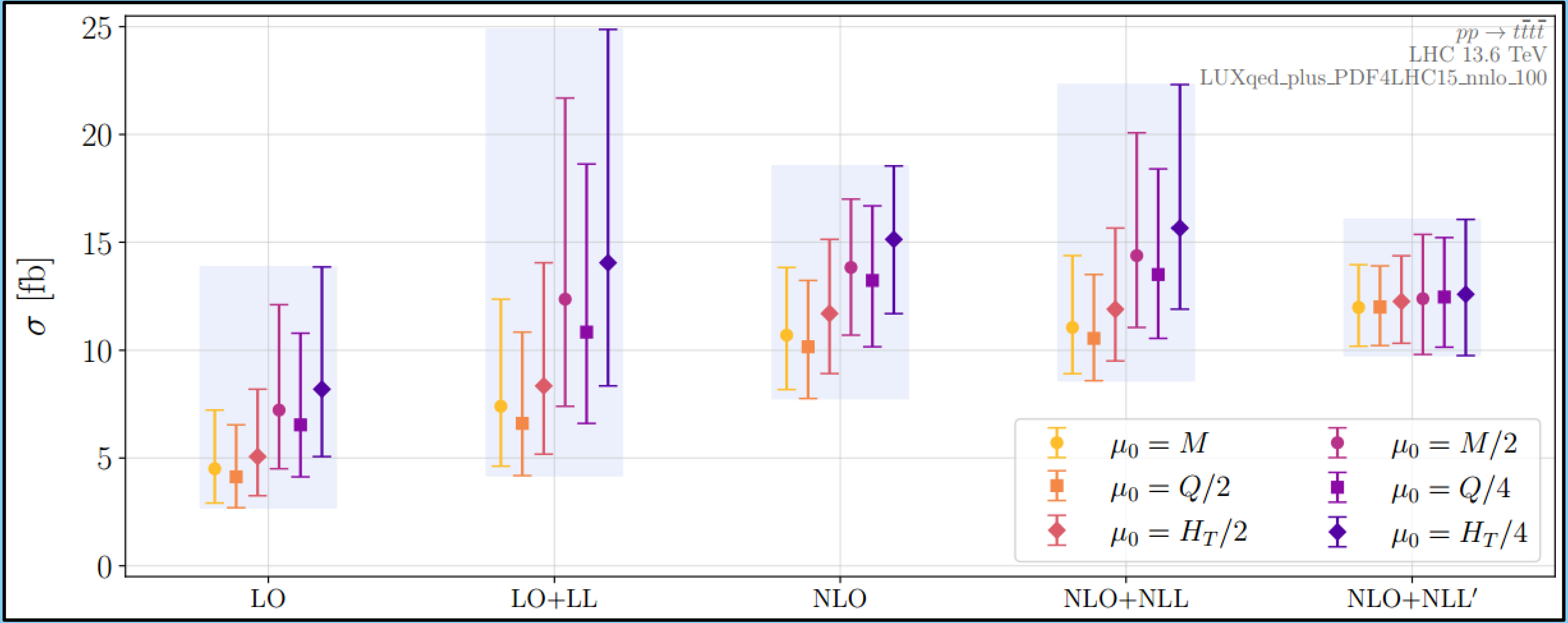
NLO+NLL’:

- displays better convergence
- lower overall scale uncertainty
- central values differ at most 3%

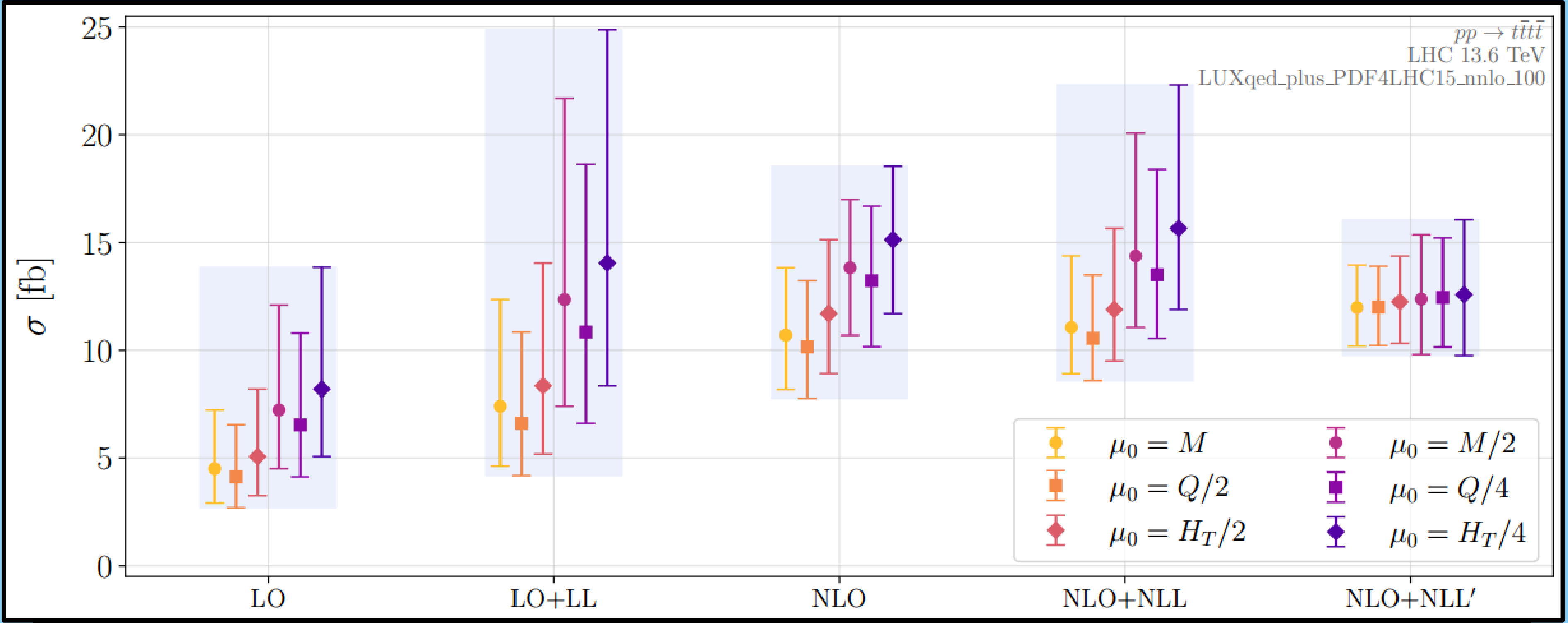
NLO:

- central values differ up to 36 %

Total cross section



Total cross section



$\sqrt{S}$ [TeV]	μ_0	NLO [fb]	NLO+NLL [fb]	$\mathcal{K}^{\mathrm{NLL}}$	NLO+NLL′ [fb]	$\mathcal{K}^{\mathrm{NLL}'}$
13.6	$M/2$	$13.83^{+23.0\%}_{-22.6\%}$	$14.38^{+39.6\%}_{-23.1\%}$	1.04	$12.38^{+24.1\%}_{-20.8\%}$	0.90
	$Q/2$	$10.16^{+30.1\%}_{-23.6\%}$	$10.55^{+27.9\%}_{-18.6\%}$	1.04	$12.00^{+15.8\%}_{-14.9\%}$	1.18
	$H_T/2$	$11.70^{+29.3\%}_{-23.8\%}$	$11.89^{+31.7\%}_{-20.1\%}$	1.02	$12.25^{+17.3\%}_{-15.7\%}$	1.05

Total cross section

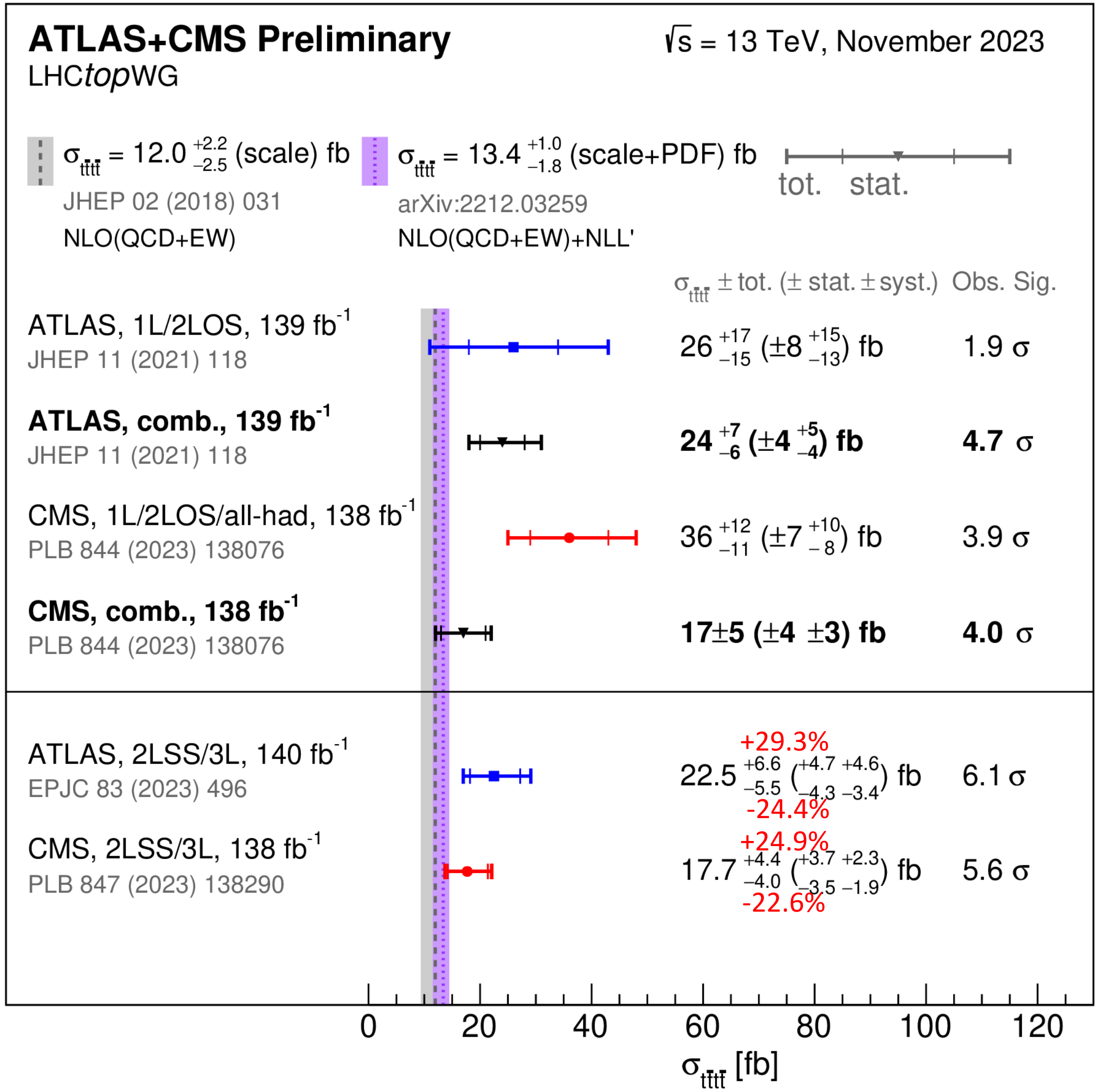
$\sqrt{S}$ [TeV]	μ_0	NLO+NLL′ [fb]
13	$M/2$	$10.43^{+23.6\%}_{-20.8\%}$
	$Q/2$	$10.16^{+15.7\%}_{-14.8\%}$
	$H_T/2$	$10.35^{+17.1\%}_{-15.7\%}$

$\mu_0 = Q/2$:

- 1.8 σ from CMS
- 2.2 σ from ATLAS

$\mu_0 = M/2$:

- 1.5 σ from CMS
- 2.0 σ from ATLAS



5

SUMMARY AND CONCLUSIONS

I presented the most accurate QCD predictions for $t\bar{t}t\bar{t}$ to date. The NLO results have been combined with NLL’ **(NLO+NLL’)**, and thus include all-order corrections in the soft gluon emission limit.

NLO+NLL′ [fb]	
$12.38^{+24.1\%}_{-20.8\%}$	0.9
$12.00^{+15.8\%}_{-14.9\%}$	1
$12.25^{+17.3\%}$	

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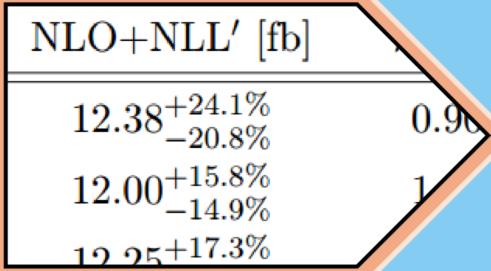

NLO+NLL′ [fb]
$12.38^{+24.1\%}_{-20.8\%}$
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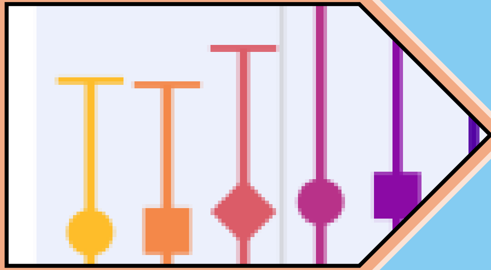
The NLL’ corrections reduce the theoretical uncertainty and improve the convergence of the predictions.



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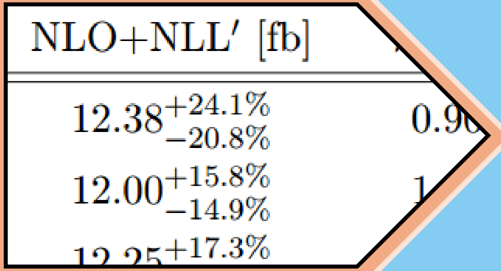

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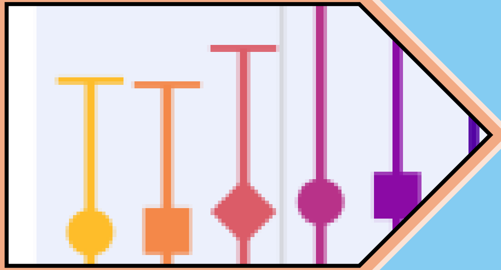
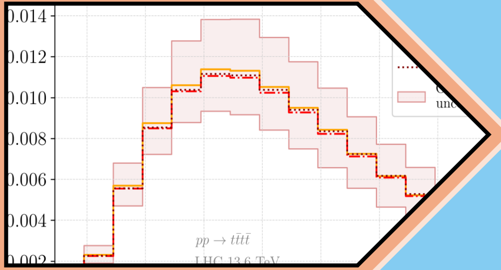


For the first time, soft-gluon corrections to the invariant mass distribution Q of the $t\bar{t}t\bar{t}$ system have been obtained.



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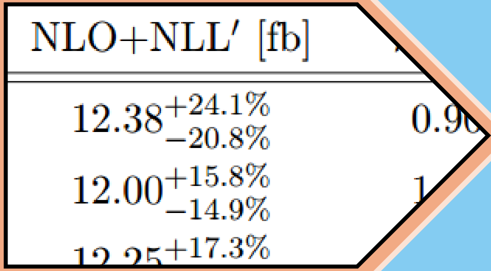

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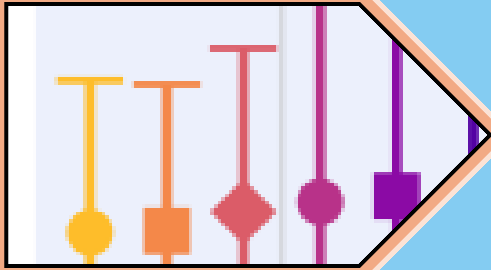

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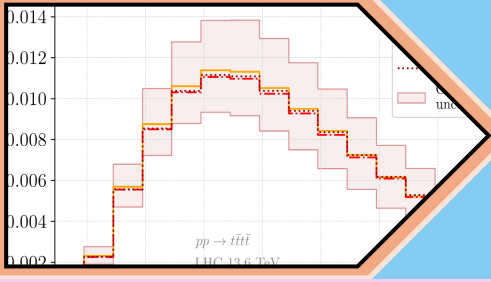

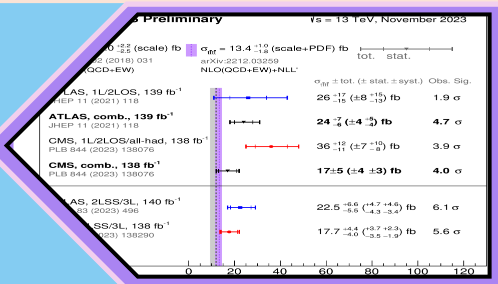

The new theoretical predictions are in agreement with the experimental results. However, both the theoretical uncertainty and the experimental error are still quite large. With HL-LHC, further effort from theory side is needed.

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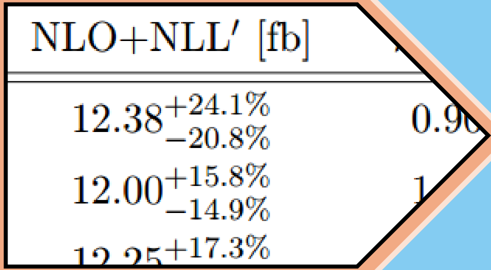

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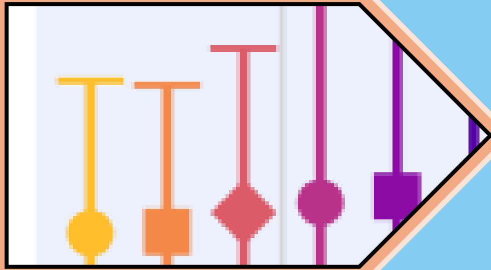

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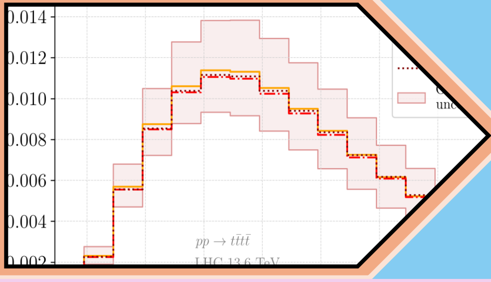

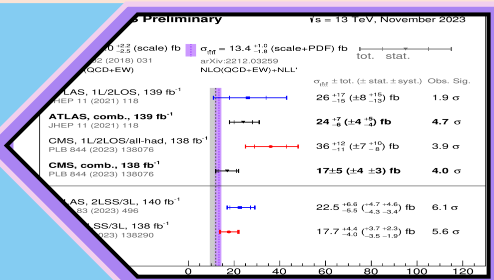

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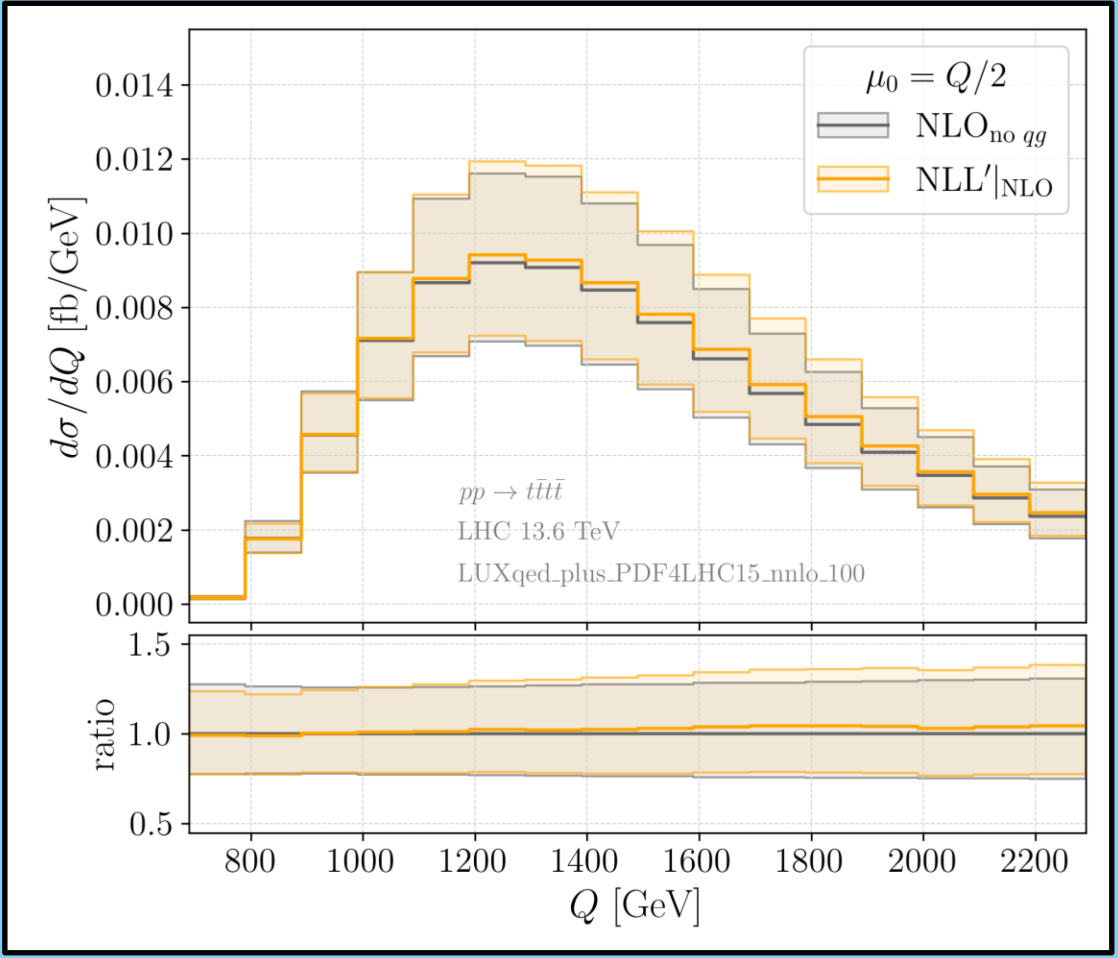
$$\Gamma^{(2)}\,,\ g_3$$

Next step: performing the calculation at **NLO+NNLL** accuracy.

BACKUP SLIDES

Approximate NLO



μ_0	$\mathrm{NLO}_{\mathrm{QCD}}$ [fb]	$\mathrm{NLO}_{\mathrm{no}\ qg}$ [fb]	$\mathrm{NLL}'\vert_{\mathrm{NLO}}$ [fb]
$M/2$	$13.13^{+25.2\%}_{-24.5\%}$	$13.05^{+20.2\%}_{-21.1\%}$	$13.45^{+21.6\%}_{-21.9\%}$
$Q/2$	$9.38^{+33.3\%}_{-25.8\%}$	$9.77^{+28.1\%}_{-23.9\%}$	$9.92^{+28.7\%}_{-24.1\%}$
$H_T/2$	$10.88^{+32.3\%}_{-25.8\%}$	$11.22^{+26.0\%}_{-23.7\%}$	$11.44^{+27.0\%}_{-24.0\%}$

- NLL' expanded reproduces $NLO_{no\ qg}$ reliably, both at the differential and integrated level.
- qg contribution to the cross section is very small.
- Differences between $NLL'|_{NLO}$ and $NLO_{no\ qg}$ do not exceed 3%.
- Differences between $NLL'|_{NLO}$ and NLO are at most 6%.