Electroweak Baryogenesis from Non-Equilibrium QFT

Martin Schmied

Technische Universität München

17th July 2025

Contents

Introduction

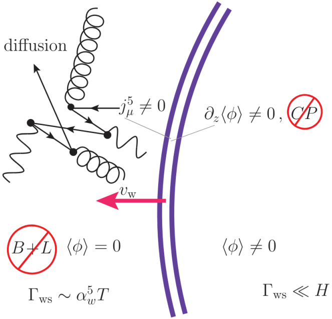
- Observed matter-antimatter asymmetry:

$$5.8 \cdot 10^{-10} \leq \frac{n_{\mathrm{B}}}{n_{\gamma}} \leq 6.6 \cdot 10^{-10}$$

- Could be explained by initial conditions, another explanation more intriguing
- Sakharov conditions:
  - Baryon number violation
  - C and CP violation
  - Deviation from the thermal equilibrium

Overview of electroweak baryogenesis

- Electroweak symmetry breaking through a first order phase transition – extension of the SM necessary
- CP-violating current generated in the bubble wall by the gradients of the Higgs expectation value
- Sphalerons generate baryon-plus-lepton number from the produced charges
- The bubble captures the baryon-plus-lepton number, and sphaleron processes freeze out



Classical example

- Classical point particle with spacetime-dependent mass $m(x)$ and momentum $p^\mu = m\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$:

$$S = \int_{\tau_\mathrm{A}}^{\tau_\mathrm{B}} \mathrm{d}\tau \left[-m(x)c^2\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} p^\mu = c^2 \frac{\mathrm{d}m}{\mathrm{d}x_\mu}$$

- Apply the Liouville theorem to a distribution function $g(x, p)$:

$$\frac{\mathrm{d}}{\mathrm{d}\tau} g(x,p) = \frac{1}{m} p^\mu \frac{\partial g(x,p)}{\partial x^\mu} + \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} \frac{\partial g(x,p)}{\partial p^\mu} = 0 \qquad (*)$$

- Insert distribution of the quasi-particle form:

$$g(x,p) = 2\pi\delta(p^2 - m^2) f(x, \boldsymbol{p})$$

Classical example

- Take the zeroth moment of eq. $(*)$ $\int \frac{2\mathrm{d}p^0}{(2\pi)}$[eq. $(*)$] to get a kinetic equation:

$$u^\mu \frac{\partial f(x, \boldsymbol{p})}{\partial x^\mu} + \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} \frac{\partial f(x, \boldsymbol{p})}{\partial p^\mu} = 0,$$

with $u^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$

Ansatz in QFT

- Described by Wightman functions:

$$\mathrm{i}\Delta^{>}(x,y) = \left\langle \phi(x)\phi^{\dagger}(y) \right\rangle$$

- Consider its Wigner transform:

$$\Delta^{>}(x,k) = \int \mathrm{d}^4 r \mathrm{e}^{\mathrm{i}k\cdot r} \Delta^{>}\left(x + \frac{r}{2}, x - \frac{r}{2}\right)$$

- Ansatz:

$$\begin{aligned}
\mathrm{i}\Delta^{>}(x,k) &= 2\pi\delta(k^2 - m^2)[\theta(k^0)(1 + f(\boldsymbol{k},x)) + \theta(-k^0)\bar{f}(-\boldsymbol{k},x)] \\
&= \frac{\pi}{\omega(\boldsymbol{k})}\delta(k^0 \mp \omega(\boldsymbol{k}))[\theta(k^0)(1 + f(\boldsymbol{k},x)) + \theta(-k^0)\bar{f}(-\boldsymbol{k},x)] \\
\omega(\boldsymbol{k}) &= \sqrt{\boldsymbol{k}^2 + m^2}
\end{aligned}$$

- Corrections necessary if going beyond 2nd order in derivatives of the mass

Kadanoff-Baym equation

- Schwinger-Dyson equation for the Wightman functions on CTP (Schwinger-Keldysh closed time-path method), also called Kadanoff-Baym equation:

$$
\begin{aligned}
\left[-\partial^2 - M^2\right] \Delta^{>}(x, y) - \int \mathrm{d}^4 z (\Pi^{\mathrm{H}}(x, z)\Delta^{>}(z, y) + \Pi^{>}(x, z)\Delta^{\mathrm{H}}(z, y)) \\
= \frac{1}{2} \int \mathrm{d}^4 z (\Pi^{>}(x, z)\Delta^{<}(z, y) - \Pi^{<}(x, z)\Delta^{>}(z, y))
\end{aligned}
$$

Kadanoff-Baym equation

- Perform a Wigner transform:

$$\left[-\frac{1}{4}\partial^2 + \mathrm{i}k\cdot\partial + k^2 - M^2 \mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k}\right]\Delta^> - \mathrm{e}^{-\mathrm{i}\diamond}\{\Pi^H\}\{\Delta^>\} - \mathrm{e}^{-\mathrm{i}\diamond}\{\Pi^>\}\{\Delta^H\}$$
$$= \frac{1}{2}\mathrm{e}^{-\mathrm{i}\diamond}(\{\Pi^>\}\{\Delta^<\} - \{\Pi^<\}\{\Delta^>\})$$
$$\diamond\{G(k,x)\}\{F(k,x)\} = \frac{1}{2}\left(\frac{\partial G(k,x)}{\partial x^\mu}\frac{\partial F(k,x)}{\partial k_\mu} - \frac{\partial G(k,x)}{\partial k^\mu}\frac{\partial F(k,x)}{\partial x_\mu}\right)$$

- Set the collision term to zero:

$$\left[-\frac{1}{4}\partial^2 + \mathrm{i}k\cdot\partial + k^2 - M^2 \mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k}\right]\Delta^{<,>} = 0$$

Time dependent case

- Time dependent mass matrix:

$$M^2 = \begin{bmatrix} m_1^2 & \mathrm{e}^{\mathrm{i}\varphi(t)} v(t) \\ \mathrm{e}^{-\mathrm{i}\varphi(t)} v(t) & m_2^2 \end{bmatrix}$$

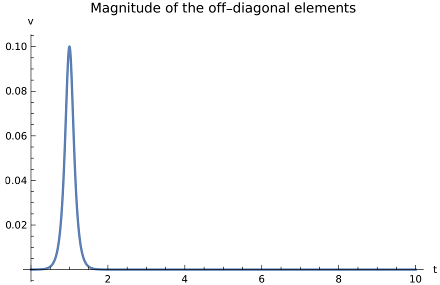
- Take moments of the Kadanoff-Baym equation
- Under which conditions do charges get generated?
- Numerical solution for selected off-diagonal elements

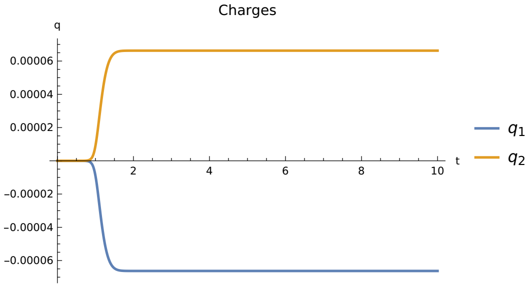
Time dependent case

- For example:

$$M^2 = \begin{bmatrix} 0.9 & \frac{e^{i\pi t}}{10\cosh(10(t-1))} \\ \frac{e^{-i\pi t}}{10\cosh(10(t-1))} & 1.1 \end{bmatrix}$$

$$\boldsymbol{k}^2 = 1$$





Solving the equation

- Mass matrix:

$$M^2 = \begin{bmatrix} m_1^2 & \mathrm{e}^{\mathrm{i}\varphi} v(z) \\ \mathrm{e}^{-\mathrm{i}\varphi} v(z) & m_2^2 \end{bmatrix}$$

- Take the hermitian and antihermitian parts of the Kadanoff-Baym equation and expand up to order 2 in derivatives, $\partial_t \to 0$, $k^x\partial_x + k^y\partial_y \to 0$, [2]:
  - Kinetic equation (hermitian part):

$$\mathrm{i}k^z\partial_z\Delta^> + \frac{1}{2}[\Delta^>, M^2] - \frac{\mathrm{i}}{4}\{\partial_{k^z}\Delta^>, \partial_z M^2\} + \frac{1}{16}[\partial_z^2 M^2, \partial_{k^z}^2\Delta^>] = 0$$

  - Constraint equation (antihermitian part):

$$-\frac{1}{4}\partial^2\Delta^> + k^2\Delta^> - \frac{1}{2}\{M^2, \Delta^>\} + \frac{\mathrm{i}}{4}[\partial_{k^z}\Delta^>, \partial_z M^2] + \frac{1}{16}\{\partial_z^2 M^2, \partial_{k^z}^2\Delta^>\} = 0$$

- Expand the off-diagonal elements of the Wightman functions:

$$\Delta_{ij}^> = \Delta_{ij}^{>,(0)} + \Delta_{ij}^{>,(1)} + \Delta_{ij}^{>,(2)} + \mathcal{O}(v'^3, v''v', v''')$$

Solving the equation

- Solve the kinetic equation order by order for the off-diagonal elements up to order 1 in derivatives
- Substitute into the constraint equation, and solve it up to order 1 in derivatives and order 2 in v to obtain an ansatz for the diagonal elements:

$$\mathrm{i}\Delta_{ii}^{>}(x,k) = \frac{\pi}{\omega_{ii}(\boldsymbol{k})}\delta(k^0 \mp \omega_{ii}(\boldsymbol{k}))[\theta(k^0)(1+f_{ii}(\mathbf{k},x)) + \theta(-k^0)\bar{f}_{ii}(-\mathbf{k},x)]$$

$$\omega_{ii}(\boldsymbol{k}) = \sqrt{\mathbf{k}^2 + m_i^2} + \frac{m_i\delta m_i}{\sqrt{\mathbf{k}^2+m_i^2}}$$

$$\delta m_1 = \frac{v^2}{2m_1(m_1^2-m_2^2)} + \mathcal{O}(v^4)$$

$$\delta m_2 = \frac{v^2}{2m_2(m_2^2-m_1^2)} + \mathcal{O}(v^4)$$

Solving the equation

- Insert the off-diagonal elements and the ansatz into diagonal components of the kinetic equation at order 1 in derivatives
- Integrate over positive and negative k^0 to get equations for the distribution functions of particles and antiparticles, respectively
- Insert the Bose-Einstein distribution

$$\overset{(-)}{f}_{ii}(\boldsymbol{k}, z) = (\exp[\beta\gamma_\mathrm{w}(\omega_{ii}(\boldsymbol{k}) + v_\mathrm{w} k^z - \overset{(-)}{\mu}{}^i)] - 1)^{-1}$$

with the wall velocity v_w and expand up to the first order in v_w and $\overset{(-)}{\mu}{}^i$

- Subtract the equation for antiparticles from the equation for particles to derive a Boltzmann equation for the deviation $\delta f_{ii} = f_{ii} - \bar{f}_{ii}$ with a source $\mathcal{S}_{ii}$:

$$(v_{z,ii}\partial_z + F_{ii}\partial_{k^z})\delta f_{ii} + \mathcal{S}_{ii} = 0$$

- $\mathcal{S}_{ii} \neq 0$, but $\mathcal{S}_{ii} = 0$ for vector density $\delta f^{v}_{s,ii}$ of fermions

Sources

[1] B. Garbrecht, "Why is there more matter than antimatter? Calculational methods for leptogenesis and electroweak baryogenesis," Progress in Particle and Nuclear Physics 110 (2020), 103727, doi:10.1016/j.ppnp.2019.103727 [arXiv:1812.02651]

[2] T. Prokopec, M. G. Schmidt and S. Weinstock, "Transport equations for chiral fermions to order h bar and electroweak baryogenesis. Part 1," Annals Phys. 314 (2004) 208, doi:10.1016/j.aop.2004.06.002 [hep-ph/0312110]

[3] B. Garbrecht, B. Ilyas, C. Tamarit, G. White, "Kinetic Theory in spacetime dependent background and a simple recipe for calculating CP-Violating Sources for Electroweak Baryogenesis," not published yet

[4] B. Garbrecht, T. Prokopec, M. G. Schmidt, "Particle number in kinetic theory," Eur.Phys.J. C38 (2004) 135-143, doi:10.1140/epjc/s2004-02007-0 [arXiv:hep-th/0211219]

Summary

- Use Wightman functions in Wigner space
- Kinetic and constraint equations from the Kadanoff-Baym equation
- Solve for the off-diagonal elements
- Obtain an ansatz for the diagonal elements
- Take the zeroth moment
- Insert the Bose-Einstein distribution to get a Boltzmann equation and extract a source