Topological Defects to One-Loop Order

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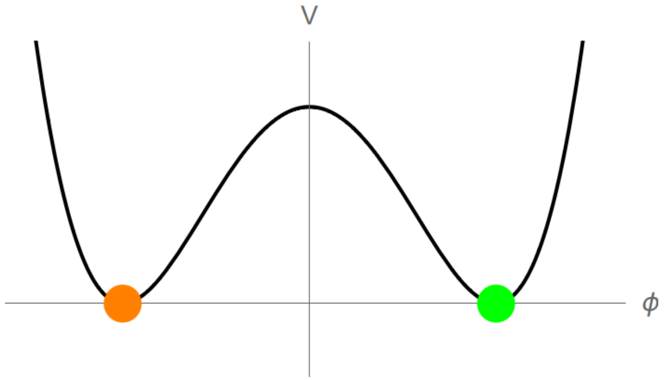
IMPRS Recruiting Workshop

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

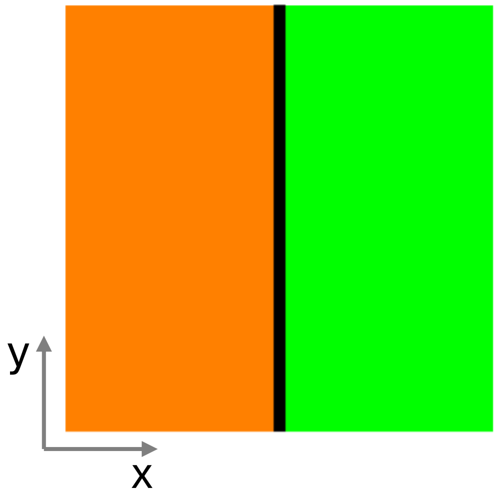
MAX-PLANCK-INSTITUT FÜR PHYSIK

Introduction



$$\pi_0(\mathcal{M}) \neq \mathbb{1}$$

Domain Walls



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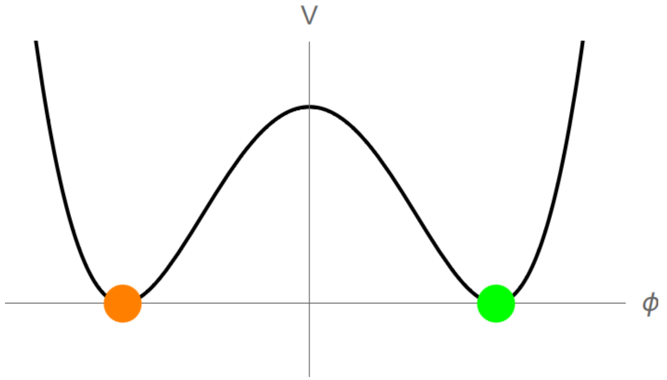

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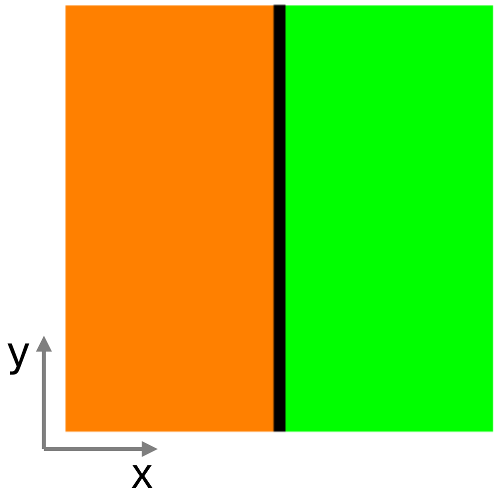
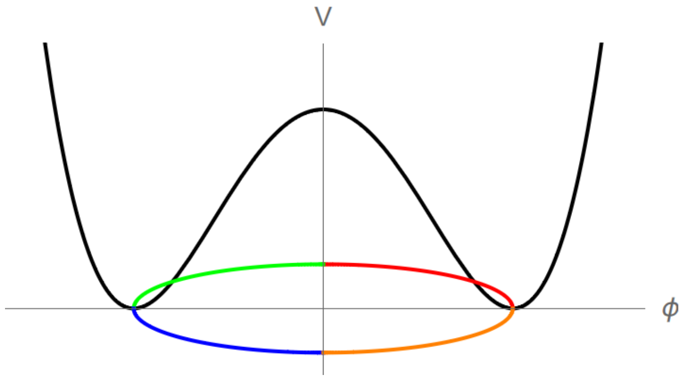
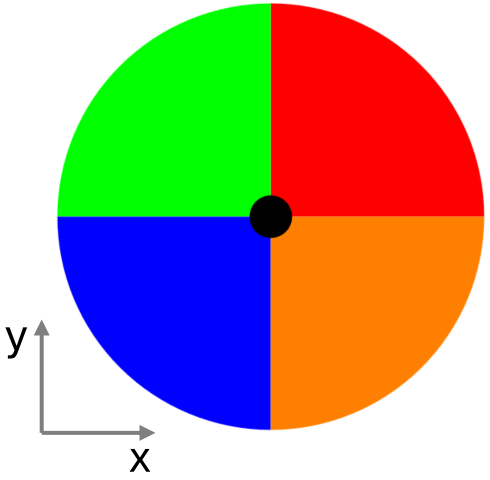




$$\pi_1(\mathcal{M}) \neq \mathbb{1}$$

Strings



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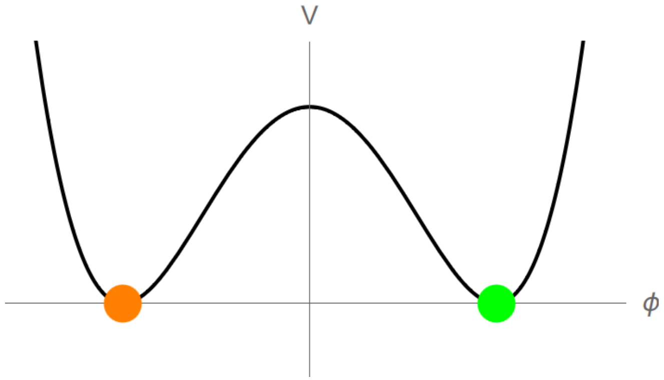

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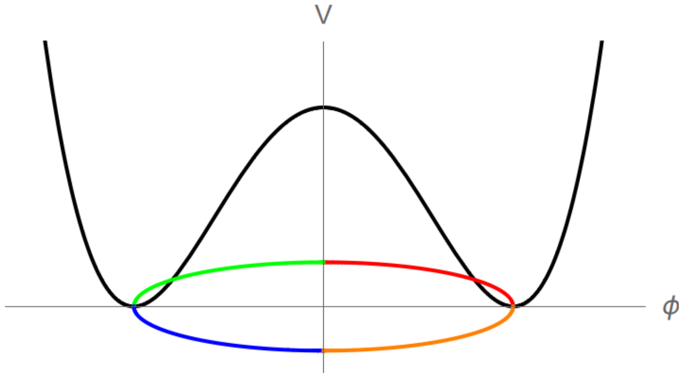

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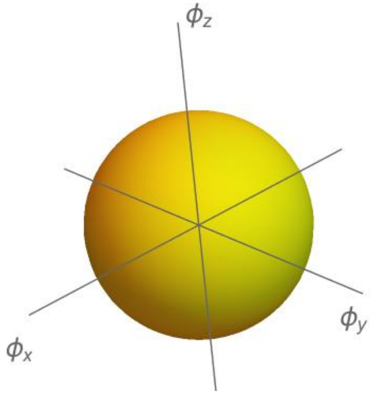
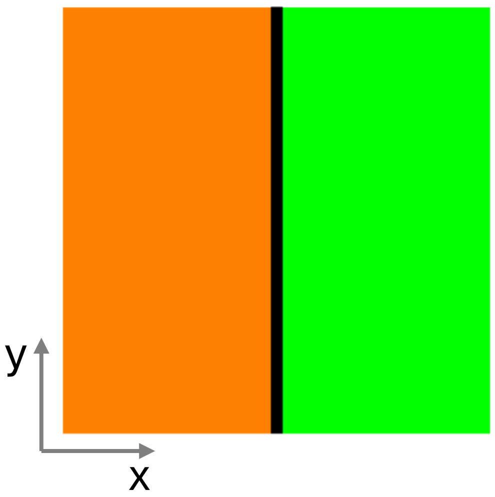
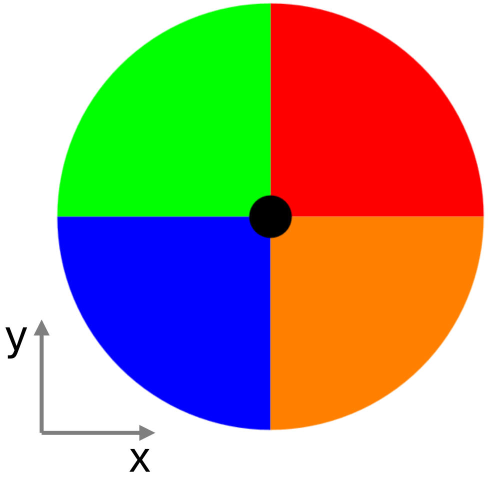
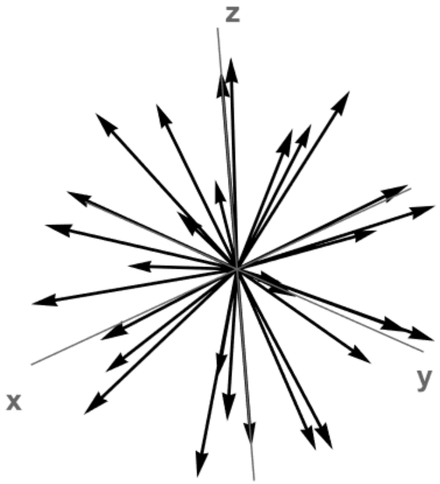


$$\pi_2(\mathcal{M}) \neq \mathbb{1}$$

Monopoles







‘t Hooft-Polyakov Magnetic Monopoles

SU(2) gauge theory

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \frac{1}{2}(D_\mu\phi)^a(D^\mu\phi)^a - \frac{m^2}{2}\phi^a\phi^a - \frac{\lambda}{4!}(\phi^a\phi^a)^2$$

$$(D_\mu\phi)^a = \partial_\mu\phi^a + g\varepsilon_{abc}W^b_\mu\phi^c$$

Symmetry:

$$\phi = \phi^a T^a$$

$$\phi \to U\phi U^\dagger$$

$$U^\dagger = U^{-1}$$

‘t Hooft 1974
Polyakov 1974

't Hooft-Polyakov Magnetic Monopoles

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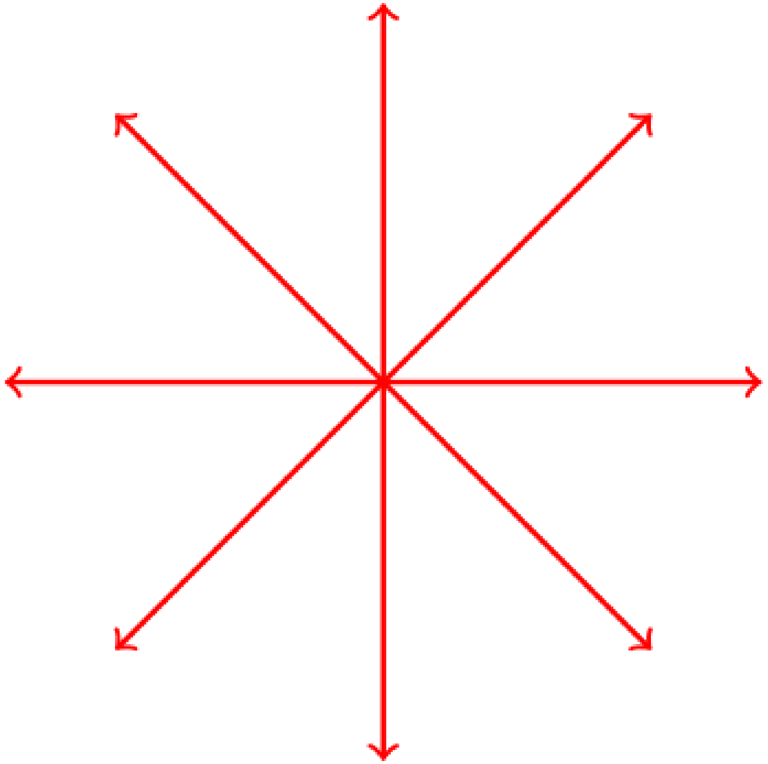
Symmetry:

$$\phi = \phi^a T^a \qquad \phi \to U\phi U^\dagger \qquad U^\dagger = U^{-1}$$

$$m^2 < 0$$

Spontaneous symmetry breaking:

$$\mathrm{SU}(2) \to \mathrm{U}(1)$$



Monopole configuration:

$$\phi = v\begin{pmatrix}\cos(\varphi)\sin(\theta)\\ \sin(\varphi)\sin(\theta)\\ \cos(\theta)\end{pmatrix}$$

U(1) gauge field yields **magnetic charge**

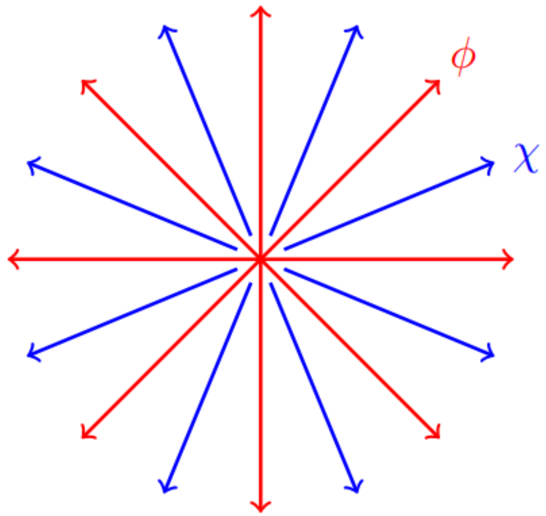
't Hooft 1974
Polyakov 1974

The Model

Second field: $$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \frac{1}{2}(D_\mu\phi)^a(D^\mu\phi)^a - \frac{m^2}{2}\phi^a\phi^a - \frac{\lambda}{4!}(\phi^a\phi^a)^2$$
$$+\frac{1}{2}(D_\mu\chi)^a(D^\mu\chi)^a - \frac{n^2}{2}\chi^a\chi^a - \frac{\gamma}{4!}(\chi^a\chi^a)^2$$

$$m^2,\ n^2 < 0$$

Monopoles in both fields
on top of each other
BUT only one mag. charge



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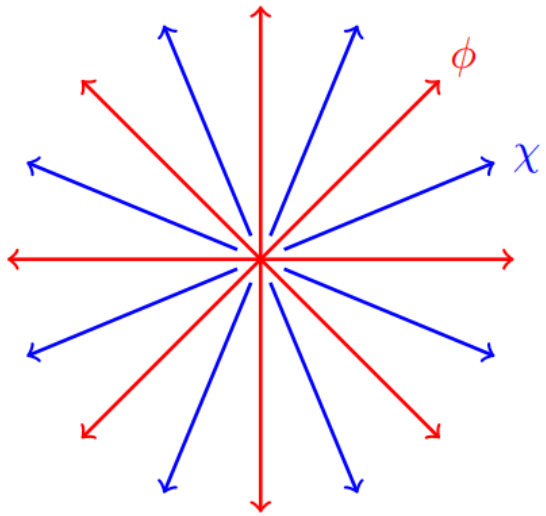
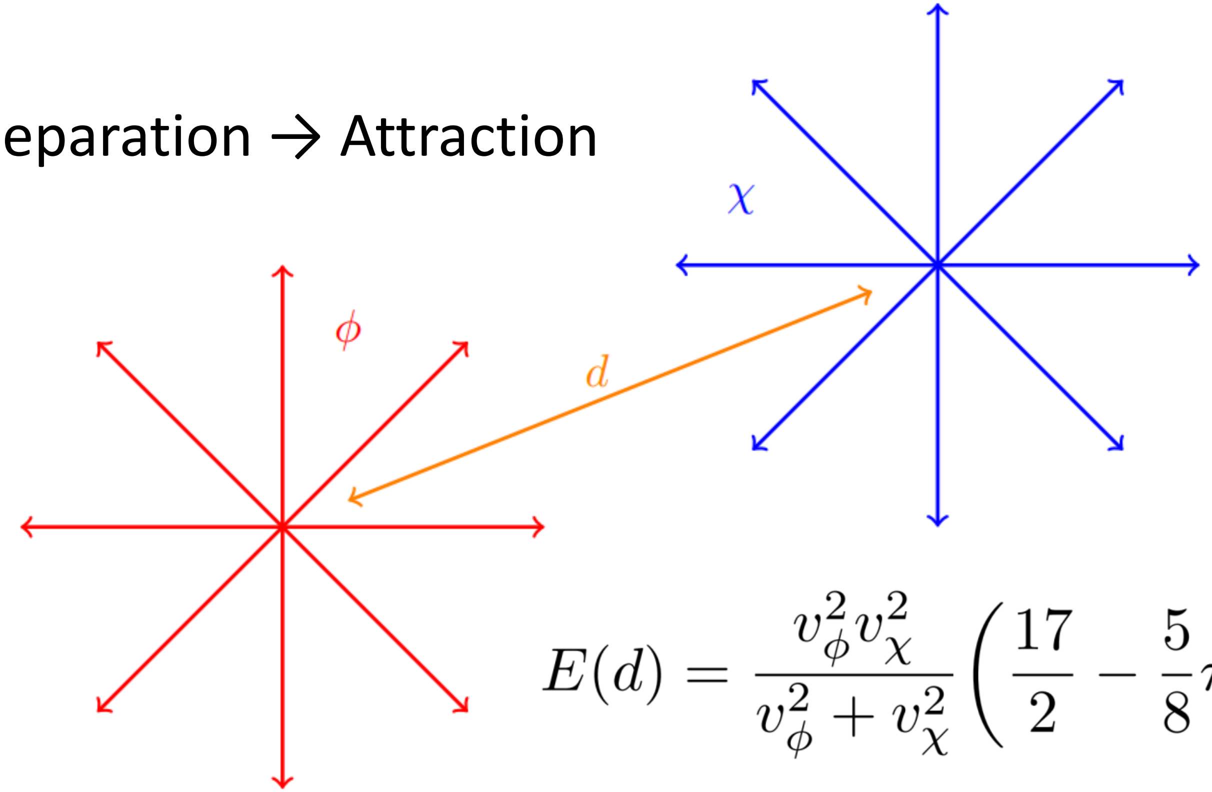


Separation → Attraction



$$E(d) = \frac{v_\phi^2 v_\chi^2}{v_\phi^2 + v_\chi^2}\left(\frac{17}{2} - \frac{5}{8}\pi^2\right)d$$

Effective Potential

$$V_{eff} := \sum \begin{matrix} \text{Diagrams with external scalars } (k_\mu = 0) \\ \text{external lines} \rightarrow \text{factor of } \phi^a/\chi^a \end{matrix}$$

Coleman and Weinberg 1973

Effective Potential

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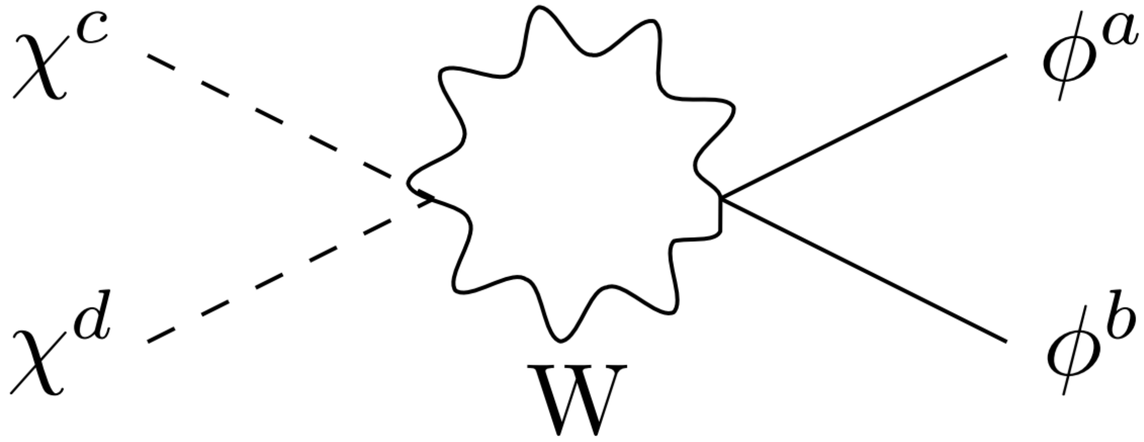
$$(D_\mu \phi)^a (D^\mu \phi)^a \supset W_\mu^a W^{\mu b} \phi^a \phi^b$$



Coleman and Weinberg 1973

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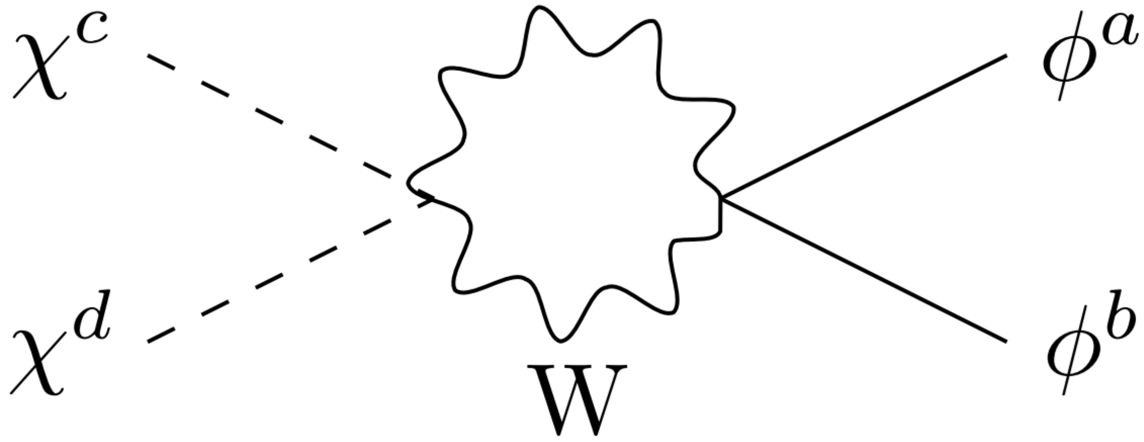

New interaction: $\phi^a\phi^b\chi^c\chi^d\delta_{ac}\delta_{bd} = \phi^a\chi^a\phi^b\chi^b =: (\phi\chi)^2$

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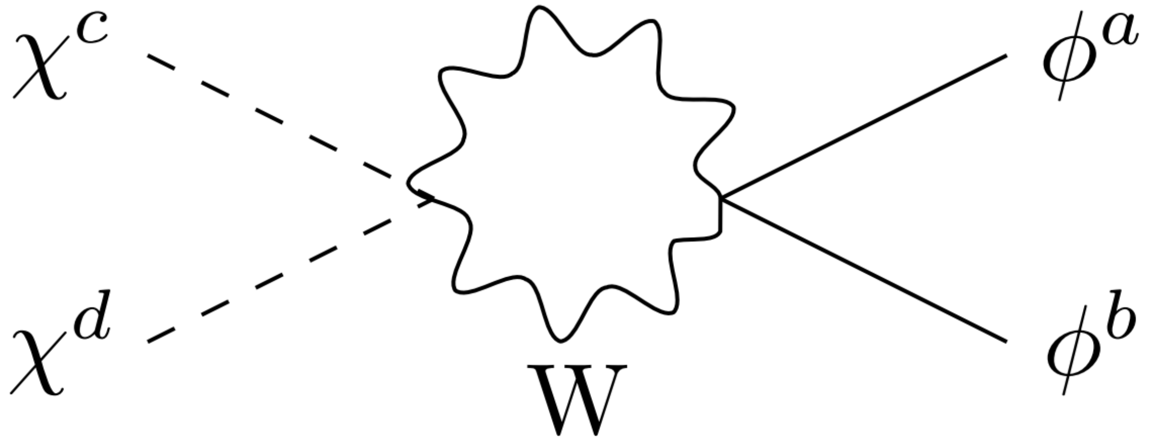

New interaction: $\phi^a\phi^b\chi^c\chi^d\delta_{ac}\delta_{bd} = \phi^a\chi^a\phi^b\chi^b =: (\phi\chi)^2$

Angle dependence!

Coleman and Weinberg 1973

Effective Potential to One-Loop Order

$$
\begin{aligned}
V_{eff} =& \frac{m^2}{2}\phi^2 + \frac{n^2}{2}\chi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\gamma}{4!}\chi^4 \\
&+ \frac{1}{64\pi^2}\Bigg[(\lambda m^2\phi^2 + m^4)\ln\left(\frac{\lambda\phi^2}{2m^2}+1\right) + (\frac{2}{3}\lambda m^2\phi^2 + 2m^4)\ln\left(\frac{\lambda\phi^2}{6m^2}+1\right) - \frac{5}{6}\lambda m^2\phi^2 \\
&\quad - \phi^4\left(5640g^4 + \lambda^2\Big(\frac{11}{72} + \frac{4752m^8 + 14688\lambda m^6 M^2 + 8728\lambda^2 m^4 M^4 + 1816\lambda^3 m^2 M^6 + 121\lambda^4 M^8}{(12m^4 + 8\lambda m^2 M^2 + \lambda^2 M^4)^2}\Big)\right) \\
&\quad + \lambda^2\phi^4\left(\frac{1}{4}\ln\left(\frac{\lambda\phi^2 + 2m^2}{\lambda M^2 + 2m^2}\right) + \frac{1}{18}\ln\left(\frac{\lambda\phi^2 + 6m^2}{\lambda M^2 + 6m^2}\right)\right) + (\phi \leftrightarrow \chi,\ \lambda \leftrightarrow \gamma,\ m \leftrightarrow n)\Bigg] \\
&+ \frac{3g^4}{16\pi^2}\Bigg[(\phi^4 + \chi^4 + \frac{1}{2}\phi^2\chi^2)\ln\left(\frac{\phi^2+\chi^2}{M^2}\right) + (\frac{1}{2}\phi^4 + \frac{1}{2}\chi^4 + (\phi\chi)^2)\ln\left(\frac{\phi^2\chi^2 - (\phi\chi)^2}{M^4}\right) \\
&\quad + \frac{1}{2}(\phi^2+\chi^2)\sqrt{4(\phi\chi)^2 + (\phi^2-\chi^2)^2}\,\ln\left(\frac{\phi^2+\chi^2+\sqrt{4(\phi\chi)^2+(\phi^2-\chi^2)^2}}{\phi^2+\chi^2-\sqrt{4(\phi\chi)^2+(\phi^2-\chi^2)^2}}\right) - 5(\phi\chi)^2 - 7\phi^2\chi^2\Bigg]
\end{aligned}
$$

Angle dependent terms

Effective Potential to One-Loop Order

$$V_{eff} = G(\phi^2, \chi^2) + F(\phi^2, \chi^2, (\phi\chi)^2)$$

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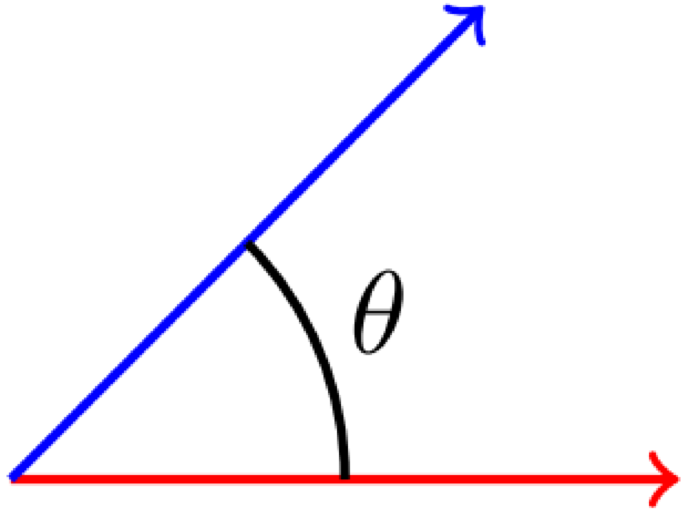
$$\langle\phi\rangle^2 = v_\phi^2, \quad \langle\chi\rangle^2 = v_\chi^2 \quad \rightarrow \quad (\phi\chi)^2 = v_\phi^2 v_\chi^2 \cos^2(\theta)$$



Effective Potential to One-Loop Order

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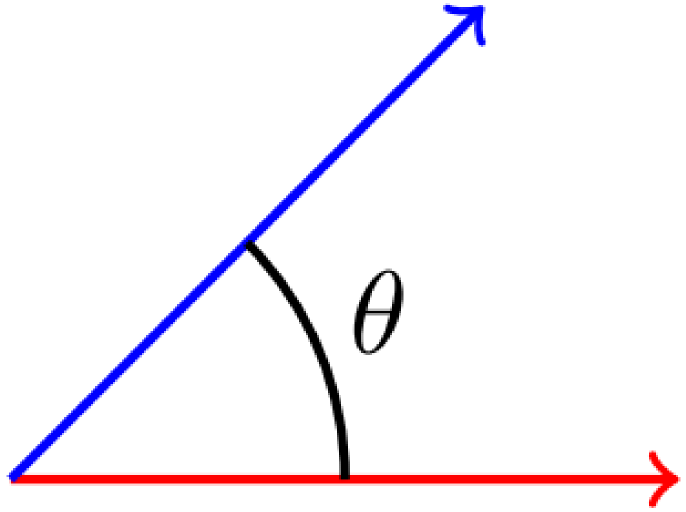
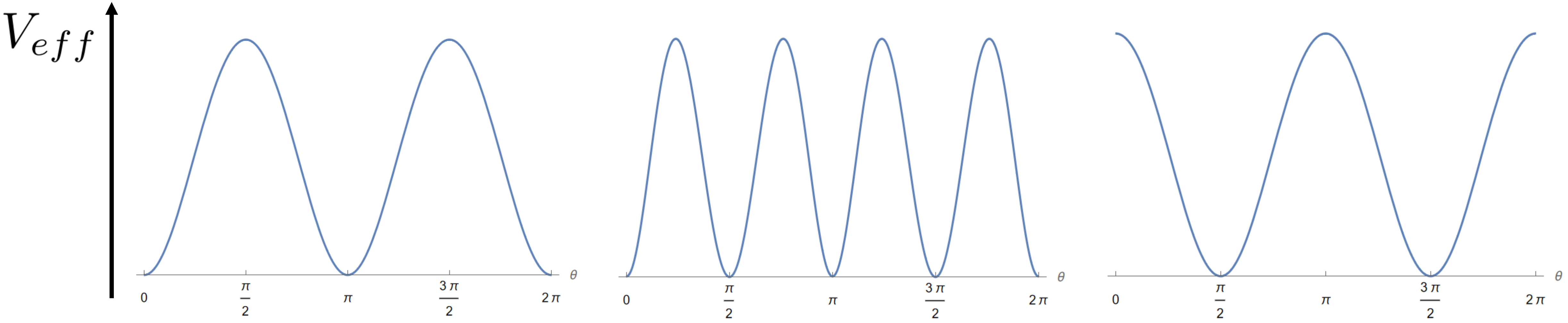


$$V_{eff} = F\big(\cos^2(\theta)\big) + \text{const}$$

Which angle minimizes the potential?

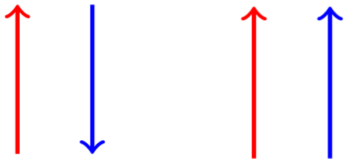
Angle Dependence

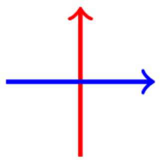


$$\frac{v_\phi}{v_\chi} \sim 1$$

$$\frac{v_\phi}{v_\chi} = r_{crit} > 1$$

$$\frac{v_\phi}{v_\chi} > r_{crit}$$

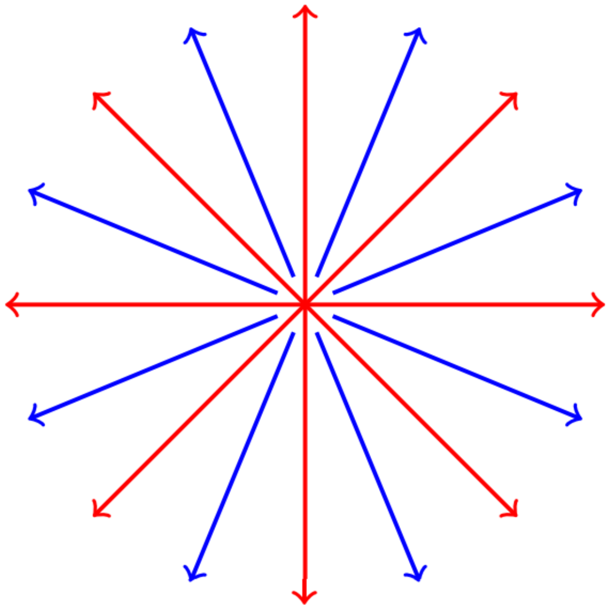


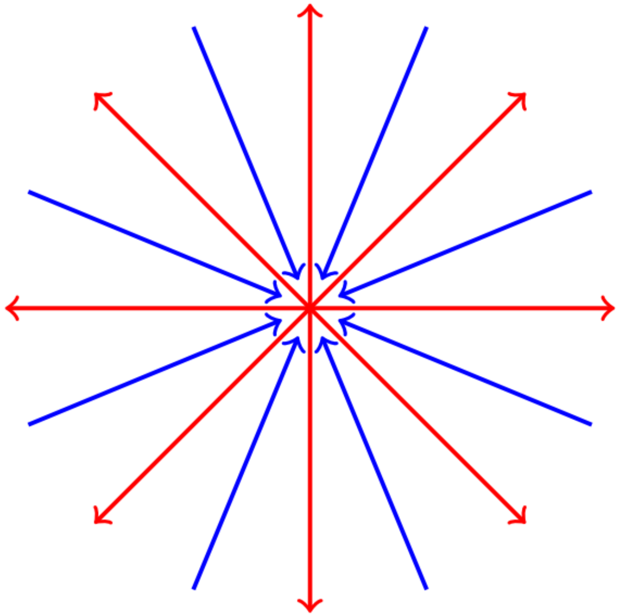


Angle Dependence – (Anti-)Parallel

If monopole in ϕ

$\Longrightarrow$ (anti-)monopole in χ





Not charge 2 monopole!

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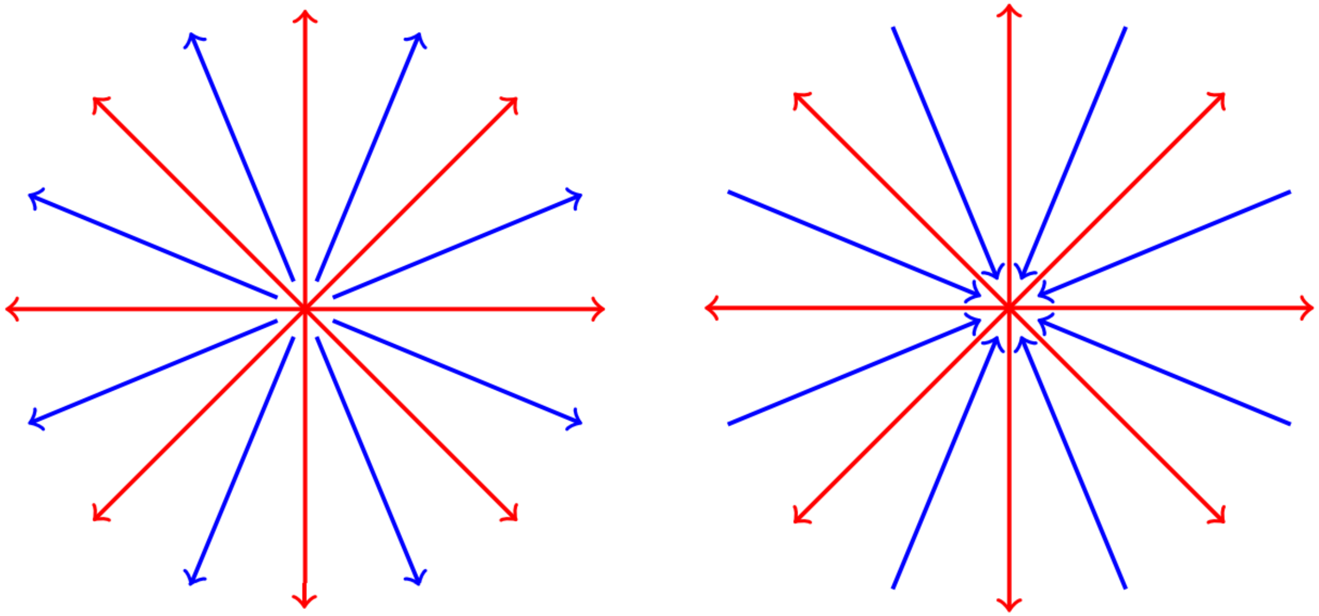
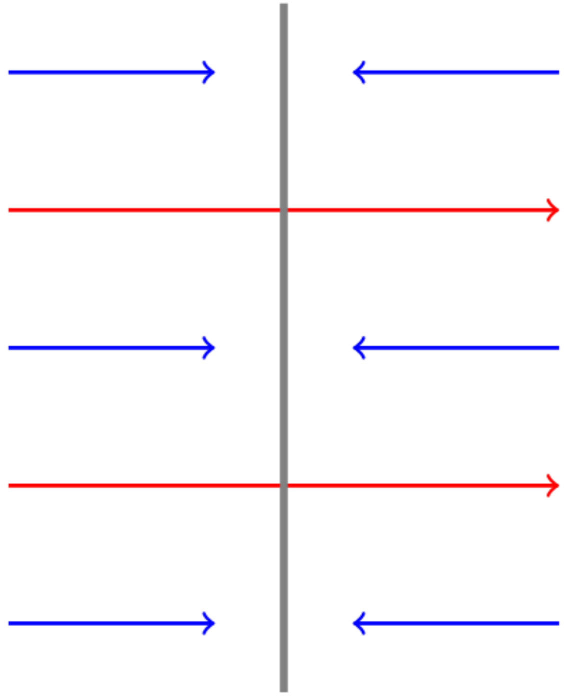


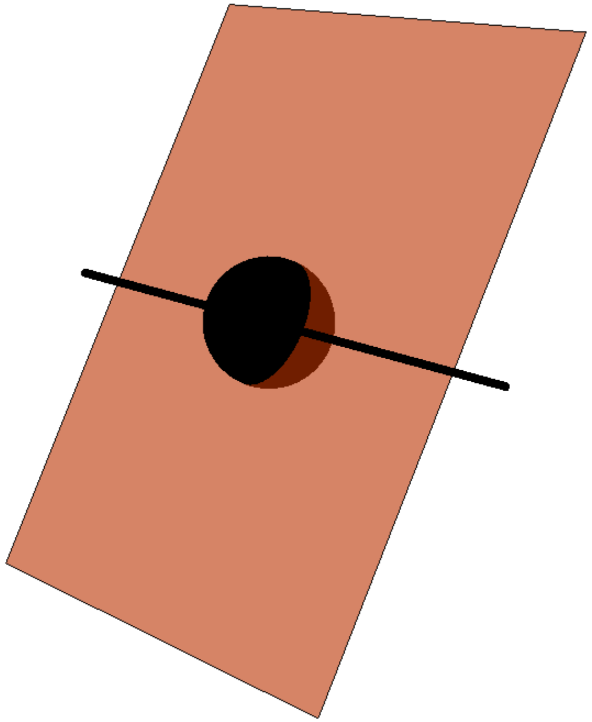
Not charge 2 monopole!

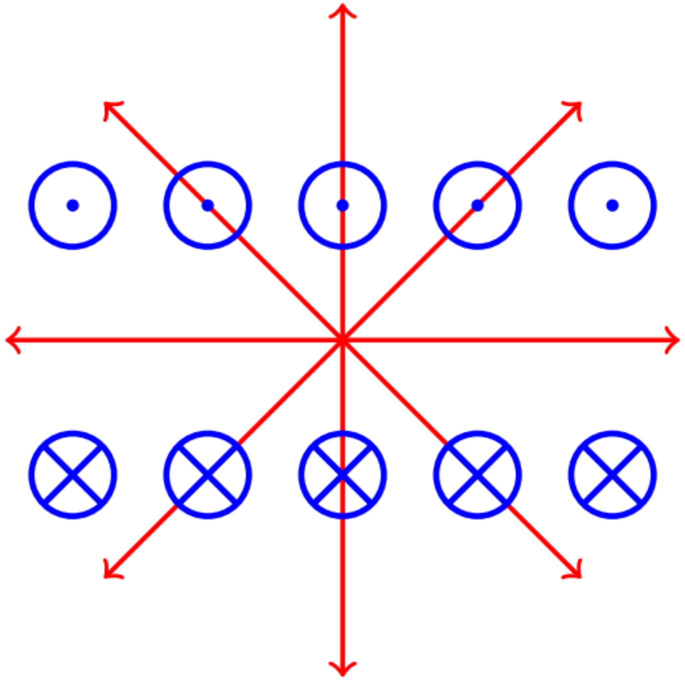
Domain walls between the parallel and antiparallel regimes can form



Angle Dependence – Orthogonal

If monopole in ϕ $\implies$ string in χ





Vilenkin and Shellard 1994

Angle Dependence – Orthogonal

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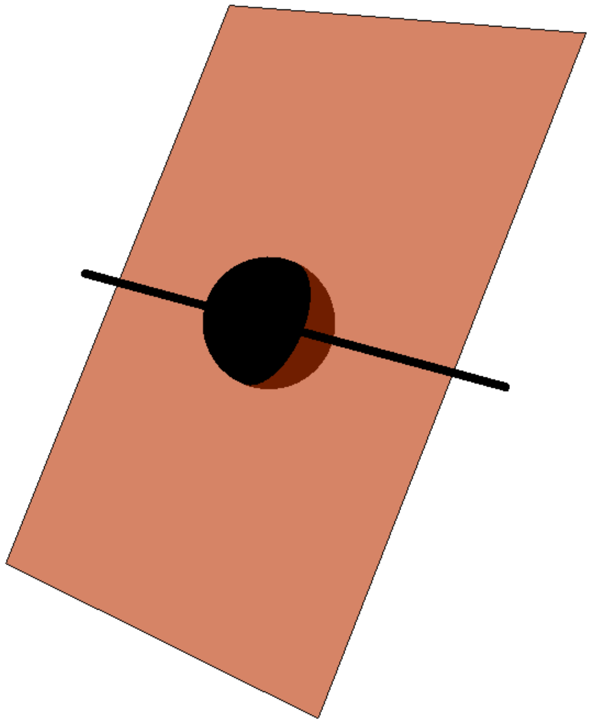
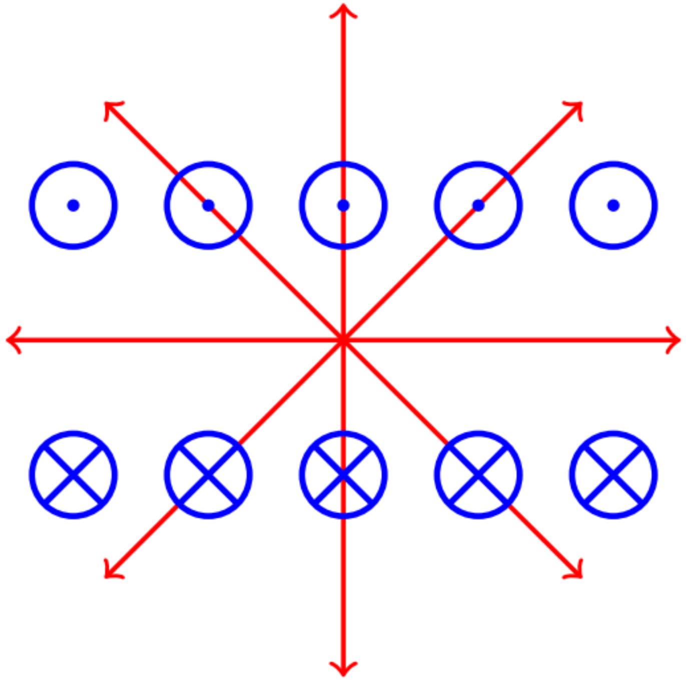
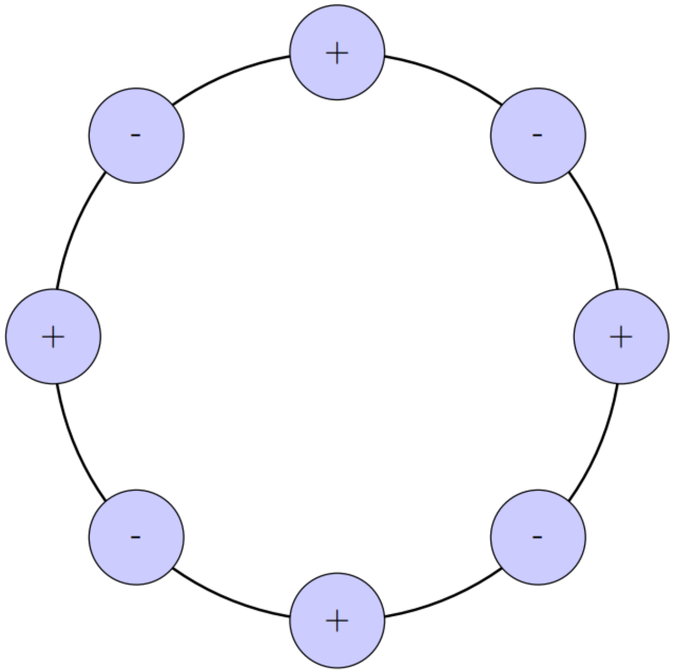






Monopole Necklace:
Monopoles connected by strings

Vilenkin and Shellard 1994

Summary

- Quantum corrections introduce new interactions

  → Non-trivial angle dependence

  → New types of topological defects

- Similar effect in other models:

  – 2x fundamental
  – fundamental + adjoint
  – Fermions instead of gauge fields

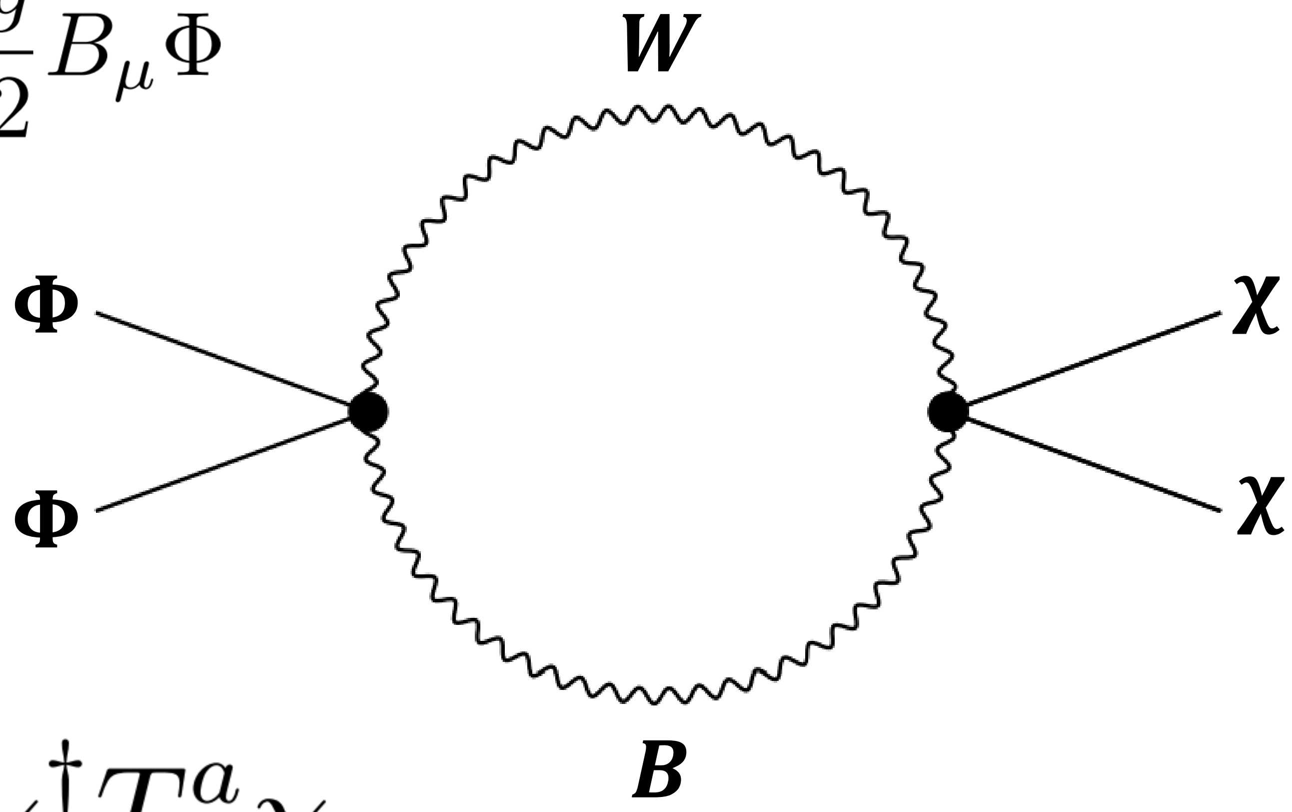
Thank You

2x Fundamental

$$D_\mu \Phi = \partial_\mu \Phi - igW_\mu^a T^a \Phi - i\frac{\tilde{g}}{2} B_\mu \Phi$$

$$\mathcal{L} \supset g\tilde{g} W_\mu^a B^\mu \Phi^\dagger T^a \Phi$$

$$\sim \Phi^\dagger T^a \Phi \; \chi^\dagger T^a \chi$$



Adjoint + Fundamental

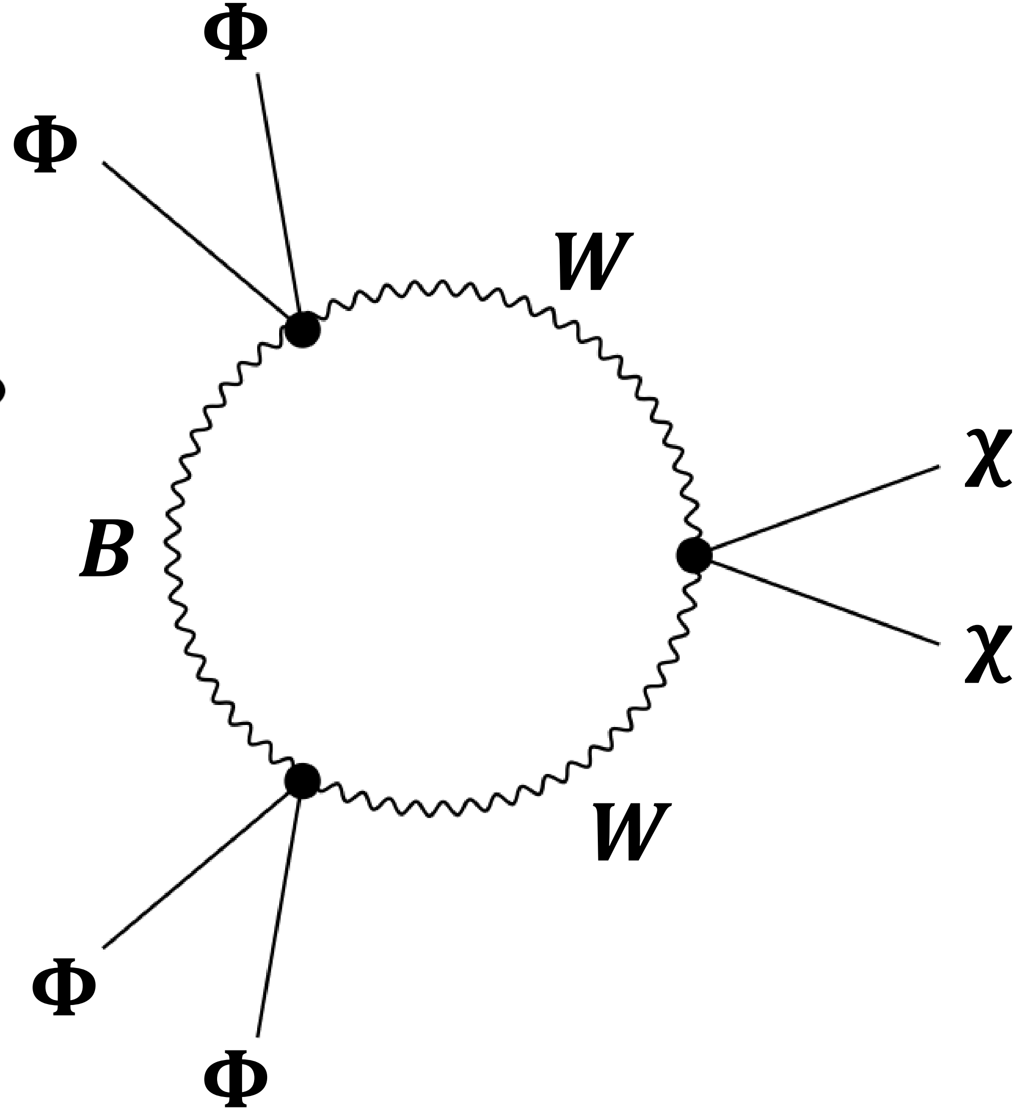
$$D_\mu \chi^a = \partial_\mu \chi^a + g\varepsilon_{abc} W_\mu^b \chi^c$$

$$D_\mu \Phi = \partial_\mu \Phi - igW_\mu^a T^a \Phi - i\frac{\tilde{g}}{2} B_\mu \Phi$$

$$\mathcal{L} \supset g^2 W_\mu^a W^{\mu b} \chi^a \chi^b$$

$$\mathcal{L} \supset g\tilde{g} W_\mu^a B^\mu \Phi^\dagger T^a \Phi$$

$$\sim \Phi^\dagger T^a \Phi \; \chi^a \chi^b \; \Phi^\dagger T^b \Phi = \left(\Phi^\dagger \chi \Phi\right)^2$$



Fermions Instead of Gauge Fields

$$\mathcal{L} \supset \bar{\psi}_1^a \chi^a \psi_2$$

$$\sim \phi^a \chi^a$$