Mass Effects in Resonance Decays at (N)NLO

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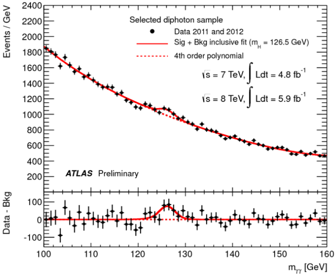
Motivation

Why Higgs Boson?

2012 : experimental discovery of the Higgs Boson at LHC. [ATLAS/CMS '12]

2026 : expected end of Run 3 for LHC at the start of July

2030 : expected start of Run 4 at LHC (High-Luminosity Phase) in June

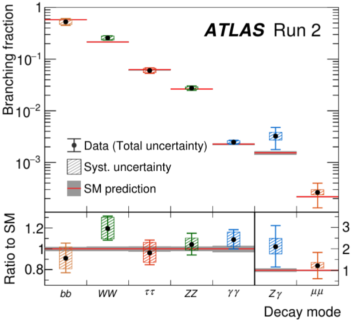


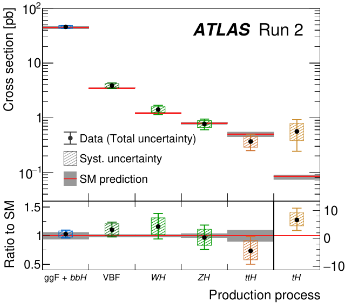
- Improve the precision of measurements of couplings to vector bosons and quarks t, b.
- Investigate the self-coupling and couplings to other quarks.
- Search for signatures of BSM physics.

Motivation

Why $H \to b\bar{b}$?

- Most abundant decay channel, with $BR \sim 58\%$, precision $\sim 15\%$ [ATLAS '22]
- Used for Higgs boson *tagging*
- Relatively easy to study





Main production processes used:

- VH $\to$ vector boson gives a clear signature
- VBF $\to$ presence of clearly separated jets

Objectives

$\Longrightarrow$ Reproduce result for NLO partial decay width analytically [Braaten,Leveille '80]

$\Longrightarrow$ Reproduce result for NLO partial decay width numerically

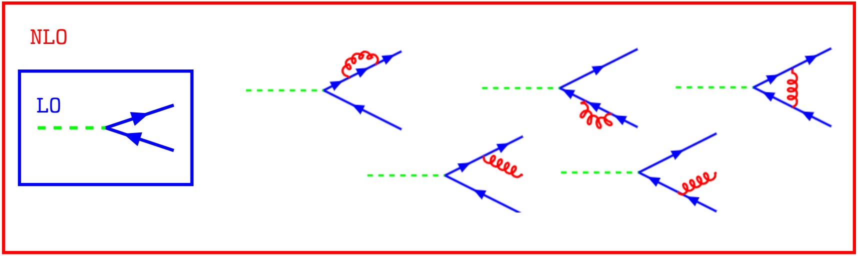
$\Longrightarrow$ Implementation of the algorithm in the `POWHEG` framework
[Nason '04] [Frixione,Nason,Oleari '07] [Alioli,Nason,Oleari,Re '10]

$\Longrightarrow$ Interface with `Pythia8` Parton Shower [Sjöstrand et al. '14]

$\Longrightarrow$ Observe QCD NLO corrections (LO+PS vs. NLO+PS)

$\Longrightarrow$ Observe mass effects (NLO+PS vs. `MiNLO'`+PS) [Bizon,Re,Zanderighi '20]

Analytical NLO Result



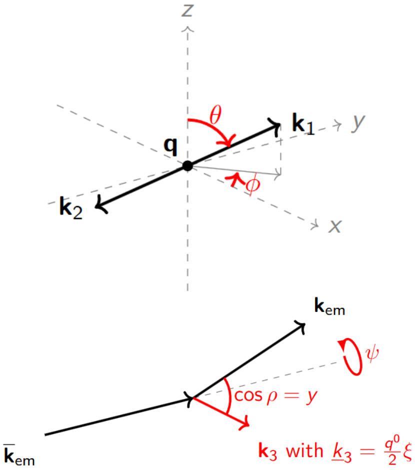
$\Longrightarrow$ Partial decay width at NLO [Braaten,Leveille '80]

$\Longrightarrow$ Reproduced the existing result using `Mathematica` and Scalar 1-Loop integrals [Ellis,Zanderighi '11]

$\Longrightarrow$ Sizeable NLO QCD corrections:

$$\Gamma_{\mathrm{NLO}} \,/\, \Gamma_{\mathrm{LO}} \simeq 0.66$$

Phase Space Parameterization



- $\Longrightarrow$ Mapping for radiation coming from heavy quarks
  [Buonocore, Nason, Tramontano '18]
- $\Longrightarrow$ Born phase space $(\cos\theta, \phi)$
- $\Longrightarrow$ Radiation phase space $\longrightarrow$ FKS variables $\left(\tilde{\xi}, y, \psi\right)$
- $\Longrightarrow$ Parameterization to the unit cube to cancel integrable divergence

Numerical NLO Result

$\Longrightarrow$ Subtraction formalism using the *plus distribution*:

$$\int_0^1 \mathrm{d}\tilde{\xi} \left(\frac{1}{\tilde{\xi}}\right)_+ \hat{\mathcal{R}} := \int_0^1 \mathrm{d}\tilde{\xi}\, \frac{\hat{\mathcal{R}}(\tilde{\xi}) - \hat{\mathcal{R}}(\tilde{\xi}=0)}{\tilde{\xi}}$$

$$\longrightarrow \int \mathrm{d}\Phi_3 \left\{\mathcal{R}\left(\Phi_3\right) - \sum_\alpha \mathcal{C}^{(\alpha)}\left(\Phi_3\right)\right\} = \text{finite}$$

$$\longrightarrow \Gamma_{\text{NLO}} = \int \mathrm{d}\Phi_2 \left[\mathcal{B} + \mathcal{V} + \mathcal{G} + \sum_{(\alpha)} \tilde{\mathcal{C}}^{(\alpha)}\right] + \int \mathrm{d}\Phi_3 \left\{\mathcal{R}\left(\Phi_3\right) - \sum_\alpha \mathcal{C}^{(\alpha)}\left(\Phi_3\right)\right\}$$

Method	Γ_{NLO} $(10^{-3}\,\text{GeV})$	$\pm\Delta\Gamma_{\text{NLO}}$ $(10^{-8}\,\text{GeV})$	$\Delta\sigma$
ANALYTICAL	3.67831	-	-
DEFAULT	3.67828	7	0.42
ALTERNATIVE	3.67848	17	1.00
MERGE	3.67880	49	1.00

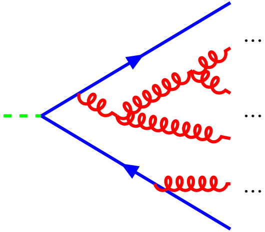
Reaching NLO+PS accuracy

$\Longrightarrow$ No IR divergences (KLN theorem [Kinoshita '62] [Lee,Nauenberg '64]); still, large logs $L := \log\left(Q/\tilde{Q}\right)$ break the fixed-order description

$\Longrightarrow$ Resummation: reorganization of the perturbative expansion

$\Longrightarrow$ Parton Shower: probabilistic procedure executing the all-order resummation

$$\sigma_{\mathrm{SMC}} = \int \underbrace{\mathcal{B}(\Phi_2)\, \mathrm{d}\Phi_2}_{\mathrm{d}\sigma_{\mathrm{LO}}} \left[\Delta(Q^2, \tilde{Q}^2) + \int \underbrace{\mathrm{d}\mathcal{P}_{\text{emission}}\left(q^2\right)}_{\frac{\mathrm{d}q^2}{q^2}\,\mathrm{d}z\,\mathrm{d}\psi\,\frac{\alpha_s}{2\pi}P(z)} \Delta(Q^2, q^2) + \dots \right]$$

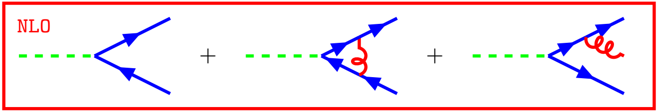


$$\Delta\left(Q^2, \tilde{Q}^2\right) := \exp\left\{ -\int_{\tilde{Q}^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \Gamma(Q, q) \right\} \approx$$

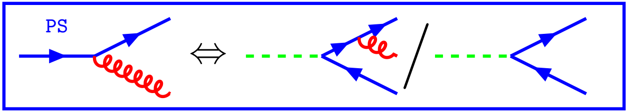
$$\approx \exp\left\{-\alpha_s L^2\right\}$$

$\Longrightarrow$ LO+LL accuracy

$$\mathcal{B}\left(\Phi_2\right) \Rightarrow \overline{\mathcal{B}}\left(\Phi_2\right) := \mathcal{B}\left(\Phi_2\right) + \mathcal{V}\left(\Phi_2\right) + \mathcal{G}\left(\Phi_2\right) + \sum_{\alpha} \int \mathrm{d}\Phi_{\mathrm{r}} \left[\mathcal{R}^{(\alpha)}\left(\Phi_3\right) - \mathcal{C}^{(\alpha)}\left(\Phi_3\right)\right]$$

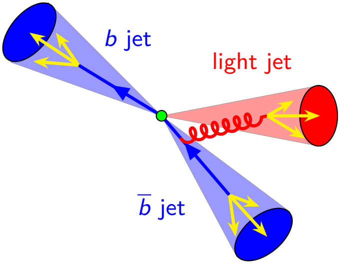


$$\sigma_{\mathrm{POWHEG}} := \int \mathrm{d}\Phi_2 \, \overline{\mathcal{B}}\left(\Phi_2\right) \left[\Delta(\Phi_2, p_T^2) + \int \mathrm{d}\Phi_3 \, \Delta\left(\Phi_2, p_T^2\right) \frac{\mathcal{R}\left(\Phi_3\right)}{\mathcal{B}\left(\Phi_2\right)}\right]$$



$$\Delta(\Phi_2, p_T^2) := \theta\left(p_T^2 - p_{T,\mathrm{min}}^2\right) \exp\left[-\int \mathrm{d}\Phi_{\mathrm{rad}} \frac{\mathcal{R}\left(\Phi_2, \Phi_{\mathrm{rad}}\right)}{\mathcal{B}\left(\Phi_2\right)} \theta\left(k_T^2\left(\Phi_3\right) - p_T^2\right)\right]$$

Jet Algorithms



$\Longrightarrow$ k_T algorithm (Durham)
[Catani et al. '91]

$\Longrightarrow$ k_T-flavour algorithm
[Banfi,Salam,Zanderighi '06]

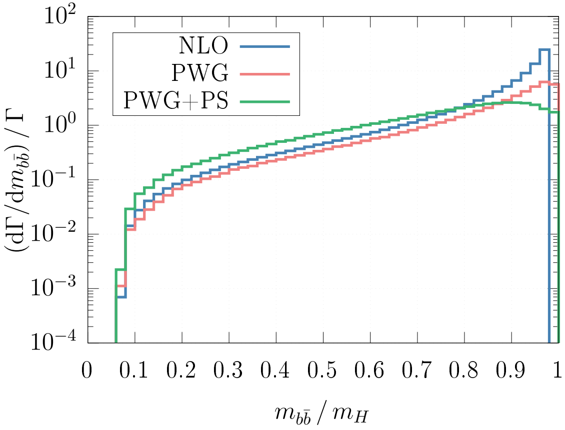
$+$ "bland" versions

$\longrightarrow$ Recombination scheme E:

$$k_{\text{recombined}} = k_i + k_j$$

Algorithm	**2 Jets [%]**	**3 Jets [%]**
k_T	88.9	86.6
k_T-flavour	95.4	94.6
Bland k_t	97.1	97.4
Bland k_t-flavour	97.2	97.8

Effects of PS Resummation



⇒ NLO: contribution of the real emission matrix element; fixed-order description breaks down in the low-hardness emission ($m_{b\bar{b}}/m_H \to 1$)

⇒ PWG: partonic events $\left(H \to b\bar{b}/b\bar{b}g\right)$

⇒ PWG+PS: final state multiplicity enhanced (hardest b and $\bar{b}$ considered)

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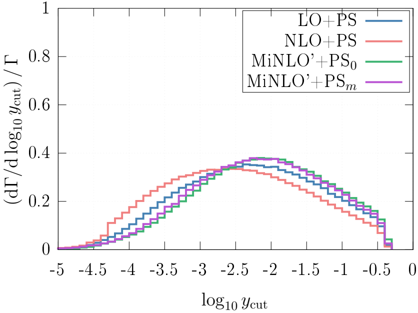
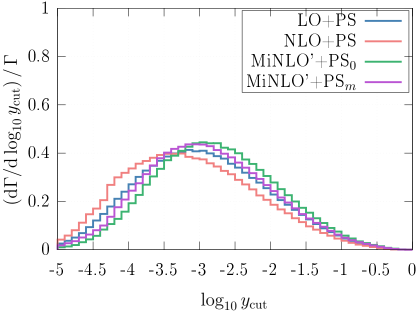
Exclusive 2- and 3-jet



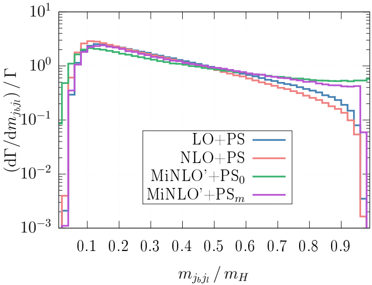


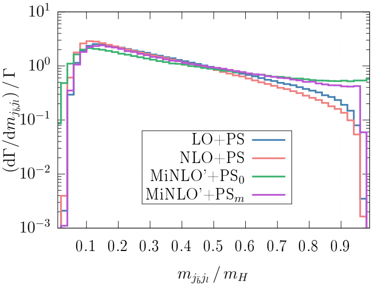
$\Longrightarrow$ NLO corrections: clear distinction between the LO+PS and NLO+PS distributions

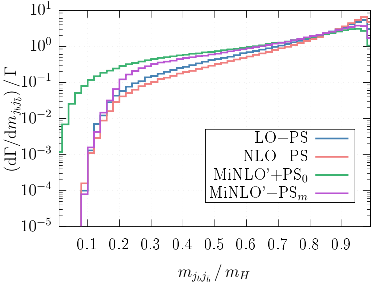
$\Longrightarrow$ Mass effects: clear distinction between the NLO+PS and `MiNLO'`+PS$_0$ distributions (to be investigated)

$\Longrightarrow$ At first sight, *reshuffling* (`MiNLO'`+PS$_m$) approximation might miss some effects

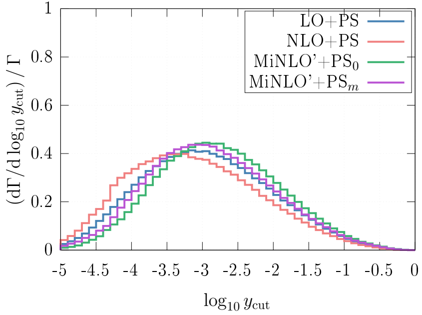
Exclusive 3-jet

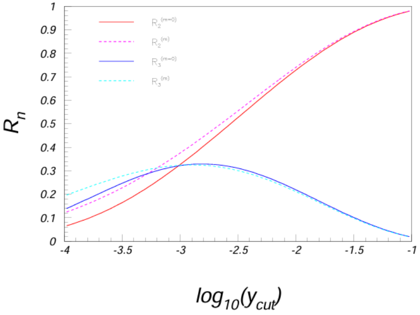






Comparison with theoretical prediction





[Krauss,Rodrigo '03]

⟹ Semi-analytical 3-jet rates for the process $Z \to b\bar{b}$ using the k_t algorithm

⟹ Mass effects bear the same result: shift towards lower recombination cut-off

Future prospects

$\Longrightarrow$ Inclusion of theoretical uncertainty bands with scale variation (in progress)

$\Longrightarrow$ Extension to additional decay channels

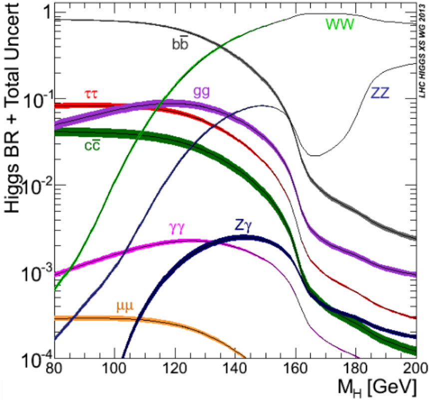
$\Longrightarrow$ Application to production processes (e.g., VH, VBF, ...)

$\Longrightarrow$ Inclusion of hadronization

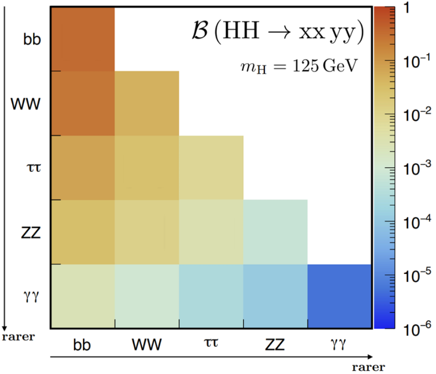
$\Longrightarrow$ Achieving NNLO+PS QCD accuracy in the massive case

Thanks for your attention!

Backup



[CERN Yellow Report 3]



[CMS-CR-2022-131]

Backup

Some results about QCD corrections to the Higgs decay:

$\Longrightarrow$ NLO massive [Braaten,Leveille '80]

$\Longrightarrow$ NNLO massless [Anastasiou,Herzog,Lazopoulos '12] [Del Duca et al. '15]

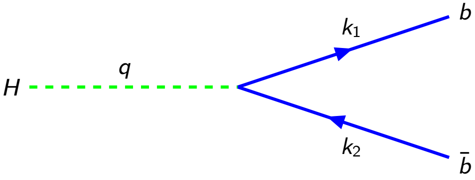
$\Longrightarrow$ N^3LO massless [Mondini,Schiavi,Williams '19] [Mondini,Williams '19]

$\Longrightarrow$ NNLO massive [Bernreuther,Chen,Si '18] [Behring,Bizon '20] [Primo et al. '19]

$\Longrightarrow$ NNLO+PS massless [Bizon,Re,Zanderighi '20]

Backup

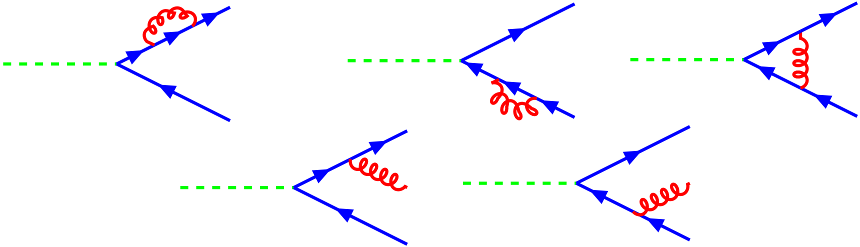
A single diagram at LO:



$$\mathrm{d}\Phi_2 = \frac{1}{32\pi^2}\beta_0\,\mathrm{d}\Omega\,, \quad \beta_0 := \sqrt{1-4\frac{m_b^2}{m_H^2}}$$

$\Longrightarrow$ Analytical results at LO:

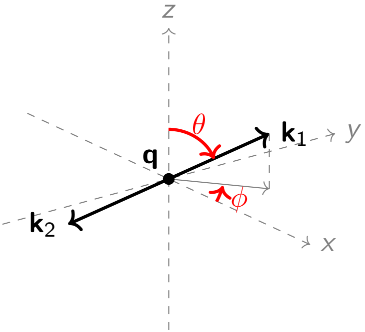
$$\Gamma_{\mathrm{LO}} := \int \mathrm{d}\Phi_2\,\mathcal{B}(\Phi_2) = \frac{3}{8\pi}\beta_0^3 g^2 m_H\,, \quad g := \frac{m_b}{v}$$



$$\Gamma = \Gamma_0 \left(1 + \frac{\alpha_s}{2\pi}\left\{6 - \frac{3}{4}\frac{1+\beta_0^2}{\beta_0^2} + 12\log\frac{m_b}{m_H} - 8\log\beta_0 + \left(\frac{5}{\beta_0} - 2\beta_0 + \right.\right.\right.$$

$$\left. + \frac{3}{8}\frac{\left(1-\beta_0^2\right)^2}{\beta_0^3}\right)\log\left(\frac{1-\beta_0}{1+\beta_0}\right) + \frac{1+\beta_0^2}{\beta_0}\left[4\log\left(\frac{1-\beta_0}{1+\beta_0}\right)\log\left(\frac{1+\beta_0}{2\beta_0}\right) - \right.$$

$$\left.\left.\left. -2\log\left(\frac{1+\beta_0}{2}\right)\log\left(\frac{1-\beta_0}{2}\right) + 8Li_2\left(\frac{1-\beta_0}{1+\beta_0}\right) - 4Li_2\left(\frac{1-\beta_0}{2}\right)\right]\right\}\right) \approx 0.66\,\Gamma_0$$

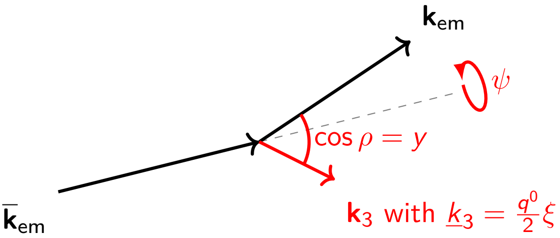


$\Longrightarrow$ Parameterization of Φ_2 in terms of the unit square:

$$\cos\theta = 2\left(\frac{1}{2} - X_{\mathrm{b}}^{(1)}\right)$$
$$\phi = 2\pi X_{\mathrm{b}}^{(2)}$$

We generate $H \to b\bar{b}$ events once having reconstructed $(\mathbf{k}_1, \mathbf{k_2})$ in terms of $(\cos\theta, \phi)$.

Backup



$\Longrightarrow$ Parameterization of Φ_3 in terms of the unit cube:

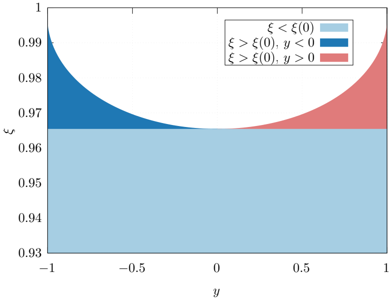
$$\tilde{\xi} = 1 - \left(1 - X_{\mathrm{r}}^{(1)}\right)^2$$

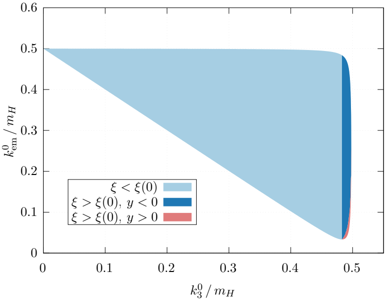
$$y = \frac{1}{\beta_0}\left[1 - (1+\beta_0)\exp\left\{-X_{\mathrm{r}}^{(2)}\log\left(\frac{1+\beta_0}{1-\beta_0}\right)\right\}\right]$$

$$\psi = 2\pi X_{\mathrm{r}}^{(3)}$$

$$J(\xi, y, \psi) = \frac{q^2}{(4\pi)^3} \frac{\underline{k}_{\rm em}^3}{\underline{k}_{\rm em}} \frac{2\xi}{k_{\rm em}^0 (2\overline{k}_{\rm em}^0 - q^0 \xi) - m_b^2 (2 - \xi)}$$

$$\tilde{\xi} \to 1 : \quad J \sim \frac{1}{\sqrt{1 - X_{\rm r}^{(1)}}}\,, \quad J_u \sim 1 - X_{\rm r}^{(1)}$$





Backup

⇒ Massive case: only soft divergences!

Subtraction of divergent terms using the *plus distribution*:

$$\tilde{\xi}^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta\left(\tilde{\xi}\right) + \left(\frac{1}{\tilde{\xi}}\right)_+ + \mathcal{O}(\epsilon)$$

$$\int_0^1 \mathrm{d}\tilde{\xi}\left(\frac{1}{\tilde{\xi}}\right)_+ \hat{\mathcal{R}} := \int_0^1 \mathrm{d}\tilde{\xi}\,\frac{\hat{\mathcal{R}}(\tilde{\xi}) - \hat{\mathcal{R}}(\tilde{\xi}=0)}{\tilde{\xi}}$$

$$\int \mathrm{d}\Phi_3 \left\{\mathcal{R}\left(\Phi_3\right) - \sum_\alpha \mathcal{C}^{(\alpha)}\left(\Phi_3\right)\right\} =$$

$$= \int \mathrm{d}\Phi_2 \int_0^{2\pi} \mathrm{d}\psi \int_{-1}^1 \mathrm{d}y \int_0^1 \mathrm{d}\tilde{\xi}\,\frac{1}{\tilde{\xi}}\left\{\frac{J\left(\tilde{\xi},y,\psi\right)}{\xi}\left[\tilde{\xi}^2\mathcal{R}\right] - \right.$$

$$\left. - \lim_{\tilde{\xi}\to 0}\frac{J\left(\tilde{\xi},y,\psi\right)}{\xi}\left[\tilde{\xi}^2\mathcal{R}\right]\right\} + \log\left[\xi^{(-)}(y)\right]\lim_{\tilde{\xi}\to 0}\left\{\frac{J\left(\tilde{\xi},y,\psi\right)}{\xi}\left[\tilde{\xi}^2\mathcal{R}\right]\right\}$$

Backup

$\Longrightarrow$ Perturbative expansions in powers of α_s is justified when

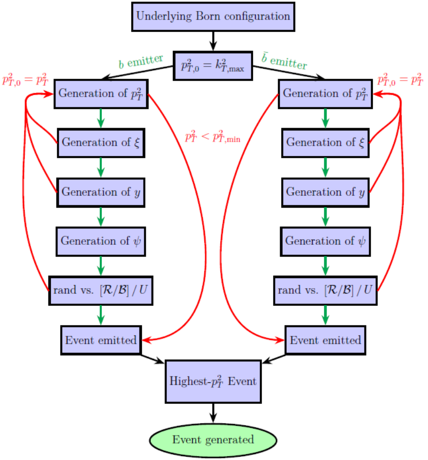
$$Q^2 \gg \Lambda^2_{\text{QCD}} \to \alpha_s(Q^2) \ll 1$$

$\Longrightarrow$ Coefficients of the expansion can potentially be large. Introducing the physical scales Q and $\tilde{Q}$, large logarithmic terms $L = \log\left(Q/\tilde{Q}\right)$ can arise

$\Rightarrow$ Resummation: reorganization of the perturbative expansion

$$\begin{aligned} \mathrm{d}\sigma =& 1 + \alpha_s\left(L^2 + L + 1\right) + \alpha_s^2\left(L^4 + L^3 + L^2 + L + 1\right) + \ldots \\ \longrightarrow& \exp\left[\underbrace{Lg_1\left(\alpha_s L\right)}_{\text{LL}} + \underbrace{g_2\left(\alpha_s L\right)}_{\text{NLL}} + \underbrace{\alpha_s g_3\left(\alpha_s L\right)}_{\text{NNLL}}\right] \times \text{const.}(\alpha_s) + \ldots \end{aligned}$$

Backup



Modifications:

- Hardness scale for massive case:

$$k_T^2 := 2\frac{k_3^0}{k_{em}^0} k_{\mathrm{em}} \cdot k_3 =$$
$$= \frac{q^2}{2}\xi^2 \left(1 - \beta\, y_{\mathrm{phys}}\right)$$

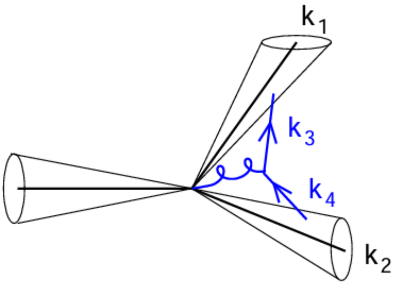
- Upper bound function:

$$U'(\xi, k_T^2) := \frac{N}{\xi k_T^2 (1 - k_T^2/q^2)}$$
$$U(\xi, y) = \frac{\partial k_T^2}{\partial y} U'\left(\xi, k_T^2(y)\right)$$

Backup

k_T algorithm

$$y_{ij}^{(\mathrm{D})} = \frac{2(1-\cos\theta_{ij})}{s}\min\{E_i^2, E_j^2\}$$



[Banfi,Salam,Zanderighi '06]

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k_T-flavour algorithm

$$y_{ij}^{(\mathrm{F})} := \frac{2\,(1-\cos\theta_{ij})}{s}\begin{cases}\min\{E_i^2, E_j^2\}\,, & \text{softer is flavourless}\\ \max\{E_i^2, E_j^2\}\,, & \text{softer is flavoured}\end{cases}$$

$\Rightarrow$ Infrared-safe way of recombining jet flavour starting from NNLO

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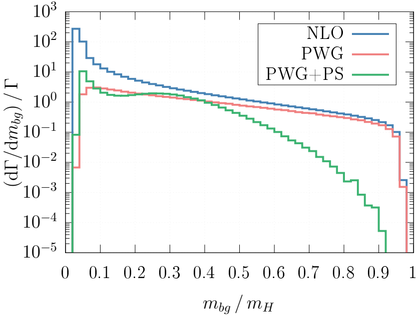
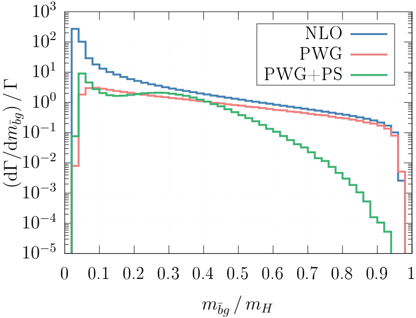

NLO
PWG
PWG+PS
$(\mathrm{d}\Gamma/\mathrm{d}m_{bg})\,/\,\Gamma$
$m_{bg}\,/\,m_H$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


NLO
PWG
PWG+PS
$(\mathrm{d}\Gamma/\mathrm{d}m_{\bar{b}g})\,/\,\Gamma$
$m_{\bar{b}g}\,/\,m_H$
10^3
10^2
10^1
10^0
10^{-1}
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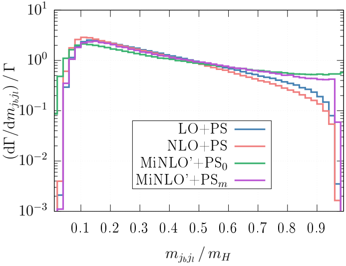
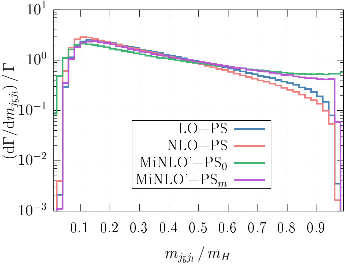
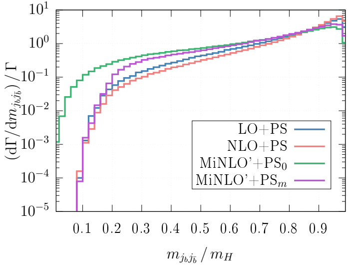
10^{-3}
10^{-4}
10^{-5}
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$(\mathrm{d}\Gamma/\mathrm{d}m_{j_b j_l})\,/\,\Gamma$
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
$m_{j_b j_l}\,/\,m_H$
LO+PS
NLO+PS
MiNLO'+PS_0
MiNLO'+PS_m


10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$(\mathrm{d}\Gamma/\mathrm{d}m_{j_{\bar b} j_l})\,/\,\Gamma$
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
$m_{j_{\bar b} j_l}\,/\,m_H$
LO+PS
NLO+PS
MiNLO'+PS_0
MiNLO'+PS_m


10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$(\mathrm{d}\Gamma/\mathrm{d}m_{j_b j_{\bar b}})\,/\,\Gamma$
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
$m_{j_b j_{\bar b}}\,/\,m_H$
LO+PS
NLO+PS
MiNLO'+PS_0
MiNLO'+PS_m