Concepts of Experiments at Future Colliders II

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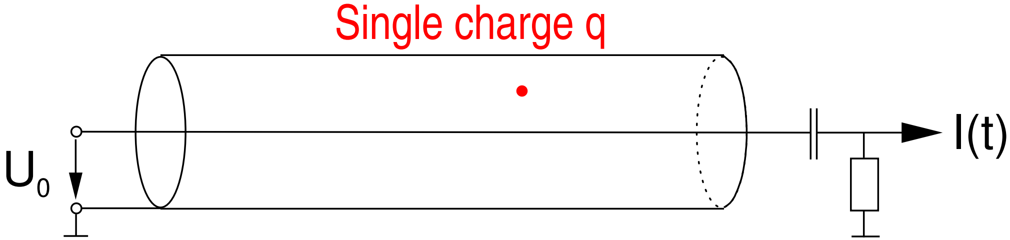
Topics of the lecture in the summer semester

1. Fundamentals of electronic readout of particle detectors.
2. Fundamentals of statistical treatment of experimental data.
3. Reconstruction of pp collision events.
4. Trigger concepts for experiments at hadron colliders.

Fundamentals of electronic readout of particle detectors



Derivation of the Shockley-Ramo theorem

- Theorem about the current induced on an electrode by a moving charge q.
- Derivation using an electrostatic approximation, i.e. neglecting the magnetic field created by the moving charge.
- If we know the electric field of the configuration depicted above, we can compute the charge density on the electrode and how it changes when the charge q is moving.
- As shown by Shockley and Ramo one does not need to know the exact form of this field, but just some of its properties.

Shockley-Ramo theorem for a cylindrical drift tube

$$\begin{aligned} rot\vec{E} &= 0, \text{ hence } \vec{E} = \nabla\phi. \\ \nabla \cdot \vec{E} &= -\Delta\phi = \frac{\rho}{\epsilon_0}. \end{aligned}$$

Electrodes are conductors $\Rightarrow$ ϕ=const on the surfaces of the electrodes.

Configuration 1: $q = 0$

- Electric field $E_{q=0}$, potential $\phi_{q=0}$.
- $\phi_0|_{anode\ \ wire} = U_0$, $\phi_0|_{tube\ \ wall} = 0$.
- $-\Delta\phi_0 = 0$ between the electrodes, i.e. within the gas volume.

Configuration 2: $q \neq 0$, all electrodes grounded

- $\phi_q|_{anode\ \ wire} = 0$, $\phi_q|_{tube\ \ wall} = 0$.
- $-\Delta\phi_q = \frac{q}{\epsilon_0}\delta(\vec{x} - \vec{x}_q)$.

Configuration 3: $q \neq 0$, $\phi|_{anode\ \ wire} = U_0$, $\phi|_{tube\ \ wall} = 0$

- $\phi = \phi_0 + \phi_q$.
- $-\Delta\phi = \frac{q}{\epsilon_0}\delta(\vec{x} - \vec{x}_q)$ with $q \neq 0$, $\phi|_{anode\ \ wire} = U_0$

Shockley-Ramo theorem for a cylindrical drift tube

Green's second identity

$$\int_V \left(\phi_0 \nabla^2 \phi_q - \phi_q \nabla^2 \phi_0\right) dV = \int_{\partial V} \left(\phi_0 \nabla \phi_q - \phi_q \nabla \phi_0\right) \cdot d\vec{S}.$$

Choice of V

Volumne between the electrodes without a small ball $B_\epsilon(\vec{x}_q)$ around q.

Consequences

- $\phi_0, \phi_q = 0$ in V $\Rightarrow \int_V \left(\phi_0 \nabla^2 \phi_q - \phi_q \nabla^2 \phi_0\right) dV = 0.$
- $\int_{\partial V} (\dots) \cdot d\vec{S} = \int_{wire} + \int_{wall} + \int_{\partial B_\epsilon(\vec{x}_q)} (\dots) \cdot d\vec{S}.$
- $0 = U_0 \int_{wire} \nabla \phi_q \cdot d\vec{S} + \phi(\vec{x}_q) \int_{\partial B_\epsilon(\vec{x}_q)} \nabla \phi_q \cdot d\vec{S} + \phi_q(\partial B_\epsilon(\vec{x}_q)) \int_{\partial B_\epsilon(\vec{x}_q)} \nabla \phi_0 \cdot d\vec{S}.$
- Hence with $\vec{E_{0/q}} = -\nabla \phi_{0/q}$ we get $U_0 Q_{wire} = -q \phi_0(\vec{x}_q)$.

$$I_{wire} = \frac{dQ_{wire}}{dt} = -q \frac{1}{U_0} \nabla \phi_0(\vec{x}_q) \cdot \frac{d\vec{x}_q}{dt}.$$

Shockley-Ramo theorem for a cylindrical drift tube

Theorem

The induced current I by a given electrode due to the movement of a charge q equals

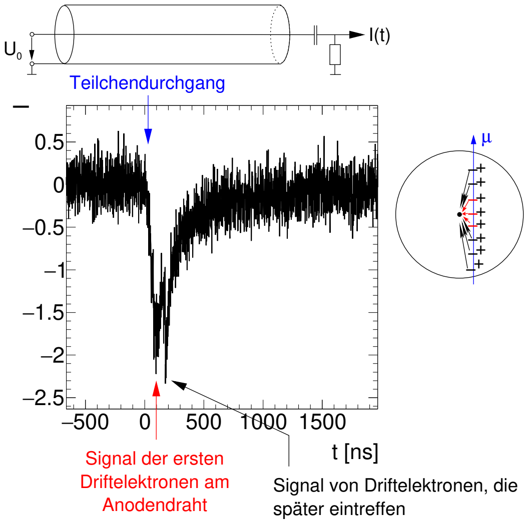
$$I = E_v qv$$

where v is the instantaneous velocity of the charge and E_v is the component in the direction v of that electric field which would exist at the charge's position under the following circumstances: charge removed, given electrode raised to unite potential, all other electrodes grounded.

Consequences

- Avalanche electrons give a large, but very short current because of their small drift distance to the anode wire.
- Ions give currents over a longer time interval. As they are created close to the anode wire, I is initally large and becomes smaller with the drift towards the tube wall.

Introductory example: cylindrical drift tube



- Particle detectors provide current or voltage pulses, which contain information about particle passage or deposited energy.
- To obtain this information, they must be processed electronically.

Analog and digital signals

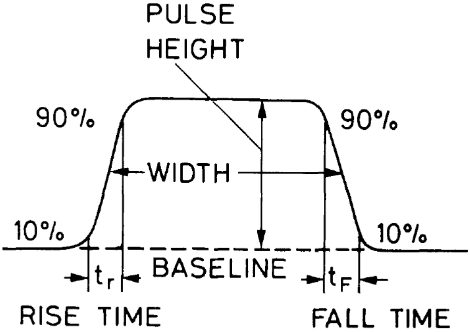
Analog signal: Information contained in the continuous variation of electrical signal properties, e.g., pulse height, pulse duration, or pulse shape.

Digital signal: Information stored in discrete form.

Example. TTL (Transistor-Transistor Logic):
Logical 0: Signal between 0 and 0.8 V.
Logical 1: Signal between 2 V and 5 V.

Advantage of a digital signal: No information loss with small signal disturbances.

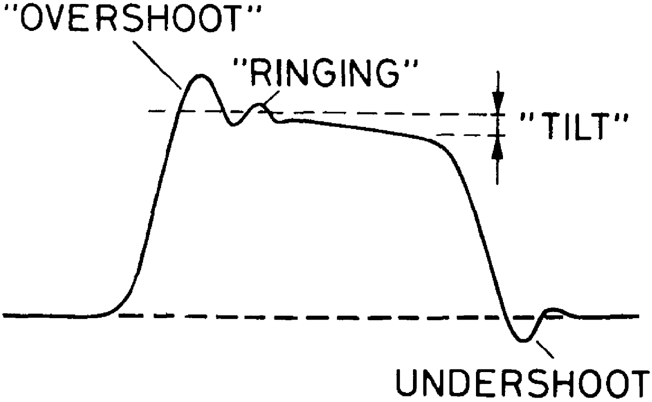
Characteristics of a signal pulse



Slow Signal: $t_A \gtrsim 100$ ns.

Fast Signal: $t_A \lesssim 1$ ns.

Deformed rectangular pulse



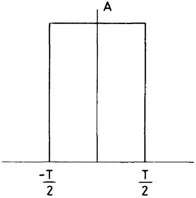
Fourier decomposition of a signal

Temporal evolution of a signal: $s(t)$.

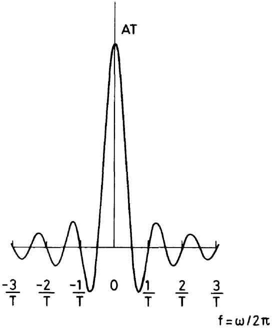
Fourier transform: $\hat{s}(\omega)$.

Example of an ideal rectangular pulse

$$s(t) = \begin{cases} A \text{ for } t \in [-\frac{T}{2}, \frac{T}{2}], \\ 0 \text{ otherwise.} \end{cases}$$



$$\begin{aligned} \underline{\hat{s}(\omega)} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}} \int_{-T/2}^{T/2} A \cdot e^{-i\omega t} dt \\ &= \frac{A}{\sqrt{2\pi}} \frac{-i}{\omega} e^{-i\omega T} \Big|_{-T/2}^{T/2} = \underline{\frac{AT}{\sqrt{2\pi}} \frac{\sin\left(\frac{\omega T}{2}\right)}{\omega T/2}}. \end{aligned}$$



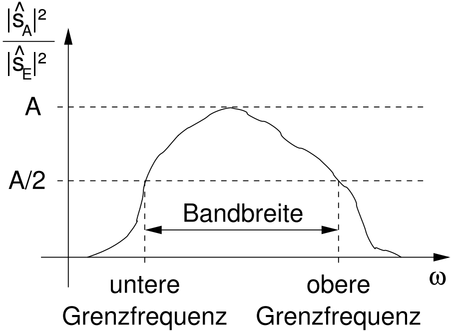
Attenuation and bandwidth

Attenuation

Attenuation [dB]:$=10\cdot\log_{10}\left(\frac{|\hat{s_A}|^2}{|\hat{s_E}|^2}\right)$.

$$-3\ \text{dB} = 10\cdot\log_{10}\left(\frac{|\hat{s_A}|^2}{|\hat{s_E}|^2}\right) \Leftrightarrow \frac{|\hat{s_A}|^2}{|\hat{s_E}|^2} = 10^{-\frac{3}{10}} = \frac{1}{2}.$$

Bandwidth



Drude's model of electrical conduction in metals

Metals are electrical conductors. In an ideal conductor, the conduction electrons experience no resistance. In a real conductor, they collide with the atomic nuclei.

Assumptions

- Neglect of interaction between the conduction electrons.
- Free electron motion between collisions with atomic nuclei.
  - Non-accelerated motion in between collisions.
- Elastic collisions between conduction electrons and atomic nuclei. The conduction electrons are not heated by the collisions.

Electron movement in the Drude model

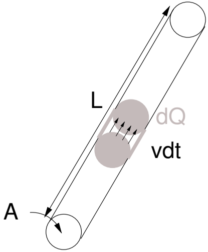
Equation of motion of a conduction electron:

$$m_e \cdot \frac{d\vec{v}}{dt} = -e\vec{E}.$$

τ: Average time between two collisions off atoms.

$$<\vec{v}> = -\frac{e}{m_e}\vec{E}\cdot\tau + \underbrace{<\vec{v}_0>}_{=0\ (in\ \ therm.\ \ equ.)} = -\frac{e}{m_e}\tau\cdot\vec{E}.$$

n: Conduction electron density.
L: Length of the real conductor.
A: Cross section of the real conductor.



$$dQ = -n\cdot e|\vec{v}|\cdot dt\cdot A \ \Leftrightarrow\ I = \frac{dQ}{dt} = -nev\cdot A = \frac{ne^2\tau}{m_e}\cdot A\cdot E.$$

Hence

$$\vec{j} = \frac{ne^2\tau}{m_e}\cdot\vec{E} =: \sigma\cdot\vec{E}.$$

σ: electric conductivity.

Ohm's law

Voltage between the ends of the conductor:

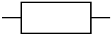
$$U = L \cdot \underbrace{E}_{=\frac{I}{\sigma \cdot A}} = \frac{L}{\sigma \cdot A} \cdot I =: R \cdot I \text{ (Ohm's Law).}$$

Ohmic resistance

$$R = \frac{L}{\sigma \cdot A} =: \rho \cdot \frac{L}{A}.$$

ρ: specific resistance (unit: Ωcm).

Schematic symbols for an ohmic resistance:

 (DIN)

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Passive electronic components – capacitance

$$C = \frac{Q}{U} \quad \Rightarrow \quad \text{No current flow at DC voltage.}$$

Current flow at AC voltage:

$$\frac{dU}{dt} = \frac{\frac{dQ}{dt}}{C} = \frac{I}{C}.$$

Transition to frequency representation:

$$U(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \hat{U}(\omega) e^{i\omega t} d\omega, \;\; I(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \hat{I}(\omega) e^{i\omega t} d\omega.$$

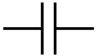
$$\frac{dU}{dt} = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} i\omega \hat{U}(\omega) e^{i\omega t} d\omega = \frac{I(t)}{C} = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \frac{1}{C} \hat{I}(\omega) e^{i\omega t} d\omega,$$

leading to $i\omega \hat{U}(\omega) = \frac{1}{C}\hat{I}(\omega)$, thus $\boxed{\hat{U}(\omega) = \frac{1}{i\omega C}\hat{I}(\omega)}$.

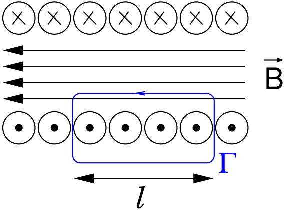
Capacitance – impedance and schematic symbol

$$\boxed{\hat{U}(\omega) = \frac{1}{i\omega C}\hat{I}(\omega).}$$

Impedance: $Z_C = \frac{1}{i\omega C}$.

Schematic symbol: 

Reminder: Field inside an ideal coil



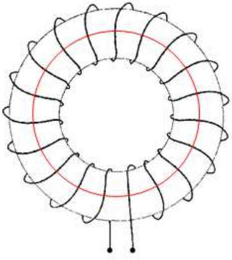
$\frac{dN}{dl}$: Number of turns per unit length.
Ampére's law:

$$\oint_{\Gamma} \vec{B} \cdot d\vec{s} = l \cdot B = \mu_0 \cdot I \cdot \frac{dN}{dl} \cdot l.$$

$$B = \mu_0 \frac{dN}{dl} \cdot I =: \frac{1}{A} L \cdot I.$$

A: Cross-sectional area of the coil.
L: Inductance.

Ideal toroidal coil



- B exists only inside the coil.
- If the coil is made of an ideal conductor, $\vec{E}$ inside the conductor is 0. Otherwise, an infinitely large current would flow through the conductor.

$\Rightarrow$ $U_{ab} = 0.$

- With alternating current, because $\frac{dI}{dt} \neq 0$, $\frac{\partial B}{\partial t} \neq 0$, resulting in a non-zero electromotive force.

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

$$U_{ab} = \oint \vec{E} \cdot d\vec{s} = \int_A \text{curl } \vec{E} d\vec{A} = -\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = -\frac{\partial}{\partial t} B \cdot A = -\frac{\partial}{\partial t} \frac{1}{A} LIA = -L\frac{dI}{dt}.$$

In the frequency domain, we have $\boxed{\hat{U}(\omega) = -i\omega L\hat{I}(\omega)}$.

Inductance – impedance and circuit symbol

$$\boxed{\hat{U}(\omega) = -i\omega L \hat{I}(\omega).}$$

Impedance: $Z_L = -i\omega L$.

Circuit Symbol:  (DIN)

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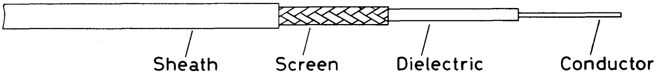
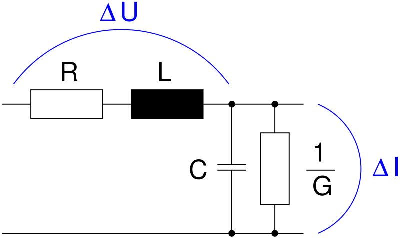
Remark. In the frequency domain, the behavior of a circuit containing the mentioned passive elements can be calculated in a similar manner to a circuit containing ohmic resistances, by using impedances.

Explanatory example: signal transmission via a coaxial cable



Due to their shielding, coaxial cables do not emit electromagnetic waves. However, they can intercept electromagnetic interference from the surroundings through their shielding.

Equivalent circuit diagram for a Δz length segment of a coaxial cable



R, L, C, $\frac{1}{G}$ represent resistance, inductance, capacitance, and conductance per uni t length, respectively.

In an ideal cable, R and G are both equal to 0.

Derivation of the general wave equation for a coaxial cable

$$\Delta U = -(R \cdot \Delta z) \cdot I - (L \cdot \Delta z) \cdot \frac{\partial I}{\partial t}.$$

$$\Delta I = -(G \cdot \Delta z) \cdot U - (C \cdot \Delta z) \cdot \frac{\partial U}{\partial t}.$$

Dividing by Δz and taking the limit as $\Delta z \to 0$ yields

$$\frac{\partial U}{\partial z} = -R \cdot I - L \cdot \frac{\partial I}{\partial t},$$

$$\frac{\partial I}{\partial z} = -G \cdot U - C \cdot \frac{\partial U}{\partial t}.$$

Wave equation for a coaxial cable

$$\frac{\partial U}{\partial z} = -R \cdot I - L \cdot \frac{\partial I}{\partial t}, \quad \left| \frac{\partial}{\partial z} \cdot \right.$$
$$\frac{\partial I}{\partial z} = -G \cdot U - C \cdot \frac{\partial U}{\partial t}. \quad \left| \frac{\partial}{\partial t} \cdot \right.$$

$$\frac{\partial^2 U}{\partial z^2} = -R \cdot \frac{\partial I}{\partial z} - L \frac{\partial^2}{\partial z \partial t} I,$$
$$\frac{\partial^2}{\partial z \partial t} I = -G \cdot \frac{\partial U}{\partial t} - C \cdot \frac{\partial^2 U}{\partial t^2}.$$

$$\boxed{\frac{\partial^2 U}{\partial z^2} = LC \frac{\partial^2 U}{\partial t^2} + (LG + RC) \frac{\partial U}{\partial t} + RGU.}$$

Ideal cable: R=0, G=0. $\boxed{\frac{\partial^2 U}{\partial z^2} = LC \frac{\partial^2 U}{\partial t^2}}$

(Wave equation with $v = \frac{1}{\sqrt{LC}}$).

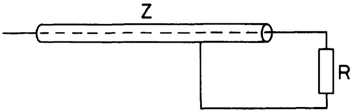
Properties of a coaxial cable

- In a real cable, G is very close to 0.
- In a real cable, $R \neq 0$ leads to dispersion. In practice, the cables used are usually so short that dispersion can be neglected, so $R = 0$ can be assumed.
- $L = \frac{\mu}{2\pi} \ln \frac{b}{a}$ [H/m], $C = \frac{2\pi\epsilon}{\ln \frac{b}{a}}$ [F/m].

  $$\Rightarrow v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}.$$

  Thus, the choice of dielectric determines v.
- Characteristic impedance: $Z := \frac{dU}{dI} = \sqrt{\frac{L}{C}}$.
  The characteristic impedance depends on the geometry of the cable, i.e., its inner and outer diameter as well as the dielectric used.

Reflections at the ends of the cables



$$U(t,x) = f(x - vt) + g(x + vt),$$

representing an incoming + reflected wave.

Input signal: U_E, I_E. $Z = \frac{U_E}{I_E}$.
Reflected signal: U_R, I_R, $Z = \frac{U_R}{I_R}$.
Voltage drop across the resistor R: $U_E + U_R$.
Current through R: $I_E + I_R$.

$$\Rightarrow R = \frac{U_E + U_R}{I_E - I_R} = \frac{U_E\left(1 + \frac{U_R}{U_E}\right)}{I_E\left(1 - \frac{I_R}{I_E}\right)} = Z\frac{1+\rho}{1-\rho}$$

with the reflection coefficient $\rho := \frac{U_R}{U_E} = \frac{I_R}{I_E}$. It holds $\rho = \frac{R-Z}{R+Z}$.

- Open cable: $R = \infty$. $\rho = 1$. Complete reflection at the cable end.
- Short-circuited cable: $R = 0$. $\rho = -1$. Reflection with opposite amplitude.
- Terminated cable: $R = Z$. $\rho = 0$. No reflection.