Concepts of Experiments at Future Colliders II

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Recapitulation of the previous lecture

Processing of analog detector signals

- The analog signals from particle detectors are usually very small.

  Example: MDT drift tube filled with Ar/CO_2 (93:7) at 3 bar.
  $\frac{dE}{dx} = 7.5$ keV/cm$\widehat{\approx} 7.5/0.03 = 250$ Electron ion pair/cm.
  At a gas gain of 20,000 this corresponds a total charge of only ~ 1 pC.

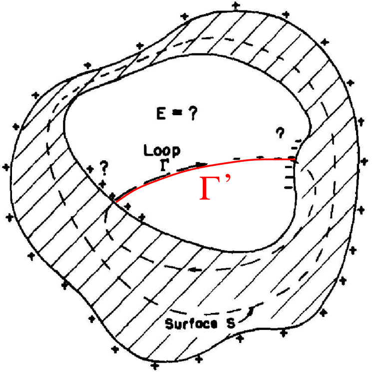
$\Rightarrow$ Protection of small signals by a Faraday cage.

$\Rightarrow$ Amplification of signals.

$\Rightarrow$ Transmission of unamplified signal over as short as possible distances.

A Faraday cage in electrostatics

- No electric field inside a conductors, otherwise there would be a current.
- The electric field in a region perfectly enclosed by a conducting cavity equals 0.
  Proof by contradiction.



(Fig.5-12 from Feynman lectures Vol 2)

If E were non-zero inside the cavity, there would be a path Γ' for which $\int_{\Gamma'} \vec{E} \cdot d\vec{s} \neq 0$.

Since $\vec{E} = 0$ inside the conductor, then $\oint_{\Gamma} \vec{E} \cdot d\vec{s} = \int_{\Gamma'} \vec{E} \cdot d\vec{s} \neq 0$, which contradicts rot $\vec{E} = 0$.

Functioning of a Faraday cage in alternating fields

1. Equation of motion underlying the Drude model

$$m_e \frac{d\vec{v}}{dt} = -\frac{m_e}{\tau}\vec{v} - e\vec{E}.$$

Considering $\vec{E}(t,\vec{x}) = \vec{E}(\omega,\vec{x})e^{-i\omega t}$, then $\vec{v}(t,\vec{x}) = \vec{v}(\vec{x})e^{-i\omega t}$, and we obtain

$$\vec{v}(\vec{x}) = \frac{-e\tau}{m_e}\frac{1}{1-i\omega\tau}\vec{E}(\omega,\vec{x}),$$

leading to

$$\vec{j} = -ne\vec{v} = \frac{e^2\tau}{m_e}\frac{1}{1-i\omega\tau}\vec{E} =: \underbrace{\frac{\sigma_0}{1-i\omega\tau}}_{=:\sigma(\omega)}\vec{E}$$

Functioning of a Faraday cage in alternating fields

2. Maxwell's equations for electromagnetic fields in conductors

$$\text{div}\,\vec{E} = 0. \quad \text{div}\,\vec{B} = 0. \quad \text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}. \quad \text{rot}\,\vec{B} = \frac{1}{c^2\epsilon_0}\vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}.$$

$$\text{rot}(\text{rot}\,\vec{E}) = \text{grad}(\underbrace{\text{div}\,\vec{E}}_{=0}) - \Delta\vec{E} = \text{rot}\left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\text{rot}\,\vec{B}.$$

Now, utilizing $\vec{j} = \sigma(\omega)\vec{E}$ for $\vec{E}(t,\vec{x}) = \vec{E}(\omega,\vec{k})e^{-i(\omega t - \vec{k}\cdot\vec{x})}$, we obtain

$$|\vec{k}|^2 = \frac{\omega^2}{c^2}\left[1 + i\frac{\sigma(\omega)}{\epsilon_0\omega}\right].$$

$\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau} \underset{\omega\tau\gg 1}{\rightarrow} \frac{i\sigma_0}{\omega\tau}$, thus

$$|\vec{k}|^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\sigma_0}{\epsilon_0\omega^2\tau}\right) = \frac{\omega^2}{c^2}\left(1 - \frac{ne^2}{\epsilon_0\omega^2}\right),$$

which is negative for $\omega < \frac{ne^2}{\epsilon_0}$. Then, $|\vec{k}|$ is imaginary and the electric field exponentially decreases with increasing penetration into the conductor.

Recapitulation of the previous lecture

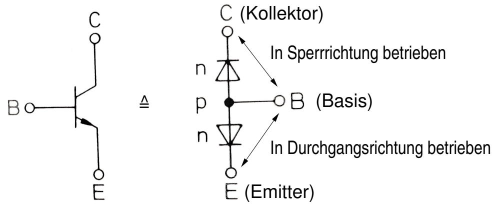
Functioning of a Faraday cage in alternating fields

Conclusions

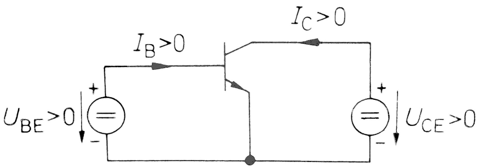
- Even alternating fields can be shielded by a Faraday cage if their frequency does not become too high.
- For example, choosing aluminium or brass as sufficiently thick material for the Faraday cage, one can shield fields up to the gigahertz range.

Bipolar transistor as an example of a signal amplifier

A bipolar transistor is an npn or pnp junction with 3 terminals.



Polarity of an npn transistor



Increasing U_{BE} reduces the voltage between the base and collector, causing diode BC to conduct more and thus allowing more current to flow from the emitter than has flowed into the base.

Input and out put characteristics of a bipolar transistor

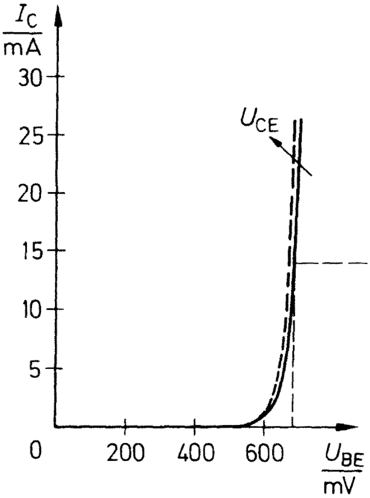


Abb. 4.5 Übertragungskennlinie

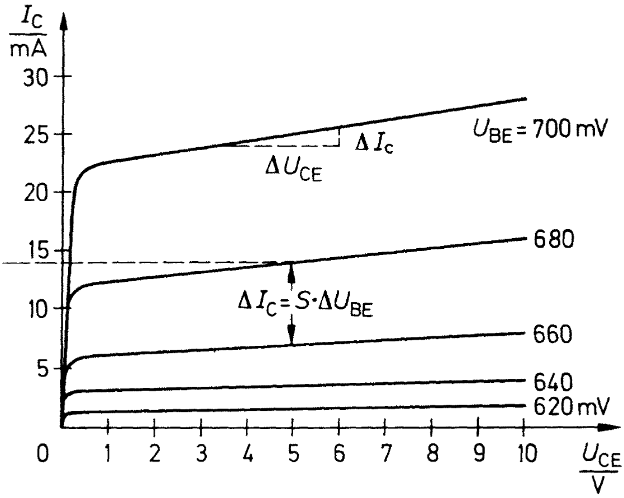


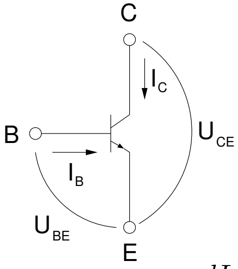
Abb. 4.6 Ausgangskennlinienfeld

Tietze, Schenk, Halbleiterschaltungstechnik, 1993

Concept of small-signal amplification

- A bipolar transistor is a current amplifier with the current amplification $B = \frac{I_C}{I_B}$.
- The value of B depends on the values of the applied voltages.
- In practice, one is interested in the amplification of small signals. To achieve this, these small signals are superimposed on a DC voltage that sets the operating point of the transistor.
- Since B fluctuates from one transistor to another, the amplification is determined by the circuitry of the transistor, as explained in the following examples.

Basic equations for small-signal amplification



Goal: Amplification of small, time-varying signals.

$$dI_B = \left.\frac{\partial I_B}{\partial U_{BE}}\right|_{U_{CE}} \cdot dU_{BE} + \left.\frac{\partial I_B}{\partial U_{CE}}\right|_{U_{BE}} \cdot dU_{CE},$$

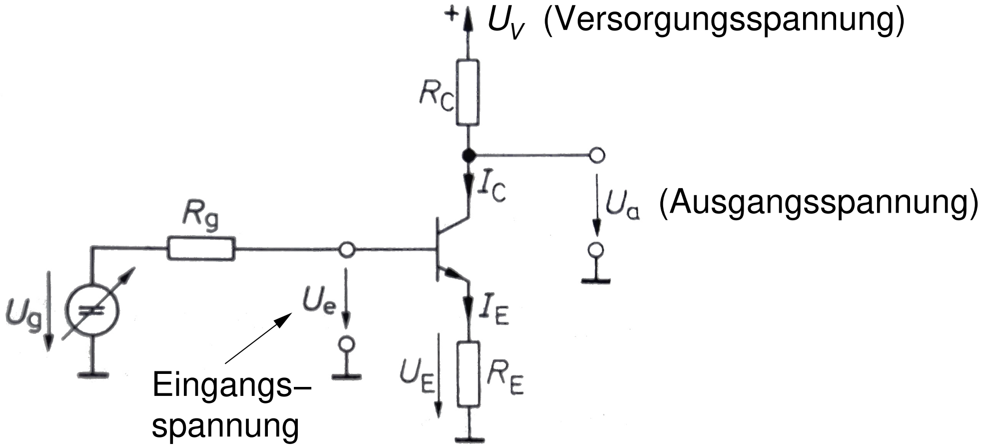
$$dI_C = \left.\frac{\partial I_C}{\partial U_{BE}}\right|_{U_{CE}} \cdot dU_{BE} + \left.\frac{\partial I_C}{\partial U_{CE}}\right|_{U_{BE}} \cdot dU_{CE}.$$

- $\frac{1}{r_{BE}} := \frac{\partial I_B}{\partial U_{BE}}|_{U_{CE}}$ is small. $\frac{\partial I_B}{\partial U_{CE}}|_{U_{BE}} \approx 0$.
- Slope $S := \frac{\partial I_C}{\partial U_{BE}}|_{U_{CE}}$ is large. $\frac{1}{r_{CE}} := \frac{\partial I_C}{\partial U_{CE}}|_{U_{BE}}$ is small.

$$\Rightarrow dI_B = \frac{1}{r_{BE}} \cdot dU_{BE},$$

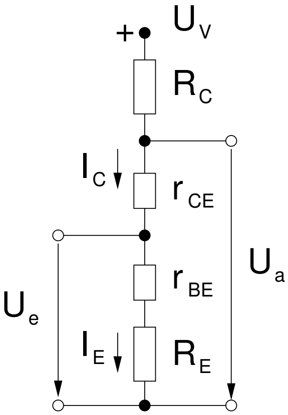
$$dI_C = S \cdot dU_{BE} + \frac{1}{r_{CE}} \cdot dU_{CE}.$$

1st Example: Emitter circuit with current feedback



Calculation of small-signal amplification

Equivalent circuit for calculating the small-signal amplification $A := \frac{dU_a}{dU_e}$



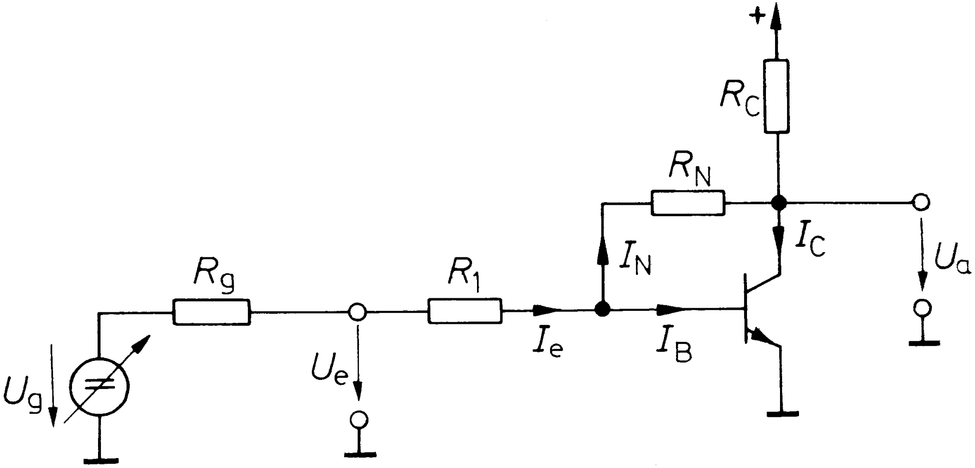
$$dI_E = \frac{dU_e}{r_{BE} + R_E} \underset{r_{BE} \ll R_E}{\approx} \frac{dU_e}{R_E}.$$

$$dI_C = \frac{d(U_V - U_a)}{R_C} \underset{dU_V = 0}{=} -\frac{dU_a}{R_C}.$$

$$dI_E = dI_C \Rightarrow A = \frac{dU_a}{dU_e} = -\frac{R_C}{R_E}.$$

The circuit is inverting with a small-signal amplification that depends only on the configuration of the transistor, namely R_C and R_E.

2nd Example: Emitter circuit with voltage feedback



Calculation of small-signal amplification

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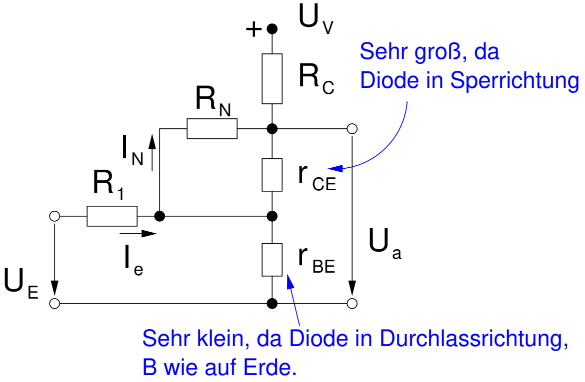


$$dU_e = R_1 dI_e, \; dU_a = R_N dI_N = -R_N dI_E.$$

$$\Rightarrow A = \frac{dU_a}{dU_e} = \frac{-R_N}{R_1}.$$

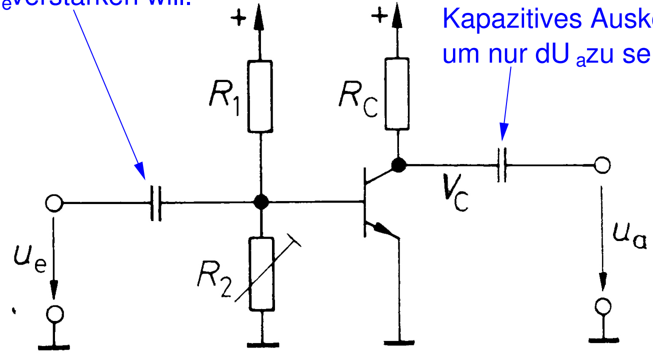
The circuit is inverting with a small-signal amplification that depends only on the configuration of the transistor, namely R_N and R_1.

Recapitulation of the previous lecture

Operating point adjustment

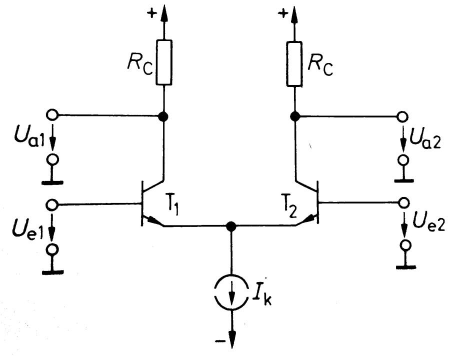
Kapazitives Einkoppeln des Signals, um den Arbeitspunkt nicht zu verschieben. Möglich, da man nur dU_e verstärken will.

Kapazitives Auskoppeln, um nur dU_a zu sehen.



Spannungsteiler zur Fest–legung des Arbeitspunktes des Transistors

Operation of a differential amplifier



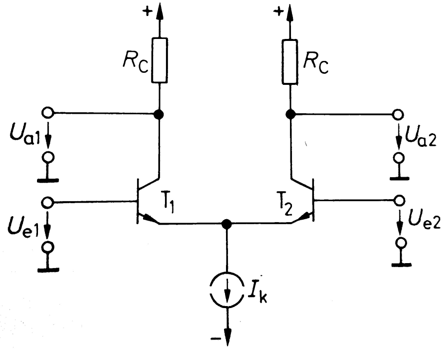
- Constant current source at the emitter. $\Rightarrow dI_k = 0$.
- Internal resistance of the constant current source: r_k.

- $I_k = I_{C1} + I_{C2} \Rightarrow dI_{C1} = -dI_{C2}$.
- So $dU_{a1} = -dU_{a2}$.
- Also
  $dU_{e1} = dU_{BE1} = -dU_{BE2} = -dU_{e2}$.
- $U_D := U_{e1} - U_{e2}$.
  $$dU_{e1} = d(U_{e1} - U_{e2} + U_{e2}) = dU_D + dU_{e2} = dU_D - dU_{e1},$$
  thus $dU_D = \frac{1}{2} dU_{e1}$.

$\Rightarrow$ Differential amplification $A_D = \frac{dU_{a1}}{dU_D}$

$$A_D = \frac{dU_{a1}}{2dU_{BE1}} = -\frac{1}{2} S(R_C || r_{CE}).$$

Since S is large, A_D is also large.



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- So $dU_{a1} = -dU_{a2}$.
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- $U_D := U_{e1} - U_{e2}$.
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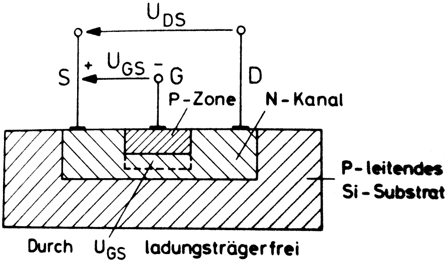
$$A_D = \frac{dU_{a1}}{2dU_{BE1}} = -\frac{1}{2}S(R_C||r_{CE}).$$
Since S is large, A_D is also large.

Besides the differential amplification, there is also a much smaller common-mode amplification $A_{CM} := \frac{dU_{a1}}{d(U_{e1}+U_{e2})/2} = -\frac{1}{2}\frac{R_C}{r_k}$, which immediately follows from the formula for the amplification of the emitter circuit with current feedback.

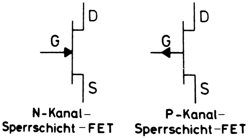
Construction of an n-channel junction field-effect transistor



S: Source.

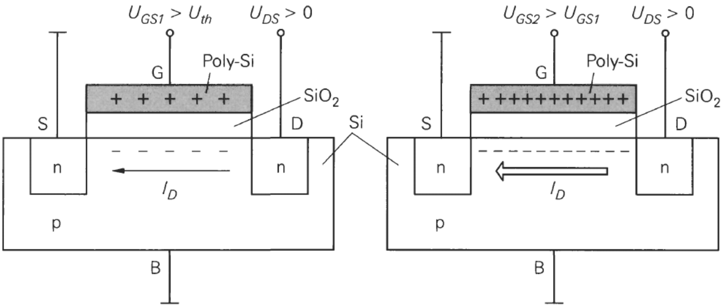
G: Gate.

D: Drain.



- Control of the size of the charge carrier-free zone via the value of the voltage U_{GS}.
- Thickness of the charge carrier-free zone determines the resistance between drain and source.
- Advantage of field-effect transistors over bipolar transistors: Lower power consumption, as the control is done via the applied electric field and not via a current.

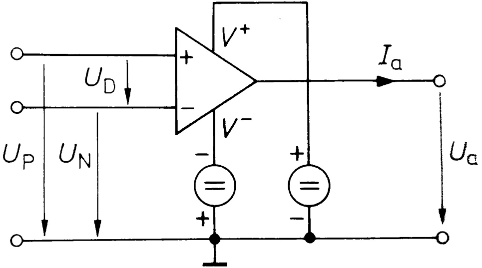
Metal-oxid-semiconductor field-effect transistor



- Structure forms a capacitor from gate terminal, dielectric, and bulk terminal.
- Application of positive voltage between gate and bulk charges the capacitor.
- Electric field causes migration of minority carriers (electrons in p-silicon) to the junction and recombination with majority carriers (defect electrons in p-silicon), known as depletion.
- Space charge region forms at the junction with negative space charge.
- At threshold voltage U_{th}, displacement of majority carriers becomes significant, limiting recombination.
- Accumulation of minority carriers results in near-inversion of p-doped substrate close to the oxide, known as strong inversion
- Increased gate voltage induces band bending of conduction and valence bands at the junction in band model.
- Fermi level shifts closer to the conduction band than the valence band, inverting the semiconductor material.
- Formed thin n-type conducting channel connects source and drain n-regions, allowing charge carriers to flow (almost) unimpeded from source to drain.

Operational amplifiers

- Operational amplifiers are broadband differential amplifiers with high gain and high input impedance.
- Operational amplifiers are available as integrated circuits made of bipolar and field-effect transistors.

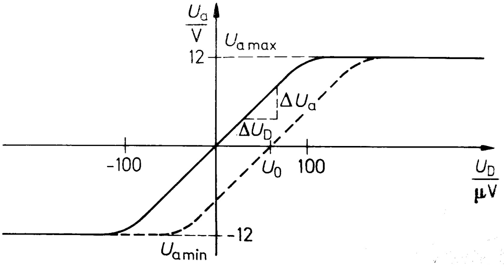


- Input stage designed as a differential amplifier, hence two inputs (+ and -).
- Positive and negative supply voltage required to drive the inputs and outputs positively and negatively.

- Open-loop gain:

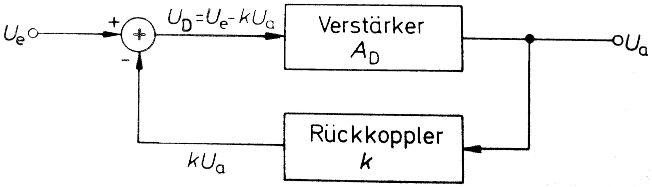
$$A_D := \frac{dU_a}{dU_D}.$$

Characteristic of an operational amplifier



- Offset voltage U_0 adjustable in most operational amplifiers.
- Linear dependency of U_a on U_D in a small range of U_D around U_0.
- Constant output voltage outside of this range (amplifier saturation).

Principle of negative feedback

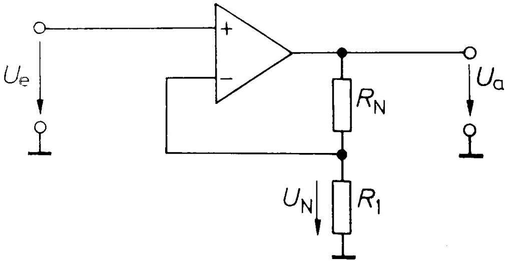


- $U_a = A_D(U_e - kU_a) \Leftrightarrow U_a = \frac{A_D}{1+kA_D} U_e \underset{A_D \to \infty}{\approx} \frac{1}{k} U_e$.
- $U_P = U_e$, $U_N = kU_a$, $|U_a|$¡const. Thus,

$$|U_P - U_N| = \frac{U_a}{A_D} \underset{A_D \to \infty}{\to} 0,$$

i.e., $\boxed{U_P = U_N}$.

Non-inverting amplifier

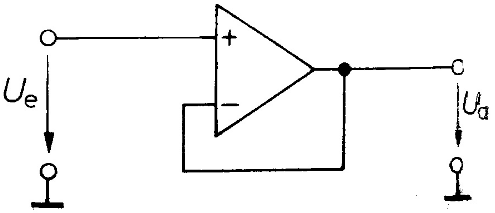


$$U_e = U_P = U_N = \frac{R_1}{R_1 + R_N} U_a$$

$$\Leftrightarrow \quad U_a = \left(1 + \frac{R_N}{R_1}\right) U_e.$$

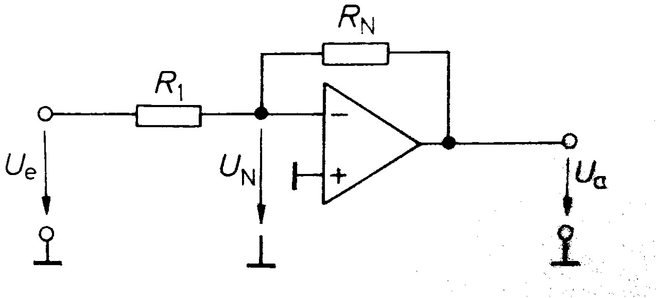
- Amplification is positive.
- Value of the amplification is fully determined by the choice of R_N and R_1.

Voltage follower



- $U_a = U_e$.
- Small output impedance, i.e., behaves like a voltage source.
- Use of this circuit as an impedance converter.

Inverting amplifier

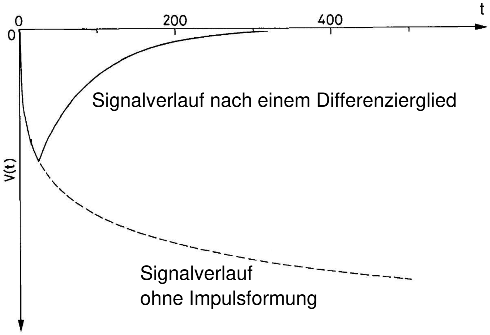


$$\color{blue}{U_P = U_N} = 0.$$

$$\Rightarrow \quad \underline{U_a} = R_N \cdot I_N = R_N(-I_1) = -R_N \frac{U_e}{R_1} = \underline{-\frac{R_N}{R_1} U_e}.$$

- Amplification is negative.
- Value of the amplification is fully determined by the choice of R_N and R_1.

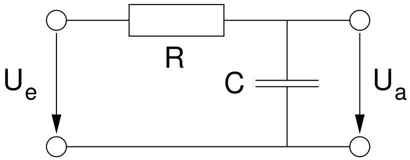
Introductory Example: Signal Pulse of a Cylindrical Drift Tube



Pulse shaping with a differentiator

- Retains the information of the signal start time.
- Significantly reduces the dead time of the tube compared to the case without pulse shaping.

Low-Pass

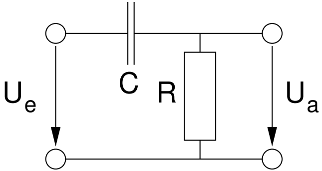


$$U_a = \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} U_e = \frac{1}{1 + i\omega RC} U_e.$$

$\omega \to 0$: $U_a \to U_e$.

$\omega \to \infty$: $U_a \to 0$.

High-Pass

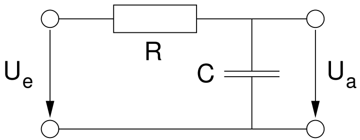


$$U_a = \frac{R}{R + \frac{1}{i\omega C}} U_e = \frac{1}{1 + \frac{1}{i\omega RC}} U_e.$$

$\omega \to 0$: $U_a \to 0$.

$\omega \to \infty$: $U_a \to U_e$.

Low-Pass



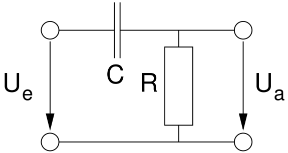
$$U_a = \frac{1}{1+i\omega RC} U_e.$$

3dB Cutoff Frequency

$$\frac{1}{|1+i\omega RC|^2} = \frac{1}{2} \Leftrightarrow \omega = \frac{1}{RC}.$$

$\omega \gg \frac{1}{RC}$: $U_a \approx \frac{1}{i\omega RC} U_e = \frac{\hat{U}_e(\omega)}{i\omega RC} e^{i\omega t}$, so
$U_a \approx \frac{1}{RC} \int U_e dt$.
Integrating above the cutoff frequency.

High-Pass



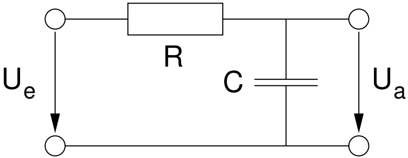
$$U_a = \frac{1}{1+\frac{1}{i\omega RC}} U_e.$$

3dB Cutoff Frequency

$$\frac{1}{\left|1+\frac{1}{i\omega RC}\right|^2} = \frac{1}{2} \Leftrightarrow \omega = \frac{1}{RC}$$

$\omega \ll \frac{1}{RC}$:
$U_a \approx i\omega RC U_e = i\omega RC \hat{U}_e e^{i\omega t}$, so
$U_a \approx RC \frac{dU_e}{dt}$.
Differentiating above the cutoff frequency.

Behavior of a low pass filter



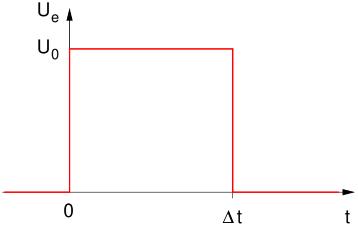
1st possibility: Use of complex impedances and a Fourier transformation from the frequency to the time domain.

2nd possibility: Solving the following differential equation.

$$U_a = \frac{Q}{C} \Rightarrow \frac{dU_a}{dt} = \frac{1}{C} I.$$

$$U_e = U_R + U_a = R \cdot I + U_a = RC\frac{dU_a}{dt} + U_a.$$

Low pass: behavior with a rectangular pulse



$$U_e(t) = \begin{cases} U_0 & (t \in [0, \Delta t]), \\ 0 & \text{otherwise.} \end{cases}$$

$t \leq 0$: $0 = RC\frac{dU_a}{dt} + U_a$, hence $U_a = 0$.

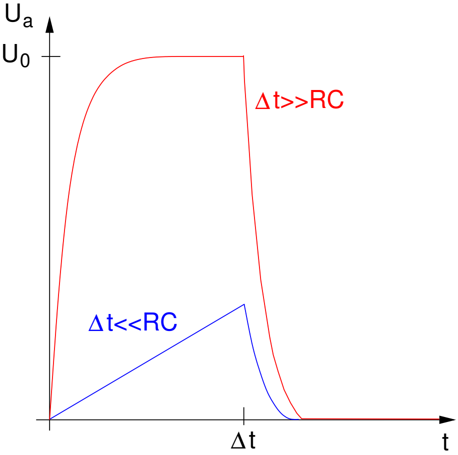
$t \in (0, \Delta t)$:
$$U_0 = RC\frac{dU_a}{dt} + U_a$$

$$\Leftrightarrow \quad U_0 - U_a = RC\frac{dU_a}{dt} \quad \Leftrightarrow \quad \int_0^t \frac{1}{RC}dt' = \int_0^{U(t)} \frac{dU_a}{U_0 - U_a}$$

$$\Leftrightarrow \quad -\frac{t}{RC} = \ln\frac{U_0 - U_a(t)}{U_0} \quad \Leftrightarrow \quad e^{-\frac{1}{RC}t} = \frac{U_0 - U_a(t)}{U_0}$$

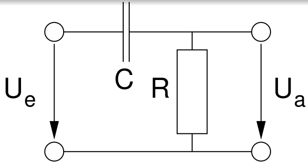
$$\Leftrightarrow \quad U_a(t) = U_0(1 - e^{-\frac{1}{RC}t}).$$

$t \geq \Delta t$: $\frac{dU_a}{dt} = -\frac{1}{RC}U_a$, hence $U_a(t) = U_a(\Delta t)e^{-\frac{1}{RC}(t-\Delta t)}$.



$\Delta t \gg RC$: $U_a(t \to \Delta t - 0) \approx U_0$.

$\Delta t \ll RC$: $U_a(t) \approx U_0 \frac{t}{RC}$ for $t \in (0, \Delta t)$.



$$U_a = R \cdot I = RC\frac{d(U_e - U_a)}{dt} = RC\frac{dU_e}{dt} - RC\frac{dU_a}{dt}.$$

Choose U_e as before, as a rectangular pulse.

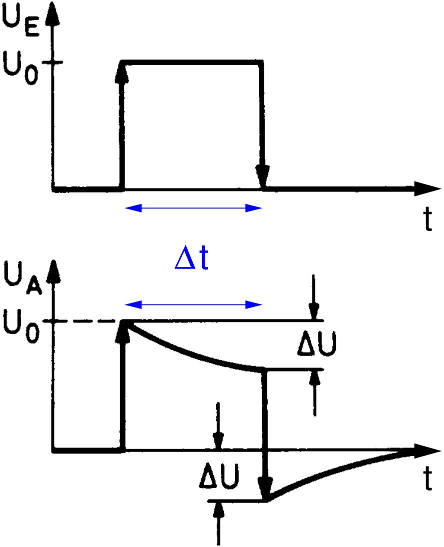
$t \leq 0$: $U_a(t) = 0$.

$t \in (0, \Delta t)$: $U_a(t) = -RC\frac{dU_a}{dt}$, hence $U_a(t) = U_a(0)e^{-\frac{t}{RC}} = U_0 e^{-\frac{t}{RC}}$.

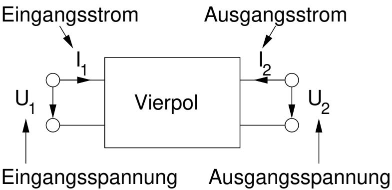
$\epsilon \to 0+0$: $t \in [\Delta t, \Delta t + \epsilon)$: $U_e(t) = U_0\left(1 - \frac{t-\Delta t}{\epsilon}\right)$, hence $\frac{dU_e}{dt} = -\frac{U_0}{\epsilon}$.

$$\begin{aligned}
& U_a + \frac{RC}{\epsilon}U_0 = -RC\frac{dU_a}{dt} \\
\Leftrightarrow \quad & \epsilon U_a + RCU_0 = -\epsilon RC\frac{dU_a}{dt} \\
\underset{\epsilon\to 0}{\Leftrightarrow} \quad & U_0 = -\epsilon\frac{dU_a}{dt},\ U_0\epsilon = -\epsilon\left[U_a(\Delta t + \epsilon) - U_a(\Delta t)\right] \\
\Leftrightarrow \quad & U_a(\Delta t + \epsilon) = U_a(\Delta t) - U_0 = U_0\left(e^{-\frac{\Delta t}{RC}} - 1\right)
\end{aligned}$$

$t \geq \Delta t$: $U_a(t) = U_0\left(e^{-\frac{\Delta t}{RC}} - 1\right)e^{-\frac{t-\Delta t}{RC}}$.



Bipolar pulse shaping possible with a high pass.



Low-pass, high-pass, and similar circuits with a total of four connections are called four-poles. Using so-called four-pole equations, one can easily calculate the behavior of circuits composed of many four-poles.

Two of the four quantities are freely selectable, the other two depend on these. For example, $U_1 = U_1(I_1, I_2)$, $U_2 = U_2(I_1, I_2)$.

$$\begin{aligned} dU_1 &= \left.\frac{\partial U_1}{\partial I_1}\right|_{I_2} dI_1 + \left.\frac{\partial U_1}{\partial I_2}\right|_{I_1} dI_2, \\ dU_2 &= \left.\frac{\partial U_2}{\partial I_1}\right|_{I_2} dI_1 + \left.\frac{\partial U_2}{\partial I_2}\right|_{I_1} dI_2. \end{aligned}$$

If the four-pole consists only of linear, passive components, then even $\frac{\partial U_k}{\partial I_\ell} = \frac{U_k}{I_\ell}$ holds.

For the calculation of the behavior of a chain of four-poles, the chain form is useful, where the input or output variables are expressed as functions of the output or input variables:

$$
\begin{aligned}
dU_1 &= \left.\frac{\partial U_1}{\partial U_2}\right|_{I_2} dU_2 + \left.\frac{\partial U_1}{\partial I_2}\right|_{U_2} dI_2, \\
dI_1 &= \left.\frac{\partial I_1}{\partial U_2}\right|_{I_2} dU_2 + \left.\frac{\partial I_1}{\partial I_2}\right|_{U_2} dI_2.
\end{aligned}
$$

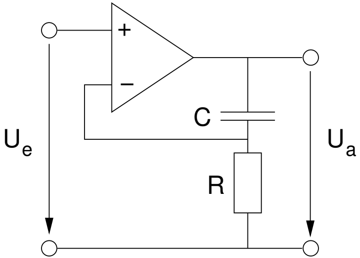
$$
d\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = A \cdot d\begin{pmatrix} U_2 \\ I_2 \end{pmatrix}.
$$

To obtain the behavior of a four-pole consisting of a chain of four-poles, one only needs to multiply the production of the matrices A_k of the individual four-poles with each other.

Pulse shaping with low and high pass filters

For pulse shaping of detector signals, one connects low and high pass filters of different time constants (RC) in series. To separate the passes, an operational amplifier with capacitive coupling of the signals can be used.

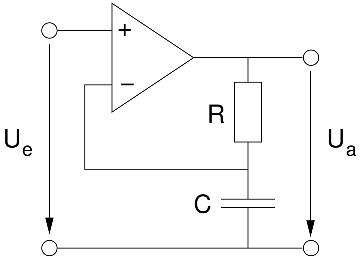
Low Pass



$$U_a = \left(1 + \frac{1}{i\omega RC}\right) U_e.$$

Amplification of low frequencies.

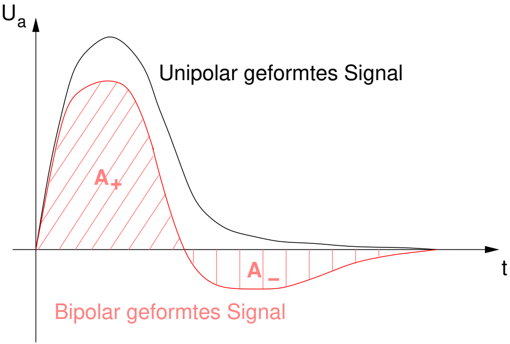
High Pass



$$U_a = (1 + i\omega RC)\, U_e.$$

Amplification of higher frequencies.

Unipolar and bipolar pulse shaping



$$|A_+| = |A_-|$$

Disadvantage of unipolar signal shapes: Drift of the pulse baseline due to the superposition of successive pulses at high signal rates.

Remedy for this problem: Use of bipolar pulse shaping, which on average does not shift the pulse baseline.